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# Multiplexing Efficiency of MIMO Antennas

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**Abstract**—A simple and intuitive metric of multiplexing efficiency is proposed for evaluating the performance of MIMO antennas in the spatial multiplexing mode of operation. Apart from gaining valuable insights into the impact of antenna efficiency, efficiency imbalance and correlation on multiplexing performance, the metric is particularly useful for antenna engineers whose goal is to achieve the optimum antenna system design. Experimental results involving prototype mobile terminals highlight the effectiveness of our proposal.

**Index Terms**—MIMO systems, antenna measurements, channel capacity.

## I. INTRODUCTION

Despite intense academic research in multiple-input multiple-output (MIMO) technology for over two decades [1], and its recent adoption in major wireless standards, performance characterization of multiple antenna terminals is a subject of current interest [2]. Depending on the signal-to-noise ratios (SNRs) of the received signals, different MIMO modes are required to optimize the system performance. For the low SNR regime, diversity techniques are applied to mitigate fading and the performance gain is typically expressed as diversity gain (in decibel, dB) [3]. Such a measure is convenient for antenna engineers, since performance improvement is translated into a tangible power gain, or equivalently, an increase in coverage area. On the other hand, higher SNRs facilitate the use of spatial multiplexing (SM), *i.e.*, the transmission of parallel data streams, and information theoretic capacity in bits per second per Hertz (bits/s/Hz) is the performance measure of choice [4]. In [5], a metric based on difference in capacity is proposed for performance comparison of multiple antenna terminals in a reverberation chamber (RC). The motivation is that the proposed capacity difference metric does not require the *absolute values* of the actual received SNRs in a RC, which cannot be estimated in common RC setups with vector network analyzers [5]. However, capacity is a system level metric that is less intuitive to antenna engineers who would prefer a power related measure, such as the diversity gain. Moreover, since SM is the primary mechanism for increasing

the spectral efficiency of MIMO systems, it is important to consider it explicitly in antenna design.

This Letter introduces *multiplexing efficiency* as a power related metric for the SM mode of operation in MIMO systems and derives its approximate closed form expression. The unique features of the expression are both its simplicity and the valuable insights it offers with respect to the performance impact of non-ideal behaviors of multiple antennas. An example application of the metric is demonstrated for two realistic mobile terminal prototypes.

## II. MULTIPLEXING EFFICIENCY METRIC

Considering a  $M \times M$  MIMO channel  $\mathbf{H}$ , the instantaneous channel capacity with no channel information at the transmitter (*i.e.*, equal transmit power allocation) can be expressed as [6]

$$C = \log_2 \det \left( \mathbf{I}_M + \frac{\rho_T}{M} \mathbf{H} \mathbf{H}^H \right), \quad (1)$$

where the SNR  $\rho_T$  is defined by  $\rho_T = P_T / \sigma_n^2$ .  $P_T$  denotes the transmit power and  $\sigma_n^2$  is the noise power at the receiver. Since the interest here is in antenna design, the reference propagation environment of independent and identically distributed (iid) Rayleigh fading channel  $\mathbf{H}_w$  is assumed, *i.e.*, the entries of  $\mathbf{H}_w$  are zero mean circularly symmetric complex Gaussian random variables. Without loss of generality, the case of receive antennas is examined. Then, the MIMO channel is given by

$$\mathbf{H} = \mathbf{R}^{\frac{1}{2}} \mathbf{H}_w, \quad (2)$$

where  $\mathbf{R}$  denotes the receive correlation matrix which fully describes the effects of the antennas on the channel, *i.e.*, it characterizes the efficiency, efficiency imbalance and correlation among the receive antennas. Specifically,

$$\mathbf{R} = \mathbf{\Lambda}^{\frac{1}{2}} \bar{\mathbf{R}} \mathbf{\Lambda}^{\frac{1}{2}}, \quad (3)$$

where  $\bar{\mathbf{R}}$  is a normalized correlation matrix whose diagonal elements are 1 and  $(i, j)$ th ( $i \neq j$ ) element  $\bar{\mathbf{R}}(i, j)$  denotes the complex correlation coefficient between the 3D radiation patterns of the  $i$ th and  $j$ th antenna ports.  $\mathbf{\Lambda}$  denotes a diagonal matrix given by

$$\mathbf{\Lambda} = \text{diag}[\eta_1, \eta_2, \dots, \eta_M], \quad (4)$$

where  $\eta_i$  is the total efficiency of the  $i$ th antenna port.

