# A Non-linear Drum-Boiler Turbine Model

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ABSTRACT

The changing structure of the Swedish electric power production system has increased the interest in dynamics of thermal power plants. Computer simulation using a dynamic model is one way to investigate the dynamics and evaluate control strategies. In this report a non-linear drum boiler-turbine model is presented. The model is derived from basic physical laws. The purpose of the model is to design and simulate control systems for output power, drum pressure, drum level, and steam temperatures. The model is based on previous work by Karl Eklund [1].

TABLE OF CONTENTS

# 1. INTRODUCTION

The interest in the dynamics of thermal power plants for electric energy production has grown considerably in Sweden during the last years. The reason is the changing structure of the Swedish electric power production system. The energy consumption during 1973 was $62 \cdot 10^3$ GWh. The energy production in hydro power stations was $42 \cdot 10^3$ GWh and in thermal power stations was $16 \cdot 10^3$ GWh. The future demand for increased electric energy production will be covered by nuclear power units. New hydro power units will be installed to cover the load peaks.

Today the thermal power units do not participate in the automatic generation control. In the future the fossile power units have to contribute significantly in the automatic generation control. The installed power of the fossile units in Sweden is 9000 MVA and 97% of them are equipped with drum boilers. These facts motivate the study of drum boilers.

Computer simulation using a dynamic model of the unit is one way to investigate the dynamic properties of the unit and to evaluate possible control strategies.

The changing operating conditions thus motivate the study of models valid for both small and large load changes. The scope of the work is to derive, if necessary nonlinear, models for a drum boiler-turbine unit. The models are to be used for design of a coordinated controller for the unit.

There are two different approaches to the problem of modelling an industrial process. One approach is to use basic physical laws and construction data. Another approach is to use data from dynamic tests and process identification. It is also possible to use a combination; using basic physical laws in order to find the structure of the model and data from dynamic tests in order to estimate parameters in the model. The choice of method depends on a priori knowledge of the process, on available data from dynamic tests, on the possibilities to do dynamic tests and of the purpose of the model. Eklund [1] has investigated the same unit in detail and concluded that it is possible to use basic physical laws in order to model a drum boiler-turbine unit. Eklund also concluded that such a model describes the dynamics of the process quite will. He also pointed out the weak points in his model and how to improve the model. The model presented in this report relies very heavily on Eklund's PhD thesis. The model developed in this report is a nonlinear model derived from basic physical laws. The reason for choosing a nonlinear model is a desire to study large load changes. The reason for deriving the model from basic physical laws is the wide range of applicability.

The boiler-turbine unit was conceptually divided into the following sections: feedwater flow, superheated steam flow, heat flows, economizer, drum, primary superheater, first attemperator, secondary superheater, second attemperator, tertiary superheater, high-pressure turbine, reheater, intermediate-pressure turbine, low-pressure turbine, condensor, low-pressure feedwater preheater, deaerator, and high-pressure preheater.

The sections are organized as follows: Every section starts with a general description. Then the properties of the subsystem are summarized. The assumptions are then stated and commented upon. The inputs, states, variables, outputs, and parameters are then enumerated. Then the basic physical laws governing the subsystem are stated and manipulated. Finally the algebraic and differential equations of the computer program are given.

The considered unit is the largest of five units of the Öresundsverket power plant. The plant belongs to Sydsvenska Kraft AB. It is situated in Malmö, Sweden. This unit was also considered by Eklund [1].

The unit P16-G16 was taken into operation in 1964. The drum boiler was built by Steinmüller and the turbine by Stal-Laval AB.

The alternators were manufactured by ASEA. Fig. 1.1 shows a drawing of the entire power plant.

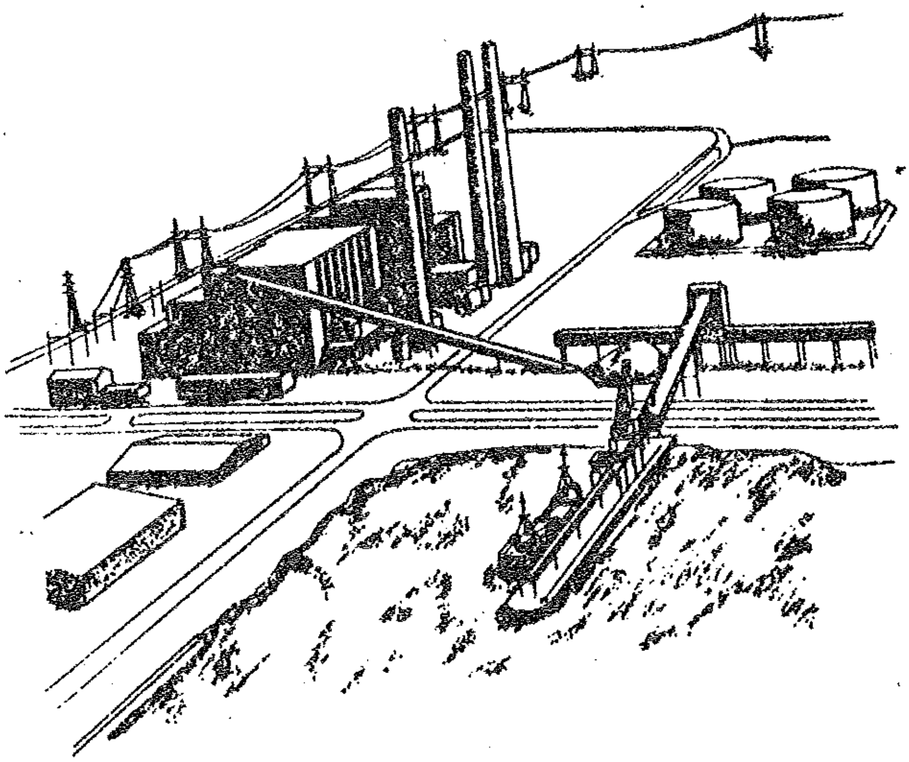

Fig. 1.1 - The Öresundsverket power plant.

The size of the unit is defined by the following data valid at maximum output power:

| | |
|---|---|
| output power | 160 MW |
| steam flow | 140 kg/s |
| drum pressure | 15.0 MPa |
| superheated steam pressure | 13.0 MPa |
| reheated steam pressure | 3.2 MPa |
| superheated steam temperature | 535 °C |
| reheated steam temperature | 535 °C |
| feedwater temperature | 300 °C |

A schematic diagram of the thermal unit P16-G16 is shown in fig. 1.2. In the combustion chamber heat is produced by oil-firing. The heat is transferred to the economizer, the risers, the superheaters and the reheater. The feedwater from the economizer is evaporated in the drum system. The pressure of the steam in the drum can be changed by changing the fuel flow.

The drum pressure determines the temperature of the steam entering the superheaters. The temperature is rised to about 535$^{\circ}$C in the superheaters. The temperature of the superheated steam can be changed by changing the spray flows of the attemperators. The mass flow rate of steam can be changed by changing the position of the control valve. In the high-pressure turbine the steam expands to about 3.2 MPa thereby developing maximally 45 MW. The temperature of the steam drops to about 350$^{\circ}$C and the steam is returned to the reheater.

In the reheater the temperature of the steam is rised to about 535$^{\circ}$C. After expansion in the intermediate- and low-pressure turbine which maximally give 115 MW the steam is condensed to water in the condensor. The condensate is pumped through the low-pressure feedwater preheater (F1, F2 and F3) by the condensate pump. The feedwater is pumped through the high-pressure feedwater preheater (F4, F5, F6 and F7) and the economizer into the drum by the feedwater pump. The mass flow rate of feedwater can be changed by changing the speed of the feedwater pump. The temperature of the feedwater is rised by steam extracted at seven points at the turbine. The temperature of the feedwater is also rised in the economizer by heat from the combustion gases.

There are two equal steam paths comprising three superheaters and two spray attemperators each. Under normal operating conditions the thermal state of steam in the two paths are roughly equal. The two paths are lumped together in the model.

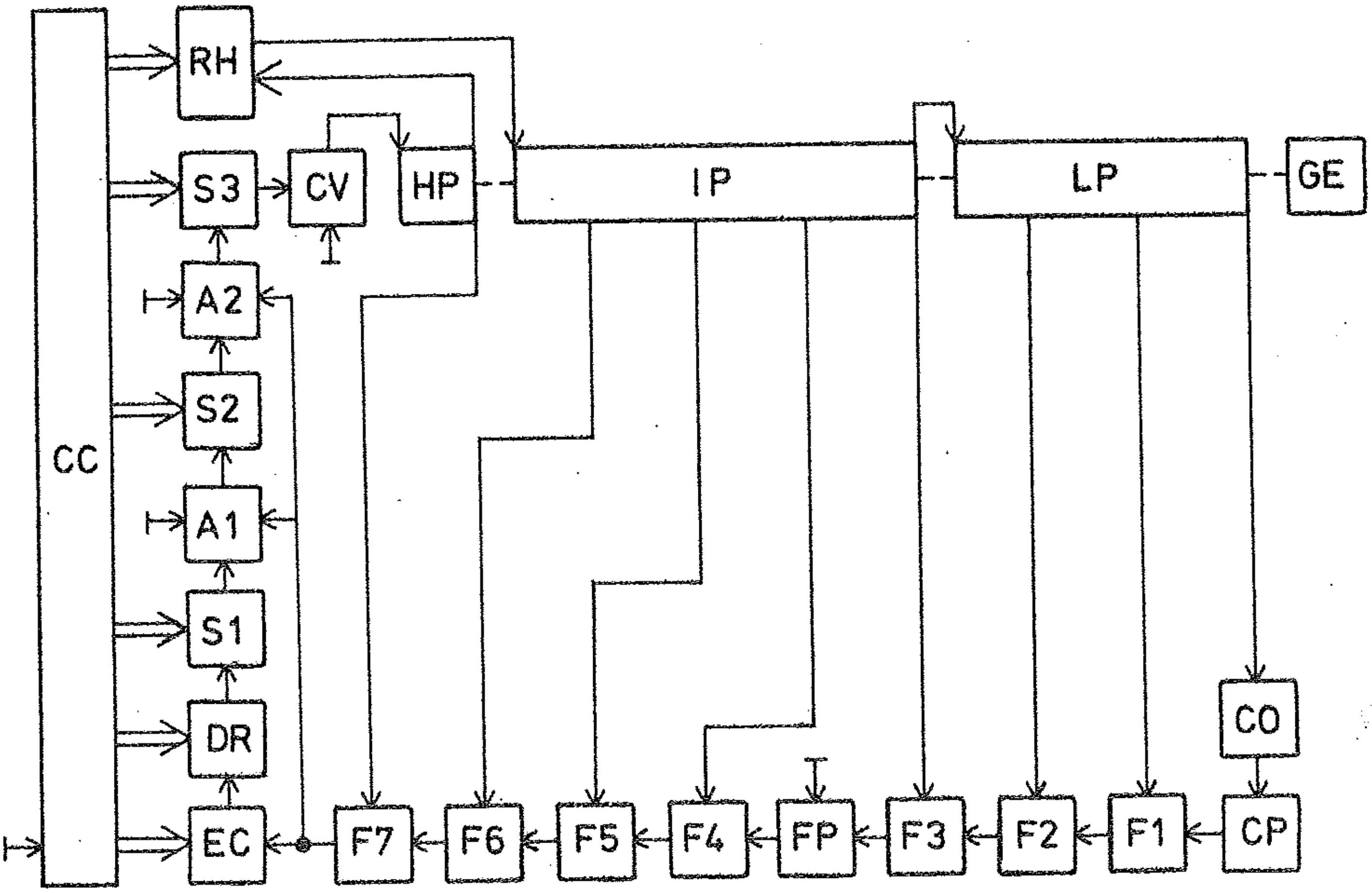


Fig. 1.2 - A schematic diagram of the thermal unit, P16-G16.

CC = combustion chamber
EC = economizer
DR = drum system
S1 = superheater 1
A1 = attemperator 1
S2 = superheater 2
A2 = attemperator 2
S3 = superheater 3
CV = control value
HP = high-pressure turbine
RH = reheater
IP = intermediate-pressure turbine
LP = low-pressure turbine
GE = generator
CO = condensor
CP = condensate pump
F1 = feedwater preheater 1
F2 = feedwater preheater 2
F3 = feedwater preheater 3
FP = feedwater pump
F4 = feedwater preheater 4
F5 = feedwater preheater 5
F6 = feedwater preheater 6
F7 = feedwater preheater 7

## 2. NOMENCLATURE

There are more than 500 variables and parameters in the model of the thermal power plant. There is a need for a simple and systematic method of assigning symbols to these variables and parameters. It is a decided advantage if similar symbols could be used both in the report and in the computer programs.

It was decided to code the model in FORTRAN. This decision prevented the distinction between small and capital letters. It was also decided to run the program under SIMNON [2]. This decision limited the number of characters in the symbols to five. The number of subsystems is 16 and this prevents a unique correspondence between a single number and a subsystem.

The following viewpoints were considered important:

1) The symbols shall contain information about the type of variable or parameter (area, enthalpy, mass, pressure, heat flow, density, temperature or volume).

2) The symbols shall contain information about the subsystem (drum, superheater, turbine, condensor, feedwater preheater or economizer).

3) The symbol shall contain information about the medium (metal, steam or water).

The following scheme for the symbols was finally chosen:

In the report: $a_{aax}$
In the computer program: AAAX

where a (A) represents an alphabetic character and x (X) represents a number.

The first alphabetic character corresponds uniquely to the type of the variable or parameter. The second alphabetic character corresponds uniquely to the subsystem. The third alphabetic character corresponds uniquely to the medium. The number indicates the position in the subsystem. Usually "1" corresponds to the input and "2" corresponds to the output. The drum system and the feedwater preheater are exceptions and the correspondence is given in sections 10, 20 and 22.

The meaning of the first three alphabetic characters are given below:

The first alphabetic character

A (A) = area
a (A) = normalized area
b (B) = heat flow coefficient
c (C) = specific heat
D (D) = diameter
$\eta$ (E) = efficiency
f (F) = friction coefficient
h (H) = enthalpy
L (L) = length
m (M) = mass
N (N) = shaft power
p (P) = pressure
Q (Q) = heat flow
$\rho$ (R) = density
s (S) = normalized position of the servo of a valve
T (T) = temperature
V (V) = volume
w (W) = mass flow rate
z (Z) = level

The second alphabetic character

a (A) = deaerator
b (B) = burners
c (C) = condensor
d (D) = drum system
e (E) = economizer
f (F) = feedwater preheater
h (H) = high-pressure turbine
i (I) = intermediate-pressure turbine
$\ell$ (L) = low-pressure turbine
p (P) = primary superheater
r (R) = reheater
s (S) = secondary superheater
t (T) = tertiary superheater
v (V) = steam control valve
w (W) = feedwater control valve

The third alphabetic character

c (C) = condensate
$\ell$ (L) = liquid
m (M) = metal
o (O) = oil
s (S) = steam
w (W) = water

To describe the dynamics of the subsystems and the interconnections between them the following nomenclature is used:

Inputs (to a subsystem): Process variables determined completely or partially outside the subsystem.

<u>States (of a subsystem)</u>: Process variables, which vary continuously with time. There is a certain degree of freedom in the choice of the states. The following rules for the choice of states can be given:

1) The knowledge of all states in the whole process and of all inputs to the process shall make it possible to compute all inputs to the subsystems. This means that the inputs of the subsystems may depend on states in several subsystems (including the own subsystem) and the inputs to the process. The inputs to the process are assumed to be completely determined outside the process.

2) The knowledge of the inputs to a subsystem and the states of the subsystem must make it possible to compute all variables and outputs of the subsystem as well as the time derivative of all state variables of the subsystem.

<u>Variables (of a subsystem)</u>: Process variables used to compute the time derivatives of the state variables or to compute the outputs of the subsystem and process variables, which are monitored or measured in the protection and control system.

<u>Outputs (of a subsystem)</u>: Process variables used as inputs in some other subsystem(s).

<u>Parameters (of a subsystem)</u>: Coefficients of the equations. The variation of the parameters during the simulations are insignificant. The parameters are not necessarily constants. The heat transfer coefficients may vary considerably during the life-time of the process. In such cases it is assumed that it is sufficient to simulate the process with several <u>constant</u> values of the heat transfer coefficients.

## 3. MODELLING

There are many problems associated with boiler-turbine units which may be investigated by computer simulation using a mathematical model of the dynamics. The complexity and the type of the model is determined by the problem under investigation. It is not unreasonable to say that every problem requires its own model although several problems may be investigated using the same model. The purpose of this model is to make it possible to design a coordinated unit control system by simulation.

The task of such a control system is to increase the output power during disturbances, increase the damping of tie-line oscillations and to adjust the output power to meet the varying demand. The setpoint of the output power shall be adjustable from the control room of the unit and from the dispatch center. In order to form the reference value of the output power the set point value shall be adjusted with respect to locally measured network frequency. In order to prevent unnecessary stresses the loading rate shall be limited to adjustable values, one to be used during normal operation, and one to be used during disturbances. The choice of higher loading rates can be done manually from the control room of the unit, automatically based on locally measured network frequency, or from the dispatch center.

The control of the output power cannot be treated as an isolated problem. The manipulation of the control valve or the fuel flow in order to control the output power will also affect other important process variables. Some of the most important are the pressure of the steam in the drum, the level of the water in the drum, and the temperature of the steam. These variables can be controlled by manipulating the position of the steam control valve, the mass flow rate of fuel, the position of the feedwater control valve, and the position of the spray attemperator valves. The possibility of controlling the output power by varying the mass flow rates of the steam extracted from the turbine will also be investigated.

The purpose of the model can be expressed more exactly by mentioning some problems, which cannot be investigated using the model. The model is not intended for the investigation of:

1) The specific fuel consumption.

2) The thermal stresses of individual tubes of the heat-exchangers.

3) The control of levels in the condensor, the feedwater preheaters and the deaerator.

4) The control of the pressure of the steam in the deaerator.

5) The control of the speed of the feedwater pump.

6) The performance of the servos of the feedwater control valve, the fuel control valves, the steam control valve, and the spray attemperator valves.

7) The determination of transient overspeed of the turbine after load rejection.

The fact that the model, in its present form, does not make it possible to investigate these problems does not mean that it is impossible to extend the model so that it becomes possible.

It was decided to exclude the possibility of accurate calculations of the steady-state values, of investigations of isolated dynamical problems and of very rapid phenomena. A phenomenon is said to be rapid in this context if it is associated with a time constant shorter than one second. This means that some differential equations are replaced by algebraic equations. The sampling intervals of the available experimental results were 10 seconds. The conclusion is that the dynamics associated with time constants smaller than 1 second are not represented in the model, that dynamics associated with angular frequencies over 0.3 rad/s are represented in the model but may be in error due to the lack of experimental results, that dynamics associated with angular frequencies under 0.3 rad/s are represented fairly accurate in the model, and that the model may contain steady-state errors with a magnitude less than about 10%.

Some of the components, especially the heat-exchangers, are described accurately by nonlinear partial differential equations. The standard technique is to replace the partial differential equations by a number of ordinary differential equations. The approximation error decreases if the number of differential equations are increased. The approximation error depends also on the way the spatial derivatives are approximated (forward, central or backward differences). These problems are very pronounced in models describing the dynamics of heat-exchangers and superheaters. The problems have, among many others, been investigated by Enns [4], Isermann [10, 11] and Thal-Larsen [5]. Eklund [1] investigated the necessary number of sections of models describing the dynamics of the superheaters and the reheater of the P16-G16 unit, Öresundsverket. He used backward differences and concluded that there was no significant difference between first order models and fourth order models.

The models of the subsystems of the boiler-turbine unit are treated in sections 5 to 22 of the report. The sections are organized in a uniformly way. The assumptions used in order to develop the model of the dynamics are given under the title "Assumptions". The motivations are given under the title "Comments". The assumptions are sometimes motivated by a desire to reduce the number of state variables and the number of arithmetic operations. Generally the differential equations associated with rapid dynamic phenomena are replaced by algebraic equations. At least in one case a differential equation was retained in order to avoid an iterative solution of an implicit algebraic equation. The differential equation caused no trouble during the simulation due to a fairly long (~10 seconds) time-constant. In some other case it was also desirable to retain the differential equation but it caused too much trouble during the simulation and had to be replaced by an implicit algebraic equation. Some of the assumptions are simplifying approximations, possibly introducing steady-state errors, but introduced in order to avoid time-consuming iterative solution of the algebraic equations. Some of these approximations can be motivated directly by estimating the steady-state error. Some of the approximations can only be motivated indirectly by a comparison of the model responses and the experimental responses. Sometimes the assumptions are motivated by a desire to retain physical variables in the model.

## 4. SIMULATION

The boiler-turbine model is simulated completely digitally. One reason is the availibility of a very flexible interactive simulation program. The program is developed at the Institute of Automatic Control, LTH, by H. Elmqvist [2]. The program is designed to integrate nonlinear differential equations of the form:

$$\frac{dx}{dt} = f(x,u,p,t) \tag{1a}$$

$$y = g(x,u,p,t) \tag{1b}$$

where

x is a vector of states,
u is a vector of inputs,
y is a vector of outputs,
p is a vector of parameters, and
t is time.

The right-hand side of (1) can be defined by statements written in an ALGOL-like high-level language or by subroutines written in FORTRAN. Initially it was felt that some nonlinear algebraic equations required iterative solution. There was also a need for general routines defining the thermodynamic state of steam and water. It was more convenient to write them and to interface them with subroutines, written in FORTRAN. This motivates the decision to define the model by subroutines written in FORTRAN. The subroutines defining the boiler-turbine model are given in Appendices A - D. The main subroutine is named BOILR. The states, inputs, outputs and parameters are defined in the subroutines DCL1 and DCL2. The initial value of the vectors of states and parameters are given in subroutine BPAR. The derivatives and the outputs are computed in the subroutines DRUM, TURB and COND.

