Incommensurability and Vagueness

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Abstract: This paper casts doubts on John Broome’s view that vagueness in value comparisons crowds out incommensurability in value. It shows how vagueness can be imposed on a formal model of value relations that has room for different types of incommensurability. The model implements some basic insights of the ‘fitting attitudes’-analysis of value.

1. Preliminaries
Two items are commensurable in value if and only if one of them is better than the other or if they are equally as good. They are incommensurable if none of these relations obtains between them (cf. Raz 1985-86). Given incommensurability, not even a purely ordinal measure is available for comparison: We cannot represent the relationship between the items by assigning a number to each that specifies the position of that item in the value ordering.¹

Is it plausible to expect the existence of incommensurabilities? If the domain of items contains objects of different ontological kinds, examples of incommensurabilities will of course be plentiful. It doesn’t make sense to say that a state of affairs is better than a person, or that a property and an event are equally as good. But what if the compared items belong to one and the same ontological category? Even then incommensurabilities will be easy to find if some items do not fall under the kind of value we happen to be interested in. It doesn’t make sense to inquire whether Mozart was a better scientist than Shakespeare, or whether Michelangelo and Phidias were equally good composers. Assume, however, that we focus on cases in which the objects that are being compared do fall under the value under consideration. Is it plausible to expect incommensurabilities even in cases like this?

To find incommensurabilities in cases of this kind, Ruth Chang (2002b) suggests we consider pairs of objects that realize the value in question (i.e., that fall under ‘the covering consideration’, as she puts it) but do so in importantly different ways – so different that we would deny that one of them is better and the other worse. As an example, think of two great artists, such as Mozart (x) and Michelangelo (y). Each of them demonstrated a very high degree of artistic excellence, but none of them, I take it, could be said to have been a better artist than the other. To exclude the remaining possibility that they were equally good as artists, Chang suggests we can use ‘The

¹ This does not exclude, of course, the possibility that some more complex numerical representation of their value relationship might still be possible; say, by assigning vectors of numbers to items. So, to that extent, referring to the relationship in question as ‘incommensurability’ is somewhat arbitrary. An alternative label could be ‘incomparability’, but even this label is problematic. Furthermore, I am going to use ‘incomparability’ in a more restricted sense below.
Small-Improvement Argument’. The argument goes like this: We are asked to envisage a third artist, \(x^+\), very similar to \(x\) but slightly better than the latter. \(x^+\) is a fictional figure – a slightly improved version of Mozart – perhaps Mozart who lived a little longer and had time to compose yet another Requiem and a couple of operas. Now, the idea is that \(x^+\) could well be a better artist than \(x\) without thereby being better than \(y\). Such a ‘one-sided’ improvement would have been impossible, however, if \(x\) and \(y\) had been equally good: Anything better than one would then have been better than both. That betterness and equal goodness are related to each other in this way is a conceptual truth.

To take another example, suppose we compare two different holiday options: a trip to Australia (\(x\)) and a trip to South Africa (\(y\)). If none of them is better than the other, as we might well believe, then they aren’t equally good either, since a slight improvement of one option, say, a small discount on the trip to Australia (\(x^+\)), is not enough to make it better than the other option. Or, again, suppose we compare two different careers, one of a philosopher and the other of an architect. If we deny that one career is better than the other, then we would normally judge that the former wouldn’t be better the latter even if it were slightly improved.

Essentially the same form of argument can be found in de Sousa (1974), Broome (1978), Parfit (1984, p. 431), Sinnott-Armstrong (1985) and Raz (1985/86). In his classical paper, Joseph Raz refers to the possibility of a ‘one-sided’ improvement as ‘the mark of incommensurability’ and presents the argument as follows: “We have here a simple way of determining whether two options are incommensurate given that it is known that neither is better than the other. If it is possible for one of them to be improved without thereby becoming better than the other or if there can be another option which is better than the one but not better than the other then the two original options are incommensurate.” (Raz 1985/86, p. 121)

While this argument if taken in abstracto is compelling, it is open to two objections. There is an epistemic objection to begin with: When we try to apply the argument to real-life cases, we need to take into consideration the possibility that our comparative judgments might be mistaken. Thus, in the case of Mozart and Michelangelo, we have been taking for granted that the former wasn’t a better artist than the latter, or vice versa. But is it really something we can know for certain? As Chang (2002, pp. 668f) puts it:

How can we be sure that our judgments in these cases are correct? ... The items for which the Small-Improvement Argument most plausibly holds are evaluatively very different – they are items that bear very different aspects of the covering consideration – and figuring out how evaluatively very different items compare is, generally speaking, no easy matter. Perhaps in these hard cases, all we can rationally judge is that we are uncertain as to which relation holds.

So, perhaps, \(x\) after all is better or worse than \(y\), or they might even be equally as good (in which case \(x^+\) is better than both). It is only that we cannot know which of these value relations obtains, if any.

The second objection is semantic in nature and has to do with the potential vagueness in comparative value claims. Even if it isn’t true that \(x\) is better than \(y\), this needn’t mean that it is false. There is yet another possibility: The claim in question might instead be neither true nor

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2 While Raz thought that the possibility of a ‘one-sided’ improvement was a mark of incommensurability, he wasn’t prepared to say that this mark would always be available: ‘it may be the case that neither [option] is better than the other nor are they of equal value, and yet any conceivable option which is better than the one is better than the other. The same could be the case for any option worse than one of them.’(Raz, ibid.)
false. In other words, it might be indeterminate (vague) whether $x$ is better than $y$. The same applies to the other two relations. Perhaps it is indeterminate whether $y$ is better than $x$ and likewise indeterminate whether these items are equally good. Note, however, that while for each of these relations it might be indeterminate whether it obtains or not, it could still be determinate that one of them does obtain between $x$ and $y$. In other words, it could still be determinately true that $x$ and $y$ are not incommensurable.

Given this possibility of vagueness in value comparisons, the Small-Improvement Argument loses its force. The introduction of $x^+$ does not allow us to definitely rule out the possibility of $x$ and $y$ being equally good, as long as we cannot definitely establish that $x^+$ is not better than $y$.

