



LUND UNIVERSITY

Smoke Flow in Buildings: Entrainment around the Corner (Hydraulic Model)

Thomas, Philip H

1992

[Link to publication](#)

Citation for published version (APA):

Thomas, P. H. (1992). *Smoke Flow in Buildings: Entrainment around the Corner (Hydraulic Model)*. (LUTVDG/TVBB--3067--SE; Vol. 3067). Department of Fire Safety Engineering and Systems Safety, Lund University.

Total number of authors:

1

General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

Lund University . Sweden

Institute of Technology

Department of Fire Safety Engineering

ISSN 1102-8246

ISRN LUTVDG/TVBB--3067--SE

Brandteknik
Lunds Tekniska Högskola
Biblioteket

Philip H. Thomas

**Smoke Flow in Buildings:
Entrainment around the Corner
(Hydraulic Model)**

Research financed by the Swedish Fire Research Board (BRANDFORSK)

Lund, June 1992

Lund University . Sweden
Institute of Technology
Department of Fire Safety Engineering
ISSN 1102-8246
ISRN LUTVDG/TVBB--3067--SE

Brandteknik
Lunds Tekniska Högskola
Biblioteket

Philip H. Thomas

**Smoke Flow in Buildings:
Entrainment around the Corner
(Hydraulic Model)**

Research financed by the Swedish Fire Research Board (BRANDFORSK)

Lund, June 1992

Smoke flow in buildings: Entrainment around a Corner

P H Thomas
Department of Fire Safety Engineering
Lund University, Sweden

Summary

In an engineering treatment of the rotation of a layer of hot gas emerging from under a balcony Morgan and Marshall [1] introduced the concept of a horizontal Equivalent Gaussian Source for the resulting vertical plume.

They found it necessary to suppose that in the rotation the entrainment was more than 5 times the amount to be expected using conventional entrainment coefficients for vertical plumes.

In this report two versions of an alternative model are given and it is suggested that the apparently high entrainment is due to the way equivalence is defined, i.e. it is in part real in part fictitious and in part an effect of temperature not in the Morgan and Marshall model.

1. Introduction

It is now practical to analyse flow problems by computational fluid dynamics and this means that some of the analyses can be refined, modified or validated against such calculations. This paper deals with a problem in smoke flow still awaiting such an analysis: it does however offer an explanation of an unresolved discrepancy between an empirical model and data.

Morgan and Marshall [1] have interpreted data on horizontal flows emerging from under a balcony as if there was considerable extra entrainment when the layer rotated into an upward plume. They originally proposed a model, which as they themselves recognized, was "ad hoc": to fit data to it required assuming an entrainment coefficient over 5 times that found by Lee and Emmons [2] for a vertical plume, i.e. 0.90 instead of 0.16.

In horizontal stratified flows one would expect a lower value so the result presents a problem in interpretation: no explanation has as yet been offered of why the value of 0.90 is so far above what would be expected for a situation intermediate between a near zero entrainment coefficient and 0.16 for vertical flow.

2. Theory

2.1 The Theory of Morgan and Marshall

Morgan and Marshall wished to produce a model for the flow of a layer around a corner with no account being taken of the conditions in a room which resulted in a layer of hot gas moving under a flat horizontal surface. Their result would therefore be suitable for general application.

Continuity of thermal energy, i.e. convected heat was assumed and the effect of buoyancy on the rotating flow was described in terms of the change from potential to kinetic energy, the resulting upward flow being described in terms of a plume from an Equivalent Source having Gaussian distributions of velocity and temperature rise across a horizontal section.

The value of an arbitrary coefficient of entrainment on one side of the rotating flow was obtained by matching the equations to data. It was this arbitrary coefficient which proved unexpectedly large.

There are two defects in the theory. The assumption of the conservation of potential and kinetic energy is plausibly correct in the limit of no entrainment when, at any level 'z' there would be acceleration due to buoyancy. However, in

the turbulent system there is a continuous dissipation of energy: in plume theory momentum and buoyancy equations are used, not energy.

Secondly, making a Gaussian Source equivalent to a "top hat" or triangular distribution means one cannot have equivalence both in average temperature rises and in peak values. If average temperature rises are conserved (in the absence of entrainment) one would have to postulate a rise in peak temperature! Peak temperature rises would be less than "theoretically" required and would imply a fictitious entrainment.

Matching therefore takes place over a distance over which the average temperature rise falls sufficiently. Any temperature measurements within this region would be misleading if interpreted as being for a Gaussian plume.

2.2 The Velocity Distribution

The discussion of the flow out of openings customarily assumes the velocity is low in the neutral plane where inflow and outflow pass each other, see Fig. 1. The gases flowing out at A have a velocity due to the reduction in hydrostatic pressure from B to A but there is a vena contracta since some fluid has to move down from C and cannot change direction sharply.

