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Overcoming the Nyquist Limit in Blood Flow Velocity Estimation

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Abstract—Spectral Doppler ultrasound imaging typically consists of a spectrogram, showing the velocity distribution of the blood, and a brightness (B-) mode image allowing the operator to navigate. It is desirable to have both high spectral and velocity resolution, so that details in the blood flow can be traced, as well as a high B-mode frame rate to allow for tracking of movements and to adjust the position of the transducer. The blood flow signal is often sampled 1) using alternating transmissions for blood flow estimation and for B-mode imaging, or, 2) by acquiring a full Doppler spectrum and then parts of the B-mode image. The former has the disadvantage that it halves the sampling rate, making it likely that aliasing will occur when imaging fast moving blood or deeply positioned vessels; the latter that gaps appears in the spectrogram, and that if the frame rate of the B-mode images is slow, it will be difficult to track movements. Adaptive methods have been implemented to circumvent such problems, but even so, to get an acceptable frame rate of the B-mode images, the number of transmissions for Doppler estimation will be limited, restricting the spectral resolution. Alternatively, one may use an irregularly spaced emission pattern, but existing work on the topic is limited and generally suffers from poor resolution and spurious velocity components resulting from the irregular sampling pattern. In this paper, we examine the BIAA algorithm, showing that this approach allows for an accurate velocity estimate even from irregularly sampled measurements. Using an irregular emission pattern, with half the emissions used to form the B-mode image, the remaining emissions are found to yield accurate velocity estimates without reducing the maximally measurable velocity and without the spurious velocity components. Moreover, we show that the approach will allow for the same maximal velocity without aliasing as if all emissions would have been used for the velocity estimation.

Index Terms—Nyquist limit; Irregular sampling; Doppler ultrasound

I. INTRODUCTION

SPECTRAL Doppler is an effective tool in medical ultrasound as it allows for non-invasive estimation of velocities in blood vessels. The operator does not only get an image of the blood vessel but also of the flow dynamics in it, allowing for the diagnosis of several blood related diseases, such as arteriosclerosis [1]. The data is obtained by focusing the ultrasound transducer array along a single direction and

sampling data at the depth of interest. The velocity of the moving blood can be estimated by illuminating the same image line repeatedly, and hereby follow the motion of the blood. Taking out a single sample from each pulse emission produces a signal sampled at the pulse repetition frequency, f_{prf} , which yields a sinusoidal signal with a frequency of

$$f_p = \frac{2v_z}{c} f_c, \quad (1)$$

where v_z is the blood velocity along the ultrasound direction, $c = 1540$ m/s is the speed of propagation, and f_c the emitted ultrasound (center) frequency (typically 3-10 MHz) [2]. The velocity can then be found by estimating the spectral content of the signal (this signal is often referred to as the *slow-time signal*, as opposed to the *fast-time signal* which refers to the sampling of the pulse emission). The spectrum is then commonly estimated using Welch's method (see, e.g., [2], [3]) which has several well-known limitations. It requires, for example, a large number of transmissions (about 100) to form an acceptable estimate of the spectrum. This will effectively reduce the temporal resolution, making it difficult, for instance, to see the details of the rapid acceleration phases of the cardiac cycle. Moreover, as the same system is used to acquire the B-mode images necessary to navigate within the blood vessel, the B-mode and the velocity estimation transmission have to be interleaved. Typically, every second transmission is a B-mode acquisition, thereby resulting in a reduction of the measurable velocity range by a factor of two due to the Nyquist frequency being halved [2]. For vessels with high flow dynamics, one may instead choose to use blocks of B-mode transmissions, but this will then result in holes in the blood velocity spectrogram [4]. However, making us of recent advances in sparse signal estimation, it is possible to form spectrum estimates using irregular transmission schemes where a high B-mode frame rate is kept without losing performance in the velocity estimation. This method for velocity estimation is named Blood Iterative Adaptive Approach, or BIAA, and was originally presented in [5] as well as with a more computationally effective version in [6]. In this paper, we discuss how the (uniform sampling) Nyquist limit can be overcome by using an irregular transmission scheme and give an overview of the BIAA spectrum estimation method that can

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handle this kind of sampling schemes. We will also review an alternative definition of the Nyquist limit for irregularly sampled sequences.

