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Olsson, Henrik; Åström, Karl Johan

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221 00 Lund
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Henrik Olsson
Karl Johan Åström

Department of Automatic Control
Lund Institute of Technology
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Department of Automatic Control
Lund Institute of Technology
P.O. Box 118
S-221 00 Lund Sweden

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Author(s)
Henrik Olsson and Karl Johan Åström

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Abstract

Friction is an important aspect of high quality servomechanisms. Failure to account for the effects of friction can lead to tracking errors and even oscillations when small movements are demanded. In this report we will briefly review some friction models and propose a new model that captures several important aspects of friction. The goal is to obtain a model that can be used in simulation studies and to form a basis for adaptive friction compensation.

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1. Introduction

Friction is an important aspect of many control systems, high quality servomechanisms as well as simple pneumatic and hydraulic systems. Failure to account for the effects of friction can lead to tracking errors and oscillations. In this report we will briefly review some friction models and propose a new model that captures several important aspects of friction. A nice and more complete review of friction models can be found in [Ehr91]. The goal is to obtain a model that can be used in simulation studies and to form a basis for adaptive friction compensation.

2. A Review of Friction Models

Friction is a natural phenomenon that is quite hard to model and it is not yet completely understood. In this section we discuss friction modelling. The model is restricted to describe the behaviour of friction as a function of relative velocity and time. Temperature, age and lubrication are other parameters that affect friction significantly. These effects are not considered in the model. Another effect that is not accounted for in the model is that the magnitude of friction is different for different directions of motion.

The most classical description of friction got its name from Charles de Coulomb but it was known already to Leonardo Da Vinci. This model which is called *Coulomb friction*, assumes that the friction force is constant, proportional to the normal force and opposed to the motion. There is a dimensionless friction coefficient μ which is the ratio between friction force and normal force which is approximately constant for a given application. A typical value for lubricated surfaces is $\mu = 0.1$. Mathematically Coulomb friction can be described as

$$F_{fr} = F_c \operatorname{sign}(v)$$

where $F_c = \mu N$ and N is the normal force.

In addition to Coulomb friction it is common to add *viscous friction* which is a friction force proportional to the relative velocity, i.e.

$$F_{fr} = F_v v$$

Viscous friction was introduced when the dynamics of fluid flow began to be developed. Later studies showed that viscous friction force is not necessarily proportional to velocity. There are applications where the friction force is proportional to the square root of the relative velocity.

The friction force acting on a body at rest is called *static friction* or *stiction*. It counteracts the forces applied to the body but is limited to a level called the *break-away force*. This is the force necessary to initiate motion of a body at rest and is typically greater then the force required to maintain motion.

Friction models including Coulomb, viscous and static friction were used already during the 19th century, [Mor33, Rey86].

When looking into the finer details of friction a few observations can be made. During transition between rest and motion, the friction force is in many cases decreasing with increased relative velocity. This is known as the *Stribeck effect*, [Str02]. It is an example of so called negative viscous friction. The

Stribeck effect is caused by increased fluid lubrication. It can be described by adding the term

$$F_{fr} = F_s e^{-(v/v_s)^2} \operatorname{sign}(v)$$

to the other friction forces.

Another observation is that small displacements due to both plastic and elastic deformation in fact occurs during stiction. The behaviour very much resembles that of a connection with a stiff spring with damper. The behaviour is known as the *Dahl effect*, [Dah77]. Observations show a relative displacement in the order of 1-10 microns.

Much of the variation in friction force is due to the changing nature of lubrication. The changes are however not instantaneous when relative velocity varies. It takes time to build up a lubrication layer. The delay could be in the range of milli-seconds to tenths of milli-seconds. The phenomenon is sometimes referred to as *frictional lag*, see for example [HS90]. The frictional lag leads to a *hysteresis* effect when friction is described as a function of velocity. The size of the hysteresis depends on the rate of change of the velocity.

There is hardly any lubrication at low relative velocities. The friction forces are built up by shear forces when asperities on the two bodies make contact with each other. The asperities and their sizes are not evenly distributed over the surfaces. The friction force at low velocities therefore has a random character. This is captured in the *bristle model* by Haessig and Friedland, [HF90]. In this model friction is described by bristles which are randomly located on the surfaces. Each pair of bristles that make contact contributes to a small part of the friction force.

