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Value Superiority[1]

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Let's say that A and B are two types of goods such that more of A or B is better than less. A is *strongly superior* to B if and only if any amount of A is better than any amount of B. It is *weakly superior* to B if and only if some amount of A is better than any amount of B. We shall first discuss some different ways in which these relations of superiority can be relevant to the aggregation of welfare. We shall then give a precise and generalised statement of Strong and Weak Superiority. Lastly, we shall prove a number of general results concerning these two relations, one of which can be used as an argument against the existence of value superiority or, alternatively, as an argument against superiority being a radical difference in value. The result in

[1] This paper draws on Arrhenius (2005) and Arrhenius & Rabinowicz (2005). We would like to thank John Broome, Erik Carlson, Roger Crisp, Iwao Hirose, Karsten Klint Jensen, and Jonas Olson for helpful comments. Thanks also to the Collège d'études mondiales for being such a generous host during some of the time when this paper was written. Financial support from the Swedish Research Council as well as from Riksbankens Jubileumsfond and Fondation Maison des sciences de l'homme through the *Franco-Swedish Program in Economics and Philosophy* is gratefully acknowledged.

question is, roughly, that if one holds that some type of good A is strongly or weakly superior to another type of good B and that type B can be reached from type A by a long enough sequence of slight worsenings, then one is committed to holding that there are two types of goods C and D in this sequence such that C is weakly superior to D although goods of type C are only marginally better than goods of type D.

I. Introduction

Let's say that A and B are two types of goods such that more of A or B is better than less. We are going to discuss the following two relations that can obtain between A and B:

> *Strong Superiority* (roughly): Any amount of A is better than any amount of B.[2]

> *Weak Superiority* (roughly): A sufficient amount of A is better than any amount of B.[3]

It is easy to find examples of these relations in the literature, sometimes under the labels "higher goods" or "discontinuity in value". For example, already in the 19th century Franz Brentano claimed that "[i]t is quite possible for there to be a class of goods which could be

[2] In Arrhenius & Rabinowicz (2005), this condition (or rather a version of it) is referred to simply as "Superiority".

[3] The distinction between these two relations goes back to Griffin (1986), who refers to strong superiority as "trumping" ("*any* amount of *A*, however small, outranks *any* amount of *B*, however large", p. 83) and to weak superiority as "discontinuity" ("enough of *A* outranks any amount of *B*", p. 86)

increased *ad indefinitum* but without exceeding a given finite good".[4] Likewise, W. D. Ross asserted that "[w]ith respect to pleasure and virtue, it seems to me much more likely to be the truth that *no* amount of pleasure is equal to any amount of virtue, that in fact virtue belongs to a higher order of value, beginning at a point higher on the scale of value than pleasure ever reaches…"[5] Similar views have been proposed by, among others, Roger Crisp, Jonathan Glover, James Griffin, Rem Edwards, Noah Lemos, Derek Parfit, and John Skorupski.[6] Its lineage goes back to at least Francis Hutcheson in the early 18th century and John Stuart Mill in the mid-19th century.[7]

Superiority in value can be contrasted with

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4 Brentano (1907). If "finite good" here stands for a finite amount of a good, then this is just a statement of weak superiority. But if Brentano by a finite good means a good – a positively valuable object – whose *value* is finite, then he postulates more than just the existence of weak superiorities; he prefigures a more specific claim we are going to argue for in this paper, namely the claim that superiority in value (whether weak or strong) does not require infinite betterness (see below, section VII).

5 Ross (1930), p. 150.

6 See Crisp (1988), p. 188; Crisp (1992), p. 151; Glover (1977), p. 71; Griffin (1986), pp. 85-6; Edwards (1979), pp. 69-72; Lemos (1993), Parfit (1986), pp. 161-4; Skorupski (1999), pp. 94-101. See also Feit (2001), Klint Jensen (1996), pp. 90-1; Portmore (1999); Riley (1993, 1999, 2008. 2009).

7 Hutcheson (1968 [1755]), pp. 117-18 (Hutcheson completed his treatise in 1738 but it was published posthumously in 1755. Cf. Edwards (1979), p. 71.); Mill (1998), p. 56. For another early source with a racist twist, see Rashdall (1907), pp. 238-9. Newman (1885), vol. I, p. 204, makes a similar claim about pain and sin. Hutcheson's pioneering position might be questioned. Brentano (1969 [1907]), p. 157) ascribes to Pascal the view that "there are classes of goods that can be ranked in the following way: the smallest of any of the goods that are to be found in the higher class will always be superior to the totality of goods which are to be found in the lower class."

The Archimedean Property of Value (roughly): This property holds for goods of type B relative to goods of type A if and only if for any amount of A there is some amount of B which is at least as good.

This is like the Archimedean property of the real numbers: For any positive numbers x and y, there is a natural number n such that $nx \geq y$. The Archimedean property seems to capture the way we usually think about the aggregation of goods. Let's say that you are considering two holiday packages. The first is a week in Stockholm, the other a week in Copenhagen. You have a preference for Stockholm. It is possible, however, to better the Copenhagen-package by adding some extra days. It seems plausible that there is such a bettering, other things being equal, that would reverse your preference in such a way that you would prefer the Copenhagen holiday. If Stockholm-days is taken as one type of good (type A) and Copenhagen-days as another type of good (type B), then this case is an illustration of the Archimedean Property of Value. One might think that this feature is a general property of goods, that all types of goods satisfy the Archimedean Property of Value. Weak Superiority is a denial of this claim. Strong Superiority entails Weak Superiority. They are thus both versions of what we call *non-Archimedeanism*.

Superiority and non-Archimedeanism are structural features that can be true of many kinds of orderings. We shall discuss some different ways in which these ideas can be applied to the aggregation of welfare. It is important to separate these different applications of the superiority idea, since they will yield quite distinctive views with varying intuitive support.

First, however, we shall describe some of the problems that have motivated the recent interest in non-Archimedeanism.

II. Intrapersonal and Interpersonal Repugnant Conclusions

During the last thirty years or so, non-Archimedeanism has again become popular in connection with theories of welfare and population ethics. What is the reason behind this renaissance?

Derek Parfit has brought attention to a problem for classical Total Utilitarianism. This view tells us to maximise the welfare in the world and implies what Parfit calls the Repugnant Conclusion:

> *The Repugnant Conclusion*: For any population consisting of people with very high positive welfare, there is a better population in which everyone has a very low positive welfare, other things being equal.[8]

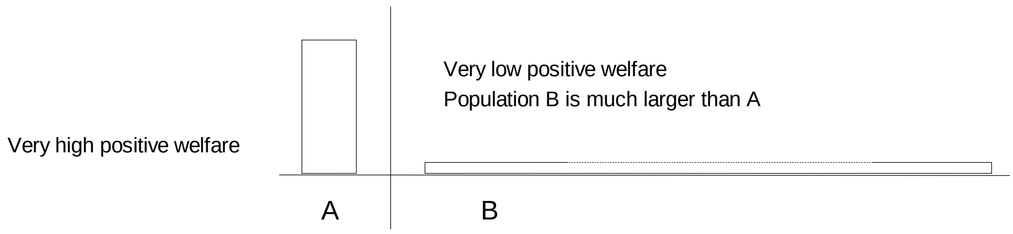

Diagram 1.