The following are mutually compatible claims:

(i) It is indeterminate whether $x$ is equally as good as $y$.
(ii) It is determinate that $x^+$ is better than $x$.
(iii) It is indeterminate whether $x^+$ is better than $y$.

In addition, these three claims are jointly compatible with it being determinate that $x$ and $y$ are commensurable.

The epistemic objection and the vagueness objection make it difficult to conclusively argue, in particular cases, that a case under consideration indeed is an instance of incommensurability. Chang (2002b) tries to answer both objections, but it is disputable whether she succeeds. My objectives in this paper are different: I would like to address the relationship between incommensurability and vagueness in a more general way. The point of departure will be an argument by John Broome who has questioned whether vagueness and incommensurability can at all peacefully co-exist with each other (cf. Broome 1997 and 2004). On his view, the former crowds out the latter, so to speak: If we allow for potential vagueness in value comparisons, as we obviously should, we cannot at the same time find room for incommensurabilities. If Broome is right, then friends of incommensurability would be in serious trouble. I will argue, however, that Broome’s argument is not convincing: The principle it rests on is open to criticism (section 2). I will then sketch a general modelling of value relations that makes ample room for incommensurability (section 3) and show how this modelling can be injected with vagueness (section 4).

In what follows, I am going to interpret vagueness on the supervaluationist lines. Thus, a sentence shall be said to be vague (indeterminate) if and only if it comes out as true on some plausible sharpenings (on some ‘precisifications’) of the predicates it contains and as false on other plausible sharpenings. Supervaluationism is a controversial theory and it has its share of

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3 As it should be clear, I am here rejecting the epistemic interpretation of vagueness, on which a vague sentence does have a definite truth-value, but that truth-value is inaccessible to knowledge. (For different variants of such an epistemicist account, cf. Williamson (1994) and Sorensen (2001).) Thus, I take the vagueness objection and the epistemic objection to be distinct from each other.

4 In what follows, I will use expressions of the form ”It is indeterminate whether …” as synonymous with ‘It is neither true nor false that …’. ‘It is determinate that …’ will stand for ‘It is true that …’. Note that ‘It is indeterminate whether …’ is not the negation of ‘It is determinate that …’. Instead, this sentence negates ‘It is determinate whether …’, which is a short for the disjunction ‘It is determinate that … or determinate that not …’. I will for the most part freely move between object-language formulations such as ‘It is determinate that …’ or ‘It is indeterminate whether …’ and the meta-linguistic constructions ‘The sentence “…” is true’ or ‘The sentence “…” is neither true nor false’. I hope this sloppiness in usage will not lead to misunderstandings.

5 There are close similarities between my views on the interrelation between incommensurability and vagueness and those of Mozaffar Qizilbash. See his (2007). But the account of incommensurability to be presented here is quite different from Qizilbash’s proposal.
problems. However, it is a simple theory and it is easy to work with. Also, it is an approach favoured by Broome himself, which makes it convenient to use it when his argument is being examined. I do not think, however, that the main conclusions to be reached in what follows crucially depend on the particular theory of vagueness that is being assumed.

2. Does vagueness crowd out incommensurability?

Rather than going through Broome’s argument in full generality, I will present one of its applications, in population ethics. If the argument fails in this application, then it cannot be universally valid.

In *Weighing Lives*, Broome critically examines and finally rejects what he calls “the intuition of neutrality”. He puts this idea as follows: ‘We think intuitively that adding a person to the world is very often ethically neutral. We do not think that just a single level of wellbeing is neutral …’ (Broome 2004, p. 143; for an extended discussion of this idea, see also Broome 2005.) That is, there is on this view a range of well-being levels, let’s call it the **neutral range**, such that, everything else being equal, adding an extra person with a well-being level within that range is neutral from the ethical point of view. The idea behind the intuition of neutrality goes back to Jan Narveson’s well-known criticism of total-sum utilitarianism: ‘We are in favour of making people happy, we are neutral about making happy people.’ (Narveson 1973) This intuition has been attractive to many philosophers, Broome himself included. Only gradually has he come to the conclusion that it cannot be maintained, after all.

The intuition of neutrality can be given either a deontological or an axiological interpretation. That is, it can be understood as addressing either the question whether we ought to add extra people to the world or the question about the value of such an addition. Broome focuses on the axiological interpretation. On that reading, the intuition states that there is a range of well-being levels—‘the neutral range’—such that adding a person with a well-being level within that range does not make the world **either better or worse** ceteris paribus.

As Broome points out, the neutrality range is not supposed to be unbounded. It is certainly bounded from below: Adding *un*happy people makes the world worse. And, at least according to some friends of the neutrality intuition (though perhaps not according to radicals as Narveson), it may also be bounded from above: Adding *very* happy people might make the world better. Still, within such boundaries, the neutral range is broad enough to contain a fair number of distinct well-being levels. In what follows, I shall refer to them as the **neutral levels** of well-being.

Does the neutrality of additions mean that the world with an added person is **equally as good** as the world without that addition? As Broome points out, this equal-goodness interpretation of neutrality cannot be upheld, as long as there is more than one neutral level. The argument goes like this: Adding a person with a well-being at a higher neutral level would be better, ceteris paribus, than adding the same person with a lower neutral level. This follows from what Broome

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6 For criticisms of supervaluationism, see especially Williamson (1994), ch. 5. For an extended defense, see Keefe (2000).
8 This section draws on a material from Rabinowicz (forthcoming-a).
9 In its more general form, the principle claims that adding *any* number of people at well-being levels within the neutral range is, ceteris paribus, ethically neutral.
10 ‘Interpreted axiologically, in terms of goodness, the intuition is that if a person is added to the population of the world, her addition has no positive or negative value in itself.’ (Broome 2004, pp. 145f) Cf. also Broome (2005), p. 402: ‘The intuition is that adding a person to the population is ethically neutral. By this, I really mean that a world that contains an extra person is neither better nor worse than a world that does not contain her.’
calls the principle of personal good: If worlds A and B have the same population and some people in A have higher levels of well-being than in B while everyone else has the same level of well-being in both worlds, then A is better than B (cf. Broome 2004, p. 58). But then, if one of the worlds with the expanded population is better than the other, by the principle of personal good, they obviously cannot both be equally as good as the original world, without this expansion. Thus, the equal-goodness interpretation of neutrality is untenable.

