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Analytical model of beach erosion and overwash during storms

M. Larson DSc, C. Donnelly PhD, J. A. Jiménez PhD and H. Hanson DSc

During severe storms high waves and water levels may greatly impact the sub-aerial portion of the beach inducing significant morphological change at elevations that the waves can not reach under normal conditions.

Morphological formations such as dunes and barrier islands may suffer from direct wave impact and erode. Overwash occurs if the wave run-up and/or the mean water level are sufficiently high allowing for water and sediment to pass over the beach crest, which in turn causes flooding and deposition of sediment shoreward of the crest. An analytical model of sub-aerial beach response to storms was developed based on impact theory, including overwash, and the evolution of schematised dunes was investigated. Furthermore, the analytical model was applied to the case of schematised barrier islands exposed to extensive overwash. After validation using field data, the analytical model was employed at two coastal sites, namely Ocean City on the United States east coast and the Ebro Delta on the Spanish Mediterranean coast, in order to calculate quantities for assessing the storm impact on beaches, such as eroded volume, overwash volume, beach crest reduction, and contour-line retreat. These quantities were subsequently analysed to derive empirical probability distribution functions to be utilised in different types of risk assessment concerning flooding and erosion in coastal areas.

NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>dune or barrier height</td>
</tr>
<tr>
<td>$T$</td>
<td>wave or swash period</td>
</tr>
<tr>
<td>$T_o$</td>
<td>peak spectral wave period</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$u_o$</td>
<td>bore front speed in the swash</td>
</tr>
<tr>
<td>$u_k$</td>
<td>initial bore front speed at the start of uprush</td>
</tr>
<tr>
<td>$V$</td>
<td>dune or barrier volume</td>
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<tr>
<td>$V_o$</td>
<td>initial dune or barrier volume</td>
</tr>
<tr>
<td>$V_D$</td>
<td>dune or barrier volume (analytic model)</td>
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<tr>
<td>$V_{over}$</td>
<td>initial dune or barrier volume (analytic model)</td>
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<tr>
<td>$x_B$</td>
<td>overwash volume</td>
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<tr>
<td>$x_{B_k}$</td>
<td>cross-shore location of shoreward limit of dune or barrier</td>
</tr>
<tr>
<td>$x_c$</td>
<td>initial cross-shore location of shoreward limit of dune or barrier</td>
</tr>
<tr>
<td>$x_o$</td>
<td>cross-shore location of seaward limit of dune or barrier</td>
</tr>
<tr>
<td>$x_{xo}$</td>
<td>initial cross-shore location of seaward limit of dune or barrier</td>
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<td>$z_o$</td>
<td>elevation of dune or barrier foot above still water level</td>
</tr>
<tr>
<td>$\beta_B$</td>
<td>beach slope on shoreward dune or barrier side</td>
</tr>
<tr>
<td>$\beta_o$</td>
<td>beach slope on seaward dune or barrier side</td>
</tr>
<tr>
<td>$\Delta R$</td>
<td>run-up height above the beach crest</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time period</td>
</tr>
<tr>
<td>$\Delta V$</td>
<td>volume of eroded sediment</td>
</tr>
<tr>
<td>$\Delta V_B$</td>
<td>volume of eroded sediment going over the beach crest</td>
</tr>
<tr>
<td>$\Delta V_{E}$</td>
<td>volume of eroded sediment during a storm event</td>
</tr>
<tr>
<td>$\Delta V_o$</td>
<td>volume of eroded sediment from seaward side of dune or barrier</td>
</tr>
<tr>
<td>$\Delta W$</td>
<td>weight of eroded sediment</td>
</tr>
<tr>
<td>$\Delta R_B$</td>
<td>change in shoreward limit of dune or barrier</td>
</tr>
<tr>
<td>$\rho$</td>
<td>water density</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>sediment density</td>
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I. INTRODUCTION

1.1 Background

During severe storms high waves and water levels may greatly impact the sub-aerial portion of the beach inducing significant morphological change at elevations where the waves can not
reach under normal conditions. Coastal dunes may suffer direct wave impact and erode, increasing the likelihood of breaching and subsequent flooding of low-lying areas behind the dunes. Overwash occurs if the wave run-up and/or the mean water level are sufficiently high allowing for water and sediment to pass the beach crest, which in turn causes flooding and deposition of sediment shoreward of the crest. In the case of overwash, severe lowering of the crest may take place, increasing the probability of flooding and breaching as the natural defence offered by sub-aerial morphological formations (e.g. dunes) is weakened.

A barrier island is another type of morphological formation that is vulnerable to high waves and water levels. Typically, these formations are regularly exposed to overwash because the crest tends to be at a low elevation. The shoreward transport of sediment in connection with overwash is an important element in the sediment budget for a barrier island and this transport is also thought by many researchers to be the cause of the onshore migration that many such islands are experiencing. As the crest of a barrier island is typically low-lying, the mean water level during a storm might exceed the crest elevation leading to inundation overwash. The case when the mean water level is below the crest but the run-up passes over it is often denoted as run-up overwash. Other morphological features such as spits, or shore-attached shoals that pierce the water surface, may exhibit frequent overwash and display similarities to barrier islands in terms of the response, albeit at a smaller scale.