The organization of the subroutines can be illustrated by fig. 4.1. The inputs of the process as well as the inputs and outputs of the subsystems are represented by arrows. The states of the subsystems are given within the blocks. Some crucial variables: enthalpy of feedwater leaving the high-pressure feedwater preheater, mass flow rate of feedwater leaving the economizer, mass flow rate of water entering the spray attemperators, mass flow rate of saturated steam leaving the drum, and mass flow rate of extraction steam leaving the turbines have to be computed before the inputs of the subsystems are known. To compute them it was decided to call the subroutines DRUM, TURB and COND twice.

In the early stages of modeling it was also decided to use only SI-units. Later it proved to be necessary to scale down the pressure variables from $\sim 100 \cdot 10^5$ to 100 due to accuracy tests in the integration routine. The rescaling of integration state variables to SI-units has to be done before the pressure variables could be used. This fact also motivated the two calls of the subroutines DRUM, TURB and COND.

The computations were organized as follows:

<u>First call to DRUM</u>

Rescaling of pressure state variable.

<u>First call to TURB.</u>

Rescaling of pressure state variable.

<u>First call to COND</u>

Rescaling of pressure state variables. Computation of feedwater enthalpy.

<u>Second call to DRUM</u>

Computation of mass flow rate of feedwater leaving the high-pressure feedwater preheater.

Computation of pressures along the feedwater circuit.

Computation of mass flow rate of superheated steam entering the high-pressure turbine.

Computation of pressures along the superheated steam circuit.

Computation of mass flow rates of feedwater entering the spray attemperators.

Computation of mass flow rate of superheated steam leaving the superheaters.

Computation of mass flow rate of feedwater leaving the economizer.

Computation of derivatives and outputs of the subsystems: economizer, drum, and superheaters.

Second call to TURB

Computations of derivatives and outputs of the subsystems: high-pressure turbine, reheater, intermediate-pressure turbine, and low pressure turbine.

Second call to COND

Computations of derivatives and outputs of the subsystems: condensor, low-pressure feedwater preheater, deaerator, and high-pressure feedwater preheater.

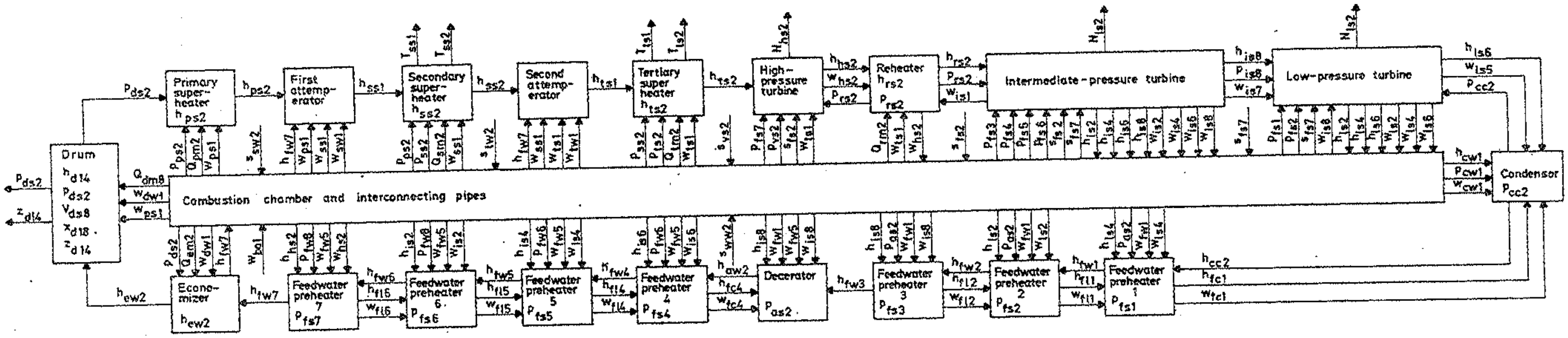


Fig. 4.1 - Block diagram of the boiler-turbine model.

## 5. THE FEEDWATER FLOW

Condensate is delivered to the suction of the feedwater pump from the deaerator. The feedwater flow from the pump can be regulated by the feedwater control valve. The attemperator spray flow is extracted after the feedwater control valve.

### Properties

The feedwater flow depends on the characteristics of the feedwater pump, the friction coefficient of the high-pressure feedwater preheater, the area of the feedwater valve, the friction coefficient of the economizer, and the pressure in the drum. The feedwater flow also depends on the area of the spray attemperator valves and the steam pressure after the primary and the secondary superheater. The feedwater flow is about 130 kg/s at maximum output power. The sum of the maximum spray flows is about 15 kg/s.

### Assumptions

The acceleration term of the Bernoulli equation can be neglected (A5.1).

The mass flow rate of feedwater leaving the high-pressure preheater is equal to the mass flow rate of feedwater entering the economizer (A5.2).

The characteristics of the feedwater pump can be approximated by a constant pressure behind a friction term, which is proportional to the square of the mass flow rate of feedwater (A5.3).

Comments

Assumption (A5.1) is physically well founded because the time constant of the corresponding differential equation is only about 0.2 second and it is not intended to model the dynamics of rapid phenomena. McDonald et. al. [7] have used the same assumption.

Assumption (A5.2) is an approximation motivated by a desire to avoid the solution of a system of nonlinear algebraic equations. The calculated feedwater flow is in error due to the neglected spray attemperator flows. The neglected spray attemperator flows are about 12% of the feedwater flow and the pressure drop of the economizer is about 20% of the total pressure drop of the feedwater line. The error of the pressure drop of the economizer is hence about 25% and the error of the total pressure drop is about 5%. The conclusion is that the error of the total feedwater flow is about 2%. But the position of the feedwater valve is adjusted in order to obtain the desired feedwater flow and the assumption introduces an error of the position of the feedwater control valve and of the pressure drop in the economizer. These errors correspond to a variation of 2% of the total feedwater flow.

Assumption (A5.3) is an approximation motivated by a desire to simplify the calculations and a lack of detailed knowledge of the characteristics of the feedwater pump. Often there is a regulator, which changes the speed of the feedwater pump. This means that the pressure behind the pressure drop in the feedwater valve changes. The regulator is designed to keep the pressure drop in the feedwater valve fairly constant. This means that the control of the feedwater flow is easier than it is assumed here. The conclusion is that the assumption simplifies the calculation of the feedwater flow but makes the control problem more severe.

Inputs

$p_{ds2}$ = pressure of the steam in the drum, (PDS2), [Pa],

$p_{fw5}$ = internal pressure of the feedwater pump, (PFW5), [Pa], and

$s_{ww2}$ = normalized position of the servo of the feedwater valve, (SWW2).

Variables

$a_{ww1}$ = normalized area of the feedwater valve, (AWW1).

Outputs

$p_{fw6}$ = pressure of the feedwater leaving the feedwater pump, (PFW6), [Pa],

$p_{fw8}$ = pressure of the feedwater leaving the high-pressure feedwater preheater, (PFW8), [Pa],

$p_{ww2}$ = pressure of the feedwater leaving the feedwater valve, (PWW2), [Pa],

$w_{fw5}$ = mass flow rate of feedwater entering the feedwater pump (WFW5), [kg/s].

Parameters

$a_{ww1b}$ = base value of the normalized position of the servo of the feedwater valve, (AWW1B),

$a_{ww1m}$ = selector of the normalized area of the area of the feedwater valve, (AWW1M),

$f_{fw5}$ = pressure drop coefficient of the feedwater pump, (FFW5), [Pa/(kg/s)$^2$],

$f_{fw7}$ = pressure drop coefficient of the high-pressure feedwater preheater, (FFW7), [Pa/(kg/s)$^2$],

$f_{ww1}$ = pressure drop coefficient of the feedwater valve, (FWW1), [Pa/(kg/s)$^2$], and

$f_{ew1}$ = pressure drop coefficient of the economizer, (FEW1), [Pa/(kg/s)$^2$].

Basic Physical Equations

According to assumption (A5.3) the discharge pressure of the feedwater pump can be written:

$$p_{fw6} = p_{fw5} - f_{fw5} w_{fw5}^2$$

The pressure drop in the high-pressure feedwater preheater, feedwater valve, and the economizer can be computed from Bernoulli equation using assumption (A5.1).

The pressure drop in the high-pressure feedwater preheater is given by:

$$p_{fw6} - p_{fw8} = f_{fw7} w_{fw5}^2$$

The pressure drop in the feedwater valve is given by:

$$p_{fw8} - p_{ww2} = f_{ww1} (w_{fw5}/a_{ww1})^2$$

The pressure drop in the economizer is given by

$$p_{ww2} - p_{ds2} = f_{ew1} w_{dw1}^2$$

or by the use of assumption (A5.2)

$$p_{ww2} - p_{ds2} = f_{ew1} w_{fw5}^2$$

Algebraic Equations

The mass flow rate of feedwater is computed on line 104-17 in subroutine DRUM, Appendix A39 and on line 324-26 in subroutine REGU, Appendix A56. The area of the feedwater valve is given by:

$$a_{ww1} = \begin{cases} s_{ww2}^2 & \text{if } a_{ww1m} = 0 \\ a_{ww1b}^2 & \text{if } a_{ww1m} = 1 \end{cases}$$

The mass flow rate of feedwater entering the feedwater pump is given by

$$w_{fw5} = \frac{\Delta p}{|\Delta p|} \sqrt{|\Delta p|/f_r}$$

where

$$\Delta p = p_{fw5} - p_{ds2}$$

and

$$f_r = f_{fw5} + f_{fw7} + f_{ww1}/a_{ww1}^2 + f_{ew1}$$

The pressure of the feedwater leaving the feedwater pump is given by:

$$p_{fw6} = p_{fw5} - f_{fw5}w_{fw5}^2$$

The pressure of the feedwater leaving the high-pressure feedwater preheater is given by

$$p_{fw8} = p_{fw6} - f_{fw7}w_{fw5}^2$$

The pressure of the feedwater leaving the feedwater valve is given by:

$$p_{ww2} = p_{fw8} - f_{ww1}(w_{fw5}/a_{ww1})^2$$

## 6. THE STEAM FLOW

Saturated steam leaves the drum and enters the primary superheater, where the temperature of the steam is rised by heat from the furnace. Superheated steam leaves the primary superheater and passes the first attemperator, the secondary superheater, the second attemperator, the tertiary superheater, and the control valve before it enters the high-pressure turbine. Feedwater extracted after the feedwater control valve enters the spray attemperators.

### Properties

The steam flow depends on the pressure in the drum, the pressure drop coefficients of the superheaters, the area of the control valve, the pressure drop coefficient of the control valve, and the characteristic of the high-pressure turbine. The steam flow also depends on the area of the spray attemperator valves and the pressure after the feedwater valve. The steam flow is about 130 kg/s at maximum output power. The maximum value of the first spray flow is about 9 kg/s. The maximum value of the second spray flow is about 6 kg/s.

### Assumptions

The acceleration term of the Bernoulli equation can be neglected (A6.1).

The steam obeys the ideal gas law (A6.2).

The steam temperature is constant (A6.3).

The mass flow rate of steam entering the primary, secondary, and tertiary superheaters are equal when the mass flow rate of steam entering the tertiary superheater is computed (A6.4).

Comments

Assumption (A6.1) is physically well founded because the time constant of the corresponding differential equation is less than 0.2 second and it is not intended to model the dynamics of rapid phenomena. Eklund [1] and McDonald et.al. [7] have used the same approximation.

Assumption (A6.2) is crude if the steam is saturated or slightly superheated but reasonable in other cases. The accuracy of the approximation is discussed by Traupel [8].

Assumption (A6.3) is an approximation motivated by a desire to avoid the solution of nonlinear algebraic equations. The assumption means that the variation of the pressure drop in the superheaters with the temperature is neglected. The error is equal to the relative variation of the temperature (in deg K) of the steam. The error is estimated to be less than 10%.

Assumption (A6.4) is also an approximation motivated by a desire to avoid the solution of nonlinear algebraic equations. The calculated steam flow is in error due to the neglected spray attemperator flows. This means that the error of the primary superheater steam flow is about 12% and the error of the error of the secondary superheater steam flow is about 5%. The pressure drops of the primary and the secondary superheaters are both about 5% of the drum pressure. The error of the pressure drop of the primary and the secondary superheater is about 25% and 10% respectively. The error of the total pressure drop is hence about 1.25 + 0.5 = 1.75%. The conclusion is that the error of the tertiary superheater steam flow is about 1%. But the position of the steam control valve is adjusted in order to obtain the desired steam flow and the assumption introduces an error of the position of the steam control valve and of the pressure drop of the primary and secondary superheaheater. These errors correspond to a variation of 1% of the tertiary superheater steam flow.

Inputs

$p_{ds2}$ = pressure of the steam in the drum, (PDS2), [Pa],

$p_{ww2}$ = pressure of the feedwater leaving the feedwater valve, (PWW2), [Pa],

$s_{sw2}$ = normalized position of the spray flow valve of the first attemperator, (SSW2),

$s_{tw2}$ = normalized position of the spray flow valve of the second attempterator, (STW2), and

$s_{vs2}$ = normalized position of the control valve, (SVS2).

Variables

$a_{sw1}$ = normalized area of the spray flow valve of the first attemperator, (ASW1),

$a_{tw1}$ = normalized area of the spray flow valve of the second attemperator, (ATW1), and

$a_{vs1}$ = normalized area of the control valve, (AVS1).

Outputs

$p_{ps2}$ = pressure of the steam leaving the primary superheater, (PPS2), [Pa],

$p_{ss2}$ = pressure of the steam leaving the secondary superheater, (PSS2), [Pa],

$p_{ts2}$ = pressure of the steam leaving the tertiary superheater, (PTS2), [Pa],

$p_{vs2}$ = pressure of the steam leaving the control valve, (PVS2), [Pa],

$w_{ps1}$ = mass flow rate of steam entering the primary superheater, (WPS1), [kg/s],

$w_{ss1}$ = mass flow rate of steam entering the secondary superheater, (WSS1), [kg/s],

$w_{ts1}$ = mass flow rate of steam entering the tertiary superheater, (WTS1), [kg/s],

$w_{sw1}$ = mass flow rate of feedwater entering the first attemperator, (WSW1), [kg/s], and

$w_{tw1}$ = mass flow rate of feedwater entering the first attemperator, (WTW1), [kg/s].

Parameters

$a_{sw1b}$ = base value of the normalized position of the spray flow valve of the first attemperator, (ASW1B),

$a_{tw1b}$ = base value of the normalized position of the spray flow valve of the second attemperator, (ATW1B),

$s_{vs2b}$ = base value of the normalized position of the control valve, (SVS2B),

$a_{sw1m}$ = selector for the normalized position of the spray flow valve of the first attemperator, (ASW1M),

$a_{tw1m}$ = selector for the normalized position of the spray flow valve of the second attemperator, (ATW1M),

$s_{vs2m}$ = selector for the normalized position of the control valve, (SVS2M),

$f_{ps1}$ = pressure drop coefficient of the primary superheater, (FPS1), $[(Pa)^2/(kg/s)^2]$,

$f_{ss1}$ = pressure drop coefficient of the secondary superheater, (FSS1), $[(Pa)^2/(kg/s)^2]$,

$f_{ts1}$ = pressure drop coefficient of the tertiary superheater, (FTS1), $[(Pa)^2/(kg/s)^2]$,

$f_{vs1}$ = pressure drop coefficient of the control valve, (FVS1), $[(Pa)^2/(kg/s)^2]$,

$f_{hs1}$ = pressure coefficient of the high pressure turbine, (FHS1), $[Pa/(kg/s)]$,

$f_{sw1}$ = pressure drop coefficient of the spray flow valve of the first attemperator, (FSW1), $[Pa/(kg/s)^2]$, and

$f_{tw1}$ = pressure drop coefficient of the spray flow valve of the second attemperator, (FTW1), $[Pa/(kg/s)^2]$.

Basic Physical Equations

Consider the steam flow of a nozzle with area, A [$m^2$]. The upstream pressure, $p_1$ [Pa], and the downstream pressure, $p_2$ [Pa], are related to the mass flow rate, w [kg/s], by the following equation:

$$p_1 - p_2 = \frac{0.5}{\rho}\left(\frac{w}{A}\right)^2$$

where $\rho$ [$kg/m^3$] is the density of the steam. This equation is valid under subsonic conditions.

If the steam obeys the ideal gas law

$$p/\rho = 0.5(p_1+p_2)/\rho = RT$$

we have

$$p_1^2 - p_2^2 = RT\left(\frac{w}{A}\right)^2$$

If the temperature is constant we have:

$$p_1^2 - p_2^2 = \frac{RT}{A_{max}^2}\left(\frac{A_{max}\ w}{A}\right)^2 = f\left(\frac{w}{a}\right)^2$$

where a is the normalized area. The experimental relation between pressure drop and steam flows are given in fig. 6.1. The experimental results are obtained from Larsson and Öhbom [3].

The relation between pressure and steam flow of the high-pressure turbine is given in fig. 6.2. It is clear from this figure that a straight line is a reasonable approximation.

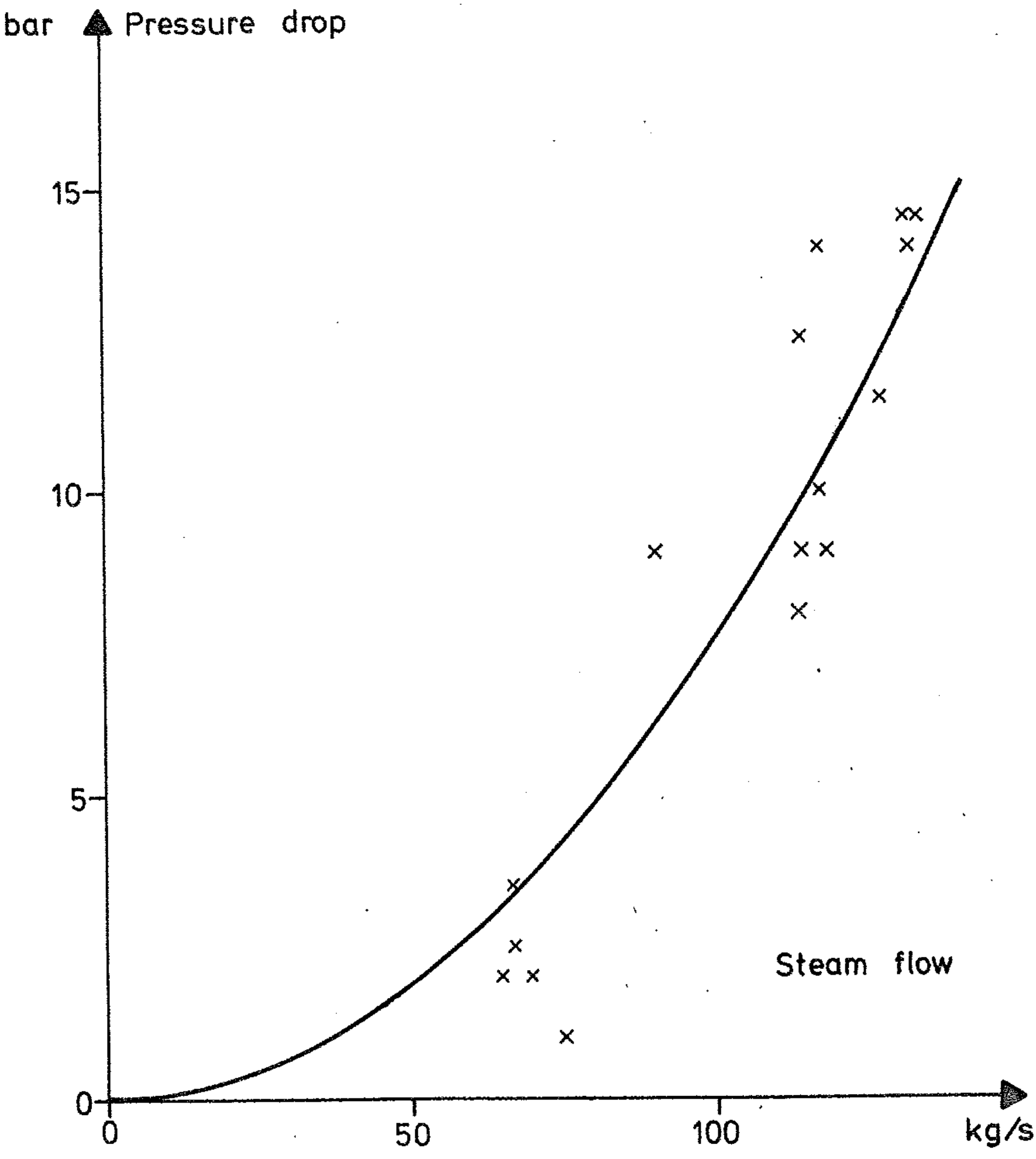


Fig. 6.1 - Relation between pressure drop in the superheaters and the steam flow. The crosses (×) represent experimental results and the full line (—) represents the approximation used in the model.

