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# Multi-Echelon Inventory Control with Consideration of Emissions and Service Differentiation

LINA JOHANSSON
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Lina Johansson

DOCTORAL DISSERTATION

Thesis advisors: Dr. Peter Berling, Prof. Johan Marklund, Dr. Fredrik Olsson
Faculty opponent: Prof. Sandra Transchel

To be presented, with the permission of the Faculty of Engineering LTH of Lund University, for public criticism in the KC:E lecture hall on Friday, the 30th of October 2020 at 10:15.

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**Abstract**

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*To design mathematical models for evaluation and control of multi-echelon distribution systems with stochastic demand, in order to develop efficient methods for inventory control, with emphasis on the impact of various initiatives towards green supply chains. More specifically, this includes emissions related to system design decisions and service differentiation.*

This doctoral thesis consists of four scientific papers preceded by a summarizing introduction. Common for all papers is that they focus on inventory control in divergent multi-echelon inventory systems with one central warehouse supplying an arbitrary number of retailers or local warehouses. The research provides methods for determining control variables for efficient control in the different settings analysed in Papers I - IV, respectively. From a methodological perspective, the research belongs to the inventory control area in the field of operations research/management science.

Paper I considers a situation where customers have an acceptable waiting time for which they are willing to wait for a requested item, but it is critical not to exceed it. This type of time limits may be present in for example spare part inventory systems in the food industry where it is crucial to repair broken-down machines before the product perishes. Thus, a time based service level or a non-linear backorder cost reflecting this is considered. The backorder cost structure is extended to include general backorder costs as a function of the customers' waiting time. Paper I presents an exact method for finding optimal base-stock levels to minimize the total system cost under a service constraint or using a backorder cost. Furthermore, the model allows for quantification of emissions due to production waste from perished products as a consequence of delayed deliveries.

Paper II extends Paper I by allowing for emergency replenishments to prevent perishable products from going to waste because the acceptable waiting time is exceeded. The emergency replenishments are assumed to be faster than normal replenishments but at the cost of more emission intense transports. The model allows for emissions from production waste and the added emissions from the emergency replenishments to be quantified and compared.

Paper III presents efficient heuristics for finding optimal reorder levels and shipment intervals in an inventory system with time based shipment consolidation and lumpy demand. Shipments from the central warehouse is dispatched periodically to the retailers at fixed time intervals as a means to increase the utilization of the transports. Thereby the number of transports needed is lowered with the aim to reduce the connected costs and emissions. Both a fill rate constraint model as well as a backorder cost model is considered. The aim of the presented heuristics is to be applicable to real life inventory systems, and they are tested for this with good results.

Paper IV considers a setting with direct customer demand present not only at the retailers but also at the central warehouse. This type of inventory system may be found in an omni-channel distribution system with online sales, and it accentuates the need for service differentiation at the central warehouse. A heuristic for setting reorder points throughout the system and to decide how much stock to reserve at the central warehouse for the direct customer demand in order to achieve efficient control of the system is developed. The heuristic is designed to be implementable in practice, and it is evaluated using real life data from two different companies showing good results.

To summarize, this doctoral thesis provides mathematical models and develops efficient methods for inventory control with consideration of emissions and service differentiation.

**Key words**
Inventory, Multi-Echelon, Divergent, Stochastic, Sustainability, Service differentiation

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*Ad utrumque paratus*

# Contents

# Populärvetenskaplig sammanfattning

Ett ökat intresse för hållbarhetsfrågor och miljömässiga aspekter, samt en komplexare lagersituation för försörjningskedjor med ett ökat inslag av e-handel skapar nya utmaningar för effektiv styrning av materialflöden och lagersystem i olika försörjningskedjor. I denna avhandling studeras olika aspekter av dessa utmaningar för distributionssystem som omfattar ett centrallager och ett antal återförsäljare eller lokallager.

Problemställningarna som studeras är motiverade av de utmaningar som företagen vi samarbetat med står inför, men som också många andra företag möter. I två av forskningsartiklarna har de framtagna metoderna för lagerstyrning kunnat utvärderas för och jämföras med verklig företagsdata, med goda resultat. Fokus har även legat på att ta fram metoder som medger implementation i verkliga system, både vad gäller att de är konceptuellt tilltalande och beräkningsmässigt genomförbara.

Ett centralt begrepp i denna avhandling är service, det vill säga förmågan att leverera produkter till kunder utan fördröjning. I de fyra forskningsartiklarna som avhandlingen omfattar tas olika former av service i beaktande, vilka uppmärksammar fler än de traditionella servicemåtten. Målsättningen med de metoder som tagits fram är att ett förutbestämt krav på service till slutkunder med osäker efterfrågan ska uppfyllas, samtidigt som kostnaderna minimeras.

Ekonomiska och miljömässiga hållbarhetsaspekter adresseras i avhandlingen genom att utveckla och utvärdera stokastiska modeller för styrning av kopplade lagersystem där kostnader och emissioner av olika slag kan beaktas. En sådan situation som studeras i de två inledande forskningsartiklarna är motiverad av ett samarbete med ett företag som levererar produktionsutrustning och tillhörande reservdelar till livsmedelsindustrin. Ett stopp i produktionen hos någon av företagets kunder, på grund av brist på en kritisk komponent till en fallerande maskin, kan innebära att en stor mängd produkter måste kasseras ifall komponenten som krävs för att få igång produktionen igen inte uppbringas inom en viss tid. Att behöva kassera exempelvis stora mängder mejeriprodukter innebär, förutom en stor kostnad, även ett avsevärt koldioxidutsläpp då produkter går till spillo. Detta utsläpp är en konsekvens av lagerstyrningsbesluten i systemet. Vanligtvis ligger fokus på utsläpp kopplat till enbart transporter. En viktig aspekt som analyseras är dock att balansera koldioxidutsläpp från transporter och kassation, till exempel genom att kunna expressleverera reservdelar. Expressleveranserna sker då med snabbare, men också mer emissionsintensiva transportslag, för att kunna förhindra kassation och därmed tillhörande utsläpp.

En annan viktig aspekt när det gäller att reducera transportemissioner är att genom godskonsolidering uppnå en hög fyllnadsgrad hos de fordon som används. En ökad fyllnadsgrad innebär att färre transporter behövs och därmed lägre transportemissioner totalt. Samtidigt leder konsolidering till längre ledtider och förändrade förutsättningar för lagerstyrningen i systemet. En vanligt förekommande situation som studeras i den tredje forskningsartikeln är tidsbaserad skeppningskonsolidering av leveranser från ett centrallager till ett antal återförsäljare. Detta innebär att transporter från centrallagret görs med givna tidsintervall till grupper av återförsäljare längs förutbestämda rutter. I studien jämförs en nyutvecklad metod för styrning av verkliga system med denna typ av konsolidering med en vanligt förekommande okoordinerad lösningsmetod som idag används av ett av samarbetsföretagen. Den numeriska studien visar att den nya metoden erbjuder en förbättrad service som når de uppsatta servicemålen samtidigt som lagerkostnaderna sänks med 20 %.

En av logistikutmaningarna som den ökande e-handeln för med sig är högre krav på integrerad styrning av lagersystem där flera distributionskanaler med olika servicekrav ska tillfredsställas på ett kostnadseffektivt sätt. Ett allt vanligare upplägg är till exempel att ett centrallager både levererar direkt till e-handelskunder och till ett nätverk av återförsäljare. Den avslutande forskningsartikeln i denna avhandling introducerar en ny metod för mer effektiv lagerstyrning av sådana system där kunders efterfrågevolymer varierar kraftigt, något som är en vanlig, men svårhanterlig situation. Den nya metoden ger en avsevärt bättre styrning jämfört med redan existerande metoder. Den numeriska studien, baserad på data från två olika företag, visar att med bibehållen service till de olika kundsegmenten kan en kostnadsbesparing på upp till 27 % uppnås för systemet som helhet.

# Abstract

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To summarize, this doctoral thesis provides mathematical models and develops efficient methods for inventory control with consideration of emissions and service differentiation.

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To my friends who are cheering me on.

To my family, my parents Birgitta and Gert, Christian, and my grandmother Svea.

