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Abstract

In this paper, a set of formulae are derived to approximately tune a PI controller to meet specified gain and phase margins for the class of processes with integration. Expressions to approximate the gain and phase cross-over frequencies as well as the attainable gain and phase margins are also given. These quantities are useful for assessing the closed-loop behaviour. From the results obtained, an interesting observation can be made. The gain and phase margins for this class of processes with PI control are largely decoupled.

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1. Introduction

PI controllers are common in the process industries. Several methods have been proposed for the tuning of PI controllers. The Ziegler-Nichols, (1942) and Cohen Coon, (1953) formulae are among the more well known ones. These tuning formulae have been derived to give quarter amplitude damping for load disturbances. This is considered to be too oscillatory, particularly for setpoint response. Manual fine tuning is a trial and error process – laborious and time consuming.

In this paper, a set of formulae are derived to approximately tune a PI controller to meet specified gain and phase margins for the class of processes with integration. The gain and phase margins serve as a measure of robustness. It is known from classical control that the phase margin is related to the damping of the system. It can therefore also serve as a performance measure.

Apart from the gain and phase margins, a third parameter, the normalized dead-time, is used to characterize the process. The normalized dead-time is used elsewhere for assessing the performance of the Ziegler-Nichols tuned PID controller as well as for characterizing processes (Åström, 1990; Åström et al., 1990). It has also been used successfully to tune PID controllers (Hang et al., 1990).

The paper is organized as follows. Section 2 introduces the concept of the normalized dead-time. A set of formulae to calculate parameters of the PI controller to give specified gain and phase margins are derived in section 3. Assessment of PI control is discussed in section 4. Conclusions are presented in section 5 and acknowledgements in section 6.

2. The Normalized Dead-time

The normalized dead-time of a self-regulating process is defined as

$$\Theta = \frac{L}{T}$$

where L and T are the apparent dead-time and apparent time constant respectively. (See Figure 1)

Many industrial processes can be modeled by a second order plus dead-time model. For a process with integration, it takes the form

$$G_p(s) = \frac{e^{-sL}}{s(sT+1)} \tag{1}$$

A normalized dead-time can also be defined for this class of processes. It is shown in Åström et al., (1990) that such a process can be characterized by the normalized dead-time of $sG_p(s)$. For a process given by equation (1), the normalized dead-time is thus given by $\Theta = L/T$. The normalized dead-time can be estimated from the impulse response.

With PI control, the results in this paper show that for $\Theta \geq 1$ the model can be further simplified to

$$G_p(s) = \frac{1}{s}e^{-sL} \tag{2}$$

where the normalized dead-time $\Theta = \infty$. Gain and phase margins tuning formulae will be derived for the processes of equations (1) and (2). It will be shown that these formulae can also be applied to more general integrating processes.

The formulae derived for the process of equation (2) can be applied to the first order self-regulating process with very small dead-time. To get some insight into this, consider the first order process

$$G_p(s) = \frac{1}{(s+\lambda)}e^{-sL}$$

The normalized dead-time is

$$\Theta = \lambda L$$

If

$$\lambda = 0$$

then

$$G_p(s) = \frac{1}{s}e^{-sL}$$

and

$$\Theta = 0$$

Hence an integrator with dead-time may be regarded as an approximation of the first order process with very small normalized dead-time.

3. Gain and Phase Margin

The transfer function of the PI controller is given by

$$G_p(s) = k_c(1 + \frac{1}{T_i})$$

For a process with transfer function $G_p(s)$, the gain margin, A_m, the phase margin, ϕ_m, the gain crossover frequency, ω_g, and the phase crossover frequency, ω_p, can be obtained from the following equations.