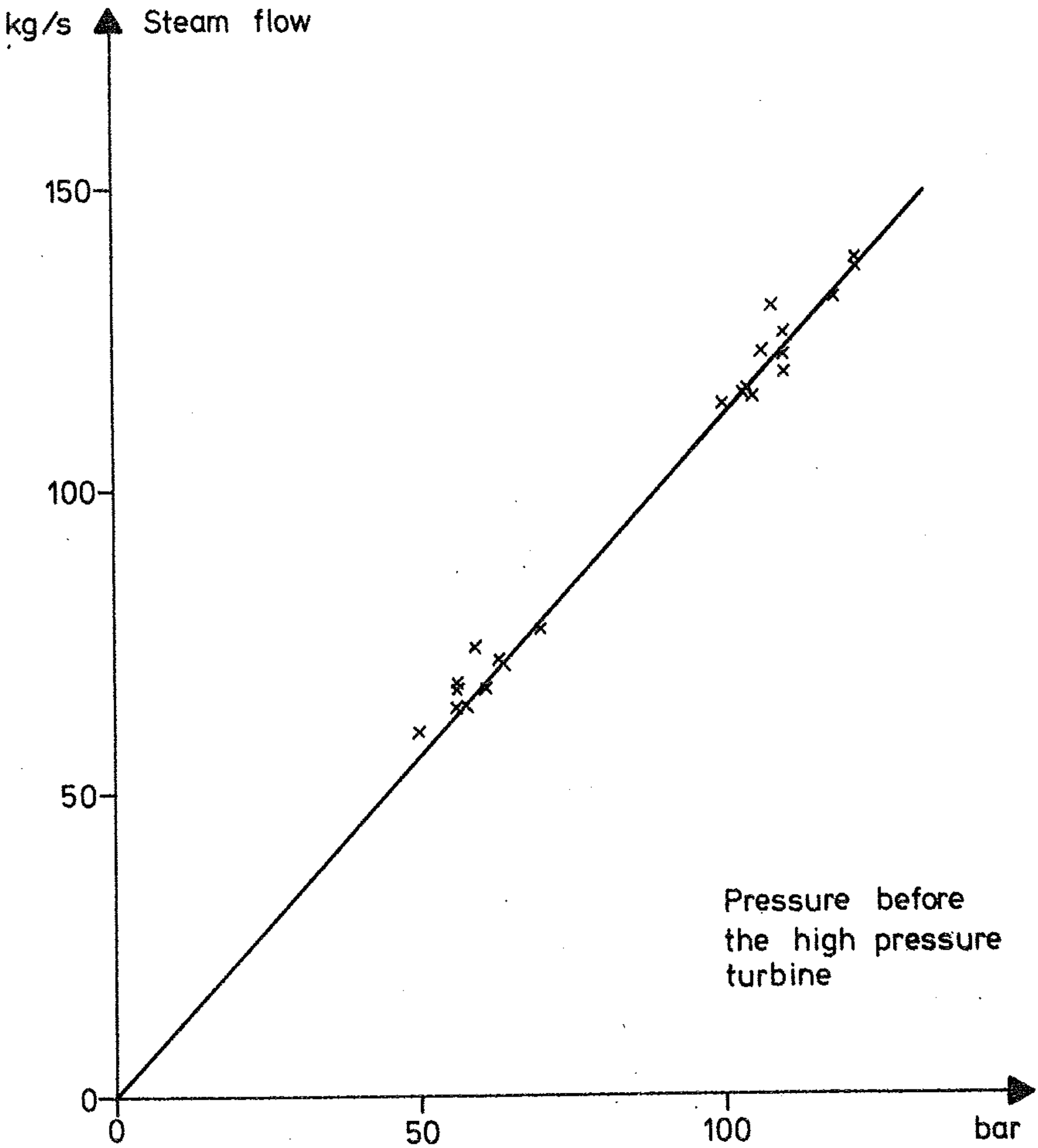


Fig. 6.2 - Relation between pressure before the high pressure turbine and the steam flow. The crosses (×) represent experimental results and the full line (—) represents the approximation used in the model.

The pressure drops of the primary, secondary, and tertiary superheater are given by:

$$p_{ds2}^2 - p_{ps2}^2 = f_{ps1} w_{ps1}^2 \tag{6.1}$$

$$p_{ps2}^2 - p_{ss2}^2 = f_{ss1} w_{ss1}^2 \tag{6.2}$$

$$p_{ss2}^2 - p_{ts2}^2 = f_{ts1} w_{ts1}^2 \tag{6.3}$$

The pressure drop in the control valve is given by:

$$p_{ts2}^2 - p_{vs2}^2 = f_{vs1} (w_{ts1}/a_{vs1})^2 \tag{6.4}$$

The pressure after the control valve is given by

$$p_{vs2} = f_{ns1} w_{ts1} \quad \text{or}$$

$$p_{vs2}^2 = f_{ns1}^2 w_{ts1}^2 \tag{6.5}$$

The pressure drops of the spray flow valve of the first and the second attemperator are given by:

$$p_{ww2} - p_{ps2} = f_{sw1} (w_{sw1}/a_{sw1})^2$$

$$p_{ww2} - p_{ss2} = f_{tw1} (w_{tw1}/a_{tw1})^2$$

The normalized areas of the spray flow valve of the first and the second attemperator are given by:

$$a_{sw1} = s_{sw2}^2$$

$$a_{tw1} = s_{tw2}^2$$

The normalized area of the control valve is a piecewise linear function of the normalized position of the control valve servo. This relation is shown in fig. 6.3. It is obtained from Larsson and Öhbom [3]. The relation between steam flow and normalized control valve position is given in fig. 6.4.

Algebraic Equations

The steam flow is computed on the lines 118-51 in subroutine DRUM, Appendix A39 and A40, and on the lines 327-34 in subroutine REGU, Appendix A56.

The normalized areas of the spray flow valve of the first and of the second attemperator are given by:

$$a_{sw1} = \begin{cases} s_{sw2}^2 & \text{if} \quad a_{sw1m} = 0 \\ s_{sw1b} & \text{if} \quad a_{sw1m} = 1 \end{cases}$$

$$a_{tw1} = \begin{cases} s_{tw2}^2 & \text{if} \quad a_{tw1m} = 0 \\ s_{tw1b} & \text{if} \quad a_{tw1m} = 1 \end{cases}$$

The normalized area of the control valve is given by:

$$a_{vs1} = \begin{cases} f(s_{vs2}) & \text{if} \quad a_{vs1m} = 0 \\ f(s_{vs2b}) & \text{if} \quad a_{vs1m} = 1 \end{cases}$$

where the function f(·) is defined by the subroutine VALVE.

To compute the mass flow rate of steam entering the tertiary superheater assumption (A6.4) is used. Addition of equations

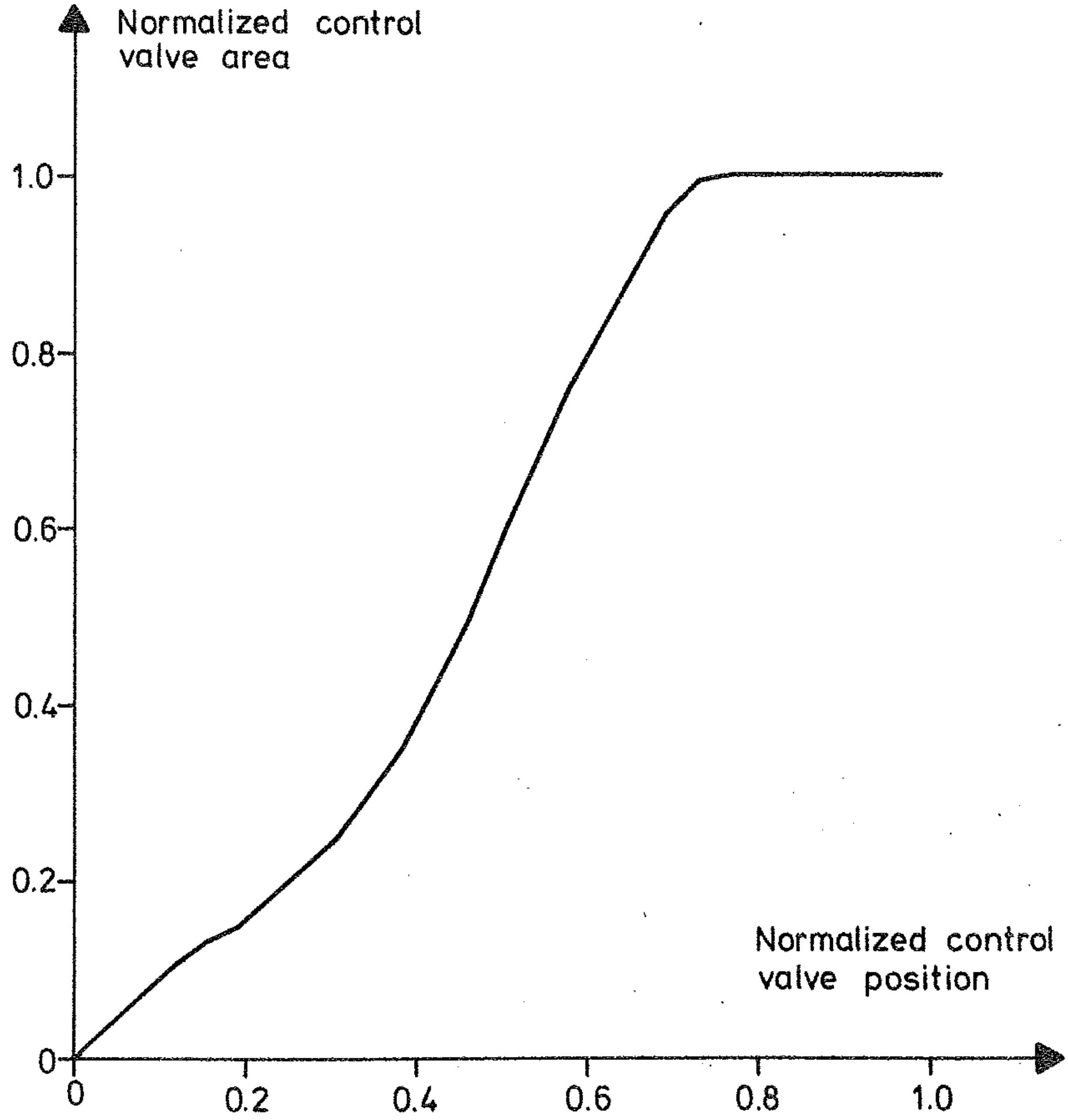


Fig. 6.3 - Relation between the normalized area of the control valve and the normalized position of the control valve.

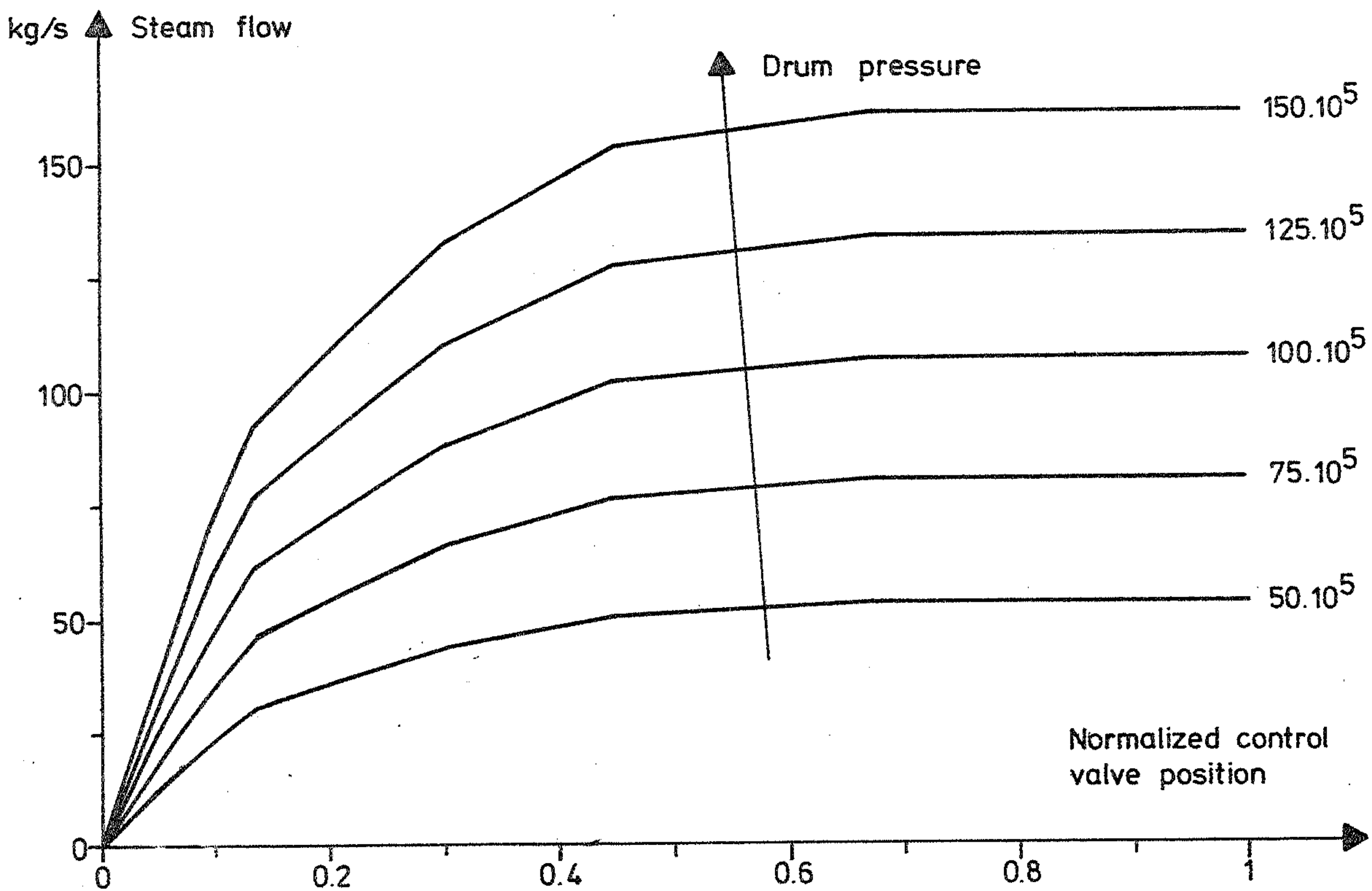


Fig. 6.4 - Relation between steam flow and normalized control valve position with drum pressure as parameter.

(6.1), (6.2), (6.3), (6.4), and (6.5) gives:

$$w_{ts1} = \sqrt{\frac{p_{ds2}}{f_{ps1} + f_{ss1} + f_{ts1} + f_{vs1}/a_{vs1}^2 + f_{hs1}^2}}$$

The pressure of the steam leaving the control valve is given by:

$$p_{vs2} = f_{hs1} w_{ts1}$$

The pressure of the steam leaving the tertiary superheater is given by:

$$p_{ts2} = \quad p_{vs2}^2 + f_{vs1}(w_{ts1}/a_{vs1})^2$$

The pressure of the steam leaving the secondary superheater is now given by:

$$p_{ss2} = \quad p_{ts2}^2 + f_{ts1} w_{ts1}^2$$

The mass flow rate of feedwater entering the second attemperator is given by:

$$w_{tw1} = a_{ts1} \sqrt{\left|p_{ww2} - p_{ss2}\right| / f_{tw1}}$$

and the mass flow rate of steam entering the secondary superheater by:

$$w_{ss1} = w_{ts1} - w_{tw1}$$

The pressure of the steam leaving the primary superheater is given by:

$$p_{ps2} = p_{ss2}^2 + f_{ss1}w_{ss1}^2$$

The mass flow rate of feedwater entering the first attemperator is given by

$$w_{sw1} = a_{sw1}\sqrt{|p_{ww2} - p_{ps2}|/f_{sw1}}$$

The mass flow rate of steam entering the primary superheater is given by:

$$w_{ps1} = w_{ss1} - w_{sw1}$$

# 7. THE HEAT FLOWS

The heat flow to the economizer, the risers, the superheaters, and the reheater are produced by oil-firing in twelve burners. The number of burners in operation depends on the load but can be varied to a certain extent. The oil needed for the combustion first passes the oil-heater. The air needed for combustion first passes the air-preheater. The heat flow to the air-preheater is taken from the combustion gases, which have passed the risers, the superheaters, the reheater, and the economizer.

## Properties

The heat is transferred to the risers and the primary superheater by both radiation and convection. The heat is transferred to the economizer, secondary superheater, tertiary superheater, and the reheater by convection. The heat flows to the metal walls depend on the temperatures of the combustion gases and the temperatures of the metal walls, as well as the velocities of the combustion gases.

The determination of the heat flows under these circumstances is a complicated problem.

## Assumption

The heat flows are functions of the fuel flow only (A7.1).

Comment

Assumption (A7.1) is an approximation motivated by a desire to simplify the model. Eklund [1] assumed that the heat flows were proportional to (linear functions of) the fuel flow. His assumption did not introduce any serious errors. Assumption (A7.1) is a refinement of the assumption of Eklund [1] motivated by a desire to cover a larger operating interval. McDonald et.al. [7] did not neglect the influence of the metal temperature on the heat flows.

Inputs

$w_{bol}$ = mass flow rate of fuel flow (WBO1), [kg/s].

Outputs

$Q_{em2}$ = heat flow to the economizer, (QEM2), [kJ/s],

$Q_{dm8}$ = heat flow to the risers, (QDM8), [kJ/s],

$Q_{pm2}$ = heat flow to the primary superheater, (QPM2), [kJ/s],

$Q_{sm2}$ = heat flow to the secondary superheater, (QSM2), [kJ/s],

$Q_{tm2}$ = heat flow to the tertiary superheater, (QTM2), [kJ/s],

$Q_{rm2}$ = heat flow to the reheater, (QRM2), [kJ/s].

Parameters

$b_{em0}$ = zero order coefficient of the polynomial approximating the heat flow to the economizer, (BEM0), [kJ/s],

$b_{em1}$ = first order coefficient of the polynomial approximating the heat flow to the economizer, (BEM1), [(kJ/s)/(kg/s)],

$b_{em2}$ = second order coefficient of the polynomial approximating the heat flow to the economizer, (BEM2), $[(kJ/s)/(kg/s)^2]$,

$b_{dm0}$ = zero order coefficient of the polynomial approximating the heat flow to the risers, (BDM0), [kJ/s],

$b_{dm1}$ = first order coefficient of the polynomial approximating the heat flow to the risers, (BDM1), [(kJ/s)/(kg/s)],

$b_{dm2}$ = second order coefficient of the polynomial approximating the heat flow to the risers, (BDM2), $[(kJ/s)/(kg/s)^2]$,

$b_{pm0}$ = zero order coefficient of the polynomial approximating the heat flow to the primary superheater, (BPM0), [kJ/s],

$b_{pm1}$ = first order coefficient of the polynomial approximating the heat flow to the primary superheater, (BPM1), [(kJ/s)/ /(kg/s)],

$b_{pm2}$ = second order coefficient of the polynomial approximating the heat flow to the primary superheater, (BPM2), $[(kJ/s)/ /(kg/s)^2]$,

$b_{sm0}$ = zero order coefficient of the polynomial approximating the heat flow to the secondary superheater, (BSM0), [kJ/s],

$b_{sm1}$ = first order coefficient of the polynomial approximating the heat flow to the secondary superheater, (BSM1), [(kJ/s)/(kg/s)],

$b_{sm2}$ = second order coefficient of the polynomial approximating the heat flow to the secondary superheater, (BSM2), $[(kJ/s)/(kg/s)^2]$,

$b_{tm0}$ = zero order coefficient of the polynomial approximating the heat flow to the tertiary superheater, (BTM0), [kJ/s],

$b_{tm1}$ = first order coefficient of the polynomial approximating the heat flow to the tertiary superheater (BTM1), [(kJ/s)/(kg/s)],

$b_{tm2}$ = second order coefficient of the polynomial approximating the heat flow to the tertiary superheater, (BTM2), $[(kJ/s)/(kg/s)^2]$,

$b_{rm0}$ = zero order coefficient of the polynomial approximating the heat flow to the reheater, (BRM0), [kJ/s],

$b_{rm1}$ = first order coefficient of the polynomial approximating the heat flow to the reheater, (BRM1),[(kJ/s)/(kg/s)],

$b_{rm2}$ = second order coefficient of the polynomial approximating the heat flow to the reheater, (BRM2).

Basic Physical Equations

Using assumption (A7.1) we can write:

$$Q_i = f_i(w_{bol})$$

From acceptance tests presented in Larson and Öhbom [3] the heat flows were computed for ten different values of fuel flow. The computed heat flows are given in table 7.1.

| $w_{bol}$ | $Q_{ew2}$ | $Q_{ds8}$ | $Q_{pm2}$ | $Q_{sm2}$ | $Q_{tm2}$ | $Q_{rm2}$ |
|---|---|---|---|---|---|---|
| kg/s | MW | MW | MW | MW | MW | MW |
| 5.25 | 6.9 | 116.5 | 36.5 | 8.5 | 13.4 | 27.8 |
| 6.53 | 12.8 | 140.0 | 46.2 | 14.1 | 16.0 | 35.4 |
| 4.36 | 5.3 | 98.3 | 29.4 | 6.8 | 10.7 | 22.2 |
| 9.28 | 30.9 | 177.7 | 67.5 | 20.7 | 22.0 | 49.1 |
| 9.31 | 30.5 | 177.7 | 68.1 | 22.1 | 21.9 | 48.8 |
| 9.75 | 34.2 | 182.5 | 71.1 | 23.4 | 22.9 | 53.9 |
| 7.83 | 20.8 | 160.4 | 57.2 | 18.2 | 18.6 | 42.7 |
| 1.61 | 2.9 | 26.4 | 7.1 | 1.4 | 3.9 | 6.8 |
| 9.17 | 30.3 | 175.8 | 66.9 | 21.1 | 21.5 | 49.3 |
| 9.67 | 34.0 | 181.4 | 71.1 | 23.2 | 21.2 | 51.9 |

Table 7.1 - Fuel flow and heat flows.

It was decided to approximate the measurements given in table 7.1 by polynomials:

$$Q_i = \sum_{j=0}^{n} b_{ij} w_{bol}^{i}$$

The coefficients were determined by the least-squares method.

This means that the coefficients were chosen in order to minimize the expression

$$J_i(b_{i0}, b_{i1}, \ldots, b_{in}) = \sum_{k=1}^{N} \left( Q_{ik} - \sum_{j=0}^{n} b_{ij} w_{balk}^{j} \right)^2$$

where $Q_{ik}$ and $w_{bolk}$ are heat flow and fuel flow from test k and N is the number of tests. The computer program and the results for n = 1, 2, 3, and 4 are given in Appendix E.

The estimated standard deviation, $\sigma_i$

$$\sigma_i = \sqrt{\min_{b_i} J_i(b_i)/N}$$

and the maximum error, $\varepsilon_i$

$$\varepsilon_i = \max_k \left| Q_{ik} - \sum_{j=0}^{n} b_{ij} w_{bolk}^j \right|$$

for different n:s are given in table 7.2.

| Subsystem | n = 1 | | n = 2 | | n = 3 | | n = 4 | |
|---|---|---|---|---|---|---|---|---|
| | σ | ε | σ | ε | σ | ε | σ | ε |
| | MW | MW | MW | MW | MW | MW | MW | MW |
| Economizer | 3.6 | 6.9 | 0.4 | 0.7 | 0.4 | 0.6 | 0.3 | 0.6 |
| Drum | 7.7 | 15.0 | 0.4 | 0.6 | 0.3 | 0.5 | 0.3 | 0.5 |
| Primary Superheater | 0.4 | 0.7 | 0.3 | 0.5 | 0.3 | 0.4 | 0.3 | 0.4 |
| Secondary Superheater | 0.9 | 1.8 | 0.7 | 1.2 | 0.6 | 1.2 | 0.5 | 1.0 |
| Tertiary Superheater | 0.6 | 1.4 | 0.4 | 1.1 | 0.4 | 1.0 | 0.4 | 1.0 |
| Reheater | 0.8 | 1.3 | 0.7 | 1.5 | 0.7 | 1.5 | 0.6 | 1.0 |

Table 7.2 - Approximation error for different degree n of the approximating polynomial.