## List of Publications

This thesis is based on the following publications, referred to by their Roman numerals:

I **Quantifying Sustainable Control of Inventory Systems with Non-Linear Backorder Costs**

**L. Johansson**, F. Olsson
(2017) Annals of Operations Research, 259, pp. 217–239

II **Age-Based Inventory Control in a Multi-Echelon System with Emergency Replenishments**

**L. Johansson**, F. Olsson
(2018) European Journal of Operational Research, 265, pp. 951–961

III **Controlling Distribution Inventory Systems with Shipment Consolidation and Compound Poisson Demand**

**L. Johansson**, D. R. Sonntag, J. Marklund, G. P. Kiesmüller
(2020) European Journal of Operational Research, 280, pp. 90–101

IV **Controlling Inventories in Omni-Channel Distribution Systems with Variable Customer Order Sizes**

P. Berling, **L. Johansson**, J. Marklund
Department of Industrial Management and Logistics
Faculty of Engineering LTH, Lund University
(2020) Submitted

Related publications:

**Multi-Echelon Inventory Control with General Backorder Cost Structures and Emissions**

**L. Johansson**
Department of Industrial Management and Logistics
Faculty of Engineering LTH, Lund University
(2016) Licentiate Thesis

# Introduction

The introduction of this doctoral thesis will give a general background of the research presented and how the appended scientific papers relate to the overall research objective and to each other. Section 1 provides a background to the research and why the problems studied are chosen. Section 2 defines the overall research objective and the objectives for the four appended scientific papers. To give the reader an understanding of the context of the performed research, Section 3 discusses the research methodology used and Section 4 presents a brief overview of inventory control theory. Section 5 presents an overview of the appended scientific papers and their modelling features. The papers are summarized one by one in Section 6, presenting their respective main contributions. Section 7 summarizes the research contributions of the doctoral thesis and outline possible future research directions. This introductory chapter is partly based on the licentiate thesis Johansson (2016), which includes earlier versions of Paper I and Paper II.

## 1 Background

The measurable increase of the global temperature and increased concern for the environment have accentuated the need to incorporate more sustainability aspects in decision making, including inventory control decisions. Optimal decisions for economic sustainability have been widely investigated within inventory control theory, which has resulted in numerous methods for minimizing the total cost of the inventory system at hand. The addition of goals for environmental sustainability complicates matters and leaves many open questions. This doctoral thesis addresses some of these questions by providing models and decision support tools for inventory control that include other than the purely economic aspects. The research constituting the basis for this doctoral thesis is motivated by discussions and collaborations with industry partners. However, the problems studied are not unique to the partner companies.

Environmental sustainability has received increased attention lately with a focus on reducing the carbon footprint. Emissions from transports account for a large share of the global carbon footprint and have, quite naturally, been the focus of the operations research literature on sustainable supply chain management. Many of the strategies for decarbonizing supply chains (World Economic Forum; 2009) targets transport emission, e.g. the use of better vehicles, despeeding, optimizing transportation routes, increased utilization of vehicles, and increased amount of home deliveries. Many of these strategies have an impact on the premises of the inventory control system. This thesis studies how to account for this in the modelling and the decision process when versions of two of these strategies, increased vehicle utilization and increased home deliveries, are implemented. Transportation is not the only source of emission in the supply chain, though. For example, the amount of emissions from perishable products going to waste due to a machine breakdown is directly affected by the control of spare parts inventory. This dependency under various service definitions is also studied in this thesis.

The study of emissions due to lack of spare part availability and better methods to remedy this stems from a collaboration with a supplier of machines and spare parts to the dairy industry. In this industry, a stop in production due to a machine failure might lead to an entire batch of dairy products going to waste. This will happen if the required critical spare part cannot be provided within a certain amount of time, due to the perishable nature of the product. Not being able to fulfil the demand within this time window is of course costly, both in economical terms, but also in the form of emissions from the products being perished. The expected production waste in this situation is a consequence of the inventory control policy, and this type of indirect effects and how to handle them are studied in the first two papers of this thesis (Paper I and II).

Shipment consolidation is one method suggested to increase vehicle utilization and to reduce the carbon footprint of transports. This thesis investigates the impact of time based consolidation, i.e. the impact of shipping goods from the central warehouse to the retailers at fixed time intervals rather than as demand occurs. This leads to improved utilization of the vehicles by consolidating orders. In general, the utilization will increase with the shipping interval, which thus has a positive effect on the transport emissions and costs. Less frequent transport will, on the other hand, increase the length and variability of the retailers' lead times. Longer and more uncertain lead times result in more shortages and a need for more safety stock, which increase the inventory related costs. Inventory control methods that strike the balance between these costs in such a system, i.e. deciding on the shipment intervals and the inventory policy, are found in the third paper of this thesis (Paper III).

An increasing trend to use an online sales channel to supplement existing sales channels in an omni-channel network can be seen. Lately, this trend has been accentuated by the Covid-19 situation. This trend will lead to more home deliveries, which has been identified as an opportunity for the reduction of carbon emissions. However, it will also affect how to efficiently control the inventory system if the central warehouse is used to serve external as well as internal customers (that is, online customers and own retail stores). This is the case since the optimal service to provide typically differs significantly between the two with external customers expecting a higher service compared to the retailers. This stresses the need for integrated inventory control of different channels in order to achieve the required customer service levels in an efficient manner. Methods for doing this are developed in the last paper of this thesis (Paper IV).

# 2 Research Objectives

The overall research objective in this doctoral thesis can be stated as follows:

*To design mathematical models for evaluation and control of multi-echelon distribution systems with stochastic demand, in order to develop efficient methods for inventory control, with emphasis on the impact of various initiatives towards green supply chains. More specifically, this includes emissions related to system design decisions and service differentiation.*

In this doctoral thesis, "design mathematical models for evaluation and control" means developing mathematical models for determining the expected costs and, in relevant cases, the expected emissions and service levels in the inventory systems considered to be able to find optimal, or near-optimal, control variables.

The common features of the inventory systems throughout this doctoral thesis are that they can be described as "multi-echelon distribution systems with stochastic demand". That is, they consist of a central warehouse at the upper echelon supplying a number of local warehouses (often referred to as retailers) at the lower echelon, and the system faces stochastic customer demand. This is a common structure of inventory systems in practice, as well as in the literature, and it is present at all of the companies that have been collaborating partners during the thesis project. The inventory systems considered do differ with regards to, for instance, the topology, type of stochastic demand, replenishment policies, cost structure, and service requirements.

The implication of "efficient methods for inventory control" is the ambition that the developed methods should be conceptually and computationally tractable enough to be implementable in practice. Sometimes, this entails heuristics rather than exact solution methods. The heuristics should provide a near-optimal solution within reasonable time, and a practitioner should be able to grasp the underlying heuristic idea. Optimality is here defined by the objective to minimize the costs associated with the inventory system, given a certain inventory policy, while meeting a number of predetermined constraints on the system. The control variables considered are reorder levels and shipment intervals.

The "emphasis on the impact of various initiatives towards green supply chains" stems from the focus on the implications some of the suggested methods for supply chain decarbonization have on the inventory system, as well as the evaluation of the environmental cost or benefit of different solutions when allowing for an extended time based service definition. The "service differentiation" is linked to finding methods for evaluation and control of the system under an extended definition of the service level with the additional dimension of time, as well as finding the optimal differentiation of the service level between different channels when customer demand occurs at the retailer level as well as at the central warehouse.

The work in this doctoral thesis consists of four appended scientific papers with their specific aims linked to the research objective of the doctoral thesis.

An important objective in Paper I is to see how policy decisions affect emissions under time based service constraints, or non-linear backorder costs if a pure cost framework is used, in situations where the customers have an acceptable waiting time. Hence, the sought-after model should include the possibility to evaluate specific expected emissions connected to not fulfilling demand in time. That is, the model should

- Include a time window in which the customers acceptable waiting time lies, to better reflect possible service contracts for spare parts, and to further extend the model to consider general backorder cost structures.
- Provide a tool for quantifying the expected emissions connected to production waste as a consequence of inventory control decisions and the resulting service fulfilment.

In line with the aim of the focal company, we have used $CO_2$ emissions to model the environmental impact, although any type of emissions or environmental impact associated with the consequences of such a shortage may be handled with the model.

This work is continued in Paper II, where the objective is to evaluate the effect of including emergency delivery options. The model in Paper I is extended with the possibility of faster, but more emission intense emergency replenishments (compared to normal replenishments). This leads to a trade-off between extra costs and $CO_2$ emissions from transportations and the prevention of shortage costs and emissions emanating from the products perished when the spare part is not provided in time. The aim of Paper II is to design a model that makes it possible to

- Evaluate the consequence of allowing for emergency supply, in terms of expected costs and emissions, that with a certain probability can prevent a spare part from arriving after the acceptable waiting time and thereby prevent production waste emissions.

- Develop a heuristic for determination of the base-stock levels at the retailers and the central warehouse to minimize the expected total cost of the inventory system.

In Paper III, the problem of finding the optimal shipping intervals under a time based shipment consolidation and the corresponding inventory control policy under lumpy demand, i.e. the customer orders vary stochastically in time and quantity, is investigated. There exists a model for an exact solution for this problem, but it is computationally too complex to be used for inventory systems with more than a few retailers. The aim of Paper III is to develop two efficient heuristics, one under service level constraints and one using backorder costs, that are applicable to real demand data that finds

- How often shipments from the central warehouse should be dispatched to groups of retailers.
- The corresponding retailer base-stock levels and the reorder point at the central warehouse in order to minimize the expected total system cost, while meeting the target fill rate (in relevant cases).

