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Identification of Ship Steering Dynamics*

K. J. ÅSTRÖM† and C. G. KÄLLSTRÖM†

Maximum likelihood identification is applied to determine ship steering dynamics of a freighter and a tanker using free steering experiments on full scale ships.

Summary—System identification techniques are applied to determine ship steering dynamics. The parameters of a linear continuous time model are determined using discrete time measurements. The parameters are estimated using the maximum likelihood method. Applications to measurements on a freighter and a tanker are given.

1. INTRODUCTION

THE dynamics of ship steering is of interest when evaluating manœuvrability and it is a necessity when designing autopilots, navigation and steering systems. Ship dynamics is traditionally determined from first principles using Newton's laws of motion. The main difficulty is to determine the hydrodynamic forces acting on the hull. These are usually obtained from tests with scale models. Two types of tests are common, free steering experiments, where the rudder is changed and the motion observed, and captive tests, where the scale model is forced into a specific motion and the forces acting on the scale model are measured. The results obtained from scale model tests are scaled in order to obtain mathematical models that are relevant for full scale ships. Special dynamics experiments are sometimes made during delivery tests in order to evaluate the steering and manœuvring ability.

In this paper the problem of determining ship steering dynamics is approached from the point of view of system identification. Free steering experiments on full scale ships are considered. The precise nature of the rudder perturbation is not critical as long as the perturbations excite the ship motion sufficiently. The data obtained in the experiment are used to estimate the parameters in a mathematical model of the ship. The structure of the mathematical model is determined from the steering equations of the ship. The results indicate that it is possible to obtain models for ship steering dynamics using the proposed scheme.

The paper is organized as follows. A review of ship steering dynamics is given in Section 2. Disturbances are discussed in Section 3. The problem of identifiability is analysed in Section 4. The analysis shows which parameters can be determined from different types of measurements. In particular it is shown that all linear hydrodynamic derivatives cannot be determined from a free steering experiment. The parameters of a complete state model can, however, be determined provided that both the heading angle and the sway velocity are measured. The parameters of a transfer function model relating heading angle to rudder motion can, of course, always be determined. The parameter estimation problem is discussed in Section 5. This problem concerns the estimation of parameters of a stochastic differential equation using discrete time observations. The equations for the likelihood function are given in a form which is convenient for numerical computations. The estimation scheme developed allows for non-uniform sampling and missing measurements. The procedure has been tested on data obtained from experiments on a freighter, the M/S *Atlantic Song*, and on a tanker, the T/T *Sea Splendour*. The experiments are described in Section 6. Mathematical models having different structure were fitted to the experimental data. Both estimation of parameters in a

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black box model and parameters of models having the correct physical structure are discussed in Section 6. The results from the tanker experiment are particularly interesting, because a large manœuvre was made during the experiment. The results obtained from this experiment indicate that the nonlinear effects are important for a model which should be valid for such manœuvres.

2. SHIP DYNAMICS

The equations describing ship dynamics are well known. They are obtained from Newton's laws expressing conservation of linear and angular momentum. See refs. [1–7]. The main difficulty when deriving the equations is to describe the hydrodynamic forces acting on the hull. The forces are in general complicated functions of the ship's motion, i.e. the time history of the velocity, angular velocity and the rudder motion. They also depend on trim and draught. In shallow water and close to shore the forces will also depend on the topography.

If a ship is considered as a rigid body it has 6 degrees of freedom corresponding to translations in 3 directions and rotation around 3 axes. The equations of motion are conveniently expressed using a co-ordinate system fixed to the ship. The hydrodynamic forces are easy to describe in such a co-ordinate system because the symmetry of the hull can be exploited. Neglecting sensor and actuator dynamics, the ship can thus be modelled as a 12-order system. Additional dynamics are also introduced by the rudder servo. In many cases it has, however, been shown that the model can be simplified. It is customary at least for tankers and similar ships to neglect the coupling between the yaw motion and the pitch and roll motions. See ref. [4], p. 62. Since the yaw motion is often sufficient to discuss steering and autopilot design, the following treatment will be limited to this motion only.

Equations of motion

To describe the equations of motion the co-ordinate system fixed to the ship shown in Fig. 1 is used.

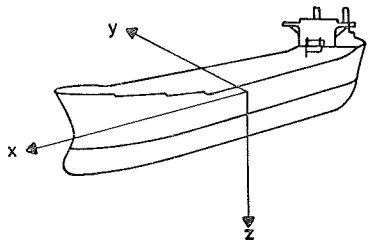

FIG. 1. Definition of co-ordinates fixed to the ship. Translation along the co-ordinate axes are called surge, sway and heave and rotation around the co-ordinate axes are called roll, pitch and yaw, respectively.

Let v be the projection of the ship's velocity on the y-axis, and r the component of the angular velocity on the z-axis. See Fig. 2. The projection of the ship's velocity on the x-axis is assumed to be constant and equal to u_0. The equations for the yaw motion are then given by the laws of conservation of linear and angular momentum, ref. [1], p. 21

$$\left.\begin{aligned} m\left(\frac{\mathrm{d}v}{\mathrm{d}t}+ru_0+x_G\frac{\mathrm{d}r}{\mathrm{d}t}\right) &= Y, \\ I_z\frac{\mathrm{d}r}{\mathrm{d}t}+mx_G\left(\frac{\mathrm{d}v}{\mathrm{d}t}+ru_0\right) &= N, \end{aligned}\right\} \tag{2.1}$$

where m is the mass of the ship, I_z its moment of inertia about the z-axis, Y the component of the hydrodynamic forces on the y-axis, N the z-component of the torque due to the hydrodynamic forces and x_G the x co-ordinate of the centre of mass. It is assumed that the centre of mass is located in the x–z plane. The hydrodynamic force Y and the torque N are complicated functions of the motion. It is usually assumed that

$$\left.\begin{aligned} Y &= Y(v, r, \delta, \dot{v}, \dot{r}), \\ N &= N(v, r, \delta, \dot{v}, \dot{r}), \end{aligned}\right\} \tag{2.2}$$

where δ is the rudder deflection. The functions Y and N will also depend on trim and draught. This is not introduced explicitly in the following which means that the results will hold for one loading condition only.

Stationary solutions

Assuming that the rudder is kept constant at the centre position the steady-state solution to the equations of motion is given by

$$\left.\begin{aligned} f(v,r) &= Y(v,r,0,0,0)-mru_0 = 0, \\ g(v,r) &= N(v,r,0,0,0)-mx_G ru_0 = 0. \end{aligned}\right\} \tag{2.3}$$

For a ship which is symmetric around its centre plane the force Y and the torque N will vanish for a motion with $v = 0$ and $r = 0$. The stationary solution to (2.3) is then given by $v = 0$ and $r = 0$. Depending on the properties of the functions Y and N there may, however, also be other solutions. These are easily obtained from a graph of the functions f and g. See Fig. 3. The case of one stationary point Q only as shown in Fig. 3(A) is most common. For very large tankers the case shown in Fig. 3(B) can, however, occur. In such a case the solution $v = 0$ and $r = 0$, point Q in Fig. 3(B), which corresponds to a straight line motion is unstable while the solutions $v = v_0$, $r = -r_0$ and $v = -v_0$, $r = r_0$, point P_1 and P_2 respectively in Fig. 3(B), which correspond to circular motions are stable. A ship with these properties

cannot be kept on a straight course with zero rudder. It will either go into a port yaw or into a starboard yaw and the motion will tend to a stationary circular motion.

