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A Generalized Method of Moments Detector for Block Fading SIMO Channels

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***Abstract*—In this letter we apply the Generalized Method of Moments (GMM), widely used in econometrics, to receivers operating with imperfect channel state information (CSI) of single-input-multiple-output (SIMO) block-fading channels where a single pilot symbol is used. The GMM results in the standard maximum ratio combining (MRC) receiver, but with an improved channel estimate. Although not our goal at the outset, this result reveals an inherent capability of the GMM to improve *any* channel estimate through filtering of the initial channel estimate with a matrix that is constructed from the received signals. The filtering involves a matrix inverse of size $\min\{T, M\}$, where M is the number of receive antennas, and $T+1$ is the coherence interval of the channel. The gain over an MRC receiver, using a scaled version of the pilot observation as channel estimate, lies in the range 0.1-3 dB depending on the system configuration. A coherence interval of about 5 symbol intervals is sufficient to reach these gains.**

***Index Terms*—Generalized Method of Moments, Block-Fading.**

I. INTRODUCTION

We study receiver design for Single-input Multiple-output (SIMO) transmissions in block fading channels in the face of additive Gaussian noise where the channel is a-priori unknown. Such systems have been widely studied and the interplay between the amount of training and payload data is today well understood [1], [2]. In this paper we allocate one time slot of the coherence interval to training data. The literature on possible receiver designs is vast. The Generalized Likelihood Ratio Test optimal detector has been studied in [3] and has cubic complexity in the block length $T+1$. If the statistics of the channel are known, then the true Maximum-Likelihood (ML) detector operates on the basis of the conditional probability but has exponential complexity in $T+1$ [4]. Another method is iterative joint channel estimation and data detection [5] which can approach the performance of the ML detector but with much less computational complexity.

In this paper we investigate to what extent the Generalized Method of Moments (GMM) can be utilized for SIMO receiver design with imperfect channel state information (CSI). The GMM is a relatively new method that has been widely and successfully applied within econometrics [6]. However, applications of the GMM to the field of communications are scarce. To the best of the authors' knowledge, it was first applied, with great success, in [7] for reciprocity calibration of base stations in a large-scale multiuser Multiple-input Multiple-output (MIMO) setup. Noteworthy, [7] rediscovered the GMM as it did not identify the method as an instance of the GMM.

The contributions of this letter are as follows.

- We apply the GMM to SIMO systems with imperfect CSI and investigate its potential for receiver design.
- We find that the GMM improves the quality of *any* channel estimate by filtering it with a matrix that is constructed explicitly from the received signals.
- We show that the GMM receiver performs maximum ratio combining (MRC) using the improved channel estimate.
- We show that the complexity overhead compared with an MRC receiver that forms a channel estimate based on the pilot observation is small.

Altogether, the GMM detector has low complexity, yields signal-to-noise ratio (SNR) gains, and can be expressed in an easily understandable manner. Our current GMM detector cannot easily be extended to MIMO systems; we mention where the problem for MIMO occurs later. For a single-input single-output systems GMM detection does not offer any gain.

II. SYSTEM MODEL

We consider a block fading SIMO system with coherence time $T+1$ symbol times in additive Gaussian noise. The received signal vector at time k can be expressed as

$$\boldsymbol{y}_k = \boldsymbol{h}x_k + \boldsymbol{n}_k, \quad 0 \leq k \leq T \tag{1}$$