Finally in Paper IV, a heuristic for controlling an omni-channel network where there exists a demand at retailer level as well as a direct upstream demand at the central warehouse is developed. More specifically, the work strives to develop efficient implementable heuristics that provides a way to determine

- How much inventory should be reserved for the direct upstream demand at the central warehouse, i.e. to what extent shall the service level between the different channels de differentiated.
- The choice of the reorder point for the general non-reserved stock at the central warehouse and the reorder levels at the retailers.

# 3 Research Methodology

This doctoral thesis follows a traditional operations research/management science research methodology, i.e. focused on mathematical modelling of operational decision problems. The mathematical model shall represent the most important aspects of the system, but cannot represent all aspects of the real system. The aim is to capture the most central features that are under investigation so that the model can provide support for the decisions in the real system, see Figure 1. Mathematical models can be either purely analytical, in the form of a simulation model or combinations thereof. The mathematical model describes the system by variables that characterize the system and logical relations representing the interactions between these variables. See e.g. Hillier and Lieberman (2015), Law and Kelton (2000) and Axsäter and Marklund (2010). In this doctoral thesis, both analytical models (exact as well as approximate) and discrete event simulation models have been used. As the objective for this doctoral thesis is to develop mathematical models for inventory control of the studied systems and quantitative decision tools, other choices of methods, such as for example case studies or surveys, are not suitable.

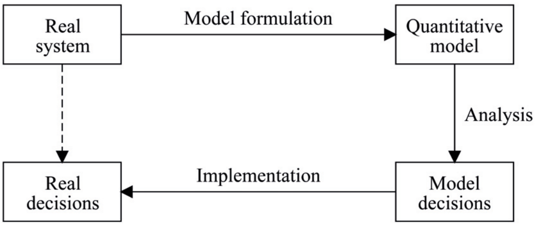


**Figure 1:** The use of quantitative models for decision making (Axsäter and Marklund; 2010).

Building a mathematical model is an iterative process often characterized by three steps. The first step is to formulate and structure the problem at hand. The key is to capture the essence of the problem while still making it possible to analyse and solve. The appropriate objective should also be determined and stated mathematically. Assumptions regarding the characteristics of the system are defined. In the next step, the problem is described mathematically and solved. To do this, approximations might have to be used. The third step is to validate the model and test if it properly describes the real system as intended. The assumptions made might need to be reviewed and the modelling might need to return to previous steps. Given that the mathematical model is valid, it can be used to analyse the real problem, and the decisions that are good for the model can be expected to work well for the real system.

One advantage of mathematical modelling is that the model is a concise and quantitative description of the problem. The model may allow for an understanding of the relation between different aspects of the problem and provide a decision support tool when decisions cannot be made directly from the real system. A drawback with a mathematical modelling approach is that the model of course is an idealization of the true problem. Hence, simplifying assumptions and approximations often have to be made, particularly in analytical models, which affect the generality of the model. One challenge is to find an appropriate level of detail. A model with too few details lacks in validity, while an overly detailed model is too complex to analyse. The level of detail in this doctoral thesis is partly influenced by the aim to derive implementable models.

In this doctoral thesis, emphasis is put on analytical models, and simulation models have been used to validate the results of the analytical models. This is the case since simulation models offer a better flexibility and ability to capture complex systems. Simulation models are thus a useful tool for evaluating a problem for a given scenario. However, simulations typically do not give the same type of insights as analytical models, and optimization through simulation is very time consuming. As stated in the research objective, this doctoral thesis concerns inventory systems with stochastic demand, specifically Poisson and compound Poisson demand. Stochastic discrete event simulation models fit well with these demand processes, and such models have been used to validate the results.

# 4 Fundamental Inventory Control Theory

A short introduction to inventory control theory is given in this section, to give an overview of fundamental definitions and concepts within the field that are relevant for the appended scientific papers. For a general overview of inventory theory, see e.g. Axsäter (2015) and references therein.

The stock points of an inventory system can be organized in several ways. The simplest form of an inventory system is a single-echelon system consisting of a single stock point as presented in Figure 2. Systems where

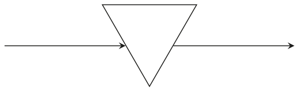

**Figure 2:** A single-echelon inventory system.

several inventory locations are connected in a structure with more than one echelon of stock points are called multi-echelon systems. A serial system consists of stock points connected one after each other, where each stock point has at most one immediate predecessor and one immediate successor. For an example with three stock points, see Figure 3.

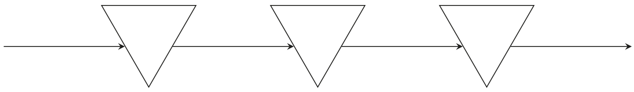

**Figure 3:** A three-echelon serial inventory system.

In more complex systems, a stock point can have more than one immediate predecessor or successor. Two such systems are assembly systems and divergent distribution systems. In an assembly system, each stock point can have several immediate predecessors, but only one immediate successor, making the flow of items converge as in the example in Figure 4. This type of systems is typically connected to the assembling of products where stock points upstream hold components that are combined and put together downstream to form the final product.

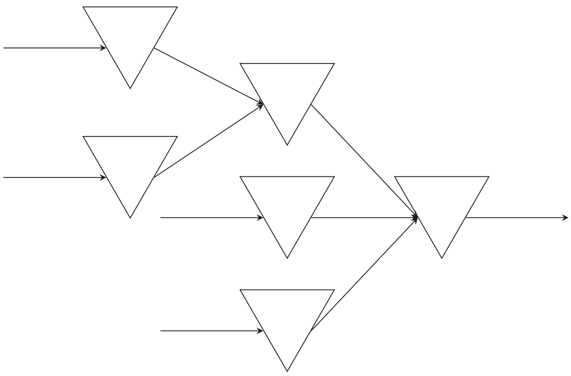

**Figure 4:** A three-echelon assembly inventory system.

In contrast, a stock point in a divergent distribution system has only one immediate predecessor, but it can have several immediate successors, see an example in Figure 5. Divergent systems can be found in distribution inventory systems where a product is distributed from a central warehouse to a retailer, via a local distribution centre. The same structure can also be found when a raw material is used for producing multiple end products. The focus of this doctoral thesis lies on distribution inventory systems with two echelons. The flow of products in this type of system originates from an outside supplier assumed to have ample stock that delivers items to a central warehouse who, in turn, delivers items to the local retailers. This type of system is often referred to as a one-warehouse-multiple-retailer (OWMR) system.

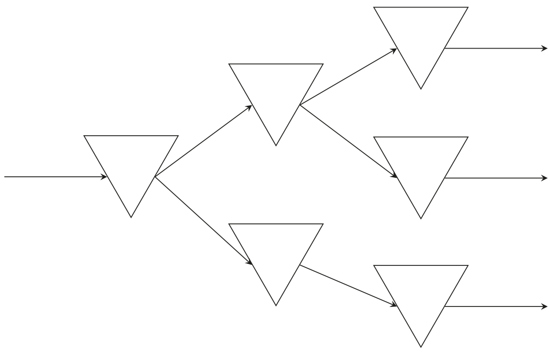

**Figure 5:** A three-echelon divergent distribution inventory system.

To model the stochastic demand of the customers requesting items from the inventory system, it is common to use a standard distribution. A commonly used process is the Poisson process. That is, customers arrive according to a Poisson process and place an order for one item each. The inter-arrival times between consecutive customer arrivals are independent and exponentially distributed, and the process is referred to as memoryless, i.e. the time until the next customer arrival is independent of when the last customer arrived. This gives the Poisson process analytical advantages. Poisson demand is often suited to model low demand items. If customer order sizes differ, a compound Poisson process can be used where the number of requested items varies according to the compounding distribution. For Poisson demand, the variance-to-mean ratio of the demand under a certain time period is equal to one, whereas compound Poisson demand can model demand with a variance-to-mean ratio larger than or equal to one. The compound Poisson, or Poisson, process may be computationally demanding, and it may be favourable to use the normal distribution. Especially for high demand items, the normal demand assumption may be better suited.

Two important concepts within the field is the inventory level and the inventory position. The inventory level is defined as the stock on hand minus the backorders. That is, a positive inventory level means that there are items in stock, while a negative inventory level means that there are unfulfilled demand where customers are waiting for their items. The inventory position is defined as the stock on hand minus the backorders plus the outstanding orders. To control the inventory system, it must be inspected. This can be done either periodically, at regular time intervals, or continuously. In a continuous review system, the inventory situation is known at all times, whereas for a periodic review system, it is only known at the review epochs, leaving it unknown in between inspections.