A. The Nyquist Frequency

The Nyquist-Shannon sampling theorem states that a *uniformly sampled* signal can be perfectly recovered if the signal does not contain frequencies larger than half the sampling frequency, the so-called Nyquist frequency. However, for an irregularly sampled signal, the corresponding Nyquist frequency is not that clearly defined. One possible definition is through the spectral window, as suggested in [7] (see also [8]), defined at a frequency, f , as

$$W(f) = \left| \frac{1}{N} \sum_{n=1}^N e^{i2\pi f t_n} \right|^2, \quad (2)$$

where $\{t_n\}_{n=1}^N$ are the sampling instances. We note that $W(0) = 1$ and $W(f) \leq 1$, $\forall f \neq 0$. For regular sampling schemes, W is a periodic function with a period of two times the Nyquist frequency, i.e.,

$$W(f) = W(f + 2Pf_{Nyq}), \quad (3)$$

where $P \in \mathbb{Z}$ and f_{Nyq} denotes the Nyquist frequency for a regularly sampled signal. We therefore define the Nyquist frequency for an irregularly sampled signal, \hat{f}_{Nyq} , as the smallest frequency for which $W(2\hat{f}_{Nyq}) \approx 1$.

II. METHOD

The noise-free (slow-time) data acquired from ultrasound transmission n at depth k , for blood scatterers with (axial) velocity v , is generally modeled as [2], [9]

$$\tilde{x}_k(n) = \alpha_v^{(k)} e^{j\phi_c k + j\psi_v n}, \quad (4)$$

where $\alpha_v^{(k)}$ is the (complex-valued) amplitude of the sinusoidal signal at frequency ψ_v (at depth k), which is directly related to v , as

$$\psi_v = -\frac{2\omega_c}{cf_{prf}}v = -\frac{2v}{c}\omega_c T_{prf}, \quad (5)$$

where $\omega_c = 2\pi f_c$ and $T_{prf} = f_{prf}^{-1}$ is the time between pulse repetitions. Furthermore, ϕ_c is the demodulating frequency, relating the samples along each emission (the so-called fast-time), defined as $\phi_c = \omega_c/f_s$, where f_s is the sampling frequency. We can rewrite the signal in (4), describing it as a sum of the contributions from all frequency grid points $\{\psi_{v_m}\}_{m=1}^M$ (with v_m denoting the m :th (axial) velocity),

$$x_k(n) = e^{j\phi_c k} \sum_{m=1}^M \alpha_{v_m}^{(k)} e^{j\psi_{v_m} n} + w_k(n), \quad (6)$$

where $w_k(n)$ denotes a residual term consisting of all signals at velocities different from the M considered velocities as well as additive noise. From (5) and (6), it is clear that the spectral density of $x_k(n)$ with respect to ψ_v is equivalent to the blood velocity distribution at the examined location. The problem of estimating the blood velocity distribution can thus be seen to

be equivalent to the estimation of $|\alpha_v|^2$ for all velocities of interest. We can also exploit the fact that the blood flow profile is rather smooth over the neighboring depths, so that $\alpha_v^{(k)}$, for $k = k_1, \dots, k_K$, is almost constant. We therefore form

$$y(n) = \frac{1}{K} \sum_{k=k_1}^{k_K} e^{-j\phi_c k} x_k(n), \quad (7)$$

in order to increase the signal to noise ratio (SNR) of the processed signal.

A. The BIAA Method

Assuming uniform pulse emissions for either velocity estimation or B-mode imaging, the slow-time measurements may be viewed as exhibiting a reoccurring block structure, such that each block consists of the pattern of velocity and B-mode transmissions, typically having the form

$$\mathbf{y}_{N_g}(p) = [y(pN_s) \quad \dots \quad y(pN_s + N_g - 1)]^T, \quad (8)$$

where $N_s = N_g + N_m$ and N_g and N_m denote the number of velocity emissions (given samples) and the number of B-mode emissions, here simply treated as missing samples, respectively. For the traditional case with every second emission being a B-mode acquisition, $N_g = N_m = 1$, but more general sampling patterns can also be used [4], [5]. The measurements used for velocity estimation at time p is then formed as the concatenation of the N_b most recent sub-blocks, i.e.,

$$\mathbf{z}_N(p) = [\mathbf{y}_{N_g}^T(p - N_b + 1) \quad \dots \quad \mathbf{y}_{N_g}^T(p)]^T, \quad (9)$$

with the last sub-block being the most current measurements, and where $N = N_b N_g$ denotes the total number of available measurements in the observation window. Clearly, N will be limited by the stationarity of the examined blood velocity signal, bounding how many emissions that may be used to form the resulting blood velocity spectral estimate.