$$arg G_c(j\omega_p)G_p(j\omega_p) = -\pi \tag{3}$$

$$A_m = \frac{1}{|G_c(j\omega_p)G_p(j\omega_p)|} \tag{4}$$

$$|G_c(j\omega_g)G_p(j\omega_g)| = 1 \tag{5}$$

$$\phi_m = arg G_c(j\omega_g)G_p(j\omega_g) + \pi \tag{6}$$

Using these equations and making a series of approximations, formulae for the k_c/k_u and T_i/t_u ratios for the processes of equations (1) and (2) will be derived. The ultimate gain and period of the process are denoted by k_u and t_u respectively. The formulae are derived to give good results for $2 \leq A_m \leq 5$ and $45° \leq \phi_m \leq 75°$ which is sufficient for most purposes. A rule of thumb is $A_m = 4$ and $\phi_m = 60°$ (Shinners, 1964). Notice that our formulae do not admit phase margin of less than 45°. This means for example that Ziegler-Nichols like tunings which give phase margins as low as 30° cannot be dealt with. The formulae derived for the process of equation (2) are applicable to processes with $\Theta \geq 1$ as well as to a first order self-regulating process with $\Theta \leq 0.05$. On the other hand, the formulae derived for the process of equation (1) are applicable when $0.5 < \Theta < 1$.

Integrator With Dead-time

In this section, tuning formulae for the process of equation (2) will be derived. When solving the equations numerically, it was observed that $\omega_p T_i$ was of the order of 8 – 18 and $\omega_g T_i$ was of the order of 2 – 5 for $2 \leq A_m \leq 5$ and $45° \leq \phi_m \leq 75°$. It is therefore reasonable to approximate $\sqrt{\omega_p^2 T_i^2 + 1}$ by $\omega_p T_i$ and $\sqrt{\omega_g^2 T_i^2 + 1}$ by $\omega_g T_i$. By making these approximations and approximating the arctan function, it is possible to derive analytical expressions for the controller parameters.

For the process of equation (2), the loop transfer function with PI control is

$$G_c(s)G_p(s) = \frac{k_c(sT_i + 1)}{s^2 T_i} e^{-sL} \tag{7}$$

Hence

$$|G_c(j\omega)G_p(j\omega)| = \frac{k_c}{\omega^2 T_i} \sqrt{\omega^2 T_i^2 + 1}$$

and

$$arg G_c(j\omega)G_p(j\omega) = -\pi + \arctan \omega T_i - \omega L$$

It follows from equation (3) that the phase crossover frequency is given by

$$\arctan \omega_p T_i - \omega_p L = 0 \tag{8}$$

The arctan function is approximated by

$$\arctan x \approx 0.8x, \qquad |x| \leq 1 \tag{9}$$

The approximation error is less than 0.07. The identity

$$\arctan x = \frac{\pi}{2} - \arctan \frac{1}{x} \tag{10}$$

is used when $|x| > 1$. Hence

$$\omega_p \approx \frac{\pi}{4L} + \sqrt{(\frac{\pi}{4L})^2 - \frac{0.8}{T_i L}} \tag{11}$$

The gain margin of the system is given by

$$A_m = \frac{\omega_p^2 T_i}{k_c \sqrt{\omega_p^2 T_i^2 + 1}} \tag{12}$$

which can be approximated as

$$A_m \approx \frac{\omega_p}{k_c} \tag{13}$$

for $\omega_p^2 T_i^2 \gg 1$.

The gain crossover frequency is given by

$$\frac{k_c}{w_g^2 T_i}\sqrt{\omega_g^2 T_i^2 + 1} = 1 \tag{14}$$

For $\omega_g^2 T_i^2 \gg 1$ we get approximately

$$\omega_g \approx k_c \tag{15}$$

The phase margin is given by

$$\phi_m = \arctan \omega_g T_i - \omega_g L \tag{16}$$

Using approximation (9) and identity (10) for $\arctan \omega_g T_i$ gives

$$\phi_m \approx \frac{\pi}{2} - \frac{0.8}{\omega_g T_i} - \omega_g L \tag{17}$$