In order to obtain a reliable estimate of the multiplexing capability of the antennas, it is noted that at high SNRs, the instantaneous capacity of (1) can be written as [6]

$$C \approx C_0 + \log_2 \det(\mathbf{R}), \quad (5)$$

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where  $C_0$  denotes the capacity of the ideal iid Rayleigh channel at high SNR,

$$C_0 = \log_2 \det \left( \frac{\rho_T}{M} \mathbf{H}_w \mathbf{H}_w^H \right), \quad (6)$$

which is achieved with ideal antennas in uniform 3D angular power spectrum (APS), *i.e.*,  $\mathbf{R} = \mathbf{I}_M$ . Ideal antennas are 100% efficient and completely orthogonal to one another in radiation pattern (either in space and/or polarization). The high SNR approximation is also used in [5] to obtain capacity difference for comparing different multiple antenna terminals. By assuming the noise floor and propagation loss to be the same when measuring the channel in the RC for different terminals, it is shown that these parameters do not appear in the capacity difference metric [5] at sufficiently high SNR.

Since  $\log_2 \det(\mathbf{A}) \leq 0$  and  $\log_2 \det(\bar{\mathbf{R}}) \leq 0$  (see also [6]) in (5), non-ideal antenna effects will result in a constant degradation in the channel capacity over SNR, relative to that of the iid channel. In order to translate this capacity gap into a power related measure, we can apply the following equality

$$\det(\mathbf{R}) = \det(\det(\mathbf{R})^{\frac{1}{M}} \mathbf{I}_M), \quad (7)$$

to (5), which can then be rewritten as

$$C \approx \log_2 \det \left( \frac{\rho_T}{M} \det(\mathbf{R})^{\frac{1}{M}} \mathbf{H}_w \mathbf{H}_w^H \right). \quad (8)$$

Comparing (8) with (6), we conclude that at high SNRs, the capacity  $C$  in (5) with non-ideal antennas is equivalent to that of ideal antennas in iid channel with the SNR

$$\rho_0 = \rho_T (\det \mathbf{R})^{\frac{1}{M}}. \quad (9)$$

In this context, the multiplexing efficiency is defined as

$$\eta_{\text{mux}} = \rho_0 / \rho_T \leq 1, \quad (10)$$

or equivalently

$$\eta_{\text{mux}} \text{ [dB]} = \rho_0 - \rho_T \leq 0, \quad (11)$$

which measures the loss of efficiency in SNR (or power, assuming the noise power  $\sigma_n^2$  is the same) when using a real multiple-antenna prototype in an iid channel (with SNR  $\rho_T$ ) to achieve the same capacity as that of an ideal array in the same iid channel (with SNR  $\rho_0$ ).

For high SNRs,  $\eta_{\text{mux}}$  is readily obtained from (9), *i.e.*,

$$\tilde{\eta}_{\text{mux}} = \lim_{\rho_T \rightarrow \infty} \eta_{\text{mux}} = \det(\mathbf{R})^{\frac{1}{M}}. \quad (12)$$

Substituting (3) into (12), we can rewrite

$$\tilde{\eta}_{\text{mux}} = \det(\Lambda \bar{\mathbf{R}})^{\frac{1}{M}} = \left( \prod_{k=1}^M \eta_k \right)^{\frac{1}{M}} \det(\bar{\mathbf{R}})^{\frac{1}{M}}, \quad (13)$$

which shows that the multiplexing efficiency is determined by the product of the geometric mean (or the arithmetic mean in dB scale) of the antenna efficiencies and a correlation induced term  $\det(\bar{\mathbf{R}})^{\frac{1}{M}}$ . The geometric mean term is intuitive, in that the overall efficiency should come in between the efficiencies of the constituent antennas. For the correlation induced term, its impact can be understood in that, as the correlation among the ports increases, the condition number of  $\bar{\mathbf{R}}$  increases. This in turn decreases both  $\det(\bar{\mathbf{R}})$  and  $\det(\bar{\mathbf{R}})^{\frac{1}{M}}$ . In other words,

a higher  $\rho_T$  is needed in order for its capacity to match that of the iid case with SNR  $\rho_0$ .

In general, the definition (10) is still valid, even when the high SNR assumption is not satisfied. However, the resulting expression for  $\eta_{\text{mux}}$  is more involved, and is a function of  $\rho_T$  (or equivalently,  $\rho_0$ ). In other words, the constant capacity gap seen in (5) may not apply.