In diagram 1, the width of each block represents the number of people whereas the height represents their lifetime welfare. All the lives in the diagram have positive welfare, or, as we also could put it, all the people have lives worth living. The A-people have very high welfare whereas the B-people have very low positive welfare. The reason for this could be that in the B-lives

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[8] See Parfit (1984), p. 388. Our formulation is more general than Parfit's. The *ceteris paribus* clause in the formulation is meant to rule out that the compared populations differ in any axiologically relevant aspect apart from individual welfare levels. Although it is through Parfit's writings that this 'repugnant' implication of Total Utilitarianism has become widely discussed, it was already noted by Henry Sidgwick (1907), p. 415, before the turn of the century. For other early sources of the Repugnant Conclusion, see Broad (1930), pp. 249-50, McTaggart (1927), pp. 452-3, and Narveson (1967).

there are, to paraphrase Parfit, only enough ecstasies to just outweigh the agonies, or that the good things in those lives are of uniformly poor quality, e.g., eating potatoes and listening to Muzak.[9] However, since there are many more people in B, the total sum of welfare in B is greater than in A. Hence, Total Utilitarianism ranks B as better than A --- an example of the Repugnant Conclusion.[10]

One way of avoiding this implication but retaining the idea of maximisation is to invoke a form of non-Archimedeanism and claim that there is a number of lives with very high welfare, which is better than any number of lives with very low positive welfare, although the addition of more lives with very low positive welfare always makes a population better.

Diagram 1 could also be taken to represent an intrapersonal version of the Repugnant Conclusion. The width of each block would then stand for the length of a life, and the height for the well-being at a certain time. For example, block A could represent a life of a hundred years in which every year is of a very high quality, and block B could stand for a much longer life in which every year is of very low quality.[11] If the B-life is long enough, then the total welfare of that life will be greater than that of the A-life and thus this life will be ranked as better by a

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[9] See Parfit (1984), p. 388 and Parfit (1986), p. 148. For a discussion of different interpretations of the Repugnant Conclusion see Arrhenius (2000b, 2014) and Parfit (1984, 2014).

[10] Notice that problems like this are not just problems for utilitarians or those committed to welfarism, the view that welfare is the only value that matters from the moral point of view, since we have assumed that other axiologically relevant aspects are equal (cf. fn. 8). Hence, other values and considerations are not relevant for the value comparison of populations A and B. This is thus a problem for all moral theories according to which welfare matters at least when all other things are equal, which arguably is a minimal adequacy condition for any moral theory.

[11] There are other versions of the interpersonal case in which the compared lives may be equally long but one life contains some amount of A-goods and the other an arbitrarily large amount of B-goods, where some of the B-goods come more or less simultaneously. An example of the latter case is appreciation from enthusiastic readers.

maximising theory of welfare. Again, this implication could be blocked by invoking a form of non-Archimedeanism. This is what Parfit suggests:

> I could live for another 100 years, all of an extremely high quality. Call this *the Century of Ecstasy*. I could instead live for ever, with a life that would always be barely worth living … the only good things would be muzak and potatoes. Call this *the Drab Eternity*. - - - I claim that, though each day of the Drab Eternity would be worth living, the Century of Ecstasy would give me a better life. ---Though each day of the Drab Eternity would have some value for me, *no* amount of this value could be as good for me as the Century of Ecstasy.[12]

Likewise, in his influential discussion of intrapersonal aggregation of welfare, James Griffin has proposed that there can be what he calls "discontinuity" among prudential values (welfare) of the form "enough of A outranks any amount of B" (this is what we call "Weak Superiority").[13] Discontinuity entails, he explains

> … the suspension of addition; … we have a positive value that, no matter how often a certain amount is added to itself, cannot become greater than another positive value, and cannot, not because with piling up we get diminishing value or even disvalue …, but because they are the sort of value that, even remaining constant, cannot add up to some other value. - - - … [I]t is more plausible that, say, fifty years at a very high level of well-being – say, the level which makes possible satisfying personal relations, some understanding of what makes life worth while,

[12] Parfit, (1986), pp. 161-4, emphasis in original.

[13] Griffin (1986), p. 85.

> appreciation of great beauty, the chance to accomplish something with one's life – outranks any number of years at the level just barely living – say, the level at which none of the former values are possible and one is left with just enough surplus of simple pleasure over pain to go on with it.[14]

We take it that most people find Griffin's and Parfit's examples rather persuasive. To establish whether the non-Archimedean idea involved in the examples is tenable, however, we need to spell it out in more detail and distinguish among different varieties of it.

III. Aggregation of Welfare and Non-Archimedeanism

Roughly, a person's welfare has to do with how well her life is going for her. To specify a person's welfare is to specify how good or bad her life is for her. Let's call those things that are good or bad for people 'welfare components'. Here are some components that have been proposed in the literature: pleasure or pain; satisfied or frustrated desires; autonomy or its absence; greater or lesser accomplishments; true or false beliefs; satisfying or dissatisfying personal relationships; experiences of beautiful or ugly objects, and so forth.

The kind of non-Archimedeanism expressed in the quotes above from Griffin and Parfit concerns the relationship between orderings of welfare components and the ordering of lives in respect to welfare. However, the route from welfare components to the welfare of lives can take different paths and, which is important, some form of non-Archimedean property can appear at different points along this path. Above, we only stated the Archimedean property for value but we can of course also ascribe this property to other orderings, for example, the ordering of pleasures. For example, we could order all experiences, including collections of experiences, in terms of how pleasurable they are (how much pleasure these experiences contain

[14] Griffin (1986), pp. 85-6.

taken as a whole), i.e., by the relation "is at least as pleasurable as". We could then ask whether this ordering is Archimedean. If it is, then for any two kinds of experiences A and B, and for any collection of experiences of kind A, there is some collection of experiences of kind B that is more pleasurable.

Even if the ordering of pleasures is Archimedean, we could still ask how such an ordering relates to the welfare of a life. We may assume that it is always better for a person to enjoy a greater amount of pleasure rather than less, other things being equal.[15] However, we could ask whether for any kinds of pleasures A and B, and for any amount of pleasure of kind A, there is always some amount of pleasures of type B which is *better* for a person. Here we are asking whether the Archimedean property applies to the contributive value of pleasure to the welfare of a life. And we might deny this. Hence, it is possible that the ordering of experiences by the relation "is at least as pleasurable as" is Archimedean despite the fact that the betterness ordering of pleasurable experiences is not.

On the other hand, we could question whether the very ordering of pleasure is Archimedean. We could start by looking at *atomic* experiences of pleasures, understood as the shortest possible experiences of pleasure that are the building blocks of all other experiences of pleasure. We could then investigate how the ordering of these atomic experiences relates to the ordering of all experiences, including collections of pleasurable experiences, in terms of the relation "is at least as pleasurable as". We could then ask whether for any two kinds of atomic experiences A and B, and for any collection of A-experiences, there is always some collection of B-experiences which, taken together, are *more pleasurable* (or, to put it differently, constitute a greater amount of pleasure). Here we are asking whether the non-Archimedean property appears in the aggregation of atomic pleasures to the overall pleasure in a life.