Coastal engineers are often faced with estimating the impact on the beach of severe storms in terms of, for example, eroded volume, recession distance, and overwash volumes. Several numerical models have been developed for this purpose, but they are in general time-consuming to apply and require a significant amount of data at a high level of detail. Furthermore, only a limited number of models include the possibility to simulate overwash. As an alternative to a numerical approach an analytical model might be employed, requiring limited input data. The use of an analytical model involves a high degree of schematisation concerning the forcing, boundary, and initial conditions, but if the model captures the main governing physics, reliable estimates of storm impact on beaches are obtained that could be used in preliminary studies. Another application for analytical models is to derive long-term statistical properties of key parameters quantifying the morphological impact of storms. Long time series of wave and water level data, together with beach profile and sediment characteristics, may serve as input for computing time series of such parameters (e.g. eroded volume, recession distance, and overwash volume). Subsequently, different types of statistical analysis can be used to estimate the probability of a certain morphological impact on the beach in connection with a storm. This is an important step in a risk-based strategy to manage coastal erosion and flooding.

1.2 Objectives
The main objectives of this study were: (a) to develop an analytical model of erosion due to wave impact and sediment transport in the overwash; (b) to validate the model with high-quality field data; and (c) to develop methods to employ the model for estimating the statistical properties of morphological response in connection with storms based on long time series of input data on waves and water level. In this paper the term validation is used for the process of confirming that a general functional relationship with the physical quantities involved is suitable to describe a set of data, including developing appropriate values for any empirical coefficients that appear in the relationship based on the data.

1.3 Procedure
The transport formula developed by Larson et al. was employed to compute dune or barrier island erosion due to wave impact. The eroded sediment is transported offshore or over the beach crest (overwash) depending on the wave and water level conditions in relation to the crest elevation. A simple analytical model was derived for a triangular sub-aerial cross-section, which is often a good characterisation of dunes and barrier islands. The model includes three variables to describe the beach response, namely the crest height, the seaward (front) beach foot location, and the shoreward (rear) beach foot location. Three sand conservation equations govern the beach response.

A high-quality database on sub-aerial profile response due to wave impact and overwash in the field was compiled to validate the transport formulae employed and the analytical solutions to describe beach evolution. The field data included pre- and post-storm profiles and the associated wave and water level forcing, as well as sediment characteristics, for several major storm events. Furthermore, two long-term time series of wave and water-level data encompassing 50–70 years at a resolution of 1–3 h were employed to calculate dune erosion and overwash transport for statistical analysis. These data originated from Ocean City, Maryland, USA, and the Ebro Delta, Spain. The long-term wave and water-level data were used as input to the analytical model to calculate a time series of morphological impact parameters, primarily eroded volume, overwash volume, and recession distance. Empirical distribution functions were derived from these times series as a basis for assessing the probability of exceedance for a specific event.

2. ANALYTICAL MODEL OF SUB-AERIAL BEACH RESPONSE
The analytical model of sub-aerial beach response has two main transport components, namely erosion of the dune face due to wave impact and transport over the beach crest due to overwash. Transport relationships for these processes, together with conservation equations for sediment, make it possible to solve for the evolution of the sub-aerial profile cross-section. In the following, the transport relationships employed for wave impact erosion and overwash are discussed. Then, the analytical model to simulate the effect of overwash and erosion of the seaward side of the beach (the source of the overwash deposits) on the sub-aerial profile shape is developed.

2.1 Transport due to wave impact
Larson et al. developed a sediment transport formula for the erosion from a dune face due to wave impact based on work by Fisher et al. (see also Overton et al. and Nishi and Kraus). This model was combined with the sediment continuity equation and then solved analytically for simple cases with regard to geometry and forcing. The basic assumption in estimating dune erosion from wave impact theory is that there is a linear relationship between the impact (force $F$ on the dune due to change in the momentum flux of the bores impacting the dune) and the weight ($AW$) of the sediment volume eroded from the dune according to $AW = C_e F$, where $C_e$ is an empirical coefficient. Figure 1 gives a definition sketch of a wave...
impacting a dune, resulting in the release of material from the dune face. The weight of the released (eroded) volume ($\Delta V$) is given by $\Delta W = \Delta V \rho_s (1 - \phi) g$, where $\rho_s$ is the density of the sediment, $\phi$ is the sediment porosity, and $g$ is the acceleration due to gravity. The swash force ($F$) is a result of the change in the momentum of the bores hitting the dune and it may be estimated from $F \sim \rho u_0^2 h_0 \Delta T / T$, where $\rho$ is the water density, $u_0$ the speed of the bore, $h_0$ the height of the bore, $\Delta T / T$ the number of incoming waves during the period $\Delta t$, and $T$ the period at which waves hit the dune (taken to be approximately equal to the incident wave period).