Table 7.2 shows how the approximation errors decrease with n. The most significant decrease is obtained when n is changed from one to two. It was thus decided to use a second order polynomial. The heat flows and the approximating polynomials are given in figs. 7.1 to 7.6.

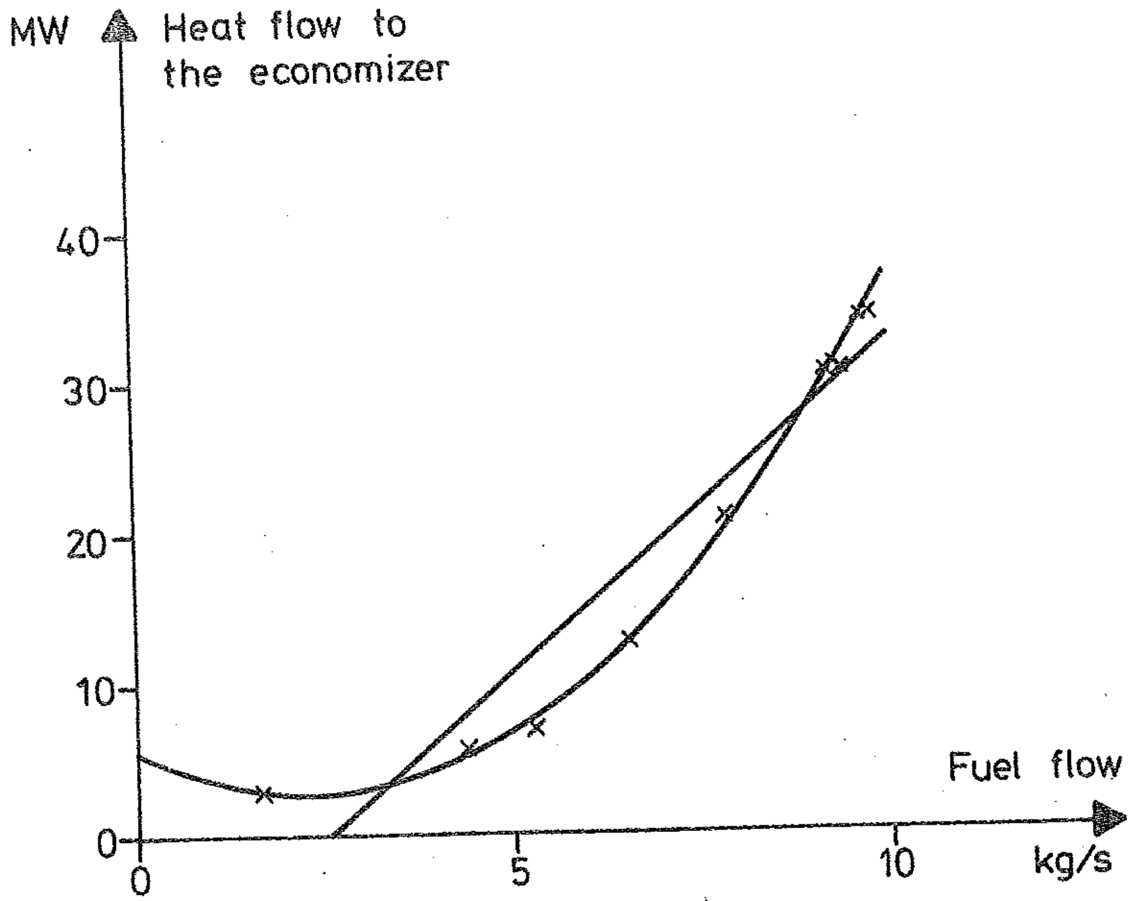


Fig. 7.1 - Heat flow to the economizer. The crosses (×) denote experimental results and the full lines (—) denote the first and second order approximations.

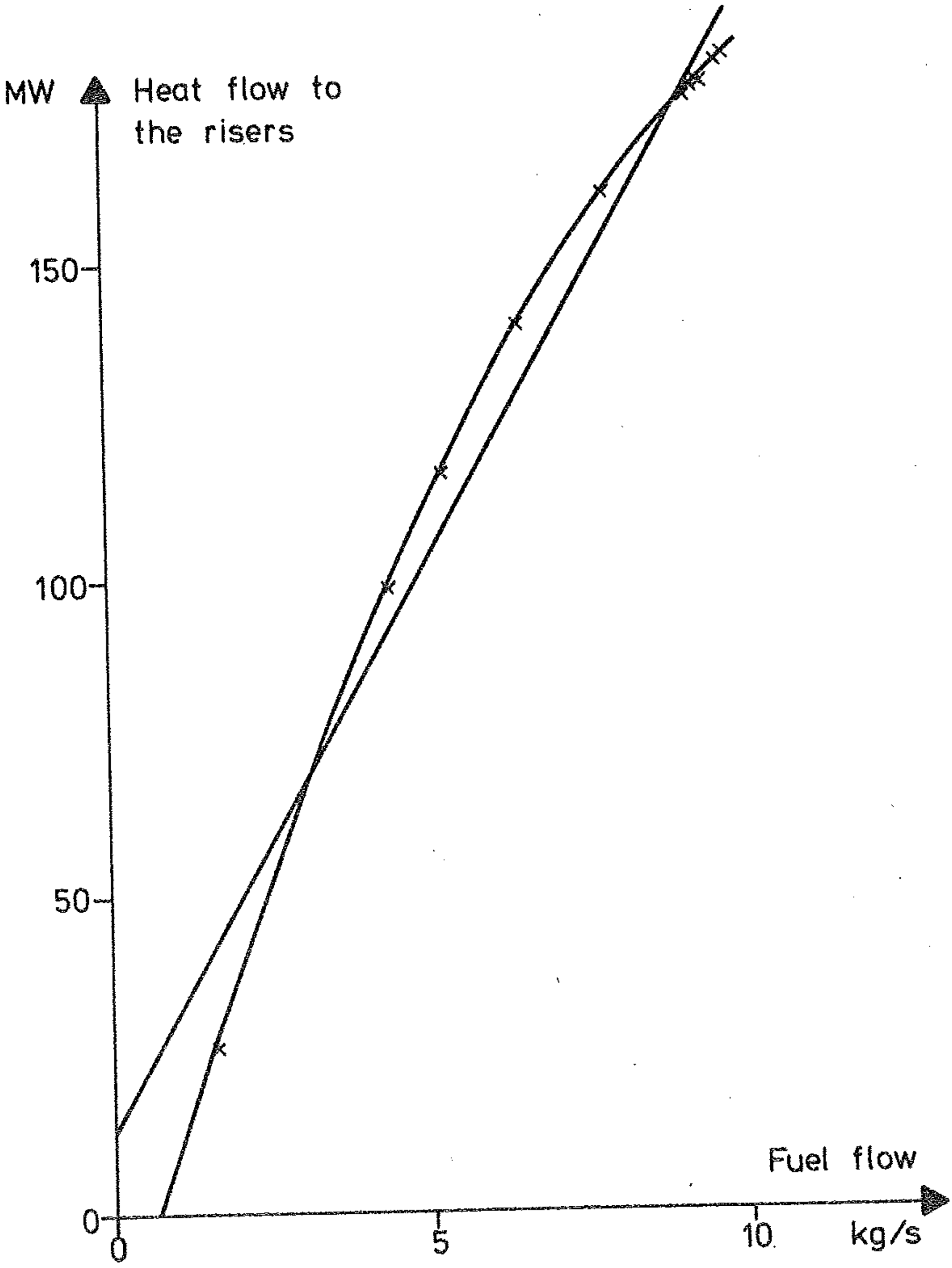


Fig. 7.2 - Heat flow to the risers. The crosses (×) denote experimental results and the full lines (—) denote the first and second order approximations.

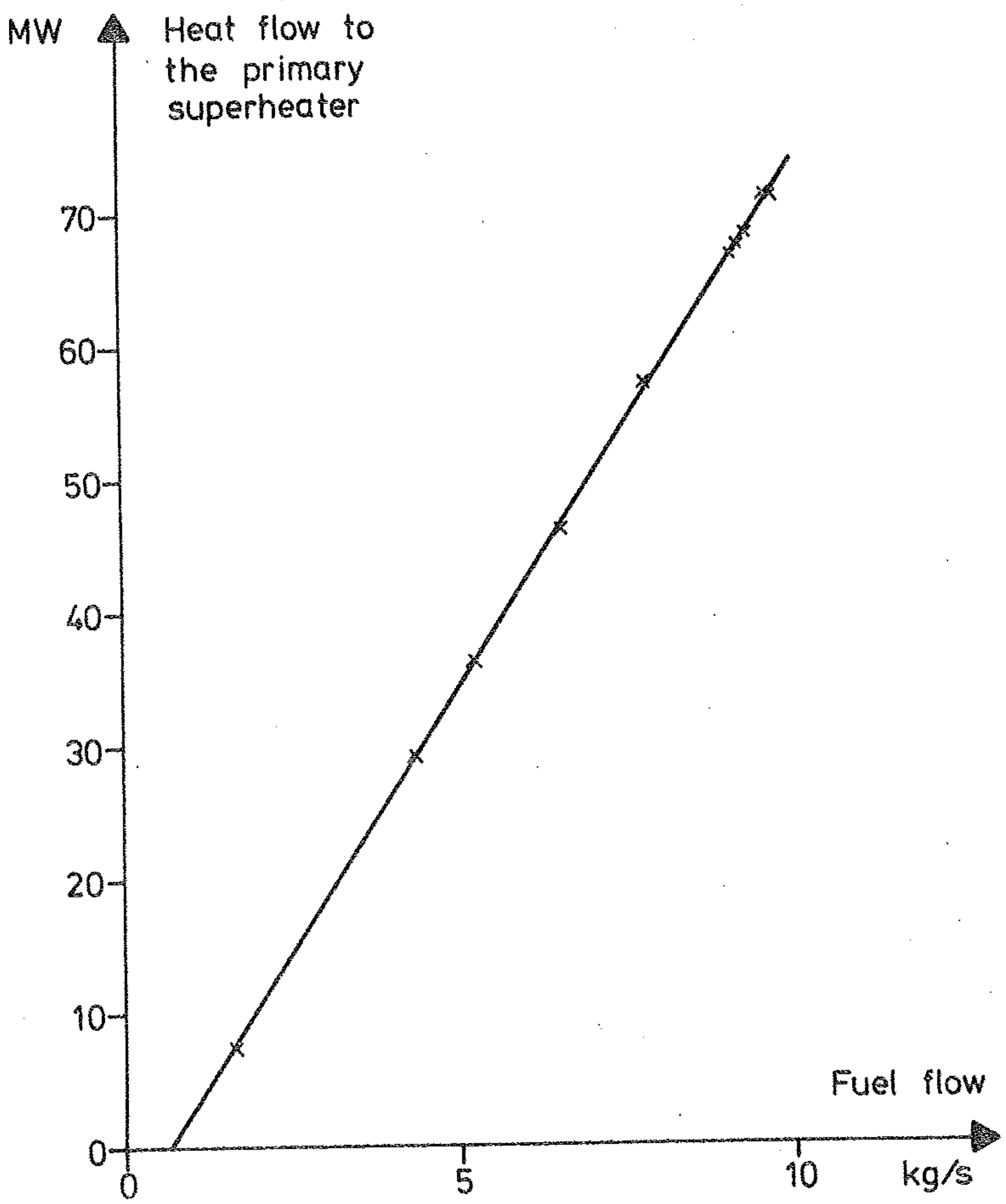


Fig. 7.3 - Heat flow to the primary superheater. The crosses (x) denote experimental results and the full lines (—) denote the first and second order approximations.

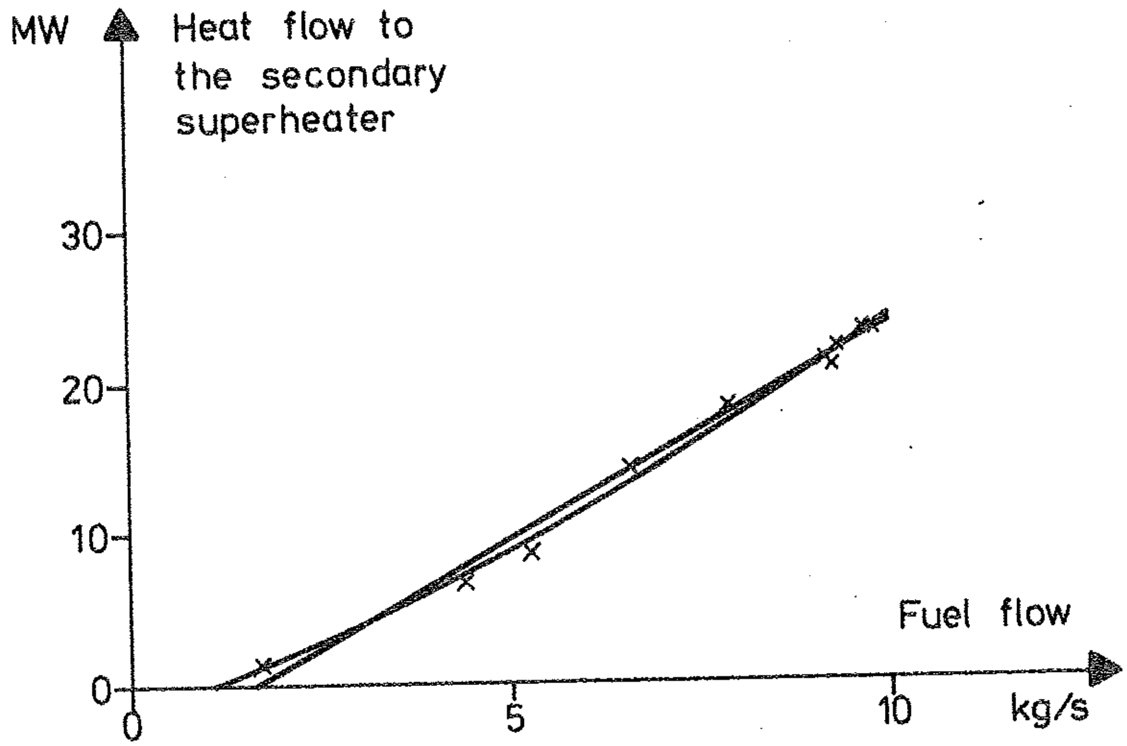


Fig. 7.4 - Heat flow to the secondary superheater. The crosses (×) denote experimental results and the full lines (—) denote the first and second order approximations.

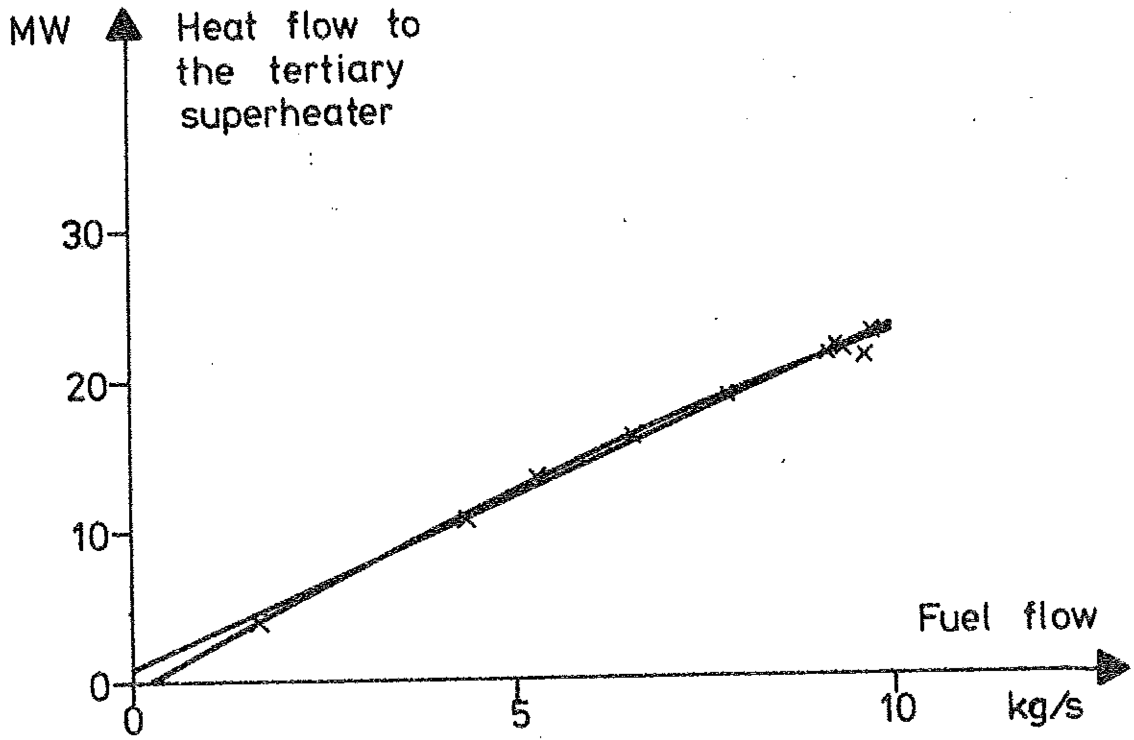


Fig. 7.5 - Heat flow to the tertiary superheater. The crosses (×) denote experimental results and the full lines (—) denote the first and second order approximations.

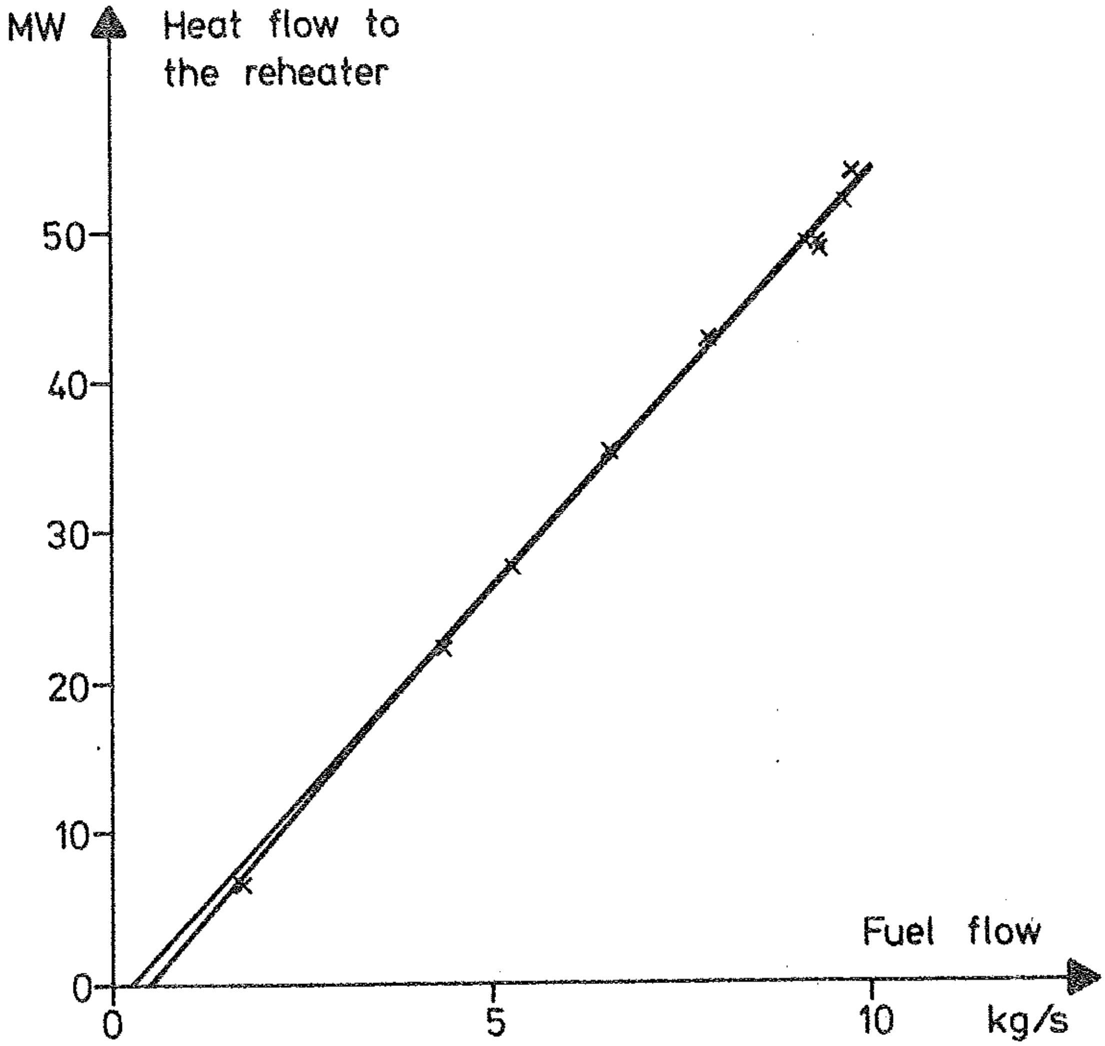


Fig. 7.6 - Heat flow to the reheater. The crosses (×) denote experimental results and the full lines (—) denote the first and second order approximations.