where $x_0 = 1$ is a known inserted training symbol, $\{x_k\}_{k=1}^{T}$ are multi-level QAM data symbols with unit average energy, $\boldsymbol{h}$ is a random $M \times 1$ vector representing the communication channel, and $\boldsymbol{n}_k$ is complex Gaussian noise with covariance matrix $N_0\boldsymbol{I}$. The vector $\boldsymbol{h}$ is unknown to the receiver, and we also assume that the receiver does not know the joint density of the entries of $\boldsymbol{h}$. For later use, we assemble the vectors $\boldsymbol{y}_k$ into a matrix $\tilde{\boldsymbol{Y}} = [\boldsymbol{y}_0 \ \boldsymbol{Y}]$ where $\boldsymbol{Y} = [\boldsymbol{y}_1 \ \ldots \ \boldsymbol{y}_T] \ \mathbb{C}^{M\times T}$ is containing the data observations. Similarly, $\boldsymbol{x}$ denotes the vector of data symbols $\boldsymbol{x} = [x_1 \ \ldots \ x_T]$ and $\tilde{\boldsymbol{x}} = [1 \ \boldsymbol{x}]$. With that, we have $\boldsymbol{Y} = \boldsymbol{h}\boldsymbol{x} + \boldsymbol{N}$ where $\boldsymbol{N} = [\boldsymbol{n}_1 \ \ldots \ \boldsymbol{n}_T]$.

We analyze two corner cases: ***(i)*** detection with known SNR $\gamma = \mathbb{E}[\|\boldsymbol{h}\|^2]/MN_0$ (measured per receive antenna)[1], and ***(ii)*** detection with unknown SNR. The former case corresponds to a scenario where the statistics of the channel do not change among the transmitted blocks, such that the SNR can be estimated with zero asymptotic error. The latter corresponds to a case where the transmitted blocks are sparse so that the SNR changes abruptly between two blocks. This occurs, e.g., often in machine-to-machine communications, where single antenna nodes transmit a small packet of control data at a very low periodicity [8]. Indeed, if the SNR is not known

[1]Here, $\mathbb{E}[\cdot]$ is the expectation operator, and $\|\cdot\|$ is the Frobenius norm.

to the receiver, it can be estimated from the received data block. This is, however, out of scope for this letter, and we assume the SNR to be either unknown or perfectly known, which represents the two corner cases.

A. Benchmark Detectors

A standard way to implement a detector for signals of the form (1) is to estimate the channel as $\hat{\boldsymbol{h}} = \beta \boldsymbol{y}_0$. In the case that the SNR is known, then $\beta = \gamma(\gamma+1)^{-1}$ so that minimum mean square error (MMSE) channel estimation is obtained. If the SNR is not known, a least squares (LS) estimation is performed which implies that $\beta = 1$. Once the channel has been estimated, the detector proceeds as if the estimate is correct and performs MRC, i.e.

$$\hat{\boldsymbol{x}} = \frac{1}{\|\hat{\boldsymbol{h}}\|^2} \hat{\boldsymbol{h}}^{\mathrm{H}} \boldsymbol{Y}. \tag{2}$$

III. The Generalized Method of Moment Detector

The GMM explores a particular structure of the system model, more specifically the *moment conditions* [6]. Inspection of (1) indicates the moment condition

$$\mathbb{E}[\boldsymbol{f}_{k,\ell}] \triangleq \mathbb{E}[\boldsymbol{y}_k x_\ell - \boldsymbol{y}_\ell x_k] = \boldsymbol{0}, \tag{3}$$

where $0 \le k < \ell \le T$, and $\boldsymbol{0}$ has zeros in all of its entries. For MIMO, a condition similar to (3) is not available which limits the application of the GMM to SIMO. With $\boldsymbol{g} \triangleq \left[\boldsymbol{f}_{0,1}^{\mathrm{T}} \cdots \boldsymbol{f}_{T-1,T}^{\mathrm{T}}\right]^{\mathrm{T}}$, the GMM detector is given by

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \boldsymbol{g}^H \boldsymbol{W} \boldsymbol{g}. \tag{4}$$