Equating swash force and weight of eroded volume, and considering the process of dune erosion to be continuous, although in reality it is often discrete depending on the prevailing failure mechanism, an average rate of dune erosion ($q_D$) is derived

$$q_D = \frac{dV}{dt} = -C_s \frac{u_0^4}{g^2 T}$$

where $C_s$ is an empirical transport coefficient and a minus sign was introduced since the dune volume must decrease with time ($t$). In order to arrive at Equation 1 it was assumed that the speed of the bore is related to the bore height according to $u_0 \sim \sqrt{g h_0}$ (e.g. as shown in Cross and Miller). The bore speed in front of the dune face ($u_d$) is estimated as

$$u_d^2 = u_0^2 - 2 g z_0$$

where $u_0$ is the speed of the bore as it starts its travel up the foreshore and $z_0$ is the elevation difference between the dune foot and the beginning of the swash (see Figure 1). This estimate of $u_0$ is obtained by regarding the bore as a slug of water moving along the foreshore (friction neglected; as, for example in Waddell and Hughes). At the limit of the run-up ($R$) the speed $u_0$ should be zero, implying that $u_0^2 = 2 g R$, which means that predictive formulas for $R$ may be used to derive $u_0$. By substituting in the expression for $u_0$ in Equation 1, using $R$ instead of $u_0$, the following equation is obtained for the dune response

$$\frac{dV}{dt} = -4 C_s \frac{(R - z_0)^2}{T}$$

For the case when $R$ and $z_0$ are constants, the following solution to Equation 2 is obtained

$$V = V_0 - 4 C_s \frac{(R - z_0)^2}{T} t$$

where the initial condition $V = V_0$ at $t = 0$ was employed. Thus, the eroded volume $\Delta V_e = V_0 - V$ after time $t$ may be determined to be $\Delta V_e = 4 C_s (R - z_0)^2 / T$.

### 2.2 Transport in overwash

Coastal overwash is the flow of water and sediment over the crest of the beach (e.g. dune or barrier island) that does not directly return to the ocean from where it originated. Only a few formulas have been developed to estimate sediment transport by overwash and they are typically based on the flow of water over the crest. In more sophisticated formulations different approaches are employed depending on the type of overwash regime – that is, whether run-up or inundation overwash occurs. In most cases run-up overwash occurs before and after inundation overwash during a specific event, implying that a fine resolution in time is needed to model the processes in detail. Thus, such modelling requires a numerical approach, whereas in an analytical description considerable simplifications are employed to arrive at closed-form solutions. These simplifications would involve schematised geometries, constant or representative forcing (waves and water level), and a limited number of parameters to describe the evolution of the beach in response to overwash.

### 2.3 Analytical solution of profile cross-section evolution

Dune and barrier island cross-sections can often be schematised using a triangular shape (see Figure 2). In the case of a dune the seaward and shoreward slopes are steeper than for a barrier island. Waves impacting the seaward side of the beach will cause erosion and a certain portion of the eroded volume ($\Delta V_o$) will be transported across the crest with the overwash ($q_o$), whereas the rest is transported offshore ($q_d$). Figure 2 illustrates a triangular beach cross-section subject to erosion from wave impact and overwash during a time step, $\Delta t$ (variables studied change a quantity $\Delta$ during this short period). The washover volume ($\Delta V_B$) is assumed to be deposited on the shoreward side maintaining the slope ($\beta_B$), and a constant slope is also maintained during erosion on the seaward side ($\beta_d$). The seaward and shoreward locations of the beach foot are denoted as $x_b$, whereas the beach crest height is $x$. These three variables uniquely specify the profile change taking place because of erosion and overwash.

Appendix 1 provides a brief derivation of the governing equations for the evolution of the triangular beach that are based on conservation equations for sediment volume. The analytical solution to these equations under the assumption of constant values of $q_d$ and $q_o$ is given by

$$s = \frac{2 V_0}{u_0} \sqrt{1 - \frac{q_o t}{V_0}}$$

Figure 2. Schematic of a beach cross-section subject to wave impact and overwash

![Figure 1. Definition sketch for analytical model of dune erosion](image)
where $V_D$ is the barrier (dune) volume ($V_D = (x_B - x_o)s/2$), $l_B$ the barrier (dune) width ($l_B = x_B - x_o$), and the second subscript (o) denotes the value at $t = 0$. The overwash volume during a specific event ($V_{over}$) is defined by the material that is transported over the crest

$$V_{over} = V_D - \frac{1}{2} (x_B - x_o)(x_c - x_o)\tan\beta_o$$

where

$$x_c = \frac{x_o\tan\beta_o + x_B\tan\beta_B}{\tan\beta_o + \tan\beta_B}$$

If $x_o > x_B$, that is, the front face of the dune or barrier moves past its initial shoreward limit – the overwash volume is given by $V_{over} = V_D$.

3. DATA EMPLOYED

3.1. Storm impact database

Tinh performed numerical modelling of coastal overwash using an extensive high-quality database of severe storms impacting the east coast of the USA. Ten events where overwash occurred were identified and detailed on data waves, water levels, and profiles were compiled. The database covered five different storms and one to four sites for each storm, including both dunes and barrier islands. Table 1 summarises the hydrodynamic data measured and estimated for the storms, whereas Table 2 yields measured dune and overwash volumes based on the profile surveys.