If we nevertheless want to hold on to the neutrality intuition in its axiological version, the only remaining option is to assume that the worlds with added people at neutral levels of well-being are all incommensurable with the original world, ceteris paribus. Some such worlds can then still be better than others (just as x is better that x, in the examples in the previous section, even though both these items might be incommensurable with a third item y). Thus, the principle of personal good won’t need to be compromised.

It is at this point that vagueness considerations become relevant for Broome’s criticism of the intuition of neutrality. To reject the incommensurability interpretation of neutrality, Broome appeals to a general principle that is supposed to govern vague comparatives. Broome’s principle states, for every comparative ‘Fer than’ and for all x and y, that if it is determinate that y is not Fer than x, then it is determinate whether or not x is Fer than y. Or, in Broome’s formulation:

**Collapsing Principle:** For all x and y, if it is false that y is Fer than x and not false that x is Fer than y, then it is true that x is Fer than y.

The idea is that if it is not ruled out that x is Fer than y but it is ruled out that this relation obtains in the opposite direction, then it appears that x must be F to a higher degree than y, which in Broome’s view allows us to conclude that x indeed is Fer than y. Thus, the principle collapses, so to speak, a potentially vague comparative relation between two items into a determinate relation, provided it is determinate that this relation doesn’t hold in the opposite direction between the items in question.

It can be shown that Collapsing Principle implies

**Symmetry of Indeterminacy:** If it is indeterminate whether x is Fer than y, then it is indeterminate whether y is Fer than x.

In fact, Symmetry of Indeterminacy not only follows from the Collapsing Principle; it also entails it. The two principles are equivalent.

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11 It should be noted, however, that this objection from vagueness is just one of several criticisms that Broome in his (2004) directs against the incommensurability interpretation of the intuition of neutrality, and it is not even the most important one. His main objection is that this interpretation makes neutrality ‘greedy’, as he puts it: It makes neutrality spread, in a dramatic way, to a very broad spectrum of inter-world comparisons. For a discussion, see Rabinowicz (forthcoming-a).

12 Proof by reductio: Suppose (i) it is indeterminate whether x is Fer than y, but (ii) it is not indeterminate whether y is Fer than x. (i) implies that it either is determinate that y is Fer than x (Case 1), or it is determinate that y is not Fer than x (Case 2). In Case 1, ‘y is Fer than x’ comes out as true on all plausible sharpenings of ‘Fer than’, and consequently, by the asymmetry of comparatives, ‘x is not Fer than y’ comes out as true on all the sharpenings. This means however that it is determinate that x is not Fer than y, which contradicts (i). In Case 2, on the other hand, Collapsing Principle implies that it must either be true or false that x is Fer than y. This, however, again contradicts (i).

13 The proof of this entailment is given in Carlson (2004). Suppose, for reductio, that Collapsing Principle is false, i.e., that (i) it is false that y is Fer than x, but (ii) it is neither true nor false that x is Fer than y. (i) means that it is indeterminate whether x is Fer than y, which – by the Symmetry of Indeterminacy – implies that it is indeterminate whether y is Fer than x, contrary to what we have assumed in (i).
Now, if applied to ‘better than’, Symmetry of Indeterminacy yields

**Symmetry of Indeterminacy for Betterness**: If it is indeterminate whether \( x \) is better than \( y \), then it is indeterminate whether \( y \) is better than \( x \).

This principle plays a crucial role in Broome’s argument against the incommensurability interpretation of the intuition of neutrality, which I am now going to present.

Consider, say, the upper boundary of the neutrality range. It is reasonable to expect that this boundary will be vague.\(^{14}\) In other words, for some well-being levels that lie at the upper boundary, it should be indeterminate whether worlds with added persons at those levels are incommensurable with the original world or better. Let \( l^- \) be one of such levels. However, the vagueness at the boundary should not spread out too much; in particular, there should be some well-being levels that determinately lie within the neutrality range. Adding a person with a life at such a determinately neutral level would give us, everything else being equal, a world that is determinately incommensurable with the original world, if neutrality is interpreted as incommensurability. Let \( l \) be one of such levels, where \( l < l^- \).

Now, let \( A \) be the original world, let \( B \) be a world with an extra person, \( S \), added (ceteris paribus) to the population of \( A \) at level \( l \), and let \( C \) be a world with the same extra person added, but at level \( l^- \) instead. From our assumptions about \( l^- \) and \( l \) it follows that

(i) It is indeterminate whether \( C \) is better than \( A \)

But

(ii) It is determinate that \( B \) is incommensurable with \( A \).

(iii) entails that

(iii) It is determinate that \( A \) is not better than \( B \).

Since the Principle of Personal Good is supposed to be determinately true, it follows that

(iv) It is determinate that \( C \) is better than \( B \).

It is determinate that ‘better than’ is transitive: The transitivity of betterness is a conceptual truth. Therefore, we can derive the following conclusion from (iii) and (iv) using the supervaluationist account:

(v) It is determinate that \( A \) is not better than \( C \).

Proof: By supervaluationism, (iii) and (iv) imply that, on all plausible sharpenings of ‘better than’, it is true that \( C \) is better than \( B \) and \( A \) is not better than \( B \). By the transitivity of betterness, this implies that on none of these sharpenings is it true that \( A \) is better than \( C \). That is, on all plausible sharpenings, it is true that \( A \) is not better than \( C \), which implies (v) by supervaluationism.

However, (v) is incompatible with (i), given Symmetry of Indeterminacy for Betterness. If it is indeterminate whether \( C \) is better than \( A \), as stated in (i), then—it by this symmetry principle—it should also be indeterminate whether \( A \) is better than \( C \). But this contradicts (v).