Stock is refilled to a stock point by replenishments orders, placed according to an ordering policy that controls how replenishment decisions are made. The replenishment policy specifies when an order should be placed and the quantity to be ordered The replenishment policy is most commonly based on the inventory position. One such policy often used in practice and in theory is the $(R, Q)$ policy. This policy is a batch-ordering policy that will trigger an order when the observed inventory position declines to or below $R$ with an order quantity of $Q$ units. If needed, multiple batches are ordered to assure that the inventory position is above $R$ after ordering. An alternative ordering policy is the $(s, S)$ policy where an order is placed when the inventory position falls to or below $s$. In contrast to the $(R, Q)$ policy, the order quantity is not fixed. Instead, the policy triggers an order to bring the inventory position up to the maximum level $S$. The ordered quantity is thus dependent on the value of the inventory position when the order is placed.

A special case of both the $(s, S)$ and the $(R, Q)$ policy is the so called base-stock policy, which is a one-for-one policy. That is, when the inventory position drops below the base-stock level, the number of items that have been demanded are ordered to make the inventory position reach the base-stock level again. In case of continuous review, a new item is ordered as soon as an item is taken out of stock. The base-stock level, or order-up-to level, is often denoted $S$, and the policy may also be referred to as an $(S-1, S)$ policy, since an order is placed at the reorder point $S-1$, bringing the inventory position to $S$. The base-stock policy is equivalent to an $(s, S)$ policy with $s = S-1$, and to an $(R, Q)$ policy with $R = S-1$ and $Q = 1$.

It has been shown that the optimal policy structure for single-echelon systems, under very general conditions, is of the $(s, S)$ type. For multi-echelon systems, the structure of an optimal ordering policy is more complicated, and optimality results are scarce. For serial systems there exist some results, and for assembly systems as well, since it has been shown that, under certain conditions, an assembly system can be decomposed into a number of serial systems. For divergent systems, there are currently no general optimality results. The structure of an optimal policy would be very complex, and the majority of the existing literature has thus focused on optimizing system variables under reasonable and simple ordering policies.

The service a customer receives can be measured in different ways. Two common measures are the so called fill rate and ready rate. The fill rate is defined as the fraction of demand that can be filled directly from stock on hand. The ready rate is the fraction of time with positive stock on hand. When the demand is continuous or one-for-one, these two service levels are equal. For compound Poisson on the other hand, they differ. Service measures are common in practice, and companies often specify a target service that should be met in relation to their customers. The fill rate is wide spread in industry and also the service level the collaborating companies are using.

When finding the optimal solution, the objective is often to minimize the overall total cost of the inventory system. This total cost can consist of several components. Holding inventory is associated with a cost for the warehouse, handling, tied up capital etc. The holding cost is often a cost per unit and time unit. When placing an order, there may be a set-up cost for the ordering. When using target service levels, service constraints are added to the optimization problem specifying the service that must be fulfilled while finding the lowest cost. An alternative to setting service targets is to use penalty costs. Orders that cannot be fulfilled incur a penalty cost. Traditionally, this cost is either in the form of a fixed cost per unit, or a backorder cost per unit and time unit also reflecting the amount of time the order is waiting before being fulfilled. Using penalty costs instead of service constraints allows for simpler comparisons between different solutions and choices of control variables. A pure cost problem can be evaluated in only one dimension, while a cost problem under a service constraint is two-dimensional where both the expected cost and the expected service affect the quality of a solution.

The replenishment orders that the retailers place at the central warehouse are generally sent directly to the retailer if there is stock on hand. If there is not, the items are backordered. When the central warehouse has stock on hand again, the backorders will be filled. The order in which the backorders are handled may differ, and different allocation policies may be used. For example, one retailer may be prioritized compared to another, and then it is preferable to fill backorders from that retailer first, before tending to the others. The simplest policy is to allocate the refilled stock to the oldest backorder first, that is in a first come-first served basis. This is often referred to as an FCFS policy. An alternative to backorders, when the customer will wait for its ordered items until they arrive, is that the demand is lost. That is, the customer cancel its order. This is denoted lost sales, and in that case an allocation policy is no longer of interest.

Emergency replenishments can be used to prevent long waiting times when normal replenishments are not fast enough and customer demand cannot be satisfied directly. The emergency replenishments can be sent from different stock points, such as the central warehouse, an outside supplier, or from another retailer at the same echelon. This can prevent lost sales or backorders at the expense of an emergency shipment cost, but naturally this assumes that the emergency shipment is faster than normal ones. In the special case where the emergency replenishments are sent from another stock point at the same echelon, they are denoted lateral transshipments.

In order to achieve more efficient transports and reduce the number of shipments, orders can be consolidated. Instead of sending orders immediately as they are requested, several orders are shipped together. This increases the utilization of the vehicles, and thereby reduces the number of shipments needed. This will save both transport costs as well as lowering the transport emissions. In principle, there are two basic types of shipment consolidation, and a mix thereof. Time based shipment consolidation means that shipments are sent with a fixed time interval, for example once a week. Alternatively, shipment consolidation can be quantity based. Then, a shipment is sent when a certain quantity is reached, for example the quantity corresponding to a full load carrier. The two types can also be combined in time and quantity based consolidation.

# 5 Overview of Modelling Features in the Papers

This section presents an overview of the key characteristics of the problems and models in each of the four appended scientific papers. The aim is not to provide a complete list of modelling features, but to highlight some of the common aspects as well as central differences between the models and to summarize the scope of the research. Model assumptions are further explained and motivated in Section 6 and in Papers I–IV.

All of the inventory models analysed in this doctoral thesis share the same basic structure of a divergent distribution system with two echelons where a central warehouse at the upper echelon serves a number of non-identical local warehouses or retailers at the lower echelon. The central warehouse replenishes its stock from an outside supplier. This is a so called one-warehouse-multiple-retailer system, as depicted in Figure 6.

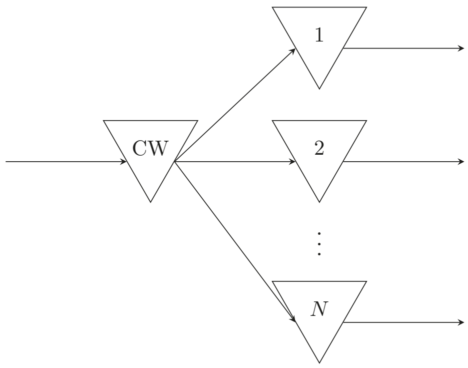


**Figure 6:** The considered inventory system.

Common for all models is that they assume continuous review, complete backordering, FCFS allocation, linear holding costs at all locations, constant transportation times within the system, and a constant lead time from the outside supplier.

To highlight how the papers relate to each other, their different modelling features are presented in Table 1.

**Table 1:** Overview of modelling features in Papers I–IV.

| | | I | II | III | IV |
|---|---|---|---|---|---|
| Demand structure | Poisson | x | x | | |
| | Compound Poisson | | | x | x |
| Replenishment policy | $(S-1, S)$ at all locations | x | x | | |
| | $(R, Q)$ at CW and $(S-1, S)$ at retailers | | | x | |
| | $(R, Q)$ at all locations | | | | x |
| Service measure / penalty cost | Fill rate | x | | x | x |
| | Time window service level | x | | | |
| | Linear backorder cost | x | | x | |
| | Piecewise constant backorder cost | x | x | | |
| | General backorder cost | x | | | |
| Emission quantification | | x | x | | |
| Tested on real data | | | | x | x |
| Exact solution | | x | | | |

In all papers, commonly used replenishment policies are assumed, and the control variables of the system are optimized given these policies. The overall objective is to minimize the total system cost, possibly under service constraints.

All papers consider service measures, but to various degrees. Paper III and Paper IV consider a traditional fill rate, where all provided solutions should meet the specified target fill rate. Paper I considers a more general service constraint, i.e. a time window based service level. This service level reflects the acceptable customer waiting time. This time window based service level is discussed as an option in Paper II.

The alternative to a service level, i.e. a penalty cost, is also considered in all papers. Directly in Papers I–III, and as a part of the heuristics in Paper IV. Similar to a more general time based service measure, more general backorder costs as functions of the customer waiting time are considered in Paper I and Paper II, whereas a standard linear backorder cost per unit and time unit is considered in Paper III (and indirectly in

Paper IV).

Environmental considerations have inspired the problem formulation in all papers, and Paper I and Paper II also provide quantification of the considered emissions.

The problems and models studied in the appended papers are all motivated by our industry partners and the challenges that they, as well as many other companies, face. Two of the papers, Paper III and Paper IV, include real data in their numerical studies, allowing the developed heuristics to be tested on it.