Rabinowicz, [Rab58], made another observation regarding static friction. He found that a connection between the magnitude of the static friction and the time spent in the stiction phase can be of help when predicting the behaviour of stick-slip motion.

As already mentioned the friction force depends strongly on the relative velocity. It is therefore natural to divide the relative velocity into different regions each with a different description of the friction. The classical division has a stiction zone for zero velocity and Coulomb and viscous friction for non-zero velocity. This model may be difficult to simulate and it is therefore common to broaden the stiction zone by saying that stiction occurs in an interval around zero velocity. This is done in *Karnopp's model*, [Kar85], where it can be recommended to let the velocity interval depend on the desired accuracy. A more elaborate model could include a stiction zone, a low velocity zone were the Stribeck effect is dominating and then a high velocity zone where the friction might be described with the classical models. It would however be nice to a have smooth transitions between the zones instead of sharp boundaries so as to reconcile more with the dynamics of friction.

Adaptive friction compensation has so far been based mainly on friction models describing Coulomb friction and viscous friction, [GW74, CdWÅB87, ELK92]. This is satisfactory in situations when velocity reversals are not frequently occurring. For high precision servos with frequent velocity reversals it would be useful to have a model that describes the behaviour for low velocities more accurately. Some attempts have been made in order to get a better compensation when reversing velocity by incorparating dynamics such as the Dahl effect into the underlying friction model, [Wal84, ELK92]. However, the results are not totally satisfying.

3. A New Model

The starting point of this model is a simplified version of the bristle model, [HF90]. Instead of describing the friction with a number of bristles, each acting as a spring, it is here assumed that the stiction at rest behaves as a single spring and damper. At higher velocities the friction is described by Coulomb and viscous friction. A variable s is introduced. Its purpose is to tell what type of friction that we can expect to occur. The slip variable s should be one when the system slips and zero when there is no slipping. Instead of having a sharp boundary between the two behaviours, s will be continuous so that the transitions are smooth. A key problem is to decide the quantities that s should depend on. We have chosen to let it depend on the relative velocity v. As described in the previous section there is experimental evidence that there is some lag in establishing friction. To account for the lag we assume that there is a time constant associated with the slip variable. Our slip variable, s_v, is therefore modeled by

$$\tau_{del}\frac{ds_v}{dt} = -s_v + f(v) \tag{1}$$

where $f(v)$ is a function which is close to 1 for large values of the velocity v and almost zero for small values.

An auxiliary variable x is also introduced. This variable represents the relative displacement in the stiction phase. In the model proposed this variable is driven towards zero when slipping occurs. The variable is modeled by

$$\frac{dx}{dt} = (1 - s_v)v - s_v\frac{1}{\tau_r}x \tag{2}$$

where the last term of the right hand side makes x go to zero when slipping occurs. The rate is governed by the parameter τ_r. The relative velocity is the variable v. Let the saturation function be defined as

$$\mathrm{sat}(F_0, x) = \begin{cases} F_0, & \text{if } x \geq F_0; \\ x, & \text{if } -F_0 < x < F_0; \\ -F_0, & \text{if } x \leq -F_0. \end{cases}$$

The stiction force F_s is given by

$$F_s = (1 - s_v)\mathrm{sat}(F_{s_0}, k_s x + d_s v) \tag{3}$$

It is thus a weighted and saturated value of the force described by the spring. The spring constant is k_s and d_s the damping constant. The break-away force is F_{s_0}.

