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Holmqvist, Anders; Magnusson, Fredrik; Stenström, Stig

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Scale-up Analysis of Continuous Cross-flow Atomic Layer Deposition Reactor Designs

A. Holmqvist^{a,*}, F. Magnusson^b, S. Stenström^a

Abstract

This paper presents the development of a non-dimensional model of a continuous cross-flow atomic layer deposition (ALD) reactor with temporally separated precursor pulsing and a structured model-based methodology for scaling up the substrate dimensions. The model incorporates an ALD gas-surface reaction kinetic mechanism for the deposition of thin ZnO films from Zn(C₂H₅)₂ and H₂O precursors that was experimentally validated in our previous work (Holmqvist et al., 2012, 2013a). In order to maintain dynamic similarity, a scaling analysis was applied based on the dimensionless numbers, appearing in non-dimensionalized momentum and species mass conservation equations, that describe the convective laminar flow, mass transfer and heterogeneous reaction. The impact on these dimensionless numbers and, more importantly, the impact on the limit-cycle deposition rate and its relative uniformity was thoroughly investigated when linearly scaling up the substrate dimensions. In the scaleup procedure, the limit-cycle precursor utilization was maximized by means of dynamic optimization, while ensuring that identical deposition profiles were obtained in the scaled-up system. The results presented here demonstrated that the maximum precursor yields were promoted at higher substrate dimen-

^a Department of Chemical Engineering, Lund University, P.O. Box 124, SE-221 00 Lund, Sweden

^b Department of Automatic Control, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden

^{*}Corresponding author. Tel.: +46 46 222 4925; fax: +46 46 222 4526 Email addresses: anders.holmqvist@chemeng.lth.se (A. Holmqvist), fredrik.magnusson@control.lth.se (F. Magnusson), stig.stenstrom@chemeng.lth.se (S. Stenström)

sions. Limit-cycle dynamic solutions to the non-dimensionalized model, computed with a collocation discretization in time, revealed that it is a combination of the degree of precursor depletion in the flow direction and the magnitude of the pressure drop across the reactor chamber that governs the extent of the deposition profile non-uniformity. A key finding of this study is the identification of optimal scaling rules for maximizing precursor utilization in the scaled-up system while maintaining fixed absolute growth rate and its relative uniformity. Keywords: Atomic layer deposition, Scale-up analysis, Dynamic optimization, Mathematical modeling, Limit-cycle kinetics, Numerical analysis

1. Introduction

- Atomic layer deposition (ALD) is a gas-phase deposition process that can
- produce conformal thin films with controlled uniform thickness in the nanome-
- ter range (George, 2010). This attribute is inherent to the sequential self-
- 5 terminating (Puurunen, 2005) ALD gas-surface reactions (Masel, 1996), in
- 6 which the non-overlapping alternating injection of chemical precursor species
- separated by intermediate purge steps prevents reactions in the gas phase (Mi-
- 8 ikkulainen et al., 2013). The deposition process depends strongly on two char-
- acteristic time scales (see, for example, (Adomaitis, 2010; Granneman et al.,
- 2007)): the time scale of underlying reactor-scale mass transport (Aarik et al.,
- 2006; Jur and Parsons, 2011; Mousa et al., 2012), and the time scale of the het-
- erogeneous gas-surface reactions (Ritala and Leskelä, 2002; Yanguas-Gil and
- 13 Elam, 2014).
- 14 Conventional thermal ALD is a special modification of the chemical vapor
- deposition (CVD) technique. One of the essential advantages of ALD is that
- its self-terminating nature enables uniform coating of substrates with large sur-
- face areas (Levy and Nelson, 2012; Sundaram et al., 2010), and it is thus easier
- to scale up the process of ALD than that of CVD (Yanguas-Gil and Elam,
- 19 2012). In this study, the geometrical scale up of the substrate dimensions in
- 20 cross-flow, low-volume ALD reactor designs with temporal precursor pulsing was

investigated. Such reactor designs are of major interest for the equipment used to manufacture substrates of large surface area (Henn-Lecordier et al., 2011). Such substrates are subject to stringent uniformity constraints (Cleveland et al., 2012), where the fundamental requirement for growth uniformity is the attainment of surface saturation. This, in turn, requires adequate precursor delivery (Knoops et al., 2011; Ylilammi, 1995), optimal process operating conditions, and optimal reactor design (Elers et al., 2006). Non-uniform film thickness profiles in cross-flow ALD reactor designs can result from precursor depletion, which can be a concern for precursors with a low vapor pressure (Granneman et al., 2007).

Several studies on scaling up horizontal reactor designs for CVD have been 31 published, (see, for example, Dam et al. (2007) and the references cited therein). 32 However, a model-based study of dimensionless numbers with respect to scaling 33 up has never been rigorously carried out for ALD growth. The overall objective 34 of the present study, therefore, was to develop a model-based method for the geometrical scale up of the substrate dimension in cross-flow reactor designs that 36 use temporal precursor pulsing. The scale-up procedure provides a fixed abso-37 lute growth rate and relative uniformity while maximizing precursor utilization. This methodology was applied to an experimentally validated mechanism of the 39 ALD gas-surface reactions for the deposition of thin ZnO films from Zn(C₂H₅)₂ and H₂O precursors (Holmqvist et al., 2012, 2013a). The study presented here had three main objectives:

- i) To develop a structured model-based method for the geometrical scaling up
 of the substrate dimensions in continuous cross-flow ALD reactor designs,
 and to identify the scaling guidelines that are best suited to maintaining
 the limit-cycle deposition rate and its relative uniformity in the scaled-up
 system.
- ii) To investigate dynamic similarity by deriving the fully coupled compressible flow equations, along with their boundary conditions and initial conditions, of the developed reactor model in its non-dimensional form.

- 51 iii) To formulate and solve a dynamic optimization problem in order to opti-52 mize precursor utilization, subject to terminal constraints of the limit-cycle 53 deposition rate and its relative uniformity.
- This paper is organized as follows: Section 2 outlines the mechanism of the ZnO ALD gas—surface reactions. Section 3 derives the non-dimensional ALD reactor model and identifies the associated non-dimensional variables that appear. Section 4 describes the scale-up strategies and formulates the dynamic optimization problem, while Section 5 outlines the modeling and optimization framework. Section 6 presents the results from the scale-up analysis, and Section 7 presents concluding remarks.

61 1.1. Previous Modeling

Our previous work (Holmqvist et al., 2012, 2013a,b), which presents a mechanistic model of the continuous cross-flow ALD reactor system F-120 manufac-63 tured by ASM Microchemistry Ltd. (Suntola, 1992), is particularly relevant to the present article. The work of Yanguas-Gil and Elam (2012) on what is known as the "SMART" model (where "SMART" is an acronym for "Simple Model for Atomic layer deposition precursor Reaction and Transport") for the analysis of transport-reaction processes in a tubular, laminar flow reactor is also highly 68 relevant. The non-dimensional model presented in the present study is founded 69 on the dimensional model that we have previously developed (Holmqvist et al., 2012, 2013a,b), and the approximation of fully developed laminar channel flow 71 defined in a one-dimensional computational domain. Moreover, the model presented here comprises fully coupled compressible equations for the conservation 73 of mass, momentum and individual gas-phase species, while the SMART model, 74 in contrast, assumes incompressible flow. The application range of the model is expanded in this way to include the region in which the pressure of the precursor is significant, relative to that of the carrier gas, which is necessary in the scale-up analysis.

2. ALD Surface Reaction Kinetics

The predictive capability of the developed physically-based model to decou-80 ple the effects of precursor partial pressure, exposure times, process manipulated 81 variables, and the dynamics of each exposure period on the limit-cycle spatially 82 dependent substrate film thickness profile is essential for the purposes of this investigation. For this reason, an experimentally validated gas-surface reaction mechanism for the deposition of ZnO films from $\rm Zn(C_2H_5)_2$ and $\rm H_2O$ precursors 85 was incorporated into the model developed during the present study, in order to obtain as accurate a model as possible. The experimental investigation was conducted in the F-120 reactor system from ASM Microchemistry Ltd. (Suntola, 1992) and the estimated rate coefficients from ex situ X-ray reflectivity (XRR) thickness profile measurements are reported in Holmqvist et al. (2013a), 90 which contains also details of the film characterization and data preprocessing. Consider a simple ZnO ALD gas-surface reaction kinetic mechanism, with 92 the overall reaction stoichiometry given by:

$$\operatorname{Zn}(C_2H_5)_2 + \operatorname{H}_2O \longrightarrow \operatorname{ZnO} + 2C_2H_6$$
 (R1)

and only encompassing the primary irreversible and sequential elementary gas surface reactions for the $\rm Zn(C_2H_5)_2$ and $\rm H_2O$ precursors on a normally hydroxylated surface. Such a reactions were defined in Holmqvist et al. (2013a) as:

$$\nu(-OH)\langle s \rangle + Zn(C_2H_5)_2\langle g \rangle \xrightarrow{k_1^{\text{fwd}}} (-O-)_{\nu}Zn(C_2H_5)_{2-\nu}\langle s \rangle + \text{(R2a)}$$

$$\nu C_2H_6\langle g \rangle$$

$$(-O-)_{\nu} Zn(C_2H_5)_{2-\nu} \langle s \rangle + H_2O \langle g \rangle \xrightarrow{k_2^{\text{fwd}}} (-O-)Zn(-OH)_{\nu} \langle s \rangle +$$

$$(R2b)$$

$$(2-\nu)C_2H_6 \langle g \rangle$$

Here $\langle s \rangle$ and $\langle g \rangle$ denote surface and gaseous species, respectively, and $\nu = 1.37$ (Elam and George, 2003) is the average number of hydroxyl groups that react with each $Zn(C_2H_5)_2$ molecule. Thus, the ZnO deposition half-reactions (Reactions (R2a and R2b)) were not broken down further into the elemental

Table 1: A summary of gaseous and fractional surface coverage species in Reactions (R2a-R2d), and their abbreviations.

Gaseous species $(\langle g \rangle)$	α	Surface species $(\langle s \rangle)$	κ
$\operatorname{Zn}(\mathrm{C}_2\mathrm{H}_5)_2$	A	(-OH)	A*
$\mathrm{H}_2\mathrm{O}$	B	$(-O-)_{\nu} Zn (C_2H_5)_{2-\nu}$	B*
C_2H_6	C	(-O)	C*
N_2	P		

adsorption and reaction steps during each precursor exposure (as is done in, 108 for example, Elliott (2012); Ren (2009); Travis and Adomaitis (2013a,b,c)). 109 Thereby, the ligand elimination was assumed to proceed without intermediate adsorption adducts or their transition states being formed, and that this is the 111 rate-limiting step. 112

The irreversible half-reactions defined in Reaction (R2) subject to constant 113 activation energies govern a growth per cycle (GPC) that increases strictly with 114 the deposition temperature, and hence, Reaction (R2) cannot describe the sharp 115 decrease in GPC that occurs at elevated temperatures (see, for example, Yousfi 116 et al. (2000)). This phenomenon is generally attributed to the gradual reduction of the surface hydroxyl groups through the recombination reaction (Reaction (R2c)) (Deminsky et al., 2004; Matero et al., 2000; Rahtu et al., 2001): 119

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$$2(-OH)\langle s \rangle \xrightarrow{k_3^{rev}} (-O)\langle s \rangle + H_2O\langle g \rangle$$
 (R2c)

where the hydroxyl groups may be reformed on the oxide surface during ex-123 posure to H₂O through the reverse reaction (Reaction (R2d)). It is, however, 124 noteworthy that the GPC that is obtained, which is governed by the limit-125 cycle ZnO ALD kinetics proposed in Reactions (R2), has a convex temperature dependence. Finally, the abbreviations for the gaseous and fractional surface 127 species in Reactions (R2a–R2d) used in this paper are listed in Table 1.

3. Physical Modeling

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A one-dimensional representation of the square duct reaction chamber (with dimensions $(L \times W \times H)$ 5.0×5.0×0.2 (cm)) of the reactor system constituted the spatial domain, $z \in [0, L]$, in the present study, with its z-axis coincident with the flow direction. Further details of the reactor system are given in Holmqvist et al. (2012, 2013b) and in Baunemann (2006); Yousfi et al. (2000). The original experimental configuration is denoted as the *Reactor* $\mathscr A$ in the following scale-up analysis.

3.1. Nominal State and Algebraic Variables

For this paper a non-dimensionalized, physically-based model was used to 138 calculate the spatially and temporally dependent concentration and deposition 139 profiles for the original $Reactor \mathcal{A}$ and have subsequent been applied to the scaled-up $Reactor \mathcal{B}$, while keeping certain dimensionless numbers in the dif-141 ferential equations and boundary conditions the same for both systems. In this way, the scale-up method strives to preserve dynamic similarity. However, in or-143 der to maintain constant dimensionless numbers, the underlying nominal state 144 and algebraic variables $[\hat{\rho}, \hat{v}_{\zeta}, \hat{w}_{\alpha}, \hat{p}]$ must be expressed in terms of the process 145 manipulated variables, $\mathbf{u} = [\dot{Q}_{\beta}, \dot{V}_{\mathrm{VP}}, \dot{Q}_{\alpha}, \Delta \tau_{\alpha}]$ and $\forall \alpha \in \{A, B\}$. Thus, the 146 length, $\zeta \in [\zeta_0, \zeta_{\text{end}}]$, and time, $\tau \in [\tau_0, \tau_f]$, are scaled by the nominal values L and L/\hat{v}_{ζ} , respectively. Additionally, the nominal mass averaged velocity, \hat{v}_{ζ} , 148 density, $\hat{\rho}$, and pressure, \hat{p} , of the carrier gas as well as the nominal precursor 149 mass fraction, $\hat{\omega}_{\alpha}$, are conveniently expressed by the reactor dynamic material balances of the continuous stirred-tank reactor (CSTR) model: 151

$$\left[\left[\frac{\hat{v}_{\zeta}}{L}\right]\right]\frac{d\hat{\rho}}{d\tau} = \frac{1}{V} \sum_{\forall \alpha} \dot{Q}_{\alpha} \rho_{\text{STP},\alpha} \Pi_{\alpha}(\tau, \Delta \tau_{\alpha}) - \frac{\dot{V}_{\text{VP}}}{V} \hat{\rho} + \sum_{\forall \alpha} S_{\alpha}$$
(1a)

$$\begin{bmatrix} \frac{\hat{v}_{\zeta}}{L} \end{bmatrix} \frac{d\hat{\rho}\hat{\omega}_{\alpha}}{d\tau} = \frac{1}{V}\dot{Q}_{\alpha}\rho_{\text{STP},\alpha}\Pi_{\alpha}(\tau,\Delta\tau_{\alpha}) - \frac{\dot{V}_{\text{VP}}}{V}\hat{\rho}\hat{\omega}_{\alpha} + S_{\alpha} \tag{1b}$$

where S_{α} denotes the net mass consumption owing to the heterogeneous gas surface reactions (see Section 2). See Travis and Adomaitis (2013a) for further details of how the reactor dynamic material balances are derived. A smooth rectangular function was used to model the non-overlapping precursor injections in a cyclic time sequence. This function, $\Pi_{\alpha}(\tau, \Delta \tau_{\alpha}) \in [0, 1]$, was composed of superposed continuously differentiable logistic functions, $L(\tau)$:

$$\Pi_{\alpha}(\tau, \Delta \tau_{\alpha}) = L(\bar{\tau} - \bar{\tau}_{0,\alpha}) - L(\bar{\tau} - \bar{\tau}_{f,\alpha}) \tag{2a}$$

$$L(\bar{\tau}) = [1 + \exp(-\delta \tau_{\alpha} \bar{\tau})]^{-1}$$
 (2b)