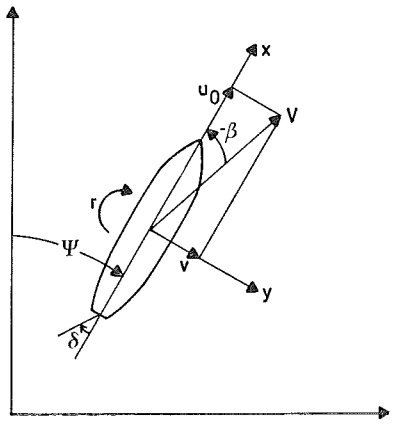

FIG. 2. Variables used to describe the linearized yaw motion of a ship. Notice that different conventions for the sign of δ are used in the literature.

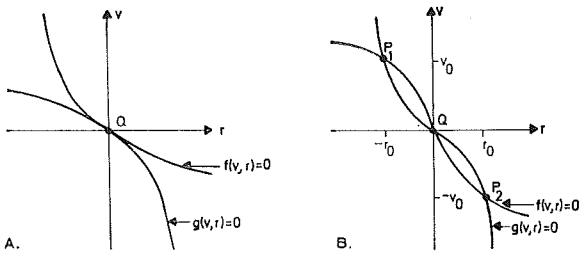

FIG. 3. The determination of the stationary motions as the intersections of the curves $f(v, r) = 0$ and $g(v, r) = 0$ is illustrated. In case A the curves intersect at the origin only but in case B there are 3 stationary solutions.

Normalization and linearization

It is customary to normalize the equations by introducing dimension free quantities. This can be done in several different ways. In the 'prime' system, which is most common, the length unit is the length of the ship, L, the time unit is L/V, where V is the ship's speed, and the mass unit is $\rho L^3/2$, where ρ is the mass density of water. The normalized variables are denoted by introducing a 'prime' on the non-normalized variables.

To linearize the equations it is necessary to introduce the partial derivatives of the force Y and the torque N. The partial derivative

$$Y_v = (\partial/\partial v)\, Y(v, r, \delta, \dot{v}, \dot{r}),$$

where the right-hand side is evaluated at arguments zero, is called a hydrodynamic derivative. The derivatives Y_r, Y_δ, $Y_{\dot{v}}$, $Y_{\dot{r}}$, N_v, N_r, N_δ, $N_{\dot{v}}$ and $N_{\dot{r}}$ are defined analogously.

Linearization of (2.1) around the stationary solution $v = 0$, $r = 0$ and normalization gives

$$\begin{bmatrix} m' - Y_{\dot{v}}' & m'x_G' - Y_{\dot{r}}' \\ m'x_G' - N_{\dot{v}}' & I_z' - N_{\dot{r}}' \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t'} \begin{bmatrix} v' \\ r' \end{bmatrix} = \begin{bmatrix} Y_v' & Y_r' - m' \\ N_v' & N_r' - m'x_G' \end{bmatrix} \begin{bmatrix} v' \\ r' \end{bmatrix} + \begin{bmatrix} Y_\delta' \\ N_\delta' \end{bmatrix} \delta, \quad (2.4)$$

where all parameters and variables are dimension free. Notice that it has been assumed that $u_0/V = 1$.

The derivatives $Y_{\dot{v}}'$ and $N_{\dot{r}}'$ are negative. Notice that they appear in the equations in the same way as the mass and the inertia. These terms are therefore sometimes called added mass and added inertia. In more accurate representations they will depend on the frequency of the excitation.

The hydrodynamic derivatives can be determined from hydrodynamic theory. See refs. [7] and [8]. They can also be determined from scale model tests. Descriptions of different model tests and comparisons with theory are given in refs. [6], [7] and [9]. Examples of normalized hydrodynamic derivatives for different ships are shown in Table 1.

State equations

The normalized equations of motion (2.4) are easily converted to standard state space notation by solving for the derivatives $\mathrm{d}v'/\mathrm{d}t'$ and $\mathrm{d}r'/\mathrm{d}t'$. This gives the following model for the yaw motion of the ship

$$\frac{\mathrm{d}}{\mathrm{d}t'} \begin{bmatrix} v' \\ r' \\ \psi \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v' \\ r' \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix} \delta, \quad (2.5)$$

where the heading ψ defined by $\mathrm{d}\psi/\mathrm{d}t' = r'$ has also been introduced as an extra variable. The heading ψ is shown in Fig. 2.

The parameters of (2.5) are remarkably similar for different ships as can be seen in Table 1. The numerical values will, however, change with trim and draught.

The linearized yaw motion of a ship can thus be described as a third-order dynamical system where the state variables can be chosen as

- v' the sway velocity, i.e. the component of the ship velocity on the y-axis in the co-ordinate system fixed to the ship,
- r' the ship's angular velocity about the z-axis,
- ψ the deviation in heading angle.

Other state variables are sometimes chosen. The angle of attack, i.e. β in Fig. 2, can be used instead of the sway velocity v. It is sometimes necessary to introduce additional dynamics to account for the

properties of the rudder servo. In the examples which follow this is represented simply as a time delay.

TABLE 1. EXAMPLES OF NORMALIZED HYDRODYNAMIC DERIVATIVES AND DIMENSION FREE PARAMETERS OF STATE MODELS FOR DIFFERENT SHIPS

Ship	Mine sweeper	Merchant ship Ser. 60, block coeff 0.70	Merchant ship Mariner class	Tanker 190 000 dwt	Tanker 210 000 dwt
Length L (m)	55	160	161	305	310
Speed V (m/s)	4.0	7.8	7.7	8.2	4.1
$m'-Y_{\dot{v}}'$		0.0229	0.01546	0.03139	0.03590
$m'x_G'-Y_{\dot{r}}'$		0.00039	0.00026	0	0.00113
$m'x_G'-N_{\dot{v}}'$		0.00048	0.00012	0	0.00089
$I_z'-N_{\dot{r}}'$		0.0012	0.00083	0.00192	0.00210
Y_v'		-0.0222	-0.01160	-0.01873	-0.02012
$Y_r'-m'$		-0.0076	-0.00526	-0.01169	-0.00880
N_v'		-0.00567	-0.00291	-0.00701	-0.00675
$N_r'-m'x_G'$		-0.0034	-0.00184	-0.00359	-0.00346
Y_δ'		0.00211	0.00278	0.00323	0.00490
N_δ'		-0.00105	-0.00133	-0.00152	-0.00245
a_{11}	-0.863	-0.895	-0.693	-0.597	-0.466
a_{12}	-0.482	-0.286	-0.304	-0.372	-0.196
a_{21}	-5.25	-4.37	-3.41	-3.66	-3.02
a_{22}	-2.45	-2.72	-2.17	-1.87	-1.56
b_{11}	0.175	0.108	0.207	0.103	0.176
b_{21}	-1.38	-0.92	-1.63	-0.80	-1.24
Ref.	[2]	[5]	[7], [9]	[10]	[11]

Input–output relations

Let the rudder angle δ be the input and the heading angle ψ the output. The input–output relation can then be represented by the transfer function

$$G_1(s) = \frac{b_1 s + b_2}{s(s^2 + a_1 s + a_2)} = \frac{K'(1+sT_3')}{s(1+sT_1')(1+sT_2')}, \quad (2.6)$$

where

$$\left.\begin{aligned} a_1 &= -a_{11} - a_{22}, \\ a_2 &= a_{11}a_{22} - a_{12}a_{21}, \\ b_1 &= b_{21}, \\ b_2 &= a_{21}b_{11} - a_{11}b_{21}. \end{aligned}\right\} \quad (2.7)$$

This model of ship dynamics has been used by several authors when investigating steering, manœuvrability and autopilots as discussed in refs. [3] and [12]. It has been proposed by Nomoto [13] that the transfer function (2.6) should be approximated with

$$G_1(s) = \frac{K'}{s(1+sT_s')}. \quad (2.8)$$

There are also several procedures proposed to determine K' and T_s' in (2.8) from experimental data. These parameters have also been determined for many different ships.