It follows from (1),(2) and (3) that the steady state relation between static friction force F_s and velocity v is

$$F_s = (1 - f(v))\mathrm{sat}(F_{s_0}, k_s\frac{\tau_r v(1 - f(v))}{f(v)} + d_s v) \tag{4}$$

The total friction force is then given by

$$F = F_s + F_c + F_v$$

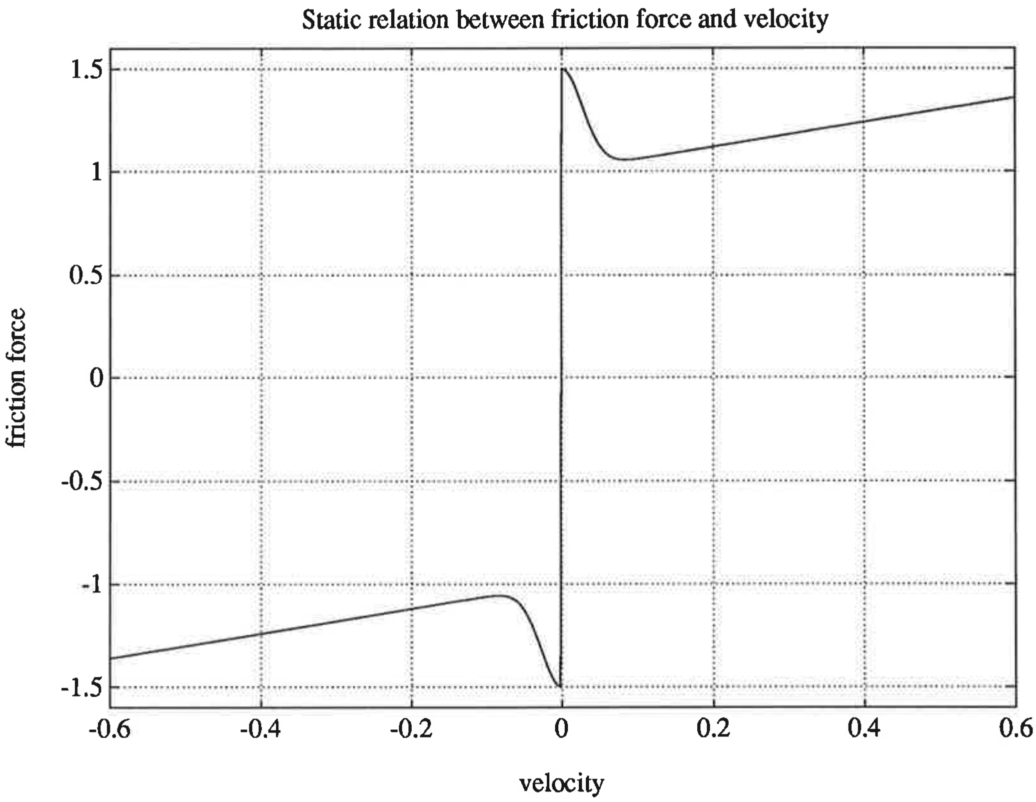

Figure 1. Static relation between friction force and velocity for the proposed model.

where F_c is Coulomb friction

$$F_c = s_v F_{c_0} \operatorname{sign}(v)$$

and F_v is viscous friction

$$F_v = s_v F_{v_0} v$$

i.e. weighted versions of classical Coulomb and viscous friction. The static relation between the total friction force and velocity is

$$F_{F_T} = f(v)(F_{c_0} \operatorname{sign}(v) + F_{v_0} v) + (1 - f(v)) \operatorname{sat}(F_{s_0}, k_s \frac{\tau_r v(1 - f(v))}{f(v)} + d_s v) \quad (5)$$

The shape of this function depends on the function $f(v)$ as well as on the parameters. Figure 1 shows a plot of the static relation between friction force and velocity when the function $f(v)$ is chosen as

$$f(v) = 1 - e^{-(v/v_s)^2}$$

The curve shows that the model exhibits the desired Stribeck effect.

Parameter values

The parameter values of the friction model depend on the application. It would be desirable to have as few unknown parameters as possible in the model. Some values could probably be determined experimentally once and for all, while others must be estimated continuously. As described in Section

2, reasonable values of frictional lag and pre-sliding displacement have been determined experimentally.

The spring constant k_s can be determined from measurements of break-away force, F_{s_0}, and maximum presliding displacement, δ, using the equation

$$k_s = \frac{F_{s_0}}{\delta}$$

Measurements of the static relation relation between friction force and velocity can indicate the value of v_s used to determine the slip variable. The damping constant d_s can be related to k_s and normal force. For high relative velocities the model is linear in the parameters and therefore the viscous friction coefficient F_{v_0} and the parameter F_{c_0} for Coulomb friction could be obtained by conventional estimation methods in the high velocity region. It remains, however, to experimentally try out what parameters that can be assigned a fixed value without deterioating the ability to compensate well for friction and what parameters need to be estimated continuously during friction compensation.