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where $\bar{\tau} = N_{\Delta\tau} - \lfloor N_{\Delta\tau} \rfloor$ is the normalized cycle time, $N_{\Delta\tau} = \tau/\Delta\tau$ is the cycle number, $\Delta\tau = \Delta\tau_A + \Delta\tau_B + 2\Delta\tau_P$ denotes a complete ALD cycle, $\delta\tau_\alpha$ is a parameter that influences the maximum derivative of the function, and $\Delta\tau_\alpha = (\bar{\tau}_{f,\alpha} - \bar{\tau}_{0,\alpha})\Delta\tau$. Imposing stationarity on Eq. (1) under non-reactive conditions, i.e. prescribing $S_\alpha := 0$, allows to define the following explicit relationships:

$$\hat{\rho} := \frac{\dot{Q}_{\beta} \rho_{\text{STP},\beta}}{\dot{V}_{\text{VP}}} \tag{3a}$$

$$\hat{\omega}_{\alpha} := \frac{\dot{Q}_{\alpha} \rho_{\text{STP},\alpha}}{\dot{Q}_{\alpha} \rho_{\text{STP},\alpha} + \dot{Q}_{\beta} \rho_{\text{STP},\beta}} \tag{3b}$$

The nominal density, $\hat{\rho}$, in Eq. (3a) is defined for the carrier gas purge by prescribing $\Pi_{\alpha}(\tau, \Delta \tau_{\alpha}) := 0$, whereas the nominal gas-phase mass fraction, $\hat{\omega}_{\alpha}$, in Eq. (3b) is defined for the precursor pulse by prescribing $\Pi_{\alpha}(\tau, \Delta \tau_{\alpha}) := 1$ and $\forall \alpha \in \{A, B\}$. Likewise, \hat{p} and \hat{v}_{ζ} are defined for the carrier gas purge by means of the equation of state, where:

$$\hat{p} = \frac{\hat{\rho}}{M_{\beta}}RT := \frac{1}{\dot{V}_{\text{VP}}} \frac{\dot{Q}_{\beta} \rho_{\text{STP},\beta}}{M_{\beta}} RT \tag{3c}$$

$$\hat{v}_{\zeta} := \frac{1}{A'} \frac{\dot{Q}_{\beta} \rho_{\text{STP},\beta}}{M_{\beta}} RT \frac{1}{\hat{p}} := \frac{\dot{V}_{\text{VP}}}{A'}$$
(3d)

where $\rho_{\text{STP},\beta}$ is the density of the carrier gas species β at standard temperature and pressure (STP).

It is noteworthy that the CSTR model was exploited in this study due to its inherent formalism, which describes the correlation between the mass flow of the α th precursor, \dot{Q}_{α} , and that of the carrier gas, \dot{Q}_{β} , (which are both specified upstream of the reaction chamber), and the volumetric flow rate through the vacuum pump, $\dot{V}_{\rm VP}$, (which is specified downstream of the reaction chamber). Accordingly, the CSTR model conveniently correlates the impact of the free design variables, **u**, on the nominal state and algebraic variables needed to derive the equations that describe the spatially distributed reactor model in its non-dimensional form. The non-dimensionalized spatially distributed reactor model was subsequent utilized in the scale-up analysis.

192 3.2. Spatially Distributed ALD Reactor Model

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The isothermal and variable-density gas flow in the viscous regime can be described by fully coupled, compressible (Bird et al., 1960) equations for the conservation of mass, momentum, and individual gas-phase species. This is the type of gas flow encountered in low-volume, continuous cross-flow ALD reactor designs with temporal precursor pulsing. The governing equations defined in the non-dimensionalized spatial, $\zeta \in [\zeta_0, \zeta_{\rm end}]$, and temporal, $\tau \in [\tau_0, \tau_f]$, domains are, in an appropriate non-dimensional form:

$$\frac{\partial \rho}{\partial \tau} = -\frac{\partial}{\partial \zeta} \left(\rho v_{\zeta} \right) + \sum_{\forall \alpha} \left[\frac{L}{\hat{\rho} \hat{v}_{\zeta} \hat{\omega}_{\alpha}} \right] S_{\alpha} \tag{4}$$

$$\frac{\partial \rho v_{\zeta}}{\partial \tau} = -\frac{\partial}{\partial \zeta} \left(\rho v_{\zeta} v_{\zeta} + \mathscr{P} \right) + \left[\left[\frac{\hat{\mu}}{\hat{\rho} \hat{v}_{\zeta} L} \right] \right] \frac{4}{3} \frac{\partial}{\partial \zeta} \left(\mu \frac{\partial v_{\zeta}}{\partial \zeta} \right) - \left[\left[\frac{\hat{\mu}}{\hat{\rho} \hat{v}_{\zeta} L} \right] \right] \Phi_{\zeta}$$
 (5)

$$\frac{\partial \rho \omega_{\alpha}}{\partial \tau} = -\frac{\partial}{\partial \zeta} \left(\rho v_{\zeta} \omega_{\alpha} \right) + \left[\left[\frac{\hat{\mathcal{D}}_{\alpha\beta}}{\hat{v}_{\zeta} L} \right] \right] \frac{\partial}{\partial \zeta} \left(\rho \mathcal{D}_{\alpha\beta} \frac{\partial \omega_{\alpha}}{\partial \zeta} \right) + \left[\left[\frac{L}{\hat{\rho} \hat{v}_{\zeta} \hat{\omega}_{\alpha}} \right] \right] S_{\alpha}$$
 (6)

where $\mathscr{P} = p[\![\hat{p}/(\hat{\rho}\hat{v}_{\zeta}\hat{v}_{\zeta})]\!]$ denotes the characteristic modified pressure. Two dimensionless numbers dominate the gas flow and mass transfer:

$$Re := \left[\frac{\hat{\rho}\hat{v}_{\zeta}L}{\hat{\mu}} \right] \tag{7}$$

Pe :=
$$\left[\frac{\hat{v}_{\zeta}L}{\hat{\mathcal{D}}_{\alpha\beta}}\right]$$
 (8)

where Re is the Reynolds number, which describes the ratio between the inertial forces and the viscous forces, and Pe is the Peclet number, which describes the ratio between the convective mass transport and the diffusive mass transport. Here, ρ and μ are the non-dimensional density and the dynamic viscosity of the gas mixture at a certain nominal pressure and temperature. The non-dimensional pressure, p, is governed by the equation of state, and (making use

of Eq. (3a) is given by:

$$p = \left(\frac{\hat{\rho}}{\hat{p}}RT\right)\rho\sum_{\forall\alpha}\frac{\hat{\omega}_{\alpha}\omega_{\alpha}}{M_{\alpha}}$$

$$:= \rho\sum_{\forall\alpha}\left(\hat{\omega}_{\alpha}\frac{M_{\beta}}{M_{\alpha}}\right)\omega_{\alpha}$$
(9)

The non-dimensional transport coefficients, $\mathcal{D}_{\alpha\beta}$ and μ_{α} , in Eqs. (4–8) were determined from the Chapman-Enskog kinetic theory of dilute gases (Hirschfelder et al., 1964; Reid et al., 1988), and the non-dimensional viscosity for the multicomponent mixture of gases, μ , was determined from the semi-empirical mixing formula (Wilke, 1950). The transport coefficients were converted to non-dimensional forms by determining the corresponding values at the aforementioned nominal values determined by Eq. (3). Finally, the last term in Eq. (5) is given by:

$$\Phi_{\zeta} = 12(L/H)^2 \mu v_{\zeta} \tag{10}$$

and originates from the shear stress, $-\mu \partial v_{\zeta}(y)/\partial y$, and has been derived by assuming that the flow in a square duct is fully developed and laminar, with a velocity distribution given by $v_{\zeta}(y) = v_{\zeta,\max}(1-[2y/H]^2)$, with $y \in (H/2)[-1,1]$ and $v_{\zeta,\max} = (3/2)v_{\zeta}$ (Bird et al., 1960).

3.2.1. Boundary Conditions

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Analogous to inlet and outlet flow rates of the CSTR model (see Eqs. (1 and 3)), the boundary conditions to the set of partial differential equations (PDEs) (Eqs. (4–6)) prescribes the precursor and carrier gas mass flow as a standard volumetric flow rate at the inlet, and the volumetric flow rate through the vacuum pump, $\dot{V}_{\rm VP}$, at the outlet. Thus, the inlet, $\zeta = \zeta_0$, mass fluxes for each component α and for the gas mixture, along with a Neumann condition on

the velocity, are given by the equations: 239

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$$(\rho v_{\zeta})\Big|_{\zeta=\zeta_0} = \frac{1}{\hat{\rho}\hat{v}_{\zeta}} \frac{1}{A'} \sum_{\forall \alpha} \dot{Q}_{\alpha} \rho_{\text{STP},\alpha} \Pi_{\alpha}(\tau, \Delta \tau_{\alpha})$$
(11)

$$\frac{\partial v_{\zeta}}{\partial \zeta}\Big|_{\zeta=\zeta_{0}} = 0$$

$$(\rho v_{\zeta}\omega_{\alpha})\Big|_{\zeta=\zeta_{0}} = \frac{1}{\hat{\rho}\hat{v}_{\zeta}\hat{\omega}_{\alpha}} \frac{1}{A'} \dot{Q}_{\alpha}\rho_{\text{STP},\alpha}\Pi_{\alpha}(\tau, \Delta\tau_{\alpha})$$
(12)

$$(\rho v_{\zeta} \omega_{\alpha})\Big|_{\zeta=\zeta_{0}} = \frac{1}{\hat{\rho}\hat{v}_{\zeta}\hat{\omega}_{\alpha}} \frac{1}{A'} \dot{Q}_{\alpha} \rho_{\text{STP},\alpha} \Pi_{\alpha}(\tau, \Delta \tau_{\alpha})$$
(13)

Further, the outlet boundary condition, $\zeta = \zeta_{\rm end}$, prescribes that the diffusive mass is zero along with a Dirichlet condition on the velocity: 244

$$v_{\zeta}\Big|_{\zeta=\zeta_{\text{end}}} = \frac{1}{\hat{v}_{\zeta}} \frac{\dot{V}_{\text{VP}}}{A'}$$

$$\frac{\partial \omega_{\alpha}}{\partial \zeta}\Big|_{\zeta=\zeta_{\text{end}}} = 0$$
(14)

$$\frac{\partial \omega_{\alpha}}{\partial \zeta}\Big|_{\zeta=\zeta_{\text{end}}} = 0 \tag{15}$$

3.3. Gas-phase Species Flux at the Growth Surface

The heterogeneous ALD gas-surface reactions (Reaction (R2)) lead to a net 248 mass consumption at the substrate surface. The molar reaction rate of the ith elementary reaction is described by the general reversible Langmuir formalism 250 (Holmqvist et al., 2012): 251

$$r_i = \left[\frac{\hat{\rho}\hat{\omega}_{\alpha}}{M_{\alpha}}RT \right] \rho \omega_{\alpha} k_i^{\text{fwd}} \Lambda^{n_i^{\text{fwd}}} \left(1 - \sum_{\mathbf{k}, \ell} \theta_{\ell} \right)^{n_i^{\text{fwd}}} - k_i^{\text{rev}} \Lambda^{n_i^{\text{rev}}} \theta_{\kappa}^{n_i^{\text{rev}}}$$
(16)

where Λ is the maximum molar concentration of surface sites per unit area available for deposition, the subscript ℓ represents all κ th surface species with 254 which the α th gaseous species cannot undergo a reaction, and n_i^{fwd} and n_i^{rev} 255 are the orders of the forward and reverse reactions, respectively. The partial 256 pressure, p_{α} , of the α th precursor in Eq. (16) can be expressed in terms of the 257 mass fraction, ω_{α} , and the density of the gas mixture, ρ , using the equation of state, to give: 259

$$\hat{p}_{\alpha}p_{\alpha} := \left[\left[\frac{\hat{\rho}\hat{\omega}_{\alpha}}{M_{\alpha}}RT \right] \rho\omega_{\alpha} \right]$$
(17)

Consequently, the source term, S_{α} , in the species-continuity equation (Eq. (6)) 261 states that the total gas-phase mass flux of the α th species at the growth surface is balanced by the net consumption or production of mass per unit area:

$$\begin{bmatrix}
\frac{L}{\hat{\rho}\hat{v}_{\zeta}\hat{\omega}_{\alpha}}
\end{bmatrix} S_{\alpha} = \begin{bmatrix}
\frac{L}{\hat{\rho}\hat{v}_{\zeta}\hat{\omega}_{\alpha}}
\end{bmatrix} \left(\frac{A}{V}\right) M_{\alpha} \sum_{i=1}^{N_{i}} \xi_{\alpha,i} r_{i} \tag{18}$$

$$:= \sum_{i=1}^{N_{i}} \xi_{\alpha,i} \left[\begin{bmatrix} \frac{L}{\hat{v}_{\zeta}} \left(\frac{A}{V}\right) R T k_{i}^{\text{fwd}} \Lambda^{n_{i}^{\text{fwd}}} \end{bmatrix} \rho \omega_{\alpha} \left(1 - \sum_{\forall \ell} \theta_{\ell}\right)^{n_{i}^{\text{fwd}}} - \begin{bmatrix} \frac{L}{\hat{v}_{\zeta}} \frac{M_{\alpha}}{\hat{\rho}\hat{\omega}_{\alpha}} \left(\frac{A}{V}\right) k_{i}^{\text{rev}} \Lambda^{n_{i}^{\text{rev}}} \end{bmatrix} \theta_{\kappa}^{n_{i}^{\text{rev}}} \right]$$

where $\xi_{\alpha,i}$ denotes the stoichiometric coefficient corresponding to the α th species, and the appearing surface Damköhler numbers, $\mathrm{Da}_{\alpha,i}^{\mathrm{fwd}}$ and $\mathrm{Da}_{\alpha,i}^{\mathrm{rev}}$, are the ratios between the molar growth rate of the ALD film at the substrate and the speed of convective transport of the growth limiting species. Thus, $\mathrm{Da}_{\alpha,i}^{\mathrm{fwd}}$ and $\mathrm{Da}_{\alpha,i}^{\mathrm{rev}}$ are given by:

$$\mathrm{Da}_{\alpha,i}^{\mathrm{fwd}} := \left[\frac{L}{\hat{v}_{\zeta}} \left(\frac{A}{V} \right) RT k_{i}^{\mathrm{fwd}} \Lambda^{n_{i}^{\mathrm{fwd}}} \right]$$
 (19)