Solving (11), (13), (15) and (17) for k_c and T_i gives

$$k_c \approx \frac{\pi(A_m - 1) + 2\phi_m}{2L(A_m^2 - 1)} \tag{18}$$

and

$$T_i \approx \frac{0.8}{k_c(\frac{\pi}{2} - k_c L - \phi_m)} \tag{19}$$

Introduce the dimension-free parameters

$$\alpha_k = \frac{k_c}{k_u}$$

and

$$\alpha_i = \frac{T_i}{t_u}$$

where

$$k_u = \frac{\pi}{2L}$$

and

$$t_u = 4L$$

The derivation of k_u and t_u are shown in the appendix. Expressions (18) and (19) now becomes

$$\alpha_k \approx \frac{\pi(A_m - 1) + 2\phi_m}{\pi(A_m^2 - 1)} \tag{20}$$

and

$$\alpha_i \approx \frac{0.8}{\alpha_k \pi(\pi - \alpha_k \pi - 2\phi_m)} \tag{21}$$

Numerical Results

When the gain and phase margins are specified, α_k and α_i are calculated from equations (20) and (21). The controller gains are then obtained from the ultimate gain and period of the process. Since several approximations were used to derive equations (20) and (21), we will investigate the accuracy of the approximations. This is done by specifying the gain and phase margins and calculating the controller parameters using the design procedure. The true gain and phase margins with the chosen controller are then computed numerically from the exact formulae. The results of these calculations for different processes are given in Table 1-5. The entry $\hat{\omega}_p$ will be explained later.

Table 1. Results of approximation for $\frac{1}{s}e^{-s}$ ($\Theta = \infty$ $k_u=\frac{\pi}{2}$ $t_u=4$)

Specified				Actual				
A_m	ϕ_m	k_c	T_i	A_m	ω_p	ϕ_m	ω_g	$\hat{\omega}_p$
3	45	0.49	5.5	2.9	1.5	41	0.52	1.5
5	45	0.29	5.5	4.9	1.4	42	0.33	1.5
5	60	0.31	12	5.0	1.5	57	0.32	1.5

Table 2. Results of approximation for $\frac{1}{s(s+1)}e^{-s}$ (Θ=1.0 k_u=1.1 t_u=7.3)

Specified				Actual				
A_m	ϕ_m	k_c	T_i	A_m	ω_p	ϕ_m	ω_g	$\hat{\omega}_p$
3	45	0.36	10	2.8	0.78	35	0.35	0.81
5	45	0.21	10	4.6	0.78	41	0.23	0.81
5	60	0.22	22	4.8	0.83	53	0.23	0.84

Table 3. Results of approximation for $\frac{1-2s}{s(s+1)^3}$ (Θ=1.0 k_u=0.33 t_u=19)

Specified				Actual				
A_m	ϕ_m	k_c	T_i	A_m	ω_p	ϕ_m	ω_g	$\hat{\omega}_p$
3	45	0.10	26	2.9	0.31	39	0.11	0.32
5	45	0.062	26	4.8	0.31	41	0.071	0.32
5	60	0.064	56	4.9	0.33	55	0.067	0.33

Table 4. Results of approximation for $\frac{1}{s(s+1)^{15}}$ (Θ=1.1 k_u=0.12 t_u=59)

Specified				Actual				
A_m	ϕ_m	k_c	T_i	A_m	ω_p	ϕ_m	ω_g	$\hat{\omega}_p$
3	45	0.036	82	2.9	0.10	40	0.038	0.099
5	45	0.022	82	4.8	0.10	43	0.025	0.099
5	60	0.022	180	4.9	0.10	57	0.023	0.10

Table 5. Results of approximation for $\frac{1}{(s+1)}e^{-0.05s}$ (Θ = 0.05 k_u=32 t_u=0.20)

Specified				Actual				
A_m	ϕ_m	k_c	T_i	A_m	ω_p	ϕ_m	ω_g	$\hat{\omega}_p$
3	45	10	0.28	2.9	30	46	11	30
5	45	6.0	0.28	4.9	30	51	6.8	30
5	60	6.2	0.60	5.0	31	66	6.4	31