4. Validation

It is important that a model captures the essential behaviour of friction that has been observed in experiments. In this section we will show how the proposed model acts in a number of simulations. The parameters of the friction model have been chosen as, $F_{s_0} = 1.5$, $F_{c_0} = 1$, $F_{v_0} = 0.6$, $k_s = 10000$, $ds = 1100$, $\tau_{del} = 0.01$, $\tau_r = 0.001$ and $v_s = 0.04$ in all simulations.

Friction force during uni-directional motion and sinusoidal velocity variations

A simulation of the friction force during a forced motion with velocity $v = 0.06 + 0.04\sin(\omega t)$ with $\omega = 4$ and $\omega = 20$ was done. The result is seen in figure 2. The figure shows the friction force as a function of relative velocity. A hysteresis effect is clearly visible in the simulation. This is caused by the frictional lag modeled by the time constant τ_{del}. The hysteresis effect is more pronounced for rapid velocity changes, i.e. increasing with the frequency of the sinusoidal velocity variation.

Friction force during bi-directional motion and sinusoidal velocity variations

In this experiment a forced motion with velocity reversals was simulated. The velocity was $v = 0.1\sin(25t)$. Figure 3 shows hysteresis as a function of relative displacement.

Presliding displacement phenomenon

Courtney-Pratt and Eisner found that there is a small displacement when the applied force is less than the break-away force. To investigate if this is supported by our model a sinusoidal force, $F_{ext} = 1.4\sin(8t)$, was applied in a simulation. Figure 4 shows the friction force as a function of displacement. Note that no true sliding occurs. The simulation agrees with the Dahl effect.

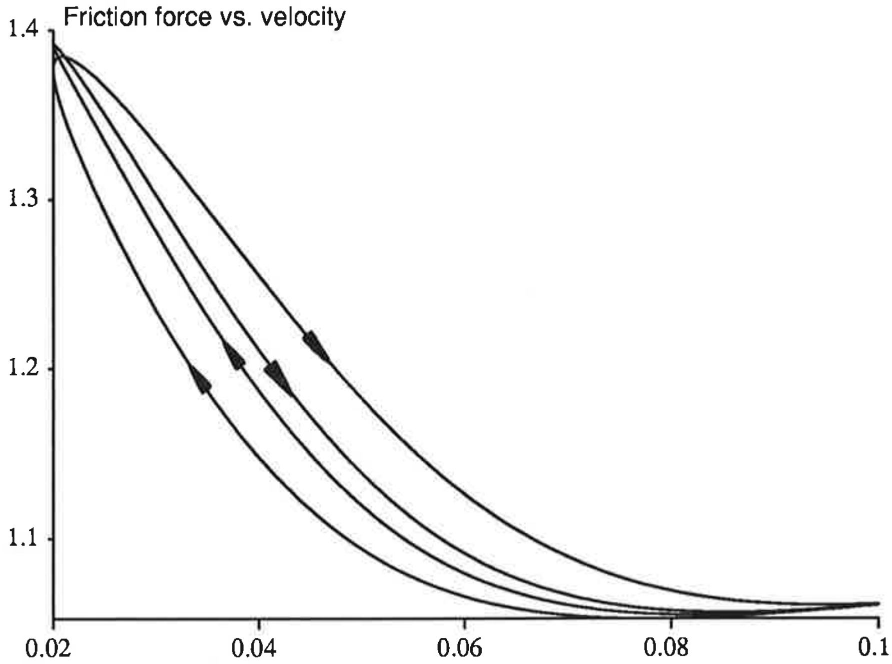

Figure 2. Relation between friction force and velocity. The velocity is sinusoidal with two different frequencies. The one with the higher frequency shows the largest hysteresis.