3. MODELS FOR DISTURBANCES

If available physical knowledge should be exploited when modelling the system, it is also desirable to try to describe the disturbances by physical models. The motion of a ship is influenced by variations in wind and waves. There may also be forces which are not accounted for in the simple models described in Section 2, for example forces due to coupling with pitch and roll motions and forces due to the fact that the ship is elastic. In this section it is attempted to give a simple model for the wind forces, assuming that the only effect of the wind is an additive force Y_w and an additive torque N_w in the model (2.1). The wind force will depend on the relative wind as seen from the ship. Let the relative wind vector w_{rel} have the angle θ with the nominal heading as shown in Fig. 4. The equations of motion (2.1) then become

$$\left.\begin{aligned} m\left(\frac{dv}{dt} + ru_0 + x_G\frac{dr}{dt}\right) &= Y + Y_w(\theta,\psi), \\ I_z\frac{dr}{dt} + mx_G\left(\frac{dv}{dt} + ru_0\right) &= N + N_w(\theta,\psi). \end{aligned}\right\} \quad (3.1)$$

If it is assumed that the wind has a constant component and the equations are linearized there will be terms proportional to ψ which are not present in

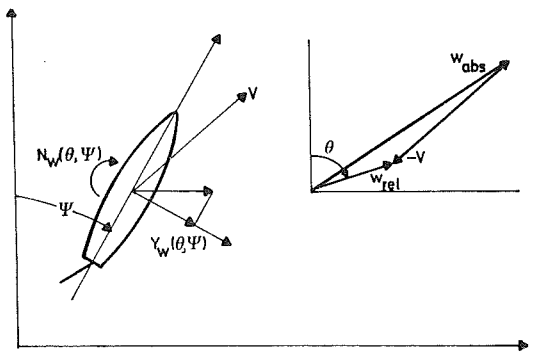

FIG. 4. The influence of wind on the ship motion.

the linear equation (2.4). The coefficients of these terms will depend on the velocity and the direction of the wind. There will also be additive terms in the linear model which depend on the fluctuations of the wind.

State equations

Reducing the linearized and normalized version of equation (3.1) to state form the following model is obtained

$$\frac{\mathrm{d}}{\mathrm{d}t'}\begin{bmatrix} v' \\ r' \\ \psi \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v' \\ r' \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix} \delta + \begin{bmatrix} e_1 \\ e_2 \\ 0 \end{bmatrix}, \quad (3.2)$$

where e_1 and e_2 are functions of the wind forces. Notice that it follows from the assumption that the effect of the wind only gives additive terms in (3.1) that the elements a_{11}, a_{12}, a_{21}, a_{22}, b_{11} and b_{21} remain the same as in the model (2.5) with no disturbances. The parameters a_{13} and a_{23} will, however, be highly dependent on wind velocity and direction. Conceivably the wind may also influence as an added mass and added inertia.

Transfer functions

It is of interest to see how the transfer function relating heading angle to rudder is changed when the effect of the wind is taken into account. The transfer function is easily obtained from (3.2). It is given by

$$G_1(s) = \frac{b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}, \quad (3.3)$$

where

$$\left.\begin{aligned} a_1 &= -a_{11} - a_{22}, \\ a_2 &= a_{11}a_{22} - a_{12}a_{21} - a_{23}, \\ a_3 &= a_{11}a_{23} - a_{13}a_{21}, \\ b_2 &= b_{21}, \\ b_3 &= a_{21}b_{11} - a_{11}b_{21}. \end{aligned}\right\} \quad (3.4)$$

Notice that in the presence of wind the transfer function relating the heading angle to the rudder will not necessarily contain an integrator.

Turbulence spectra

Much theoretical and experimental work has been devoted to the description of wind fluctuations. It has been found that at least at high altitudes the wind fluctuations can be described as homogeneous isotropic turbulence. See refs. [14] and [15]. It has been found that the lateral component can be described by the spectral density

$$\Phi(\omega) = \frac{\sigma^2 L}{2\pi} \frac{1 + 3(\omega L)^2}{[1 + (\omega L)^2]^2} \quad (3.5)$$

and the longitudinal component by the density

$$\Phi(\omega) = \frac{\sigma^2 L}{\pi} \frac{1}{1 + (\omega L)^2}, \quad (3.6)$$

where σ is the intensity, L is the scale of the turbulence and ω is the inverse wave length in metres. The scale of the turbulence is of the order of 150–1500 m at high altitudes [16].

The nature of the turbulence close to the ground differs from that at high altitudes. Equations (3.5) and (3.6) can be considered as crude approximations but the scale and the intensity will vary with height and the properties of the ground. A number of measurements have been carried out. See refs. [17–19]. The experimental data indicates that the scale L is approximately proportional to altitude. The formula

$$L = 0{\cdot}9h$$

is proposed as an approximation valid up to $h = 300$ m in ref. [15].

The ships to be discussed in Section 6 have lengths of 200–400 m. Their time constants are of the order of 20 and 200 sec. The velocities are about 10 m/sec. Both the ratio of the ship's length to the turbulence scale and the ratio of the distance travelled in a time constant to the turbulence scale is much larger than one. In the model (3.2) it is thus not unreasonable to assume that the disturbances e_1 and e_2 are realizations of white noise.

4. IDENTIFIABILITY

In Section 2 three linear models were given for the dynamics describing the yaw motion of a ship, the equations of motion (2.4), the state model (2.5) and the transfer function model (2.6). The equations of motion (2.4) give a model with 13 parameters m', x_G', I_z' and 10 hydrodynamic derivatives. The state model (2.5) has 6 unknown parameters a_{11}, a_{12}, a_{21}, a_{22}, b_{11} and b_{21} and the transfer function (2.6) is characterized by 4 parameters. Before it is attempted to estimate the parameters of these models, the problem of identifiability will be analysed. See ref. [20]. It will thus be investigated if the parameters of the models can be determined from the data of an input–output experiment. To be precise a parametric model of a system is *parameter identifiable* if the parameters can be determined uniquely from the input–output relation.

The transfer function model

The input–output pairs rudder angle, heading angle will now be considered. It is obvious that the transfer function model (2.6) with parameters a_1, a_2, b_1 and b_2 is an identifiable model except in the case when there is a pole zero cancellation.

The state model

The state model (2.5) is not identifiable since it follows from equations (2.7) that there is a 2-dimensional family of parameters that will correspond to the same input–output relation. Notice,

however, that the parameter b_{21} can be determined uniquely.

It is thus not possible to determine the parameters of the state model (2.5) from data obtained from an experiment where the rudder angle is perturbed and the heading angle is observed. Assume, however, that the sway velocity v' is also observed in the same experiment. The transfer function relating the sway velocity v' to rudder angle δ is given by

$$G_2(s) = \frac{c_1 s + c_2}{s^2 + a_1 s + a_2} = \frac{K_v'(1+sT_v')}{(1+sT_1')(1+sT_2')}, \quad (4.1)$$

where

$$\left.\begin{aligned} c_1 &= b_{11}, \\ c_2 &= a_{12} b_{21} - a_{22} b_{11} \end{aligned}\right\} \quad (4.2)$$

and the parameters a_1 and a_2 are given by (2.7). By straightforward algebra it can be shown that the parameters a_{11}, a_{12}, a_{21}, a_{22}, b_{11} and b_{21} can be solved from equations (2.7) and (4.2) if at least

$$c_2 b_1 \neq c_1 b_2. \quad (4.3)$$

This condition is equivalent to the state model (2.5) being controllable. The state model (2.5) is thus identifiable under the condition (4.3) if the input–output pairs relating rudder angle to heading angle and sway velocity are available.