Da_{$$\alpha,i$$} rev $Da_{\alpha,i}^{\text{rev}} := \left[\frac{L}{\hat{v}_{\zeta}} \frac{M_{\alpha}}{\hat{\rho} \hat{\omega}_{\alpha}} \left(\frac{A}{V} \right) k_i^{\text{rev}} \Lambda^{n_i^{\text{rev}}} \right]$ (20)

276 3.4. Growth Surface State Dynamics

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The characteristics of the surface reaction, in particular the probability that
the reaction will proceed through the formation of adsorbed species, depends
on the properties of the exposed adsorbent surface. The molar reaction rate
per unit surface area (Eq. (16)) and the non-dimensional surface Damköhler
numbers (Eqs. (19–20)) allow to determine the spatial and temporal fractional
surface coverages:

$$\frac{\partial \theta_{\kappa}}{\partial \tau} = \left[\frac{L}{\Lambda \hat{v}_{\zeta}} \right] \sum_{i=1}^{N_{i}} \xi_{\kappa,i} r_{i} \tag{21}$$

$$:= \sum_{i=1}^{N_{i}} \xi_{\kappa,i} \left[\frac{\hat{\rho} \hat{\omega}_{\alpha}}{M_{\alpha}} \left(\frac{V}{A} \right) \frac{1}{\Lambda} \right] \left[\operatorname{Da}_{\alpha,i}^{\text{fwd}} \rho \omega_{\alpha} \left(1 - \sum_{\forall \ell} \theta_{\ell} \right)^{n_{i}^{\text{fwd}}} - \operatorname{Da}_{\alpha,i}^{\text{rev}} \theta_{\kappa}^{n_{i}^{\text{rev}}} \right]$$

$$0 = \sum_{\forall \kappa} \frac{\partial \theta_{\kappa}}{\partial \tau} \tag{22}$$

where θ_{κ} is the fractional surface coverage of the κ th surface species, $\kappa \in \{A*, B*, C*\}$ (see Table 1). The non-dimensional term that appears in Eq.

(21) represents the precursor excess number and relates the maximum precursor molar density inside the reactor per unit adsorption site:

$$\gamma_{\alpha} := \left[\frac{\hat{\rho}\hat{\omega}_{\alpha}}{M_{\alpha}} \left(\frac{V}{A} \right) \frac{1}{\Lambda} \right] \tag{23}$$

292 3.5. Model Form and Size

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The equations of the spatially distributed ALD reactor model that describe 293 the gas-phase and growth surface state dynamics (see Sections 3.2–3.4), con-294 stitute a system of non-linear partial differential algebraic equations (PDAEs). 295 In this study, the PDAE system was approximated using the method-of-lines (Davis, 1984; Schiesser, 1991) and the finite volume method (FVM). The first-297 order spatial derivative of the non-dimensional density, ρ , in Eq. (4) and of the gas-phase mass fractions, ω_{α} , in Eq. (6) have been approximated using a first-299 order downwind discretization scheme, while a first-order upwind discretization 300 scheme was utilized to approximate the non-dimensional mass average velocity, v_{ζ} , in Eq. (5). The resulting non-linear index-1 differential-algebraic equation 302 (DAE) system can be written collectively as: 303

$$\mathbf{0} = \mathbf{F}(\tau, \dot{\mathbf{x}}(\tau), \mathbf{x}(\tau), \mathbf{u}(\tau), \mathbf{w}(\tau), \boldsymbol{\beta})$$
 (24a)

$$\mathbf{0} = \mathbf{F}_0(\tau_0, \dot{\mathbf{x}}(\tau_0), \mathbf{x}(\tau_0), \mathbf{u}(\tau_0), \mathbf{w}(\tau_0), \boldsymbol{\beta})$$
 (24b)

$$\mathbf{0} = \mathbf{C}_{eq}(\tau_0, \tau_f, \mathbf{x}, \mathbf{u}, \mathbf{w}, \boldsymbol{\beta}) \tag{24c}$$

$$\mathbf{x}(\tau_0) = \mathbf{x}_0 \tag{24d}$$

where ${f F}$ is the DAE that represents the dynamics of the system, ${f F}_0$ repre-308 sents the DAE augmented with additional initial conditions, $\mathbf{C}_{\mathbf{eq}}$ is a point equality-constraint function (see Section 3.6), and β is the model parameter 310 vector. Finally, $\mathbf{x} = [\rho, v_{\zeta}, \omega_{\alpha}, \theta_{\kappa}]^T$, $\mathbf{w} = [p, \theta_{C*}]^T$, and $\mathbf{u} = [\Delta \tau_{\alpha}, \dot{Q}_{\alpha}, T, \dot{V}_{VP}]^T$ 311 (where $\alpha \in \{A, B, C\}$ and $\kappa \in \{A*, B*\}$) describe dependent states, algebraic 312 variables, and free design variables. Furthermore, with N_{FVM} FVM elements, 313 the number of states, N_x , is $7N_{\text{FVM}}$ and the number of algebraic variables, N_w , 314 is $2N_{\text{FVM}}$. The number of FVM elements is a compromise between accuracy 315 and computational complexity, and should be chosen such that it gives adequate 316

representation of the dispersion. In this study, the number of FVM elements was set to 25.

3.6. Limit-cycle Criteria

For the purpose of this study, substrate effects on the nucleation and initial growth periods were not considered (see Section 2). For this reason, the solution strategy for the DAE system, \mathbf{F} , is to consider the limit-cycle dynamic solution that arises from the steady cyclic operation of the ALD reactor (Travis and Adomaitis, 2013a). Computation of limit-cycle solutions over the temporal horizon $[\tau_0, \tau_f]$ requires one additional important criterion: that the state variables, \mathbf{x} , return to their initial conditions at the end of the cycle, $\tau = \tau_f$:

$$\mathbf{x}(\tau_0) := \mathbf{x}(\tau_f) \tag{25a}$$

The following non-differentiated relationships must also be satisfied at $\tau \in \{\tau_0, \tau_f\}$, due to constraints that arise from the underlying assumptions on which Eqs. (4–6) and Eqs. (21–22) are based:

$$1 = \sum_{\forall \kappa} \theta_{\kappa}(\tau) \tag{25b}$$

$$1 = \sum_{\forall \alpha} \omega_{\alpha}(\tau) \tag{25c}$$

with $\kappa \in \{A*, B*, C*\}$, and $\alpha \in \{A, B, C, P\}$. Furthermore, the limit-cycle criteria in Eq. (25) are collected in the point equality-constraint function, $\mathbf{C_{eq}}$.

Section 5 presents numerical aspects of computing limit-cycle solutions.

337 4. Scale-up Analysis

In order to maintain the dynamic similarity of gas flow and deposition when scaling up an ALD reactor, the Peclet, Reynold, surface Damköhler, and precursor excess numbers should be kept fixed. In this case, the flow path lines and non-dimensional distributions of concentrations will remain the same after the scale-up operation. The non-dimensional deposition rate and its relative uniformity will also remain the same. The overall objective of the scale up analysis in the present study was to maintain a fixed absolute growth rate and its relative uniformity, while optimizing the precursor yield. Thus, the scale-up method can minimize individual precursor doses, while maintaining sufficiently high exposure levels, ensuring that the design specifications are not violated. The precursor yield can be expressed as:

$$\frac{dY_{\alpha}}{d\tau} = -\left[\left[\frac{A'\hat{\rho}\hat{v}_{\zeta}\hat{\omega}_{\alpha}}{\dot{Q}_{\alpha}\rho_{\text{STP},\alpha}\Pi_{\alpha}(\tau,\Delta\tau_{\alpha})}\right](\rho v_{\zeta}\omega_{\alpha})\right|_{\zeta=\zeta_{\text{end}}}$$
(26)

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subject to the initial value $Y_{\alpha}(\tau_0)=1$. Moreover, the spatially dependent growth rate, $m_{\rm s}$, in $\zeta\in[\zeta_0,\zeta_{\rm end}]$ is defined by:

$$\frac{\partial m_{\rm s}}{\partial \tau} = \sum_{i=1}^{N_i} \left[\left[\frac{\Lambda \Delta M_i}{\hat{m}_{\rm s}} \right] \xi_{\kappa,i} \gamma_{\alpha} \left[\mathrm{Da}_{\alpha,i}^{\mathrm{fwd}} \rho \omega_{\alpha} \left(1 - \sum_{\forall \ell} \theta_{\ell} \right)^{n_i^{\mathrm{fwd}}} - \mathrm{Da}_{\alpha,i}^{\mathrm{rev}} \theta_{\kappa}^{n_i^{\mathrm{rev}}} \right]$$
(27a)

subject to the initial value $m_{\rm s}(\tau_0)=0$. Eq. (27a) uses the conversion rate of the fractional coverage of surface species (given by Eq. (21)). The scaling factor, $\hat{m}_{\rm s}=M_{\rm s}\Lambda$, corresponds to fully saturated ALD growth, and ΔM_i is the difference in molar mass of the outermost surface species that are governed by the *i*th elementary reaction (see Reaction (R2)). Eq. (27a) allows to determine the substrate spatially averaged growth rate:

$$\frac{d\langle m_{\rm s}\rangle}{d\tau} = \frac{1}{(\zeta_{\rm end} - \zeta_0)} \int_{\zeta_0}^{\zeta_{\rm end}} \frac{\partial m_{\rm s}}{\partial \tau} d\zeta \tag{27b}$$

In addition, by means of Eq. (27), the growth rate uniformity, UF, was conveniently defined as:

$$\text{UF} = \begin{cases}
1 - \int_{\zeta_0}^{\zeta_{\text{end}}} |m_{\text{s}} - \langle m_{\text{s}} \rangle| d\zeta \left(\int_{\zeta_0}^{\zeta_{\text{end}}} m_{\text{s}} d\zeta \right)^{-1} & \exists \zeta \in [\zeta_0, \zeta_{\text{end}}] : m_{\text{s}}(\zeta) > 0 \\
1 & \text{otherwise}
\end{cases} \tag{28}$$

It is noteworthy that the uniformity metric defined in Eq. (28) describes the absolute deviation from the mean film mass accumulated, $\langle m_{\rm s} \rangle$, at normalized time τ . However, since the ALD process is an inherently forced periodic system, it is convenient to compare the performance of the existing design with that of the scaled-up design based on absolute normalized growth per cycle, $\langle m_{\rm s} \rangle (\tau_f)$, where $\tau_f = (\hat{v}_\zeta/L)\Delta t$ is the end of the time horizon, $[\tau_0, \tau_f]$. The uniformity metric, UF (τ_f) , is evaluated in a similar manner at the end of the pulse sequence.

The duration of the α th precursor pulse, $\Delta \tau_\alpha$, is not explicitly taken into account by the non-dimensional parameters defined in Eqs. (7, 8 and 19–23), but it is can, instead, be conveniently expressed by the half-cycle average substrate exposure dose for the α th adsorptive precursor:

$$\frac{d\langle \delta_{\alpha} \rangle}{d\tau} = \left[\frac{L}{\hat{v}_{\zeta}} \frac{\hat{\rho} \hat{\omega}_{\alpha}}{M_{\alpha}} RT \Delta \tau_{\alpha} \right] \frac{1}{\Delta \tau_{\alpha}} \frac{1}{(\zeta_{\text{end}} - \zeta_{0})} \int_{\zeta_{0}}^{\zeta_{\text{end}}} \rho \omega_{\alpha} d\zeta$$
 (29)

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subject to the initial value $\langle \delta_{\alpha} \rangle (\tau_0) = 0$. Moreover, the nominal α th exposure dose, ϕ_{α} , can be deduced from Eq. (29):

$$\phi_{\alpha} := \left[\frac{L}{\hat{v}_{\zeta}} \frac{\hat{\rho} \hat{\omega}_{\alpha}}{M_{\alpha}} RT \Delta \tau_{\alpha} \right]$$
(30)

However, to capture the increase in the total mass concentration during the precursor pulse, recall that $\hat{\rho}$ has been derived with $\Pi_{\alpha} := 0$ in Eq. (3), the nominal exposure dose, ϕ_{α} , must, therefore, be scaled accordingly:

$$\langle \hat{\delta}_{\alpha} \rangle = \frac{\phi_{\alpha}}{(1 - \hat{\omega}_{\alpha})} \tag{31}$$

The exposure dose of the growth surface to the α th precursor defined by Eq. 382 (29) is characterized by the time dependent, local, partial pressure (Eq. (17)) 383 during the exposure period, and during a portion of each purge period. The ALD gas-surface reactions must be coupled to the dynamic reactor transport 385 model in order to model these features (see the systematic modeling approach described in Sections 3.2 and 3.4). Thus, there are two main ways of varying 387 $\langle \delta_{\alpha} \rangle$: changing the mass flow of the precursors, \dot{Q}_{α} , and in this way changing 388 the partial pressure, or changing the duration of the pulse, $\Delta \tau_{\alpha}$. Eq. (29) thus adds a further dimension to the scale-up analysis, incorporating the effect of the dispersion of precursor pulses along the ζ -axis, and in this way enabling the precursor exposure dose to be accurately assessed in the scale-up analysis. 392

The set of process operating parameters that can be varied for the contin-

to maintain a fixed absolute growth rate, $\langle m_{\rm s} \rangle (\tau_f)$, and growth rate uniformity, UF(τ_f), is $\mathbf{u} = [\Delta \tau_{\alpha}, \dot{Q}_{\alpha}, T, \dot{V}_{\rm VP}]^T$ and $\alpha \in \{A, B, P\}$. Table 2 lists the dimensional and non-dimensional parameters that are relevant to the scale-up methodology and the ways in which they depend on the set of manipulating variables, \mathbf{u} , and the length of the reactor.