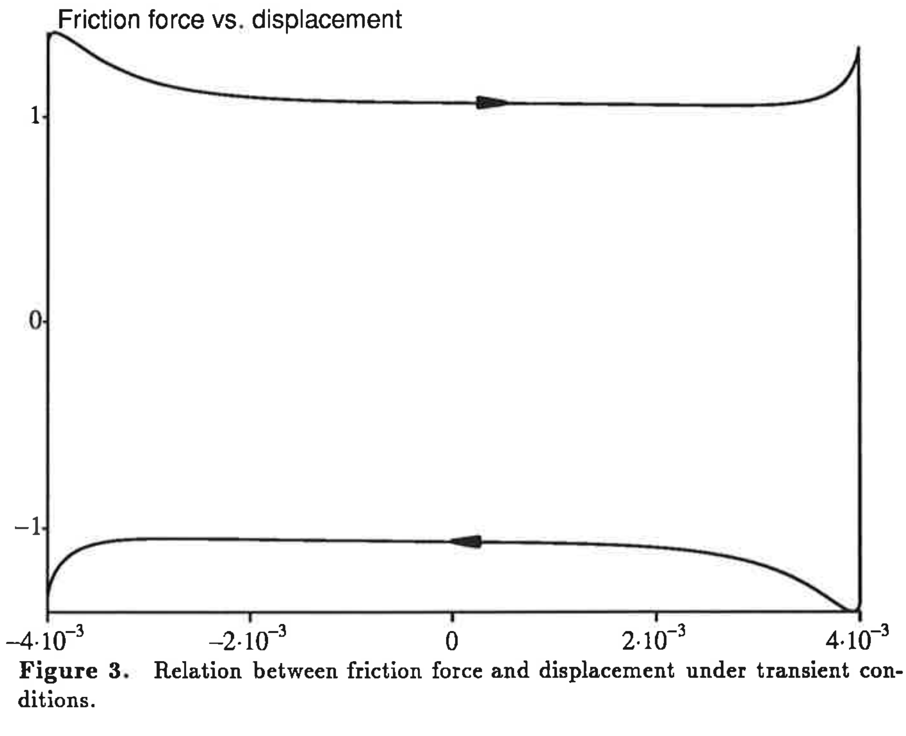

Figure 3. Relation between friction force and displacement under transient conditions.

Stick-slip motion

Simulation of stick-slip motion is another way to investigate a friction model. When discussing stick-slip motion we assume that a mass $m = 1$ is connected to a spring with spring constant $k_{pull} = 2$ and no damping. The other end is moved with constant velocity $v_{pull} = 0.1$.

To get some insight into the behaviour of the system we will do some approximate analysis. It is assumed that $s_v = 0$ in the stiction zone. The

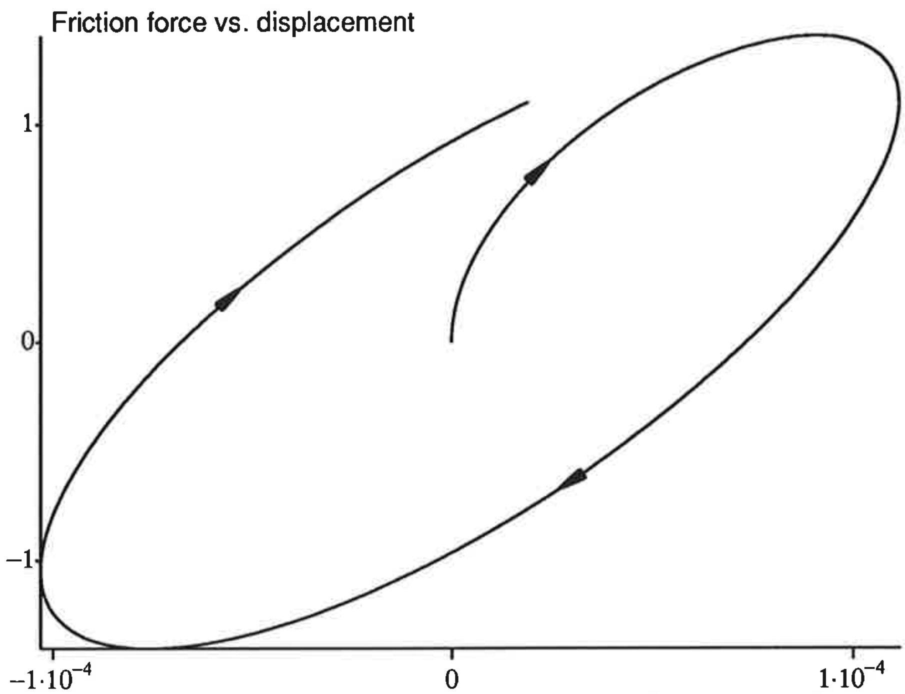

Figure 4. Presliding displacement during stiction (The Dahl effect).