Since the optimal form of the weighting matrix $\boldsymbol{W}$ depends on the unknown second-order statistics of $\boldsymbol{h}$ [6], we proceed with $\boldsymbol{W} = \boldsymbol{I}$. The cost function can thus be rewritten as

$$\begin{aligned} \boldsymbol{g}^H \boldsymbol{g} &= \sum_{k=0}^{T} \sum_{\ell=k+1}^{T} \|\boldsymbol{y}_k x_\ell - \boldsymbol{y}_\ell x_k\|^2 \\ &= \boldsymbol{x}\boldsymbol{\Psi}\boldsymbol{x}^{\mathrm{H}} - 2\mathcal{R}\{\boldsymbol{b}\boldsymbol{x}^{\mathrm{H}}\} + \|\boldsymbol{Y}\|^2, \end{aligned} \tag{5}$$

where $\boldsymbol{\Psi} = \|\tilde{\boldsymbol{Y}}\|^2 \boldsymbol{I} - \boldsymbol{Y}^{\mathrm{H}}\boldsymbol{Y}$ and $\boldsymbol{b} = \boldsymbol{y}_0^{\mathrm{H}}\boldsymbol{Y}$. The minimizer to the quadratic form (5) is

$$\begin{aligned} \hat{\boldsymbol{x}} &= \boldsymbol{b}\boldsymbol{\Psi}^{-1} \\ &= \frac{1}{\|\tilde{\boldsymbol{Y}}\|^2} \boldsymbol{y}_0^{\mathrm{H}} \boldsymbol{Y} \left[\boldsymbol{I} - \frac{\boldsymbol{Y}^{\mathrm{H}}\boldsymbol{Y}}{\|\tilde{\boldsymbol{Y}}\|^2}\right]^{-1}. \end{aligned} \tag{6}$$

As can be seen from (6), the GMM detector operates over the entire block $\boldsymbol{Y}$. It is preferable to reach an expression where the GMM detector operates over one received vector per time. To get such a form, we rewrite (6) into

$$\begin{aligned} \hat{\boldsymbol{x}} &= \frac{1}{\|\tilde{\boldsymbol{Y}}\|^2} \boldsymbol{y}_0^{\mathrm{H}} \left[\boldsymbol{I} - \frac{\boldsymbol{Y}\boldsymbol{Y}^{\mathrm{H}}}{\|\tilde{\boldsymbol{Y}}\|^2}\right]^{-1} \boldsymbol{Y} \\ &= \boldsymbol{y}_0^{\mathrm{H}} \boldsymbol{E} \boldsymbol{Y}, \end{aligned} \tag{7}$$

where

$$\boldsymbol{E} = \frac{1}{\|\tilde{\boldsymbol{Y}}\|^2} \left[\boldsymbol{I} - \frac{\boldsymbol{Y}\boldsymbol{Y}^{\mathrm{H}}}{\|\tilde{\boldsymbol{Y}}\|^2}\right]^{-1}.$$

The particular form of the GMM detector (7) can, however, be computationally overwhelming if $M > T$, since the inversion required to establish $\boldsymbol{E}$ is of size $M \times M$. As a remedy we use the matrix inversion lemma and rewrite (7) as

$$\begin{aligned} \hat{\boldsymbol{x}} &= \frac{1}{\|\tilde{\boldsymbol{Y}}\|^2} \boldsymbol{y}_0^{\mathrm{H}} \left[\boldsymbol{I} + \frac{\boldsymbol{Y}}{\|\tilde{\boldsymbol{Y}}\|} \left[\boldsymbol{I} - \frac{\boldsymbol{Y}^{\mathrm{H}}\boldsymbol{Y}}{\|\tilde{\boldsymbol{Y}}\|^2}\right]^{-1} \frac{\boldsymbol{Y}^{\mathrm{H}}}{\|\tilde{\boldsymbol{Y}}\|}\right] \boldsymbol{Y} \\ &= \boldsymbol{y}_0^{\mathrm{H}} \boldsymbol{E} \boldsymbol{Y}, \end{aligned} \tag{8}$$

where in this case we have

$$\boldsymbol{E} = \frac{1}{\|\tilde{\boldsymbol{Y}}\|^2} \left[\boldsymbol{I} + \frac{\boldsymbol{Y}}{\|\tilde{\boldsymbol{Y}}\|} \left[\boldsymbol{I} - \frac{\boldsymbol{Y}^{\mathrm{H}}\boldsymbol{Y}}{\|\tilde{\boldsymbol{Y}}\|^2}\right]^{-1} \frac{\boldsymbol{Y}^{\mathrm{H}}}{\|\tilde{\boldsymbol{Y}}\|}\right],$$

so that only a $T \times T$ inversion is needed.

