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Abstract

A Stochastic alternative to fuzzy control is proposed. The so called membership functions are used to define a probability space, and the values of the membership functions may be interpreted as probabilities. Conditioned probabilities corresponds to the IF-THEN-statements of the fuzzy controller.

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1. Introduction

For the last ten years fuzzy control has been widely applied in industry to design controllers. The first industrial application was the control of a cement kiln [Holmblad and Oestergaar, 1982]. However, fuzzy control was described already in [Mamdani, 1974]. The ideas came from [Zadeh, 1965; Zadeh, 1973]. Other early applications can be found in [Kickert and van Nauta Lemke, 1976; King and Mamdani, 1977; Tong *et al.*, 1980].

One advantage with fuzzy controllers, which they share with other simple controllers such as e.g. PID-controllers, is that they are easy to understand without knowing much mathematics. One may interpret fuzzy controllers as non-linear controllers designed from the operators know-how—the fuzzy concept is a good programming environment for the operator to automate his control actions. If the know-how is not present in terms of an operator, then the design of the controller is more difficult, [Tong, 1977]. However, attempts have been made to automatically generate fuzzy controllers. Optimal and adaptive fuzzy control has been described in [Mamdani and Baaklini, 1975; Procyk and Mamdani, 1979; Kacprzyk, 1983; Kacprzyk and Iwanski, 1987; Swierniak, 1989]. To compute an optimal controller, a model of the process is needed. This has been described in [Kandel, 1980; Pedrycz, 1981]. To obtain a model, estimation has to be performed, which has been described in [Gaines, 1979; Benlahcen and Lamotte, 1981; Pedrycz, 1981; Hirota and Pedrycz, 1983]. However, note that conventional models such as e.g. ordinary differential equations cannot be used. Thus, when the know-how is not present, the use of new complicated mathematics is necessary both for describing the models and for estimating them. Moreover, the analysis of the controller, e.g. closed loop stability or performance, is just as difficult as with PID-controllers, if not more difficult, since the controller is non-linear, [Braae and Rutherford, 1979].

The raison d'être of fuzzy logic is based on the need for imprecise statements. It was the same need that influenced the emerging of probability theory in the beginning of this century. There have been discussions about what is best—fuzzy logic or probability theory. One view has been given in [Stallings, 1977]. The question is not easy to answer. Actually many questions with respect to different features have to be answered. It is obvious that the design of fuzzy controllers is easy when operator knowledge is present. Thus any alternative to fuzzy control must have this feature as well. However, fuzzy control is difficult to use, when this knowledge is not present. Further the analysis is difficult.

In this report an alternative to fuzzy control is presented. It will be shown that most nice features of the usual fuzzy controller are preserved. Moreover, the probability-interpretation will in the future make it possible to use existing probability theory for analyzing the controller as well as for designing controllers.

2. Probability Interpretation of Membership Functions

In this section it will be shown how the so called membership function from fuzzy logic can be interpreted as a probability that can be computed from a certain probability measure. The relevant theory of probability can be found in e.g. [Kolmogoroff, 1933; Chung, 1974; Shiryayev, 1984].

Let U be a non-empty set, and let A be any subset of U, i.e. $A \subset U$. With fuzzy logic a notion of unsharpness is introduced in the sense that for any fixed $x \in U$, it is not possible to know if $x \in A$ or not.

This can also be accomplished with probability theory. Let the sample space be given by

$$\Omega = \{A : A \subset U\}$$

The events that are of interest are of the type $\{A \in \Omega : x \in A\}, \quad x \in U$. Therefore introduce the following set of events

$$\mathcal{E} = \{\{A \in \Omega : x \in A\} : x \in U\}$$

Further let the smallest σ-algebra that contains all elements of $\mathcal{E}$ be denoted by

$$\mathcal{F} = \sigma(\mathcal{E})$$

which exists by [Shiryayev, 1984, Lemma 1, §2, Ch. II]. Now $(\Omega, \mathcal{F})$ is a measurable space. Define the probability measure

$$\mathrm{P} : \mathcal{F} \to [0, 1]$$

on the measurable space via the identity

$$\mu(x) = \mathrm{P}\{A \in \Omega : x \in A\}, \quad x \in U$$

where

$$\mu : U \to [0, 1]$$

is a membership function. We have now defined a probability space $(\Omega, \mathcal{F}, \mathrm{P})$, and the value of the membership function μ can for any fixed $x \in U$ be interpreted as the probability that for a randomly chosen set $A \in \Omega$ it holds that $x \in A$. It should be stressed that x is fixed and A is random. It may very well be that the definition of the probability measure is not unique. This is no problem. What is a problem, is that $\mathcal{F}$ may be to rich for the definition to be consistent. Moreover, for P to be uniquely determined by a finite dimensional distribution function the elements of Ω may not have more than a countable number of coordinates, [Kolmogoroff, 1933]. However, it will be seen that for the purposes of control, it will be possible to find a sample space Ω simple enough and a σ-algebra $\mathcal{F}$ which is not too rich.

3. Membership Functions on the Real Line

In this section it will be seen how the definition of the probability measure given in the previous section is consistent and can be described by a two-dimensional distribution function for the case of a simple sample space.