Tables 1-5 show that the tuning formulae give approximations that are acceptable for the chosen processes. The largest error in gain margin is 0.4

which occurs in the second entry of table 2. The largest error of the phase margin is 10° which occurs in the first entry in table 2. The next largest errors are 0.2 in gain margin and 6° in phase margin. The approximations are more accurate for larger phase margins. This is because a larger phase margin results in a larger integral time, and the approximations $\omega_p^2 T_i^2 \gg 1$ and $\omega_g^2 T_i^2 \gg 1$ are better. There are, however, exceptions because of the approximation for the arctan function.

Integrator plus First Order Lag with Dead-time

Tuning formulae for the process given by equation (1) will be derived in this section. When solving the equations numerically, it was observed that $\omega_p T_i$ was of the order of $8 - 26$ and $\omega_g T_i$ of the order of $2 - 8$ for $2 \le A_m \le 5$ and $45° \le \phi_m \le 75°$. It is therefore reasonable to approximate $\sqrt{\omega_p^2 T_i^2 + 1}$ by $\omega_p T_i$ and $\sqrt{\omega_g^2 T_i^2 + 1}$ by $\omega_g T_i$. By making these approximations and approximating the arctan function, it is possible to derive analytical expressions for the controller parameters.

Equation (1) can be normalized by a change of time scale to the following form.

$$G_p(s) = \frac{e^{-s\Theta}}{s(s+1)} \tag{22}$$

where $\Theta = L/T$. The loop transfer function with PI control is

$$G_c(s)G_p(s) = \frac{k_c(sT_i + 1)}{s^2 T_i (s+1)} e^{-s\Theta} \tag{23}$$

Hence

$$|G_c(j\omega)G_p(j\omega)| = \frac{k_c}{\omega^2 T_i} \sqrt{\frac{\omega^2 T_i^2 + 1}{\omega^2 + 1}}$$

and

$$arg G_c(j\omega)G_p(j\omega) = -\pi + \arctan \omega T_i - \arctan \omega - \omega\Theta$$

It follows from equation (3) that the phase crossover frequency is determined by

$$\arctan \omega_p T_i - \arctan \omega_p - \omega_p \Theta = 0 \tag{24}$$

Using approximation (9) and identity (10) for the arctan function gives

$$\omega_p \approx \sqrt{\frac{0.8}{\Theta}(1 - \frac{1}{T_i})} \tag{25}$$

The gain margin of the system is determined from

$$A_m = \frac{\omega_p^2 T_i}{k_c} \sqrt{\frac{\omega_p^2 + 1}{\omega_p^2 T_i^2 + 1}} \tag{26}$$

which can be approximated as

$$A_m \approx \frac{\omega_p}{k_c} \sqrt{\omega_p^2 + 1} \tag{27}$$

for $\omega_p^2 T_i^2 \gg 1$. Substituting for ω_p from expression (25) gives

$$A_m \approx \frac{1}{k_c}\sqrt{(\frac{0.8}{\Theta})^2(1-\frac{1}{T_i})^2 + \frac{0.8}{\Theta}(1-\frac{1}{T_i})}$$
$$\approx \frac{1}{k_c}\sqrt{\frac{0.8}{\Theta}(1+\frac{0.8}{\Theta} - \frac{1}{T_i}(1+\frac{1.6}{\Theta}))} \quad (28)$$

where the $1/T_i^2$ term has been neglected since T_i is large.