400 4.1. Scale-up Methodology for Cross-flow ALD Reactor Designs

This paper analyses the implications of linearly scaling up an existing Re-401 actor \mathscr{A} (see Section 3) to a scaled-up Reactor \mathscr{B} by a geometric factor $\lambda =$ 402 $L_{\mathscr{B}}/L_{\mathscr{A}}$. In particular, the various dimensional and non-dimensional parameters presented in Table 2 and the design criteria defined in Eqs. (27–29) are consid-404 ered. The scale-up strategies presented here are based on a sequential method-405 ology in which the way in which carrier gas manipulated variables, $[\dot{Q}_{\beta}, \dot{V}_{\mathrm{VP}}]$, depend on the dynamic similarity of gas flow is assessed first. The way in which 407 the precursor pulse dose parameters, $[\dot{Q}_{\alpha}, \Delta \tau_{\alpha}]$ and $\forall \alpha \in \{A, B\}$, depend on the apparent ALD deposition rate and its relative uniformity is subsequently 409 assessed. 410

4.1.1. Scaling Rules for the Process Operating Conditions

In the context of the carrier gas manipulated variables, two fundamentally different strategies (see also Table 2) were investigated when the reactor was scaled up, with $L \propto \lambda$:

i) Scale-up strategy I involves the reactor being scaled up without changing 415 the carrier gas mass flow, \dot{Q}_{β} , or the flow rate through the vacuum pump, 416 \dot{V}_{VP} . The nominal pressure, \hat{p} , is unchanged when this strategy is used, 417 while $\gamma_{\alpha} \mathrm{Da}_{\alpha,i}^{\mathrm{fwd}}$, $\langle \hat{\delta}_{\alpha} \rangle \propto \dot{Q}_{\alpha} \lambda^2$ for the α th precursor, and the residence time, 418 $\hat{\tau} \propto \lambda^2$, change dramatically. The resulting increase in the product of the 419 surface Damköhler number and precursor excess number, and the nomi-420 nal precursor exposure dose with λ , are strictly positive, and give a higher 421 efficiency of the deposition process in terms of precursor utilization and de-422

Table 2: Scaling behavior of various dimensional and non-dimensional parameters. For the square duct that is being considered, the specific substrate surface area per unit reactor volume is $A/V=2L^2/(L^2H)$ (m⁻¹) and the cross-sectional area is A'=LH (m²).

Variable	Dependence	$Scale ext{-}up$	$Scale ext{-}up$
	on L , \dot{Q}_{β} ,	$strategy \ I:$	$strategy\ II:$
	$\dot{V}_{\mathrm{VP}},\dot{Q}_{lpha},$	$L \propto \lambda$	$L \propto \lambda$
	Δau_{lpha}	$\dot{Q}_{\beta} = \text{const.}$	$\dot{Q}_eta \propto \lambda^artheta$
		$\dot{V}_{\mathrm{VP}} = \mathrm{const.}$	$\dot{V}_{ m VP} \propto \lambda^{artheta}$
$\hat{v}_{\zeta} := rac{\dot{V}_{\mathrm{VP}}}{A'}$	$\propto rac{\dot{V}_{ m VP}}{L}$.	$\propto \lambda^{-1}$	$\propto \lambda^{\vartheta-1}$
$\hat{\rho} := \hat{p} \frac{M_{\beta}}{RT}$	$\propto \hat{p} \propto rac{Q_{eta}}{\dot{V}_{ m VP}}$	_	_
$\hat{\omega}_{\alpha} := \frac{\dot{Q}_{\alpha} \rho_{\text{STP},\alpha}}{\dot{Q}_{\alpha} \rho_{\text{STP},\alpha} + \dot{Q}_{\beta} \rho_{\text{STP},\beta}}$	$\propto rac{\dot{Q}_{lpha}}{\dot{Q}_{eta}}{}^a$	$\propto \dot{Q}_{lpha}$	$\propto rac{\dot{Q}_{lpha}}{\lambda^{artheta}}$
$\hat{\tau} := \frac{L}{\hat{v}_{\zeta}}$	$ \begin{array}{c} \stackrel{L}{\propto} \frac{\dot{Q}_{\beta}}{\dot{V}_{\mathrm{VP}}} \\ \propto \frac{\dot{Q}_{\alpha}}{\dot{Q}_{\beta}}^{a} \\ \propto \frac{\dot{L}^{2}}{\dot{V}_{\mathrm{VP}}} \end{array} $	$\propto \lambda^2$	$\propto \lambda^{2-artheta}$
$\hat{\mu}$	_	_	_
$\hat{\mathscr{D}}_{lphaeta}$	$\propto rac{1}{\hat{p}} \propto rac{\dot{V}_{ m VP}}{\dot{Q}_{eta}}$	_	_
$Pe := \frac{\hat{v}_{\zeta} L}{\hat{g}_{z,\beta}}$	$\propto \dot{Q}_{eta}$	_	$\propto \lambda^{artheta}$
$Re := \frac{\hat{\rho}\hat{v}_{\zeta}L}{\hat{u}}$	$\propto \dot{Q}_{eta}$	_	$\propto \lambda^{artheta}$
$\mathrm{Da}_{\alpha,i}^{\mathrm{fwd}} := \frac{L}{\hat{v}_{\ell}} \left(\frac{A}{V} \right) RT k_{i}^{\mathrm{fwd}} \Lambda^{n_{i}^{\mathrm{fwd}}}$	$\propto rac{L^2}{\dot{V}_{ m VP}}$	$\propto \lambda^2$	$\propto \lambda^{2-artheta}$
$\mathrm{Da}_{\alpha,i}^{\mathrm{rev}} := \frac{L}{\hat{v}_{\zeta}} \frac{M_{\alpha}}{\hat{\rho} \hat{\omega}_{\alpha}} \left(\frac{A}{V}\right) k_{i}^{\mathrm{rev}} \Lambda^{n_{i}^{\mathrm{rev}}}$	$\propto rac{L^2}{\dot{V}_{ m VP}} \ \propto rac{L^2}{\dot{Q}_{lpha}}^a$	$\propto rac{\lambda^2}{\dot{Q}_{lpha}}$	$\propto rac{\lambda^2}{\dot{Q}_lpha}$
$\gamma_{\alpha} := \frac{\hat{\rho}\hat{\omega}_{\alpha}}{M_{\alpha}} \left(\frac{V}{A}\right) \frac{1}{\Lambda}$	$\propto rac{\dot{Q}_{lpha}^{a}}{\dot{Q}_{eta}^{a}}^{a} \ \propto rac{\dot{Q}_{lpha}L^{2}}{\dot{Q}_{eta}\dot{V}_{\mathrm{VP}}^{a}}^{a}$	$\propto \dot{Q}_{lpha}$	$\propto rac{Q_{lpha}}{\lambda^{artheta}}$
$\gamma_{\alpha} \operatorname{Da}_{\alpha,i}^{\operatorname{fwd}} := \frac{L}{\hat{v}_{\zeta}} \frac{\hat{\rho} \hat{\omega}_{\alpha}}{M_{\alpha}} RT k_{i}^{\operatorname{fwd}} \Lambda^{n_{i}^{\operatorname{fwd}} - 1}$		$\propto \dot{Q}_{lpha} \lambda^2$	$\propto \dot{Q}_{\alpha} \lambda^{2-2\vartheta}$
$\phi_{\alpha} := \frac{L}{\hat{v}_{\zeta}} \frac{\hat{\rho} \hat{\omega}_{\alpha}}{M_{\alpha}} RT \Delta \tau_{\alpha}$	$\propto rac{\dot{Q}_{lpha}\Delta au_{lpha}L^2}{\dot{V}_{ m VP}^2}{}_a$	$\propto \dot{Q}_{\alpha} \Delta \tau_{\alpha} \lambda^2$	$\propto \dot{Q}_{lpha} \Delta au_{lpha} \lambda^{2-2lpha}$
$\langle \hat{\delta}_{\alpha} \rangle := \frac{L}{\hat{v}_{\zeta}} \frac{\hat{\rho}}{M_{\alpha}} \frac{\hat{\omega}_{\alpha}}{\hat{\omega}_{\beta}} RT \Delta \tau_{\alpha}$	$\propto rac{\dot{Q}_{lpha}\Delta au_{lpha}L^2}{\dot{V}_{ m VP}^2}$	$\propto \dot{Q}_{lpha} \Delta au_{lpha} \lambda^2$	$\propto \dot{Q}_{lpha} \Delta au_{lpha} \lambda^{2-2i}$

^aOnly valid for $\dot{Q}_{\alpha}\rho_{\text{STP},\alpha} \ll \dot{Q}_{\beta}\rho_{\text{STP},\beta}$.

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position rate. Finally, the Reynolds and Peclet numbers remain unchanged in this case, and thus dynamic similarity is preserved to some extent.

ii) Scale-up strategy II strives to maintain the reactor residence time, $\hat{\tau}$, and 425 the nominal pressure, \hat{p} , constant by scaling the carrier gas flow rate and the flow rate through the vacuum pump in parallel, as \dot{Q}_{β} , $\dot{V}_{\mathrm{VP}} \propto \lambda^{\vartheta}$ with $\vartheta \in (0,2]$. The residence time from the CSTR model is constant 428 for $\vartheta := 2$, and Scale-up strategy I is obtained in the case in which $\vartheta :=$ 0. When this strategy is used, $\gamma_{\alpha} Da_{\alpha,i}^{\text{fwd}}$, $\langle \hat{\delta}_{\alpha} \rangle \propto \dot{Q}_{\alpha} \lambda^{2-2\vartheta}$ for the α th 430 precursor and Re, Pe $\propto \lambda^{\vartheta}$. The way in which the product of the surface Damköhler number and the precursor excess number, and the nominal 432 precursor exposure dose, change, makes it clear that the efficiency of the 433 ALD process (in terms of precursor utilization and deposition rate) falls as ϑ increases. 435

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It is beneficial when using Scale-up strategy II to use a low value of the resi-436 dence time in cross-flow reactor designs, as this imposes a lower boundary onto 437 the carrier gas purge time, $\Delta t_{\rm P} \propto \hat{\tau}$ (see, for example, Jur and Parsons (2011); 438 Mousa et al. (2012)). This is needed in the ALD sequence to ensure negligi-439 ble precursor interaction, and ultimately to maximize throughput in terms of 440 the overall deposition rate per cycle time (see, for example, Holmqvist et al. 441 (2013b)). 442

The deposition temperature, T, must be considered in a complete investiga-443 tion of the carrier gas manipulated variables. Aarik et al. (2006) investigated 444 experimentally the effects of deposition temperature, while Holmqvist et al. (2013b) and Travis and Adomaitis (2013b) carried out theoretical studies. The 446 present investigation, however, is limited to $\mathbf{u} = [\dot{Q}_{\beta}, \dot{V}_{\mathrm{VP}}]$, since the tempera-447 ture depends on growth per cycle in a convex manner (Puurunen, 2005), which 448 means that its optimum value could be easily determined before the scale-up 449 analysis. The optimal deposition temperature used in this study was determined from the study by Holmqvist et al. (2013b) and set to T = 175 (°C). The de-451 sign specifications presented in Table 3 were determined in this way. The surface 452 Damköhler number for desorption, $Da_{\alpha,i}^{rev}$ and i=3, is not important at the op-453 timum operating temperature (though included in the model), and it is only 454

relevant in the high temperature region where extensive dehydroxylation takes
place (Deminsky et al., 2004; Matero et al., 2000; Rahtu et al., 2001) (see Eqs.
(R2c–R2d)). Thus, the impact of the surface Damköhler number for desorption
was not considered in the scale-up analysis.

459 4.1.2. Optimal Scaling Rules for the Precursor Exposure Dose

High uniformity is one of the key attributes of the ALD technology (Cleveland et al., 2012; Henn-Lecordier et al., 2011). Dynamic similarity is preserved to 461 some extent when the scale-up strategies in Section 4.1.1 are applied. However, 462 the growth rate, $\langle m_s \rangle (\tau_f)$, and the uniformity of the deposition rate, UF(τ_f), 463 falls as ϑ increases unless the precursor exposure dose is properly scaled. This 464 is, of course, not a desired result. Deposition rates and uniformities can be maintained by scaling the precursor pulse dose parameters, $\mathbf{u} = [\dot{Q}_{\alpha}, \Delta \tau_{\alpha}]$ and 466 $\forall \alpha \in \{A, B\}$. The scaling rules deduced from Table 2 demonstrate how the 467 free design variables depend on the metrics relevant in the scale-up strategies in a mechanistic manner, even though they are based solely on the CSTR model 469 (see Section 3.1). The half-cycle average substrate exposure dose for the α th 470 precursor (Eq. (29)), in particular, introduces a complex interdependency be-471 tween the mass flow, \dot{Q}_{α} , the pulse duration, $\Delta \tau_{\alpha}$, and the resulting local partial 472 pressure dynamics throughout the spatial domain, $\zeta \in [\zeta_0, \zeta_{end}]$. Furthermore, the uniformity of the film thickness (Eq. (27)) is inherently spatially dependent, 474 and thus cannot be reproduced by the CSTR model. 475