[15] When we say that, for example, one collection of experiences contain a greater *amount* of pleasure than another, we just mean that the former collection is more pleasurable than the other, i.e., comes higher on the "at least as pleasurable as"-ranking.

If non-Archimedeanism about pleasure is combined with the view "the more pleasure, the better", then it might be extensionally equivalent to non-Archimedeanism about the *value* of pleasure. Still, these two versions of non-Archimedeanism are different in an important way: If we hold that the non-Archimedean property appears in the ordering of pleasures, then it's still possible for us to claim that "the more pleasure, the better". This might be of importance for a hedonist who adheres to the classical utilitarian principle of maximisation. One can accept this principle while still interpret the ordering of pleasures in non-Archimedean terms and thus avoid some of the implications that are usually associated with classical utilitarianism, such as the Repugnant Conclusion, and the like.

As we mentioned above, one can instead hold that the non-Archimedean property only appears in the betterness ordering of pleasure. The following quote from Mill might be read in this way:

> … some *kinds* of pleasure are more desirable and valuable than others. It would be absurd that while, in estimating all other things, quality is considered as well as quantity, the estimation of pleasures should be supposed to depend on quantity alone. --- Of two pleasures, if …. one of the two is, by those who are competently acquainted with both, placed so far above the other that they … would not resign it for any quantity of the other pleasure which their nature is capable of, we are justified in ascribing to the preferred enjoyment a superiority in quality, so far outweighing quantity as to render it, in comparison, of small account.[16]

However, if one takes this view, then one cannot claim that "the more pleasure, the better". It is therefore on this reading somewhat surprising that Mill thought that the above

[16] Mill (1998), p. 56.

position "is quite compatible with the principle of utility" (ibid.), which he also sometimes refers to as "the Greatest Happiness Principle" (ibid, p. 55). The latter label suggests that value tracks the quantity of pleasure.[17]

Riley (2008) defines standard ethical hedonism as the view that pleasure is the sole value and that it is 'homogenous' in nature: there are no qualitatively different pleasures; the only differences concern duration and intensity. On this view, the ordering of pleasures is Archimedean. Riley claims that it is necessary for a standard hedonist to accept value additivity (which presupposes that the value ordering of pleasures also satisfies the Archimedean property).[18] As he puts it, "value additivity is built into the very meaning of standard hedonism". The crucial premise in his argument for this claim is as follows:

> …one unit of pleasure contributes just as much positive intrinsic value as every other unit of pleasure does to the value of the whole pleasure comprised of the individual pleasures. (Ibid., p. 261)

This premise is ambiguous. In a given whole composed of pleasures, each unit of pleasure does contribute just as much to the value of the whole as every other unit. That's true: there are no

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17 Roger Crisp has suggested to us that Mill might well have thought that value tracks pleasantness – the degree of pleasure – while interpreting this degree as a function not merely of the quantity of pleasure but also of its quality. This would make the quoted passage compatible with the Greatest Happiness Principle.

18 One way of stating value additivity is as follows. Let "**o**" stand for the operation of forming wholes (mereological sums). For any object *e*, let V(*e*) be the value of *e*. Then, for all non-overlapping objects *e* and *e'*, V(*e* **o** *e'*) = V(*e*) + V(*e'*). This formulation of additivity requires value to be measurable on at least a ratio scale. If we can only assume measurability on an interval scale, we need a bit more complicated formulation of additivity: For all objects *e*, *e'*, and *e"*, if *e* doesn't overlap with either *e'* or *e"*, then V(*e* **o** *e'*) - V(*e'*) = V(*e* **o** *e"*) - V(*e"*). I.e., for every *e*, size of *e*'s value contribution to a whole in which it is a part is constant.

relevant qualitative differences between pleasures on standard hedonism. Indeed, on that view, the value contribution of each unit of pleasure to a whole composed of n such units can be set to $1/n$-th of the value of the whole. However, it doesn't follow that a unit of pleasure has a fixed contributive value in the *marginalist* sense, i.e., that it contributes the same amount of value to a whole to which it is added as any other unit contributes to the whole to which *it* is added. For example, the marginal contributive value of adding one unit of pleasure to a whole consisting of five units of pleasure doesn't have to be the same as the marginal value of adding one unit to a whole consisting of ten units of pleasure. Nothing in the definition of standard hedonism as given by Riley requires this, but this is what he needs for his argument for additivity to succeed. If the marginal contributive value of an additional unit can vary, then, in particular, its marginal contribution can decrease and converge down to zero as the number of units of pleasure goes to infinity. And then one can easily obtain a non-Archimedean effect in so far as the value of pleasure is concerned, if one like Mill allows for higher pleasures along with the lower ones.

Indeed, to obtain non-Archimedean effects one doesn't need to move beyond standard hedonism. Thus, in the example discussed above, block A in diagram 1 could represent a life of a hundred years in which every moment is very pleasurable, and block B could stand for a very long life in which every moment is pleasurable to a very low degree. However long the B-life is, it might be less valuable than the A-life (albeit it might very well contain a greater amount of pleasure).[19] Pace Riley, this violation of the Archimedean property of value is perfectly compatible with standard hedonism. As we have seen, different kinds of pleasurable experiences are needed for a violation of the Archimedean property by the value ordering of pleasures. One might therefore ask how there can be such different kinds of pleasures given standard hedonism. The answer is simple: we can define kinds in terms of intensity of pleasures. In the example

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[19] In fact, one might even want to claim that the A-life is more pleasurable than the B-life, however long the latter might be. This would mean that one accepts non-Archimedeanism about the ordering of pleasure and not just about the ordering of the value of pleasure.

above, we can assume that moments of intense pleasure form the superior kind A and moments of low intensity pleasure form the inferior kind B.

Non-Archimedeanism can appear not only in the ordering of personal value, i.e., value for someone, but also in the ordering of impersonal value – value period. Thus, we can order populations of lives in terms of how good they are, that is, by the relation "is at least as good as". In this case, it is not a matter of goodness for someone but a question of goodness, period. We can ask how welfare contributes to this value. Assume that we have an ordering of lives in terms of welfare. We could ask whether for any population consisting of people with very high positive welfare, there is a population of people with slightly positive welfare which is better. Here we are asking whether the Archimedean property holds for the contributive value of welfare to the value of populations. But we could also ask whether this property holds for the aggregation of individual welfare to the overall welfare of the population. If this property is violated at one of these steps, then we can still be maximisers of goodness but avoid the Repugnant Conclusion.

Of course, the non-Archimedean property could also apply to other welfare components than pleasure. Notably, it could hold *between* welfare components of different types. For example, one could hold that no amount of trivial pleasures can outweigh the loss of one's autonomy. This could be a view about how pleasure and autonomy contribute to welfare, or to the 'impersonal' value of a life (its goodness period). The non-Archimedean property might also appear in the aggregation of welfarist and non-welfarist goods into a measure of the value of actions, people or lives. This seems to be what Ross had in mind in the passages we have quoted in the introduction and what Hutcheson had in mind when he wrote that "[t]he exercise of virtue for a short period ... is of incomparably greater value than the most lasting sensual pleasure".[20]

[20] Hutcheson (1755), pp. 118.