Algebraic Equations

The heat flows are computed on lines 163-64, 176-77, 189-90, 204-05, and 252-53 in subroutine DRUM, Appendix A40-41, and on lines 164-65 in subroutine TURB, Appendix A45, as well as on lines 322-23 in subroutine REGU, Appendix A56. The fuel flow is given by

$$w_{bol} = \begin{cases} s_{b03} & \text{if } w_{bolm} = 0 \\ w_{bolb} & \text{if } w_{bolm} = 1 \end{cases}$$

The heat flows to the economizer, the risers, the primary superheater, the secondary superheater, the tertiary superheater and the reheater are given by:

$$Q_{em2} = b_{em0} + b_{em1} w_{bol} + b_{em2} w_{bol}^2$$

$$Q_{dm8} = b_{dm0} + b_{dm1} w_{bol} + b_{dm2} w_{bol}^2$$

$$Q_{pm2} = b_{pm0} + b_{pm1} w_{bol} + b_{pm2} w_{bol}^2$$

$$Q_{sm2} = b_{sm0} + b_{sm1} w_{bol} + b_{sm2} w_{bol}^2$$

$$Q_{tm2} = b_{tm0} + b_{tm1} w_{bol} + b_{tm2} w_{bol}^2$$

and

$$Q_{rm2} = b_{rm0} + b_{rm1} w_{bol} + b_{rm2} w_{bol}^2$$

# 8. THE ECONOMIZER

Feedwater from the high-pressure preheater enters the economizer. The temperature of the feedwater is rised by heat from the combustion gases.

## Properties

The temperature of the feedwater entering the economizer is about 230°C at maximum output power. The temperature of the feedwater leaving the economizer is about 290°C at maximum output power.

## Assumptions

The economizer can be treated as a single lumped system (A8.1).

## Comments

Assumption (A8.1) was suggested by Eklund [1] and was based on his comparison of a model without any model and experimental results.

## Inputs

$h_{fw7}$ = enthalpy of the feedwater leaving the high-pressure feedwater preheater, (HFW7), [kJ/kg],

$p_{ds2}$ = pressure of the steam in the drum, (PDS2), [Pa],

$Q_{em2}$ = heat flow to the metal of the economizer, (QEM2), [kJ/s],

$w_{dw1}$ = mass flow rate of steam entering the drum, (WDW1), [kg/s].

States

$h_{ew2}$ = enthalpy of the feedwater leaving the economizer, (HEW2), [kJ/kg].

Variables

$\rho_{ew2}$ = density of the feedwater leaving the economizer, (REW2), [$kg/m^3$],

$T_{em2}$ = temperature of the metal of the economizer, (TEM2), [$^oC$],

$T_{ew2}$ = temperature of the feedwater leaving the economizer, (TEW2), [$^oC$], and

$T_{ew2h}$ = derivative of $T_{ew2}$ with respect to enthalpy, (TEW2H), [$^oC/(kJ/kg)$].

Parameters

$c_{em2}$ = specific heat of the metal of the economizer, (CEM2), [$kJ/(kg\ ^oC)$],

$m_{em2}$ = mass of the metal of the economizer, (MEM2), [kg],

$T_{ew3}$ = heat transfer coefficient of the economizer, (TEW3), [$^oC/(kJ/s)$],

$V_{ew2}$ = volume of the feedwater in the economizer, (VEW2), [$m^3$].

## Basic Physical Equations

An energy balance of the metal and the feedwater gives:

$$\frac{d}{dt}\left[c_{em2}m_{em2}T_{em2} + V_{ew2}\rho_{ew2}h_{ew2}\right] = Q_{em2} + w_{dw1}h_{fw7} - w_{dw1}h_{ew2}$$

The temperature difference between metal and steam is given by:

$$T_{em2} - T_{ew2} = T_{ew3}Q_{em2}$$

## Algebraic Equations

The equations of the economizer are written on lines 156-61 and 165-68 in subroutine DRUM, Appendix A40. The temperature of the metal is given by:

$$T_{em2} = T_{ew2} + T_{ew3}Q_{em2}$$

## Thermodynamic Equations

The feedwater leaving the economizer has the density:

$$\rho_{ew2} = RHP(h_{ew2}, p_{ds2})$$

The temperature is given by:

$$T_{ew2} = THP(h_{ew2}, p_{ds2})$$

The derivative of $T_{ew2}$ with respect to enthalpy is given by:

$$T_{ew2h} = THPH(h_{ew2}, p_{ds2})$$

Differential Equation

The derivative of the enthalpy of the feedwater with respect to time is given by:

$$\frac{d}{dt}(h_{ew2}) = \left[Q_{em2} - w_{dw1}(h_{ew2}-h_{ew1})\right]/\tau_{ew2}$$

where

$$\tau_{ew2} = c_{em2}m_{em2}T_{ew2h} + V_{ew2}\rho_{ew2}$$

## 9. THE DRUM SYSTEM

The drum system consists of the drum, the downcomers and the risers. Feedwater from the economizer enters the drum. Saturated steam from the drum enters the primary superheater. Water from the drum enters the downcomers. A mixture of water and steam from the risers enters the drum. The circulation is driven by a force caused by the difference between the density of the water in the downcomers and the density of the steam-water mixture in the risers. This force is balanced by friction losses in the downcomer-riser loop.

### Properties

The drum system includes some very intricate physical phenomena. The risers contain a boiling liquid and thus both a vapor phase and a liquid phase are present. The amount of each phase changes along the risers. The two phases have different densities and move with different velocities. The water is slightly undercooled at the riser inlets.

The position where the boiling of the water starts is not constant. This means that the specific heat flow from the metal to the mixture of steam and water is not constant. This implies that the temperature distribution of the riser tubes is not uniform. The spatial variation of temperature causes a non-constant specific heat flow from the combustion gases to the risers. The heat is transferred by radiation and convection. If the drum pressure decreases fast enough, boiling will occure in the downcomers. The circulation flow decreases and the refrigeration of the risers may become insufficient. The result is process failure.

Assumptions.

The acceleration term of the Bernoulli equation for the downcomer-riser loop can be neglected (A9.1).

In order to compute the circulation flow the difference between the mass flow rate in the downcomers and the mass flow rate in the risers can be neglected (A9.2).

The derivative of the temperature of the metal of the risers with respect to time is equal to the derivative of the saturation temperature with respect to time (A9.3).

The steam quality is linearly distributed in the risers (A9.4).

The enthalpy of the water in the drum is equal to the enthalpy of the water leaving the downcomers (A9.5).

The vapor phase is always in saturation state (A9.6).

The liquid phase is incompressible (A9.7).

The velocity of the steam bubbles and the velocity of water is equal in the risers (A9.8).

The transportation delay of steam bubbles produced in the risers can be modelled by a first order system having a time constant equal to the mean transportation time (A9.9).

Comments

Assumption (A9.1) is physically well-founded. In the early stages of the modelling process the corresponding differential equation was coded and tested. In fig. 9.1 the transient of the circulation flow is shown. The time constant associated with this differential equation is obvious less than 0.2 second.

Eklund [9] has investigated the accuracy of models of the drum-downcomer-riser loop with and without the acceleration term of the Bernoulli equation. The models were intended for the design of steady-state boiler control systems. Eklund concluded that the acceleration term could be neglected.

Assumption (A9.2) is motivated by a desire to simplify the computations. The assumption is completely justified in steady-state and does not introduce any steady-state errors. During transients the assumption may introduce errors in the circulation flow. The errors are introduced in the pressure drop of the risers due to friction, which is one of four pressure drop terms of the downcomer-riser loop. During transients there is a difference between the total steam production and the mass flow rate of steam leaving the drum. These transients are mainly introduced by rapid changes of the output power and consequently of the mass flow rate of steam leaving the drum. The upper limit of conceivable step changes of output power can be estimated to 20%. The mass flow rate of steam leaving the drum is proportional to the output power. The volumes of steam in the drum and the risers are about the same. This means that the error of the mass flow rate of steam leaving the risers can be estimated to be less than 10%. The error increases approximately linearly along the riser tubes. The mean error can be estimated to 5%. This means that the error of the pressure drop term of the risers can be estimated to about 10%. The four pressure drops are about equal, which means that the error of the total pressure drop is about 2.5%.

It is concluded that the error of the mass flow rate of water of the downcomers are less than 2%.

Assumption (A9.3) is proposed by Eklund [1] and is physically well-founded. Without the assumption a fast mode will appear in the model. This fast mode caused some trouble during the initial stage of the modelling. The fast mode is due to the

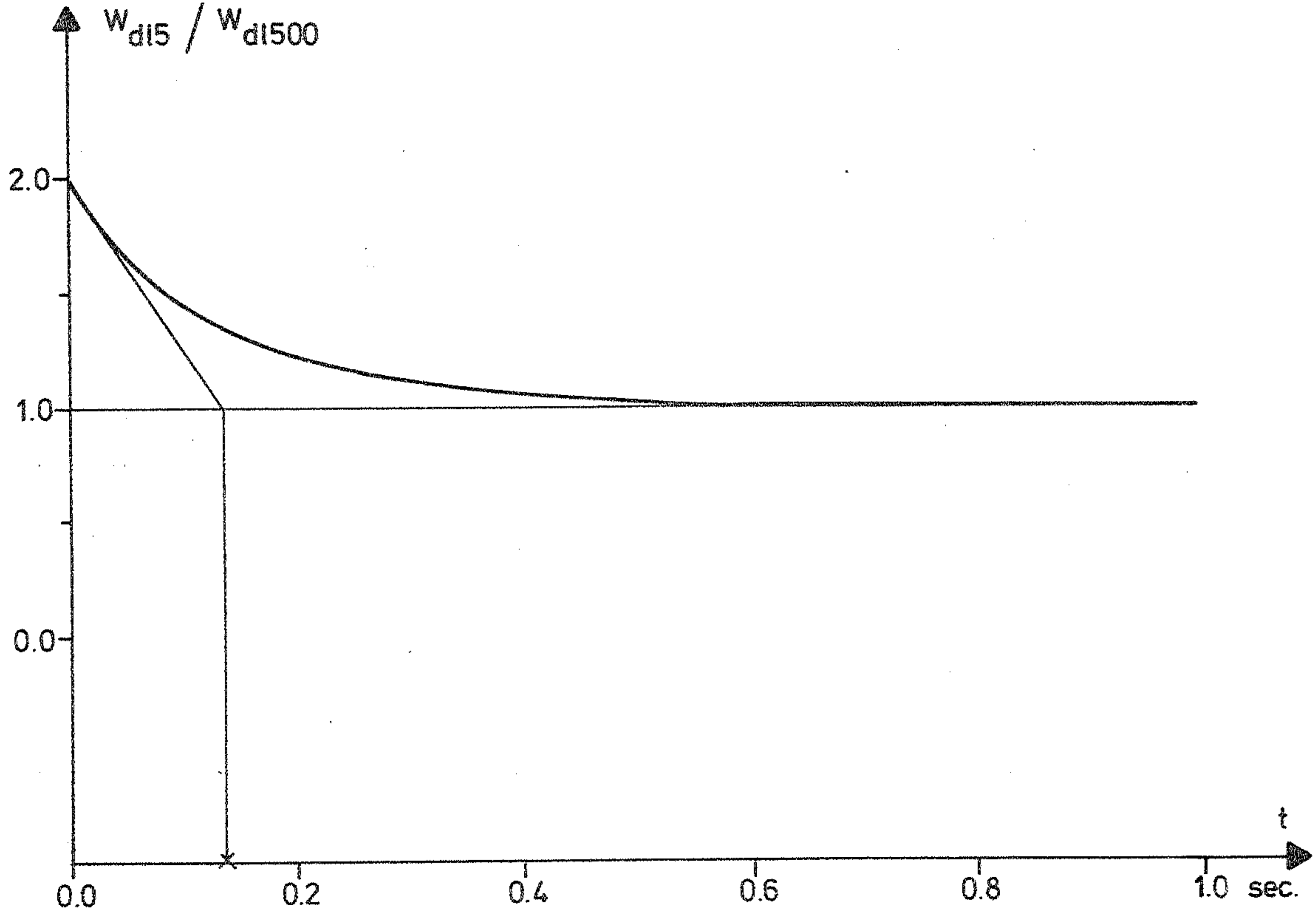


Fig. 9.1 - Transient of the normalized circulation flow.

high heat-transfer coefficient between the metal walls of the risers and the liquid in the risers. This means that the temperature difference between the metal and the liquid assumes the final value very rapidly.

Assumption (A9.4) was used by Eklund [1] and is justified if the specific heat flow is constant. The temperature of the steam-water mixture in the boiling zone of the risers is constant. This fact contributes to a constant specific heat flow. During transients there may be deviations from the linear shape.

Assumption (A9.5) was used by Eklund [1] and reduces the number of states. The errors introduced are negligible.

Assumption (A9.6) was introduced by Eklund [1] and is very close to reality.

Assumption (A9.7) was used by Eklund [1] and is also very close to reality.

Assumption (A9.8) was used by Eklund [1] and is motivated by a desire to simplify the model. There is undoubtedly a velocity difference between the vapor and the liquid phases. This is commonly modelled by introducing a slip factor. The effect of the velocity difference is a reduction of the transportation delay of the steam production.

Assumption (A9.9) is introduced in order to increase the magnitude of the non-minimum phase behaviour of the drum level obtained by Eklund [1]. He modelled the storage phenomenon of the steam-water volume of riser tubes but neglected the transport delay of the steam production. This means that he assumed that the steam quality corresponding to the mean steam quality is available immediately at the outlet of the risers. His responses of the drum level had the correct shape but the magnitude of the non-minimum phase response of the drum level will increase. The model developed here shows a better agreement with the experiments performed by Eklund [1] and assumption (A9.9) is justified indirectly.

Inputs

$h_{ew2}$ = enthalpy of the feedwater leaving the economizer, (HEW2), [kJ/kg],

$Q_{dm8}$ = heat flow to the risers, (QDM8), [kJ/s],

$w_{dw1}$ = mass flow rate of feedwater entering the drum, (WDW1), [kg/s], and

$w_{ps1}$ = mass flow rate of steam entering the primary superheater, (WPS1), [kg/s].

States

$h_{d\ell 4}$ = enthalpy of the water leaving the downcomers, (HDL4), [kJ/kg],

$p_{ds2}$ = pressure of the steam in the drum, (PDS2), [N/m$^2$],

$V_{ds8}$ = volume of steam bubbles in the risers, (VDS8), [m$^3$],

$x_{d\ell 8}$ = steam quality of the steam-water mixture leaving the risers, (XDS8), and

$z_{d\ell 4}$ = level of feedwater in the drum, (ZDL4), [m].

Variables

$h_{ds2}$ = enthalpy of the steam in the drum system, (HDS2), [kJ/kg],

$h_{ds2p}$ = derivative of $h_{ds2}$ with respect to pressure, (HDS2P), [(kJ/kg)/Pa],

$h_{dw8}$ = enthalpy of the water in the risers, (HDW8), [kJ/kg],

$h_{dw8p}$ = derivative of $h_{dw8}$ with respect to pressure, (HDW8P), [(kJ/kg)/Pa],

$\rho_{d\ell 4}$ = density of the water leaving the downcomers, (RDL4), [kg/m$^3$],

$\rho_{d\ell 8}$ = density of the steam-water mixture in the risers, (RDL8), [kg/m$^3$],

$\rho_{ds2}$ = density of the steam in the drum system, (RDS2), [kg/m$^3$],

$\rho_{ds2p}$ = derivative of $\rho_{ds2}$ with respect to pressure, (RDS2P), [(kg/m$^3$)/Pa],

$\rho_{dw8}$ = density of the water in the risers, (RDW8), [kg/m$^3$],

$T_{d\ell 4}$ = temperature of the water leaving the downcomers, (TDL4), [$^o$C],

$T_{d\ell 8}$ = temperature of the steam-water mixture in the risers, (TDL8), [$^o$C],

$T_{d\ell 8p}$ = derivative of $T_{d\ell 8}$ with respect to pressure, (TDL8P), [$^o$C/Pa],

$T_{dm8}$ = temperature of the metal of the risers, (TDM8), [$^o$C],

$V_{d\ell 4}$ = volume of the water in the drum, (VDL4), [m$^3$],

$V_{ds2}$ = volume of the steam in the drum, (VDS2), [m$^3$],

$w_{d\ell 5}$ = mass flow rate of steam entering the downcomers, (WDL5), [kg/s],

$w_{ds7}$ = total steam production in the drum system, (WDS7), [kg/s], and

$w_{dw8}$ = mass flow rate of water leaving the risers, (WDW8), [kg/s].

Parameters

$A_{d\ell 4}$ = area of the free water-surface in the drum, (ADL4), $[m^2]$,

$A_{d\ell 5}$ = area of the downcomers, (ADL5), $[m^2]$,

$A_{d\ell 7}$ = area of the risers, (ADL7), $[m^2]$,

$c_{dm8}$ = specific heat of the metal of the risers, (CDM8), $[kJ/(kg\ {}^oC)]$,

$D_{d\ell 5}$ = diameter of the downcomers, (DDL5), [m],

$D_{d\ell 7}$ = diameter of the risers, (DDL7), [m],

$f_{d\ell 5}$ = friction coefficient of the downcomers, (FDL5),

$f_{d\ell 7}$ = friction coefficient of the risers, (FDL7),

$g$ = acceleration due to gravity, (G), $[m/s^2]$,

$L_{d\ell 5}$ = length of downcomers and risers, (LDL5), [m],

$m_{dm8}$ = mass of the metal of the risers, (MDM8), [kg],

$T_{d\ell 9}$ = heat transfer coefficient of the risers, (TDL9), $[{}^oC/(kJ/kg)^{1/3}]$,

$V_{d\ell 3}$ = nominal volume of the water in the drum, (VDL3), $[m^3]$,

$V_{d\ell 6}$ = volume of the downcomers, (VDL6), $[m^3]$,

$V_{d\ell 8}$ = volume of the risers, (VDL8), [$m^3$], and

$V_{ds1}$ = nominal volume of the steam in the drum, (VDS1), [$m^3$].

Basic Physical Equations

Using assumption (A9.1) the total pressure drop of the downcomer-riser loop can be computed from Bernoulli's equation:

$$\left(f_{d\ell 5}\frac{L_{d\ell 5}}{D_{d\ell 5}} + 1\right)\frac{w^5_{d\ell 5}}{2A^2_{d\ell 5}\rho_{d\ell 4}} - gL_{d\ell 5}\rho_{d\ell 4} +$$

$$+ \left(f_{d\ell 7}\frac{L_{d\ell 5}}{D_{d\ell 7}} + 1\right)\frac{w^2_{d\ell 7}}{2A^2_{d\ell 7}\rho_{d\ell 8}} + gL_{d\ell 5}\rho_{d\ell 8} = 0$$

Assumption (A9.2) makes it possible to compute the circulation flow:

$$w_{d\ell 5} = \frac{\Delta\rho}{|\Delta\rho|}\sqrt{|2gL_{d\ell 5}\Delta\rho/F|}$$

where

$$\Delta\rho = \rho_{d\ell 4} - \rho_{d\ell 8}$$

and

$$F = \frac{f_{d\ell 5}L_{d\ell 5}/D_{d\ell 5} + 1}{A^2_{d\ell 5}\rho_{d\ell 4}} + \frac{f_{d\ell 7}L_{d\ell 5}/D_{d\ell 7} + 1}{A^2_{d\ell 7}\rho_{d\ell 8}}$$

The energy balance of the riser metal, the steam-water mixture in the risers, and the steam in the drum becomes:

$$\frac{d}{dt}\left[c_{dm8}m_{dm8}T_{dm8} + (V_{d\ell 8}-V_{ds8})\rho_{dw8}h_{dw8} + (V_{ds2}+V_{ds8})\rho_{ds2}h_{ds2}\right] =$$

$$= q_{dm8} + w_{d\ell 5}h_{d\ell 4} - w_{ps1}h_{ds2} - w_{dw8}h_{dw8} \qquad (9.1)$$

The mass balance the steam-water mixture in the risers and the steam in the drum becomes:

$$\frac{d}{dt}\left[(V_{d\ell 8}-V_{ds8})\rho_{dw8} + (V_{ds2}+V_{ds8})\rho_{ds2}\right] =$$

$$= w_{d\ell 5} - w_{ps1} - w_{dw8} \qquad (9.2)$$

Multiplication of equation (9.2) by $h_{dw8}$ and subtraction from equation (9.1) gives:

$$c_{dm8}m_{dm8}\frac{dT_{dm8}}{dt} + (V_{d\ell 8}-V_{ds8})\rho_{dw8}\frac{dh_{dw8}}{dt} +$$

$$+ (V_{ds2}+V_{ds8})\rho_{ds2}\frac{dh_{ds2}}{dt} + (h_{ds2}-h_{dw8})(V_{d\ell 8}+V_{ds2})\frac{d\rho_{ds2}}{dt} +$$

$$+ (h_{ds2}-h_{dw8})\rho_{ds2}\frac{d}{dt}(V_{d\ell 8}+V_{ds8}) =$$

$$\doteq q_{dm8} - w_{d\ell 5}(h_{dw8}-h_{dw4}) - w_{ps1}(h_{ds2}-h_{dw8}) \qquad (9.3)$$

If $dp_{ds2}/dt$ is 0.04 $10^5$ Pa/s and $|w_{dw1} - w_{ps1}|$ is 10 kg/s the terms of the left hand side of equation (9.3) are 1.6, 2.8, -0.2, 1.0 and 1.6 MW respectively. This means that the fifth term cannot be neglected. The mass balance of the water in the drum system becomes

$$\frac{d}{dt}\left\{[V_{tot} - (V_{ds2}+V_{ds8})]\rho_{dw8}\right\} = w_{dw1} - w_{ds7} \qquad (9.4)$$

The mass balance of the steam volume in the drum system becomes:

$$\frac{d}{dt}\left[(V_{ds2}+V_{ds8})\rho_{ds2}\right] = w_{ds7} - w_{ps1} \tag{9.5}$$