For these reasons, an optimization problem was formulated in order to distinguish the proper scaling factors of $\mathbf{u} = [\dot{Q}_A, \dot{Q}_B]$. The present investigation was limited to the optimization of the precursor mass flows, since Holmqvist et al. (2013b) have recently shown that the optimal precursor pulse durations, $\Delta \tau_{\alpha}$ and $\forall \alpha \in \{A, B\}$, are always at the lower boundary of the assigned range when precursor utilization and overall deposition rate per cycle time are targeted. Therefore, the precursor pulse duration in the scaled up *Reactor* $\mathscr B$ was set to that of *Reactor* $\mathscr A$ (see Table 3). Thus, in order to penalize high values of the decision variables, \mathbf{u} , the cost function was defined as:

$$\Phi(\mathbf{y}, \mathbf{u}) = -\sum_{\alpha \in \{A, B\}} \int_{\tau_0}^{\tau_f} \frac{dY_{\alpha}}{d\tau} d\tau$$
(32)

and assembles the precursor yields (Eq. (26)). The cost function was optimized while fulfilling the requirements placed on deposition rate and its uniformity by incorporating the terminal inequality constraints:

$$C_{\text{ieq}}^{\langle m_s \rangle} = \langle m_s \rangle^{\mathscr{A}}(\tau_f) - \langle m_s \rangle^{\mathscr{B}}(\tau_f)$$
(33a)

$$C_{\text{ieq}}^{\text{UF}} = \text{UF}^{\mathscr{A}}(\tau_f) - \text{UF}^{\mathscr{B}}(\tau_f)$$
(33b)

which can be collectively written as $\mathbf{C_{ieq}} = [\mathbf{C_{ieq}}^{\langle m_s \rangle}, \mathbf{C_{ieq}}^{\mathrm{UF}}]^T$. Table 3 lists the optimal design variables and resulting design criteria for $Reactor \mathscr{A}$ used in Eq. (33). Finally, the dynamic optimization problem (DOP) in the time interval $\tau \in [\tau_0, \tau_f]$ of achieving the assigned design criteria (see Eqs. (27b–29)) in the scaled-up design $Reactor \mathscr{B}$ may be stated using the cost function, Φ , as:

$$\min_{\mathbf{u}, \mathbf{x}_0} \Phi(\mathbf{y}, \mathbf{u}) \qquad (34)$$

$$\mathbf{s.t.} \quad \mathbf{0} = \mathbf{F}(\tau, \dot{\mathbf{x}}, \mathbf{x}, \mathbf{w}, \mathbf{u}, \boldsymbol{\beta})$$

$$\mathbf{0} = \mathbf{F}_0(\tau_0, \dot{\mathbf{x}}(\tau_0), \mathbf{x}(\tau_0), \mathbf{u}(\tau_0), \mathbf{w}(\tau_0), \boldsymbol{\beta})$$

$$\mathbf{y} = \mathbf{g}_{\mathbf{y}}(\mathbf{x}, \mathbf{w}, \mathbf{u}, \boldsymbol{\beta})$$

$$\mathbf{0} \geq \mathbf{C}_{\mathbf{ieq}}(\tau_0, \tau_f, \mathbf{x}, \mathbf{u}, \mathbf{w}, \boldsymbol{\beta})$$

$$\mathbf{0} = \mathbf{C}_{\mathbf{eq}}(\tau_0, \tau_f, \mathbf{x}, \mathbf{u}, \mathbf{w}, \boldsymbol{\beta})$$

$$\mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max}, \quad \mathbf{w}_{\min} \leq \mathbf{w} \leq \mathbf{w}_{\max}$$

$$\mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}, \quad \mathbf{x}(\tau_0) = \mathbf{x}_0$$

where $\mathbf{g_y}$ is the response function (Eqs. (26–28)) that governs the model output, and where $\mathbf{y} = [\langle m_{\rm s} \rangle, \mathrm{UF}, Y_{\alpha}]^T$, with $\alpha \in \{A, B\}$, is used to define the cost function and terminal inequality constraints of the DOP. An important implication of this formulation of the simultaneous optimization problem is that it enables the limit-cycle criteria (see Section 3.6) to be satisfied while maximizing the precursor yields (Eq. (26)). Thus, the initial values, \mathbf{x}_0 , are set to be free

Table 3: Reactor \mathscr{A} design specifications used in terminal inequality constraints, $\mathbf{C_{ieq}} = [\mathbf{C_{ieq}}^{\langle m_s \rangle}, \mathbf{C_{ieq}}^{\mathrm{UF}}]^T$, and those that must be satisfied in the scaled-up Reactor \mathscr{B} for each geometric factor, λ .

Design va	riables		Design criteria		
T	$1.75\cdot 10^2$	(°C)	$\langle m_{ m s} angle^{\mathscr{A}} (au_f)^a$	$9.00 \cdot 10^{-1}$	(cycle^{-1})
\dot{Q}_{lpha}	$1.83\cdot 10^1$	(sccm)	$Y_A^{\mathscr{A}}(au_f)^b$	$3.59\cdot10^{-2}$	$({\rm cycle}^{-1})$
\dot{Q}_{eta}	$5.00\cdot 10^2$	(sccm)	$Y_B^{\mathscr{A}}(au_f)^b$	$3.53\cdot10^{-2}$	$({\rm cycle}^{-1})$
$\hat{ au}\Delta au_lpha$	$2.00\cdot10^{-2}$	(s)	$\mathrm{UF}^\mathscr{A}(au_f)^a$	$9.50\cdot10^{-1}$	$({\rm cycle}^{-1})$
$\hat{ au}\Delta au_{eta}$	1.00	(s)	$\langle \delta_A angle^{\mathscr{A}} (au_f)^b$	2.68	(Pas)
$p _{\zeta=\zeta_{\mathrm{end}}}$	$3.00\cdot 10^2$	(Pa)	$\langle \delta_B angle^{\mathscr{A}} (au_f)^b$	2.68	(Pas)
$\dot{V}_{ m VP}{}^c$	$4.62\cdot10^{-3}$	$(m^3 s^{-1})$			

^aAssigned reference value in Eq. (33).

when solving Eq. (34), and varied such that the equality constraint relations are satisfied at $\tau \in \{\tau_0, \tau_f\}$ (Eq. (25)).

5. Modeling and Optimization Environment

Modelica (The Modelica Association, 2012) was used as the description lan-515 guage for the dynamic ALD process model developed in this paper. Modelica 516 is an equation-based language for complex physical models, whose underlying 517 mathematical formalism is that of DAEs. The open-source platform JModel-518 ica.org (Åkesson et al., 2010) was used for simulation and optimization of the 519 Modelica model. In the context of simulation, JModelica.org was used to compile the Modelica model into a functional mock-up unit (FMU) (Blochwitz et al., 521 2011), thus transforming it from a DAE form into an ordinary differential equa-522 tion (ODE) form. JModelica.org's interface to SUNDIALS (Hindmarsh et al., 2005) was subsequently used to simulate the model. The user interacts with the 524 various components of JModelica.org using the Python scripting language.

^bDetermined with optimized design variables from Eq. (34).

^cDetermined from Eq. (3c) and $\mathbf{u} = [T, \dot{Q}_{\beta}, p|_{\zeta = \zeta_{\text{end}}}]^T$.

526 5.1. Dynamic Optimization of DAEs Using Direct Collocation with CasADi

To enable the formulation of the DOP (see Eq. (34)) based on the model 527 (see Eq. (24)) described by Modelica code, the Modelica extension Optimica 528 (Åkesson, 2008) has been developed and integrated into JModelica.org. The 520 algorithm used in the work described in this paper to solve the DOP uses a di-530 rect and local collocation method (Biegler, 2010) on finite elements, using Radau 531 points and Lagrange interpolation polynomials (Magnusson and Åkesson, 2012). The algorithm has been implemented in Python in the JModelica.org framework, 533 using the computer algebra system with automatic differentiation (CasADi) op-534 timization package (Andersson et al., 2012). Using CasADi's symbolic syntax, it is possible to transcribe the DOP into a finite dimensional non-linear program-536 ming problem (NLP). The NLP was subsequently solved using the primal-dual 537 interior point method IPOPT v.3.10.3 (Wächter and Biegler, 2006), using MA27 538 as a linear solver. The first and second derivatives required by IPOPT are auto-539 matically and efficiently generated by CasADi, using automatic differentiation (AD) techniques. 541

The time horizon of the DOP in Eq. (34) was set to $\tau \in [0, 2.8] \cdot \hat{\tau}^{-1}$ 542 (a.u.), corresponding to a single pulse sequence (see Table 3), and the collocation 543 scheme used had 50 finite elements with three Radau points in each. The state 544 and algebraic variables were approximated using Lagrange polynomials of order 545 three and two, respectively. As all trajectories over the time horizon are solved 546 for simultaneously when using a collocation method, good initial guesses of 547 the state and algebraic variables at the collocation points are crucial. For this reason, the result of a simulation from the initial stationary point (through 549 imposing stationarity on Eq. (24) under non-reactive conditions (Section 3.1)) was used as the initial guess. 551

6. Results and Discussion

This paper describes two inherently different scale-up strategies, outlined in Section 4.1.1. It is, however, vital for both strategies that the CSTR model (Section 3.1) is valid, as it governs the scaling behavior of the non-dimensional parameters in Table 2. The validity of the CSTR model can be assessed by determining the discrepancy between $\hat{\tau} = L/\hat{v}_{\zeta}$ and the apparent spatially averaged reactor chamber residence time, defined as:

$$\langle \hat{\tau} \rangle := \left[\left[\frac{L}{\hat{v}_{\zeta}} \right] (\zeta_{\text{end}} - \zeta_0) \left(\int_{\zeta_0}^{\zeta_{\text{end}}} v_{\zeta} d\zeta \right)^{-1}$$
(35)

where v_{ζ} is governed by the compressible Navier–Stokes equation (Eq. (5)). In order for the relationship $\langle \hat{\tau} \rangle := \hat{\tau}$ to be valid, the spatially averaged normalized velocity field described by Eq. (5) must be equal to one. Thus, analyzing the Navier–Stokes equation shows that this relationship is valid only if the shear stress, Φ_{ζ} , approaches zero. A small discrepancy between $\hat{\tau}$ and $\langle \hat{\tau} \rangle$, however, is expected for low values of Φ_{ζ} . The validity of the CSTR model has been assessed when calculating the results for both scale-up strategies.

6.1. Scale-up Strategy I

Fig. 1 shows the cost function entities, $Y_{\alpha}(\tau_f)$ and $\alpha \in \{A, B\}$, and the 568 terminal inequality constraint entities, $\langle m_s \rangle (\tau_f)$ and UF(τ_f), as functions of the 569 geometric factor, λ , when applying Scale-up strategy I. The results have been determined for the optimal precursor mass flows, $\dot{Q}_A^{\mathscr{B}}:=\dot{Q}_B^{\mathscr{B}}.$ Thus, the mass 571 flows of the two precursors were set to be equal and determined from solving 572 the DOP (Eq. (34)) for each $\lambda \in [1, 20]$. The assigned design specifications for Reactor \mathcal{A} , listed in Table 3, have been retained with $\lambda = 1$, and the 574 quotient between the metrics associated with $Reactor~\mathscr{B}$ and those associated with Reactor \mathscr{A} are denoted by the superscript \mathscr{B}/\mathscr{A} . As expected from the 576 quadratic scaling behavior of $\hat{\tau}$ with λ in Table 2, the relative apparent reactor 577 residence time, $\langle \hat{\tau} \rangle^{\mathscr{B}/\mathscr{A}}$, depends in a linear manner on λ on a logarithmic scale with base two. By this means, the validity of the CSTR model is valid 579 in these conditions. In contrast, the optimal relative precursor mass flows, $\dot{Q}_{\alpha}^{\mathscr{B}/\mathscr{A}}$, depend on λ in a weakly exponential manner, in order for the inequality constraints to be fulfilled to the specified tolerances. In addition, the relative

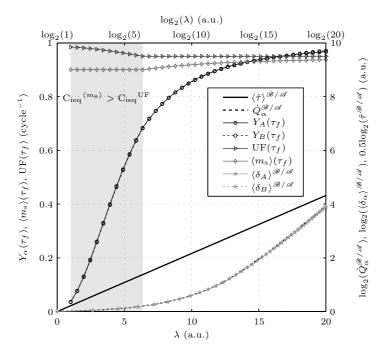


Figure 1: The effects of the geometrical factor, $\lambda \in [1, 20]$, and the optimal precursor mass flow, \dot{Q}_{α} and $\alpha \in \{A, B\}$, (which were set to be equal) on the cost function entities and the terminal inequality constraints when applying *Scale-up strategy I*. The shaded area represents the geometric scaling factors for which the terminal inequality constraint for $\langle m_s \rangle (\tau_f)$ is active.

apparent precursor exposure doses, $\langle \delta_{\alpha} \rangle^{\mathscr{B}/\mathscr{A}}(\tau_f)$ and $\alpha \in \{A, B\}$, determined from Eq. (29) with all gas—surface reactions switched off (i.e. $S_{\alpha} := 0$ and $\alpha \in \{A, B, C\}$), depend on λ in a similar manner as the optimal relative precursor mass flows.

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More importantly, Fig. 1 shows that two different regimes appear, depending on the value of the geometric factor, λ , since only one of the terminal inequality constraints, $\mathbf{C_{ieq}} = [\mathbf{C_{ieq}}^{\langle m_s \rangle}, \mathbf{C_{ieq}}^{\mathrm{UF}}]^T$, is active for each λ . The terminal inequality constraint for $\mathrm{UF}(\tau_f)$ is active for $\lambda \leq 6.5$ when the reference values of Reactor $\mathscr A$ given in Table 3 are used, whereas the constraint for $\mathrm{UF}(\tau_f)$ is active for $\lambda > 6.5$. This implies that the film thickness uniformity constraint is more easily satisfied than the deposition rate per cycle at lower substrate di-

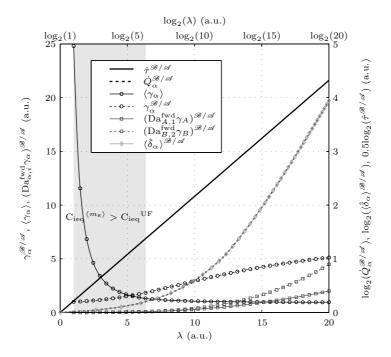


Figure 2: The effects of the geometrical factor, $\lambda \in [1, 20]$, and the optimal precursor mass flow, \dot{Q}_{α} and $\alpha \in \{A, B\}$, (which were set to be equal) on the non-dimensional parameters listed in Table 2 when applying *Scale-up strategy I*. The shaded area represents the geometric scaling factors for which the terminal inequality constraint for $\langle m_s \rangle(\tau_f)$ is active.