Let us now discuss the interpretation of the GMM detector $\hat{\boldsymbol{x}} = \boldsymbol{y}_0^{\mathrm{H}}\boldsymbol{E}\boldsymbol{Y}$. The benchmark (2) applies MRC based on $\hat{\boldsymbol{h}}$. With unknown SNR, $\hat{\boldsymbol{h}}$ coincides with $\boldsymbol{y}_0$ so that the benchmark detector reads, omitting the scaling term in (2) for now, $\boldsymbol{y}_0^{\mathrm{H}}\boldsymbol{Y}$. In view of this, we can see that the GMM detector $\hat{\boldsymbol{x}} = \boldsymbol{y}_0^{\mathrm{H}}\boldsymbol{E}\boldsymbol{Y}$ takes the LS estimate $\boldsymbol{y}_0$, *purifies* it through the matrix $\boldsymbol{E}$, and then applies MRC, i.e.

$$\hat{\boldsymbol{x}} = \hat{\boldsymbol{h}}_{\mathrm{GMM}}^{\mathrm{H}} \boldsymbol{Y} = (\boldsymbol{E}\hat{\boldsymbol{h}})^{\mathrm{H}} \boldsymbol{Y}.$$

Thus, the GMM provides a simple method for improving the LS channel estimate. In the next section we will draw on this and show that the matrix $\boldsymbol{E}$ in fact improves any channel estimate, not only the LS one, especially as T grows large. The purification of the channel estimate is done once per received block. The additional complexity compared to the benchmark detector lies in computing the matrix $\boldsymbol{E}$ and the vector $(\boldsymbol{E}\hat{\boldsymbol{h}})^{\mathrm{H}}$ once per coherence time. Thus the complexity is dominated by a matrix inversion of size $\min\{T, M\} \times \min\{T, M\}$. The GMM detector $\hat{\boldsymbol{x}} = \boldsymbol{y}_0^{\mathrm{H}}\boldsymbol{E}\boldsymbol{Y}$ is linear in $\boldsymbol{Y}$ once $\boldsymbol{E}$ has been computed. An important remark is that for $T = 1$, the GMM detector coincides with MRC based on $\hat{\boldsymbol{h}}_{\mathrm{LS}}$ and thus provides no additional benefit.

Finally, note that the matrix $\boldsymbol{Y}\boldsymbol{Y}^{\mathrm{H}}/\|\tilde{\boldsymbol{Y}}\|^2$ is not the sample covariance matrix of $\boldsymbol{Y}$ as it is normalized with its own energy, rather than with the block length. A problem with the GMM detector is that as the block length $T+1$ grows large, the matrix $\boldsymbol{E}$ converges to the all zero matrix. In other words, it is a biased estimator. For constant modulus constellations this is not an issue, but it quickly renders the GMM detector unsuitable for multi-level QAM constellations. Before discussing the bias, we turn to an SNR analysis for which the bias effect is irrelevant.

IV. SNR Asymptotic Analysis

In this section we analyze the asymptotic SNR of the GMM detector as the block length grow large. As scalings are irrelevant for SNR computations, we ignore the scaling of the matrix $\boldsymbol{E}$, so that we let the GMM detector for x_k be $\hat{x}_k = \boldsymbol{y}_0^{\mathrm{H}}(\boldsymbol{I} - \boldsymbol{Y}\boldsymbol{Y}^{\mathrm{H}}/\|\tilde{\boldsymbol{Y}}\|^2)^{-1}\boldsymbol{y}_k$. However, recalling that $\boldsymbol{y}_0$ is the LS channel estimate of $\boldsymbol{h}$, we here replace $\boldsymbol{y}_0$ with any estimate $\hat{\boldsymbol{h}}$ that has the form $\hat{\boldsymbol{h}} = \alpha\boldsymbol{h} + \boldsymbol{w}$ where $\boldsymbol{w}$ is complex Gaussian noise with covariance matrix $N_{\mathrm{w}}\boldsymbol{I}$. We will now show that the matrix $\boldsymbol{E}$ will improve *any* channel estimate regardless of N_w and α. The LS and MMSE channel estimates are then the special cases $N_\mathrm{w} = \alpha^2 N_0$, with $\alpha = 1$ and $\alpha = \gamma(\gamma+1)^{-1}$, respectively.