The gain crossover frequency is given by

$$\frac{k_c}{\omega_g^2 T_i}\sqrt{\frac{\omega_g^2 T_i^2 + 1}{\omega_g^2 + 1}} = 1 \quad (29)$$

For $\omega_g^2 T_i^2 \gg 1$ and $\omega_g^2 \ll 1$, we get approximately

$$\omega_g \approx k_c \quad (30)$$

The phase margin is given by

$$\phi_m = \arctan \omega_g T_i - \arctan \omega_g - \omega_g \Theta \quad (31)$$

Using approximation (9), identity (10) for $\arctan \omega_g T_i$ ($\omega_g Ti > 1$) and approximation (9) for $\arctan \omega_g$ ($\omega_g < 1$) gives

$$\phi_m \approx \frac{\pi}{2} - \frac{0.8}{\omega_g T_i} - \omega_g(\Theta + 0.8) \quad (32)$$

Finally, solving (28), (30) and (32) for k_c and T_i gives

$$k_c \approx \frac{-(\frac{\pi}{2} - \phi_m)(\Theta + 1.6) + \sqrt{a_3\Theta^3 + a_2\Theta^2 + a_1\Theta + a_0}}{2\Theta^2(A_m^2 - 1) - 4.8\Theta - 2.56} \quad (33)$$

where

$$a_0 = 0.64(4\phi_m^2 - 4\pi\phi_m + \pi^2 - 5)$$
$$a_1 = 0.8(4\phi_m^2 - 4\pi\phi_m + \pi^2 - 12.8)$$
$$a_2 = \phi_m^2 - \pi\phi_m + 0.25\pi^2 - 10.24 + 2.56A_m^2$$
$$a_3 = 3.2(A_m^2 - 1)$$

and

$$T_i \approx \frac{0.8}{k_c(\frac{\pi}{2} - \phi_m) - k_c^2(\Theta + 0.8)} \quad (34)$$

Introduce the dimension-free parameters

$$\alpha_k = \frac{k_c}{k_u}$$

and

$$\alpha_i = \frac{T_i}{t_u}$$

The following approximate expressions are derived in the appendix.

$$k_u \approx \sqrt{\frac{1}{\Theta^2} + 0.44\frac{1}{\Theta} - 0.2} \tag{35}$$

and

$$t_u \approx 2\pi\sqrt{\frac{\Theta}{1 - 0.28\Theta}} \tag{36}$$

Using these approximations, expressions (33) and (34) become

$$\alpha_k \approx \frac{-(\frac{\pi}{2} - \phi_m)(\Theta + 1.6) + \sqrt{a_3\Theta^3 + a_2\Theta^2 + a_1\Theta + a_0}}{k_u(2\Theta^2(A_m^2 - 1) - 4.8\Theta - 2.56)} \tag{37}$$

and

$$\alpha_i \approx \frac{0.8}{\alpha_k k_u t_u(\frac{\pi}{2} - \phi_m) - \alpha_k^2 k_u^2 t_u(\Theta + 0.8)} \tag{38}$$

Numerical Results

When the gain and phase margins are specified, α_k and α_i are calculated from equations (37) and (38). The controller gains are then obtained from the ultimate gain and period of the process. The results of the aproximations for various processes with $\Theta \approx 0.5$ are given in tables 6-8. They shows that the tuning formulae give good approximation for the chosen processes. The largest gain margin error is 0.2 and the largest phase margin error is 6°.

Table 6. Results of approximation for $\frac{1}{s(s+1)}e^{-0.5s}$ ($\Theta = 0.5$ k_u=2.2 t_u=4.8)

Specified				Actual				
A_m	ϕ_m	k_c	T_i	A_m	ω_p	ϕ_m	ω_g	$\hat{\omega}_p$
4	45	0.47	9.8	4.1	1.2	41	0.44	1.2
5	45	0.36	7.0	5.0	1.2	38	0.36	1.2
5	55	0.39	20	5.2	1.3	50	0.40	1.2

Table 7. Results of approximation for $\frac{1-s}{s(s+1)^3}$ ($\Theta = 0.57$ k_u=0.49 t_u=15)

Specified				Actual				
A_m	ϕ_m	k_c	T_i	A_m	ω_p	ϕ_m	ω_g	$\hat{\omega}_p$
4	45	0.11	26	4.1	0.39	46	0.11	0.39
5	45	0.084	21	5.2	0.38	42	0.093	0.38
5	55	0.090	51	5.2	0.40	57	0.092	0.40