mensions, when optimizing precursor yields. Moreover, the extent of the region in which $C_{\rm ieq}^{\langle m_s \rangle} > C_{\rm ieq}^{\rm UF}$ is smaller at higher values of the assigned reference 595 $\langle m_{\rm s} \rangle^{\mathscr{A}}(\tau_f)$ of Reactor \mathscr{A} . The maximum growth rate per cycle, in particular, 596 is obtained for $\langle m_s \rangle^{\mathscr{A}}(\tau_f) := 1$, at which value the film thickness uniformity is 597 equal to one. Finally, Fig. 1 also shows that the optimal precursor yield ap-598 proaches unity as $\lambda \to 20$. This implies that the precursor utilization increases with scale up when applying Scale-up strategy I. 600 One of the conditions imposed by Scale-up strategy I is that the reactor 601 is scaled up without changing any of the operating parameters, \dot{Q}_{β} and \dot{V}_{VP} . Further, dynamic similarity will be maintained to some extent as Re and Pe re-603 main unchanged, whereas the remaining dimensionless numbers in Table 2 will

change. Fig. 2 shows these dimensionless numbers as functions of $\lambda \in [1, 20]$. 605 The way in which the relative reactor chamber residence time, $\hat{\tau}^{\mathscr{B}/\mathscr{A}}$, and the 606 relative apparent precursor exposure doses, $\langle \hat{\delta}_{\alpha} \rangle^{\mathscr{B}/\mathscr{A}}(\tau_f)$ and $\alpha \in \{A, B\}$ (Eq. 607 (30)), depend on the geometrical factor resemble those of their apparent counter-608 parts shown in Fig. 1. The product of the relative surface Damköhler number and the precursor excess number, $(\mathrm{Da}_{\alpha,1}^{\mathrm{fwd}}\gamma_{\alpha})^{\mathscr{B}/\mathscr{A}}$ and $\alpha\in\{A,B\}$, increases 610 strictly with λ when the optimal precursor mass flows are used. This metric is 611 essential since it is a major factor in determining the fractional surface coverage 612 (Eq. (21)), and the resulting deposition rate (Eq. (27a)). Thus, the increase 613 in $\mathrm{Da}^{\mathrm{fwd}}_{\alpha,1}\gamma_{\alpha}$ with λ and \dot{Q}_{α} causes the inherent benefits in precursor utilization when the substrate dimension is scaled up, as shown in Fig. 1. The excess 615 number, γ_{α} , however, (as defined in Eq. (23)) is only valid for a specific time 616 instant, and thus cannot describe the entire molar amount of precursors injected 617 during a single pulse. The total molar amount of precursors injected per molar 618 unit adsorption site, $\langle \gamma_{\alpha} \rangle$, was defined for this reason as:

$$\langle \gamma_{\alpha} \rangle := \frac{\dot{Q}_{\alpha} \rho_{\text{STP},\alpha} \hat{\tau} \Delta \tau_{\alpha}}{\Lambda A M_{\alpha}} \tag{36}$$

Fig. 2 shows that $\langle \gamma_{\alpha} \rangle$ decreases strictly with λ when the optimal precursor mass flows are used. Moreover, approximately 25 times the saturation molar amount is required for the terminal inequality constraints to be satisfied at $\lambda=1$, in these conditions. In contrast, the value of $\langle \gamma_{\alpha} \rangle$ asymptotically approaches unity as $\lambda \to 20$, and ultimately promotes the increase in precursor utilization. Finally, Eq. (36) allows to calculate the maximum theoretical precursor yield from the inverse of $\langle \gamma_{\alpha} \rangle$, i.e. $\max Y_{\alpha}(\tau_f) := \langle \gamma_{\alpha} \rangle^{-1}$.

6.1.1. Film Thickness Uniformity

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Fig. 3 shows the true implication of optimizing the precursor yields in scaleup studies. It is evident that optimizing the cost function of precursor utilization, subject to the inequality constraint of film thickness uniformity, creates a strong coverage gradient towards the trailing edge of the substrate as $\lambda \to 20$. This is a consequence of the cross-flow ALD reactor design, which means that

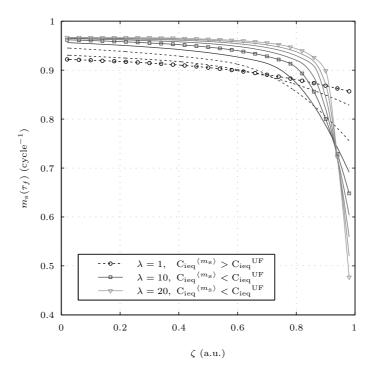


Figure 3: Film thickness profiles as functions of the non-dimensionalized spatial coordinate variable, $\zeta \in [0,1]$, sampled for $\lambda \in [1,20]$ when applying *Scale-up strategy I.* (--) indicates the geometric scaling factors for conditions in which the terminal inequality constraint for $\langle m_s \rangle(\tau_f)$ is active.

the trailing edge of the substrate is exposed to a more depleted precursor flow 634 than the leading edge, and that the cross-substrate film deposition rate is al-635 ways inhomogeneous to some extent. Accordingly, the optimal precursor yields approach unity for high values of the geometric factor (see Fig. 1), and only a 637 very small amount of the injected precursor dose is available for reaction close 638 to the trailing edge. This causes the deposition rate to approach zero rapidly in 639 this region. In contrast, the gradients in this region are significantly gentler for 640 low values of λ , such as those associated with low precursor yields and values of $\langle \gamma_{\alpha} \rangle \gg 1$. It is, however, noteworthy that the terminal inequality constraint for 642 deposition rate uniformity is satisfied for all the profiles shown in Fig. 3. Recall 643 that the uniformity metric that was defined in Eq. (28) relates the absolute deviation of the deposition rate to its spatially averaged mean value. In order to enforce gentler gradients of $m_{\rm s}(\tau_f)$ towards the trailing edge of the substrate, it is suggested that UF (τ_f) can be redefined to describe simply the relative accumulated mass, $m_{\rm s}|_{\zeta=\zeta_{\rm end}}$, at $\zeta=\zeta_{\rm end}$, since the deposition rate will always be a minimum here in the cross-flow reactor design. This formulation overcomes the limitation of a spatially averaged metric (Eq. (28)), while still providing the physical interpretation of a perfectly uniform profile for UF (τ_f) := 1.

6.1.2. Gas-phase Limit-cycle Dynamics

Fig. 4 shows the limit-cycle solution for the gas-phase state and algebraic 653 variables when using Scale-up strategy I and a geometric factor of $\lambda = 20$. It 654 is evident that the state and algebraic variables conform to periodic boundary conditions over the time horizon $[\tau_0, \tau_f]$. The markers indicate the locations 656 of the Radau collocation points for $\zeta \in \{1/2, N_{\text{FVM}} - 1/2\} \cdot (\zeta_{\text{end}} - \zeta_0)/N_{\text{FVM}}$ 657 (i.e. the centers of the first and last FVM elements). The CSTR model dynamic reactor gas-phase material balances (Eq. (1)) have been extended to incorporate 659 the instantaneous formulation of the surface-state dynamics (Eq. (21)) and the accumulated mass deposited (Eq. (27a)), in order to make it possible to 661 compare the results from the complete CSTR model with those obtained from 662 the spatially distributed PDAE model (Eq. (24)). Dashed lines in Fig. 4 show the results from the extended CSTR model. 664

Fig. 4a shows that the nominal velocity, \hat{v}_{ζ} , (Eq. (3d)) is prescribed at $\zeta = \zeta_{\rm end}$ through Eq. (14). The output from the CSTR model follows that 666 of the distributed PDAE model (Eq. (24)) at the outlet, as expected, but it 667 should be remembered that the center of the last FVM element is located at $\zeta = (N_{\rm FVM} - 1/2) \cdot (\zeta_{\rm end} - \zeta_0)/N_{\rm FVM}$. The pressure effects propagate instan-669 taneously throughout the spatial domain, $\zeta \in [\zeta_0, \zeta_{end}]$, during the precursor 670 exposure periods (which are indicated by shaded rectangles) (Fig. 4c), whereas 671 the density of the gas mixture propagates with the mass average velocity (Fig. 672 4b). During the subsequent purge period, the chamber pressure drops in the manner of a first-order dynamical system with a single time constant given by 674

the ratio of chamber volume to pumping speed (Travis and Adomaitis, 2013a,b).

The base-line pressure, \hat{p} (Eq. (3c)), is approached during the subsequent purge

for $\zeta = (N_{\rm FVM} - 1/2) \cdot (\zeta_{\rm end} - \zeta_0)/N_{\rm FVM}$, but it is not fully attained in the

conditions studied here.

More importantly, the spatial distributions of v_{ζ} and ρ (Figs. 4a and 4b) result from the pressure drop across the reactor chamber (Fig. 4c). This pressure 680 drop is ultimately governed by the compressible formulation of the continuity 681 equation and the Navier-Stokes equation (Eqs. (4-5)), and it is determined by 682 the chamber dimensions and process operating parameters $[\dot{V}_{\rm VP}, \dot{Q}_{\beta}, T]$. The 683 resulting pressure drop across the reactor chamber that originates from the non-zero shear stress, Φ_{ζ} , in Eq. (5) implies that the cross-substrate film thick-685 ness deposition rate (Eq. (27a)) is always inhomogeneous despite the injected 686 precursor exposure dose, $\langle \delta_{\alpha} \rangle$. Consequently, it is a combination of the degree of precursor depletion in the flow direction and the magnitude of the pressure 688 drop across the reactor chamber that governs the extent of the deposition profile non-uniformity (see Fig. 3). Moreover, it is noteworthy that Eq. (10) governs 690 that the pressure drop across the reactor chamber is higher for low-volume re-691 actor designs with high aspect ratios, $L/H \gg 1$, and for high carrier gas linear 692 velocities. 693

Finally, the results from the distributed PDAE model (Eq. (24)) in Fig. 4d 694 shows that the α th precursors are separated at all positions of the spatial do-695 main, $\zeta \in [\zeta_0, \zeta_{end}]$. Especially, the high precursor utilization at this geometric 696 factor ensures that negligible amounts of precursor remain after each exposure period in the gas phase, at the start of the subsequent precursor exposure dose 698 period. Undesirable CVD conditions are in this way avoided, as recently in-690 vestigated by Travis and Adomaitis (2013a). However, Fig. 1 shows that the 700 number of reactor chamber volumes purged during the carrier gas purge period, 701 $(\hat{\tau}\Delta\tau_{\beta})/\langle\hat{\tau}\rangle$ (cf. Eq. (35)), decreases linearly with the apparent residence time from that of the base-case Reactor \mathscr{A} when applying Scale-up strategy I. Thus, 703 when the large precursor doses that are associated with lower yields are used, 704 excess precursors remain in the gas phase and the carrier gas purge may be

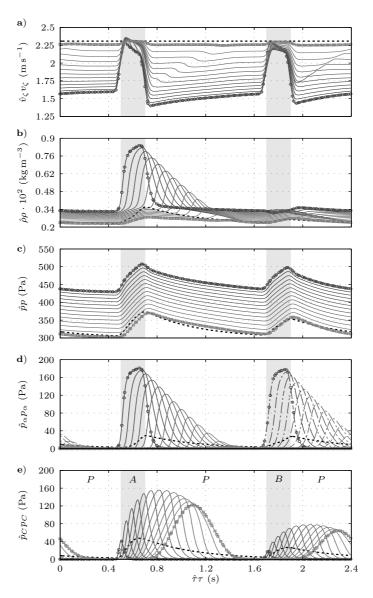


Figure 4: Scaled-up Reactor $\mathcal B$ gas-phase dynamics for a single-pulse horizon $\tau \in [\tau_0, \tau_f]$ and $\lambda = 20$ when using Scale-up strategy I. The limit-cycle solution is spatially resolved for $\zeta = (\jmath - 1/2)(\zeta_{\rm end} - \zeta_0)/N_{\rm FVM}$ and $\jmath \in \{1,3,\cdots,N_{\rm FVM}\}$. $(-\circ -)$ indicates the state and algebraic variables for $\jmath = 1$, while $(-\Box -)$ indicates the corresponding variables for $\jmath = N_{\rm FVM}$. $(-\cdot)$ indicates the limit-cycle solution from the CSTR model. The shaded areas indicate the precursor pulse interval endpoints.

insufficient. It is expected that this phenomenon will be more pronounced at shorter purge periods. In addition, the limit-cycle solution obtained from the CSTR model depicted in Fig. 4d shows that non-negligible amounts of precursors remain in the gas phase from the previous precursor exposure at the start of the subsequent precursor exposure for the given residence time. This will become more evident when examining the accumulated mass trajectory in Section 6.1.3.

713 6.1.3. Growth Surface Limit-cycle Dynamics

Fig. 5 shows the limit-cycle solutions for the growth surface state and al-714 gebraic variables obtained using Scale-up strategy I and a geometric factor of 715 $\lambda = 20$. The spatially distributed mass gain trajectory (Fig. 5a) determined from Eq. (27) can be physically interpreted by means of the underlying chem-717 ical composition of the growth surface (Figs. 5b-5d). In particular, the net 718 contribution from the irreversible reactions (R2a-R2b) to Eq. (27b) is the 719 degree of saturation of the fractional surface coverage onto which the respec-720 tive precursors can adsorb, and the difference in molar masses, ΔM_i (where 721 $i \in \{1, 2, \cdots, 4\}$), of the adsorptive precursors and the associated number of 722 ligands that desorb from the growth surface. Thus, the difference in molec-723 ular mass between the initial and terminal surface species in Reaction (R2a), $\Delta M_1 = M_{\rm Zn(C_2H_5)_2} - \nu M_{\rm C_2H_6}$, results in a net mass increase, whereas that of 725 Reaction (R2b), $\Delta M_2 = M_{\rm H_2O} - (2 - \nu) M_{\rm C_2H_6}$, results in the net contribution to Eq. (27) from this half-reaction being less than zero, when $\nu = 1.37$. The 727 significant difference in net mass contribution to Eq. (27) from each precursor 728 half-reaction is reflected also in the trajectory of the deposition rate uniformity (Fig. 5a). This trajectory falls at the start of the $Zn(C_2H_5)_2$ precursor period 730 and passes through a minimum, as the travelling wave of the precursor propa-731 gates across the substrate. The uniformity subsequently rises when the entire 732 substrate has been exposed. The corresponding effect of the subsequent H₂O 733 precursor exposure is not as pronounced, which is a consequence of the small difference in molecular mass between the initial and terminal surface species in 735

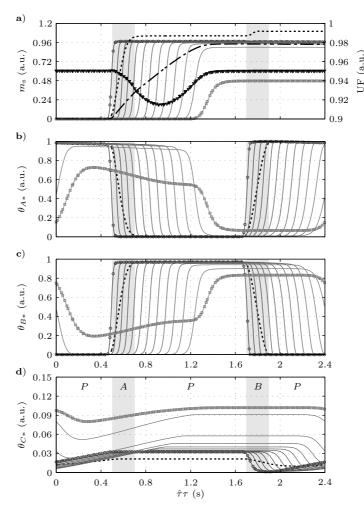


Figure 5: Scaled-up Reactor \mathscr{B} film-growth dynamics for a single-pulse horizon $\tau \in [\tau_0, \tau_f]$ and $\lambda = 20$ when applying Scale-up strategy I. The limit-cycle solution is spatially resolved for $\zeta = (j-1/2)(\zeta_{\rm end} - \zeta_0)/N_{\rm FVM}$ and $j \in \{1,3,\cdots,N_{\rm FVM}\}$. $(-\circ -)$ indicates the state and algebraic variables for j=1, while $(-\Box -)$ indicates the corresponding variables for $j=N_{\rm FVM}$. (--) indicates the limit-cycle solution from the CSTR model, while $(-\cdot -)$ indicates the substrate spatially averaged deposition rate, and $(-\nabla -)$ the uniformity of the film thickness. The shaded areas indicate the precursor pulse interval endpoints.