Thus, something has to give. If we hold on to the Principle of Personal Good and to the Symmetry of Indeterminacy for Betterness, we must reject one of the premises in the argument,

\(^{14}\) What about the lower boundary of the neutral range? It is arguable that this boundary is located at the point where the added person’s life stops being worth living. However, the borderline between a life that is worth living and one that is not isn’t very sharp. Which means that the lower boundary will be vague as well.
either (i) or (ii). If we are unwilling to give up the idea that the neutral range is vague at its boundaries, we must retain (i), which means that it is (ii) that has to go. It cannot be determinate that \( B \) is incommensurable with \( A \).

As can easily be seen, we could construct an analogous argument starting out from the vagueness of the lower boundary of the neutral range instead. If it is indeterminate whether the original world \( A \) is better than a world in which person \( S \) is added at a well-being level lying at the lower boundary, then—given the Principle of Personal Good and the Symmetry of Indeterminacy for Betterness—it cannot be determinate that world \( B \) is incommensurable with \( A \). The conclusion is thus that indeterminacy at any of the boundaries crowds out determinate incommensurability in the remaining area of the range. More generally, the following result has been established:

For all items \( x \) and \( y \), if for some \( z \) that is determinately better or determinately worse than \( y \), it is indeterminate whether \( x \) is incommensurable than \( z \), then it is not determinate that \( x \) is incommensurable with \( y \).

But then, if determinate incommensurability is pushed out in this way, incommensurability as such perhaps should be ruled out altogether.\(^{15}\)

The argument presented above would be disarmed if we could make it plausible that Symmetry of Indeterminacy for Betterness is not a universally valid principle. And, as a matter of fact, this was already done by Carlson (2004). Here is one of his counter-examples to the symmetry principle: Suppose we compare philosophers. Among these, Alf and Beth are identical in all respects that make one a good philosopher, with the possible exception of just one feature: Alf possesses rhetorical skills that Beth lacks. However, it is indeterminate whether this feature is relevant for the comparison at hand. While it is determinate that rhetorical skills do not make one a worse philosopher, it is indeterminate whether they positively contribute to philosophical excellence. Consequently, since Alf and Beth are identical in other relevant respects, it is indeterminate whether Alf is a better philosopher than Beth, but determinate that Beth is not a better philosopher than Alf. This contradicts the Symmetry of Indeterminacy for Betterness.

Carlson’s other counter-examples follow the same pattern. In all of them, two items, \( x \) and \( y \), which are being compared, are identical in all value-relevant respects, with the possible exception of just one feature, \( P \), which is exhibited by \( x \) but not by \( y \). It is indeterminate, though, whether \( P \) is relevant for the value comparison. More precisely, while it is determinate that \( P \) does not contribute negatively to the value of the item, it is indeterminate whether it makes any positive value contribution. Consequently, it is indeterminate whether \( x \) is better than \( y \), even though it is determinate that \( y \) is not better than \( x \). Which contradicts the Symmetry of Indeterminacy for Betterness.

I have found these counter-examples convincing.\(^{16}\) But, in all of them, what is indeterminate is whether one item, \( x \), is better than the other, \( y \), or equally as good. What amounts to the same, while it is indeterminate whether \( x \) is better than \( y \), it is determinate that \( x \) is either better than \( y \) or equally as good as \( y \). (That disjunction comes out as true on every plausible sharpening of the list

\(^{15}\) Note, however, that there is this lacuna in Broome’s argument: What his argument shows, if correct, is not that vagueness crowds out incommensurability, but only that it crowds out determinate incommensurability.

\(^{16}\) Broome, however, is not as impressed. He questions whether there could be this kind of indeterminacy about value-making features. He doesn’t think it could be indeterminate whether a certain feature does or does not contribute to the value of an item. See Broome (forthcoming).
of value-relevant respects of comparison.) This gives rise to a query: Could one present an objection analogous to Carlson’s but in which indeterminacy concerns not the choice between betterness and equal goodness, but instead the choice between betterness and incommensurability? Clearly, this is what is needed to undermine Broome’s claim that vagueness and incommensurability cannot co-exist. However, as far as I can see, one does not have to look far in order to find cases of this kind. The example of an addition of an extra person to the world exhibits exactly the structure we need. Consider again worlds $A$ and $C$. The latter differs from the former in just one respect: In $C$, an extra person is added at a well-being level $l^+$, with $l^+$ lying at the upper boundary of the neutrality range. It is indeterminate whether this feature of $C$ is relevant for value comparison. While it is determinate that it does not make the world worse, it is indeterminate whether it makes it better, just as in Carlson’s examples. But this time what is indeterminate is whether $C$ is better than $A$ or incommensurable with $A$.

3. Modeling Value Relations
Here I will sketch a general account of value relations, which I develop in greater detail elsewhere. The approach I use relies on the so-called ‘fitting-attitudes analysis’ of value, which goes back to Brentano (1969 (1889)) and Ewing (1947 and 1959). Later it was adopted by such philosophers as McDowell (1985), Wiggins (1987), Gibbard (1990 and 1998), and Scanlon (1998), among others. On this format of analysis, an item is valuable if and only if it has features that make favoring it fitting or appropriate. ‘Fitting’, ‘appropriate’, ‘ought’, etc, stand for the normative component in this type of analysis, ‘favoring’ is a place-holder for a pro-attitude, the features of the object that make favoring appropriate are its ‘value-making’ properties, and different ways of favoring—desire, admiration, cherishing, etc.—correspond to values of different kinds: desirability, admirability, preciousness, etc. For ‘better than’, the relevant way of favoring has commonly been taken to be preference: ‘... we define “better” as “what ought to be preferred”’ (Ewing 1959, p. 85) In other words, an item is better than another if and only if preferring it is required.

Equal goodness can be analyzed analogously, in terms of indifference (i.e. equi-preference):

Two items are equally good if and only if it is required to be indifferent between them.