Let's summarise this section. The non-Archimedean property can appear at different stages in the path from goods to welfare and to the value of people, lives, populations, worlds, and actions. It can appear in the contributive value of atomic experiences of pleasure to the pleasantness of aggregates of such experiences, in the contributive value of pleasure to the welfare of a life, in the contributive value of the welfare of lives to the value of populations and worlds, in the aggregation of welfarist and non-welfarist goods into a measure of the value of actions, people, and lives, and so forth. Note that these possibilities are logically independent. For example, one could hold that 'trivial' pleasures can always outweigh 'high quality' pleasures when it comes to the welfare of a life but not when it comes to the contributive value of a life to the value of a population. One might think that a life with some amount of high quality pleasures has a higher contributive value to a population than any number of lives with any amount of trivial pleasures, although the latter lives may have a higher degree of welfare.

IV. The General Structure of Superiority in Value

As we have seen above, Superiority and non-Archimedeanism are structural features that can be true of many kinds of orderings. In what follows, however, we are going to focus on superiority in value, and more specifically in goodness rather than in goodness for someone, but our results are applicable to all orderings that exhibit these features. The relation "is at least as good as" that we shall focus on can be taken as a place holder for other possible ordering relations, such as those that we have discussed above.

Thus, suppose a domain of objects is ordered by the relation "is at least as good as". Assume that this relation is a weak order, that is, transitive and complete in the domain under consideration. The completeness assumption, according to which all objects are ordered by the relation in question, is problematic, but we just make it for the sake of simplicity.[21] Assume that the domain is closed under concatenation, by which we mean the operation of forming 'conjunctive' wholes out of any finite set of objects. Such wholes are themselves objects in the domain. We also take it that for any object *e* in the domain and for any number *m*, the domain contains a whole composed of *m* mutually non-overlapping '*e*-objects', by which we mean objects of the same type as *e*. We take object *types* to be understood in such a way that any two representatives of the same type are equally good and interchangeable in every whole without influencing the value of the whole in question. Intuitively, we might think of objects of the same type as being identical in all value-relevant respects.[22] In what follows, statements such as '*m e*-objects are better than *k e'*-objects' should be read as claims about complex objects obtained by this kind of "same type"-concatenation: 'A whole composed of *m e*-objects is better than a whole composed of *k e'*-objects.'

It will also simplify matters if we suppose that all the objects in the domain are positively valuable, by which we mean that for any object *e* and any *m*, *m+1 e*-objects are better than *m e*-objects. In other words, concatenating objects of the same type is value increasing. But we allow that the value of the objects in the domain may otherwise vary, and quite dramatically sometimes. Now we can define strong and weak superiority in a precise manner:

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[21] For an account of the "is at least as good as"-relation, which analyses it in terms of required preference-or-indifference and explains why this relation might well not be complete, see Rabinowicz (2008), (2012).

[22] Object types should be distinguished from object *kinds*. Two wholes composed of objects of the same type can be said to be of the same kind. Thus, a whole composed of *m e*-objects is of the same kind as a whole composed of *n e*-objects, for any *m*, *n*, and *e*.

Strong Superiority: An object *e* is *strongly superior* to an object *e'* if and only if *e* is better than any number of *e'*-objects.

Weak Superiority: An object *e* is *weakly superior* to an object *e'* if and only if for some number *m*, *m e*-objects are better than any number of *e'*-objects.[23]

In other words, *e* is strongly superior to *e'* if it is better than any whole composed of *e'*-objects, however large. It is weakly superior to *e'* if a sufficient number of *e*-objects are better that any whole composed of *e'*-objects, however large. Consequently, if *e* is weakly superior to *e'*, then a whole composed of a sufficient number of *e*-objects is strongly superior to *e'*. Thus, the existence of weak superiorities entails the existence of strong superiorities in the domain, given closure under concatenation.

Both superiority and weak superiority involve violations of the Archimedean Property for betterness ordering:

The Archimedean Property of Value (exact formulation): For any object *e* and any positively valuable object *e'*, there is a number *k* such that *k e'*-objects are at least as good as *e*.

Along with these two kinds of superiority relations between objects in the domain, we could define the corresponding relations between object *types*, one being that any object of a certain type is better than any number of objects of another type, and the other being that a

[23] Note that, according to this definition, *e* may be weakly superior to *e'* even though *e'* is better than *e*. If this sounds unnatural, one might always add to the definition an additional requirement that *e* must be better than *e'* in order to be weakly superior to it. However, for simplicity's sake, we prefer to work with a more austere definition of weak superiority.

sufficient number of objects of one type is better than any number of objects of another type. In what follows, however, we shall restrict our attention to superiority relations between objects.

A different perspective on superiority relations would involve thinking of objects as exhibiting various value-relevant *attributes*, each of which can be present in an object in varying degrees. As an example, think of an object as a possible outcome that can be characterised in terms of such value-relevant attributes as, say, (the levels of) achievement, satisfaction, freedom, etc. We could then study superiority relations between attributes, rather than between objects themselves (or between object types). An attribute may be said to be superior to another attribute *relative* to an object e if and only if any improvement of e with respect to the former attribute is better than any change of e with respect to the latter attribute. Correspondingly, an attribute is weakly superior to another attribute relative to e if and only if some improvement of e with respect to the former attribute is better than any change of e with respect to the latter attribute. Apart from these superiority relations, which are relative to a specific reference-point (object e), one can also study global superiority relations between attributes, which hold for all reference-points. This is a plausible way of understanding views such as the one expressed by Ross in the passage quoted in the introduction, and the one expressed by Hutcheson in the quote in section III. Such relative and global relations are discussed in Broome & Rabinowicz (2005) where it is shown that the results we are about to prove for superiority relations between objects in a large measure extend to the corresponding relationships between attributes.[24]

[24] Can one also define superiority relations for values accruing to *states of affairs*? We are indebted to Iwao Hirose for alerting us to this question. At present, however, we don't see how this could be done, if it can be done at all.

V. Strong Superiority without Abrupt Breaks

By a 'decreasing sequence' $e_1, \ldots, e_n$, we shall in what follows mean a sequence of objects such that e_1 is better than e_2, e_2 is better than e_3, …, and e_{n-1} is better than e_n. It is a common view that there could not exist decreasing sequences in which the first element is strongly superior to the last one but no element is strongly superior to the one that immediately follows. What often lies behind this view is the belief that e_1 can be strongly superior to e_n only if e_1 is *infinitely* better than e_n.[25] But if the latter is the case, then a decreasing sequence that starts with e_1 and ends with e_n must at some point involve an infinite drop in value. In other words, it must at some point reach an element e_i such that e_i is infinitely better than e_{i+1}. Which implies that e_i must be strongly superior to its immediate successor e_{i+1}.[26]

However, the claim that strong superiority between the first and the last element of the decreasing sequence implies strong superiority between some adjacent elements in the sequence

[25] Cf. Crisp (1992); Riley (1993) (1999); Parfit (1986). Riley (2008) also ascribes this view to Mill.