Reaction (R2b). It is also evident from Fig. 5a that the terminal inequality constraints for $\langle m_{\rm s} \rangle (\tau_f)$ and UF(τ_f) are satisfied.

As the travelling wave of the ${\rm Zn}({\rm C_2H_5})_2$ precursor propagates through the

reactor, the partial pressure, p_A , becomes lower in the direction of the flow (Fig. 739 4d). As discussed in Section 6.1.1 and 6.1.2, this phenomenon originates from 740 the pressure drop (cf. Eq. (10)) across the reactor chamber and the conver-741 sion of available surface OH ligands through Reaction (R2a). As this reaction 742 continues, the coverage of $\theta_{A*} \to 0$ as the growth surface saturates with B*. The subsequent H₂O precursor exposure and half-reaction (see Reaction (R2b)) 744 proceed in an analogous manner, ultimately resulting in the reformation of A*. 745 The degree of saturation at each position in $[\zeta_0, \zeta_{end}]$ is governed by the par-746 tial pressure of precursors in the vicinity of the growth surface (see Eq. (16)). 747 Thus, as $p_{\alpha} \to 0$ in the region close to the trailing edge of the substrate, the 748 conversion rate of surface species falls significantly as the reaction rate $r_i \to 0$. 749 This causes the sharp decrease in deposition rate shown in Fig. 5a and in Fig. 750 3. Likewise, the trailing edge of the substrate is also subject to the most severe 751 dehydroxylation (see Fig. 5d), due to the low H₂O precursor dose level in this 752 region, which limits the rate of Reaction (R2d). The dehydroxylation reaction 753 continues throughout the purge period following H₂O exposure, which reduces 754 the ligand density of surface OH groups, and ultimately the overall deposition 755 rate. 756

Fig. 5a shows that there is a clear distinction between the accumulated 757 mass trajectory, $\langle m_s \rangle$, determined from the spatially distributed PDAE model 758 (Eq. (24)) and that determined from the CSTR model. As expected, the CSTR 759 model predicts an instantaneous net mass increase at the start of each precursor 760 exposure period, whereas the PDAE model predicts a net mass increase that is related to the propagation of the travelling wave of precursors throughout the 762 spatial domain. In addition, the mass gain trajectory from the CSTR model 763 clearly shows the implication of the coexistence of precursors in the gas phase, 764 as previously described in Section 6.1.2. Recall that the maximum growth rate 765 per cycle is obtained for $\langle m_s \rangle(\tau_f) := 1$, and a net mass decrease is expected from the H₂O half-reaction (Reaction (R2b)). A net mass increase in $\langle m_s \rangle$ is, 767 however, predicted by the CSTR model during the H₂O precursor exposure, 768 and its terminal value exceeds one (which means that more than a single monolayer is deposited). However, the model does not describe the gas-phase CVD reactions, instead this phenomenon arises from the adsorption of the remaining $Zn(C_2H_5)_2$ precursor in the gas-phase onto the newly formed OH ligands on the growth surface, which are, in turn, converted instantaneously throughout Reaction (R2b). In contrast, the PDAE model does not predict undesirable CVD conditions, and the accumulated mass trajectory that the model predicts agrees with that expected to arise in true ALD conditions.

777 6.2. Scale-up Strategy II

Fig. 6 shows the cost function entities, $Y_{\alpha}(\tau_f)$ and $\alpha \in \{A, B\}$, and the 778 terminal inequality constraint entities, $\langle m_s \rangle(\tau_f)$ and UF (τ_f) , as functions of 779 the geometric factor, λ , sampled for $\vartheta \in (0,2]$ when using Scale-up strategy 780 II. The results have been calculated with the minimum precursor mass flows, $\dot{Q}_A^{\mathscr{B}} := \dot{Q}_B^{\mathscr{B}}$, that satisfy the terminal inequality constraints (Eq. (33)) for 782 each value of λ and ϑ . In order to maintain the nominal mass fraction for 783 the α th precursor (Eq. (3b)) in the scaled-up Reactor \mathcal{B} , it is necessary that 784 $d\log_2(\dot{Q}_{\alpha}^{\mathscr{B}/\mathscr{A}})/(d\log_2(\lambda)) := \vartheta$ (see also Table 2). However, there is a clear 785 discrepancy between the optimal precursor mass flows, $\dot{Q}_{\alpha}^{\mathscr{B}/\mathscr{A}}$, and $\dot{Q}_{\alpha}^{\mathscr{B}/\mathscr{A}}:=\lambda^{\vartheta}$ for all $\vartheta \in [0,2]$ shown in Fig. 6d. For this reason, the scaling rule for the 787 nominal mass fraction, i.e. $\dot{Q}_{\alpha}^{\mathscr{B}/\mathscr{A}} := \lambda^{\vartheta}$, is considered not adequate to achieve 788 the assigned design criteria in the scaled-up design $Reactor \mathcal{B}$. 789

Fig. 6c shows that the highest precursor yields, $Y_{\alpha}(\tau_f)$, are obtained when 790 scaled-up Reactor \mathscr{B} is operated at the lowest possible carrier gas mass flow, \dot{Q}_{β} . 791 As was the case for Scale-up strategy I, precursor utilization increases strictly 792 with λ when using Scale-up strategy II. However, for $\vartheta = 2$ (which corresponds 793 to maintaining constant reactor residence time, $\hat{\tau}$, as indicated in Table 2) ap-794 proximately the same yield is obtained for all $\lambda \in \{1, 20\}$. This is a consequence 795 of the way in which the product of the surface Damköhler number and precursor excess number, $\gamma_{\alpha} \mathrm{Da}_{\alpha,i}^{\mathrm{fwd}} \propto \dot{Q}_{\alpha} \lambda^{2-2\vartheta}$ for the α th precursor, depends on ϑ . 797 Thus for $\vartheta = 2$, the inherent increase in this non-dimensional metric, ultimately promotes the reaction rate through Eq. (21) and the resulting deposition rate

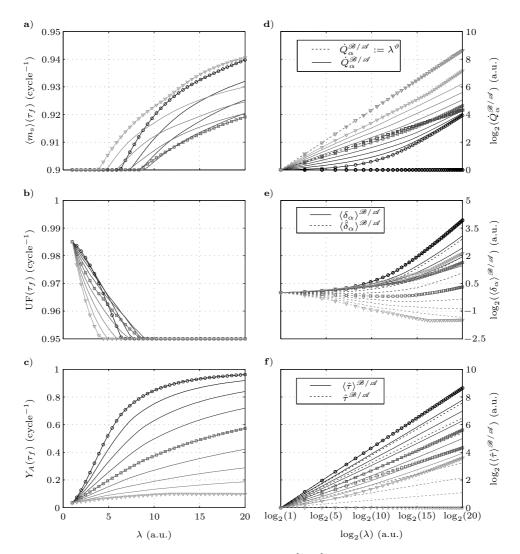


Figure 6: The effects of the geometrical factor, $\lambda \in [1,20]$, the optimal precursor mass flow, \dot{Q}_{α} and $\alpha \in \{A,B\}$, (which are set to be equal) on the cost function entities and terminal inequality constraint entities when using *Scale-up strategy II*, with $\vartheta \in (0,2]$. (o) indicates the solution for $\vartheta = 0.0$ (i.e. that of *Scale-up strategy II*), while (\square) indicates that for $\vartheta = 1.0$, and (\triangledown) that for $\vartheta = 2.0$.

through Eq. (27), with the geometric scaling factor is lost.

More importantly, Figs. 6e and 6f show clearly that the discrepancies between the apparent reactor residence time, $\langle \hat{\tau} \rangle$, and the apparent precursor

exposure dose, $\langle \delta_{\alpha} \rangle$, from their respective non-dimensional counterparts, $\hat{\tau}$ and 803 $\langle \hat{\delta}_{\alpha} \rangle$, increases as $\vartheta \to 2$. These discrepancies arise from the pressure drop 804 across the spatial domain, $\zeta \in [\zeta_0, \zeta_{end}]$, (see Section 6.1.2 and Fig. 4c) and 805 result in the spatial distribution of v_{ζ} and ρ . For this reason, a higher averaged 806 linear flow rate, $\langle v_{\ell} \rangle$ is necessary to maintain a constant residence time, $\hat{\tau}$, as $\lambda \to 20$. Thus, larger pressure drops are obtained as $\lambda \to 20$ and as the linear 808 flow rate increases, since $\Phi_{\zeta} \propto v_{\zeta}$ in Eq. (10), subject to \dot{Q}_{β} , $\dot{V}_{\rm VP} \propto \lambda^{\vartheta}$. This gives the larger discrepancies shown in Figs. 6e and 6f under these conditions. 810 In particular, $\hat{\tau}$ is maintained for the scaled-up Reactor \mathscr{B} proposed in Table 2, 811 whereas its apparent value, $\langle \hat{\tau} \rangle$, increases strictly with the geometric factor. In 812 contrast, these values, and those of the precursor exposure dose, coincide over 813 the entire range of $\lambda \in [0, 20]$ when using Scale-up strategy I. In conclusion, the 814 non-dimensional variables in Table 2 and the CSTR model become less valid 815 as $\vartheta \to 2$. Thus, the results reported here clearly motivate the utility of the 816 spatially distributed PDAE model in combination with dynamic optimization 817 methods for maximizing the precursor utilization in the scaled-up system while 818 maintaining fixed absolute growth rate and its relative uniformity. 819

Fig. 6a shows that $\langle m_s \rangle (\tau_f)$ depends on ϑ in a weakly convex manner for 820 each value of $\lambda \in [10, 20]$. Fig. 7 shows this more clearly, where $\langle m_s \rangle (\tau_f)$ is 821 plotted as a function of ϑ and sampled for $\lambda \in [10, 20]$. The terminal inequality 822 constraint for the deposition rate uniformity is active at these geometric factors. 823 Figs. 7a and 7c show the noteworthy result that the highest values of both 824 $\langle m_s \rangle (\tau_f)$ and $Y_A(\tau_f)$ are obtained for $\lambda = 20$ in the entire range, $\vartheta \in (0,2]$. The magnitude of the normalized deposition rate per cycle is governed to a 826 large extent by the apparent precursor exposure dose, $\langle \delta_{\alpha} \rangle$. Thus, it can be 827 concluded that the weak convex dependence of relative apparent exposure dose 828 with ϑ , shown in Fig. 7b underlies the dependence of $\langle m_s \rangle (\tau_f)$ shown in Fig. 829 7a. The magnitude of the apparent precursor exposure dose (Eq. (30)), in turn, arises from the spatially distributed pressure across the reactor and the mass 831 fraction of the injected precursor pulse, $\hat{\omega}_{\alpha}$. In this context, larger pressure 832 drops are obtained as $\vartheta \to 2$ and as the linear flow rate increases, since $\Phi_{\zeta} \propto v_{\zeta}$, 833

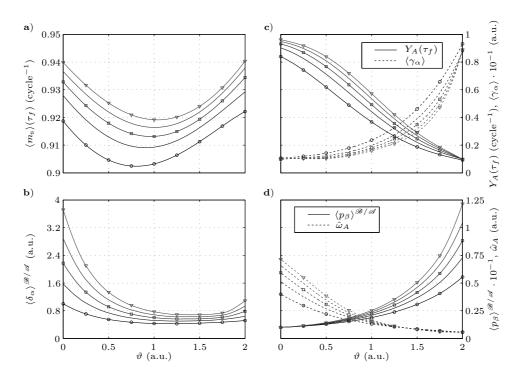


Figure 7: The effects of the geometrical factor, $\lambda \in \{10, 12.5, 15, 17.5, 20\}$, and the optimal precursor mass flow, \dot{Q}_{α} and $\alpha \in \{A, B\}$, (which were set to be equal) on the cost function entities and terminal inequality constraints when using *Scale-up strategy II* with $\vartheta \in (0, 2]$. (\circ) indicates the solution for $\lambda = 10$, (\square) for $\lambda = 15$, and (\square) for $\lambda = 20$.

subject to \dot{Q}_{β} , $\dot{V}_{\rm VP} \propto \lambda^{\vartheta}$. This is clearly shown in Fig. 7d, where the spatially 834 averaged stationary carrier gas pressure, $\langle p_{\beta} \rangle$, is plotted as a function of ϑ . In 835 addition, the precursor mass fraction decreases with ϑ , as a consequence of the 836 minimal precursor mass flows, which must ensure that the terminal inequality 837 constraints (see Eq. (33)) are satisfied, and the assigned carrier gas mass flow, 838 $\dot{Q}_{\beta} \propto \lambda^{\vartheta}$, determined from the scaling rules in Table 2 (Fig. 7d). Therefore, 839 the compromise between the steeper pressure gradient and the lower precursor 840 mass fractions causes the observed convex dependence of $\langle \delta_{\alpha} \rangle$ on $\vartheta \in [0,2]$. Moreover, Fig. 7b makes it clear that significantly higher molar amounts of 842 precursors must be injected per molar unit adsorption site, $\langle \gamma_{\alpha} \rangle$, as $\vartheta \to 2$, and 843 the precursor yield falls accordingly. Thus, the film thickness profiles shown in

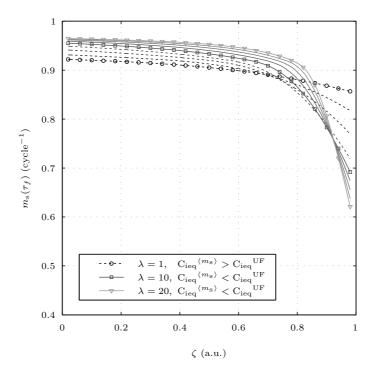


Figure 8: Film thickness profiles as functions of the non-dimensionalized spatial coordinate variable, $\zeta \in [0,1]$, sampled for $\lambda \in [1,20]$ when using *Scale-up strategy II* with $\vartheta = 0.5$. (--) indicates the geometric scaling factors for those profiles for which the terminal inequality constraint for $\langle m_s \rangle(\tau_f)$ is active.

Fig. 8 for $\vartheta=0.5$ are more uniform for higher geometric factors than those obtained when using Scale-up strategy I (Fig. 3), since the overall precursor yields are lower when Scale-up strategy II is used. It is, however, important to remember that the negative pressure gradient in the ζ -direction (Fig. 4c) also influences the deposition rate through the precursor partial pressure: it lowers the driving force for the reaction closer to the trailing edge of the substrate, and thereby contributes to the formation of a non-uniform deposition profile.