Normativity admits of two levels: the stronger level of requirement (‘ought’) and the weaker level of permission (‘may’). Requirement and permission are dual notions; something is required if and only if its absence is not permissible. In symbols, $O X \iff \neg P \neg X$. Or, equivalently, something is permissible if and only if its absence is not required: $P X \iff \neg O \neg X$. Requirement entails permission, $O X \Rightarrow P X$, but not vice versa.

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17 This section draws on the material from Rabinowicz (2008) and (forthcoming-b).
18 Scanlon (1998, p. 97) calls this analysis “the buck-passing account of value”, since it transfers the reason to favor from the object’s value to its value-making properties. Some of the difficulties facing this format of analysis are discussed in Rabinowicz and Ronnow-Rasmussen (2004) and (2006). One such difficulty - “The Wrong Kind of Reasons Problem” - is that the pro-attitude might be required not because of the features that make the object valuable, but rather because the pro-attitude itself would be valuable, or because such attitude would be appropriate for purely deontological reasons, not having to do with the value of its object. Cases like this must somehow be excluded for the analysis to be acceptable. Another difficulty is that there is a danger of circularity in this approach if either the normative component (fittingness) or the attitudinal component themselves need to be analyzed in terms of the concept of value.
As was first noted by Joshua Gert (2004), the presence of two levels of normativity opens up for a broader range of possible value relations, and thus makes room for incommensurability. In particular, the somewhat elusive and controversial notion of parity, due to Ruth Chang (cf. her introduction to Chang 1997, and her 2002a and 2002b), can now be easily defined. It seems too restrictive to assume that for any pair of items there exists a unique appropriate preferential attitude. For some pairs, the preference for one item and the opposing preference for the other item might both be permissible. It is here that parity comes in:

Two items, \( x \) and \( y \), are on a par if and only if it is
(i) permissible to prefer \( x \) to \( y \), and (ii) permissible to prefer \( y \) to \( x \).

If \( x \) and \( y \) are on a par, neither of them is better than the other: we are not required to prefer \( x \) to \( y \) or \( y \) to \( z \). Nor are they equally good: Equi-preference for \( x \) and \( y \) is not being required. Thus, parity is a paradigm case of incommensurability.

Preferential attitudes are on this approach understood as choice dispositions rather than as judgments of some kind. On the choice-dispositional interpretation of preference, to prefer one item to another is to be disposed to opt for the former rather than the latter when one has to make a choice between them. Indifference is also a type of choice disposition: To be indifferent is to be equally prepared to make either choice. However, for some pairs of items we might lack any choice disposition. If necessary, we would make a choice, but not because we are so disposed. In the case of indifference, the subject smoothly proceeds to a decision. But in the absence of a disposition to choose, we typically experience the choice problem as internally conflicted. We can see reasons on each side, but we cannot (or at least will not) balance them off. If we have to, we choose, but without the conflict of reasons being resolved. It seems, then, that not all of our choices are manifestations of choice dispositions.

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19 That preference for \( x \) over \( y \) and the opposing preference for \( y \) over \( x \) are both permissible does not mean of course that it is permissible to have both at the same time. But it is permissible to have each.
20 Gert’s own analysis of parity is more restrictive; in fact, it is much too restrictive, as is shown in Rabinowicz (2008). For a very different analysis of parity, which does not presuppose ‘fitting attitudes’-approach to value, see Carlson (2007).
21 In particular, interpreting preference for \( x \) over \( y \) as a judgment that \( x \) is better than \( y \) would make our analysis of betterness (in terms of required preferences) circular. What’s even more worrying, it would create serious problems for the analysis of parity. If I know that two items are on a par, I know that neither of them is better than the other. The analysis of parity implies that I am permitted to prefer any of the items. However, it would obviously be inappropriate of me to judge it to be better than the other item, knowing that that they are on a par. (I owe this point to Andrew Reisner.) Possibly, however, we might allow for the possibility that preferring involves something akin to an unreliable perception of value: Insofar as I prefer \( x \) to \( y \), \( x \) appears as better than \( y \). Such quasi-perceptual appearances might well be discounted by the subject: Even if I very well know that \( x \) is not better than \( y \), it might still appear to me as better. The interpretation of preferences and desires as fallible perceptions of value is developed in Graham Oddie’s recent book (Oddie 2005). A similar idea, though applied to emotions, is discussed but then finally rejected by D’Arms and Jacobson (2003) under the label of ”quasijudgmentalism”.
22 In principle, it might always be possible to trace back one’s choices to psychophysical dispositions to react to stimuli. But here I am trying to get at a stronger sense of a choice disposition – the sense in which such a disposition is present only if my choice would be reason-based. (Indifference does not preclude choice in this qualified sense. When two options come out as equal in my balancing of reasons, my choice can be said to be reason-based even though I could just as well have chosen the other option instead.) In this stronger sense, of course, not everything one chooses is due to a choice disposition, since not every choice is a reasoned one. It is arguable that if the notion of preference used in the analysis of value relations is to be understood in choice-dispositional terms, then it should refer to choice dispositions in this stronger sense.
Assuming that choice dispositions in this sense can be absent, their absence or presence might be subject to normative assessments. If the absence of a choice disposition with regard to a pair of items is what is being required, then—I would suggest—we have a case of radical incomparability. That is,

$x$ and $y$ are **incomparable** if and only if it is required not to prefer one to the other or be indifferent.

We could also define a less demanding relation that applies when the absence of choice disposition is permissible (but not necessarily required):

$x$ and $y$ are **weakly incomparable** if and only if it is permissible not to prefer one to the other or be indifferent.

Weak incomparability might be a rather widespread phenomenon, but what about radical incomparabilities? These are, to be sure, easy to find, if the domain under consideration contains items from different ontological categories. It is arguable that neither preference nor indifference makes sense if one item is, say, a person, and the other item is a state of affairs or a character feature. The same applies to pairs of items that do not fall under the value under consideration (see above, section 1). But what about items that do fall under this value? Can they ever be radically incomparable? Well, logically it’s possible, but it is unclear whether this possibility has any real instantiations. The most promising examples might be cases of tragic dilemmas, such as Sophie’s Choice. It is arguable that when you have to choose which of your children is to be saved, preferring one of the options is as impermissible as being indifferent.

