Data-driven stability analysis and enforcement for Loewner Data-Driven Control

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1

Data-driven stability analysis and enforcement for Loewner Data-Driven Control

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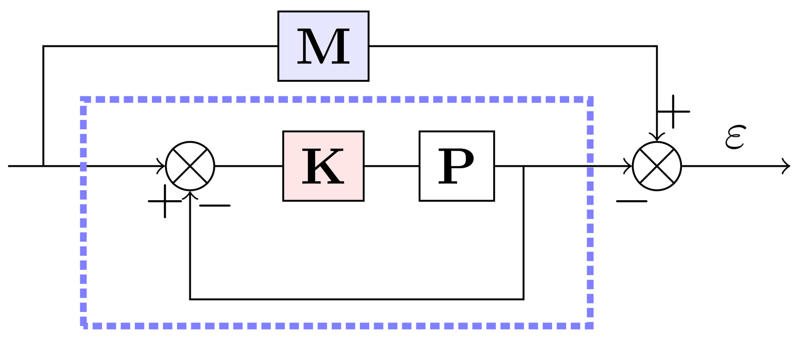
Loewner Data-Driven Control: General formulation



Input data

- Frequency-domain data from the plant $\mathbf{P}$: $\{\omega_i, \Phi_i\}$, $i = 1 \dots N$.
- Reference model $\mathbf{M}$.

Proposed methodology

1. Computation of the ideal controller $\mathbf{K}^\star$ frequency-response:
$$\mathbf{K}^\star(\imath\omega_i) = \Phi_i^{-1}\mathbf{M}(\imath\omega_i)(I-\mathbf{M}(\imath\omega_i))^{-1}.$$
2. Interpolation and reduction of the ideal controller $\mathbf{K}^\star$ through the Loewner framework.

A simple example

$$\mathbf{P}(s) = \frac{0.03616(s-140.5)(s-40)^3}{(s^2+1.071s+157.9)(s^2+3.172s+1936)} \quad \mathbf{M}(s) = \frac{1}{0.01s^2+0.25s+1}$$

$$\mathbf{K}^\star(s) = k\frac{(s^2+1.071s+157.9)(s^2+3.172s+1936)}{s(s+10)(s-140.5)(s-40)^3}$$

→ The reference model should be achievable by the plant.

$$\begin{cases} \mathbf{y}_{z_i}^T\mathbf{P}(z_i) = 0 \\ \mathbf{y}_{p_j}\mathbf{P}(p_j) = \infty \end{cases} \Rightarrow \begin{cases} \mathbf{y}_{z_i}^T\mathbf{M}(z_i) = 0 \\ \mathbf{M}(p_j)\mathbf{y}_{p_j} = \mathbf{y}_{p_j} \end{cases}.$$

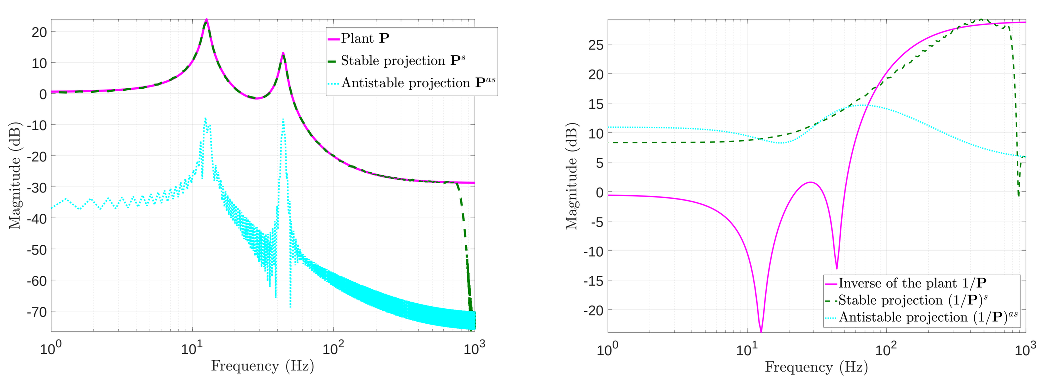
→ A data-driven closed-loop stability analysis is needed.