friction force then becomes

$$F = k_s x + d_s v$$

Equation (1) simplifies to

$$\frac{dx}{dt} = v$$

and the equation of motion of the mass becomes

$$m\frac{d^2x}{dt^2} = F_{ext} - F_{fr} = F_{ext} - k_s x - d_s\frac{dx}{dt}$$

In the stiction zone the motion is thus an oscillation with undamped natural frequency

$$\omega = \sqrt{\frac{k_s}{m}}$$

and relative damping

$$\zeta = \frac{d_s}{2m\omega}$$

When there is slipping we have $s_v = 1$ and the friction force becomes

$$F = F_{c_0}\,\mathrm{sign}(v) + F_v v$$

In this region we thus have a motion with Coulomb and viscous friction.

Figure 5 shows a simulation of stick-slip motion. The upper part of figure 5 shows the position of the end of the spring and the mass. The lower part shows the friction force. The mass is initially at rest. The force increases linearly because of the spring action in the friction model. When the force reaches 1.5 the mass starts to slip. The friction force drops rapidly to the Coulomb level. It increases due to the increasing velocity. The increasing

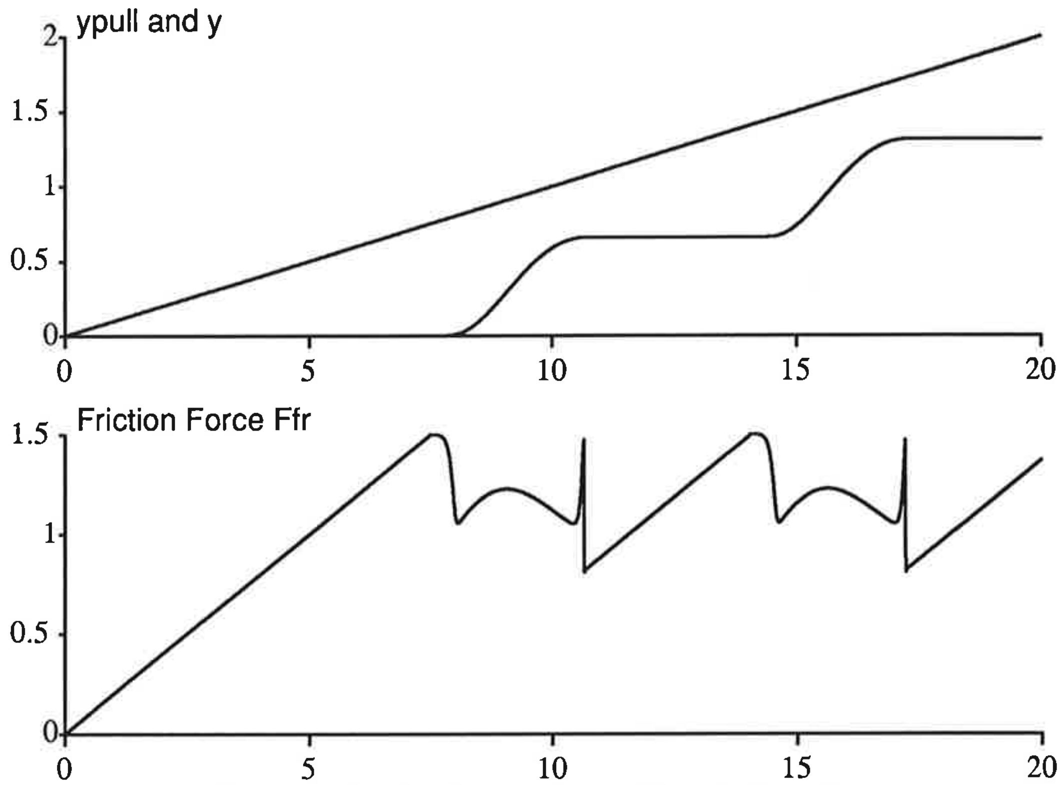

Figure 5. Results of an experiment with stick-slip motion.

velocity causes the spring to contract, the velocity decreases and the friction force also decreases. When the velocity becomes sufficiently small, the friction increases and the mass sticks because of the Stribeck effect.