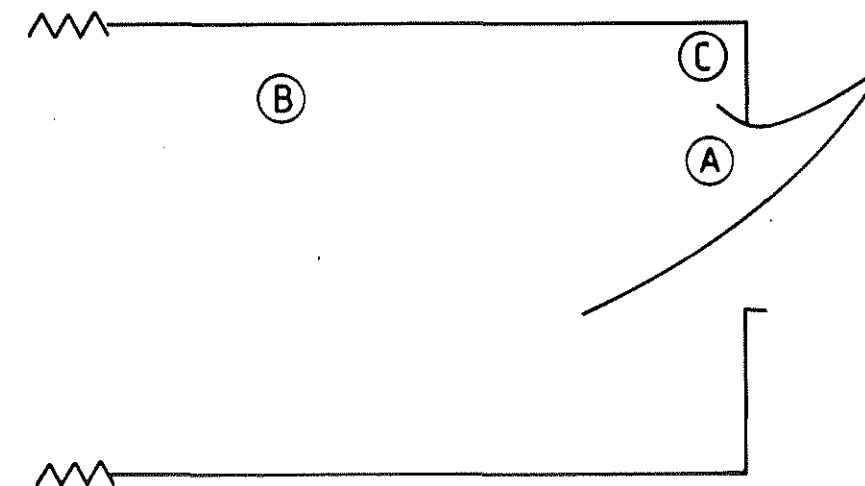


Figure 1

This effect does not arise in the same way when the layer flows out from under a flat surface as in Fig. 2.

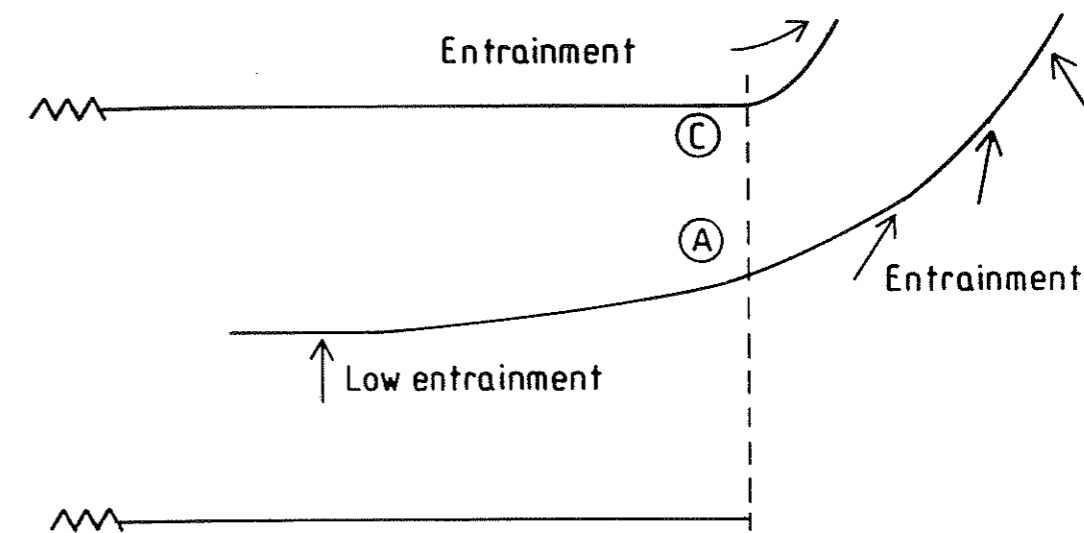


Figure 2

There is no stagnant region upstream, the only pressure drop being associated with friction or a change in cross-section.

This type of flow would have a lower velocity at C than at A. In view of these different processes giving rise to different velocity distributions across the layer it is plausible, in the absence of other information, to assume uniformity as did Morgan and Marshall.

2.3

Alternative Theory

We shall describe two models (based not on energies, but on momentum and buoyancy) one integral and one differential, but we first will need to compare a plume with "top hat" distributions of velocity and temperature rise with a Gaussian Lee and Emmons plume. This will permit an exploration of what is meant by an Equivalent Gaussian Source.

2.3.1 Gaussian and Top Hat Plumes

Lee and Emmons assumed a distribution of a small density difference and hence of a small temperature rise across the plume viz

$$\theta = \theta_c e^{-\frac{x^2}{\lambda^2 b^2}}$$

and of velocity

$$w = w_c e^{-\frac{x^2}{b^2}}$$

where the suffix c denotes centre line, peak values, b is a characteristic width of plume (which depends on height) and λ is a parameter found experimentally to be 0.9.