The procedure for deriving the exact  $\eta_{\text{mux}}$  for a given instantaneous realization of  $\mathbf{H}_w$  is given as follows:

- Equate the iid capacity  $C_{\text{iid}}$  (of SNR  $\rho_0$ ) with (1), and by the property of determinant

$$\underbrace{\log_2 \det \left( \mathbf{I}_M + \frac{\rho_0}{M} \mathbf{H}_w \mathbf{H}_w^H \right)}_{C_{\text{iid}}} = \underbrace{\log_2 \det \left( \mathbf{I}_M + \frac{\rho_T}{M} \mathbf{R} \mathbf{H}_w \mathbf{H}_w^H \right)}_C \quad (14)$$

- Since the function  $\log_2(\cdot)$  is monotonic

$$\det \left( \mathbf{I}_M + \frac{\rho_0}{M} \mathbf{H}_w \mathbf{H}_w^H \right) = \det \left( \mathbf{I}_M + \frac{\rho_T}{M} \mathbf{R} \mathbf{H}_w \mathbf{H}_w^H \right) \quad (15)$$

- Introduce (10) in (15) and solve for  $\eta_{\text{mux}}$  numerically.

Whereas  $\tilde{\eta}_{\text{mux}}$  in (12) does not depend on either the channel realization or the exact SNR,  $\eta_{\text{mux}}$  is influenced by both factors. In practice, MIMO performance is typically characterized by ergodic capacity, which is calculated from a large number of Monte Carlo realizations of the channel matrix  $\mathbf{H}$ . Thus, it is more appropriate to derive  $\eta_{\text{mux}}$  based on the ergodic capacity. This is achieved by first taking the expectation on both sides of (14)

$$\mathbb{E} \{ C_{\text{iid}} \} = \mathbb{E} \{ C \}. \quad (16)$$

However, there is no known exact closed form solution for (16). Instead, for a given SNR  $\rho_T$ , the ergodic capacity for the non-ideal antennas is calculated from Monte Carlo simulations, and the required SNR  $\rho_0$  for the ideal antennas to offer the same ergodic capacity can then be obtained by a parametric search (*e.g.*, by decreasing  $\rho_0$  progressively from a starting guess of  $\rho_0 = \rho_T$ ). Hence,  $\eta_{\text{mux}}$  can be calculated numerically from the given  $\rho_T$  and the corresponding solution  $\rho_0$  using (10).

One way to get around this cumbersome approach is to take the upper bound from Jensen's inequality on both sides of (16), which after some manipulations yields

$$(1 + \rho_0)^M = \det(\mathbf{I}_M + \rho_T \mathbf{R}), \quad (17)$$

Then, substituting  $\rho_0 = \eta_{\text{mux}} \rho_T$  into (17) and solving for  $\eta_{\text{mux}}$ , we obtain

$$\eta_{\text{mux}} = \left( \det(\mathbf{I}_M + \rho_T \mathbf{R})^{\frac{1}{M}} - 1 \right) / \rho_T, \quad (18)$$

which can be shown to converge to (12) in the limit of high SNRs. However, it is noted that the closed form solution (18) is obtained using the upper bounds for Jensen's inequality, which is in fact a loose bound, as can be seen in [7]. Moreover, the calculation involves taking the roots of a polynomial in  $\rho_T$  and does not readily offer useful insights into the impact of non-ideal multiple antennas, as is possible with (13). In addition, since  $\mathbf{R}$  is deterministic, it can be shown that the high SNR

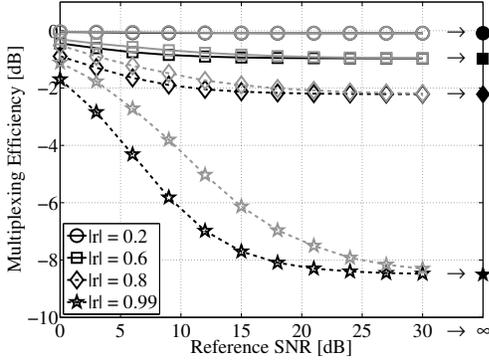
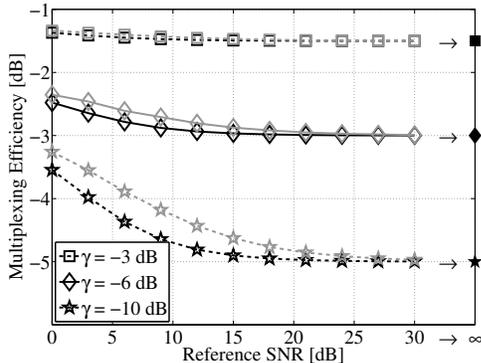
(a) Correlation  $|r|$ (b) Imbalance  $\gamma$ 