Addition of equations (9.4) and (9.5) gives:

$$- (\rho_{dw8}-\rho_{ds2})\frac{d}{dt}(V_{ds2}-V_{ds8}) =$$

$$= - (V_{ds2}+V_{ds8})\frac{d}{dt}(\rho_{ds2}) + w_{dw1} - w_{ps1} \tag{9.6}$$

Elimination of $\frac{d}{dt}(V_{ds2}+V_{ds8})$ between equation (9.3) and equation (9.6) and use of assumption (A9.3) gives:

$$\Big[c_{dm8}m_{dm8}T_{d\ell 8p} + (V_{d\ell 8}-V_{ds8})\rho_{dw8}h_{dw8p} +$$

$$+ (V_{ds2}+V_{ds8})\rho_{ds2}h_{ds2p} + (h_{ds2}-h_{dw8})(V_{d\ell 8}+V_{ds2})\rho_{ds2p} +$$

$$+ (h_{ds2}-h_{dw8})\rho_{ds2}\rho_{ds2p}/(\rho_{ds8}-\rho_{ds2})\Big]\frac{d}{dt}(p_{ds2}) =$$

$$= q_{dm8} - w_{d\ell 5}(h_{dw8}-h_{dw4}) - w_{ps1}(h_{ds2}-h_{dw8}) -$$

$$- (h_{ds2}-h_{dw8})\rho_{ds2}(w_{ps1}-w_{dw1})/(\rho_{dw8}-\rho_{ds2}) \tag{9.7}$$

In order to obtain the steam production the term $\frac{d}{dt}(V_{ds2}+V_{ds8})$ is eliminated between equations (9.4) and (9.5):

$$\frac{(V_{ds2}+V_{ds8})}{\rho_{ds2}}\rho_{ds2p}\frac{d}{dt}(p_{ds2}) = \frac{w_{ds7} - w_{ps1}}{\rho_{ds2}} + \frac{w_{dw1} - w_{ds7}}{\rho_{dw8}}$$

and finally:

$$(\rho_{dw8}-\rho_{ds2})w_{ds7} = \rho_{dw8}w_{ps1} - \rho_{ds2}w_{dw1} +$$

$$+ \rho_{dw8}(V_{ds2}+V_{ds8})\rho_{ds2p}\frac{d}{dt}(p_{ds2})$$

The steam production is not available in the drum immediately. There is a transport delay from the position where the steam is produced and to the riser outlet. The storage time-constant

$$\tau_{d\ell 8} = V_{d\ell 8}\rho_{d\ell 8}/w_{d\ell 5}$$

is typical for the delay of a steam bubble produced at the riser inlet. The delay of a steam bubble produced at the riser outlet is zero. The transport delay in the risers is modelled by a first order exponential lag with time-constant $\tau_{d\ell 8}/2$. Using assumption (A9.4) we have:

$$0.5x_{d\ell 8} = V_{ds8}\rho_{ds8}/(V_{d\ell 8}\rho_{d\ell 8})$$

The derivative of the steam quality at the outlet of the risers are, using assumption (A9.9), given by:

$$\frac{d}{dt}(x_{d\ell 8}) = \frac{2V_{d\ell 5}}{V_{d\ell 8}\rho_{d\ell 8}}\left(\frac{2V_{ds8}\rho_{ds8}}{V_{d\ell 8}\rho_{d\ell 8}} - x_{d\ell 8}\right)$$

The mass balance of the steam bubbles in the risers reads:

$$\frac{d}{dt}(V_{ds8}-\rho_{ds2}) = w_{ds7} - x_{d\ell 8}w_{d\ell 8} \qquad (9.8)$$

The mass balance of the water in the risers becomes:

$$\frac{d}{dt}[(V_{d\ell 8}-V_{ds8})\rho_{dw8}] = w_{d\ell 5} - w_{ds7} - (1-x_{d\ell 8})w_{d\ell 8} \qquad (9.9)$$

Elimination of $w_{d\ell 8}$ between equations (9.8) and (9.9) gives:

$$\left[x_{d\ell 8}\rho_{dw8} + (1-x_{d\ell 8})\rho_{ds2}\right]\frac{d}{dt}(V_{ds8}) =$$

$$= w_{ds7} - x_{d\ell 8}w_{d\ell 5} - (1-x_{d\ell 8})V_{ds8}\rho_{ds2p}\frac{d}{dt}(p_{ds2})$$

The mass balance of the water in the risers reads:

$$\frac{d}{dt}\left[(V_{d\ell 8}-V_{ds8})\rho_{dw8}\right] = w_{d\ell 5} - w_{dw8} - w_{ds7}$$

which gives

$$w_{dw8} = w_{d\ell 5} - w_{ds7} + \rho_{dw8}\frac{d}{dt}(V_{ds8})$$

The derivative of the drum level with respect to time is given by

$$\rho_{d\ell 4}V_{d\ell 4z}\frac{d}{dt}(z_{d\ell 4}) = w_{dw1} + w_{dw8} - w_{dw5}.$$

The energy balance of the water in the drum and the downcomers reads:

$$\frac{d}{dt}\left[(V_{d\ell 4}+V_{d\ell 6})\rho_{d\ell 4}h_{d\ell 4}\right] = w_{dw1}h_{dw1} + w_{dw8}h_{dw8} - w_{d\ell 5}h_{d\ell 4}$$

or

$$(V_{d\ell 4}+V_{d\ell 6})\rho_{d\ell 4}\frac{d}{dt}(h_{d\ell 4}) = w_{dw1}h_{dw1} + w_{dw8}h_{dw8} - w_{d\ell 5}h_{d\ell 4}$$

Algebraic Equations

The equations of the drum systems are coded on lines 212 - 49 and on lines 254 - 94 in subroutine DRUM.

The density of the steam-water liquid in the riser is given by:

$$\rho_{d\ell 8} = \frac{\left[V_{ds8}\rho_{ds2} + (V_{d\ell 8}-V_{ds8})\rho_{dw8}\right]}{V_{d\ell 8}}$$

The volume of the water in the drum is given by

$$V_{d\ell 4} = V_{d\ell 3} + A_{d\ell 4}z_{d\ell 4}$$

The volume of the steam in the drum is given by

$$V_{ds2} = V_{ds1} - A_{d\ell 4}z_{d\ell 4}$$

The circulation flow is given by:

$$w_{d\ell 5} = \frac{\Delta\rho}{|\Delta\rho|}\sqrt{|2\Delta\rho g L_{d\ell 5}/F|}$$

where

$$\Delta\rho = \rho_{d\ell 4} - \rho_{d\ell 8}$$

$$F = \frac{f_{d\ell 5}L_{d\ell 5}/D_{d\ell 5} + 1}{A_{d\ell 5}^2\rho_{d\ell 4}} + \frac{f_{d\ell 7}L_{d\ell 5}/D_{d\ell 7} + 1}{A_{d\ell 7}^2\rho_{d\ell 8}}$$

The temperature of the metal of the risers is given by:

$$T_{dm8} = T_{d\ell 8} + T_{d\ell 9}\sqrt[3]{Q_{dm8}}$$

The total steam production is given by:

$$w_{ds7} = \Big[\rho_{dw8}(V_{ds2}+V_{ds8})\rho_{ds2p}\frac{d}{dt}(p_{ds2}) + + \rho_{dw8}w_{ps1} - \rho_{ds2}w_{dw1}\Big]/(\rho_{dw8}-\rho_{ds2})$$

The mass flow rate of water leaving the risers are given by:

$$w_{dw8} = w_{d\ell 5} - w_{ds7} + \rho_{dw8}V_{ds8t}$$

Thermodynamic Equations

The enthalpy of the steam in the drum system is given by:

$$h_{ds2} = HSP(p_{ds2})$$

The derivative of $h_{ds2}$ with respect to pressure is given by

$$h_{ds2p} = HSPP(p_{ds2})$$

The enthalpy of the water in the risers is given by:

$$h_{dw8} = HWP(p_{ds2})$$

The derivative of $h_{dw8}$ with respect to pressure is given by:

$$h_{dw8p} = HWPP(p_{ds2})$$

The density of the water leaving the drum is given by:

$\rho_{d\ell 4} = RHP(h_{d\ell 4}, p_{ds2})$

The density of the steam in the drum system is given by:

$\rho_{ds2} = RSP(p_{ds2})$

The derivative of $\rho_{ds2}$ with respect to pressure is given by:

$\rho_{ds2p} = RSPP(p_{ds2})$

The density of the water in the risers is given by:

$\rho_{dw8} = RWP(p_{ds2})$

The temperature of the steam-water mixture in the risers is given by:

$T_{d\ell 8} = TLP(p_{ds2})$

The derivative of $T_{d\ell 8}$ with respect to pressure is given by:

$T_{d\ell 8p} = TLPP(p_{ds2})$

The temperature of the water leaving the downcomers is given by:

$T_{d\ell 4} = THP(h_{d\ell 4}, p_{ds2})$

Differential Equations

The derivative of the pressure of steam with respect to time is given by:

$$\frac{d}{dt}(p_{ds2}) = \big(Q_{dm8} - w_{d\ell 5}(h_{dw8}-h_{d\ell 4}) - w_{ps1}(h_{ds2}-h_{dw8}) -$$

$$- \rho_{ds2}(h_{ds2}-h_{dw8})(w_{ps1}-w_{dw1})/(\rho_{dw8}-\rho_{ds2})\big)/\tau_{ds2}$$

where

$$\tau_{ds2} = m_{dm8}c_{dm8}T_{d\ell 8p} + (V_{d\ell 8}-V_{ds8})\rho_{dw8}h_{dw8p} +$$

$$+ (V_{ds2}+V_{ds8})\rho_{ds2}h_{ds2p} +$$

$$+ (h_{ds2}-h_{dw8})(V_{ds2}+V_{ds8})\rho_{ds2p} +$$

$$+ (h_{ds2}-h_{dw8})\rho_{ds2}/(\rho_{dw8}-\rho_{ds2})\rho_{ds2p}$$

The derivative of the steam quality of the steam-water mixture leaving the risers is given by:

$$\frac{d}{dt}(x_{d\ell 8}) = \left\{2\,\frac{V_{ds8}\rho_{ds8}}{V_{d\ell 8}\rho_{d\ell 8}} - x_{d\ell 8}\right\}\frac{2w_{d\ell 5}}{V_{d\ell 8}\rho_{d\ell 8}}$$

The derivative of the steam volume in the risers with respect to time is given by:

$$\frac{d}{dt}(V_{ds8}) = \frac{\left[w_{ds7} - x_{d\ell 8}w_{d\ell 5} - (1-x_{d\ell 8})V_{ds8}\rho_{ds2p}\frac{d}{dt}(p_{ds2})\right]}{\left[x_{d\ell 8}\rho_{dw8} + (1-x_{d\ell 8})\rho_{ds2}\right]}$$

The derivative of the enthalpy of the water leaving the downcomers is given by:

$$\frac{d}{dt}(h_{d\ell 4}) = \frac{\left[w_{dw8}(h_{dw8}-h_{d\ell 4}) - w_{dw1}(h_{d\ell 4}-h_{ew2})\right]}{(V_{d\ell 4}+V_{d\ell 6})\rho_{d\ell 4}}$$

The derivative of the level of the water in the drum is given by:

$$\frac{d}{dt}(z_{d\ell 4}) = (w_{dw1}+w_{dw8}-w_{d\ell 5})/(A_{d\ell 4}\rho_{d\ell 4})$$

# 10. THE PRIMARY SUPERHEATER

Saturated steam from the drum enters the primary superheater. The heat flow from the furnace rises the temperature of the steam. Superheated steam leaves the primary superheater.

## Properties

The temperature of the steam entering the primary superheater is about 300-340$^{o}$C. The temperature of the steam leaving the primary superheater is about 450-480$^{o}$C. The mass flow rate of steam at maximum output power is about 120 kg/s and the corresponding heat flow from the furnace is about 70 MW.

## Assumptions

The primary superheater can be treated as a single lumped system (A10.1).

The steam flow is incompressible (A10.2).

The difference between the temperatures of the metal and the steam can be computed from static relations (A10.3).

The derivative of the temperature of the metal with respect to time can be computed with $h_{ps1}$ and $w_{ps1}$ constant (A10.4).

## Comments

Assumption (A10.1) was investigated in detail by Eklund [1] and he concluded that it is very reasonable.

Assumption (A.10.2) was used by Eklund [1] and is physically well-founded.

Assumption (A10.3) was used by Eklund [1] and is a reasonable assumption.

Assumption (A.10.4) is an approximation motivated by a desire to simplify the calculations.

Inputs

$p_{ds2}$ = pressure of the steam in the drum, (PDS2), [Pa],

$p_{ps2}$ = pressure of the steam leaving the primary superheater, (PPS2), [Pa],

$Q_{pm2}$ = heat flow to the metal of the primary superheater, (QPM2), [kJ/s], and

$w_{ps1}$ = mass flow rate of steam entering the primary superheater, (WPS1), [kg/s].

States

$h_{ps2}$ = enthalpy of the steam leaving the primary superheater, (HPS2), [kJ/kg].

Variables

$h_{ps1}$ = enthalpy of the steam entering the primary superheater, (HPS1), [kJ/kg],

$\rho_{ps2}$ = density of the steam leaving the primary superheater, (RPS2), [kg/m$^3$],

$T_{pm2}$ = temperature of the metal of the primary superheater, (TPM2), [$^{o}$C],

$T_{ps2}$ = temperature of the steam leaving the primary superheater, (TPS2), [$^{o}$C], and

$T_{ps2h}$ = derivative of $T_{ps2}$ with respect to enthalpy, (TPS2H), [$^{o}$C/(kJ/kg)].

Parameters

$c_{pm2}$ = specific heat of the metal of the primary superheater, (CPM2), [kJ/(kg $^{o}$C)],

$m_{pm2}$ = mass of the metal of the primary superheater, (MPM2), [kg],

$T_{ps3}$ = heat transfer coefficient of the primary superheater, (TPS3), [$^{o}$C/(kJ/s)], and

$V_{ps2}$ = volume of the steam in the primary superheater, (VPS2), [$m^3$].

Basic Physical Equations

The energy balance of the metal and the steam becomes:

$$\frac{d}{dt}(m_{pm2}c_{pm2}T_{pm2} + V_{ps2}\rho_{ps2}h_{ps2}) =$$

$$= Q_{pm2} + w_{ps1}h_{ps1} - w_{ps2}h_{ps2}$$

The mass balance of the steam becomes:

$$\frac{d}{dt}(V_{ps2}\rho_{ps2}) = w_{ps1} - w_{ps2} = 0$$

The temperature of the metal is given by:

$$T_{pm2} = T_{ps2} + T_{ps3}w_{ps1}(h_{ps2}-h_{ps1})$$

Using assumption (A10.4) the derivative of the temperature of the metal can be computed:

$$\frac{d}{dt}(T_{pm2}) = \frac{d}{dt}(T_{ps2}) + T_{ps3}w_{ps1}\,\frac{d}{dt}(h_{ps2})$$

The energy balance now becomes:

$$\left[m_{pm2}c_{pm2}(T_{ps2h}+w_{ps1}T_{ps3}) + V_{ps2}\rho_{ps2}\right]\frac{d}{dt}(h_{ps2}) =$$

$$= Q_{pm2} - w_{ps1}(h_{ps2}-h_{ps1})$$

Algebraic Equation

The equations of the primary superheater are coded on lines 169-181 in subroutine DRUM, Appendix A40. The temperature of the metal is given by

$$T_{pm2} = T_{ps2} + T_{ps3}Q_{pm2}$$

Thermodynamic Equations

The enthalpy of the steam entering the primary superheater is given by:

$$h_{ps1} = HSP(p_{ds2})$$

The density of the steam leaving the primary superheater is given by:

$$\rho_{ps2} = RHP(h_{ps2}, P_{ps2})$$

The temperature of the steam leaving the primary superheater is given by:

$$T_{ps2} = THP(h_{ps2}, P_{ps2})$$

The derivative of $T_{ps2}$ with respect to enthalpy is given by:

$$T_{ps2h} = THPH(h_{ps2}, P_{ps2})$$

Differential Equations

The derivative of $h_{ps2}$ with respect to time is given by:

$$\frac{d}{dt}(h_{ps2}) = \left(Q_{pm2} - w_{ps1}(h_{ps2} - h_{ps1})\right)/\tau_{ps2}$$

where

$$\tau_{ps2} = m_{pm2}c_{pm2}(T_{ps2h} + w_{ps1}T_{ps3}) + V_{ps2}\rho_{ps2}$$

## 11. THE FIRST ATTEMPERATOR

There is a spray attemperator after the primary superheater. The steam is cooled by injecting water into the steam flow. The water evaporates and reduces the temperature of the steam leaving the attemperator. The mass flow rate of water is varied by varying the position of the spray flow valve. The spray flow valve is manipulated in order to control the temperature of the steam leaving the attemperator.

### Properties

The temperature of the steam entering the first attemperator is about 450-480$^{o}$C. The maximum spray flow of the first attemperator is about 9 kg/s. The reduction of the temperature of the steam passing the first attemperator is less than 30$^{o}$C. The volume and the mass of the first attemperator are very small (less than 1 $m^3$ and $1.0 \cdot 10^3$kg).

### Assumption

The mass storage and the energy storage of the first attemperator can be neglected (A11.1).

### Comments

Assumption (A11.1) was used by Eklund [1] and by McDonald et. al. [7]. If the pressure changes from $100 \cdot 10^5$ Pa to $150 \cdot 10^5$ Pa the stored mass in 1.0 $m^3$ changes 18.0 kg, which shall be compared with the mass flow rate of steam (130 kg/s). If the temperature changes 20 $^{o}$C the stored energy of $1 \cdot 10^3$ kg metal changes about 10 MWs, which shall be compared with the cooling power (approximately 20 MW). It is concluded that assumption (A11.1) is physically well-founded because the purpose is not to model rapid dynamic phenomena.

Inputs

$h_{fw7}$ = enthalpy of the feedwater leaving the feedwater preheater, (HFW7), [kJ/kg],

$h_{ps2}$ = enthalpy of the steam leaving the primary superheater, (HPS2), [kJ/kg],

$w_{ps1}$ = mass flow rate of the steam entering the primary superheater, (WPS1), [kg/s],

$w_{ss1}$ = mass flow rate of the steam entering the secondary superheater, (WSS1), [kg/s], and

$w_{sw1}$ = mass flow rate of the spray water entering the first attemperator, (WSW1), [kg/s].

Output

$h_{ss1}$ = enthalpy of the steam entering the secondary superheater, (HSS1), [kJ/kg].

Basic Physical Equation

Using assumption (A11.1) the energy balance of the first attemperator becomes:

$$0 = w_{ps1}h_{ps2} + w_{sw1}h_{fw7} - w_{ss1}h_{ss1}$$

Algebraic Equation

The equation of the first attemperator is coded on lines 187-188 in subroutine DRUM, Appendix A40. The enthalpy of the steam leaving the attemperator is given by:

$$h_{ss1} = (w_{ps1}h_{ps2} + w_{sw1}h_{fw7})/w_{ss1}$$

## 12. THE SECONDARY SUPERHEATER

The steam enters the secondary superheater after leaving the first attemperator. The heat flow from the furnace rises the temperature of the steam in the secondary superheater.

### Properties

The temperature of the steam entering the secondary superheater is about 450°C. The temperature of the steam is rised about 50°C in the secondary superheater. The mass flow rate of steam at maximum output power is about 130 kg/s and the corresponding heat flow is about 25 MW.

### Assumptions

The secondary superheater can be treated as a single lumped system (A12.1).

The steam flow is incompressible (A12.2).

The difference between the temperatures of the metal and the steam can be computed from static relations (A12.3).

The derivative of the temperature of the metal with respect to time can be computed with $h_{ss1}$ and $w_{ss1}$ constant (A12.4).

### Comments

Assumptions (A12.1 to A12.4) are similar to assumptions (A10.1 to A10.4) and the same comments are applicable.

Inputs

$h_{ss1}$ = enthalpy of the steam entering the secondary superheater, (HSS1), [kJ/kg],

$p_{ps2}$ = pressure of the steam leaving the primary superheater, (PPS2), [Pa],

$p_{ss2}$ = pressure of the steam leaving the secondary superheater, (PSS2), [Pa],

$Q_{sm2}$ = heat flow to the metal of the primary superheater, (QSM2), [kJ/s], and

$w_{ss1}$ = mass flow rate of steam entering the secondary superheater, (WSS1), [kg/s].

States

$h_{ss2}$ = enthalpy of the steam leaving the secondary superheater, (HSS2), [kJ/kg].

Variables

$\rho_{ss2}$ = density of the steam leaving the secondary superheater, (RPS2), [kg/m$^3$],

$T_{sm2}$ = temperature of the metal of the secondary superheater, (TSM2), [$^o$C],

$T_{ss1}$ = temperature of the steam entering the secondary superheater, (TSS1), [$^o$C],

$T_{ss2}$ = temperature of the steam leaving the secondary superheater, (TSS2), [$^{o}$C], and

$T_{ss2h}$ = derivative of $T_{ss2}$ with respect to enthalpy, (TSS2H), [$^{o}$C/(kJ/kg)].

Parameters

$c_{sm2}$ = specific heat of the metal of the secondary superheater, (CSM2), [kJ/(kg$^{o}$C)],

$m_{sm2}$ = mass of the metal of the secondary superheater, (MSM2), [kg],

$T_{ss3}$ = heat transfer coefficient of the secondary superheater, (TSS3), [$^{o}$C/(kJ/s)], and

$V_{ss2}$ = volume of the secondary superheater, (VSS2), [m$^3$].