We seek to examine the conditions for an equivalence to be plausible and compare the equations for Gaussian and top hat plumes, necessarily assuming similarity and following the far field equations of Lee and Emmons [2] and Morton [3]. Here the following derivation follows that of Mitler [4].

Gaussian (after Lee and Emmons)	Top Hat (two sided)
Mass $\frac{dm'}{dz} = 2E_G w_c \rho_0$ (1A)	$\frac{dm'}{dz} = 2E_T w_c \rho_0$ (1B)
Momentum	
$\frac{dM'}{dz} = \frac{g \theta_c b \sqrt{\pi} \lambda \rho_0}{T_0}$ (2A)	$\frac{dM'}{dz} = \frac{2g \theta_c b \rho_0}{T_0}$ (2B)

where E is the entrainment coefficient m' and M' being mass and momentum respectively per unit length.

Convection (the suffix c
being now dropped)

$$\left[\frac{\sqrt{\pi} \lambda}{(1+\lambda^2)^{1/2}} \right] \theta w c_p \rho_0 b = Q' \quad (3A)$$

where Q' is the heat flow
per unit length

$$\text{where } m' = \sqrt{\pi} w \rho_0 b \quad (4A)$$

$$M' = \frac{\sqrt{\pi} w^2 \rho_0 b}{\sqrt{2}} \quad (5A)$$

$$2\theta w c_p \rho_0 b = Q' \quad (3B)$$

$$\text{where } m' = 2wb\rho_0 \quad (4B)$$

$$M' = 2w^2 b \rho_0 \quad (5B)$$

It may be recalled that omitting the fluctuating components of w and θ is permissible so long as they are proportional (similar) to the mean values, the proportionality constant is in effect absorbed into the experimentally determined E .

Here we have not separated the velocity and temperature distributions for the top hat plume, so our comparison properly ought to be based on $\lambda=1$. From the above we now have:

Gaussian $\lambda=1$

$$\frac{dm'}{dz} = 2\sqrt{2}\rho_0 M'/m' E_G \quad (6A)$$

$$\frac{dM'}{dz} = \frac{gQ'}{c_p T_0} \frac{m'}{M'} \quad (7A)$$

Top Hat

$$= 2\rho_0 \frac{M'}{m'} E_T \quad (6B)$$

$$= \frac{gQ'}{c_p T_0} \frac{m'}{M'} \quad (7B)$$

and there is clearly an equivalence in m' and M' for a given Q' and z if [4]

$$E_T = \sqrt{2} E_G \quad (8)$$

3. An Approximate Integral Model

We consider the flow described as in Fig. 2 and Fig. 3 with "top hat" profiles.

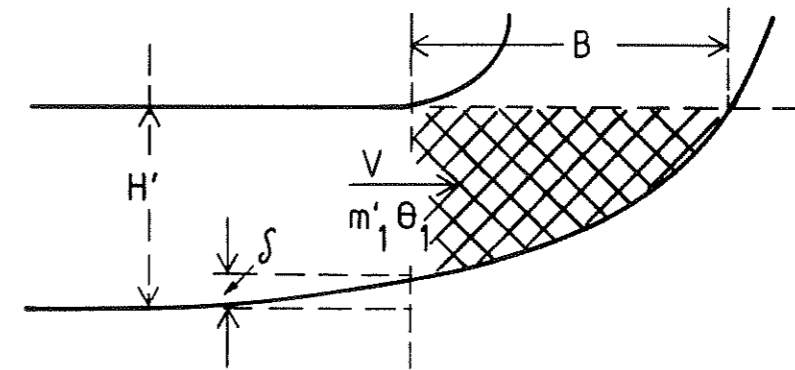


Figure 3

Continuity of convected heat gives

$$m'_1 \theta_1 = m'_2 \theta_2 \quad (9)$$

where m'_1 is the mass of gas flowing under the unit length of edge. It has a temperature rise θ_1 and a velocity V and its specific heat is c_p — assumed constant.

Equating the gain in vertical momentum to the buoyancy gives

$$m'_2 w_2 \doteq \frac{k \rho_0 g (\theta_1 + \theta_2) B H'}{2(T_0 + \theta_1)} \quad (10)$$

where w_2 is the mean vertical velocity, where $\theta_1 - \theta_2$ is provisionally assumed small compared with θ_1 and θ_2 , and where the mean density has been taken, for convenience, as $\rho_0 \frac{T_0}{T_0 + \theta_1}$ and the shaded area A is written as $k B H'$

where $1 > k > 1/2$

If the shaded area were a square	k would be	1.0
If it were a triangle	k would be	0.5
If it were a quarter of an ellipse	k would be	$\frac{\pi}{4} = 0.785$