The Folly Beach data were obtained during Hurricane Hugo and beach profiles surveyed at four locations were employed. Garden City Beach was also hit by Hugo and profile surveys from one location were available. Data from Santa Rosa Island were compiled for two hurricanes, namely Opal and Georges, and from one location. Finally, two major winter storms attacked Assateague Island during January and February 1998, and profile surveys performed at three sites before and after the two storms were available. The run-up heights given in Table 1 were computed with the formula developed by Larson et al. in conjunction with the wave impact erosion model.

<table>
<thead>
<tr>
<th>Profile name</th>
<th>Change in dune/barrier volume: $m^3/m$</th>
<th>Overwash volume: $m^3/m$</th>
<th>$d_{50}$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Folly Beach 2801</td>
<td>$-0.78$</td>
<td>4.75</td>
<td>0.17</td>
</tr>
<tr>
<td>Folly Beach 2815</td>
<td>$-1.37$</td>
<td>11.89</td>
<td>0.17</td>
</tr>
<tr>
<td>Folly Beach 2823</td>
<td>$-1.09$</td>
<td>5.58</td>
<td>0.17</td>
</tr>
<tr>
<td>Folly Beach 2883B</td>
<td>$-1.21$</td>
<td>4.19</td>
<td>0.17</td>
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<tr>
<td>Garden City 4930</td>
<td>$-2.47$</td>
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<td>0.44</td>
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<tr>
<td>Santa Rosa Island, Opal</td>
<td>$-1.60$</td>
<td>5.7</td>
<td>0.26</td>
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<td>Santa Rosa Island, Georges</td>
<td>$-1.11$</td>
<td>5.71</td>
<td>0.26</td>
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<td>Assateague Island, GPS1</td>
<td>$-1.89$</td>
<td>50.1</td>
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<td>101.4</td>
<td>0.3</td>
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<tr>
<td>Assateague Island, GPS4</td>
<td>$-1.74$</td>
<td>91.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 1. Hydrodynamic conditions during studied storms: $H_{rms}$ root-mean-square wave height; $T_{\omega}$ peak spectral wave period; $\Delta R$, run-up height above beach crest; after Tinh.

<table>
<thead>
<tr>
<th>Profile</th>
<th>$H_{rms}$: m</th>
<th>$T_{\omega}$: s</th>
<th>Water depth: m</th>
<th>Max. water level: m</th>
<th>run-up above SWL: m</th>
<th>Crest above SWL: m</th>
<th>Max. $\Delta R$: m</th>
<th>Surge duration: h</th>
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</thead>
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<tr>
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<td>10.79</td>
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<tr>
<td>Folly Beach 2815</td>
<td>4.78</td>
<td>10.79</td>
<td>16</td>
<td>2.13</td>
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<td>2.33</td>
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<td>Folly Beach 2823</td>
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<td>2.02</td>
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<td>Folly Beach 2883B</td>
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<td>5.38</td>
<td>1.69</td>
<td>3.69</td>
<td>69</td>
</tr>
</tbody>
</table>

Table 2. Dune volume before and after storm and overwash volume: $d_{50}$ median grain size; after Tinh.
3.2. Long-term data sets

Long-term data sets of hydrodynamics were used to simulate the statistical properties of the morphological response in connection with storms. Two detailed high-quality data sets were employed, one from Ocean City, Maryland on the east coast of the USA and another from the Ebro Delta on the Mediterranean coast of Spain.

In the simulations schematised sub-aerial beach cross-sections were used based on profile surveys from the study areas.

Following the method described by Irish et al. a unique, detailed data set on the nearshore hydrodynamics off the coast of Ocean City (MD) was derived based on measurements and numerical modelling. Hourly values on waves, water levels, and wind were available from January 1930 to December 1999. The primary input variables used in the simulations were root-mean-square wave height, peak spectral wave period, storm surge level and tidal elevation. The hydrodynamic time series included several major storms that occurred along the United States east coast including the September 1933 storm that opened Ocean City inlet, the Ash Wednesday storm of March 1962 and the Halloween storm of October 1991. North of Ocean City inlet, prominent dunes are present with a height of about 2 m above the dune foot, which is located at an elevation of approximately 3 m above mean sea level. The dunes are partly man-made, constructed and maintained during major beach nourishment operations that started at the end of the 1980s. In contrast, south of the inlet a low-lying barrier island is present with a crest height of about 3 m above mean sea level.