Table 8. Results of approximation for $\frac{1}{s(s+1)^6}$ ($\Theta = 0.5$ k_u=0.33 t_u=23)

Specified				Actual				
A_m	ϕ_m	k_c	T_i	A_m	ω_p	ϕ_m	ω_g	$\hat{\omega}_p$
4	45	0.072	45	4.2	0.25	48	0.074	0.25
5	45	0.055	34	5.3	0.25	43	0.061	0.25
5	55	0.060	91	5.3	0.26	59	0.061	0.25

4. Assessment of PI Control

In this section, we will discuss some of the equations in the derivation of the tuning formulae that are useful for the assessment of the closed-loop behaviour. Firstly, notice that the gain and phase margins cannot be chosen independently. In order to obtain positive controller gains, the conditions $\alpha_k > 0$ and $\alpha_i > 0$ must be satisfied. The following inequalities are obtained from expressions (20), (21) for $\Theta \geq 1$ and (37), (38) for $0.5 \leq \Theta < 1$.

$$\phi_m < \frac{\pi}{2}(1 - \frac{1}{A_m}), \qquad \text{for } \Theta \geq 1 \tag{39}$$

and

$$\phi_m < \frac{\pi}{2} - \frac{1}{A_m\Theta}\sqrt{0.8(\Theta + 0.8)^3}, \qquad \text{for } 0.5 \leq \Theta < 1 \tag{40}$$

Figure 2 shows the critical cases for $\Theta = 0.5$ and $\Theta = 1$. They correspond to the gain and phase margins obtained with a P controller. The valid region is below the curve. For a given gain margin, the PI controller will be less stable than a P controller since it has smaller phase margin. Expressions (39) and (40) give a measure of the robustness that can be obtained by PI control. Since robustness and performance are related, they also give the achievable performance of a PI controller. For example, if it is desired to specify a gain margin of 3 and a phase margin of 45° for a system with $\Theta = 0.5$ then Figure 2 shows that this is not possible with PI control. A PID or PD controller may have to be used.

The crossover frequencies are good indications of the bandwidth of the system. The phase crossover frequency can be estimated from expressions (11) and (25) if ω_p is first expressed in terms of the ultimate period of the process. This gives

$$\hat{\omega}_p \approx \frac{1}{t_u}(\pi + \sqrt{\pi^2 - \frac{3.2}{\alpha_i}}), \qquad \text{for } \Theta \geq 1 \tag{41}$$

and

$$\hat{\omega}_p \approx \frac{1}{t_u}\sqrt{\frac{3.2\pi^2}{1 - 0.28\Theta} - \frac{2\pi}{\alpha_i}\sqrt{\frac{\Theta}{1 - 0.28\Theta}}}, \qquad \text{for } 0.5 < \Theta < 1 \tag{42}$$

The results of the estimation are also given in tables 1-8. It is interesting to note from equations (15) and (30) that $\omega_g \approx k_c$. This relationship is also verified by the results in the tables. Notice that the transfer function of a process with an integrator has dimension s^{-1}. Hence k_c has dimension frequency. Finally it follows from equation (13) that for $\Theta \geq 1$, $A_m \approx \omega_p/\omega_g$.

The results in tables 1-8 shows that to a certain extent A_m is independent of T_i and ϕ_m independent of k_c. For example, in table 1 for the first and second entries when T_i is kept constant but k_c varies, the actual ϕ_m is approximately constant whereas the actual A_m varies. On the other hand, for the second and third entries when k_c is approximately constant but T_i varies, the actual A_m is approximately constant whereas the actual ϕ_m varies. One consequence of this result is that tuning of the controller is easy because adjustment of gain and phase margins are decoupled. The gain margin can be adjusted by k_c and the phase margin by T_i. Another consequence is that since the phase margin of the system is insensitive to changes in k_c, it is also insensitive to changes in the process gain. Therefore the phase margin is robust with respect to uncertainty in the process gain. The design method thus gives a loop transfer function which resembles Bode's ideal loop gain (Bode, 1945). This is clearly seen from the Nyquist curve in Figure 3.