[26] It does not really matter in the present context whether one interprets strong superiority as infinite or *lexical* betterness (for the latter interpretation, see Feit (2001). On the lexical view, a strongly superior object carries a value that is finite but belongs to a higher order than the value of an inferior object. These two constructions – the infinitistic and the lexical one - are closely related interpretations of the same idea. In particular, both imply that if the first element of the decreasing sequence is strongly superior to the last one, then at some point in the sequence, strong superiority relation must set in between the adjacent elements. (Parenthetically, however, one should add that the infinitistic and the lexical interpretations of strong superiority are not equivalent. In fact, the former lends itself to some counter-intuitive implications that the latter avoids. To see this, consider an object e that is strongly superior to some other object in the domain. On the lexical interpretation, there is no problem in requiring that same-type concatenation is value-increasing: two e-objects are better than one, three are better than two, and so on. But on the infinitistic interpretation, according to which the strongly superior e has an infinite value, it is difficult to understand how two e-objects can be more valuable than one.)

is incorrect. At least, it is incorrect in all domains in which *weak superiority does not collapse into strong superiority*. More precisely, the following can easily be proved:

> *Observation 1*: Consider any two objects *e* and *e'* such that *e* is better than *e'*. If *e* is weakly superior to *e'*, without being strongly superior to it, then the domain must contain a finite decreasing sequence of objects in which the first element is strongly superior to the last one, but no element is strongly superior to its immediate successor.

For the proof, see Appendix 1.

VI. Independence

While Observation 1 is provably true, it still seems somehow counter-intuitive. Many people seem to have a strong intuition that a decreasing sequence in which the first element is strongly superior to the last one must contain an element that is strongly superior to its immediate successor.[27] As we have seen, the belief that seems to lie behind this intuition is that strong

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[27] Ryberg (2002), p. 419, claims that "[if] there is a discontinuity between the values … at each end of the continuum [i.e at each end of a descending sequence in which each successive element is only marginally worse than the preceding one], then at some point discontinuity must set in": at some point in the sequence there must be an element that is discontinuous with the one that immediately follows. From this claim, call it (R), he draws the conclusion that discontinuities don't exist, since it is counterintuitive to suppose that a discontinuity in value could exist between objects that only marginally differ in value. Unlike Griffin (1986), Ryberg by "discontinuity" means something like strong superiority: "According to the *discontinuity view* we can have a (lower) pleasure which, no matter how often a certain unit of it is added to itself, cannot become greater in value than a unit of another (higher) pleasure" (p. 415). As

superiority in value requires infinite betterness. However, this assumption appears to rest on a presupposition that value is additive. If value is additive, then piling up the less valuable objects would sooner or later result in a whole that is at least as good as the strongly superior object, *unless* the value of that object is infinitely large by comparison. A similar presupposition of additivity seems to lurk, for example, behind the following statement of Jonathan Riley:

> Given the hedonist claim that happiness in the sense of pleasure (including the absence of pain) is the sole ultimate end and test of human conduct, there are only two logical possibilities: either qualitative differences [between pleasures] may be reduced to finite amounts of pleasure (for example, one unit of higher pleasure might be deemed equivalent to ten units of lower pleasure), in which case the quality/quantity distinction is epiphenomenal because pleasure is at bottom homogeneous stuff; or qualitative differences are equivalent to *infinite* quantitative differences, in which case pleasure is a heterogeneous phenomenon consisting of irreducibly plural kinds or dimensions arranged in a hierarchy. The second alternative is embodied in my interpretation. (Riley, 'On Quantities', p. 292, our emphasis)

Riley's *tertium non datur* is apparently grounded in a belief that if a higher pleasure only has a finite value, then that value sooner or later would be reached, if we started piling up lower pleasures. This belief is correct if value additivity is assumed, but without such an assumption it is false.

Observation 1 shows, however, claim (R) is false on this interpretation. For a discussion of Ryberg's argument, see Rabinowicz (2003), who suggests that that the assumption of (R) is based on the fallacy of identifying superiority with "infinite betterness".

To make room for strong superiority between the extrema of a decreasing sequence without strong superiority setting in at any point in the sequence, we must give up the infinitistic interpretation of superiority, which in turn requires giving up value additivity. We must allow that the aggregated value of several objects of the same type need not be the sum of the values each of them has on its own. That the value of a whole may differ from the sum of the values of its parts is of course an idea that should be familiar to post-Moorean value theorists.[28]

Actually, giving up additivity is not enough; we must be even more radical. If we want to have a sequence in which the first element is strongly superior to the last one, without any element in the sequence being strongly superior to its immediate successor, we must – as we shall see (cf. Observation 2 below) - reject the idea that the value of the whole has to be a monotonically increasing function of the value of its parts. That is, we must give up the *independence axiom* for the betterness ordering:

[28] Cf. Moore (1993 [1903]), sections 18-21, *et passim*. Moore was not the first philosopher to make this point. Another standard reference is Brentano (1969 [1989]). To be more precise, Moore does assume a form of additivity when he suggests that 'the value on the whole' is the sum of (i) the values of the parts *plus* (ii) 'the value of the whole, *as a whole*'. (The latter may be either positive or negative.) But it can be argued that this form of additivity is a purely arithmetical construct. His 'value of the whole, as a whole' could simply be interpreted as the arithmetical difference between 'the value on the whole' and the sum of the values of the parts, independently considered. Moore himself points out that the value of a whole, as a whole, may be 'expressed' as such a difference (ibid., section 129), but he seems to ascribe to it some independent significance. For an in-depth discussion of this issues, see Eric Carlson's chapter on organic unities in this *Handbook*.

Independence: An object e is at least as good as e' if and only if replacing e' by e in any whole results in a whole that is at least as good.[29]

Independence implies that the ordering of the contributions that different objects would make to the value of a given whole is context-independent. In other words, this ordering does not depend on the other parts the whole is composed of. Clearly, value additivity presupposes Independence, but the latter might hold even in the absence of the former. If additivity fails but Independence holds, the ordering of the contributions that would be made by different objects will be context independent but the size of their contributions might vary.

In fact, just because one assumes Independence, there is no need to postulate that strongly superior objects possess an *infinite* value.[30] Thus, in this respect, Independence is less demanding than value additivity.

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[29] Qualification: If e'' is a whole in which e' is replaced by e, the restriction on the replacement is that e and *e''-minus-e'* are disjoint: No part of the former is a part of the latter.

[30] It is a standard theorem of measurement theory that if a weak ordering of a domain (i.e., any complete and transitive relation on the domain) divides that domain into a countable number of equivalence classes, then this ordering can always be represented by a function that assigns real numbers to objects in the domain. (Cf., for example, Theorem 6.3 in Aleskerov *et al.* (2007), p. 206.) This applies to the relation "is at least as good as", which is assumed to be a weak ordering on the domain we consider. If this ordering divides the domain into a countable number of equivalence classes (i.e. a countable number of maximal classes comprised of equally good objects), then there is a value function V on the domain such that for all objects e and e', $V(e) \geq V(e')$ if and only if e is at least as good as e'. Note that, for all e, $V(e)$ is a real number, i.e., that each e is assigned a *finite* value on this representation. This holds even if (i) the domain is closed under value increasing same-type concatenation, (ii) some objects in the domain are strongly superior to other objects in the domain, and (iii) the "at least as good as"–ordering satisfies Independence.