- Data $\{\omega_i, \Phi_i\}_{i=1}^N$
- Performance specifications

→ Construction of an achievable reference model $\mathbf{M}$ → Definition of the associated ideal controller $\mathbf{K}^\star$ → Controller $\mathbf{K}$ identification → Data-driven stability analysis and reduction of the controller $\mathbf{K}_r$

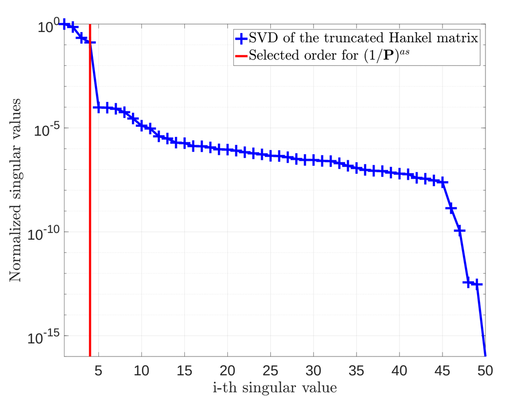
Choice of the reference model

1) Projection of the available data to determine the nature of $\mathbf{P}$:



→ The system is stable but Non-Minimum Phase (NMP).

2) Principal Hankel Components technique to determine the number of NMP zeros and obtain an estimate of the instabilities.



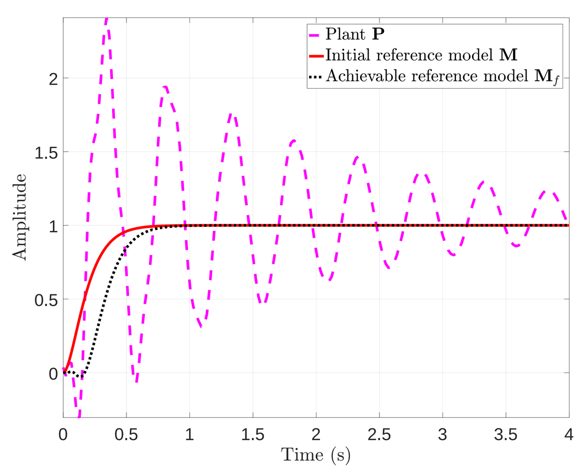
True z_i	Estimated $\tilde{z}_i$
140.5	140.58
40	41.3+2$\imath$
40	41.3+2$\imath$
40	37.4

3) Construction of an achievable reference model $\mathbf{M}_f$.