We form the gapped block Fourier vector, taking into account the missing samples, as

$$\bar{\mathbf{f}}_{\psi_{v_m}} = \mathbf{a}_{N_s, N_b} \otimes \mathbf{a}_{1, N_g}, \quad (10)$$

where \otimes denotes the Kronecker product and

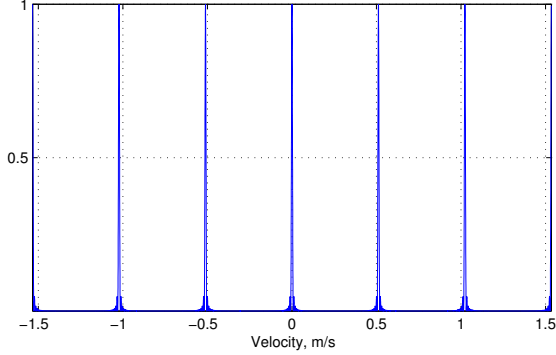
$$\mathbf{a}_{\ell_1, \ell_2} = [1 \quad e^{j\psi_{v_m} \ell_1} \quad \dots \quad e^{j\psi_{v_m} \ell_1(\ell_2 - 1)}]^T. \quad (11)$$

The BIAA algorithm, as given in [6], is then formed, using the measurements up to time p , by iteratively estimating

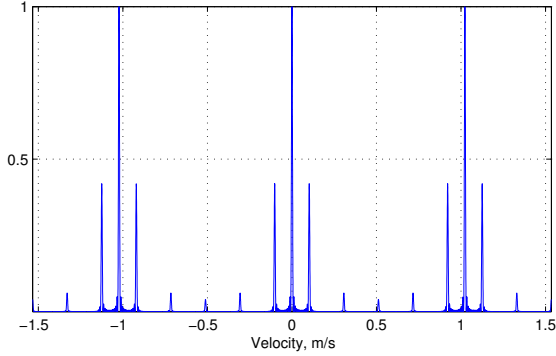
$$\hat{\alpha}_{p, v_m} = \frac{\bar{\mathbf{f}}_{\psi_{v_m}}^H \mathbf{R}_N^{-1}(p) \mathbf{z}_N(p)}{\bar{\mathbf{f}}_{\psi_{v_m}}^H \mathbf{R}_N^{-1}(p) \bar{\mathbf{f}}_{\psi_{v_m}}}, \quad (12)$$

$$\mathbf{R}_N(p) = \sum_{m=1}^M |\hat{\alpha}_{p, v_m}|^2 \bar{\mathbf{f}}_{\psi_{v_m}} \bar{\mathbf{f}}_{\psi_{v_m}}^H, \quad (13)$$

until practical convergence (typically 10 to 15 iterations), with $(\cdot)^H$ denoting the conjugate transpose. In the interest of brevity, the details for the more computationally efficient, as well as for the time-recursive, implementations of the BIAA algorithm are omitted here (see [5], [6] for these details). The



(a) Emission pattern $v b$.



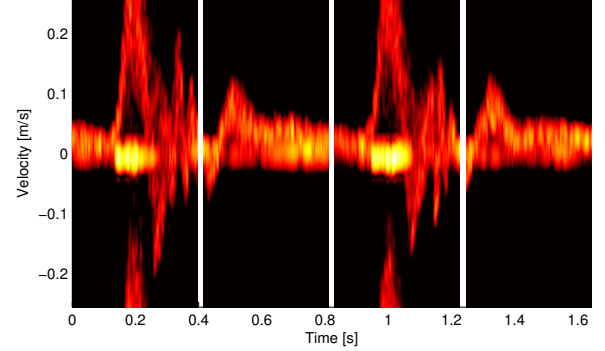
(b) Emission pattern $v v v v b b b b b$.

Fig. 1: The Nyquist limit for (a) transmission pattern $[v b]$ and (b) $[v v v v b b b b b]$. Using the former pattern, aliasing will occur for velocities larger than around 0.25 m/s whereas the latter pattern allows for the double, provided the estimation method can handle irregular sampling.