Detailed time series of hydrodynamic data was forecasted for many stations around the Mediterranean Sea within the European Union-sponsored Hipocas project. In the present study such data were used from two stations in the coastal area outside the Ebro Delta. Information on waves, water level, and wind was available every 3 h from January 1958 to December 2001. The station closest to the Ebro Delta (2056046) was used for the waves, but as the water level was not forecasted at 2056046, another station (2056045) was used to obtain these data. Wave heights and periods employed in the present study were based on corrected values from the Hipocas forecast, which originally employed the wave prediction model (WAM) model that tended to under-predict wave growth. Correction factors were developed from comparison with measured data on wave distributions and applied to the data. Profile surveys have been carried out along three different stretches in the Ebro Delta that display different morphological characteristics, namely La Marquesa Beach on the northern side, Illa de Buda on the eastern side, and Trabucador barrier beach south of the delta. La Marquesa Beach has a sub-aerial profile shape that involves dune-like features with crest elevations up to 2-5 m, whereas Trabucador beach constitutes a low-lying barrier island with a crest elevation of about 1 m and a width of about 150 m. Illa de Buda exhibits both areas with barrier island and dune features.

4. VALIDATION OF ANALYTICAL MODEL

The data from Tinh were used to investigate erosion due to wave impact when overwash occurs as well as the relationship between offshore and overwash transport. These quantities are important to specify when applying the analytical model of dune/barrier island evolution (Equations 4 to 6). Ideally, frequent measurements of the sub-aerial beach profile would be available during a storm to validate the solution and to determine various coefficient and parameter values. However, even in the most detailed data sets from the field, typically only the pre- and post-storm profiles are obtainable and so the validation has to be made on the basis of recorded volume changes, such as the eroded volume and the overwash volume.

The eroded volume from the seaward side of the beach was assumed to be the sum of the measured overwash volume and the overall reduction in beach volume, which is assumed to correspond to the volume lost offshore. This simple picture neglects any complex depositional patterns that may occur during a storm, but at the peak of the storm when large waves attack the beach crest area under elevated storm surge, the estimate should give the correct order of magnitude. Furthermore, no consideration is made for three-dimensional effects, which might be important if overwash occurs along a limited portion of the beach crest. As the analytical model is two-dimensional and only describes the evolution of a profile cross-section, no longshore effects are included and the model will be most suitable when the forcing and the profile shape is rather uniform along the shore. If there is marked longshore variability, with lateral spreading of water and sediment taking place on the landward side of the beach crest, the calculated profile response will exceed the expected one, and some correction might be needed.

During overwash a portion of the uprushing wave will pass over the crest and the impact from the waves might thus be reduced in comparison with the case when the entire wave is stopped on the seaward side of the beach (as studied by Larson et al.). Thus, a correction factor given by the ratio between the beach crest height and the run-up height above the beach foot elevation was introduced to take this effect into account. This ratio will only be applied if \( R - z_c > s \); that is, if the waves overtop the crest. For overwash conditions, the erosion due to wave impact is expressed as (compare Equation 2)

\[
\frac{dV}{dt} = -4C_s \left( \frac{R - z_o}{T} \right)^2 \frac{s}{R - z_o} = -4C_s \left( \frac{R - z_o}{T} \right) s
\]

In the evaluation of the field data (see Tables 1 and 2), hydrodynamic conditions at the peak of the storm (i.e. maximum wave height and storm surge level) were employed as characteristic values. These conditions should be the most important for shaping the profile during overwash, although the forcing is overestimated through this selection of representative conditions. A more sophisticated approach would take into account the detailed time series of the waves and water level, but under such conditions feedback from the morphology should also be included requiring a numerical solution, which is outside the scope of the present study. Average transport rates for the storm events were obtained by dividing volume changes with the surge duration, which was computed on the basis of the time period that the run-up exceeded the beach crest. This also provided a coarse description of the actual transport rates during the event that are expected to vary significantly in time; however, the available data limited the resolution of the processes, as previously pointed out.

Figure 3 shows the observed average transport from the dune face during the events summarised in Tables 1 and 2 as a
function of the modified impact parameter, \((R - z_s)s/T\). A linear fit to the data is also included following Equation 9 (because of the logarithmic scale the line appears curved). The best-fit line corresponds to a \(C_s\)-value of about \(4 \times 10^{-3}\), yielding a coefficient of determination of \(r^2 = 0.55\). The datapoint corresponding to the largest transport rate influences the fit markedly, and if this point is removed from the database the dotted line results for which \(C_s = 2 \times 10^{-4}\) \((r^2 = 0.85)\). The removed data point involves a high, narrow dune that was completely destroyed during the storm event, resulting in the largest observed eroded volume (for a profile at Santa Rosa Island during Hurricane Opal).

The values obtained on \(C_s\) are in agreement with Larson et al., although somewhat higher than what was obtained for the field data in that study. Thus, considering the uncertainties in the analysis, Figure 3 indicates that the simple correction introduced to take into account the effect of the overwash on erosion from wave impact produces satisfactory results. A more complex relationship between the transport due to wave impact and the impact parameter and/or the correction factor would possibly yield a better agreement between the theoretical curve and the data. At present, due to the limited data employed for validation, a linear relationship was deemed to yield sufficiently good agreement for the purposes here. In Figure 3 the water level was assumed to reach the front face of the beach during the peak of the storm, implying that \(z_s = 0\).