Fig. 1. Multiplexing efficiency vs. reference SNR with respect to changes in (a) antenna correlation ( $\eta_1 = \eta_2 = 1$ ) and (b) efficiency imbalance ( $r = 0$  and  $\eta_1 = 1, \eta_2 = \gamma$  in  $\mathbf{\Lambda}$ ). The darker curves denote  $\eta_{\text{mux}}$  derived from the capacity upper bounds (21) and the lighter curves denote the exact  $\eta_{\text{mux}}$  obtained from Monte Carlo simulations, respectively. The limits  $\tilde{\eta}_{\text{mux}}$  from (20) are also shown with solid markers.

solution (12) is equally valid for ergodic capacity, in spite of having been derived based on instantaneous capacity.

### III. CASE STUDY: $2 \times 2$ MIMO

For two receive antennas, the antenna efficiency and normalized correlation matrices of  $\mathbf{R} = \mathbf{\Lambda}^{\frac{1}{2}} \bar{\mathbf{R}} \mathbf{\Lambda}^{\frac{1}{2}}$  are given by

$$\mathbf{\Lambda} = \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 1 & r \\ r^* & 1 \end{bmatrix}, \quad (19)$$

where  $r$  denotes the complex correlation coefficient between the two antennas. The approximate closed form expression (12) at high SNR can then be written as

$$\tilde{\eta}_{\text{mux}} = \sqrt{\eta_1 \eta_2 (1 - |r|^2)}. \quad (20)$$

For comparison, the expression (18) for  $\eta_{\text{mux}}$  that is derived based on the upper bound of ergodic capacity and without assuming high SNR can be simplified as

$$\eta_{\text{mux}} = \left( \sqrt{\eta_1 \eta_2 (1 - |r|^2) \rho_T^2 + (\eta_1 + \eta_2) \rho_T + 1} - 1 \right) / \rho_T. \quad (21)$$

It is easily verified that (21) converges to  $\tilde{\eta}_{\text{mux}}$  at high SNRs.

As mentioned earlier, the impact of correlation and efficiency imbalance on  $\tilde{\eta}_{\text{mux}}$  can be studied separately using (13),



Fig. 2. Photo of the two terminal prototypes P1 and P2.

TABLE I  
PERFORMANCE CHARACTERISTICS OF PROTOTYPES P1 AND P2.

		P1	P2
Correlation $ r $		0.80	0.19
Efficiency	$\eta_1$	-4.7 dB	-3.9 dB
	$\eta_2$	-5.2 dB	-4.2 dB
Multiplexing Efficiency $\eta_{\text{mux}}$		-7.2 dB	-4.2 dB

which is illustrated as solid markers in Fig. 1. For comparison, the impact of correlation and efficiency imbalance on (21) and the exact  $\eta_{\text{mux}}$  from Monte Carlo simulations are also given for different SNRs. From Fig. 1(a), it is observed that the multiplexing efficiency is relatively insensitive to low to moderate values of correlation, with the decrease in efficiency of lower than 1 dB for correlation of up to 0.6. However, as the correlation increases beyond 0.6, the multiplexing efficiency decreases more severely. This observation is consistent with the rule of thumb that the influence of correlation on diversity gain becomes significant for correlation of above 0.7. In addition, the rate of convergence of (21) to the limiting value  $\tilde{\eta}_{\text{mux}}$  with SNR decreases significantly when the correlation is increased. Nevertheless, convergence is achieved at 30 dB SNR even for the highly unlikely extreme correlation of 0.99. This indicates that the approximate closed form expression of  $\tilde{\eta}_{\text{mux}}$  is accurate for practical prototypes (as is confirmed by the later examples) at commonly used reference SNR values, *e.g.*,  $\rho_0 = 20$  dB. In any case, Fig. 1(a) also reveals that the approximate solution is a conservative estimate, which gives a lower bound to the exact  $\eta_{\text{mux}}$ . Fig. 1(b) confirms that at sufficiently high SNR and with  $|r| = 0$ , the multiplexing efficiency is the arithmetic average of the individual antenna efficiencies (in dB scale), as indicated by (13).