# 6 Summary of Papers

In this section, a summary of the four appended scientific papers follows. The problem at hand and the inventory systems that are studied are presented, including the assumptions that are made. A brief description of the modelling approach is given, as well as some concluding remarks. The main contributions will be summarised in Section 7. For more details, the reader is referred to the appended papers. Note that the notation in Sections 6.1–6.4 follows the notation in Papers I–IV, respectively, and that the notation differ between the papers.

## 6.1 Paper I - Quantifying Sustainable Control of Inventory Systems with Non-Linear Backorder Costs

Paper I studies a two-echelon inventory system with one central warehouse and a number of local sites or retailers. As the focus in the paper is on expensive spare parts with low demand, all locations are assumed to use continuous review base-stock policies, which is often used in practice for this type of products. To find the optimal base-stock levels, the decision is usually based on either a linear backorder cost or on a certain target fill rate. Considering a linear backorder cost, the cost (per unit and time unit) is assumed to be proportional to the customers' waiting time, but in many practical cases this might not be true. Instead, the backorder cost may be a non-linear function of the waiting time. For example, there may exist contracts where the spare parts provider is obliged to pay a fixed penalty fee if the customer demand is not fulfilled within a certain time window, otherwise no cost is incurred. Similarly, a service criteria with the corresponding characteristics can be defined. In Paper I, we assume that there is an acceptable waiting time for which a customer finds it acceptable to wait, motivated by our underlying case and focal company.

In the studied inventory system, demand occurs only at the local sites and according to independent Poisson processes where all unsatisfied demand is backordered. We assume that customer demand should be fulfilled within a certain time window, alternatively that a significant backorder cost is incurred if it is not. Two types of backorder cost structures are considered and described in greater detail below. The cost structure also includes holding costs per unit and time unit at all locations.

The local sites replenish their stock from the central warehouse, and the transportation times are assumed to be constant. However, delays due to stockouts at the central warehouse may occur, making the lead times stochastic. The central warehouse, in turn, replenishes its stock from an outside supplier assumed to have ample stock and constant lead time.

We assume that backorders at the retailers and at the central warehouse are filled according to the first come-first served (FCFS) rule. This is not an optimal allocation policy. As an example, consider a situation where two customers are waiting at one of the local sites. The first customer has been waiting longer than the acceptable waiting time, while the second customer's waiting time has not yet reached the time limit. Then, it would be more cost efficient to assign an incoming item to the second customer instead of applying the FCFS rule. However, FCFS allocation is commonly used in theory and in practice. In our system, it is reasonable from a practical point of view. The use of another rule might lead to customers already exceeding the time limit ending up with very long waiting times when they are not being prioritized.

We consider time window based service constraints at all local sites. The time window based service level is defined as the probability that an arriving customer's time in backorder $Y_i$ will not exceed the predefined acceptable waiting time limit $\omega_i$. In models similar to ours, it is common that the service level definition is the so called fill rate, i.e. the fraction of demand that can be satisfied immediately from stock on hand. This is, of course, a special case of our service level definition with $\omega_i = 0$. Let us denote the service level $\beta_i$, $i = 1, \ldots, N$, where $N$ is the number of local sites, and require that the service level should be at least $\ell_i$ within $\omega_i$ units of time. Then, we have

$$\beta_i = \mathbf{P}(Y_i \leq \omega_i) \geq \ell_i, \ \forall i \in \{1, \ldots, N\}.$$

This service level definition can be generalized to include several time limits with different target levels defined for different intervals of the waiting time.

The first backorder cost structure considered is a piecewise constant backorder cost with similar characteristics as the time window based service level. Let us denote the backorder cost at location $i$ by $B_i$. In its simplest form, this stepfunction consists of only one step,

$$B_i(Y_i) = \begin{cases} 0 & \text{if } Y_i \leq \omega_i, \\ b_i & \text{if } Y_i > \omega_i, \end{cases}$$

where $b_i$ is a positive constant. In this case, for a waiting time up to the acceptable time limit $\omega_i$, no penalty will have to be paid, but when the limit is exceeded a large fixed cost will be incurred. The backorder cost thus resembles the time window based service level. This cost structure can be generalized, in analogy to the service level, by including more steps and time limits that, when exceeded, will lead to a step by step increasing cost. With $K$ different time limits, $\omega_i^{(j)}$, $j = 1, 2, \ldots, K$, we get the following backorder cost structure

$$B_i(Y_i) = \begin{cases} 0 & \text{if } Y_i \leq \omega_i^{(1)}, \\ b_i^{(j)} & \text{if } \omega_i^{(j)} < Y_i \leq \omega_i^{(j+1)}, \ j = 1, \ldots, K-1, \\ b_i^{(K)} & \text{if } Y_i > \omega_i^{(K)}. \end{cases}$$

That is, if an arriving customer has to wait longer than a prescribed time limit $\omega_i^{(j)}$, the service provider has to pay a fixed penalty fee $b_i^{(j)}$. Here, we assume that $\omega_i^{(1)} < \omega_i^{(2)} < \ldots < \omega_i^{(K)}$ and $b_i^{(1)} < b_i^{(2)} < \ldots < b_i^{(K)}$.

The cost structure can be generalized to any general non-linear cost function that depends on the customers' time in backorder. As an example, we consider an exponentially increasing backorder cost

$$B_i(Y_i) = c_i \cdot a_i^{Y_i},$$

where $a_i > 1$ and $c_i > 0$ are constants.

The analysis provides an exact evaluation of the expected total cost and optimization of the base-stock levels. Furthermore, the model makes it possible to quantify expected emissions associated with not satisfying demand within the given time window.

To model the system, we utilize information about the timing of outstanding orders for the warehouse and the retailers. We start by finding the limiting ages of the oldest unit at all locations. The age is defined as the time that has passed since the unit was ordered. Given the stochastic delay at the warehouse, we know if the oldest unit is in stock or not by looking at its age. By noting that the ages are Erlang distributed, we can derive the density functions of the age of the oldest unit at all locations. Using this information, we can find the distribution of the stochastic delay at the warehouse and thereby the probability functions for the inventory levels at the retailers and the probabilities that an arriving customer has to wait longer than the acceptable waiting time limit. This is an extension of the analysis in Graves (1985) and Axsäter (1990) by the derivation of closed form expressions of the probability distributions of the inventory levels and the customer waiting times. The expected inventory level at the central warehouse is easily determined since the demand is Poisson and the lead time is constant.

The expression for the expected cost is formulated using the expected positive inventory levels, $\mathbf{E}[IL_i^+]$, for all locations $i = 0, \ldots, N$ (for finding the expected total holding cost) and the expected total backorder cost, $EB$, that depends on the three cases studied,

$$EC = h_0\mathbf{E}[IL_0^+] + \sum_{i=1}^{N} h_i\mathbf{E}[IL_i^+] + EB,$$

where $h_0$ is the holding cost at the central warehouse, and $h_i$ is the holding costs at the local sites, $i =, \ldots, N$.

A cost optimization procedure is given for all three studied cases (i.e. time window service constraints, piecewise constant backorder costs, and general non-linear backorder costs) where upper and lower bounds for the base-stock levels at the central warehouse and the local sites are derived. To illustrate the effect of the service constraint and the backorder cost structures, we consider some numerical examples. We note that in some cases when the backorder cost is significantly larger for long waiting times compared to short waiting times, the inventory should be pushed upstream to the central warehouse. This is contrary to most results for multi-echelon inventory models with a traditional fill rate constraint. Using a traditional fill rate without considering the acceptable waiting time in the service measure may cause increased costs. In the numerical examples in the paper, the cost increase can be up to 35 %.

In addition to study different backorder costs and service levels, we take a first step towards quantifying how policy decisions affect the expected emissions from a production waste perspective. In a spare parts inventory system, the emissions of transporting an item or holding it in inventory may not be so large, but not delivering them promptly may have serious consequences on both costs and emissions. The underlying motivating case concerns the emissions of $CO_2$ as an effect of discarding dairy products, thus this type of emissions is studied in the numerical examples. However, the model is not limited to this type of emissions. It could handle any type of emissions associated with not fulfilling customer demand in time.

## 6.2 Paper II - Age-Based Inventory Control in a Multi-Echelon System with Emergency Replenishments

Paper II considers the same two-echelon spare parts distribution system with one central warehouse and $N$ local sites as Paper I. Base-stock policies are applied at all locations, and demand occurs at the retailers according to independent Poisson processes with customer arrival rate $\lambda_i$, $i = 1, 2, \ldots, N$. There are predefined waiting time limits $\omega_i$ for all local sites $i = 1, 2, \ldots, N$, for which customers find it acceptable to wait for an item, and all unmet demand is backordered. The time limits are reflected in the backorder costs in the form of stepfunctions, where a significant fixed backorder cost is incurred if the customer waiting time exceeds the stipulated time level. Otherwise, there is no cost when the customer receives the unit within the given time window.