What, then, about comparability in value? In one sense,

$x$ and $y$ are **comparable** if and only if they are not incomparable.

In this sense comparability and weak incomparability are not mutually exclusive. In a stronger sense, though,

$x$ and $y$ are **fully comparable** if and only they are not weakly incomparable.

According to Chang (introduction to 1997, 2002a, 2002b), parity is a form of comparability. Our definitions confirm this claim: If two items are on a par, then they are comparable. However, they need not be fully comparable. For some $x$ and $y$, it might be permissible to prefer $x$ to $y$, to have the opposite preference, or to have no choice disposition at all with regard to these two items. (In fact, the example of Mozart and Michelangelo might be a case in point.)

To model different value relations it is helpful to use a simple formal framework. For a given domain of items, we can specify **permissible preference orderings** of that domain. Different value relations between items in the domain are then defined in terms of the class of such permissible orderings.\(^{23}\) In general, a preference ordering specifies for each pair of items $x, y$ in the domain, which of the four possible preferential states obtains with regard to $x$ and $y$: either $x$ is preferred to $y$, or $y$ is preferred to $x$, or $x$ and $y$ are equi-preferred (indifference), or none of the above is the case, i.e., there obtains a preferential gap with regard to $x$ and $y$. An ordering is said to be

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\(^{23}\) Gert’s formal modeling is different: It assigns to each item an interval of permissible preference strengths and then defines value relations between items in terms of relations between the corresponding intervals (cf. Gert 2004). As is shown in Chang (2005) and in Rabinowicz (2008), this interval modeling is inadequate, for several reasons, the main one being the impossibility of representing certain structures of betterness relationships between items in terms of relations between intervals.
complete if it doesn’t contain any such gaps. Now, some preference orderings of the domain are permissible, while others are not. Let \( K \) be the class of all permissible orderings. \( K \) may be assumed to be non-empty, i.e., at least one ordering of the items in the domain should be permissible. Since we need to make room for value incomparabilities and thus to allow for preferential gaps, the orderings in \( K \) are not assumed to be complete. We do assume, however, that all the orderings in \( K \) are well-behaved at least in the following sense: In every such permissible ordering, ‘weak preference’, i.e. preference-or-indifference, is a transitive and reflexive relation.

In terms of \( K \), we can define the relation of betterness on the domain as the intersection of permissible preferences:

\( (B) \) \( x \) is better than \( y \) if and only if \( x \) is preferred to \( y \) in every ordering in \( K \).

This is just another way of saying that \( x \) is better than \( y \) if and only preferring \( x \) to \( y \) is required. Equal goodness, parity, incomparability, etc. are definable analogously:

\( (E) \) Two items are equally good if and only if they are equi-preferred in every ordering in \( K \).

\( (P) \) \( x \) and \( y \) are on a par if and only if \( K \) contains both an ordering in which \( x \) is preferred to \( y \) and an ordering in which \( y \) is preferred to \( x \).

\( (I) \) \( x \) and \( y \) are incomparable if and only if every ordering in \( K \) contains a gap with regard to \( x \) and \( y \).

\( (WI) \) \( x \) and \( y \) are weakly incomparable if and only if some ordering in \( K \) contains a gap with regard to \( x \) and \( y \).

To exemplify how this works, consider again the example with three items, \( x \) (Mozart), \( x' \) (Mozart‘) and \( y \) (Michelangelo). Suppose that only three preference orderings of these items are permissible. In each column below, which represents one such ordering, the items are ordered from the most preferred at the top to the least preferred at the bottom. Equi-preferred items are placed on the same level. In this simplified example, permissible preference orderings don’t have any gaps. Obviously, this need not be the case in general.

\[
\begin{array}{ccc}
  \text{P1} & \text{P2} & \text{P3} \\
  x' & y & x'' \\
  x & x' & x, y \\
  y & x & y \\
\end{array}
\]

The intersection of \( P1 \), \( P2 \) and \( P3 \) gives us exactly the betterness structure of our example: \( x' \) is better than \( x \), since it comes above \( x \) in every permissible ordering, and no other betterness relationships obtain between these three items, just as we have stipulated. As is easily seen, \( x \) and \( y \) are on a par with each other (\( x \) comes above \( y \) in \( P1 \), while \( y \) comes above \( x \) in \( P2 \), and so are \( x' \) and \( y \).

\[24\] The intersection modeling is based on an old idea, which goes back to Sen (1973), ch. 3. (See also Atkinson 1970.) But there is a difference in the interpretations: Unlike our modeling, Sen’s ‘intersection approach’ is not meant to provide an analysis of betterness in terms of permissible preference orderings. Instead, he takes it to be a construction of the relation of definite betterness from a class of value orderings, each of which reflects different value commitments or different respects in which comparisons between items can be made. Also, on his approach, incompleteness only shows up in the intersection, but not in the underlying orderings, as our modeling allows.
Does the modeling add in some way to our previous informal analysis of value relations? By letting $K$ be non-empty, we have excluded situations in which nothing is rationally permissible with respect to some pairs of items, not even a preferential gap. More importantly though, the modeling allows us to derive formal properties of value relations from the corresponding conditions on permissible preference orderings. Thus, we now can prove that the relation ‘better than or as good as’ is transitive and reflexive.\(^{25}\)

We now have what we need for a general taxonomy of dyadic value relations. In the table below, each column describes one type of a value relation, by specifying all the preferential attitudes that are permissible with regard to a pair of items. There are four kinds of such attitudes to consider: preference ($>$), indifference ($\approx$), ‘dispreference’ ($<$), i.e. preference in the opposite direction, and a gap ($/$). In each column, there are plus signs for every attitude that is permissible in this value type. A column must contain at least one plus sign, since for any two items at least one kind of preferential attitude towards these items must be permissible (if $K$ is non-empty). The number of columns thus equals the number of ways one can pick a non-empty subset out of the set of four possible preferential attitudes. As there are fifteen such non-empty subsets, the table has fifteen columns. For example, in type 7, all preferential attitudes except for the gap are permissible, while in type 15, which corresponds to incomparability ($I$), gap is the only permissible attitude. In type 1, which corresponds to betterness ($B$), the only permissible attitude is preference, and so on.