The equations of motion

Analysis of equation (2.4) shows that the mapping from the parameters to the input–output relation is not an injection. First, all parameters such that the quantities $m' - Y_{\dot{v}}'$, $m'x_G' - Y_{\dot{r}}'$, $m'x_G' - N_{\dot{v}}'$, $I_z' - N_{\dot{r}}'$, $Y_r' - m'$ and $N_r' - m'x_G'$ are constant will have the same input–output relation. Secondly, it is possible to multiply (2.4) with a non-singular matrix and still have the same input–output relation. We can thus conclude that the equation of motion is not identifiable no matter what input–output pairs are known.

Models with disturbances

It is possible to extend the analysis of identifiability to the models discussed in Section 3, where wind disturbances are included. It can be shown that the linearized model (3.1) is not identifiable. The state model (3.2) is identifiable if the transfer functions relating the rudder to the heading and the sway velocity are known and if the values of the parameters are such that the system is completely observable and completely controllable. The state model is, however, not identifiable if only the input–output relation between the rudder and the heading is known. The parameters of the transfer function (3.3) are also identifiable.

5. PARAMETER ESTIMATION

Having discussed the problem of identifiability, the parameter estimation problem will now be treated. This problem is extensively discussed in the literature. See e.g. the survey [20]. Both the estimation of parameters in a state model and the estimation of the parameters of a transfer function model can be formulated as a problem of determining the parameters in the stochastic differential equation

$$dx = Ax\,dt + Bu\,dt + dw. \quad (5.1)$$

It is assumed that the initial state is a gaussian vector with mean value m and covariance R_0 and that $\{w(t), 0 \leqslant t \leqslant \infty\}$ is a Wiener process with incremental covariance $R_1\,dt$, which is assumed independent of the initial state. Assume that an input signal has been applied to the system and that the output has been observed at discrete times $t_0, t_1, \ldots, t_N$ with a measuring device which can be characterized by

$$y(t_k) = Cx(t_k) + Du(t_k) + e(t_k), \; k = 0, 1, \ldots, N. \quad (5.2)$$

The measurement errors $\{e(t_k)\}$ are assumed to be independent and gaussian with zero mean and covariance R_2. It is furthermore assumed that the measurement errors are independent of $\{w(t), 0 \leqslant t \leqslant \infty\}$ and of the initial state.

The model (5.2) implies that the measuring instruments are such that they give an output signal which is the instantaneous value of a linear combination of the state variables. The errors of measurements taken at different times are independent. Equation (5.2) is a good model when the sensor dynamics are considerably faster than the system dynamics and the measurement errors of different sampling events are uncorrelated.

In the particular case of ship dynamics the model (5.2) is reasonable because the shortest time constant of interest and the sampling interval is about 5–60 sec. All sensors have dynamics with time constants shorter than 1 sec, and the measurement errors are about 0·1° in heading, 0·02°/sec in yaw rate and 0·01 m/sec in velocity.

Problem statement

It is thus assumed that an identifiable model (5.1) and (5.2) is given and the problem is to determine the identifiable parameters from observed input–output pairs. The parameters will be determined using the maximum likelihood method.

The likelihood function

To obtain the likelihood function, i.e. the joint probability density of the observed outputs assuming all parameters known, we introduce

$$y_{t_k} = \begin{bmatrix} y(t_0) \\ y(t_1) \\ \vdots \\ y(t_{k-1}) \\ y(t_k) \end{bmatrix}, \quad (5.3)$$

i.e. a vector consisting of all outputs observed up to and including time t_k.

Assume that the probability distribution of y_{t_k} has a density $p(y_{t_k})$. It then follows from the definition of conditional probabilities that

$$p(y_{t_k}) = p[y(t_k)\,|\,y_{t_{k-1}}]\,p(y_{t_{k-1}}). \tag{5.4}$$

Repeated use of this formula gives the following formula for the likelihood function

$$L = p(y_{t_k}) = p[y(t_k)\,|\,y_{t_{k-1}}]\,p[y(t_{k-1})\,|\,y_{t_{k-2}}] \ldots p[y(t_1)\,|\,y(t_0)]\,p[y(t_0)]. \tag{5.5}$$

The likelihood function can thus be conveniently written as a product of conditional densities.

In the particular case of a model described by (5.1) and (5.2) all random variables are gaussian and the conditional density is also gaussian. The logarithm of the likelihood function can then be written as

$$-\log L = \frac{1}{2}\sum_{k=0}^{N} \log\det R(t_k) + \frac{1}{2}\sum_{k=0}^{N} \varepsilon^{\mathrm{T}}(t_k)\,R^{-1}(t_k)\,\varepsilon(t_k) + \text{const}, \tag{5.6}$$

where

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k\,|\,t_{k-1}) \tag{5.7}$$

and $\hat{y}(t_k\,|\,t_{k-1})$ denotes the conditional mean of $y(t_k)$ given $y_{t_{k-1}}$ and $R(t_k)$ the conditional covariance. We have

$$\hat{y}(t_0\,|\,t_{-1}) = Cm + Du(t_0). \tag{5.8}$$

Also notice that $\{\varepsilon(t_k), k = 0, \ldots, N\}$ are the residuals or the innovations of the output process. The conditional mean $\hat{y}(t_k\,|\,t_{k-1})$ and the conditional covariance $R(t_k)$ are easily determined recursively through the Kalman–Bucy filtering theory. See e.g. ref. [21]. We have

$$\left.\begin{aligned}
\hat{y}(t_k\,|\,t_{k-1}) &= C\hat{x}(t_k\,|\,t_{k-1}) + Du(t_k),\\
\hat{x}(t_k\,|\,t_k) &= \hat{x}(t_k\,|\,t_{k-1}) + K(t_k)\,\varepsilon(t_k),\\
(\mathrm{d}/\mathrm{d}t)\,\hat{x}(t\,|\,t_k) &= A\hat{x}(t\,|\,t_k) + Bu(t),\\
&\qquad t_k \leqslant t \leqslant t_{k+1},\\
K(t_k) &= P(t_k\,|\,t_{k-1})\,C^{\mathrm{T}}\\
&\quad \times [R_2 + CP(t_k\,|\,t_{k-1})\,C^{\mathrm{T}}]^{-1},\\
P(t_k\,|\,t_k) &= P(t_k\,|\,t_{k-1})\\
&\quad - K(t_k)\,CP(t_k\,|\,t_{k-1}),\\
(\mathrm{d}/\mathrm{d}t)\,P(t\,|\,t_k) &= AP(t\,|\,t_k) + P(t\,|\,t_k)\,A^{\mathrm{T}} + R_1,\\
&\qquad t_k \leqslant t \leqslant t_{k+1},\\
R(t_k) &= R_2 + CP(t_k\,|\,t_{k-1})\,C^{\mathrm{T}}.
\end{aligned}\right\} \tag{5.9}$$

The computation of the likelihood function is thus easily done recursively. A description of the program LISPID, which performs the calculations, is given in ref. [22]. This program is a general identification program for linear systems with many different options. The program which is written in FORTRAN consists of 52 subroutines. Including comments the program size is 9200 statements. The program without any data storage requires a core of 64 k cells on the UNIVAC 1108, if no segmentation is used. It requires a core of 25 k cells using segmentation and overlays. Additional memory space to store data is necessary.

Numerical aspects

The maximum of the likelihood function is found by an optimization routine. It is, however, extremely tedious to compute the gradient analytically, so only optimization techniques using the values of the loss function have been tried.