Basic Physical Equations

The energy balance of the metal and the steam becomes:

$$\frac{d}{dt}(m_{sm2}c_{sm2}T_{sm2} + V_{ss2}\rho_{ss2}h_{ss2}) =$$

$$= Q_{sm2} + w_{ss1}h_{ss1} - w_{ss2}h_{ss2}$$

The mass balance of the steam becomes:

$$\frac{d}{dt}(V_{ss2}\rho_{ss2}) = w_{ss1} - w_{ss2} = 0$$

The temperature of the metal is given by

$$T_{sm2} = T_{ss2} + T_{ss3}w_{ss1}(h_{ss2}-h_{ss1})$$

Using assumption (A12.4) the derivative of the temperature of the metal can be computed:

$$\frac{d}{dt}(T_{sm2}) = \frac{d}{dt}(T_{ss2}) + T_{ss3}w_{ss1}\,\frac{d}{dt}(h_{ss2})$$

The energy balance now becomes:

$$\left[m_{sm2}c_{sm2}(T_{ss2h}+w_{ss1}T_{ss3}) + V_{ss2}\rho_{ss2}\right]\frac{d}{dt}(h_{ss2}) =$$

$$= Q_{sm2} - w_{ss1}(h_{ss2}-h_{ss1})$$

Algebraic Equation

The equations of the secondary superheater are coded on lines 182-193 in subroutine DRUM, Appendices A40-A41. The temperature of the metal is given by:

$$T_{sm2} + T_{ss2} + T_{ss3}Q_{sm2}$$

Thermodynamic Equations

The density of the steam leaving the secondary superheater is given by:

$$\rho_{ss2} = RHP(h_{ss2},p_{ss2})$$

The temperature of the steam entering the secondary superheater is given by:

$$T_{ss1} = THP(h_{ss1},p_{ps2})$$

The temperature of the steam leaving the secondary superheater is given by:

$$T_{ss2} = THP(h_{ss2}, p_{ss2})$$

The derivative of $T_{ss2}$ with respect to enthalpy is given by:

$$T_{ss2h} = THPH(h_{ss2}, p_{ss2})$$

Differential Equations

The derivative of $h_{ss2}$ with respect to time is given by:

$$\frac{d}{dt}(h_{ss2}) = \left[Q_{sm2} - w_{ss1}(h_{ss2}-h_{ss1})\right]/\tau_{ss2}$$

where

$$\tau_{ss2} = m_{sm2}c_{sm2}(T_{ss2h}+w_{ss1}T_{ss3}) + V_{ss2}\rho_{ss2}$$

# 13. THE SECOND ATTEMPERATOR

There is a spray attemperator also after the secondary superheater. The steam is cooled by injecting water into the steam flow. The water evaporates and reduces the temperature of the steam leaving the attemperator. The mass flow rate is varied by varying the position of the spray flow valve. The spray flow valve is manipulated in order to control the temperature of the steam leaving the attemperator.

## Properties

The temperature of the steam entering the second attemperator is about 490-520$^{o}$C. The maximum spray flow of the second attemperator is about 6 kg/s. The reduction of the temperature of the steam passing the second attemperator is less than 30$^{o}$C. The volume of the second attemperator are very small (less than 1 $m^3$ and 1.0 $10^3$ kg).

## Assumption

The mass storage and the energy storage of the second attemperator can be neglected (A13.1).

## Comment

Assumption (A13.1) is similar to assumption (A11.1) and the same comment is applicable.

Inputs

$h_{fw7}$ = enthalpy of the feedwater leaving the feedwater preheater, (HFW7), [kJ/kg],

$h_{ss2}$ = enthalpy of the steam leaving the secondary superheater, (HSS2), [kJ/kg],

$w_{ss1}$ = mass flow rate of the steam entering the secondary superheater, (WSS1), [kg/s],

$w_{ts1}$ = mass flow rate of the steam entering the tertiary superheater, (WTS1), [kg/s], and

$w_{tw1}$ = mass flow rate of the spray water entering the second attemperator, (WTW1), [kg/s].

Output

$h_{ts1}$ = enthalpy of the steam entering the tertiary superheater.

Basic Physical Equation

Using assumption (A13.1) the energy balance of the second attemperator becomes:

$$0 = w_{ss1}h_{ss2} + w_{tw1}h_{fw7} - w_{ts1}h_{ts1}$$

Algebraic Equations

The equation of the second attemperator is coded on lines 202--203 in subroutine DRUM, Appendix A41. The enthalpy of the steam leaving the attemperator is given by:

$$h_{ts1} = (w_{ss1}h_{ss2} + w_{tw1}h_{fw7})/w_{ts1}$$

## 14. THE TERTIARY SUPERHEATER

The steam enters the tertiary superheater after leaving the second attemperator. The heat flow from the furnace rises the temperature of the steam in the tertiary superheater.

### Properties

The temperature of the steam is rised about $50^{\circ}C$ in the tertiary superheater. The temperature of the steam leaving the superheater is about $540^{\circ}C$. The mass flow rate of steam at maximum output power is about 130 kg/s and the corresponding heat flow is about 25 MW.

### Assumptions

The tertiary superheater can be treated as a single lumped system (A14.1).

The steam flow is incompressible (A14.2).

The difference between the temperatures of the metal and the steam can be computed from static relations (A14.3).

The derivative of the temperature of the metal with respect to time can be computed with $h_{ts1}$ and $w_{ts1}$ constant (A14.4).

### Comments

Assumptions (A14.1 to A14.4) are similar to assumptions (A10.1 to A10.4) and the same comments are applicable.

Inputs

$h_{ts1}$ = enthalpy of the steam entering the tertiary superheater, (HTS1), [kJ/kg],

$p_{ss2}$ = pressure of the steam leaving the secondary superheater, (PSS2), [Pa],

$p_{ts2}$ = pressure of the steam leaving the tertiary superheater, (PTS2), [Pa],

$Q_{tm2}$ = heat flow to the metal of the tertiary superheater, (QTM2), [kJ/s], and

$w_{ts1}$ = mass flow rate of steam entering the tertiary superheater, (WTS1), [kg/s].

States

$h_{ts2}$ = enthalpy of the steam leaving the tertiary superheater, (HTS2), [kJ/kg].

Variables

$\rho_{ts2}$ = density of the steam leaving the tertiary superheater, (RTS2), [kg/m$^3$],

$T_{tm2}$ = temperature of the metal of the tertiary superheater, (TTM2), [$^o$C],

$T_{ts1}$ = temperature of the metal of the tertiary superheater, (TTS1), [$^o$C],

$T_{ts2}$ = temperature of the steam leaving the tertiary superheater, (TTS2), [$^{o}$C], and

$T_{ts2h}$ = derivative of $T_{ts2}$ with respect to enthalpy, (TTS2H), [$^{o}$C/(kJ/kg)].

Parameters

$c_{tm2}$ = specific heat of the metal of the tertiary superheater, (CTM2), [kJ/(kg $^{o}$C)],

$m_{tm2}$ = mass of the metal of the tertiary superheater, (MTM2), [kg],

$T_{ts3}$ = heat transfer coefficient of the tertiary superheater, (TTS3), [$^{o}$C/(kJ/s)], and

$V_{ts2}$ = volume of the tertiary superheater, (VTS2), [$m^3$].

Basic Physical Equations

The energy balance of the metal and the steam becomes:

$$\frac{d}{dt}(m_{tm2}c_{tm2}T_{tm2} + V_{ts2}\rho_{ts2}h_{ts2}) =$$

$$= Q_{tm2} + w_{ts1}h_{ts1} - w_{ts2}h_{ts2}$$

The mass balance of the steam becomes:

$$\frac{d}{dt}(V_{ts2}\rho_{ts2}) = w_{ts1} - w_{ts2} = 0$$

The temperature of the metal is given by:

$$T_{tm2} = T_{ts2} + T_{ts3} w_{ts1} (h_{ts2} - h_{ts1})$$

Using assumption (A14.4) the derivative of the temperature of the metal can be computed:

$$\frac{d}{dt}(T_{tm2}) = \frac{d}{dt}(T_{ts2}) + T_{ts3} w_{ts1} \frac{d}{dt}(h_{ts2})$$

The energy balance now becomes:

$$\left[m_{tm2} c_{tm2} (T_{ts2h} + w_{ts1} T_{ts3}) + V_{ts2} \rho_{ts2}\right] \frac{d}{dt}(h_{ts2}) =$$

$$= Q_{tm2} - w_{ts1} (h_{ts2} - h_{ts1})$$

Algebraic Equation

The equations of the tertiary superheater are coded on lines 197-201 and 206-211 in subroutine DRUM, Appendix A41. The temperature of the metal is given by:

$$T_{tm2} = T_{ts2} + T_{ts3} Q_{tm2}$$

Thermodynamic Equations

The density of the steam leaving the tertiary superheater is given by:

$$\rho_{ts2} = \mathrm{RHP}(h_{ts2}, p_{ts2})$$

The temperature of the steam entering the tertiary superheater is given by:

$$T_{ts1} = THP(h_{ts1}, p_{ts1})$$

The temperature of the steam leaving the tertiary superheater is given by:

$$T_{ts2} = THP(h_{ts2}, p_{ts2})$$

The derivative of $T_{ts2}$ with respect to enthalpy is given by:

$$T_{ts2h} = THPH(h_{ts2}, p_{ts2})$$

Differential Equation

The derivative of $h_{ts2}$ with respect to time is given by:

$$\frac{d}{dt}(h_{ts2}) = [Q_{tm2} - w_{ts1}(h_{ts2} - h_{ts1})]/\tau_{ts2}$$

where

$$\tau_{ts2} = m_{tm2}c_{tm2}(T_{ts2h} + w_{ts1}T_{ts3}) + V_{ts2}\rho_{ts2}$$

## 15. THE HIGH-PRESSURE TURBINE

The superheated steam passes the control valve and enters the high-pressure turbine. In the high-pressure turbine the energy of the steam is partly converted to mechanical energy. Steam to the seventh stage of the feedwater preheater is extracted after the high-pressure turbine.

### Properties

The temperature of the superheated steam is about 535°C. The steam expands to about $32 \cdot 10^5$ Pa in the high-pressure turbine at maximum output power. The temperature of the steam leaving the high-pressure turbine is about 350°C.

### Assumption

The expansion in the high-pressure turbine can be characterized by a constant isentropic efficiency (A15.1).

### Comment

Assumption (A15.1) is sometimes used in accurate steady-state calculations. The output power of the high-pressure turbine and the enthalpy of the steam leaving the high-pressure turbine may be erroneous at part loads due to assumption (A15.1). These errors are steady-state errors and of minor importance in a model of the dynamics.

Inputs

$h_{ts2}$ = enthalpy of the steam leaving the tertiary superheater, (HTS2), [kJ/kg],

$p_{fs7}$ = pressure of the steam in the seventh stage of the feedwater preheater, (PFS7), [Pa],

$p_{rs2}$ = pressure of the steam leaving the reheater, (PRS2), [Pa],

$p_{vs2}$ = pressure of the steam leaving the control valve, (PVS2), [Pa],

$s_{fs2}$ = normalized position of the extraction valve of the high-pressure turbine, (SFS2), and

$w_{ts1}$ = mass flow rate of the steam entering the tertiary superheater, (WTS1), [kg/s].

Variables

$a_{hs2}$ = normalized area of the extraction valve of the high-pressure turbine, (AHS2), and

$T_{hs2}$ = temperature of the steam leaving the high-pressure turbine, (THS2), [$^oC$].

Outputs

$h_{hs2}$ = enthalpy of the steam leaving the high-pressure turbine, (HHS2), [kJ/kg],

$N_{hs2}$ = mechanical output power of the high-pressure turbine, (NHS2), [kW], and

$w_{hs2}$ = extraction mass flow rate of the high-pressure turbine, (WHS2), [kg/s].

Parameters

$a_{hs2b}$ = base value of the normalized area of the extraction valve of the high-pressure turbine, (AHS2B),

$a_{hs2m}$ = selector for the normalized area of the extraction valve of the high-pressure turbine, (AHS2M),

$\eta_{hs2}$ = isentropic efficiency of the high-pressure turbine, (EHS2),

$f_{rs1}$ = pressure drop coefficient of the reheater, (FRS1), $[Pa^2/(kg/s)^2]$, and

$f_{hs2}$ = pressure drop coefficient of the extraction valve of the high-pressure turbine, (FHS2), $[Pa^2/(kg/s)^2]$.

Basic Physical Equations

The pressure drop in the reheater is given by:

$$p_{hs2}^2 - p_{rs2}^2 = f_{rs1}w_{rs1}^2 = f_{rs1}w_{ts1}^2$$

The pressure drop in the extraction valve of the high-pressure turbine is given by:

$$p_{hs2}^2 - p_{fs7}^2 = f_{hs2}w_{hs2}^2/a_{hs2}^2$$

Algebraic Equations

The area of the extraction valve of the high-pressure turbine is computed on line 340 in subroutine REGU, Appendix A56. The outputs are computed on lines 74-83 of subroutine TURB, Appendix A44.

The area of the extraction valve of the high-pressure turbine is given by:

$$a_{hs2} = \begin{cases} s_{fs2} & \text{if} \quad a_{hs2m} = 0 \\ a_{hs2b} & \text{if} \quad a_{hs2m} = 1 \end{cases}$$

The pressure of the steam at the outlet of the high-pressure turbine is given by:

$$p_{hs2} = \sqrt{p_{rs2}^2 + f_{rs1} w_{ts1}^2}$$

The enthalpy of the steam at the outlet of the high-pressure turbine is given by:

$$h_{hs2} = h_{hs2}^* + (1-\eta_{hs2})(h_{ts2}-h_{hs2}^*)$$

The mechanical output power of the high-pressure turbine is given by:

$$N_{hs2} = w_{ts1}(h_{ts2}-h_{hs2})$$

The extraction mass flow rate of the high-pressure turbine is given by:

$$w_{hs2} = a_{hs2}\sqrt{|p_{hs2}^2 - p_{fs7}^2|/f_{hs2}}$$

Thermodynamic Equations

The enthalpy of the steam after an isentropic expansion from $(h_{ts2}, p_{vs2})$ to $p_{hs2}$ is given by:

$$h^*_{hs2} = \mathrm{ISENX}(h_{ts2}, p_{vs2}, p_{hs2})$$

The temperature of the steam at the outlet of the high-pressure turbine is given by:

$$T_{hs2} = \mathrm{THP}(h_{hs2}, p_{hs2})$$

## 16. THE REHEATER

After expansion in the high-pressure turbine a minor part of the steam flow is extracted to the seventh stage of the feed-water preheater. Most of the steam enters the reheater where the temperature of the steam is rised by the heat flow from the furnace.

### Properties.

The temperature of the steam entering the reheater is about $350^{o}C$ and the mass flow rate of steam is about 125 kg/s at maximum output power. The temperature of the steam leaving the reheater is about $535^{o}C$ and the pressure is about $32 \cdot 10^{5}$ Pa. The heat flow to the reheater is about 55 MW.

### Assumptions

The reheater can be treated as a single lumped system (A16.1).

The steam flow is incompressible (A16.2).

The difference between the temperatures of the metal and the steam can be computed from static relations (A16.3).

The derivative of the metal with respect to time can be computed with $h_{rs1}$ and $w_{rs1}$ constant (A16.4).

### Comments

Assumptions (A16.1 to A16.4) are similar to assumptions (A10.1 to A10.4) and the same comments are applicable.

Inputs

$h_{hs2}$ = enthalpy of the steam leaving the high-pressure turbine, (HHS2), [kJ/kg],

$Q_{rm2}$ = heat flow from the combustion gases to the metal of the reheater, (QRM2), [kJ/s],

$w_{hs2}$ = mass flow rate of extraction steam from the high-pressure turbine, (WHS2), [kg/s], and

$w_{is1}$ = mass flow rate of steam entering the intermediate-pressure turbine, (WIS1), [kg/s],

$w_{ts1}$ = mass flow rate of steam entering the tertiary superheater, (WTS1), [kg/s].

States

$h_{rs2}$ = enthalpy of the steam leaving the reheater, (HRS2), [kJ/kg], and

$p_{rs2}$ = pressure of the steam leaving the reheater, (PRS2), [Pa].

Variables

$\rho_{rs2}$ = density of the steam leaving the reheater, (RRS2), [$kg/m^3$],

$\rho_{rs2h}$ = derivative of $\rho_{rs2}$ with respect to enthalpy, (RRS2H), [$(kg/m^3)/(kJ/kg)$],

$\rho_{rs2p}$ = derivative of $\rho_{rs2}$ with respect to pressure, (RRS2P), [$(kg/m^3)/Pa$], and

$T_{rm2}$ = temperature of the metal of the reheater, (TRM2), [$^{o}$C],

$T_{rs2}$ = temperature of the steam leaving the reheater, (TRS2), [$^{o}$C],

$T_{rs2h}$ = derivative of $T_{rs2}$ with respect to enthalpy, (RRS2H), [$^{o}$C/(kJ/kg)], and

$w_{rs1}$ = mass flow rate of steam entering the reheater, (WRS1), [kg/s].

Parameters

$c_{rm2}$ = specific heat of the reheater metal, (CRM2), [kJ/(kg $^{o}$C)],

$m_{rm2}$ = mass of the reheater metal, (MRM2), [kg],

$T_{rs3}$ = heat transfer coefficient of the reheater, (TRS3), [$^{o}$C/(kJ/s)], and

$V_{rs2}$ = volume of the reheater, (VRS2), [m$^{3}$].

Basic Physical Equations

The energy balance of the reheater becomes:

$$\frac{d}{dt}(c_{rm2}m_{rm2}T_{rm2} + V_{rs2}\rho_{rs2}h_{rs2}) =$$

$$= Q_{rm2} + w_{rs1}h_{hs2} - w_{is1}h_{rs2} \qquad (16.1)$$

The mass balance of the reheater reads

$$\frac{d}{dt}(V_{rs2}\rho_{rs2}) = w_{rs1} - w_{is1} \tag{16.2}$$

Combination of equations (16.1) and (16.2) as well as assumption (A16.4) gives:

$$(c_{rm2}m_{rm2}T_{rs2h} + V_{rs2}\rho_{rs2})\frac{d}{dt}(h_{rs2}) =$$

$$= Q_{rm2} - w_{rs1}(h_{rs2}-h_{hs2})$$

The time derivative of the pressure of the steam leaving the reheater is now given by:

$$V_{rs2}\rho_{rs2h}\frac{d}{dt}(h_{rs2}) + V_{rs2}\rho_{rs2p}\frac{d}{dt}(p_{rs2}) = w_{rs1} - w_{is1}$$

Algebraic Equations

The equations of the reheater are coded on lines 158-62 and 166-78 in the subroutine TURB, Appendix A45.

The mass flow rate of steam entering the reheater is given by:

$$w_{rs1} = w_{ts1} - w_{hs2}$$

The temperature of the metal of the reheater is given by:

$$T_{rm2} = T_{rs2} + T_{rs3}Q_{rm2}$$

Thermodynamic Equations

The density of the steam leaving the reheater is given by:

$$\rho_{rs2} = RHP(h_{rs2}, p_{rs2})$$

The derivative of $\rho_{rs2}$ with respect to enthalpy and pressure is given by:

$$\rho_{rs2h} = RHPH(h_{rs2}, p_{rs2}) \text{ and}$$

$$\rho_{rs2p} = RHPP(h_{rs2}, p_{rs2})$$

Differential Equations

The time derivative of the enthalpy of the steam leaving the reheater is given by:

$$\frac{d}{dt}(h_{rs2}) = [Q_{rm2} - w_{rs1}(h_{rs2}-h_{hs2})]/\tau_{rs2}$$

where

$$\tau_{rs2} = m_{rm2}c_{rm2}T_{rs2h} + V_{rs2}\rho_{rs2}$$

The time derivative of the pressure of the steam leaving the reheater is given by:

$$\frac{d}{dt}(p_{rs2}) = (w_{rs1}-w_{is1}-\rho_{rs2h}h_{rs2h})/(V_{rs2}\rho_{rs2})$$

## 17. THE INTERMEDIATE-PRESSURE TURBINE

The reheated steam enters the intermediate-pressure turbine. In the intermediate-pressure turbine the energy of the steam is partly converted to mechanical energy. Steam to the feed-water preheaters is extracted at four different locations of the intermediate-pressure turbine.

### Properties

The temperature of the steam entering the intermediate-pressure turbine is about $535^{o}C$ and the pressure is about $32 \cdot 10^5$ Pa at maximum output power. The temperature of the steam leaving the intermediate-pressure turbine is about $200^{o}C$ and the pressure is about $3 \cdot 10^5$ Pa at maximum output power. The intermediate-pressure turbine is conceptually divided into four sections, with extraction after every section.

### Assumptions

The expansion of every section of the intermediate-pressure turbine can be characterized by a constant isentropic efficiency, (A17.1).

The pressure of the steam entering a section of the intermediate-pressure turbine is proportional to the mass flow rate of steam into this section, (A17.2).

The mass flow rates of the extraction steam do not affect the pressures of the steam in the intermediate-pressure turbine, (A17.3).

Comments

Assumption (A17.1) is sometimes used in accurate steady-state calculations. The output power of the intermediate-pressure turbine and the enthalpy of the steam leaving the four sections of the intermediate pressure turbine may be erroneous at part loads due to assumption (A17.1). These errors are steady-state errors and of minor importance in a model of the dynamics.

Assumption (A17.2) is a reasonable approximation. Compare with fig. 6.2.

Assumption (A17.3) is motivated by a desire to simplify the calculations.

Inputs

$h_{rs2}$ = enthalpy of the steam at the output of the reheater, (HRS2), [kJ/kg],

$p_{fs3}$ = pressure of the steam in the third feedwater preheater stage, (PFS3), [Pa],

$p_{fs4}$ = pressure of the steam in the fourth stage of the feedwater preheater, (PFS4), [Pa],

$p_{fs5}$ = pressure of the steam in the fifth stage of the feedwater preheater, (PFS5), [Pa],

$p_{fs6}$ = pressure of the steam in the sixth stage of the feedwater preheater, (PFS6), [Pa],

$p_{rs2}$ = pressure of the steam at the outlet of the reheater, (PRS2), [Pa],

$s_{fs2}$ = normalized position of the servo of the high pressure feedwater preheater valves, (SFS2), and

$s_{fs7}$ = normalized position of the servo of the low pressure feedwater preheater valves, (SFS7).