$$\mathbf{M}_f = \mathbf{M}\mathbf{B}_z$$

$$\mathbf{B}_z(s) = \prod_{i=1}^{n_z} \frac{s-\tilde{z}_i}{s+\tilde{z}_i}$$

$$|\mathbf{B}_z(\imath\omega)| = 1$$



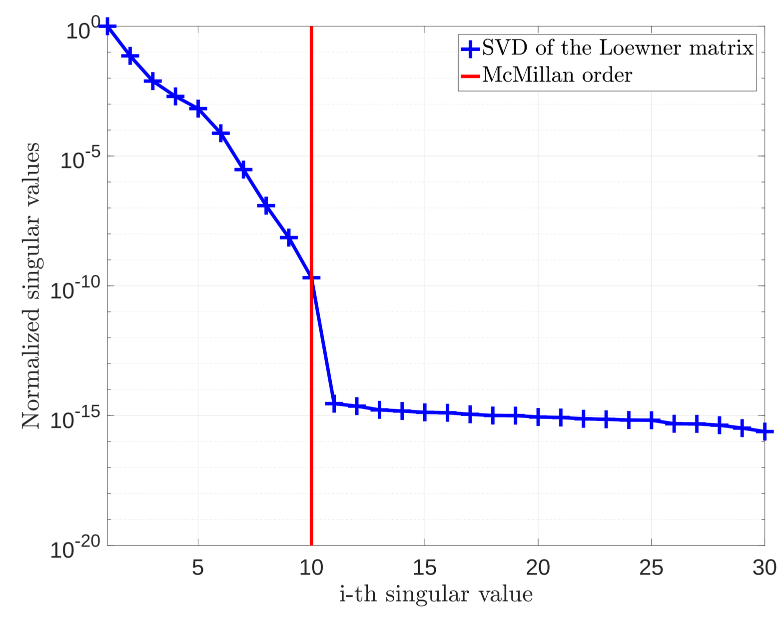
Controller identification

The objective is to obtain a rational model $\mathbf{K} = (E, A, B, C, D)$ such that:

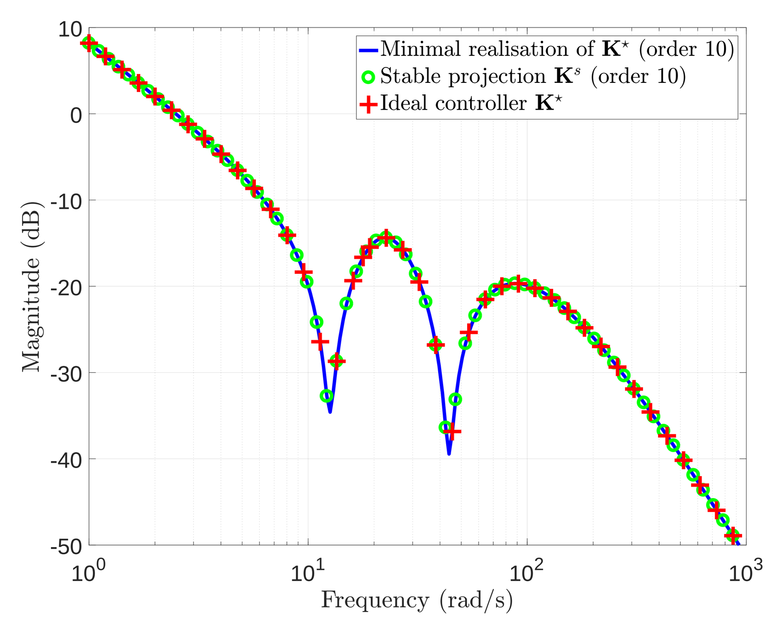
$$\forall i = 1 \dots N, \mathbf{K}(\imath\omega_i) = \mathbf{K}^\star(\imath\omega_i).$$