Many of the results below depend on \sqrt{k} which varies by $\pm 13\%$ with the above figures. We shall for convenience take

$$k = 2/3$$

$$\text{Hence } m_2' w_2 = \frac{g^k}{2} \rho_0 \frac{\theta_1}{T_0 + \theta_1} \left[1 + \frac{m_1'}{m_2'} \right] B H' \quad (11)$$

From definitions we have

$$\theta_1 = \frac{Q'}{c_p m_1'} \quad (12)$$

$$\frac{m_2'}{m_1'} = \frac{B w_2}{H' V} \quad (13)$$

where the difference between the densities of m_1' and m_2' is neglected and $\delta/H \ll 1$.

$$m_1' = \frac{\rho_0 T_0}{T_0 + \theta_1} V H' \quad (14)$$

$$\text{Hence } w_2^2 = \frac{k}{2} \frac{g}{c_p} \frac{Q'}{T_0 m_1'} \left[1 + \frac{m_1'}{m_2'} \right] H' \quad (15)$$

But from conventional theory for the flow out of openings (see also Hinkley [5])

$$\frac{m_1'}{\rho_0 \left[\frac{g Q_1'}{\rho_0 c_p T_0} \right]^{1/3}} = 2 \left[\frac{C_D}{3} \right]^{2/3} \left[\frac{T_0}{T_0 + \theta_1} \right]^{2/3} H' \quad (16)$$

$$\text{i.e. } w_2 = \frac{\sqrt{k}}{2} \left[\frac{g Q_1'}{\rho_0 c_p T_0} \right]^{1/3} \left[\frac{3}{C_D} \frac{T_0 + \theta_1}{T_0} \right]^{1/3} \left[1 + \frac{m_1'}{m_2'} \right]^{1/2} \quad (17)$$

We can write m_2' as m_1' with the addition of the entrained flow and for this we need an upward velocity averaged over the rise $H/2$. For simplicity we take the value appropriate to zero entrainment i.e. $\frac{2}{3}w_2$ (as did Morgan and Marshall) and so obtain

$$\frac{m_2'}{m_1'} = 1 + \frac{2}{3} \frac{E w_2 \rho_0 H'}{m_1'} \quad (18)$$

$$= 1 + \frac{E \sqrt{k}}{2 C_D} \left[1 + \frac{\theta_1}{T_0} \right] \left[1 + \frac{m_1'}{m_2'} \right]^{1/2} \quad (19)$$

We denote

$$\frac{E \sqrt{k}}{2 C_D} \left[1 + \frac{\theta_1}{T_0} \right] \text{ as } K$$

and $\frac{m_2'}{m_1'}$ as y

$$\text{where } y = 1 + K \left[1 + \frac{1}{y} \right]^{1/2} \quad (20)$$

for $C_D \sim 0.7$, $\theta_1 \sim T_0 \sim 300^\circ$ and $E = \sqrt{2} \times 0.16$ i.e. 0.22 and $k \sim \frac{2}{3}$ we have $K \sim 0.26$.

$$\frac{m_2'}{m_1'} \approx 1.34$$

The ratio of the peak to average temperature in a Gaussian Plume is, from Lee and Emmons

$$\frac{\theta_{\max}}{\theta} = \frac{(1 + \lambda^2)^{1/2}}{\lambda} \quad (21)$$

Here we take $\lambda = 1$ and the ratio is 1.41 instead of the 1.50 in their plume. An

equivalent Gaussian Plume , i.e. equivalent in mass momentum and heat flow requires a fictitious entrainment to explain the measured axial temperature being less than the "theoretical" and the estimate of entrainment will then be $1.4 \cdot 1.34 \approx 1.88$ i.e. 88% extra. The use of $\lambda = 0.9$ will give 100% extra.

The result is perhaps fortuitously close to Morgan and Marshall's presumption of an extra 100% entrainment around a corner. We shall, below, describe a differential version of the above model but we first need to note the ratio of w_2 to the far field velocity w_{ff} for a top hat plume.

From equation (4B) \rightarrow (7B)

$$w_{ff} = \frac{1}{(2E)^{1/3}} \left[\frac{gQ'}{\rho_o c_p T_o} \right]^{1/3} \quad (22)$$

so that

$$\frac{w_2}{w_{ff}} = \left[\frac{3}{4} \right]^{1/3} \sqrt{k} \left[\frac{E}{C_D} \frac{T_o + \theta_1}{T_o} \right]^{1/3} \left[1 + \frac{m_1}{m_2} \right]^{1/2} \quad (23)$$

which is ~ 0.83 , less than unity for the values of the constants taken here. As Morgan and Marshall state, the plume is of the retarded type but the appearance of E and C_D , both essentially experimental constants suggests that a numerical analysis is required.

Before discussing the differential model we evaluate the position of the virtual source.