5. Conclusion

In this paper a set of formulae are derived to approximately tune a PI controller to meet specified gain and phase margins for the class of processes with integration. The processes are characterized by their normalized dead-time. The approximate formulae are useful because they give analytical relations between controller parameters and specifications. Numerous numerical calculations have shown that they give a reasonable accuracy. If needed, higher accuracy can be obtained by solving the nonlinear equations iteratively. Assessment of PI control for this class of processes are also given. Quantities such as the achievable gain and phase margins as well as the crossover frequencies of the system are useful for assessing the performance of the system.

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References

ÅSTRÖM, K. J. (1990): "Assessment of Achievable Performance of Simple Feedback Loops," to appear in International Journal of Adaptive Control and Signal Processing.

ÅSTRÖM, K. J. AND T. HÄGGLUND (1988): "A new auto-tuning design," *Proc. IFAC Symp. on Adaptive Control of Chemical Processes, Lyngby, Denmark.*.

ÅSTRÖM, K. J., C. C. HANG AND P. PERSSON (1989): "Towards Intelligent PID Control," *Proc. IFAC Workshop on AI in Real Time Control, China*, pp. 38–43.

BODE, H. W. (1964): *Network Analysis and Feedback Amplifier Design*, D. Van Nostrand, New Jersey.

COHEN, G. H. AND G. A. COON (1953): "Theoretical consideration of retarded control," *Trans. ASME 75*, pp 827–834.

HANG, C. C., K. J. ÅSTRÖM AND W. K. HO (1990): "Refinements of The Ziegler-Nichols Tuning Formula," to appear in Proc. IEE Part D.

SHINNERS, S. M. (1964): *Control System Design*, John Wiley & Sons, Inc., New York, London, Sydney.

ZIEGLER, J. G. AND N. B. NICHOLS (1942): "Optimum settings for automatic controllers," *Trans. ASME, 65*, pp. 433–444.

Appendix: Derivation of the Ultimate Gain and the Ultimate Period

For

$$G_p(s) = \frac{1}{s}e^{-sL}$$

the ultimate gain and period are given by the equation

$$-\frac{\pi}{2} - \omega_u L = -\pi$$

Hence

$$t_u = 4L$$

and

$$k_u = \frac{\pi}{2L}$$

For

$$G_p(s) = \frac{1}{s(s+1)}e^{-s\Theta}, \qquad 0.5 \leq \Theta < 1$$

the ultimate gain and period are given by the equation

$$-\arctan\omega_u - \omega_u\Theta - \frac{\pi}{2} = -\pi$$

Using the approximation

$$\arctan\omega_u \approx \frac{\pi}{2} - \frac{\omega_u}{\omega_u^2 + 0.28}, \qquad |\omega_u| > 1$$