The post-processing SNR of the GMM detector is denoted by γ_GMM and is defined as follows. Define the random variable $z_k = \hat{x}_k|(x_k = 1)$. The SNR is then defined as $\gamma_\mathrm{GMM} = 2|\mathbb{E}[z_k]|^2/\mathrm{Var}[z_k]$, where $\mathrm{Var}[\cdot]$ denotes the variance of its input. We summarize our findings for the asymptotic post processing SNR $\gamma^\infty_\mathrm{GMM} = \lim_{T\to\infty}\gamma_\mathrm{GMM}$ in Proposition 1.

Proposition 1. *As $T \to \infty$, the post-processing SNR of the GMM detector $\hat{\boldsymbol{x}} = (\boldsymbol{E}\hat{\boldsymbol{h}})^\mathrm{H}\boldsymbol{Y}$, is*

$$\gamma^\infty_\mathrm{GMM} = \frac{\alpha^2\lambda^2}{\alpha\lambda(N_0+N_\mathrm{w}) + N_0 N_\mathrm{w} + \frac{N_0^3 N_\mathrm{w}(M-1)^3}{(\lambda+(M-1)N_0)^2}}.$$

Furthermore, the post-processing SNR of the benchmark detector $\hat{\boldsymbol{x}} = \hat{\boldsymbol{h}}^\mathrm{H}\boldsymbol{Y}$, γ_BM, satisfies $\gamma_\mathrm{BM} < \gamma^\infty_\mathrm{GMM}$.

Proof. Let $\boldsymbol{Q}[\sqrt{\lambda}\ \ 0\ \ \ldots\ \ 0]^\mathrm{T}$ denote the singular value decompostion of the channel vector $\boldsymbol{h}$. As $T \to \infty$, we have that $(\boldsymbol{I} - \boldsymbol{Y}\boldsymbol{Y}^\mathrm{H}/\|\tilde{\boldsymbol{Y}}\|^2)^{-1} \to \boldsymbol{Q}\boldsymbol{D}\boldsymbol{Q}^\mathrm{H}$ where

$$\boldsymbol{D} = \mathrm{diag}\left(\left[\frac{\lambda+MN_0}{(M-1)N_0}\ \frac{\lambda+MN_0}{\lambda+(M-1)N_0}\cdots\frac{\lambda+MN_0}{\lambda+(M-1)N_0}\right]\right).$$

We now have,

$$\begin{aligned}\hat{x}_k &= ([\alpha\sqrt{\lambda}\ \mathbf{0}]\boldsymbol{Q}^\mathrm{H} + \boldsymbol{w}^\mathrm{H})\boldsymbol{Q}\boldsymbol{D}\boldsymbol{Q}^\mathrm{H}(\boldsymbol{Q}[\alpha\sqrt{\lambda}\ \mathbf{0}]^\mathrm{T}x_k + \boldsymbol{n}_k)\\ &= \lambda\frac{\lambda+MN_0}{(M-1)N_0}x_k + \eta, \end{aligned} \quad (9)$$

where η is a noise variable and $\mathbf{0}$ is a $1\times(M-1)$ vector with all elements equal to zero. Based on (9) it is straightforward to derive the variance of η. By doing so, the SNR equals

$$\gamma^\infty_\mathrm{GMM} = \frac{\alpha^2\lambda^2}{\alpha\lambda(N_0+N_\mathrm{w}) + N_0 N_\mathrm{w} + \frac{N_0^3 N_\mathrm{w}(M-1)^3}{(\lambda+(M-1)N_0)^2}}.$$