1.1 The Virtual Source

Equations (6), (7) and (22) give

$$\frac{M^2 dM}{m^2 dm} = w_{ff}^3 \quad (24)$$

where for convenience M and m are used for M' and m' respectively.

$$\text{Hence } M^3 = M_2^3 + w_{ff}^3(m^3 - m_2^3)$$

where the suffix 2 refers to the level of the top of the opening

$$\text{and } 2w_{ff} \rho_0 E_T z = \int_{m_2}^m \frac{dm}{\left[1 + \frac{M_2^3 - w_{ff}^3 m_2^3}{w_{ff}^3 m^3}\right]^{1/3}} \quad (25)$$

so that for small departures from neutrality i.e.

$$M_2^3 - w_{ff}^3 m_2^3 \ll w_{ff}^3 m_2^3$$

$$2w_{ff} \rho_0 E_T z = m - m_2 - \frac{M_2^3 - w_{ff}^3 m_2^3}{6 w_{ff}^3 m_2^2} + \frac{M_2^3 - w_{ff}^3 m_2^3}{6 w_{ff}^3 m^2} \quad (26)$$

and when $m \rightarrow \infty$ and $z \rightarrow \infty$ in the far field

$$2w_{ff} \rho_0 E_T z = m - m_2 - \frac{[M_2^3 - w_{ff}^3 m_2^3]}{6 w_{ff}^3 m_2^2} \quad (27)$$

Hence the "correction" Δ required to define the virtual source is (see Fig. 3)

$$\begin{aligned} -z_0 = \Delta &= \frac{m_2}{2E_T \rho_0 A} \left\{ \frac{5}{6} + \frac{1}{6} \frac{M_2^3}{w_{ff}^3 m_2^3} \right\} \\ &= \frac{m_2}{2E_T \rho_0 w_{ff}} \left\{ \frac{5}{6} + \frac{1}{6} \left[\frac{w_2}{w_{ff}} \right]^3 \right\} \end{aligned}$$

so that from equation (23)

$$\Delta = \frac{m_2}{2E_T \rho_0 w_{ff}} \left\{ \frac{5}{6} + \frac{1}{6^{3/2}} \left[\frac{E_T}{C_D} \frac{T_H}{T_0} \right] \left[1 + \frac{m_1}{m_2} \right]^{3/2} \right\} \quad (28)$$

$\frac{M_0}{2E_T \rho_0 w_{ff}}$ is Δ_0 the correction to a neutral plume obtained by extending the truncated plume as in Fig. 4.

Hence $\frac{5}{6} \Delta_0$ is an upper bound for Δ for a retarded plume

$$\begin{aligned} \Delta &\leq \frac{5}{6} \frac{m_2}{2E_T \rho_0 w_{ff}} \\ &\leq 1.35/2E_T (2E_T)^{1/3} \cdot H \cdot Y \end{aligned}$$

where $Y = \frac{m'}{\rho_0 H \left[\frac{gQ'}{\rho_0 c_p T_p} \right]^{1/3}}$ for energy flows [5].

$$\frac{\Delta}{H} \leq \frac{5}{6} \frac{1.35}{(0.44)^{2/3}} \cdot Y \quad (29)$$

For $Y = 0.20$, $\frac{\Delta}{H}$ is between 0.46 and 0.38

Thomas [6] has analysed data obtained by Morgan and Marshall and has calculated values of $\frac{\Delta}{H}$ between 0.20 and 0.60.

Porter [7] has analysed some larger scale experiments satisfactorily with $\frac{\Delta}{H} \approx 0.50$ but to get good estimates of Δ requires more temperature data along the axis of the plume than in either of these sets of experiments: $\frac{\Delta}{H}$ varies only by 30% if entrainment around the corner is, say 100% instead of 50% of the layer flow. The role of entrainment around a corner, however measured or assessed is

only important for plumes that do not rise far. That is, it is less important for expensive smoke clearance installations when the smoke layer is kept high.