Two different algorithms have proved to be the most suitable for this kind of problem. In the first one the gradient is computed numerically using finite differences. Then a quasi-newton method is applied to find the optimum as described in ref. [23]. The other algorithm does not use numerical gradients, but gets information about the loss function by a special search pattern as discussed in ref. [24].

The actual computations are far from straightforward, and although a fast computer, a UNIVAC 1108, is used, the execution times often become rather long, especially when the observations are not equally spaced in time. The execution time also depends on the complexity of the model, the number of measurements, the number of parameters to be estimated and the accuracy of the initial parameter values.

6. EXPERIMENTS

Two sets of experiments will be considered in this paper. The first experiment was carried out on a freighter, the M/S *Atlantic Song*, using fairly primitive equipment including manual generation of test signals and manual reading of the heading angle. The idea of making such experiments was inspired by the early work of Garde and Persson [25]. The second experiment was performed on a tanker, the T/T *Sea Splendour*, using fairly sophisticated equipment including rate gyro, doppler log and an on-line process computer. The experiments are described in more detail in the following.

The Atlantic Song *experiment*

The *Atlantic Song* is a freighter of the Wallenius' Lines. It is 197 m long, it weighs 15,000 tons and has a maximum speed of 21 knots. The experiment was made in 1969, off the west coast of Denmark. The wind was about 8 Beaufort (17–20 m/sec, fresh gale) and the wave height 3·5–4 m. The

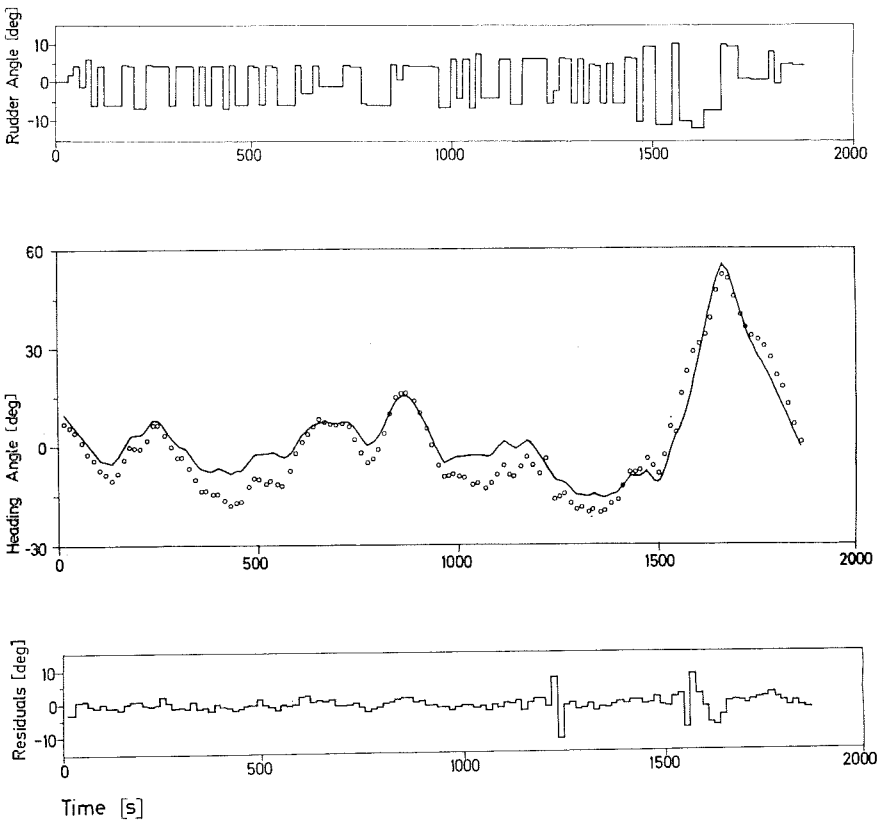

FIG. 5. Result of identification of a second-order model with the structure (6.1) to the *Atlantic Song* data (heading measurements dots, model full line). The mean values are subtracted and the output is shifted one sampling interval.

sight was poor due to a heavy snowfall. The ship had a luffing tendency, and a wind gust forced a port yaw. The impact of the waves on the bow induced sudden starboard yaws. The experiment, which lasted for about half an hour, was performed by 2 students. The speed was 18·5 knots at the beginning of the experiment and was reduced to 18 knots at the end of the experiment due to the rudder motions. A sampling interval of 15 sec was chosen based on prior knowledge of the dynamics. In the experiment the rudder angle was perturbed, and the heading angle was observed. One student was the co-ordinator during the experiment. He was standing on the bridge together with the captain. The other student was at the rudder servo in the machine room. At each sampling event the heading angle was read from the gyro compass. A command to change the rudder angle was given to the helmsman and the rudder angle was observed. The results were recorded in a table. The input was chosen as 2 periods of a PRBS signal with a length of 64 sampling intervals. The peak to peak variation was about 10°. The signal was changed somewhat to keep a reasonable course.

The input–output data recorded are shown in Fig. 5. The data have partly been analysed before. See refs. [26, 27]. A more detailed description of the results shown in this paper is given in ref. [22].

As a first attempt to analyse the data the discrete time model

$$\begin{aligned} y(t)+a_1 y(t-1)+\ldots+a_n y(t-n) \\ = b_1 u(t-1)+\ldots+b_n u(t-n) \\ +\lambda[e(t)+c_1 e(t-1)+\ldots+c_n e(t-n)] \end{aligned} \quad (6.1)$$

was fitted to the data by the maximum likelihood method. The estimation of the parameters of this model can be simplified significantly, as is described in Åström and Bohlin [28]. An interactive program IDPAC, which performs this, is described in ref. [29]. This program minimizes the loss function

$$V_n = \frac{1}{2}\sum_1^N e^2(t)$$

which is equivalent to maximizing the likelihood function [28]. Repeating the identification for different values of n the following loss functions were obtained: $V_1 = 668{\cdot}4$, $V_2 = 294{\cdot}5$ and $V_3 = 286{\cdot}9$. An attempt to determine the order by an F-test gives the test quantities $F(1\to 2) = 50{\cdot}4$ and $F(2\to 3) = 1{\cdot}0$. This indicates that a second-order model is appropriate. Akaike has in ref. [30] proposed that the order should be selected so that the

criterion

$$\mathrm{AIC} = -2\log\hat{L} + 2\nu$$

is minimal where $\hat{L}$ denotes the maximum value of the likelihood function and ν the number of estimated parameters. Akaike's criterion also gives a second-order model. The parameters are given below

$$\left.\begin{array}{ll} a_1 = -1{\cdot}64 \pm 0{\cdot}05, & c_1 = -0{\cdot}75 \pm 0{\cdot}10, \\ a_2 = 0{\cdot}66 \pm 0{\cdot}05, & c_2 = 0{\cdot}06 \pm 0{\cdot}10, \\ b_1 = -0{\cdot}11 \pm 0{\cdot}03, & V = 294{\cdot}5, \\ b_2 = -0{\cdot}19 \pm 0{\cdot}04, & \mathrm{AIC} = 560{\cdot}5. \end{array}\right\} \quad (6.2)$$

If initial values also are estimated, a second-order model is obtained as well. This model has the loss function $V = 274{\cdot}5$ and an AIC $= 555{\cdot}7$, which indicates that initial values should be estimated. The model parameters are not significantly different from the values given above.

In Fig. 5 are shown the input, the measured output, the model output and the residuals of the second-order model with estimated initial values. The residuals are far from normal. Figure 5 shows that the residuals have very large values at times close to 1250 and 1600 sec. These large residuals can be traced to bad data.