The average offshore \((q_o)\) and overwash \((q_B)\) transport rates were calculated based on the observed volume changes, as previously described. The ratio \(q_o/q_B\) should vary during a storm event depending on the relationship between the beach crest height and the run-up height \((s/R)\). However, as a first approach, the average values on the transport rates were used for each event (Tables 1 and 2) to plot \(q_o/q_B\) against \(s/R\). Figure 4 illustrates the result for the ten events together with a best-fit straight line. Although the scatter is significant due to the schematisation, there is a clear trend of decreasing importance of overwash as \(s/R\) increases. The fit for the data points where \(s/R > 0.4\) displays increased scatter with a tendency for clustering. Closer inspection showed that this clustering is not related to a particular site or event, but there seems to be a relationship with the width of the dune (barrier), where a narrow dune (barrier) implies a smaller \(q_o/q_B\) ratio. However, the number of data points is too few to further explore this relationship.

As the ratio \(s/R\) approaches 1, \(q_o/q_B\) should approach infinity as the overwash transport becomes zero (no overtopping due to run-up occurs). Thus, the linear fit has its limitations and a more general empirical equation would be

\[
\frac{q_o}{q_B} = A \frac{s/(R - z_o)}{1 - s/(R - z_o)}
\]

where \(A\) is an empirical coefficient. Figure 4 also shows the non-linear fit according to Equation 10 with \(A = 3.0\).

5. SIMULATION OF LONG-TERM IMPACT OF STORMS

5.1. Technique for assessing probability of storm impact
Storm impact on dunes and barrier islands may be characterised in terms of key morphological parameters such as eroded volume, overwash volume, duration of overwash, and recession distance. A typical method to estimate the probability of a certain impact is to calculate the morphological change caused by a storm event with a certain return period and assign the same return period to that change. However, since a complex combination of different forcing is responsible for the impact, there is not a one-to-one correspondence between the return period of a storm and its effects. Morphological change is primarily caused by a combination of wave height and period (determining the run-up height), storm surge height and duration, tidal elevation, pre-storm profile shape, and sedimentological properties. The most appropriate method to derive return periods for various types of storm impact is to simulate long time series of morphological parameters based on existing time series of hydrodynamic data, and then to perform statistical analysis on the derived data. Such simulations may require simplified models to cover long time periods or to explore many different alternatives when design options are evaluated.

In this context it is useful to employ analytical solutions for simulating long time series of morphological parameters. Such calculations can be performed for a minimum of computational effort and permit the evaluation of different alternatives when
design considerations are made (e.g. beach nourishment). By quantifying the impact in terms of morphological parameters, the implementation of simulation results into risk-based assessment might be more readily performed compared to complex numerical modelling.

5.2 Case study: Ocean City
The analytical model developed for triangular-shaped dunes and barrier islands (Equations 4 to 6) was employed to estimate eroded volume and overwash volume for two schematised profile shapes, namely a high dune and a low-crested barrier island. The idealised profile shapes were constructed based on the high dunes common north of Ocean City inlet and the low-lying barrier island south of the inlet (Assateague Island). The dune was made 2 m high with a dune foot elevation of 3 m above mean sea level (MSL), whereas the barrier island had a crest elevation of 2 m and the front-face of the island 1 m above MSL. The equilibrium volume of the dune was set to 80 m$^3$/m and of the barrier island to 200 m$^3$/m. Equation 9 was employed in the analytical solution to calculate the sediment transport from the seaward side of the dune or barrier and Equation 10 to obtain the ratio between the offshore and overwash transport.

The analytical model was used to compute the eroded volume and overwash volume during specific events. An event was defined as the time period during which the dune (or barrier island) was continuously exposed to wave impact. The eroded volume for an event was calculated as the sum of the eroded volume at each time step during the event, and the associated duration of the event was recorded. Similarly, the overwash volume and the duration of the overwash were determined when water was transported over the beach crest. It was assumed that the dune and barrier island recovered completely between the events, implying that the storms would always impact the beach at initial conditions, which were taken to represent equilibrium conditions. In order to simulate the effects of storm chronology, the recovery process between storms has to be described, for example dune build-up by wind. Such algorithms have been developed but they were not implemented in the present simulations. A representative value of $C_s = 1.7 \times 10^{-4}$ was employed on the transport rate coefficient when calculating beach erosion due to wave impact with the analytical model based on Larson et al. This value was derived from an extensive comparison with field data and it is also in good agreement with the results from this study concerning erosion from wave impact during overwash.

Figures 5 and 6 display the empirical distribution functions for the eroded volume during an event and for the event duration, respectively, for the high dune case (the probability of non-exceedance is shown for different events; the plotting position formula by Weibull was used to obtain the empirical distribution function). Approximately 1500 dune erosion events were calculated for the studied 70-year period, which implies about 20 events/year. The resolution of the event duration is 1 h, which gives the distribution function for the duration a ‘jagged’ shape at short return periods. The shape of the distribution function in Figure 5 is complex, possibly indicating that different types of processes are responsible for the eroded volume associated with different return periods. For example, it is expected that extreme storms on the United States east coast, such as hurricanes or north-easters, would have a different type of impact than more ‘normal’ storms. It should be noted that all events calculated to produce erosion were included in the analysis, although some of them were small and outside the predictive capability of the developed analytical model.