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The columns in the table correspond to *atomic* types of value relations. Groups of atomic types, such as comparability (types 1 - 14), or weak incomparability (types 8 - 15), form types in a broader sense of the word. Incommensurability is also a type in this broad sense: It covers all the atomic types from 4 to 15. Nor is parity ($P$) an atomic type, unlike the three traditional relations: better ($B$), worse ($W$), and equally good ($E$). Parity comprises atomic types from 6 to 9.

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\(^{25}\) It can be seen as a weakness in this account that it makes formal features of value relations dependent on the requirements on permissible preference orderings. As an example, consider betterness. That this relation is transitive would seem to be a conceptual truth. But on my account this feature of betterness derives from the transitivity of permissible preferences. That the latter must be transitive is a highly plausible requirement. But is it a conceptual truth? (A conceptual truth about permissible preferences?) Similar remarks apply to other formal features of value relations that are derivable only in virtue of the requirements we have imposed on permissible preference orderings (i.e. requirements that permissible orderings might conceivably violate). I am not sure how to answer this objection.
We now can see that there are other forms of comparability apart from the four that have been
distinguished by Chang (the three traditional forms plus parity). Many of the atomic types of
value relations that appear in our table have not been previously identified. As an example,
consider type 4. If \( x \) and \( y \) are related in this way, it is required to prefer \( x \) to \( y \) or be indifferent
between them. In other words, weak preference for \( x \) over \( y \) is required. But then it seems that \( x \) is
at least as good as \( y \), despite the fact that in type 4 \( x \) is neither better than nor equally as good as
\( y \). This suggests that ‘at least as good as’ should be defined as the union of types 1, 2 and 4, rather
than as the union of types 1 and 2 as the traditional analysis would have it. In other words, the
standard definition of ‘at least as good as’ as ‘better than or equally as good as’ is incorrect.
(Analogously, ‘at most as good as’ should be defined as the union of types 2, 3 and 5, rather as
the union of types 2 and 3.)

The fifteen atomic types we have listed are all conceptually possible, but some might not
represent ‘real’ possibilities. For example, whenever two items are on a par, i.e., whenever
preference for each is permissible, then—it might seem—it should also be permissible to be
indifferent between the items in question. This condition would exclude types 6 and 8. If
preferential gaps should also be permissible when parity obtains, we would have to exclude type
7 as well. Then only type 9 would be left for parity. Notice that such extra conditions importantly
differ from, say, transitivity of weak preference. The latter is a condition on each ordering in \( K \).
The extra conditions instead impose constraints on class \( K \) taken as a whole: They state that \( K \)
must contain orderings of certain kinds if it contains orderings of certain other kinds. Whether
such extra conditions are reasonable or not is difficult to tell ex ante.

4. Injecting vagueness into the model
The way to inject vagueness into our model is simple: We need to allow that class \( K \) is ‘fuzzy’ at
its boundaries. For some preference orderings in the domain it might be indeterminate whether or
not they are permissible. This will have potential repercussions for the value relations on the
domain. For some of them it might turn out that they have a partly indeterminate extension.

Let us see how this works in more detail. If \( K \) is fuzzy, it allows of different sharpenings. Let
\( \Sigma \) be the set of all admissible sharpenings of \( K \). I.e., for every \( k \in \Sigma \) \( k \) is a class of preference
orderings that represents one plausible sharpening of \( K \). Now, consider any two items \( x \) and \( y \) and
any possible type of value relation \( R \) (atomic or not). Every admissible sharpening of \( K \)
determines a particular sharpening of \( R \). It is determinate that \( xRy \) if and only if \( xRy \) comes out as
true on every such sharpening. It is determinate that it is not the case that \( xRy \) if and only if \( xRy \)
comes out as false on every such sharpening. And it is indeterminate whether \( xRy \) if and only if
\( xRy \) comes out as true on some sharpenings of \( R \) and false on its other sharpenings.

To give an example, let’s go back to our problem in population ethics. Consider again possible
worlds \( A, B \) and \( C \). \( A \) is the original world, with a certain population, \( B \) is a world with an extra
person, who is added (ceteris paribus) to the population of \( A \) at a well-being level \( l \), and \( C \) is a
world with the same extra person added, but at level \( l^+ \) instead. Since \( l < l^+ \), and the Principle of
Personal Good is supposed to be valid, it is determinate that \( C \) is better than \( B \). Now, level \( l \) lies
squarely within the neutrality range, while level \( l^+ \) lies at the upper boundary of that range. The
boundaries of the neutrality range are not sharp. Consequently, it is determinate that \( B \) is
incommensurable with \( A \), but it is indeterminate whether \( C \) is incommensurable with \( A \). It is not
ruled out that \( l^+ \) lies above the neutral range, in which case \( C \) would be better than \( A \), nor that \( l^- \)
lies within the range, in which case \( C \) would be incommensurable with \( A \). The question is: How can one represent this example in our modeling?