For the benchmark detector $\hat{x}_k = \hat{\boldsymbol{h}}^\mathrm{H}\boldsymbol{y}_k$ we obtain the SNR by replacing $\boldsymbol{D}$ with the identity matrix. This gives,

$$\gamma_\mathrm{BM} = \frac{\alpha^2\lambda^2}{\alpha\lambda(N_0+N_\mathrm{w}) + MN_0N_\mathrm{w}}.$$

The relation $\gamma^\infty_\mathrm{GMM} > \gamma_\mathrm{BM}$ is shown as follows,

$$\begin{aligned}\gamma^\infty_\mathrm{GMM} &> \frac{\alpha^2\lambda^2}{\alpha\lambda(N_0+N_\mathrm{w}) + N_0 N_\mathrm{w} + \frac{N_0^3 N_\mathrm{w}(M-1)^3}{(0+(M-1)N_0)^2}}\\ &= \gamma_\mathrm{BM}.\end{aligned}$$

∎

From the proof of Proposition 1 we can observe that as $N_0 \to 0$, we have asymptotically no SNR gain[2], i.e., $\gamma^\infty_\mathrm{GMM}/\gamma_\mathrm{BM} \to 1$. We can also deduce the following corollary.

Corollary 1. *As $T \to \infty$, the phase of the GMM channel estimate $\hat{\boldsymbol{h}}_\mathrm{GMM} = \boldsymbol{E}\hat{\boldsymbol{h}}$ is closer to the phase of the true channel than the phase of the initial estimate, that is,*

$$\left\|\frac{\hat{\boldsymbol{h}}_\mathrm{GMM}}{\|\hat{\boldsymbol{h}}_\mathrm{GMM}\|} - \frac{\boldsymbol{h}}{\|\boldsymbol{h}\|}\right\| \le \left\|\frac{\hat{\boldsymbol{h}}}{\|\hat{\boldsymbol{h}}\|} - \frac{\boldsymbol{h}}{\|\boldsymbol{h}\|}\right\|.$$

[2]This does not imply that error rates converge.

Proof. Since relative magnitudes between $\hat{\boldsymbol{h}}_\mathrm{GMM}$ and $\hat{\boldsymbol{h}}$ are irrelevant for SNR computations, the increased SNR implies that $|\hat{\boldsymbol{h}}^\mathrm{H}_\mathrm{GMM}\boldsymbol{h}|/\|\hat{\boldsymbol{h}}_\mathrm{GMM}\| \ge |\hat{\boldsymbol{h}}^\mathrm{H}\boldsymbol{h}|/\|\hat{\boldsymbol{h}}\|$. Thus, the phase of $\hat{\boldsymbol{h}}_\mathrm{GMM}$ is better aligned with the phase of $\boldsymbol{h}$ than the phase of $\hat{\boldsymbol{h}}$, which yields the inequality. ∎

V. Bias Considerations of the GMM Detector

From Corollary 1 we know that the phase of $\hat{\boldsymbol{h}}_\mathrm{GMM}$ is better aligned to the phase of $\boldsymbol{h}$ than the phase of $\hat{\boldsymbol{h}}$ is. However, the magnitude $\|\hat{\boldsymbol{h}}_\mathrm{GMM}\|$ needs not to be closer to $\|\boldsymbol{h}\|$ than the magnitude $\|\hat{\boldsymbol{h}}\|$ is. In fact, the magnitude $\|\hat{\boldsymbol{h}}_\mathrm{GMM}\|$ is typically very small. The reason is that as T grows, the GMM detector converges to the all-zero solution. To resolve this, we constrain $\|\hat{\boldsymbol{h}}_\mathrm{GMM}\|$ to be equal to $\|\hat{\boldsymbol{h}}\|$. Thus, we construct $\hat{\boldsymbol{h}}_\mathrm{GMM}$ as

$$\hat{\boldsymbol{h}}_\mathrm{GMM} = \frac{\|\hat{\boldsymbol{h}}\|}{\|\boldsymbol{E}\hat{\boldsymbol{h}}\|}\boldsymbol{E}\hat{\boldsymbol{h}}.$$