6.2.1. Gas-phase and Growth Surface Limit-cycle Dynamics

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The reason for exploring *Scale-up strategy II* was to develop a method which provides a sufficient length of the purge period to remove precursors in the gas

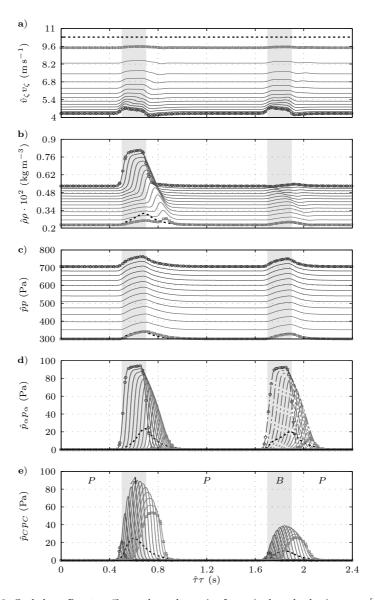


Figure 9: Scaled-up Reactor $\mathcal B$ gas-phase dynamics for a single-pulse horizon $\tau \in [\tau_0, \tau_f]$ and $\lambda = 20$ when using Scale-up strategy II with $\vartheta = 0.5$. The limit-cycle solution is spatially resolved for $\zeta = (\jmath - 1/2)(\zeta_{\rm end} - \zeta_0)/N_{\rm FVM}$ and $\jmath \in \{1, 3, \cdots, N_{\rm FVM}\}$. $(-\circ -)$ indicates the state and algebraic variables for $\jmath = 1$, while $(-\Box -)$ indicates the corresponding variables for $\jmath = N_{\rm FVM}$. $(-\cdot)$ indicates the limit-cycle solution from the CSTR model. The shaded areas indicate the precursor pulse interval endpoints.

phase remaining from the previous exposure period at the start of the subsequent 855 precursor period. This was achieved by reducing the reactor chamber residence 856 time while maintaining the nominal carrier gas pressure, \hat{p} . Fig. 9 shows the 857 limit-cycle solution for the gas-phase state and algebraic variables when using 858 Scale-up strategy II, a geometric factor $\lambda = 20$, and $\vartheta = 0.5$. The results show clearly the implications of the scaling rule for the carrier gas manipulated 860 variables, i.e. \dot{Q}_{β} , $\dot{V}_{\rm VP} \propto \lambda^{\vartheta}$, from Table 2. The travelling wave of precursors and the density of the gas mixture propagate with a significantly higher mass average 862 velocity as expected. In addition, the overall higher mass average velocity gives 863 rise to a significantly higher negative pressure gradient in the ζ -direction than that shown in Fig. 4. Moreover, at the start of the purge period, the total 865 pressure relaxes to the stationary value of the carrier gas partial pressure at each 866 position in the spatial domain, $\zeta \in [\zeta_0, \zeta_{end}]$. In particular, the total pressure at $\zeta = (N_{\text{FVM}} - 1/2) \cdot (\zeta_{\text{end}} - \zeta_0) / N_{\text{FVM}}$ relaxes to the base-line pressure, \hat{p} , and 868 this trajectory resembles that of the CSTR model.

More importantly, it is evident that the reduction in residence time, which 870 arises as a consequence of prescribing \dot{Q}_{β} , $\dot{V}_{\rm VP} \propto \lambda^{\vartheta}$ with $\vartheta = 0.5$, is suffi-871 cient to ensure that the gas-phase precursors and the reaction by-products are 872 transported out of the reactor chamber (at all positions) before the start of the 873 subsequent precursor pulse period. In addition, the stationary carrier gas par-874 tial pressure is maintained after a fraction of the purge period, when assigned 875 to $\hat{\tau}\Delta\tau_P := 1.0$ (s). Likewise, the appearance of the accumulated mass tra-876 jectory during the precursor exposures (see Fig. 10) shows that precursors do not coexist in the gas phase. These results confirm that the carrier gas pulse 878 period can be reduced for this set of \dot{Q}_{β} , $\dot{V}_{\rm VP} \propto \lambda^{\vartheta}$ and $\vartheta = 0.5$. The growth 879 rate per cycle time, $\hat{\tau}\Delta\tau$, can in this way be significantly reduced, as previously 880 shown in Holmqvist et al. (2013b). Alternatively, the reactor chamber residence 881 time can be safely increased, while preventing the undesirable CVD conditions, by choosing $\vartheta < 0.5$. In any case, there is no reason to choose $\vartheta > 0.5$, since 883 this causes a reduction in the precursor mass fractions and the partial pressures (Fig. 9d), and lowers the overall precursor yield (Figs. 6 and 7).

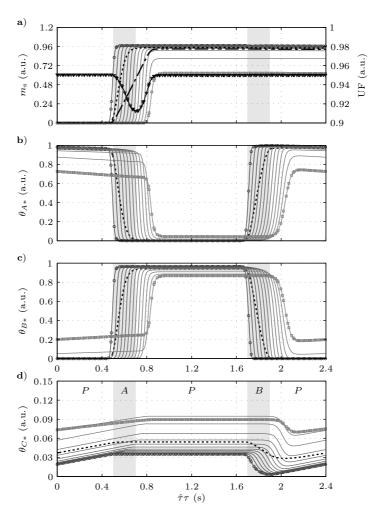


Figure 10: Scaled-up Reactor \mathscr{B} film-growth dynamics for a single-pulse horizon $\tau \in [\tau_0, \tau_f]$ and $\lambda = 20$ when using Scale-up strategy II with $\vartheta = 0.5$. The limit-cycle solution is spatially resolved for $\zeta = (\jmath - 1/2)(\zeta_{\rm end} - \zeta_0)/N_{\rm FVM}$ and $\jmath \in \{1, 3, \cdots, N_{\rm FVM}\}$. $(- \circ -)$ indicates the state and algebraic variables for $\jmath = 1$, while $(-\Box -)$ indicates the corresponding variables for $\jmath = N_{\rm FVM}$. $(- \cdot)$ indicates the limit-cycle solution from the CSTR model, while $(- \cdot -)$ indicates the substrate spatially averaged deposition rate and $(- \nabla -)$ its film thickness uniformity. The shaded areas indicate the precursor pulse interval endpoints.

86 7. Concluding Remarks

This paper presents a novel model-based methodology for scaling up continuous cross-flow ALD reactor systems that use temporally separated precursor

pulsing. The overall objective of the scale-up method was to maintain dynamic 889 similarity associated with identical absolute growth rates and to maintain uni-890 formity close to that of the base-case reactor, while maximizing precursor uti-891 lization. A one-dimensional, physically-based process model was developed that 892 integrates components that describe the reactor-scale gas-phase dynamics and surface-state dynamics with experimentally validated surface reaction kinetics 894 from previous studies (Holmqvist et al., 2012, 2013a). By this means, dynamic similarity was investigated by constructing all equations that governs the gasphase and surface-state algebraic variables, together with their boundary and 897 initial conditions, in non-dimensional form. The impact of the geometric scaling factor and the process manipulated variables on the non-dimensional variables 899 was subsequently thoroughly investigated. 900

The scale-up method developed comprises two steps: the carrier gas manip-901 ulated variables are scaled in parallel to maintain various degrees of dynamic 902 similarity in the scaled-up reactor, and the mass flow of precursors is subsequently optimized to give maximum yields under the terminal constraints of 904 absolute deposition rate and its relative uniformity. To describe accurately the 905 steady cyclic operation of the ALD reactor, the limit-cycle dynamic solution that arises in this way was discretized using a collocation scheme in time. The 907 optimization problem is fully discretized in the collocation method by approx-908 imating state algebraic and control variables by Lagrange polynomials, which 909 results in one large NLP. This NLP is solved simultaneously for all state, al-910 gebraic and control variables that describe the approximated trajectories and this ensures that the limit-cycle criteria, and the terminal design criteria, are 912 fulfilled. 913

In particular, it was demonstrated that the maximum precursor yields were promoted at higher substrate dimensions. Consequently, the trailing edge of the substrate was exposed to a lower precursor dose, and caused in this way strong deposition rate gradients in this region. Moreover, the results showed that higher carrier gas linear velocities gave rise to larger pressure drops across the reactor chamber, and thereby contributed significantly to the formation of a

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non-uniform deposition profile. In conclusion, it is a combination of the degree 920 of precursor depletion in the flow direction and the magnitude of the pressure 921 drop across the reactor chamber that governs the extent of the deposition profile 922 non-uniformity. In addition, the interaction between dose and purge periods 923 was revealed by analyzing the spatially distributed limit-cycle dynamic solution for the gas-phase precursor partial pressures, and the solution obtained for the 925 resulting accumulated mass gain trajectory. By this means, process regimes 926 were identified in which surface reactions occurred under CVD conditions. True 927 ALD conditions, associated with fully decoupled binary precursor doses, could 928 be retained by lowering the reactor chamber residence time at the expense of 929 lower precursor utilization. 930

The proposed scaling rules are based on nominal algebraic and state vari-931 ables determined from the CSTR model. The validity of the CSTR model has 932 been assessed by examining the discrepancy between the nominal and the ap-933 parent reactor chamber residence times, and between precursor exposure doses. 934 Results shown here indicate that these discrepancies are higher for low-volume 935 reactor designs with high aspect ratios, $L/H \gg 1$, and for high carrier gas linear 936 velocities. The CSTR model, however, is valid over a wider range of geometrical 937 scaling factors for reactor designs with lower aspect ratios. The results reported 938 here clearly motivate the utility of models based on PDAEs in combination with 930 dynamic optimization methods for maximizing the precursor utilization in the 940 scaled-up system while maintaining a high value of the growth rate per cycle, 941 which ensures acceptable reactor throughput. The result is a short set of optimal scaling guidelines that can be followed to maintain deposition profiles and 943 chemistry identical when adapting a laboratory-scale thin-film process to meterscale manufacturing equipment. These guidelines allow the knowledge obtained and methods developed when working with centimeter-scale substrates to be 946 directly and easily translated to larger reactors.

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952 Nomenclature

953	Roman letters		
954	A	substrate surface area	m^2
955	A'	cross section area of the reaction chamber	m^2
956	C_{eq},C_{ieq}	equality and inequality constraint vector	_
957	$\mathrm{Da}_{\alpha,i}$	surface Damköhler number	_
958	$\hat{\mathscr{D}}_{lphaeta},\mathscr{D}_{lphaeta}$	binary diffusivity	$m^2 s^{-1}, -$
959	\mathbf{F}	system of differential algebraic equations	_
960	\mathbf{g}	response function	_
961	H	reactor height	m
962	k_i	reaction rate constant	$(\text{mol m}^{-2})^{1-n_i} \text{Pa}^{-1} \text{s}^{-1},$
963			$(\text{mol m}^{-2})^{1-n_i} \text{ s}^{-1}$
964	L	reactor chamber length	m
965	M_{α}	molar mass	${\rm kgmol^{-1}}$
966	$\hat{m}_{\mathrm{s}},m_{\mathrm{s}}$	film mass increment	$\mathrm{kg}\mathrm{m}^{-2},-$
967	n_i	surface reaction order	_
968	Pe	Peclet number	_
969	\hat{p},p	pressure	Pa, -
970	\dot{Q}_{lpha}	volumetric flow rate at STP	$\mathrm{Nm^3s^{-1}}$
971	R	universal gas constant	$\mathrm{J}\mathrm{mol}^{-1}\mathrm{K}^{-1}$
972	Re	Reynolds number	_
973	r_i	surface reaction rate	$\mathrm{mol}\mathrm{m}^{-2}\mathrm{s}^{-1}$
974	S_{lpha}	source term in the general transport equation	${\rm kg}{\rm m}^{-3}{\rm s}^{-1}$
975	T	temperature	K
976	t	dimensional time	S
977	UF	film thickness uniformity factor	_
978	u	design variables	_
979	V	reactor chamber volume	m^3
980	$\dot{V}_{ m VP}$	volumetric flow rate of the vacuum pump	$\mathrm{m}^3\mathrm{s}^{-1}$
981	$\hat{v}_{\zeta}, v_{\zeta}$	linear velocity	$m s^{-1}, -$
982	\mathbf{w}	algebraic variables	_
983	x	state variables	_
984	Y_{α}	precursor yield	$cycle^{-1}$

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model output variables
      \mathbf{y}
985
                     dimensional spatial coordinate
                                                                                       \mathbf{m}
986
       Greek\ letters
      \boldsymbol{\beta}
                     model parameter vector
                     excess number
      \gamma_{\alpha}
989
      \Delta t_{\alpha}, \Delta \tau_{\alpha} pulse duration
                                                                                       s. -
990
      \hat{\delta}_{\alpha}, \, \delta_{\alpha}
                     half-cycle average precursor dose
                                                                                       Langmuir
991
                     non-dimensional spatial coordinate
      ζ
992
                     fractional surface coverage of surface species
      \theta_{\kappa}
                                                                                       \rm mol\,m^{-2}
                     maximum molar concentration of surface sites
      Λ
                     geometric scaling factor
995
                                                                                       kg m^{-1} s^{-1}, -
                     dynamic viscosity of the gas mixture
      \hat{\mu}, \mu
996
                     numbers of surface OH groups reacting
      \nu
997
                     with each Zn(C_2H_5)_2
998
      \xi_i
                     surface reaction stoichiometric coefficient
999
                     density of the gas mixture
                                                                                       \mathrm{kg}\,\mathrm{m}^{-3},\,-
      \hat{\rho}, \, \rho
1000
                     non-dimensional time
      \tau
      \Phi_{\zeta}
                     shear stress
                                                                                       Pa
1002
      Φ
                                                                                       {
m cycle^{-1}}
                     cost function
1003
      \Pi_{\alpha}
                     characteristic function of \tau and \Delta \tau_{\alpha}
1004
                     mass fraction of gaseous species
      \hat{\omega}_{\alpha}, \, \omega_{\alpha}
1005
      Subscripts and superscripts
                      nominal state and algebraic variables
1007
      0
                     initial value
1008
                     gaseous species indices
      \alpha, \beta
1009
      i
                     surface reaction index
1010
                     surface species index
      \kappa
1011
                     state and algebraic variables in Reactor \mathcal{A}
1012
                     state and algebraic variables in Reactor~\mathcal{B}
      \mathscr{B}
1013
      STP
                     state variable at STP
                     solid
1015
      \mathbf{S}
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