In the stiction zone the motion is that of a damped mass supported by two springs, one describing the static friction and one that pulls the mass. The spring constant k_s is significantly larger than the constant for the pulling spring.

Additional insight in the model is obtained from figure 6 which shows the behaviour of the system around the time when the mass gets stuck. The figure shows that the friction force increases due to the Stribeck effect. The rate of change of velocity at stiction time is almost discontinuous which explains the rapid change in friction force. The friction force is then reduced to the force needed to keep the mass at rest.

The simulation experiments indicate that the model gives a good description of stick-slip motion.

5. Summary

A new friction model has been proposed in this paper. The model is intended to be used as a basis for adaptive friction compensation in situations when there are frequent velocity reversals. The model is relatively simple and captures key behaviour of friction. One disadvantage is that the model assumes good knowledge of the velocity even for small values. Measurements are however always corrupted by noise and have a limited resolution. This means that integration of equation (2) in order to predict static friction force may be difficult. Simulation experiments covering stick-slip motion and periodic

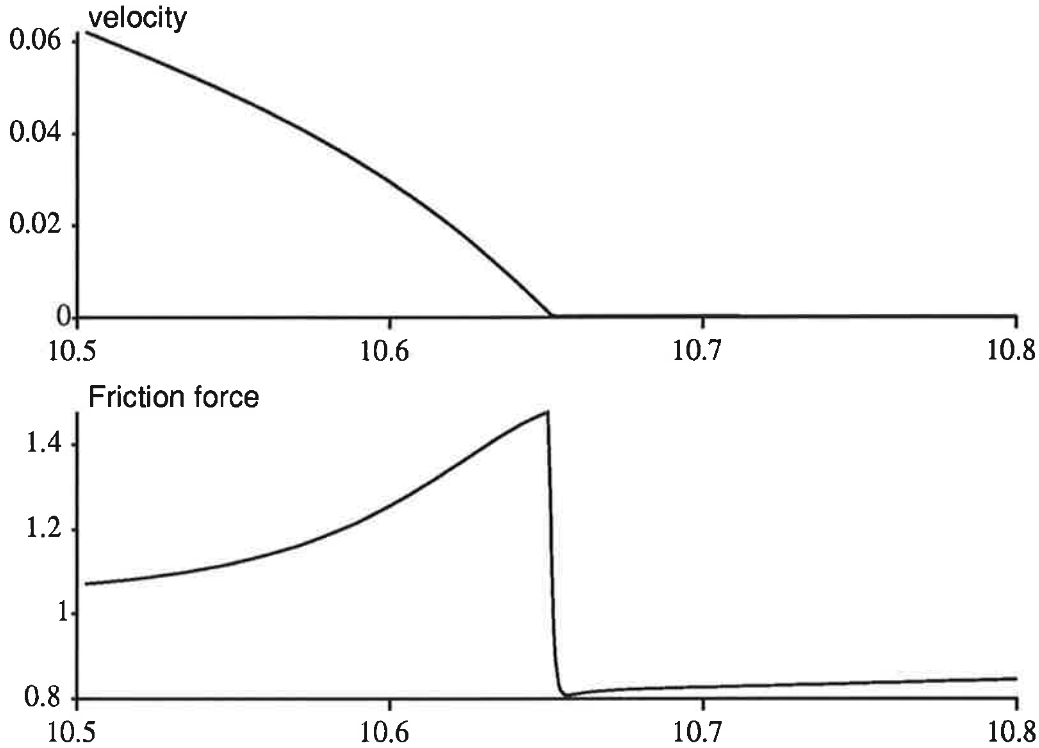

Figure 6. Velocity and friction force around time when mass get stuck.

velocity reversals show that it matches experimental data well. A detailed experimental study is in progress.

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