The GMM detector performs MRC scaling similar to (2) and is given by

$$\begin{aligned}\hat{\boldsymbol{x}} &= \frac{1}{\|\hat{\boldsymbol{h}}_\mathrm{GMM}\|^2}\hat{\boldsymbol{h}}^\mathrm{H}_\mathrm{GMM}\boldsymbol{Y}\\ &= \frac{1}{\|\hat{\boldsymbol{h}}\|\|\boldsymbol{E}\hat{\boldsymbol{h}}\|}\hat{\boldsymbol{h}}^\mathrm{H}\boldsymbol{E}\boldsymbol{Y}\end{aligned} \quad (10)$$

where $\hat{\boldsymbol{h}} = \beta\boldsymbol{y}_0$. The GMM detector (10) can be reached without knowing the SNR, in which case $\beta = 1$.

The vector $\hat{\boldsymbol{x}}$ can be modeled as $\hat{\boldsymbol{x}} = \rho\boldsymbol{x} + \tilde{\boldsymbol{n}}$ where ρ is a bias which depends on the system parameters and on the unknown channel distribution. This bias is given by the expectation $\rho = \mathbb{E}[z_k]$, where z_k is defined shortly before Proposition 1. Notice that the expectation is not dependent on k, since all z_k are statistically equivalent. Since the phase of $\hat{\boldsymbol{h}}_\mathrm{GMM}$ is better aligned with the phase of $\boldsymbol{h}$ than the phase of $\hat{\boldsymbol{h}}$ is, the bias ρ is closer to unity than the corresponding bias for the benchmark detector is. Thus, we expect superior performance with the GMM detector also for QAM constellations.

Up to this point, we have not made use of any knowledge of the SNR. If the SNR is at hand, we can seek to compensate for the bias. However, the expectation $\rho = \mathbb{E}[z_k]$ is not feasible to find in closed form, and we resort to approximations. We point out that if the receiver has access to multiple received blocks, then the bias ρ can be estimated over time. For Gaussian channels, we empirically found that a good approximation of ρ is given by

$$\rho = \frac{1}{2}\left(1 + \tanh\left(\frac{(k_1 + 10\log_{10}(\gamma))}{k_2}\right)\right). \quad (11)$$

The two parameters k_1 and k_2 depends on M and T, but can be tabulated off-line. Thus, for the case where the SNR is known to the receiver, the GMM detector becomes

$$\hat{\boldsymbol{x}} = \frac{1}{\rho}\frac{1}{\|\boldsymbol{y}_0\|\|\boldsymbol{E}\boldsymbol{y}_0\|}\boldsymbol{y}_0^\mathrm{H}\boldsymbol{E}\boldsymbol{Y}. \quad (12)$$

The reason for using $\boldsymbol{y}_0$ in (12) and not the more general notation of $\hat{\boldsymbol{h}}$ is that (11) has been computed for initial LS estimation so that $\hat{\boldsymbol{h}} = \boldsymbol{y}_0$. As MMSE estimation and LS estimation only differs with a constant, this is irrelevant for performance provided that ρ is computed for LS estimation.

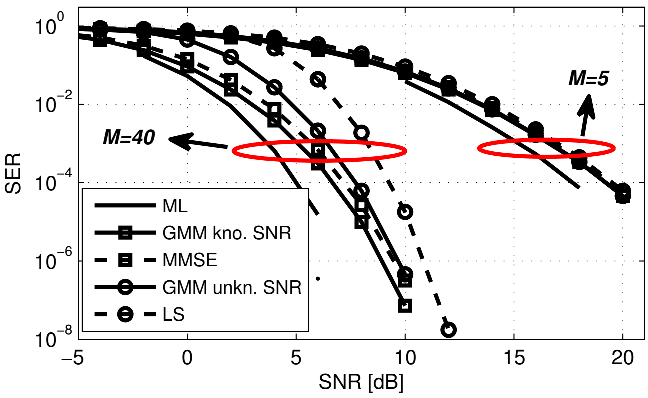

Fig. 1: Symbol error rate with 16-QAM signaling, different values of M, and $T+1=5$. The dashed curves represent the LS and MMSE (linear) detectors, and the solid marked curves represent the GMM detector when the SNR is known and unknown. The solid curve represents the receiver where joint ML detection of $\boldsymbol{h}$ and $\boldsymbol{x}$ is performed [3], [4].