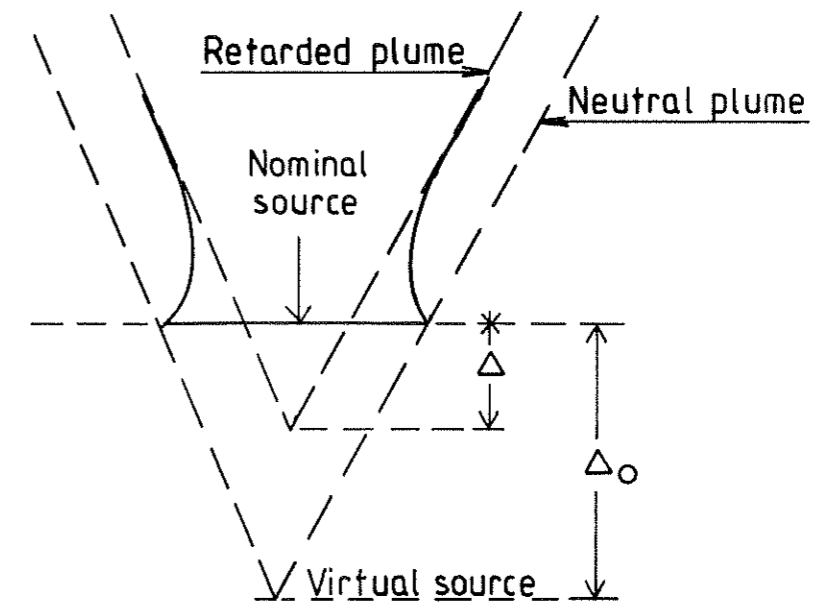


Figure 4 The definitions of Δ_0 and Δ

2 Discussion of Integral Model

Although the discussion has been in terms of zone models, perhaps now no longer necessary since these problems are better pursued by means of "field" modelling, the analysis shows how it may be possible to account for Morgan and Marshall's supposedly large entrainment. One fact is that the entrainment coefficient with "top hat" flow is higher than with Gaussian flow and the other is that matching the two flows can introduce a fictitious entrainment because matching (without thermodynamics) produces calculated (but unreal) rises in temperature (for zero entrainment) and real measured temperature rises will necessarily imply dilution. This means, in short, that fitting a plume with a Gaussian distribution down to the level of the balcony (XX) in Fig. 3 is itself the source of the problem i.e. it leads to higher than realistic entrainment coefficients.

Another source of difficulty lies in the treatment of the secondary effects of density. Consider equations (1B) to (5B), redefining

$$m = 2wb\rho_p$$

$$M = 2w^2b\rho_p$$

where $\rho_p = \rho_0 \frac{T_0}{T_0 + \theta}$, the density in the plume. Equation (1B) is unaltered.

The right hand side of equation (2B) becomes $\frac{2g\theta b\rho_0}{T_0 + \theta}$, whilst the left hand side of equation (3B) is $\frac{2\theta wc_p\rho_0 T_0}{T_0 + \theta}$.

Equations (6B) and (7B) remain unaltered and, at this level of approximation, the solutions to the equations give θ in term of $\frac{\theta}{T_0}$ not $\frac{\theta}{T_0 + \theta}$.

This suggests that in the zone where transition from layer to plume is occurring – a source of thermodynamic difficulty – the theoretical temperatures should be lower than estimated by Morgan and Marshall and a comparison with data would imply a lesser dilution than would be apparant from their theory. Lee and Emmons explicitly assume the absence of density differences except in the buoyancy term: their theory does not distinguish between ρ and ρ_0 , T and T_0 except when $\rho_0 - \rho$ and $T - T_0$ are involved.

4. A Differential Zone Model

For this approach to calculating the trajectory of the plume, equations are obtained by integrating the mass, momentum and heat conservation equations across a section at height z . The width of the plume and depth of the layer are regarded as small compared to the radius curvature. We follow Morgan and Marshall's assumption relating entrainment to vertical velocity, viz. the mass increases as

$$\frac{dm}{dz} = E\rho_0 V \sin \phi \quad (30)$$

where V is the actual velocity and ϕ is the inclination of the plume to the horizontal. Density differences are neglected except in the buoyancy term.

Horizontal momentum is conserved so

$$m' V \cos \phi = m'_1 V_1 \quad (31)$$

where m' is the local mass and m'_1 is the layer mass flow at velocity V_1 per unit length of surge. Vertical momentum and buoyancy give

$$\frac{1}{2} \frac{d}{dz} [V^2 \sin^2 \phi] = \frac{g}{T_0} \theta \quad (32)$$

for an ideal gas.

Heat conservation gives

$$m' c_p \theta = Q' \quad (33)$$

and by definition

$$\frac{dz}{dx} = \tan \phi \quad (34)$$

From equations (30), (32) and (33)

$$m' \frac{dm'}{dz} \cdot \frac{d^2 m'}{dz^2} = \rho_0^2 E^2 \frac{g Q'}{c T_0} = F \quad (35)$$

Equations (30), (31) and (34) give

$$\begin{aligned} m' \frac{dm'}{dz} &= \rho_0 E m' V \frac{dz}{dx} \\ &= B \frac{dz}{dx} \end{aligned} \quad (36)$$

Equations (35) and (36) can be used to obtain the trajectory $z(x)$ and the mass flow $m(z, m_0)$. It follows from equations (35) and (36) that

$$\frac{dz}{dx} \frac{d^2 m'}{dz^2} = \frac{F}{B}$$

so that

$$\frac{dm'}{dz} = \frac{F}{B} x \quad (37)$$

where the constant of integration is zero.