Moreover, the system also includes the possibility of fast emergency supply in cases where the customer waiting time risks to exceed the acceptable time limit. The purpose of the emergency replenishments is to reduce the impact of shortages, and therefore, the transportation time is assumed to be shorter than the given time limits. A simple and intuitive emergency replenishment rule is used. This states that an emergency replenishment is requested when an arriving customer finds a waiting time longer than the given time limit. Although an emergency replenishment is requested, it may not always be possible to arrange the shipment within the time window. Hence, a requested emergency shipment is only realized with a given probability $1 - \alpha_i$.

The cost structure in Paper II includes three types of costs. At every location, there is a holding cost per unit and time unit. The backorder costs per unit at the local sites $i = 1, 2, \ldots, N$ are, as aforementioned, assumed to be piecewise constant functions of the customers' time in backorder. As before, let us denote an arriving customer's time in backorder by $Y_i$ and let $b_i$ be a positive constant for all local sites $i$. Hence, we have the backorder costs

$$B_i(Y_i) = \begin{cases} 0 & \text{if } Y_i \leq \omega_i, \\ b_i & \text{if } Y_i > \omega_i. \end{cases}$$

That is, a constant cost of $b_i$ is incurred if the waiting time exceeds the acceptable time limit, otherwise the cost is zero. For the emergency replenishments, there is a cost of $r_i$ per unit for local site $i$.

The emergency replenishments that are requested by the local sites are shipped from the outside supplier. From a modelling perspective, the emergency replenishments can thereby be seen as lost sales. Hence, the demand structure at the central warehouse is affected and is no longer Poisson. As an approximation, we assume that this demand is still Poisson. To find the warehouse arrival rate, a fixed point equation must be solved, and we provide a proof that there exists a unique solution to this equation. Given the arrival rate at the central warehouse, we can find an expression for the expected inventory level for the central warehouse.

To find the performance characteristics for the local sites, we use the same approach as in Paper I using the stationary distributions of the age of the oldest units at the central warehouse ($X_0$) and the retailers ($X_i$). The aim is to find the expected stock on hand at the local sites and the probabilities that the waiting time limits are exceeded. Due to emergency shipments requested from the outside supplier, the inventory system can be seen as having doubly stochastic Poisson demand with state dependent intensity $\xi(X_i)$ according to

$$\xi(X_i) = \begin{cases} \alpha_i \lambda_i, & \text{for } 0 \leq X_i < L_i + z - \omega_i, \\ \lambda_i, & \text{for } X_i \geq L_i + z - \omega_i, \end{cases}$$

given a stochastic delay $Z = z$ at the warehouse, and where $L_i$ is the transportation time from the central warehouse to local site $i$. That is, demand occurs according to a Poisson process where the customer arrival rate also is a stochastic process. To find the density of the age of the oldest item at all local sites, we use the results from Olsson and Turova (2016) regarding an inventory system with doubly stochastic Poisson demand.

Emergency shipments may lead to increased environmental impact due to faster and more emission intense transports, but at the same time, they can prevent emissions stemming from product waste at the customer production site. From this point of view, there exists a trade-off between the use of emergency replenishments and the prevention of indirect emissions due to long waiting times. We assume, as an example, that the normal replenishments are made by heavy trucks and that the emergency replenishments are made by air freight emitting about five times as much $CO_2$, but the model can handle the use of other transportation modes in the same manner.

In our numerical study, we investigate a number of test problems to see how the solution procedure performs. The base-stock levels given by our model are identical to (in 93 % of the test problems) or near the optimal base-stock levels found through simulation. The costs have an average difference of 0.07 %. We also evaluate the expected emissions connected to the base-stock levels provided by the method. A comparison is made with the system without the possibility of emergency replenishments (as in Paper I) to see how the inclusion of this affects the total expected emissions. For some of the test problems, there is a reduction in expected total emissions when introducing emergency replenishments. In these cases, the emergency shipments and the use of a more emission intense transportation mode prevent enough products being discarded to render a less polluting system overall.

## 6.3 Paper III - Controlling Distribution Inventory Systems with Shipment Consolidation and Compound Poisson Demand

Our study on consolidated shipping is motivated by a problem facing one of our collaboration partners, a large European company in the construction industry. Similar to many others, they are looking for an efficient method for a time based consolidation scheme in order to reduce the environmental impact of their transports and preferably the total cost for the system. Many of the company's sheet metal products are large and bulky, and transport is expensive relative to the product value. To further reduce the number of transports, the time based shipments are made to a group of retailers, all served by a common transport in a so called milk run. Less transport has a positive effect on transport emissions as well as transport costs, but it also have an effect on the lead times. The resulting increase in the expected mean and variability of the lead times to the retailers implies a need for more safety stock and/or a poorer service which results in higher holding and/or shortage costs.

The aim of the paper is to develop computationally efficient heuristics that can be used in practice to solve the trade-offs between the different costs. That is, heuristics that optimize the periodic shipment intervals and the reorder levels at all stock points to minimize total costs for real world problems with many retailers, many retailer groups, and highly variable demand quantities. This is done both in the context of a backorder cost model and a fill rate constraint model. The latter is in coherence with the aims of the focal company, and the former is to be able to benchmark against existing methods in the literature. However, as previous work, e.g. Stenius et al. (2016), has focused on exact solutions, the resulting methods are computationally demanding and only feasible to use for small size problems with few retailers, few retailer groups, and small demand quantities, and they are thus not applicable in practice.

We consider a one-warehouse-multiple-retailer inventory system with lumpy demand in accordance with a compound Poisson distribution. This assumption fits well with the demand data from the focal company. Deliveries from the warehouse to groups of retailers are consolidated using a time based shipment consolidation policy with periodic dispatching and fixed costs per shipment. In coherence with the situation at our focal company, we assume complete backordering and first come-first served allocation. Moreover, the lead time from the outside supplier to the central warehouse and the transportation times from the warehouse to the retailers are constant.

The heuristic is based on an approximation where the compound Poisson demand is transformed into an "equivalent" Poisson demand. The approximate problem is then solved exactly using the fast recursive method in Marklund (2011). Finally, the results are re-scaled back to the original compound Poisson situation.

The approximate transformation of the compound Poisson demand to Poisson demand is obtained using an adapted version of the approach in Axsäter et al. (1994). This approach is based on the expected value, $\mu_i$, and standard deviation, $\sigma_i$, of the demand per time unit at retailer $i$. If customers at retailer $i$ arrive according to a compound Poisson process with intensity $\lambda_i$ and order size $Y_i$, these values are equal to

$$\mu_i = \lambda_i \mathbf{E}[Y_i]$$

and

$$\sigma_i = \left(\lambda_i \mathbf{E}\left[Y_i^2\right]\right)^{1/2}.$$

An "equivalent" Poisson demand, i.e. one with the same ratio between the mean value and the standard deviation of the demand per time unit, at retailer $i$ is attained by setting

$$\lambda_i^{'} = \lambda_i \frac{\mathbf{E}^2[Y_i]}{\mathbf{E}\left[Y_i^2\right]}.$$

Analogously, the transformed demand intensity at the central warehouse, $\lambda_0^{'}$, is obtained as

$$\lambda_0^{'} = \sum_{i=1}^{N} \lambda_i \frac{\mathbf{E}^2[Y_i]}{\mathbf{E}\left[Y_i^2\right]}.$$

Since $\lambda_i^{'} \leq \lambda_i$, the mean demand per time unit is scaled down from $\mu_i$ to $x_i\mu_i$ at all retailers $i$ ($i = 1, ..., N$) and from $\mu_0 = \sum_{i=1}^{N} \mu_i$ to $x_0\mu_0$ at the central warehouse (Axsäter et al.; 1994). The scaling factors equal

$$x_i = \frac{\mathbf{E}[Y_i]}{\mathbf{E}\left[Y_i^2\right]} \quad i = 1, ..., N$$

and

$$x_0 = \frac{\sum_{i=1}^{N} \lambda_i \mathbf{E}^2[Y_i]/\mathbf{E}\left[Y_i^2\right]}{\sum_{i=1}^{N} \lambda_i \mathbf{E}\left[Y_i\right]}.$$

To apply this transformation approach in our model, not only the demand has to be transformed, but also the order quantity and the cost per shipment (to preserve the relationship between the holding and shortage costs to the shipment costs). The order quantity $Q$ at the central warehouse is transformed to $Q^{'} = \lfloor x_0 Q \rceil$ ($x_0 Q$ rounded to the closest integer), since a smaller order quantity is required if demand intensity decreases. Should the closest integer be equal to zero, $Q^{'}$ is adjusted to one. The shipment cost $\omega_k$ to retailer group $k$ ($k = 1, ..., K$) has to be scaled down using the corresponding transformation factor $\kappa_k$ such that $\omega_k^{'}$ equals $\kappa_k \omega_k$, where $\kappa_k$ is determined as

$$\kappa_k = \frac{\sum_{i \in \Omega_k} \lambda_i \mathbf{E}^2[Y_i]/\mathbf{E}\left[Y_i^2\right]}{\sum_{i \in \Omega_k} \lambda_i \mathbf{E}\left[Y_i\right]} \quad k = 1, ..., K,$$

where $\Omega_k$ is the set of retailers belonging to group $k$.