### IV. EXPERIMENTAL RESULTS

To illustrate the effectiveness of the proposed metric for characterizing MIMO capability, two realistic mobile terminal prototypes are evaluated (see Fig. 2). Each of the test prototypes is fully equipped as a normal mobile terminal and has two well-matched antennas operating in the 2.45 GHz frequency band. The antennas for prototype ‘‘P1’’ is intentionally equipped with a dual-feed PIFA to achieve high correlation (for the purpose of testing) whereas prototype ‘‘P2’’ is designed with spatially separated ceramic chip antennas for low correlation. The characteristics of the antenna prototypes, including

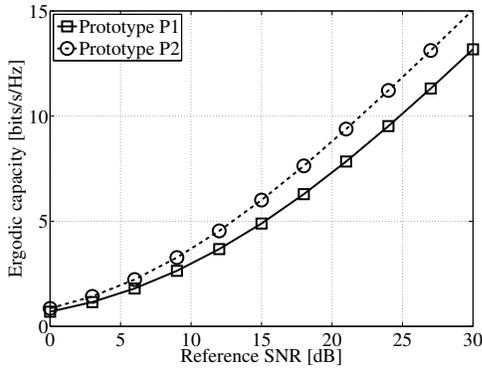


Fig. 3. Ergodic capacity of P1 and P2.

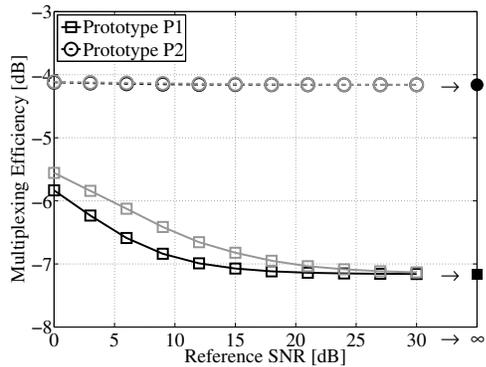


Fig. 4. Multiplexing efficiency of P1 and P2. The darker curves denote  $\eta_{\text{mux}}$  derived from the capacity upper bounds (21) and the lighter curves denote the exact  $\eta_{\text{mux}}$  obtained from Monte Carlo simulations, respectively. The limits  $\tilde{\eta}_{\text{mux}}$  from (20) are also shown with solid markers.

measured efficiencies and magnitudes of the complex pattern correlation under uniform 3D APS, are summarized in Tab. I. As can be seen in Tab. I, P1 suffers from much higher correlation, lower efficiency, and slightly higher efficiency imbalance as compared to P2.

As mentioned earlier, channel capacity is the conventional metric for evaluating and comparing the multiplexing capability of different antenna prototypes. Fig. 3 presents the ergodic capacity, which is calculated based on the antenna parameters given in Tab. I. Although the figure clearly shows that P2 has a higher ergodic capacity than P1, the absolute difference expressed in bits/s/Hz does not lend itself to a convenient interpretation and offers no direct insight into the relative influence of antenna efficiency, efficiency imbalance and correlation. In Fig. 4, the multiplexing efficiency  $\eta_{\text{mux}}$  of the two prototypes is illustrated. It is observed that the multiplexing efficiency of P2 is at  $-4$  dB, which is mainly attributed to practical limitations in antenna efficiency for fully-equipped terminal prototypes (as given in Tab. I). On the other hand, P1 has a significantly lower multiplexing efficiency of  $-7$  dB. According to Tab. I, the lower average antenna efficiency of P1 ( $-5$  dB) contributes to a loss of 1 dB in multiplexing efficiency with respect to that of P2. The correlation coefficient of 0.8 is responsible for a further 2 dB loss, which can be seen from the results shown in Fig. 1(a).

## V. CONCLUSION

In this paper, multiplexing efficiency is proposed as a simple and intuitive metric for evaluating the effectiveness of MIMO antenna terminals operating in the SM mode. Instead of comparing the ergodic capacity, the metric quantifies the performance in terms of absolute efficiency. An example highlights its utility to antenna engineers in identifying and addressing critical design parameters, which will likewise be a useful metric for testing MIMO terminals with different antenna characteristics.

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