$$\text{Hence } m' = \frac{B^2}{Fx} \frac{dz}{dx} \quad (38)$$

and differentiation gives

$$\frac{dm'}{dz} = \frac{Fx}{B} = \frac{B^2}{Fx} \cdot \frac{d^2z}{dx^2} \cdot \frac{dx}{dz} - \frac{B^2}{Fx^2} \quad (39)$$

from which

$$\frac{Fx}{B^2} \left\{ \frac{Fx}{B} + \frac{B^2}{Fx^2} \right\} = \frac{d^2z}{dx^2} / \frac{dz}{dx} \quad (40)$$

$$\therefore \frac{F}{B^2} \left\{ \frac{F}{B} x^3 + \frac{B^2}{F} \text{Ln } x \right\} = \text{Ln } \frac{dz}{dx} + p \quad (41)$$

where p is an integration constant.

$$\text{Hence } m' = m'_1 \exp \left\{ \frac{F^2 x^3}{3B^2} \right\} \quad (42)$$

$$\text{But } \frac{F^2}{3B^3} = \frac{\rho_0^3 E}{3} \left[\frac{gQ'}{\rho_0 c_p T_0} \right]^2 / m_1^3 V_1^3 = \frac{E}{3} \left[\frac{\rho_0}{m_1 V_1} \right]^3 \left[\frac{gQ'}{\rho_0 c_p T_0} \right]^2 = C \quad (43)$$

From equation (38)

$$z = \frac{Fm'_1}{B^2} \int_0^x x e^{Cx^3} dx = 3^{2/3} \frac{m'_1}{F^{1/3}} \int_0^{\Omega=C^{1/3}x} w e^{w^3} dw \quad (44)$$