The shipment intervals are determined for the Poisson situation using one of two different approaches. They are either optimized by enumeration, or they are approximated using the deterministic demand heuristic suggested in Marklund (2011). In the latter case, the shipment interval for retailer group $k$ is determined as

$$T_{P,k}^{M} = \sqrt{\frac{2\omega_k^{'}}{\sum_{i \in \Omega_k} (h_0 + h_i)\lambda_i^{'}}} \quad k = 1, ..., K,$$

where $h_0$ and $h_i$ are the holding cost per unit and time unit at the central warehouse and retailer $i$, respectively. Optimizing the shipment intervals requires a complete enumeration over all reasonable values, which can be avoided by using the heuristic. Thus, the heuristic reduces the computation time to a large extent. Comparisons of using the optimized shipment intervals and the approximate ones are made in the numerical study, showing very similar performance. Hence, the heuristic is preferable because of its computational efficiency.

To handle fill rate constraints, that are typically used in industry including our focal company, we extend the approximation method using a backorder cost into a two-step procedure. The first step includes the determination of the reorder point at the central warehouse and the shipment intervals by adapting the backorder cost heuristic. This requires a backorder cost per unit and time unit, $b_i$, that is consistent with the target fill rate, $\beta_i^*$, for each retailer $i$. A simple estimation based on the newsvendor fractile is proposed rendering

$$b_i = \frac{\beta_i^*}{1 - \beta_i^*} h_i.$$

Also for the heuristic for the fill rate constraint model, the shipment intervals are determined either by enumeration or using the deterministic demand heuristic.

The second step optimizes the retailer base-stock levels to meet the target fill rates. This is done using the original compound Poisson demand and a METRIC type approximation of the lead times to the retailers from the central warehouse, i.e. replacing the stochastic lead times by their means (Sherbrooke; 1968). The expected lead times are estimated by finding the average delay due to stock outs at the central warehouse, and the lowest base-stock level for each retailer is determined such that the target fill rate $\beta_i^*$ is achieved. The fill rate for retailer $i$ belonging to retailer group $k$ is found by calculating the expected positive inventory level, $IL_i^+$, at the beginning and the end of a shipment interval according to

$$\beta_i = \frac{\mathbf{E}[IL_i^+(t + L_i)] - \mathbf{E}[IL_i^+(t + L_i + T_k)]}{\mu_i T_k},$$

where $L_i$ denotes the expected lead time from the central warehouse to retailer $i$ in a traditional system without shipment consolidation. The difference between the expected stock on hand at the two time instances equals the expected demand satisfied from stock on hand during the interval $T_k$.

The time based consolidation allows for coordination across multiple items. A multi-item problem where the shipment intervals are restricted to be equal and synchronized for several products has also been considered and tested on the real data.

A numerical study of 256 small problem instances illustrates that the proposed methods perform well compared to the existing exact cost minimizing solutions. The approximation method for the backorder cost model provides solutions that are on average 1.7 % from the optimal cost. For the fill rate constraint model, the average deviation from target fill rates is 0.14 percentage points (pp.) and the maximum is 1.4 pp.

Moreover, using real data for 75 items from our focal company, we show that the methods are appropriate for solving real world problems that the exact methods cannot handle. The obtained policies are evaluated and compared to the company's current solutions using simulation. The results show that large improvements can be achieved, both in terms of reduced costs and better fulfilment of target fill rates. With regards to the latter, our solution renders an average deviation from target fill rates of merely 0.17 pp.

## 6.4 Paper IV - Controlling Inventories in Omni-Channel Distribution Systems with Variable Customer Order Sizes

An increasing trend of omni-channel distribution systems brings new challenges for efficient inventory control. One such challenge concerns service differentiation across channels when a central warehouse satisfies both direct upstream customer demand and replenishment orders from $N$ downstream retailers. In traditional one-warehouse-multiple-retailer (OWMR) inventory systems, where the central warehouse only replenishes downstream retailers, research has shown that the optimal solution typically entails a low service level at the central warehouse (see, for example, Axsäter et al.; 2007). This is typically not a viable solution in an omni-channel system as all customers, including those that order directly from the central warehouse, expect a high service level. The large discrepancy in service requirements for the different channels implies a need to differentiate the service level between them.

Similar to Axsäter et al. (2007), we propose to reserve some stock at the central warehouse for the direct demand. This policy is based on introducing an artificial retailer for the reserved stock for handling the direct upstream demand at the central warehouse. The artificial retailer replenishes from the warehouse using an $(S-1, S)$ policy and a transportation time of zero. Its inventory can then be interpreted as a separate stock at the warehouse, reserved for serving the direct customer demand. Everything else equal, a larger base-stock level means that more stock is reserved for the upstream demand and an increased service differentiation between the two channels. This approach transforms the problem to a more traditional OWMR problem, which has been extensively studied in the literature (Axsäter; 2015). Unlike Axsäter et al. (2007), we propose a combined stock heuristic opposed to a separate stock heuristic. That is, we do not consider only the separate stock reserved for the direct demand, but rather this stock combined with the non-reserved stock at the central warehouse when determining the reservation level $S$.

The numerical study from Axsäter et al. (2007) illustrates that under Poisson demand and linear backorder costs, the separate stock heuristic with a first come-first served allocation performs well compared to a critical level policy optimized by simulation search. However, our initial studies showed that in the real system with fill rate constraints instead of backorder costs and variable customer order sizes, the performance deteriorates. The problem with the separate stock approach is that the fill rate for the direct demand is calculated based only on the reserved inventory, rather than the total amount of inventory available at the central warehouse. As a result, more stock than necessary is reserved, which increases the holding costs.

To remedy this problem, we develop two combined stock heuristics to determine the reservation level $S$. One is based on compound Poisson demand and one is based on an adjusted normal demand. The former is more exact, but also computationally more cumbersome, which make the latter a good alternative in high demand scenarios. They both use the same heuristic idea, so to streamline the presentation here, only the formulas for the compound Poisson case is provided. The heuristics are based on the probability distribution of the combined inventory level at the warehouse, $IL_{\text{CW}}$, as a function of $S$ and the reorder point for the general stock, $R_0$. The combined inventory level consists of the non-reserved general stock (at the central warehouse), $IL_0$, and the reserved stock (at the artificial retailer), $IL_{N+1}$. To find this distribution, two distinct scenarios are considered. That is, (i) when there are non-reserved stock on hand, i.e. when $IL_0 > 0$ and by design $IL_{N+1} = S$, and (ii) when there is not, i.e. when $IL_0 \leq 0$ and $IL_{N+1} \leq S$. This renders the expression

$$\mathbf{P}(IL_{\text{CW}} = j|R_0, S) = \begin{cases} \mathbf{P}(IL_0 = j - S) & j > S \\ \mathbf{P}(IL_0 \leq 0)\mathbf{P}(IL_{N+1} = j|IL_0 \leq 0) & 0 \leq j \leq S. \end{cases}$$