Figures 7, 8 and 9 illustrate the distribution functions for eroded volume, overwash volume, and overwash duration, respectively, for the barrier island case. Due to the low elevation of the foot of the front face the island is frequently exposed to wave attack and on average erosion events would occur almost 1-5 times/day. The low crest implies that overwash takes place more than 10 times/year. As seen for the dune case, the distribution functions exhibit complex shapes indicating processes of different origin affecting the barrier island erosion and overwash.

The necessity of taking into account the combined effects of waves and water levels when assigning the probability of a specific impact is clearly illustrated by analysing the largest overwash events calculated for the high dune case. For the study period, the largest overwash event recorded was for the Ash Wednesday storm in March 1962 ($V_{over} = 13$ m$^3$/m), followed by storms in September 1933 ($V_{over} = 11$ m$^3$/m), December 1992 ($V_{over} = 10$ m$^3$/m), and November 1981 ($V_{over} = 6$ m$^3$/m). As a comparison, the maximum measured hydrodynamic parameters are...
occurred according to maximum wave height $H_{\text{rms}} = 4.4 \text{ m}$ in January 1938, and maximum water level $2.1 \text{ m}$ in September 1960. Both these extreme hydrodynamic events yielded minor overwash volumes, which were not among the 10 largest events recorded. Thus, assigning probabilities to specific morphological impact events based on analysis of individual forcing parameters will not be sufficient in many situations.

### 5.3 Case study: the Ebro Delta

The analytical solution for a triangular-shaped sub-aerial profile was also applied to two sites along the Ebro Delta representing a low-lying barrier island and a low-dune case corresponding to the conditions at Trabucador barrier beach and La Marquesa beach, respectively. The following typical elevations were selected for Trabucador, $s = 0.7 \text{ m}$ and $z_o = 0.3 \text{ m}$, and for La Marquesa beach, $s = 1.0 \text{ m}$ and $z_o = 0.5 \text{ m}$. An equilibrium volume of $60 \text{ m}^3/\text{m}$ was assigned to Trabucador and of $80 \text{ m}^3/\text{m}$ for La Marquesa beach. Otherwise the calculations were performed in the same manner as for the Ocean City simulations using the same coefficient values. Thus, it was assumed that the barrier would recover its equilibrium volume between erosive events. In this case it is assumed that constructive waves would contribute to the beach recovery rather than wind-blown transport.

Due to its low crest elevation, the Trabucador beach is exposed to frequent erosion and overwash events. The shape of the empirical distribution function for the beach erosion (Figure 10) is different in comparison with the Ocean City sites, especially regarding the extreme events. For the Trabucador there are a large number of events with eroded volumes that are small, in agreement with the Assateague Island simulations. The overwash volumes displayed in Figure 11 show similar characteristics to the eroded volume: a large number of events with small overwash volumes and no indication of a group of extreme storms with different statistical properties than the intermediate storms. In approximately 20% of the overwash cases the Trabucador beach was calculated to move shoreward some distance such that the front of the barrier passed the initial location of the back side of the barrier (barrier rollover). This is consistent with results of Jiménez and Sánchez-Arcilla on the long-term morphodynamic behaviour of this barrier beach and, also, with field observations of short-term barrier behaviour after the impact of large storms. If the barrier is assumed to breach when about 90% of the volume has eroded away, the calculations show that about 5% of the events might potentially cause breaching.
La Marquesa beach included a dune feature that was quite low, so frequent overwash was also calculated to occur at this site. Figures 12 and 13 illustrate the calculated empirical distribution functions for the eroded beach volume and the overwash volume, respectively. The curvature of the upper part of the distribution function is somewhat different for La Marquesa Beach compared to Trabucador, but overall the response is similar. It is worth noting that the number of overwash events at La Marquesa Beach was larger than at Trabucador barrier, although the latter had a significantly lower crest level. The events at Trabucadero bar have much longer durations leading to larger eroded volumes and overwash volumes. For the studied period, only one event occurred where more than 90% of beach volume was eroded away, indicating that breaching is not likely to occur for the schematised profile taken to represent La Marquesa Beach.

6. CONCLUSIONS
A simple analytical model was developed to calculate eroded volume and overwash volume from the sub-aerial portion of a beach having a triangular profile shape. This shape may represent both a dune and a barrier-island cross-section, where the latter would have considerably lower side slopes. The model assumes that the erosion from the seaward side of the beach is either lost offshore or transported over the beach crest as overwash. The impact model proposed by Larson et al. was employed to estimate the volume of sediment eroded, both for dune and barrier-island cross-sections, although this formula is primarily applicable for slopes that are steep enough to be subject to wave impact.

The impact model was also validated for the case of overwash when certain part of the impacting wave is overtopping the crest. A correction factor was introduced to reduce the impact force because of overtopping. Furthermore, the ratio between the volume lost offshore and the volume being overwashed when overtopping occurs was investigated, based on field data, and this ratio was related to the crest height divided by the run-up height over the beach foot elevation. Although the analysis of the field data involved significant uncertainties, the results indicated that the model captures the overall response of the sub-aerial profile and may provide order-of-magnitude estimates of main morphological parameters.