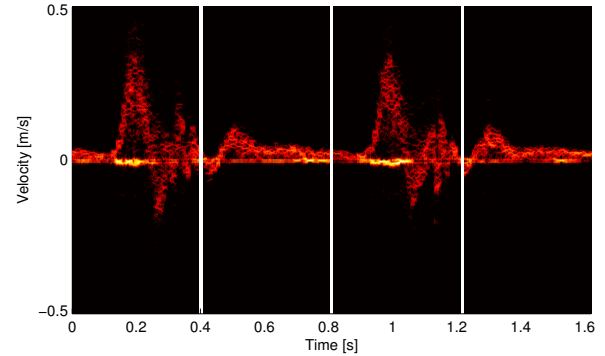
Matlab code for these implementations can be found on the second author's webpage, <http://www.maths.lth.se/matstat/staff/aj/Publications.htm>.

III. RESULTS

In order to demonstrate the usefulness of having an irregular transmission scheme for velocity estimation, we will compare the sampling patterns $[v b]$ and $[v v v v b b b b b]$, where v denotes an emission aimed at estimating the blood velocity and b an emission aimed at forming the B-mode image. We will use the *in vivo* data set originally published in [10], acquired using the experimental scanner RASMUS [11] and a B-K Medical 8804 7 MHz linear array transducer, with $f_{prf} = 9.3$ kHz. Further details on the setup is found in [10]. Assuming our data is stationary over 130 transmission instances, we will have either 65 blocks or 13 blocks of transmissions for the two cases, with an equal amount of transmissions available for velocity estimation in both cases (65 transmissions). Fig. 1 displays the spectral window computed according to (2) from which the Nyquist limit can be obtained. We see that for the commonly used emission pattern $[v b]$, wherein every second emission is aimed at forming the velocity estimate and the



(a) Welch's method.



(b) BIAA method.

Fig. 2: Spectrogram estimates using (a) Welch's method with transmission pattern $[v b]$ and (b) the BIAA method from pattern $[v v v v b b b b b]$. The figure illustrates the strengths of having an irregular transmission scheme to overcome the (regular) Nyquist limit.

other at forming the B-mode image, which is then repeated, the maximum velocity that can be estimated using this setup is around 0.25 m/s. For the pattern $[v v v v b b b b b]$, formed as five consecutive emissions aimed at forming the velocity estimate followed by five to form the B-mode image, the maximum velocity is around 0.5 m/s, i.e., twice as much. It should be stressed that each of the patterns are then repeated such that both setups are formed using the same number of emissions, with 50% of the emissions being used to form the B-mode image in both cases.

Fig. 2 shows the spectrograms produced by (a) Welch's method using 65 blocks with emission pattern $[v b]$ and (b) the BIAA method using 13 blocks with emission pattern $[v v v v b b b b b]$, both using $K = 40$ regularly spaced measurements along depths (fast-time samples) and using a dynamic range of 45 dB. Clearly, the data is aliased in Fig. 2a which is not the case in Fig. 2b. The gaps occurring in the spectrograms represent transmissions used for the necessary B-mode images during the data acquisition as the data set was obtained to evaluate the BAPES estimator [9] which requires regular emissions; thus, the B-mode emissions for this data set differ from the ones assumed in the here examined

example. If the B-mode emissions would be done according to the assumed sampling pattern over emissions, no gaps would occur.

IV. CONCLUSION

In this paper, we have demonstrated the benefits of using an irregular scheme for interleaving the B-mode image transmissions with transmissions for velocity estimation. With an irregularly spaced transmission scheme, it is possible to maintain the highest possible Nyquist limit, dictated by the pulse repetition frequency, without suffering from aliasing. We have also shown an alternative, more general, definition of the Nyquist frequency which, while still being easy to compute, also holds in the case of irregular sampling. As commonly used methods for velocity profile estimation, such as Welch's method, will not provide accurate estimates for the irregular emission patterns needed to overcome the classical Nyquist limitation, one needs to employ alternative spectral estimation methods, e.g., the BIAA method. This form of estimates allows for reliable blood velocity estimates without a reduction in the achievable velocity range, while maintaining the B-mode sampling rate and still yielding highly accurate blood velocity estimates without aliasing.

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