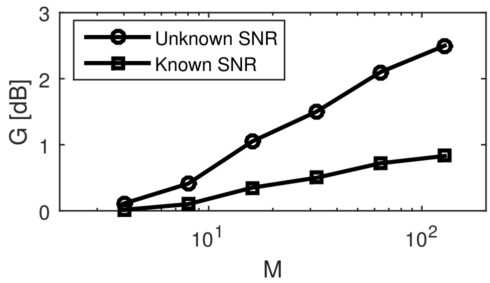

Fig. 2: Gain of the GMM receiver without SNR knowledge over the LS scheme (upper curve), and gain of the GMM receiver with SNR knowledge over the MMSE scheme (lower curve), respectively. In both cases, we use 16-QAM symbols, $T=4$, and SER $\approx 10^{-3}$.

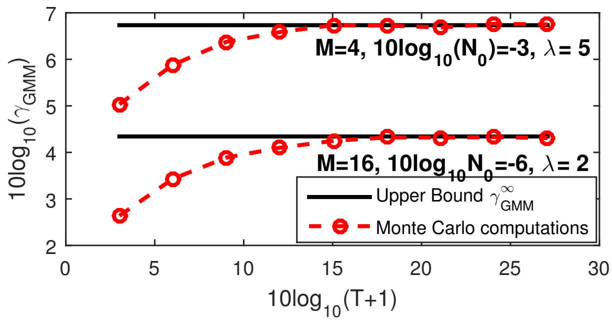

Fig. 3: Monte Carlo computations of the SNR of the GMM detector, and respective asymptotic bound, for different system parameters combinations. Here $\{x_k\}_{k=1}^{T}$ are 4-QAM symbols.

VI. Numerical Evaluations

We next provide numerical examples for the performance improvement attained by the GMM detectors over their linear counterparts, i.e., the benchmark detectors of Sec. II-A. In all presented cases, the entries of $\boldsymbol{h}$ are chosen as zero mean independent and identically distributed complex Gaussian entries with unit variance. We set $\min\{T,M\}=4$ for most simulations, and hence low additional complexity is required to perform GMM detection. Relaxing this complexity constraint trades off with better performance.

Fig. 1 shows the symbol error rate (SER) performance of different detectors. GMM detection provides SNR gains that increase with higher values of M. This dependency is shown explicitly in Fig. 2 for a given SER. Higher gains are harvested when the SNR is unknown. These gains seem linear in M which renders the method especially interesting for massive SIMO systems. A few dBs are harvested for rather small block length values. Fig. 3 shows the convergence of the post processing SNR of the GMM detector γ_{GMM} to its asymptotic bound found in Proposition 1. Estimating γ_{GMM} is done by means of Monte Carlo computations of $\mathbb{E}[z_k]$ and $\text{Var}[z_k]$. Most gains seem to be reached at moderate values of T, e.g. say $T+1=10$ for the cases of Fig. 3, but this varies depending on the parameters setting chosen. Gains close to 2 dB are reached compared to the standard LS scheme, i.e. $T=1$.

VII. Conclusions

The GMM receiver explores the structure of block fading channels to improve the channel estimate quality, i.e. it filters the raw channel estimate with a matrix constructed from the received pilots and data. The computation overhead of the GMM receiver is relatively small compared to benchmark schemes as LS and MMSE. This overhead trades off with SNR gains, that can be harvested with small block lengths. For example, gains of 1.5 dB compared to benchmark schemes are attained with a block length of five and eight receive antennas. For a fixed system setup, these gains converge asymptotically with the block length, and appear linear with the number of receive antennas which renders the method especially interesting for massive SIMO systems.

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