The exact probability distribution for $IL_0 > 0$ can, given the order quantity, $Q_0$, and the distribution of the lead time demand, $D_0(L_0)$, easily be determined as

$$\mathbf{P}(IL_0 = j) = \frac{1}{Q_0} \sum_{k=\max(R_0+1,j)}^{R_0+Q_0} \mathbf{P}(D_0(L_0) = k - j), \quad 1 \leq j \leq R_0 + Q_0$$

and from this, it follows directly that

$$\mathbf{P}(IL_0 \leq 0) = 1 - \sum_{j=1}^{R_0+Q_0} \mathbf{P}(IL_0 = j).$$

The same is not true for the probability distribution for $IL_{N+1}$, though, since the replenishment lead time, $L_{N+1}$, is stochastic. In fact, it is a function of $R_0$, which is what makes the overall problem challenging and explains the need for an efficient heuristic. Inspired by the METRIC approach by Sherbrooke (1968), we aspire to replace the stochastic lead time with a constant lead time equal to the mean value, $\hat{L}$, when $IL_0 \leq 0$. To determine this mean, the fact that delays only occur when there is no general stock on hand, i.e. $IL_0 \leq 0$, is used as a basis for the estimate

$$\hat{L} = \frac{\bar{L}}{\alpha},$$

where $\bar{L}$ is equal to the expected delay for units delivered from the general warehouse stock, and $\alpha$ is the probability of no general stock on hand. $\bar{L}$ is determined using Little's law, and $\alpha$ is the probability $\mathbf{P}(IL_0 \leq 0)$, which equals to 1 minus the ready rate. Hence, the expression for the distribution of the inventory level of the reserved stock becomes

$$\mathbf{P}(IL_{N+1} = j|IL_0 \leq 0) = \mathbf{P}(D_{N+1}(\hat{L}) = S - j).$$

Once the distribution of the combined stock on hand $\mathbf{P}(IL_{\text{CW}} = j|R_0, S)$ $\forall j \geq 0$ is determined, the fill rate for the direct upstream demand is easily obtained as

$$\gamma_{N+1}(R_0, S) = \frac{\sum_{d=1}^{d_{\max}} \sum_{j=1}^{R_0+S+Q_0} \min(j, d) f_{N+1}(d) \mathbf{P}(IL_{\text{CW}} = j|R_0, S)}{\sum_{d=1}^{d_{\max}} d\, f_{N+1}(d)},$$

where $f_{N+1}(d)$ is the probability of a customer order of size $d$. The overall objective is to minimize the cost while maintaining a target fill rate, $\gamma^*_{N+1}$, which stipulates that the reservation level, $S$, should be set as low as possible while still fulfilling this target. It is straightforward to determine the smallest $S$ that satisfies the target fill rate by increasing $S$ from zero until $\gamma_{N+1}(R_0, S) \geq \gamma^*_{N+1}$.

These combined stock heuristics can be used in conjunction with any evaluation method for traditional OWMR systems. However, the complex interdependence between $R_0$, $S$ and the other reorder points at the retailers makes exact solution procedures as for example the method in Axsäter (2000) a non-viable option for the real life problems considered, due to the excessive computation time needed. Thus, to deal with these situations, an approximate method must be used, and we extend the methods proposed in Berling and Marklund (2013, 2014). These methods use a similar five step procedure that makes use of an induced backorder cost at the central warehouse that should capture the costs that delays from the warehouse incur on the retailers, in order to optimize $R_0$. The induced backorder cost decomposes the problem into $N + 1$ single-echelon problems. (In our case, with the artificial retailer, the problem decomposes into $N + 2$ problems.) These methods are conceptually and computationally appealing, and the ERP-software provider to the companies from which we have gotten the inspiration and the real life data is working on implementing them in their software. The extended method developed in the paper consists of six steps.

The main extension to the original procedure in Berling and Marklund (2013, 2014) is the addition of a sixth step to determine $S$ in accordance with the combined stock heuristic. It is possible to use the original procedures for the artificial retailer, i.e. not considering the combined stock at the central warehouse, when determining the reservation level. We will refer to the resulting policy as a separate stock policy, which, as mentioned before, typically will reserve too much inventory for the direct upstream demand, This policy is used as a benchmark in the numerical study. In addition, the new developed heuristics considers two methods (a naïve and an iterative) for determining the induced backorder cost associated with the virtual retailer, $\beta_{N+1}$. This cost cannot be determined using the original procedures and are necessary for them to work. The naïve method sets $\beta_{N+1} = p_{N+1} = h_0\gamma^*_{N+1}/(1-\gamma^*_{N+1})$, i.e. to the backorder cost for the direct upstream demand according to a standard formula for transforming a fill rate to a backorder cost. The cost per time unit for a delay at the central warehouse cannot exceed this backorder cost, so $p_{N+1}$ may be seen as an upper bound for $\beta_{N+1}$. Thus, the naïve approximation tends to overestimate the correct value for $\beta_{N+1}$, which lead to a too high $R_0$ and too little differentiation of the service between the two channels. The iterative method attempts to find a better estimate of $\beta_{N+1}$ by applying a modified version of iterative procedure in Andersson et al. (1998).

A numerical simulation study, including both researcher generated examples and real data from two different companies, shows that the new combined stock heuristic performs very well. The fill rate targets of the direct customer demand are achieved with high accuracy. For the researcher generated data, average deviations are 0.2 percentage points (pp.) for compound Poisson demand, and −0.68 pp. for the adjusted normal approximation. For the two different test series with real data, average deviations are 1.8 pp. and 1.6 pp. respectively. At the same time, the average inventory costs in the system is significantly reduced compared to the separate stock benchmark. For the researcher generated problems, reductions are on average 8.5 % with a maximum of 16.3 %, and for the real data sets the average reductions are 5.8 % and 9.7 %, with maximums of 14.3 % and 27.7 %, respectively. Optimization through simulation search also shows that the solutions obtained with the newly developed heuristics are near-optimal.

# 7 Contributions and Future Research

To summarize the contribution of this doctoral thesis, it investigates some of the various effects that initiatives towards greener supply chains have on inventory control. Three aspects are in focus and considered in the different scientific papers.

A time window service level, as well as non-linear backorder costs, are considered in Paper I. In this paper, the $CO_2$ emissions from products perished due to machines not being repaired in time because of spare part shortages are modelled along with the total cost for the system. This provides a better understanding of the system and allows for better decisions being made. Optimization procedures for finding the base-stock levels at all stock points are presented. In Paper II this model is extended to allow for emergency deliveries of spare parts. The costs and emissions from these transports can then be balanced against the emissions from product waste, the cost for this, and the additional safety stock needed in a system without emergency deliveries. This allows for a method for determining the base-stock levels at all stock points that is presented in the paper. In Paper III, time based shipment consolidation is included in the model to develop implementable heuristics that can be used on real life data to find near-optimal shipment intervals and reorder levels. Lastly, in Paper IV, inventory control in an omni-channel setting is studied. A new heuristic, referred to as the combined stock heuristic, is developed to differentiate the service between the channels in a more efficient manner. The paper also provides a heuristic for determining reorder points at all stock points in the inventory system.

This doctoral thesis thus provides mathematical models and develops efficient methods for inventory control with consideration of emissions and service differentiation. However, much work still remains.

In Paper I and Paper II, more general backorder costs as a function of the waiting time are studied in a setting with base-stock policies at all locations and Poisson demand. It would be of interest to further consider non-linear backorder costs in a setting with batch-ordering policies and more complex demand structures. The time window service level can be extended to a more general service level definition where different target service levels may be defined for different intervals of waiting times. This is mentioned in Paper I, but not investigated. Similarly, the backorder cost in Paper II may be replaced by a time window service constraint, but this is not considered in the paper. These more general service measures would be interesting to explore.

Capacity constraints on the consolidated shipments are not considered in

Paper III. That is, all reserved items waiting to be shipped are assumed to fit on the truck. Of course, this might not always be the case, and a research stream of interest to extend upon is thus one that includes such capacity constraints into the modelling of the inventory system. Such constraints are linked to the fleet constellation, which in turn influences the transportation costs and emissions. Finding a solution that balance this against the cost and emission of inventory/shortages is a challenging endeavour for the future. Another interesting question in a time based consolidation scheme with capacity constraints is how to use excess capacity. Shall this capacity be used at all and if so, to what extent. There exist alternatives to a purely time based shipping consolidation, e.g. quantity based and time and quantity based. This leads to a more complex system from a mathematical modelling perspective, especially for controlling multiple products, that needs further investigation in order to find efficient heuristics.

In all papers, a first come-first served allocation policy is assumed. This is not optimal, as illustrated in the example in Section 6.1. The question of a better allocation scheme is accentuated if delays may occur not only because of shortages of goods, but also of transport capacity. A better allocation scheme is also of interest in an omni-channel network, and, as mentioned in Paper IV, it is a problem that has not yet been solved. Efficient heuristics for alternative allocation schemes are therefore another interesting direction for future research.

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# Scientific Publications

## Author Contributions

For each paper, the thesis author contributed in the following way:

### Paper I: Quantifying Sustainable Control of Inventory Systems with Non-Linear Backorder Costs

By implementing the analytical model and the simulation model in the necessary software and performing the numerical study, and by participating in analysing the results and writing and revising the paper.

### Paper II: Age-Based Inventory Control in a Multi-Echelon System with Emergency Replenishments

By participating in formulating the problem, by implementing the analytical model and the simulation model in the necessary software and designing and performing the numerical study, and by participating in analysing the results and writing and revising the paper.

### Paper III: Controlling Distribution Inventory Systems with Shipment Consolidation and Compound Poisson Demand

By participating in formulating the problem, developing the solution methods, implementing the analytical models and the simulation models in the necessary software, designing and performing the numerical study, analysing the results, and writing and revising the paper.

### Paper IV: Controlling Inventories in Omni-Channel Distribution Systems with Variable Customer Order Sizes

By participating in developing the solution method, implementing the simulation model in the necessary software, designing and performing the numerical study, analysing the results, and writing the paper.