which gives an error of less than 0.005, we obtain

$$t_u \approx 2\pi\sqrt{\frac{\Theta}{1 - 0.28\Theta}}$$

and

$$k_u \approx \sqrt{\frac{1}{\Theta^2} + 0.44\frac{1}{\Theta} - 0.2}$$

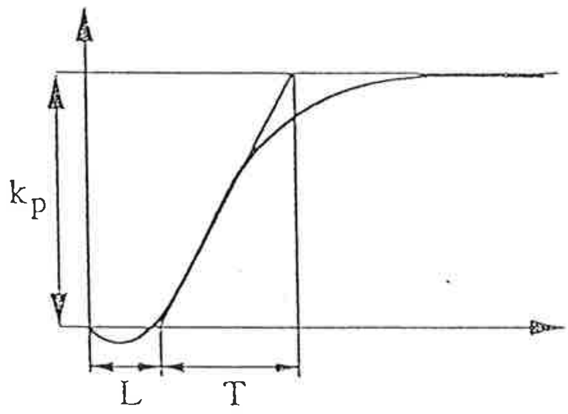

Figure 1. Apparent Dead-Time L and Apparent Time Constant T

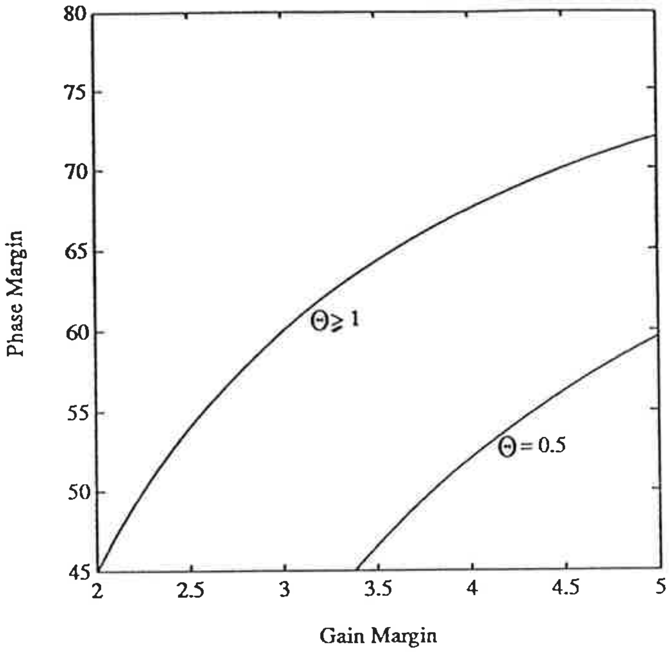

Figure 2. Region for Gain and Phase Margin Specifications

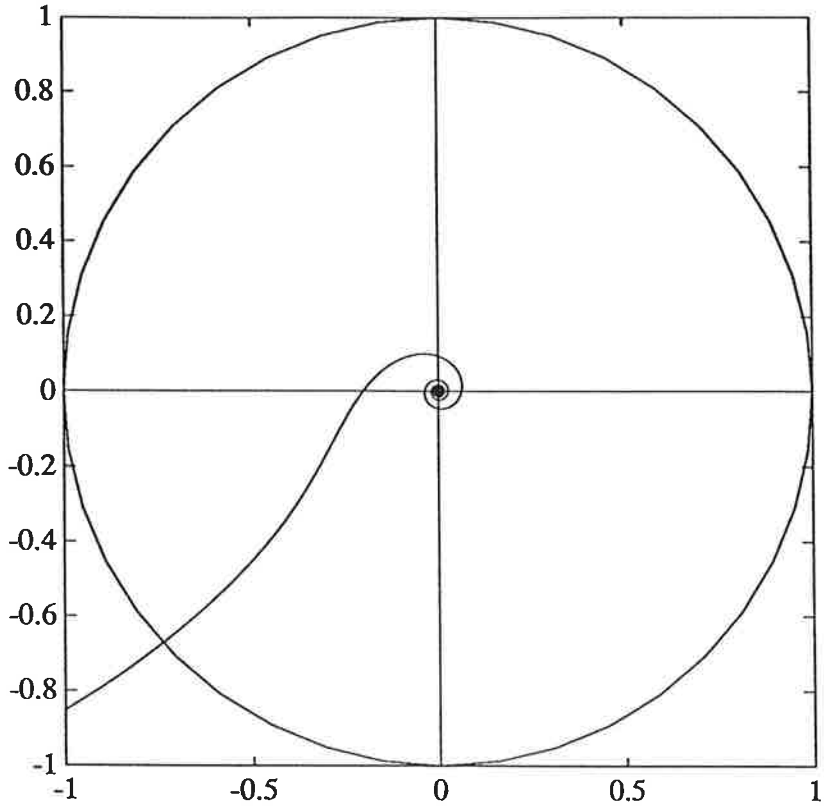

Figure 3. Nyquist curve of the loop transfer function of a PI Controller and the process $G_p(s) = \frac{1}{s}e^{-s}$