But it still is a very demanding condition. In particular, with Independence, just as with the stronger condition of value additivity, we could not have had strong superiority between the first and the last element of the sequence without strong superiority setting in at some point along the way.

> *Observation 2*: Suppose that the first element in a sequence $e_1, \ldots, e_n$ is strongly superior to the last one. Then, provided that "is at least as good as" is a complete and transitive relation on the domain under consideration, Independence implies that some element in the sequence is strongly superior to its immediate successor.

For the proof, see Appendix 2.[31]

One can also prove that, with Independence, the distinction between weak and strong superiority cannot be upheld: the former collapses into the latter.

> *Observation 3*: Assume Independence. Then, for every e and e', if e is weakly superior to e', then e is strongly superior to e'.

For the proof, due to Klint Jensen (2008), see Appendix 3.

VII. Giving up Independence

[31] Actually, for the proof of Observation 2, we do not need the full power of Independence. It is enough to assume the Independence holds from left to right, i.e., that replacing an object in a whole by another object that is at least as good always results in a whole that is at least as good.

What if we give up Independence? Then the following becomes possible: Suppose that when we start adding more and more valuable objects of the same type, the marginal value contribution of each extra object sooner or later starts to decrease, converging to zero.[32] If this decrease is sufficiently steep, then adding extra objects of the same type will never get us above a finite value limit: For any object *e* of a finite value, there will exist some finite value level v_e such that the aggregated value of an arbitrarily large number of *e*-objects is always lower than v_e. But then nothing excludes that a single object *e'* may be more valuable than any number of *e*-objects: All it takes is that the value of *e'* either equals or exceeds v_e.

The manoeuvre of letting the marginal value contribution of extra units of a given kind of good converge to zero is, of course, quite standard, even with respect to intrinsic values. In population axiology, it has been suggested as a way of avoiding the Repugnant Conclusion (cf. the discussion in section II). To avoid this conclusion, we do not have to give up the welfarist idea that the value of the world is an aggregate of the welfare levels of its inhabitants. All we need is to assume that the aggregative operation is of an appropriate kind: Adding lives with positive but low welfare increases the value of a world but it will never increase that value beyond a certain finite limit.[33]

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[32] If Independence were to hold, such a decrease in marginal contributive value would not be possible, at least as long as we consider models in which adding an object to a whole has a fix contributive value provided the whole in question does not already contain other objects of the same type. If *e* is more valuable than some object *e'* and the value of *e'* equals *x*, where *x* is some positive number higher than 0, then – if Independence were to hold – adding yet another *e* to a whole that already contains many *e*-objects but no *e'*-objects would still have to be more valuable than adding *e'*. Thus, the marginal value contribution of *e* could never fall below *x*.

[33] Cf. Parfit (1984), section 137, and Sider (1991). Parfit's application of this idea to the value of populations comes from Hurka (1983). However, neither Parfit nor Sider endorse this solution since it has other counterintuitive implications in population axiology. For a discussion, see Arrhenius (2000ab, 2014) and Arrhenius et al. (2006).

Given this convergence of value to finite limits, it is easy to account for the possibility of a decreasing sequence $e_1, \ldots, e_n$, in which (i) the first element is strongly superior to the last element, even though (ii) no element is strongly superior to the one that comes next.

As a simplest possible example, which for that reason is maximally artificial, assume that the sequence consists of three elements, e_1, e_2, e_3, with their values being, respectively, 5, 3, and 2. Suppose now, unrealistically, that the value contribution of extra objects of the same type rapidly decreases, from the very beginning, with each new contribution being half as large as the preceding one. Thus, for example, while the value of one e_3-object equals 2, the value of two such objects equals 2 + 1, the value of three e_3-objects equals 2 + 1 + ½, etc.

It is easy to see that for each object type, there is a finite value limit that cannot be exceeded by a whole composed of the objects of that type. That limit can be defined as the sum of the infinite sequence in which the first term equals the value of a single object of the type under consideration and each successive term stands for the value contribution obtained from adding another object of the same type. In the example, these limits have been chosen in such a way as to guarantee that the sequence satisfies the required conditions (i) and (ii). The value of the first element (5) exceeds the value limit for the last element (2 + 1 + ½ + … = 4). Consequently, the first element is strongly superior to the last one. But for each element in the sequence, its value is lower than the value limit for the next object in the sequence. That is, no element is strongly superior to the one that comes next. That such a construction is mathematically coherent is reassuring, since many cases of strong superiority are such that, intuitively, we take it to be possible to move from a strongly superior e_1 to an inferior e_n by a *gradually* decreasing sequence in which at no point there appears to occur a radical value loss.

Indeed, as it has been proved by Klint Jensen (2008) convergence of same-type concatenation to an upper limit is a quite pervasive phenomenon. It holds for all objects that are weakly inferior to some objects in the domain, provided that "is at least as good as" divides the object domain into a countable of equivalence classes. If the latter holds, then as we already know, there is a real-valued function V that represents "is at least as good as". I.e., for all objects e and e', $V(e) \geq V(e')$ if and only if e is at least as good as e'. Now, consider any object e such that some object is weakly superior to e. Then the sequence $V(e)$, V(2 e-objects), V(3 e-objects), …. has an upper limit. For the proof of this observation (Observation 4) due to Klint Jensen (2008), see Appendix 4.

VIII. Weak Superiority is Different

We have seen that, in the absence of Independence, we can have strong superiority between the extrema of a decreasing sequence without strong superiority setting in at any point in the sequence. It is different with weak superiority. It can be shown, without assuming Independence, that any finite sequence whose first element is strongly superior to its last element must contain some element that is *weakly* superior to the one that comes next. In other words, in a sequence in which no element is even weakly superior to its immediate successor, the first element cannot be strongly superior to the last element.

This result can be strengthened. In a sequence in which no element is weakly superior to its immediate successor, the first element cannot even be *weakly* superior to the last element. More exactly, we can prove the following:

> *Observation 5*: Suppose that "is at least as good as" is a complete and transitive relation on the domain. Then, in any finite sequence of objects in which the first element is weakly superior to the last element, there exists at least one element that is weakly superior to its immediate successor.

To establish this observation, it is enough to prove the following lemma:

Lemma 1: Suppose that "is at least as good as" is a weak order, i.e. a complete and transitive relation on the domain. For any objects *e*, *e'*, and *e"*, if *e* is weakly superior to *e"*, *e* is weakly superior to *e'* or *e'* is weakly superior to *e"*.

Note: If "is at least as good as" is transitive and complete, then, by Lemma 1, the complement of weak superiority, i.e. the relation of *not* being weakly superior, is transitive: If *e* is not weakly superior to *e'* and *e'* is not weakly superior to *e"*, then *e* is not weakly superior to *e"*. Since weak superiority by definition is asymmetric, its complement is a complete relation: *e* is not weakly superior to *e'* or *e'* is not weakly superior to *e*. Therefore, Lemma 1 implies that, given the transitivity and completeness of "is at least as good as", weak superiority is a so-called strict weak order, i.e. a relation the complement of which weakly orders the object domain.