To avoid any difficulties by trying to replace the bad data points by interpolation or other fudging of the data, only the portion 0–1200 sec will be used in the following. Notice, however, that a straightforward ML estimation of the parameters of the discrete time model (6.1), which is very inexpensive to run, is very useful in order to check the measured data.

The following results are thus based on the first 80 input–output pairs. Initial states were estimated because this gave a significant reduction of the loss function and of the AIC. It was attempted to remove both levels and trends from the data. There were no significant changes in the parameters of the second order models obtained. The following parameters were obtained

$$\left.\begin{array}{ll} a_1 = -1{\cdot}60 \pm 0{\cdot}03, & c_1 = -0{\cdot}37 \pm 0{\cdot}14, \\ a_2 = 0{\cdot}61 \pm 0{\cdot}03, & c_2 = -0{\cdot}20 \pm 0{\cdot}11, \\ b_1 = -0{\cdot}161 \pm 0{\cdot}014, & V = 17{\cdot}96, \\ b_2 = -0{\cdot}285 \pm 0{\cdot}016, & \mathrm{AIC} = 178{\cdot}9. \end{array}\right\} \quad (6.3)$$

Notice the significant reduction in loss function as compared with (6.2).

There were some difficulties in estimating a third-order model. The algorithm had difficulties in converging, and the parameter c_3 had to be fixed to zero to obtain a well-conditioned information matrix. After some attempts the following model was obtained

$$\left.\begin{array}{ll} a_1 = -1{\cdot}05 \pm 0{\cdot}05, & c_1 = 0{\cdot}18 \pm 0{\cdot}13, \\ a_2 = -0{\cdot}34 \pm 0{\cdot}07, & c_2 = -0{\cdot}18 \pm 0{\cdot}13, \\ a_3 = 0{\cdot}41 \pm 0{\cdot}04, & V = 15{\cdot}88, \\ b_1 = -0{\cdot}15 \pm 0{\cdot}01, & \mathrm{AIC} = 175{\cdot}1. \\ b_2 = -0{\cdot}38 \pm 0{\cdot}01, & \\ b_3 = -0{\cdot}13 \pm 0{\cdot}02, & \end{array}\right\} \quad (6.4)$$

A comparison with the model (6.3) gives a test value of $F = 3{\cdot}01$. Using the F-test it is thus questionable if the model (6.4) is preferable. Akaike's test indicates that the model (6.4) is better than the model (6.3). The zeros of the polynomial $A(z)$ for the model are 0·95, 0·71 and −0·61. The zeros of the polynomial B are −0·4 and −2·0. The corresponding zeros for the model (6.3) are 0·97, 0·63 and −1·8 for the A and B polynomial respectively. The zero −0·61 of the polynomial A, which is approximately cancelled by the zero −0·4 of the polynomial B, corresponds to a model of the type

$$\zeta(t+1) = -0{\cdot}6\zeta(t) + e(t).$$

A first-order model with a negative pole is typical for a case where round-off noise occurs. If the covariances of the residuals and the cross covariances between the input and the residuals are analysed, it can be concluded that the second-order model is acceptable, but that the residuals are whiter for the third-order model.

To summarize, we find that it is difficult to distinguish between the second- and the third-order models. The major improvement by going to a third-order model is that the quantization of the heading angle is modelled. Since this is not our major concern, the second-order model is accepted. Notice that the polynomial $A(z)$ of the model (6.3) has a zero very close to $z = 1$. This is expected because the model should ideally contain an integrator, as indicated in Section 2. The straightforward fitting of a model (6.1) thus gives a model of the Nomoto type with a time constant of 29·4 sec and a gain of $-0{\cdot}055\ \mathrm{sec}^{-1}$. If a model of higher order is attempted, a pole on the negative real axis is obtained resulting from an attempt to model round-off errors, which occur due to the quantization of the data.

The first 80 input–output pairs of the *Atlantic Song* experiment have also been used to estimate the parameters of the transfer functions (2.6) and (2.8) using the program LISPID. No integrator was assumed because the wind was modelled as described in Section 3. The result of the identification of the second-order model is shown in Fig. 6. The autocorrelation function of residuals and the cross correlation function between rudder angles

and residuals are shown in Fig. 7. The obtained gain and time constant, when the wind influence is eliminated, are given in Table 2, where also the result from IDPAC is shown. The third-order model obtained is given in Table 3, where the wind influence is also eliminated.

The result of an F-test shows distinctly that a second-order model is appropriate to the data. Akaike's information criterion gives the same result.

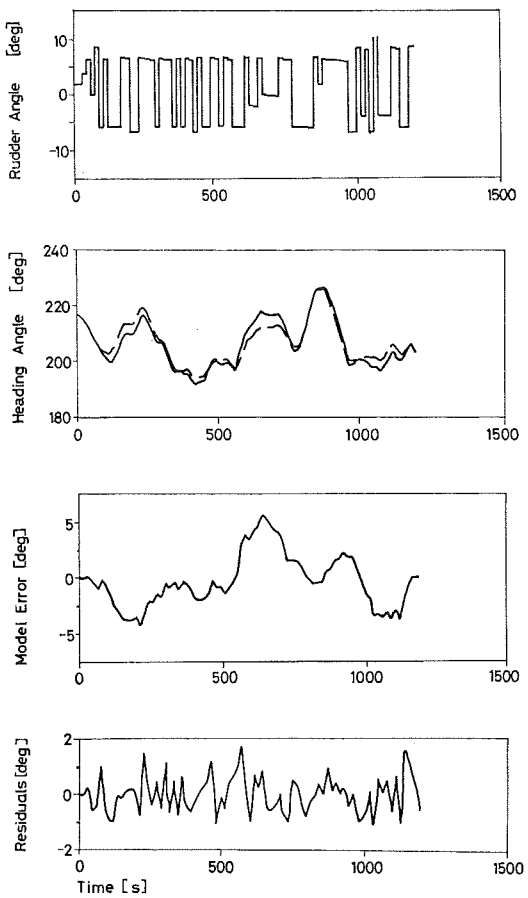

FIG. 6. Result of identification of a second-order transfer function (2.8) to the first 80 data points of the *Atlantic Song* experiment. The dashed heading is the model output, the full line is the measurements.

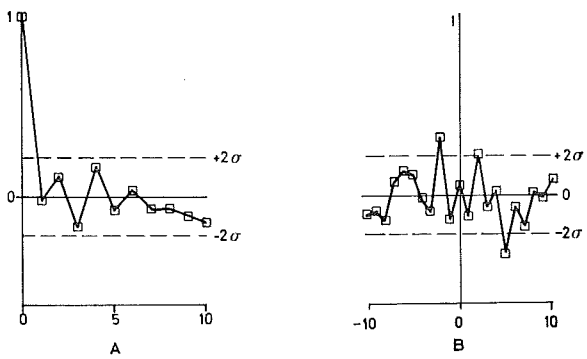

FIG. 7. The autocorrelation function of residuals (A) and the cross correlation function between rudder angles and residuals (B), where the residuals are obtained from Fig. 6.

TABLE 2. RESULT OF IDENTIFICATION OF A SECOND-ORDER MODEL TO THE FIRST 80 DATA POINTS OF THE *Atlantic Song* EXPERIMENT

		From LISPID	From IDPAC
K'		-1.62	-1.14
T_s'		1.49	1.42
K	1/s	-0.078	-0.055
T_s	s	30.9	29.4
τ	s	12.1	15.0 (fixed value)

The values of the transfer function parameters [cf. (2.8)] are given as normalized ('prime' system) and non-normalized. τ is the time lag between the rudder change and the sample event.