The analytical model was employed to simulate the long-term statistical properties of morphological impact parameters. Two detailed data sets on the hydrodynamics in the coastal zone, one from Ocean City (Maryland) in the USA and one from the Ebro Delta in Spain, were used as input to compute the morphological response of different schematised profiles. These profiles represented typical sub-aerial morphological shapes such as dune and barrier-island cross-sections. Empirical distribution functions were developed based on the computed time series of morphological parameters, for example eroded beach volume, overwash volume, and duration of erosion and overwash events. The derivation of such distribution functions may be used for risk assessment regarding beach erosion, overtopping and flooding, and breaching of dunes and barriers.

The analytical model was based on several simplifying assumptions, including longshore uniformity (only a profile cross-section was described), schematised profile shape that remains constant (triangular), and schematised wave and water level conditions. In spite of these simplifications, the model yielded satisfactory agreement with field data, presented in this paper and in previous studies. Thus, it is expected that if the basic assumptions are fulfilled, the predictions by the model will...
produce useful and reliable results for scoping-mode studies in which a quantitative overview of the subaerial beach response to storms are of interest.

APPENDIX A: ANALYTICAL SOLUTION FOR THE RESPONSE OF A TRIANGULAR DUNE OR BARRIER ISLAND TO WAVE IMPACT AND OVERWASH

The triangular dune or barrier island (denoted as the beach here) is defined by the height \( h \), the shoreward limit \( x_b \), and the seaward limit \( x_s \) (x-axis pointing onshore; see Figure 2 in the main text for a definition sketch). It is assumed that the seaward and shoreward slopes do not change with time, but the beach moves in parallel with itself in these areas. Waves impacting on the seaward side of the beach erode sediment that is either transported offshore \( q_s \) or transported over the crest \( q_b \), if overtopping occurs. A sediment volume \( \Delta V_o \) is removed by the waves from the seaward side during the time \( \Delta t \) yielding the following relationship from sediment conservation

\[
\Delta V_o = (q_o + q_b) \Delta t
\]

The sediment volume \( \Delta V_o \) deposited shoreward of the crest due to overwash is given by

\[
\Delta V_b = q_b \Delta t
\]

At the limit when \( \Delta t \) approaches zero, the following equations are obtained

\[
\frac{dV_o}{dt} = q_o + q_b
\]

\[
\frac{dV_b}{dt} = q_b
\]

The dune volume at any given time \( V_b(t) \) is calculated as

\[
V_b = \frac{1}{2} (x_b - x_s) s
\]

The change in the total dune volume with time is only a function of \( q_o \), since the overwash just implies a shoreward translation of the beach, not any loss of material

\[
\frac{dV_o}{dt} = -q_o
\]

The solution to Equation 16 for a constant \( q_o \), assuming a beach volume of \( V_{b0} \) when \( t = 0 \), is

\[
V_b = V_{b0} - q_o t
\]

From geometric relationships, \( \Delta V_b = \Delta x_b s \) and \( \Delta V_o = \Delta x_o s \), assuming small changes. Combining these relationships, noting that \( \Delta V_b = \Delta V_b - \Delta V_o \) and using Equation 16 yields

\[
\frac{d}{dt}(x_b - x_o) = -q_o \frac{s}{V_b}
\]

Using Equations 15 and 17, the following expression is obtained for \( s \)

\[
s = \frac{2V_{b0} - q_o t}{x_b - x_o}
\]

Replacing \( s \) in Equation 18, employing Equation 19 and solving gives

\[
x_b - x_o = l_d \sqrt{1 - \frac{q_o t}{V_{b0}}}
\]

where \( l_d = x_{b0} - x_{o0} \) is the initial beach width and subscript 0 denotes initial conditions at \( t = 0 \). Combining Equation 20 with Equation 19 gives the following solution for the evolution of the beach crest height

\[
s = s_0 \sqrt{1 - \frac{q_o t}{V_{b0}}}
\]

where \( s_0 = 2V_{b0}/l_d \) is the beach crest height at \( t = 0 \). From \( \Delta V_o = \Delta x_o s \) and Equation 11, the following equation results

\[
\frac{dx_o}{dt} = \frac{q_o + q_b}{s}
\]

which, combined with Equation 21, has the solution

\[
x_o = x_{o0} + l_d \left(1 + \frac{q_b}{q_o}\right) \left(1 - \sqrt{1 - \frac{q_o t}{V_{b0}}} \right)
\]

Similarly, \( \Delta V_b = \Delta x_b s \) and Equation 22 yields

\[
\frac{dx_b}{dt} = \frac{q_b}{s}
\]

which, combined with Equation 21, has the solution

\[
x_b = x_{b0} + l_d \frac{q_b}{q_o} \left(1 - \sqrt{1 - \frac{q_o t}{V_{b0}}} \right)
\]

Thus, Equations 21, 23 and 25 completely specify the time evolution of the triangular-shaped beach.

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REFERENCES

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