For the proof that Lemma 1 holds and that it entails Observation 5, see Appendix 5.[34]

Observation 5 shows that weak superiority cannot obtain between the extrema of a finite sequence without setting in at some point in that sequence. Now, if the elements in a decreasing

[34] The proof of Lemma 1 assumes that the relation 'at least as good as' is complete, which is a rather exacting requirement. What if completeness is not assumed? Well, even in the absence of completeness, we can prove a variant of Observation 5.

Definition 3: An object *e* is *minimally superior* to an object *e'* if and only if for some number *m*, no whole composed of *e'*-objects, however large, is better than *m* *e*-objects.

Observation 6: Suppose that "is at least as good as" is a transitive relation. If the first element in a finite sequence of objects is minimally superior to the last element, then there must exist some element in that sequence that is minimally superior to its immediate successor.

For the proof, see Appendix 6. For a similar result in the context of population axiology, see Arrhenius (2000b, 2014). Note that weak superiority entails minimal superiority, but not vice versa. Still, just as it is the case with these stronger relations, minimal superiority cannot obtain if "is at least as good as" has the Archimedean property.

finite sequence are chosen in such a way that each consecutive element is only *marginally* worse than the immediately preceding one, then it might seem that no element will be weakly superior to the element that comes next.[35] But then, as we just have shown, the first element will not even be weakly superior to the last element in the sequence, however long such a sequence may be. This is surprising, since one would intuitively expect that a sufficiently long series of small worsenings can sooner or later result in an element that is radically worse than the point of departure.

One interpretation of our result is that we should give up this intuition and reject the existence of superiorities in the domain. We might want to deny that a sequence of small worsenings can ever yield an element that is radically worse than the original element. Since many examples of purported superior goods do admit of series of gradual worsenings by means of which we end up with something seemingly inferior to the point of departure, this option puts into question the existence of genuine superiority relationships in the domain. On this view, alleged superiorities disappear upon reflection.

An alternative option is to accept that a series of small worsenings might lead to things that are strongly inferior to points of departure and instead to revise our pre-reflexive idea of superiority. On this second interpretation, superiority need not imply a radical difference in value. If an object *e'* is only slightly worse than another object *e*, then *e* cannot presumably be strongly superior to *e'* (unless the decrease in marginal value that takes place in same-type concatenation of *e'* is extremely rapid). But, contrary to appearances, *e* might still be weakly

[35] Cf. the Quantity Condition discussed in Arrhenius (2000b, 2014), which captures this intuition in a population context. It is shown that this condition, together with some other weak conditions, implies the Repugnant Conclusion.

superior to *e'*, even though it is better only by a small margin.[36] A sufficient number of *e*-objects might, if conjoined, form a whole that is better than *any* whole composed of *e'*-objects. Indeed, on this alternative option, strong superiority need not be a radical difference in value either. While strong superiority does seem to require a considerable difference in value, this difference, as we have seen, need not be infinitely large.

Of course, the first option might be more plausible in some domains, and the other in other domains. For example, we might find it counterintuitive that a sufficient number of pleasurable experiences of a certain intensity can make a whole that is better (or better for the subject) than any whole, *however large*, that consists of pleasurable experiences that are just *slightly* less intense. On the other hand, if some form of objective list theory of welfare is true, which might involve various perfectionist elements, then we might find it rather unproblematic that strongly inferior outcomes can be reached from the strongly superior ones by sequences of small worsenings.

Appendix 1: Strong Superiority without Abrupt Breaks

[36] On this view, then, being weakly superior to an object is not sufficient for being much better than the object in question. (Indeed, as we have seen, an object might be weakly superior to another while being *worse* than the latter.) In fact, being weakly superior is not necessary for being much better either. It is easy to construct a case in which, in a descending sequence *e*, *e'*, *e''*, (i) the value difference between the first element and the second one is larger than that between the second element and the third, (ii) the second element is weakly superior to the third, but (iii) the first element is not weakly superior to *any* of the elements that follow. While a sufficient number of *e'*-objects may be better than any number of *e''*-objects and *e* may be much better than *e'*, there might still be no number of *e*-objects that cannot be outweighed by a sufficiently large number of *e''*-objects and by a sufficient number of *e'*-objects.

Observation 1: Consider any two objects e and e' such that e is better than e'. If e is weakly superior to e', without being strongly superior to it, then the domain must contain a finite decreasing sequence of objects in which the first element is strongly superior to the last one, but no element is strongly superior to its immediate successor.

Proof: Suppose that e is better than and weakly superior to e', without being strongly superior to it. By the definition of weak superiority, there is some $m > 1$ such that m e-objects are better than any number of e'-objects. This means (cf. Section IV above) that the whole composed of m e-objects is strongly superior to e'. Now, consider the following sequence:

e_1 = the whole composed of m e-objects,

e_2 = the whole composed of $m-1$ e-objects,

…

e_{m-1} = the whole composed of 2 e-objects

$e_m = e$,

$e_{m+1} = e'$.

The first object in this sequence is strongly superior to the last one. Furthermore, since "same type"-concatenation is value increasing, each element in the sequence is better than its immediate successor. Thus, the sequence is decreasing. At the same time, no element in the sequence is strongly superior to its successor. In fact, as is easily seen, for all e_k such that $1 \leq k < m$, a whole composed of three e_{k+1} objects is better than e_k. (Such a whole consists of a larger number of e-objects than e_k and thus – by the assumption of value increasingness – must be better than e_k.) The remaining case to consider is when $k = m$, but that e_m is not superior to e_{m+1} is true by hypothesis. Consequently, none of the objects in the sequence e_1, …, e_{m+1} is strongly superior to its immediate successor, despite the fact that e_1 is strongly superior to e_{m+1}. This completes the proof.

Appendix 2: The Importance of Independence: Abrupt Breaks

> *Weak Independence*: If an object e is at least as good as e', then replacing e' by e in any whole results in a whole that is at least as good.

> *Observation 2*: Suppose that the first element in a sequence $e_1, \ldots, e_n$ is strongly superior to the last one. If "is at least as good as" is a complete and transitive relation, Weak Independence implies that some element in the sequence is strongly superior to its immediate successor.

Proof: Suppose that e_1 is strongly superior to e_n. Assume, for *reductio*, that none of the elements e_i in the sequence ($i < n$) is strongly superior to its immediate successor. By completeness, this means that for every such e_i there is some number m_i such that the whole composed of m_i e_{i+1}-objects is at least as good as e_i. Now, start with e_1 and replace it by a whole w_2 composed of m_1 e_2-objects. By assumption, w_2 is at least as good as e_1. If we replace any e_2 in w_2 by m_2 e_3-objects, Weak Independence implies that the resulting whole is at least as good as w_2. We can in this way replace every e_2-object in w_2, one after another, by m_2 e_3-objects, until we reach a whole, w_3, that is composed of $(m_1 \times m_2)$ e_3-objects. By Weak Independence, w_3 is at least as good as w_2, and thus – by transitivity – it is at least as good as e_1. Continuing in this way, from w_2 to w_3, from w_3 to w_4, and so on, using Weak Independence all the time, we finally reach a whole w_n that is composed of $(m_1 \times m_2 \times \ldots \times m_{n-1})$ e_n-objects. By transitivity, w_n is at least as good as e_1, which implies that e_1 is *not* strongly superior to e_n, which contradicts the hypothesis. □

Appendix 3: The Importance of Independence: Collapse

> *Independence*: An object *e* is at least as good as *e'* if and only if replacing *e'* by *e* in any whole results in a whole that is at least as good.