TABLE 3. RESULT OF IDENTIFICATION OF A THIRD-ORDER MODEL TO THE FIRST 80 DATA POINTS OF THE *Atlantic Song* EXPERIMENT

K'		-2.38
T_1'		8.32
T_2'		1.11
T_3'		4.61
K	1/s	-0.115
T_1	s	172.3
T_2	s	22.9
T_3	s	95.5
τ	s	12.0

The values of the transfer function parameters [cf. (2.6)] are given as normalized ('prime' system) and non-normalized. τ is the estimated time lag between the rudder change and the sample event.

The results thus show that the data from the experiment can be modelled well by a second-order model. There are very good agreements between the results obtained when fitting a discrete time model and when fitting a continuous time model. The decrease in loss function obtained when increasing the order of the continuous time model is very insignificant. The corresponding improvement in the discrete time model is more significant. The improvement corresponds to a more accurate modelling of the round-off errors. This indicates that if the experiments are performed using heading information only, it is important to have a good resolution of the measurement of heading angle.

The Sea Splendour *experiment*

The *Sea Splendour* is a tanker built for the Salén Group by Kockums Mekaniska Verkstads AB in

Malmö, Sweden. It is 329 m long, has a beam of 52 m and weighs 255,000 tons. The cargo capacity is 339,000 m³, and the maximum speed is 16 knots.

The experiments were performed in 1972 north of Stavanger, Norway. The course was 140° and the wind was blowing on starboard with a speed of about 10 m/sec. The tanker had a ballast corresponding to 50 per cent of the full capacity. The forward draught was about 10 m, and the aft draught was about 13 m during the experiment.

During the experiment, which lasted for about 50 min, the speed was between 15·5 and 16·0 knots. In the middle of the experiment the course was changed by 20°. The sampling interval was 30 sec. The input signal was chosen as a PRBS signal, but it was necessary to make many manual changes to avoid large deviations from the desired course. At every sample instant the process computer measured rudder deflection, course, yaw rate, forward velocity, bow and stern sway velocities, and printed them on a typewriter. The course was measured by a gyro compass, the yaw rate by a rate gyro, and the velocities by a doppler log. The input–output data obtained during the experiment is shown in Fig. 8. Eight consecutive readings were missed during the experiment.

Attempts to estimate the linear hydrodynamic derivatives Y_v', $Y_r'-m'$, N_v', $N_r'-m'x_G'$, Y_δ' and N_δ' of the model (2.4) failed when all data points were used. This was probably due to nonlinear effects during the course change in the middle of the experiment. Therefore, the experiment was divided into two parts, one before the yaw and one after. The linear hydrodynamic derivatives of the model (2.4) were then estimated using the second part of the data set. Parameters for the influence of the wind, instrument biases, constant terms in the force and torque equation, covariances of disturbances and measurements and initial states were also estimated. The complete model contained 23 unknown parameters. The parametrization is given in the appendix together with the estimates. The model obtained was simulated with the rudder angles from the first part of the data set as input signal. Results of both the identification and the simulation are shown in Fig. 8. The parameter values obtained from the identification are given in Tables 4, 5 and in the appendix, Table 6. The values of the acceleration derivatives and the initial estimates of the other derivatives are adjusted values from model tests with a similar tanker. A more detailed description of the results is given in ref. [22].

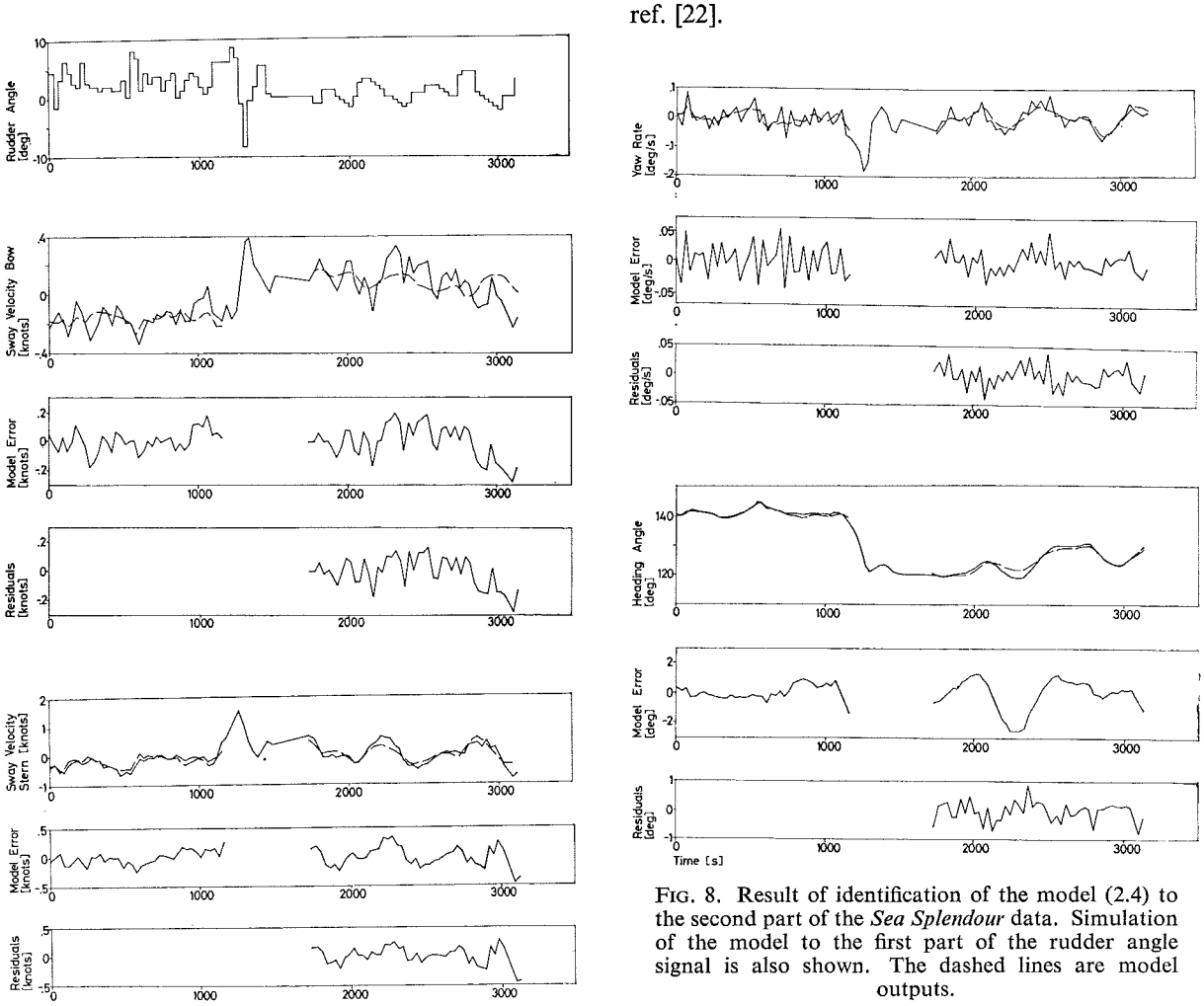

FIG. 8. Result of identification of the model (2.4) to the second part of the *Sea Splendour* data. Simulation of the model to the first part of the rudder angle signal is also shown. The dashed lines are model outputs.