Fig. 5 shows a trajectory $y(\Omega)$

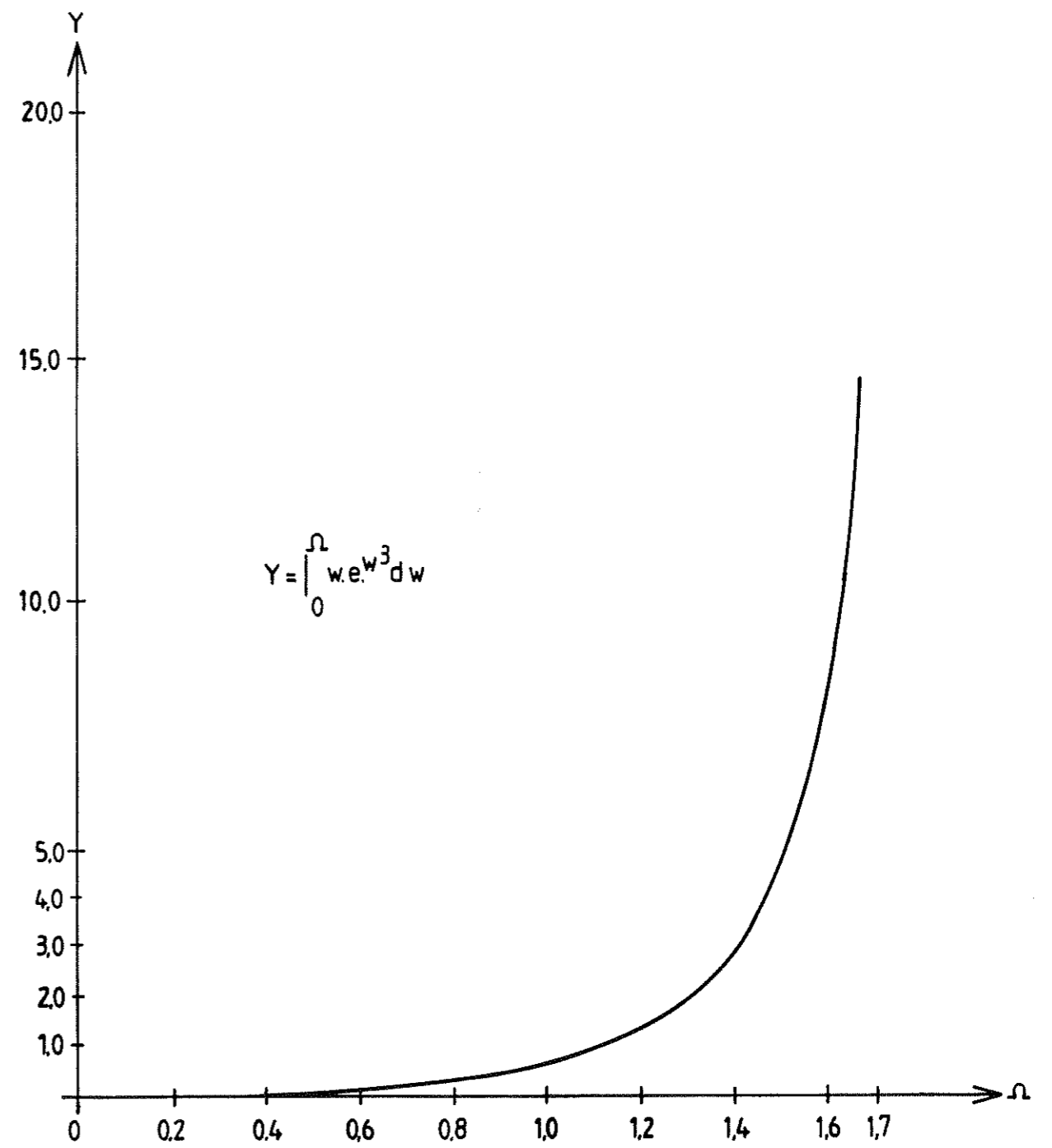


Figure 5 Idealised trajectory

$$y = \int_0^{\Omega} w e^{w^3} dw$$

appropriate for flow in free space — without a facade behind the plume. The definition of the initial conditions is inadequate since departure from the horizontal flow (i.e. $\delta \neq 0$ in Fig. 4) commences upstream of the opening. However, for our purposes and at this level of simplification it would probably not be worth while to allow for this.

We require $\frac{m_2'}{m_1'}$ when $z = H'$ for which we have to solve

$$z = H' = \frac{3^{2/3} m_1'}{\rho_0^{2/3} E^{2/3} \left[\frac{gQ'}{c_p T_0} \right]^{1/3}} \int_0^{\Omega} w e^{w^3} dw$$

But $m_1' = \rho_0 H' V_0$

$$\text{Hence } \frac{E^{2/3}}{3^{2/3}} \left[\left[\frac{gQ'}{\rho_0 c_p T_0} \right]^{1/3} / V_0 \right] = \int_0^{\Omega} w e^{w^3} dw$$

Dimensional analysis suggests the term in brackets is a constant and Hinkley obtained v i.e. $V_0 / \left[\frac{gQ'}{\rho_0 c_p T_0} \right]^{1/3}$ as 0.7.

The left hand side $\frac{E^{2/3}}{v^{2/3}}$ is then approximately 0.25 and $\Omega \div 0.66$ so that

$\frac{m_2'}{m_1'} = 1.34$ the differential and integral approximate solutions being effectively the same for the same numerical values of E and v . This dependence on only E and v , as well as presumably C_D , is made more clear in the differential solution.

For a first linear approximation we have

$$\frac{E^{2/3}}{3^{2/3}} v \div \frac{w^2}{2}$$

$$\text{and } \frac{m_2'}{m_1} = 1 + w^3 = 1 + \frac{2^{3/2}E}{3v^{3/2}}$$

Conclusion

As Mitler has shown, comparing a Gaussian plume and a Top Hat plume shows their equivalence in mass and momentum requires a 40% increase in entrainment coefficient. The definition of an Equivalent Gaussian Source presents no theoretical problems provided no significance is attached to measurements of temperature in the region where the real temperature cannot thermodynamically exceed the temperature of the layer or uniform source to which the Gaussian Source is equivalent. However based on real measurements one might deduce a fictitious dilution. It has been pointed out that modifying conventional weak plume theory to incorporate the effect of the lower plume density in the heated plume is not warranted by the Lee and Emmons theory; although some correction may be required it is not based on simple substitution of T for T_0 .

The above reasons all give grounds for thinking that the 100% extra entrainment deduced by Morgan & Marshall is unreal; a full analysis by means of computational fluid dynamics is required.

References

- [1] Morgan H.P. and Marshall N.R., "Smoke Hazards in Covered Multi-Level Shopping Malls: Some Studies Using a Model 2 Storey Mall". Current paper 48/75 BRE, Borehamwood 1975.
- [2] Lee S.-L. and Emmons H.W., Journal of Fluid Mechanics 11, 1961, p 353.
- [3] Morton B.L., Tenth Symposium (International) on Combustion. The Combustion Institute, p 973, 1965.
- [4] Mitler H. (personal communication).
- [5] Hinkley P.L., "The Flow of Gases Along an Enclosed Shopping Mall. A Tentative Theory". Fire Research Note 807, Borehamwood 1970.
- [6] Thomas P.H., Fire Safety Journal 12, 1987, p 191.
- [7] Porter A., "Larger Scale Tests to Evaluate Mass Flow of Smoke in Line Plumes" (to be published).