> *Observation 3*: Assume independence. Then, for every e and e', if e is weakly superior to e', then e is strongly superior to e'

Proof, due to Klint Jensen (2008): Suppose that (i) *e* is weakly superior to *e'*. Assume for *reductio* that (ii) *e* is not strongly superior to *e'*. (i) means that for some *m*, *m e*-objects are better than any number of *e'*-objects. (ii) means that for some *k*, *e* is not better than *k e'*-objects. Since "is at least as good as" has been assumed to be a complete relation, it follows from (ii) that *k e'*-objects are at least as good as *e*. We now want to prove, by mathematical induction, that also for all $n > 1$, *nk e'*-objects are at least as good as *n e*-objects, in contradiction to the claim made in (i). Thus, for the induction step, assume we have already established that $(n\text{-}1)k$ *e'*-objects are at least as good as $(n\text{-}1)$ *e*-objects. Then Independence implies that $(n\text{-}1)k$ *e'*-objects concatenated with *e* are at least as good as *n e*-objects. And, also by Independence, *nk e'*-objects are at least as good as $(n\text{-}1)k$ *e'*-objects concatenated with *e*. Therefore, by the transitivity of "at least as good as", *nk e'*-objects are at least as good as *n e*-objects. Which completes the proof.

Appendix 4: Convergence to Upper Limits

> *Observation 4*: Suppose that "is at least as good as" is a weak ordering that divides the object domain into a countable number of equivalence classes. Then, as we

know, there exists a real-valued function V that represents this ordering. Now, consider any object *e* such that some object is weakly superior to *e*. Then the sequence *V*(*e*), *V*(2 *e*-objects), *V*(3 *e*-objects), …. has an upper limit.

Proof (due to Klint Jensen 2008): Suppose that *e'* is weakly superior to *e*. Then there is some *m* such that *V*(*m e'*-objects) > *V*(*n e*-objects), for all *n*. Assume, for *reductio*, that the sequence *V*(e), *V*(2 *e*-objects), *V*(3 *e*-objects), . . . has no upper limit. This sequence is increasing, because we have assumed that same-type concatenation is value-increasing. But any unbounded increasing infinite value sequence approaches infinity. Therefore there must be some *n*, such that *V*(*n e*-objects) is greater than the finite value *V*(*m e'*-objects). This contradicts the hypothesis that *e'* is weakly superior to *e*.

Appendix 5: Weak Superiority

Observation 5: Suppose that "is at least as good as" is a complete and transitive relation on the domain. Then, in any finite sequence of objects in which the first element is weakly superior to the last element, there exists at least one element that is weakly superior to its immediate successor.

Proof: To establish this observation, it is enough to prove the following lemma:

Lemma 1: Suppose that "is at least as good as" is a weak order, i.e. a complete and transitive relation on the domain. For any objects *e*, *e'*, and *e"*, if *e* is weakly superior to *e"*, *e* is weakly superior to *e'* or *e'* is weakly superior to *e"*.

It is easy to see that Observation 5 entails Lemma 1, since a triple *e*, *e'*, *e"* is an example of a finite object sequence. That Lemma 1 in its turn also implies Observation 5 is easy to show. Suppose that "is at least as good as" is complete and transitive. Consider a sequence e_1, …, e_n, in which e_1 is weakly superior to e_n. By Lemma 1, (i) e_1 is weakly superior to e_2, or (ii) e_2 is weakly superior to e_n. If (i) holds, Observation 3 is established. If (ii) holds, then we consider the reduced sequence e_2, …, e_n, and repeat the argument above. I.e. either (iii) e_2 is weakly superior to e_3 or (iv) e_3 is weakly superior than e_n. If (iii), we are done, and if (iv), we consider the reduced sequence e_3, …, e_n. Continuing in this way, we finally reach a 2-membered sequence, e_{n-1}, e_n, and it is clear that Observation 5 trivially holds for such sequences.

It remains then to prove Lemma 1. Assume that (i) *e* is weakly superior to *e"*, but (ii) *e* is not weakly superior to *e'*. We need to show that, in such a case, *e'* is weakly superior to *e"*.
(i) means that there exists some number *m* such that

(1) *m e*-objects are better than any number of *e"*-objects.

(ii) implies that there is some number *m'* such that

(2) *m e*-objects are not better than *m' e'*-objects.

But then, given that "is at least as-good as" is a complete relation, (2) implies that

(3) *m' e'*-objects are at least as good as *m e*-objects.

By the transitivity of "is at least as good as", if one object is at least as good as another, which is better than some third object, then the first object is better than the third. Consequently, (3) and (1) imply that

(4) *m' e'*-objects are better than any number of *e"*-objects.

(4) implies that *e'* is weakly superior to *e"*. This completes our proof.

Appendix 6: Minimal superiority

Definition 3: *e* is *minimally superior* to *e'* if and only if for some number *m*, there is no such *k* that *k e'*-objects are better than *m e*-objects.

Observation 6: Suppose that "is at least as good as" is a transitive relation. If the first element in a finite sequence of objects is minimally superior to the last one, there must exist some object in that sequence that is minimally superior to its immediate successor.

Proof: To establish Observation 6, we prove the following lemma:

Lemma 2: Suppose that "is at least as good as" is a transitive relation. For any objects *e*, *e'* and *e"*, if *e* is minimally superior to *e"*, then *e* is minimally superior to *e'* or *e'* is minimally superior to *e"*.

Lemma 2 is implied by Observation 6 and it implies Observation 6 in exactly the same way as Lemma 1 implies Observation 5 (see above, Appendix 5). As for the proof of Lemma 2, it goes as follows:

Assume that (i) *e* is minimally superior to *e"*, but (ii) *e* is not minimally superior to *e'*. We want to show that, in such a case, *e'* is minimally superior to *e"*.

(i) means that there exists some number *m* such that

(1) for no number *k*, *k e''*-objects are better than *m e*-objects.

(ii) implies that there is some number *k'* such that

(2) *k' e'*-objects are better than *m e*-objects.

To prove that *e'* is minimally superior to *e''*, it is enough to establish that there is no such *k* that *k e''*-objects are better than *k' e'*-objects. Suppose, for *reductio*, that

(3) *k e''*-objects are better than *k' e'*-objects.

By the transitivity of "is at least as good as", the relation "is better than" is transitive as well. Therefore, (3) and (2) imply that

(4) *k e''*-objects are better than *m e*-objects.

But (4) contradicts (1), which concludes the proof.

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