Let the sample space be given by

$$\Omega = \{A : A \subset R, \quad A \text{ closed and connected}\}$$

Then for all $A \in \Omega$ it holds that $A = [a, b]$ for some $a, b \in R$. This implies that there exists a bijection between the elements of Ω and the elements of

R^2. This will make it possible to take $\mathcal{F}$ to be the σ-algebra corresponding to $\mathcal{B}(R^2)$, i.e. the Borel-σ-algebra on R^2. It is well known that a probability measure P defined on the measurable space $(R^2, \mathcal{B}(R^2))$ is uniquely defined by its two-dimensional distribution function

$$F(\xi, \eta) = \mathrm{P}\{a \leq \xi, b \leq \eta\}$$

Thus the membership function μ is uniquely determined by

$$\mu(x) = \mathrm{P}\{A \in \Omega : x \in A\} = \mathrm{P}\{a \leq x \leq b\} = \int_{\xi \leq x,\ \eta \geq x} dF(\xi, \eta), \quad x \in R$$

There remains one question to be answered. Given a membership function, is it always possible to find the two-dimensional distribution function? The answer is yes, and the proof will be done by an example. Let the membership function be the commonly used triangular one, i.e. let

$$\mu(x) = \begin{cases} x + 1, & -1 \leq x < 0 \\ -x + 1, & 0 \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Further let $F(\xi, \eta) = F_a(\xi)F_b(\eta)$, where $F_a(\xi)$ and $F_b(\eta)$ are distribution functions for rectangular distributed independent random variables on the intervals $[-1, 0]$ and $[0, 1]$ respectively. It now follows that

$$\mu(x) = F_a(x)[1 - F_b(x)], \quad x \in R$$

It is obvious that distribution functions of the type given above can be found for any membership function defined on the real numbers.

4. The Stochastic Controller

In this section it will be shown how the interpretation of the membership function as probabilities can be utilized to define an appealing controller.

Denote by $U \subset R$ and $Y \subset R$ the value sets for the scalar valued control signal and measurement signal respectively. Let

$$\begin{aligned} y_i &: Y \rightarrow Y, \quad i = 1, 2, \ldots, m \\ u_j &: U \rightarrow U, \quad j = 1, 2, \ldots, n \end{aligned}$$

be independent random variables with known distribution functions defined such that for the intervals

$$\begin{aligned} I_i^{(y)} &= [y_i, y_{i+1}], \quad i = 1, 2, \ldots, m \\ I_j^{(u)} &= [u_j, u_{j+1}], \quad j = 1, 2, \ldots, n \end{aligned}$$

it holds that

$$\begin{aligned} I_i^{(y)} \cap I_j^{(y)} &= \emptyset, \quad i \neq j \\ I_i^{(u)} \cap I_j^{(u)} &= \emptyset, \quad i \neq j \end{aligned}$$

This will as described in the previous section induce a set of membership functions

$$\mu_{y_i}(y) = \mathrm{P}\left\{y \in I_i^{(y)}\right\} = F_{y_i}(y)\left[1 - F_{y_{i+1}}(y)\right], \qquad i = 1, 2, \ldots, m, \quad y \in Y$$

$$\mu_{u_j}(u) = \mathrm{P}\left\{u \in I_j^{(u)}\right\} = F_{u_j}(u)\left[1 - F_{u_{j+1}}(u)\right], \qquad j = 1, 2, \ldots, n, \quad u \in U$$

It now holds that

$$\mu_{u_j}(u)(y) = \sum_{i=1}^{m} \mu_{y_i}(y)\mathrm{P}\left\{u \in I_j^{(u)} \middle| y \in I_i^{(y)}\right\}$$

where $P\{\cdot|\cdot\}$ are conditioned probabilities that the designer should chose. These conditioned probabilities corresponds to the IF-THEN-statements in fuzzy control. One possible way to choose the control signal u^0 given a certain measurement signal y^0 is to take the maximum-likelihood estimate of u, i.e. taking u^0 as the solution to

$$\max_j \max_u \mu_{u_j}(u)(y^0)$$

It would of course also be possible to take the mean value or the median. The results above are easily generalized to MIMO-systems by considering vector valued membership functions and matrices of conditioned probabilities in the formula for $\mu_{u_j}(u)(y)$.

5. Conclusions

It has been seen how it is possible for the membership functions used in fuzzy control to construct a probability space in such a way that the value of the membership function may be interpreted as a probability.

This has made it possible to propose a stochastic alternative to fuzzy control which is still based on membership functions. The new controller not only shares the membership functions with the fuzzy controller, it also has a similar construction as the IF-THEN-statements of the fuzzy controller. This is the conditioned probabilities. Thus the new controller have all the good properties of the fuzzy controller, which makes it easy to use to automate the operators know-how from manual control. Note that the mathematics used in the previous sections is not necessary for using the proposed controller. It is only necessary to specify the membership functions and the conditioned probabilities. The latter may be interpreted as IF-THEN-statements. Further, the probability interpretation will make the analysis and the design of e.g. optimal controllers possible without introducing new difficult mathematics that are unknown to most people.

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