TABLE 4. ESTIMATED HYDRODYNAMIC DERIVATIVES FROM IDENTIFICATION OF THE MODEL (2.4) TO THE SECOND PART OF THE *Sea Splendour* DATA (see also appendix)

$m'-Y_{\dot{v}}'$	0.0156	
$m'x_G'-Y_{\dot{r}}'$	0	
$m'x_G'-N_{\dot{v}}'$	0	
$I_z'-N_{\dot{r}}'$	0.000963	
	Initial estimates	Final estimates
Y_v'	-0.0113	-0.0146
$Y_r'-m'$	-0.00482	-0.00565
N_v'	-0.00183	-0.00168
$N_r'-m'x_G'$	-0.00238	-0.00115
Y_δ'	0.00181	0.00183
N_δ'	-0.00086	-0.00060

TABLE 5. VALUES OF NORMALIZED ('PRIME' SYSTEM) AND NON-NORMALIZED TRANSFER FUNCTION PARAMETERS [cf. (2.6) and (4.1)] COMPUTED FROM INITIAL AND FINAL ESTIMATES IN Table 4

		Initial estimates	Final estimates
K'		-0.72	-1.63
K_v'		0.47	0.76
T_1'		2.30	3.87
T_2'		0.36	0.54
T_3'		1.03	0.79
T_v'		0.21	0.32
K	1/s	-0.018	-0.040
K_v	m/s	3.8	6.2
T_1	s	92.5	155.6
T_2	s	14.6	21.5
T_3	s	41.5	31.9
T_v	s	8.3	12.9

7. CONCLUSIONS

It has been shown that ship steering dynamics can be determined from free steering experiments on full scale ships using system identification techniques. The results confirm that the transfer function relating heading to rudder angle can be determined by measurements of these 2 variables only. However, if the hydrodynamic derivatives of a linear model are desired, then it is necessary to have *a priori* knowledge or an estimate of the acceleration derivatives, and information on the sway velocity. The proposed identification method makes it possible to get insight into the nature of the disturbances. The modelling not only of measurement noise, but also of state noise has proved to be important to obtain accurate parameter estimates. The effect of the wind is assumed only to give an additive force and an additive torque. Conceivably the wind may also influence as an added mass and added inertia, but this effect is probably small. The residuals obtained from the identification procedure have proved to be extremely useful to check measured data. They also give an opportunity to judge if the assumed model is feasible.

Since free steering experiments can be performed both on scale models and on full scale ships the method offers a possibility to analyse effects due to scaling. It also makes it possible to check if hydrodynamic derivatives determined under quasi-stationary conditions can be used to describe the transient motion. Since the technique can be applied without significantly disturbing the mission of the ship, experimental data can be gathered at reasonable costs although this has partially to be paid for in terms of computer time. It is also possible to use the same identification techniques on closed loop experiments as discussed in ref. [31], where the deviation from the desired heading can be kept very small. The technique also gives possibilities to get insight into the variation of dynamical properties with loading and speed. For example, the influence of the water depth can be explored. It would also be interesting to analyse data from experiments with the same ship made under different weather and sea conditions. It is also important to check models obtained from one set of experiments against data from another experiment. Such extensions are now being carried out.

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TABLE 6. PARAMETER VALUES FROM IDENTIFICATION TO THE SECOND PART OF THE *Sea Splendour* DATA (see also Tables 4 and 5)

Linear hydrodynamic derivatives	Y_v'	(θ_1)	-0.0146
	$Y_r'-m'$	(θ_2)	-0.00565
	N_v'	(θ_3)	-0.00168
	$N_r'-m'x_G'$	(θ_4)	-0.00115
	Y_δ'	(θ_5)	0.00183
	N_δ'	(θ_6)	-0.00060
Wind parameters	θ_7		$-4.4\cdot10^{-2}$
	θ_8		$7.8\cdot10^{-2}$
Bias parameters	θ_9		$1.5\cdot10^{-2}$
	θ_{10}		$1.4\cdot10^{-4}$
	θ_{11}	knots	$8.6\cdot10^{-1}$
	θ_{12}	knots	$8.1\cdot10^{-1}$
	θ_{13}	deg/s	$-5.2\cdot10^{-4}$
Covariance matrices	$R_1(1,1)$		$3.3\cdot10^{-2}$
	$R_1(1,2)$		$1.3\cdot10^{-2}$
	$R_1(2,2)$		$5.4\cdot10^{-3}$
	$R_2(1,1)$	$(\text{knots})^2$	$1.7\cdot10^{-7}$
	$R_2(2,2)$	$(\text{knots})^2$	$1.5\cdot10^{-3}$
	$R_2(3,3)$	$(\text{deg/s})^2$	$1.0\cdot10^{-5}$
Initial state	θ_{20}	knots	$-5.3\cdot10^{-1}$
	θ_{21}	deg/s	$-4.4\cdot10^{-2}$
	θ_{22}	deg	121.0
Time lag	τ	s	6.2

APPENDIX—COMPLETE MODEL USED FOR PARAMETER ESTIMATION

The complete model, which was used for the *Sea Splendour* identification, is given below [cf. (2.4)]. It contains 23 unknown parameters, θ_1–θ_{23}. α_1 and α_2 are conversion factors from degrees to radians and from m/sec to knots, respectively. V, L, L_1 and L_2 were assigned the values 8·2 m/sec, 329·2, 147·6 and 131·1 m. The values provided of the acceleration derivatives are given in Table 4. $U1$ is an artificial input signal which consists of only figure ones to make it possible to estimate the bias parameters θ_9–θ_{13}. The estimated values of the parameters are given in Table 6.

$$\begin{bmatrix} \frac{L}{V^2}(m'-Y_{\dot v}') & \frac{L^2}{V^2}(m'x_G'-Y_{\dot r}') & 0 \\ \frac{L}{V^2}(m'x_G'-N_{\dot v}') & \frac{L^2}{V^2}(I_z'-N_{\dot r}') & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{d}v \\ \mathrm{d}r \\ \mathrm{d}\psi \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{V}\theta_1 & \frac{L}{V}\theta_2 & \theta_7 \\ \frac{1}{V}\theta_3 & \frac{L}{V}\theta_4 & \theta_7\theta_8 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ r(t) \\ \psi(t) \end{bmatrix} \mathrm{d}t$$

$$+ \begin{bmatrix} \alpha_1\theta_5 & \theta_9 \\ -\alpha_1\theta_6 & \theta_{10} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t-\tau) \\ U1 \end{bmatrix} \mathrm{d}t + \mathrm{d}w,$$

$$\begin{bmatrix} v_1(t_k) \\ v_2(t_k) \\ \bar r(t_k) \\ \bar\psi(t_k) \end{bmatrix} = \begin{bmatrix} \alpha_2 & L_1\alpha_2 & 0 \\ \alpha_2 & -L_2\alpha_2 & 0 \\ 0 & 1/\alpha_1 & 0 \\ 0 & 0 & 1/\alpha_1 \end{bmatrix} \begin{bmatrix} v(t_k) \\ r(t_k) \\ \psi(t_k) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & \theta_{11} \\ 0 & \theta_{12} \\ 0 & \theta_{13} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t_k-\tau) \\ U1 \end{bmatrix} + e(t_k),$$

$$k = 0, 1, \ldots, N,$$

$$R_1 = \begin{bmatrix} |\theta_{14}| & \sqrt{(|\theta_{14}||\theta_{15}|)}\sin\theta_{16} & 0 \\ \sqrt{(|\theta_{14}||\theta_{15}|)}\sin\theta_{16} & |\theta_{15}| & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} |\theta_{17}| & 0 & 0 & 0 \\ 0 & |\theta_{18}| & 0 & 0 \\ 0 & 0 & |\theta_{19}| & 0 \\ 0 & 0 & 0 & 0{\cdot}01 \end{bmatrix},$$

$$\begin{bmatrix} v(t_0) \\ r(t_0) \\ \psi(t_0) \end{bmatrix} = \begin{bmatrix} \theta_{20}/\alpha_2 \\ \alpha_1\theta_{21} \\ \alpha_1\theta_{22} \end{bmatrix},$$

$\tau = T|\sin\theta_{23}|$, where T is the sampling interval (30 sec).