For simplicity, let us assume that the domain of items consists of just these three worlds. Let us also suppose, as a further simplification, that permissible preference orderings on the domain do not have any gaps. Since it is determinate that \( C \) is better than \( B \), every permissible ordering in every sharpening of \( K \) must rank \( C \) above \( B \). Consequently, we only need to consider the following five orderings as possible candidates for inclusion in \( K \):

\[
P1 \quad P2 \quad P3 \quad P4 \quad P5 \\
C \quad C \quad C \quad A, C \quad A \\
A \quad B \quad A, B \quad B \quad C \\
B \quad A \\
\]

Now, since the upper boundary of the neutral range has been assumed to be fuzzy, we should have the corresponding fuzziness in \( K \). One admissible sharpening of \( K \), \( k_1 = \{P1, P2, P3\} \), corresponds to the assumption that \( l^+ \) lies above the neutral range. On that sharpening, it comes out as true that \( C \) is better than \( A \); \( C \) is ranked above \( A \) in every permissible preference ordering. On another admissible sharpening of \( K \), \( k_2 = \{P1, P2, P3, P4, P5\} \), which corresponds to the assumption that \( l^+ \) lies within the neutral range, it instead comes out as true that \( C \) is incommensurable with \( A \). More precisely, on that sharpening, \( C \) and \( A \) are on a par: In some permissible preference orderings (\( P1, P2, P3 \)), \( C \) is ranked above \( A \), while in other orderings (\( P5 \)), \( A \) is ranked above \( C \). (Since \( P4 \) belongs to \( k_2 \), it is also permissible to equi-prefer \( A \) and \( C \). Thus, the value relation between \( A \) and \( C \) is of type 7 on this sharpening.) This means, then, that it is indeterminate whether \( A \) and \( C \) are incommensurable: They are on one sharpening, but not on the other.

\( B \) and \( A \), however, are incommensurable on both sharpenings. More precisely, they are on a par: \( A \) is ranked above \( B \) in \( P1 \), \( B \) is ranked above \( A \) in \( P2 \), and these two orderings belong to both sharpenings. Thus, if \( k_1 \) and \( k_2 \) are the only admissible sharpenings of \( K \), or at least if \( P1 \) and \( P2 \) belong to every admissible sharpening, it follows that \( B \) and \( A \) are determinately on a par with each other.

It is clear, then, that the modeling we have proposed makes ample room for both incommensurability and vagueness. On the one hand, incommensurability and vagueness are mutually compatible: It can be indeterminate whether two items are incommensurable (as in the case of worlds \( A \) and \( C \) in our example). But we can also have one of them without the other. Thus, it can be determinate that two items are incommensurable (as in the case of worlds \( A \) and \( B \) and—on the other hand—it can be determinate that they are not incommensurable, even though it is indeterminate what value relation in particular obtains between them (as in Carlson’s counter-examples).26

One might think that vagueness could also be injected into the model in another way: not by making it indeterminate whether certain preference orderings are permissible (i.e. by letting \( K \) have different sharpenings), but instead by making some indeterminate preference orderings permissible. Arguably, it may sometimes be indeterminate what preferences the agent holds:

26 The case of of two philosophers – Alf and Beth – can easily be modelled on our approach. In this case, on some sharpenings of \( K \), \( K \) only contains preference orderings in which Alf ranks above Beth. On all other admissible sharpenings, on the other hand, \( K \) only contains orderings in which Alf and Beth are equi-preferred. Thus, it is indeterminate whether Alf is a better philosopher than Beth, but it is determinate that they are not incommensurable: It is determinate that either Alf is a better philosopher than Beth or they are equally as good.
whether she, for example, prefers $x$ to $y$ or is indifferent (or even has the opposite preference).

Note that indeterminacy in preference is not the same as a preference gap. In the case of a gap, the agent has no preferential attitude towards the items, while in case of indeterminacy, the attitude of the agent is indeterminate. In fact, whether the agent’s preferences contain a gap with regard to a pair of items might itself be indeterminate. Now, if indeterminate preferences are possible, then some such preferences might also be deemed permissible. This would mean that $K$ could contain some preference orderings that themselves are vague to a greater or smaller extent.

This is an alternative way of injecting vagueness into our model. Thus, to take an example, instead of letting it be indeterminate whether it is permissible to prefer world C to world A, we could assume that it is (determinately) permissible to have an indeterminate preference with regard to $A$ and $C$ and, in particular, permissible to be in a state in which it is indeterminate whether one prefers $C$ to $A$ or not. Obviously, this is another way of accounting for the apparent fuzziness in the boundaries of the neutral range.

However, if I am not mistaken, this second way of injecting vagueness into the model does not amount to another way of making value relations vague. After all, there is nothing vague on that proposal about what preferences are permissible with regard to a given pair of items. Rather, some of these permissible preferences are now taken to be vague themselves. Therefore, if statements about value relations reduce to claims about what preferences are permissible, as I have suggested, then this second way of injecting vagueness does not make the value statements vague but rather expands the range of possible types of value relations that might obtain between the items. As we have seen, different types of value relations correspond to different combinations of permissible preferential states with regard to a pair of items. If we expand the set of potentially permissible preferential states to include indeterminate states of various kinds, then the number of possible types of value relations will of course drastically increase.

This is not the place to study how such an expansion of the range of value relations would look like. The goal of this paper was to cast doubts on the view that vagueness in value comparisons crowds out incommensurability in value and to show how these two phenomena can be represented together, within a formal framework that implements some basic insights of the ‘fitting attitudes’-analysis of value. This has now been done, I think.\(^{27}\)

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**References**


\(^{27}\) The first version of this paper was presented at a symposium on John Broome’s philosophy in Canberra in 2007. Later that year, I gave a talk on this subject at a meta-ethics workshop in Leeds, and in the following year I presented it at the 8th Polish Philosophical Congress in Warsaw, at the psychology department of Purdue University in LaFayette, Indiana, and at a workshop on value in Geneva. I want to thank the participants of these events for helpful discussions. I have profited from comments by John Broome, Johan Brännmark, Krister Bykvist, Ehtibar Dzjafarov, Christian List, Kevin Mulligan, Graham Oddie, Mozaffar Qizilbash, Joseph Raz, Andrew Reisner, and Toni Rønnow-Rasmussen. I also want to thank Robert Sugden for stimulating exchanges at the final stages of preparation. I am indebted to the Swedish Research Council for a generous research grant and to the Swedish Collegium of Advanced Study (SCAS) in Uppsala for an excellent research environment.


—— forthcoming: ‘Reply to Rabinowicz’. Forthcoming in *Philosophical Issues* (a supplement to *Noûs*).


—— forthcoming-a: ‘Broome and the Intuition of Neutrality’. Forthcoming in *Philosophical Issues* (a supplement to *Noûs*).