→ Use of the Loewner pencil $[\mathbb{L}, \mathbb{L}_\sigma]$

1. Embedded order reduction

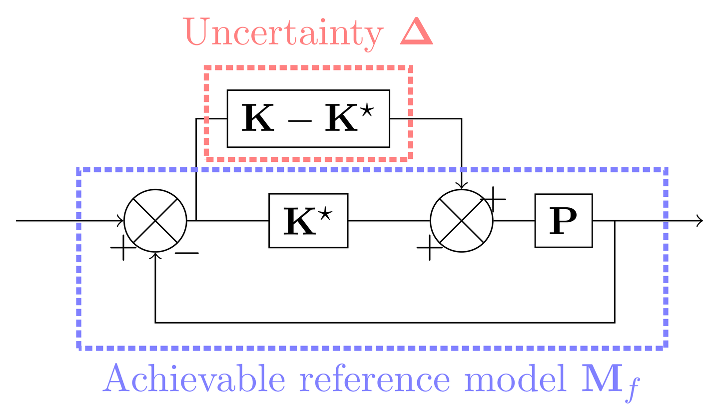


2. Stability of the identified model $\mathbf{K}$



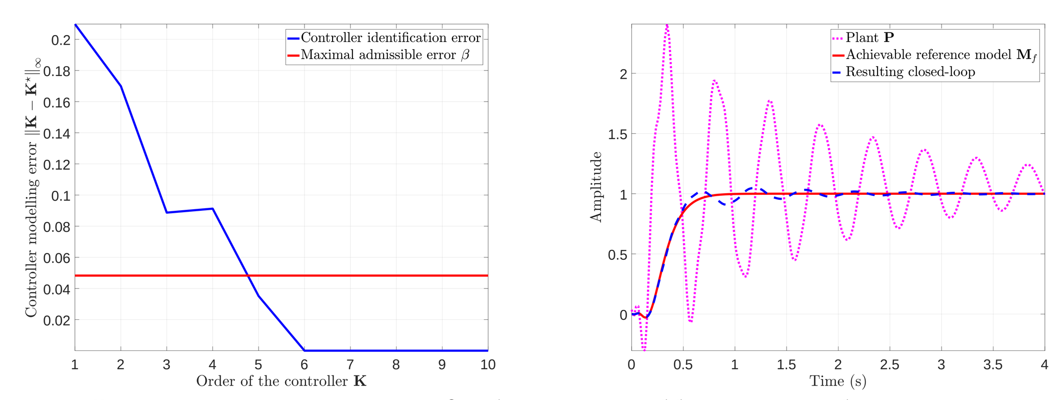
Proposed choice of $\mathbf{M}$ → no more compensation of instabilities in the open-loop!

Controller reduction



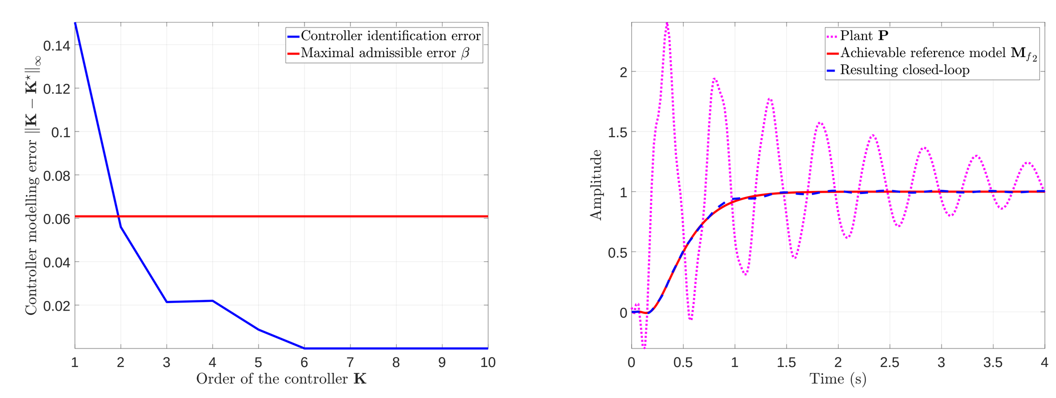
The resulting closed-loop is well-posed and internally stable for all stable Δ such that $\|\Delta\|_\infty \leq \beta$ if and only if $\|(1-\mathbf{M}_f)\mathbf{P}\|_\infty < \frac{1}{\beta}$.

→ Limiting the controller modelling error allows to ensure closed-loop internal stability!



→ Conservatism of the small-gain theorem and importance of the choice of the initial specifications

$$\mathbf{M}_2(s) = \frac{1}{0.04s^2+0.4s+1} \qquad \mathbf{M}_{f2} = \mathbf{M}_2\mathbf{B}_z$$



Structured LDDC design

General structuration of the controller:

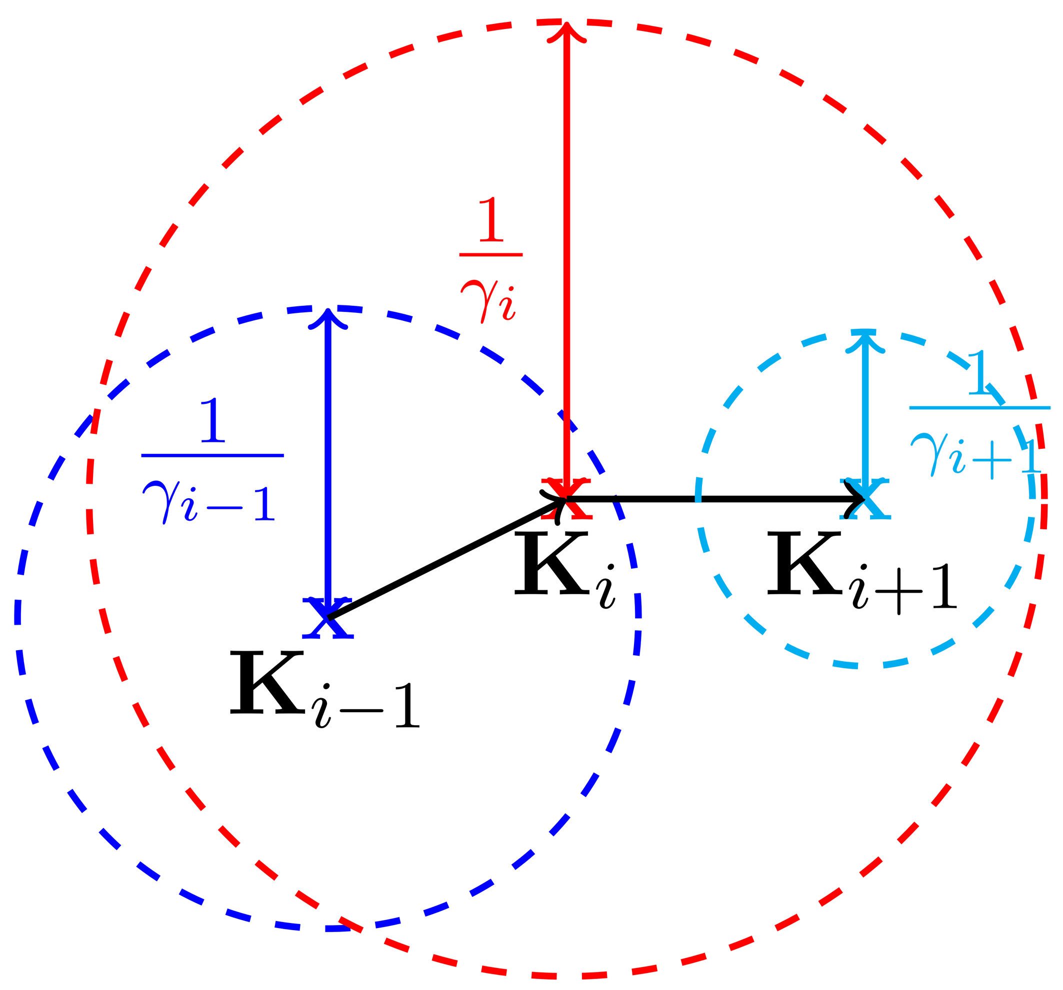
$$\mathbf{K}(s,\theta) = \frac{1}{D(s,\theta)}N(s,\theta)$$

At iteration k, problem $\mathcal{P}_k$ is solved:

$$\begin{aligned} &\min_{\theta_k} \|\mathbf{M}(\imath\omega_i) - \mathbf{H}(\theta_k, \imath\omega_i)\|_F^2 \\ \text{s.t.} \quad &\|\mathbf{K}(\theta_k) - \mathbf{K}(\theta_{k-1})\|_\infty < \tfrac{1}{\beta_{k-1}} \\ &P\theta < 0 \end{aligned}$$

where

$$\mathbf{H}(\theta_k, \imath\omega_i) = (I + \Phi_i\mathbf{K}(\theta_k, \imath\omega_i))^{-1}\Phi_i\mathbf{K}(\theta_k, \imath\omega_i)$$



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