Variables

$a_{is2}$ = normalized area of the first extraction valve of the intermediate-pressure turbine, (AIS2),

$a_{is4}$ = normalized area of the second extraction valve of the intermediate-pressure turbine, (AIS4),

$a_{is6}$ = normalized area of the third extraction valve of the intermediate-pressure turbine, (AIS6),

$a_{is8}$ = normalized area of the fourth extraction valve of the intermediate-pressure turbine, (AIS8),

$p_{is2}$ = pressure of the steam at the first extraction valve of the intermediate pressure turbine, (PIS2), [Pa],

$p_{is4}$ = pressure of the steam at the second extraction valve of the intermediate pressure turbine, (PIS4), [Pa],

$p_{is6}$ = pressure of the steam at the third extraction valve of the intermediate pressure turbine, (PIS6), [Pa],

$p_{is8}$ = pressure of the steam at the fourth extraction valve of the intermediate pressure turbine, (PIS8), [Pa],

$T_{is2}$ = temperature of the steam after the first section of the intermediate pressure turbine, (TIS2), [$^{o}$C],

$T_{is4}$ = temperature of the steam after the second section of the intermediate pressure turbine, (TIS4), [$^{o}$C],

$T_{is6}$ = temperature of the steam after the third section of the intermediate pressure turbine, (TIS6), [$^{o}$C],

$T_{is8}$ = temperature of the steam after the fourth section of the intermediate pressure turbine, (TIS8), [$^{o}$C],

$w_{is3}$ = mass flow rate into the second section of the intermediate pressure turbine, (WIS3), [kg/s], and

$w_{is5}$ = mass flow rate into the third section of the intermediate pressure turbine, (WIS5), [kg/s].

Outputs

$h_{is2}$ = enthalpy of the steam after the first section of the intermediate pressure turbine, (HIS2), [kJ/kg],

$h_{is4}$ = enthalpy of the steam after the second section of the intermediate pressure turbine, (HIS4), [kJ/kg],

$h_{is6}$ = enthalpy of the steam after the third section of the intermediate pressure turbine, (HIS6), [kJ/kg],

$h_{is8}$ = enthalpy of the steam after the fourth section of the intermediate pressure turbine, (HIS8), [kJ/kg],

$N_{is2}$ = mechanical output power of the intermediate-pressure turbine, (NIS2), [kW],

$w_{is1}$ = mass flow rate of steam into the first section of the intermediate pressure turbine, (WIS1), [kg/s],

$w_{is2}$ = extraction mass flow rate after the first section of the intermediate pressure turbine, (WIS2), [kg/s],

$w_{is4}$ = mass flow rate of extraction steam after the second section of the intermediate pressure turbine, (WIS4), [kg/s],

$w_{is6}$ = mass flow rate of extraction steam after the second section of the intermediate pressure turbine, (WIS6), [kg/s],

$w_{is7}$ = mass flow rate of steam into the fourth section of the intermediate pressure turbine, (WIS7), [kg/s],

$w_{is8}$ = mass flow rate of extraction steam after the fourth section of the intermediate pressure turbine, (WIS8), [kg/s].

Parameters

$a_{is2b}$ = base value of the normalized area of the first extraction valve of the intermediate pressure turbine, (AIS2B),

$a_{is4b}$ = base value of the normalized area of the second extraction valve of the intermediate pressure turbine, (AIS4B),

$a_{is6b}$ = base value of the normalized area of the third extraction valve of the intermediate pressure turbine, (AIS6B),

$a_{is8b}$ = base value of the normalized area of the fourth extraction valve of the intermediate pressure turbine, (AIS8B),

$a_{is2m}$ = selector for the normalized area of the first extraction valve of the intermediate pressure turbine, (AIS2M),

$a_{is4m}$ = selector for the normalized area of the second extraction valve of the intermediate pressure turbine, (AIS4M),

$a_{is6m}$ = selector for the normalized area of the third extraction valve of the intermediate pressure turbine, (AIS6M),

$a_{is8m}$ = selector for the normalized area of the fourth extraction valve of the intermediate pressure turbine, (AIS8M),

$\eta_{is2}$ = isentropic efficiency of the first section of the intermediate pressure turbine, (EIS2),

$\eta_{is4}$ = isentropic efficiency of the second section of the intermediate pressure turbine, (EIS4),

$\eta_{is6}$ = isentropic efficiency of the third section of the intermediate pressure turbine, (EIS6),

$\eta_{is8}$ = isentropic efficiency of the fourth section of the intermediate pressure turbine, (EIS8),

$f_{is1}$ = pressure drop coefficient of the first section of the intermediate pressure turbine, (FIS1), [Pa/(kg/s)],

$f_{is2}$ = pressure drop coefficient of the first extraction valve of the intermediate pressure turbine, (FIS2), $[(Pa)^2/ /(kg/s)^2]$,

$f_{is3}$ = pressure drop coefficient of the second section of the intermediate pressure turbine, (FIS3), [Pa/(kg/s)],

$f_{is4}$ = pressure drop coefficient of the second extraction valve of the intermediate pressure turbine, (FIS4), $[(Pa)^2/ /(kg/s)^2]$,

$f_{is5}$ = pressure drop coefficient of the third section of the intermediate pressure turbine, (FIS5), [Pa/(kg/s)],

$f_{is6}$ = pressure drop coefficient of the third extraction valve of the intermediate pressure turbine, (FIS6), $[(Pa)^2/ /(kg/s)^2]$,

$f_{is7}$ = pressure drop coefficient of the fourth section of the intermediate pressure turbine, (FIS7), [Pa/(kg/s)],

$f_{is8}$ = pressure drop coefficient of the fourth extraction valve of the intermediate pressure turbine, (FIS8), $[(Pa)^2/(kg/s)^2]$, and

$f_{\ell s1}$ = pressure drop coefficient of the first section of the low pressure turbine (FLS1), [Pa/(kg/s)].

Basic Physical Equations

Using assumption (A17.2) the pressure of the steam entering the intermediate-pressure turbine and the pressures of the steam leaving the four sections of the intermediate-pressure turbine are given by:

$$p_{rs2} = f_{is1} w_{is1} \tag{17.1}$$

$$p_{is2} = f_{is3}(w_{is1} - w_{is2}) \tag{17.2}$$

$$p_{is4} = f_{is5}(w_{is3} - w_{is4}) \tag{17.3}$$

$$p_{is6} = f_{is7}(w_{is5} - w_{is6}) \tag{17.4}$$

$$p_{is8} = f_{\ell s1}(w_{is7} - w_{is8}) \tag{17.5}$$

The pressure drops of the extraction valves of the intermediate-pressure turbine are given by:

$$p_{is2}^2 - p_{fs6}^2 = f_{is2} w_{is2}^2 / a_{is2}^2 \tag{17.6}$$

$$p_{is4}^2 - p_{fs5}^2 = f_{is4} w_{is4}^2 / a_{is4}^2 \tag{17.7}$$

$$p_{is6}^2 - p_{fs4}^2 = f_{is6} w_{is6}^2 / a_{is6}^2 \tag{17.8}$$

$$p_{is8}^2 - p_{fs3}^2 = f_{is8}w_{is8}^2/a_{is8}^2 \qquad (17.9)$$

<u>Thermodynamic Equations</u>

The enthalpies of the steam leaving the four sections of the intermediate-pressure turbine after an isentropic expansion should be given by:

$$h_{is2}^* = \text{ISENX}(h_{rs2}, p_{rs2}, p_{is2})$$

$$h_{is4}^* = \text{ISENX}(h_{is2}, p_{is2}, p_{is4})$$

$$h_{is6}^* = \text{ISENX}(h_{is4}, p_{is4}, p_{is6})$$

$$h_{is8}^* = \text{ISENX}(h_{is6}, p_{is6}, p_{is8})$$

The temperatures of the steam leaving the four sections of the intermediate-pressure turbine are given by:

$$T_{is2} = \text{THP}(h_{is2}, p_{is2})$$

$$T_{is4} = \text{THP}(h_{is4}, p_{is4})$$

$$T_{is6} = \text{THP}(h_{is6}, p_{is6})$$

$$T_{is8} = \text{THP}(h_{is8}, p_{is8})$$

Algebraic Equations

The areas of the extraction valves of the intermediate-pressure turbine are given by:

$$a_{is2} = \begin{cases} s_{fs2} & \text{if} \quad a_{is2m} = 0 \\ a_{is2b} & \text{if} \quad a_{is2m} = 1 \end{cases}$$

$$a_{is4} = \begin{cases} s_{fs2} & \text{if} \quad a_{is4m} = 0 \\ a_{is4b} & \text{if} \quad a_{is4m} = 1 \end{cases}$$

$$a_{is6} = \begin{cases} s_{fs2} & \text{if} \quad a_{is6m} = 0 \\ a_{is6b} & \text{if} \quad a_{is6m} = 1 \end{cases}$$

$$a_{is8} = \begin{cases} s_{fs7} & \text{if} \quad a_{is8m} = 0 \\ a_{is8b} & \text{if} \quad a_{is8m} = 1 \end{cases}$$

The mass flow rates of the steam entering, and the pressures and the enthalpies of the steam leaving the four sections of the intermediate-pressure turbine, and the mass flow rates of the extraction steam after the four sections of the intermediate-pressure turbine can be computed from equations (17.1 to 17.9) using assumption (A17.1):

$$w_{is1} = p_{rs2}/f_{is1}$$

$$p_{is2} = f_{is3}w_{is1}$$

$$h_{is2} = h^*_{is2} + (1-\eta_{is2})(h_{rs2}-h^*_{rs2})$$

$$w_{is2} = a_{is2}\sqrt{|p_{is2}^2 - p_{fs6}^2|/f_{is2}}$$

$$w_{is3} = w_{is1} - w_{is2}$$

$$p_{is4} = f_{is5}w_{is3}$$

$$h_{is4} = h_{is4}^* + (1-\eta_{is4})(h_{is2}-h_{is4}^*)$$

$$w_{is4} = a_{is4}\sqrt{|p_{is4}^2 - p_{fs5}^2|/f_{is4}}$$

$$w_{is5} = w_{is3} - w_{is4}$$

$$p_{is6} = f_{is7}w_{is5}$$

$$h_{is6} = h_{is6}^* + (1-\eta_{is6})(h_{is4}-h_{is6}^*)$$

$$w_{is6} = a_{is6}\sqrt{|p_{is6}^2 - p_{fs4}^2|/f_{is6}}$$

$$w_{is7} = w_{is5} - w_{is6}$$

$$p_{is8} = f_{\ell s1}w_{is7}$$

$$h_{is8} = h_{is8}^* + (1-\eta_{is8})(h_{is6}-h_{is8}^*)$$

$$w_{is8} = a_{is8}\sqrt{|p_{is8}^2 - p_{fs3}^2|/f_{is8}}$$

The mechanical output power of the intermediate-pressure turbine is finally given by:

$$N_{is2} = w_{is1}(h_{rs2}-h_{is2}) + w_{is3}(h_{is2}-h_{is4}) + $$
$$+ w_{is5}(h_{is4}-h_{is6}) + w_{is7}(h_{is6}-h_{is8})$$

## 18. THE LOW-PRESSURE TURBINE

Most of the steam from the fourth section of the intermediate-pressure turbine enters the low-pressure turbine. In the low-pressure turbine the energy of the steam is partly converted to mechanical energy, available on the rotating turbine-generator shaft. Steam to the feedwater preheaters are extracted at two different locations of the low-pressure turbine.

### Properties

The temperature of the steam at the inlet of the low-pressure turbine is about $200^{\circ}C$ and the pressure is about $3 \cdot 10^5$ Pa. The steam at the outlet of the low-pressure turbine is saturated and has a pressure about $0.03 \cdot 10^5$ Pa. The low-pressure turbine is conceptually divided into three sections, with extraction after the first and the second.

### Assumption

The expansion of the steam in every section of the low-pressure turbine can be characterized by a constant isentropic efficiency, (A18.1).

The pressure of the steam entering a section of the low-pressure turbine is proportional to the mass flow rate of the steam entering this section, (A18.2).

The mass flow rates of the extraction steam do not affect the pressures of the steam in the low-pressure turbine, (A18.3).

Comments

Assumption (A18.1) is sometimes used in accurate steady-state calculations. The output power of the low-pressure turbine and the enthalpy of the steam leaving the three sections of the low-pressure turbine may be erroneous at part loads due to assumption (A18.1). These errors are steady-state errors and of minor importance in a model of the dynamics.

Assumption (A18.2) is a reasonable approximation. Compare with fig. 6.2.

Assumption (A18.3) is motivated by a desire to simplify the calculations.

Inputs

$h_{is8}$ = enthalpy of the steam at the outlet of the intermediate-pressure turbine, (HIS8), [kJ/kg],

$p_{cc2}$ = pressure of the steam in the condensor, (PCC2), [Pa],

$p_{fs1}$ = pressure of the steam in the first stage of the feedwater preheater, (PFS1), [Pa],

$p_{fs2}$ = pressure of the steam in the second stage of the feedwater preheater, (PFS2), [Pa],

$p_{is8}$ = pressure of the steam at the outlet of the intermediate-pressure turbine, (PIS8), [Pa],

$s_{fs7}$ = normalized position of the servo of the low-pressure feedwater preheater valves, (SFS7),

$w_{is7}$ = mass flow rate of steam into the fourth stage of the intermediate pressure turbine, (WIS7), [kg/s], and

$w_{is8}$ = mass flow rate of extraction steam after the fourth stage of the intermediate pressure turbine, (WIS8), [kg/s].

Variables

$a_{\ell s2}$ = normalized area of the first extraction valve of the low-pressure turbine, (ALS2),

$a_{\ell s4}$ = normalized area of the second extraction valve of the low-pressure turbine, (ALS4),

$p_{\ell s2}$ = pressure of the steam after the first section of the low-pressure turbine, (PLS2), [Pa],

$p_{\ell s4}$ = pressure of the steam after the second section of the low-pressure turbine, (PLS4), [Pa],

$T_{\ell s2}$ = temperature of the steam after the first section of the low-pressure turbine, (TLS2), [$^{o}$C],

$T_{\ell s4}$ = temperature of the steam after the second section of the low-pressure turbine, (TLS4), [$^{o}$C],

$T_{\ell s6}$ = temperature of the steam after the third section of the low-pressure turbine, (TLS6), [$^{o}$C],

$w_{\ell s1}$ = mass flow rate into the first section of the low-pressure turbine, (WLS1), [kg/s], and

$w_{\ell s3}$ = mass flow rate into the second section of the low-pressure turbine, (WLS3), [kg/s].

Outputs

$h_{\ell s2}$ = enthalpy of the steam after the first section of the low-pressure turbine, (HLS2), [kJ/kg],

$h_{\ell s4}$ = enthalpy of the steam after the second section of the low-pressure turbine, (HLS4), [kJ/kg],

$h_{\ell s6}$ = enthalpy of the steam after the third section of the low-pressure turbine, (HLS6), [kJ/kg],

$N_{\ell s2}$ = mechanical output power of the low-pressure turbine, (NLS2), [kW],

$w_{\ell s2}$ = mass flow rate of extraction steam after the first section of the low-pressure turbine, (WLS2), [kg/s],

$w_{\ell s4}$ = mass flow rate of extraction steam after the second section of the low-pressure turbine, (WLS4), [kg/s],

$w_{\ell s5}$ = mass flow rate of steam into the third section of the low-pressure turbine, (WLS5), [kg/s].

Parameters

$a_{\ell s2b}$ = base value of the normalized area of the first extraction valve of the low-pressure turbine, (ALS2B),

$a_{\ell s4b}$ = base value of the normalized area of the second extraction valve of the low-pressure turbine, (ALS4B),

$a_{\ell s2m}$ = selector for the normalized area of the first extraction valve of the low-pressure turbine, (ALS2M),

$a_{\ell s4m}$ = selector for the normalized area of the second extraction valve of the low-pressure turbine, (ALS4M),

$\eta_{\ell s2}$ = isentropic efficiency of the first section of the low-pressure turbine, (ELS2),

$\eta_{\ell s4}$ = isentropic efficiency of the second section of the low-pressure turbine, (ELS4),

$\eta_{\ell s6}$ = isentropic efficiency of the third section of the low-pressure turbine, (ELS6),

$f_{\ell s2}$ = pressure drop coefficient of the first extraction valve of the low-pressure turbine, (FLS2), $[(Pa)^2/(kg/s)^2]$,

$f_{\ell s3}$ = pressure drop coefficient of the second section of the low-pressure turbine, (FLS4), [Pa/(kg/s)],

$f_{\ell s4}$ = pressure drop coefficient of the second extraction valve of the low-pressure turbine, (FLS4), $[(Pa)^2/(kg/s)^2]$,

$f_{\ell s5}$ = pressure drop coefficient of the third section of the low-pressure turbine, (FLS5), [Pa/(kg/s)].

Basic Physical Equations

Using assumption (A18.2) the pressures of the steam leaving the first two sections of the low-pressure turbine are given by:

$$p_{\ell s2} = f_{\ell s3}(w_{\ell s1} - w_{\ell s2}) \quad (18.1)$$

$$p_{\ell s4} = f_{\ell s5}(w_{\ell s3} - w_{\ell s4}) \quad (18.2)$$

The pressure drops of the two extraction valves of the low-pressure turbine are given by:

$$p^2_{\ell s2} - p^2_{fs2} = f_{\ell s2} w^2_{\ell s2} / a^2_{\ell s2} \qquad (18.3)$$

$$p^2_{\ell s4} - p^2_{fs1} = f_{\ell s4} w^2_{\ell s4} / a^2_{\ell s4} \qquad (18.4)$$

Thermodynamic Equations

The enthalpies of the steam leaving the three sections of the low-pressure turbine after an isentropic expansion should be given by:

$$h_{\ell s2} = ISENX(h_{is8}, p_{is8}, p_{\ell s2})$$

$$h^*_{\ell s4} = ISENX(h_{\ell s2}, p_{\ell s2}, p_{\ell s4})$$

$$h_{\ell s6} = ISENX(h_{\ell s4}, p_{\ell s4}, p_{cc2})$$

The temperatures of the steam leaving the three sections of the low-pressure turbine are given by:

$$T_{\ell s2} = THP(h_{\ell s2}, p_{\ell s2})$$

$$T_{\ell s4} = THP(h_{\ell s4}, p_{\ell s4})$$

$$T_{\ell s6} = THP(h_{\ell s6}, p_{cc2})$$

Algebraic Equations

The areas of the extraction valves of the low-pressure turbine are given by:

$$a_{\ell s2} = \begin{cases} s_{fs7} & \text{if} \quad a_{\ell s2m} = 0 \\ a_{\ell s2b} & \text{if} \quad a_{\ell s2m} = 1 \end{cases}$$

$$a_{\ell s4} = \begin{cases} s_{fs7} & \text{if} \quad a_{\ell s4m} = 0 \\ a_{\ell s4b} & \text{if} \quad a_{\ell s4m} = 0 \end{cases}$$

The mass flow rates of the steam entering, and the pressures and the enthalpies of the steam leaving the three sections of the low-pressure turbine, and the mass flow rates of the extraction steam leaving the first two sections of the low-pressure turbine can be computed from equations (18.1 to 18.4) using assumptions (A18.1 and A18.3).

$$w_{\ell s1} = w_{is7} - w_{is8}$$

$$p_{\ell s2} = f_{\ell s3} w_{\ell s1}$$

$$h_{\ell s2} = h^*_{\ell s2} + (1-\eta_{\ell s2})(h^*_{is8}-h_{\ell s2})$$

$$w_{\ell s2} = a_{\ell s2}\sqrt{\left|p^2_{\ell s2} - p^2_{fs2}\right|/f_{\ell s2}}$$

$$w_{\ell s3} = w_{\ell s1} - w_{\ell s2}$$

$$p_{\ell s4} = f_{\ell s4} w_{\ell s3}$$

$$h_{\ell s4} = h^*_{\ell s4} + (1-\eta_{\ell s4})(h_{\ell s2}-h^*_{\ell s4})$$

$$w_{\ell s4} = a_{\ell s4}\sqrt{\left|p_{\ell s4}^2 - p_{fs1}^2\right|/f_{\ell s4}}$$

$$w_{\ell s5} = w_{\ell s3} - w_{\ell s4}$$

$$h_{\ell s6} = h_{\ell s6}^* + (1-\eta_{\ell s6})(h_{\ell s4}-h_{\ell s6}^*)$$

The mechanical output power of the low-pressure turbine is finally given by:

$$N_{\ell s2} = w_{\ell s1}(h_{is8}-h_{\ell s2}) + w_{\ell s3}(h_{\ell s2}-h_{\ell s4}) +$$
$$+ w_{\ell s5}(h_{\ell s4}-h_{\ell s6})$$

## 19. THE CONDENSOR

The steam leaving the last section of the low-pressure turbine enters the steam side of the condensor. Sufficient heat is released to the cooling water from the steam in order to condense it. Condensate leaving the low-pressure feedwater preheater also enters the steam side of the condensor. Condensate leaves the condensor.

### Properties

The pressure of the steam in the condensor is about $0.03 \cdot 10^5$ Pa. The pressure increases with the output power and with the temperature of the cooling water. The mass flow rate of the cooling water is choosen so that the temperature of the cooling water rises about 5-10$^{o}$C at maximum output power. The specific fuel consumption decreases if the pressure of the steam in the condensor decreases. Even small errors of this pressure will influence the specific fuel consumption. The dynamics of the condensor, however, does not influence the dynamics of the whole unit. The dynamic model, with a time-constant of about 10 seconds, is motivated by a desire to simplify the calculations. An implicite algebraic equation is replaced by an explicite differential equation.

### Assumptions

The difference between the enthalpy of the condensate and the enthalpy of the cooling water is constant (A19.1).

The difference between the temperature of the condensate and the temperature of the condensor metal is constant (A19.2).

Comments

Assumption (A19.1) is a fairly crude approximation and may introduce a steady-state error of the pressure of the steam in the condensor.

Assumption (A19.2) is a reasonable approximation.

Inputs

$h_{cw1}$ = enthalpy of the cooling water entering the condensor, (HCW1), [kJ/kg],

$h_{fc1}$ = enthalpy of the condensate leaving the first stage of the feedwater preheater, (HFC1), [kJ/kg],

$h_{\ell s6}$ = enthalpy of the steam leaving the last section of the low-pressure turbine, (HLS6), [kJ/kg],

$p_{cw1}$ = pressure of the cooling water entering the condensor, (PCW1), [Pa],

$w_{cw1}$ = mass flow rate of cooling water entering the condensor, (WCW1), [kg/s],

$w_{fc1}$ = mass flow rate of condensate leaving the first stage of the feedwater preheater, (WFC1), [kg/s],

$w_{fw1}$ = mass flow rate of feedwater entering the first stage of the feedwater preheater, (WFW1), [kg/s], and

$w_{\ell s5}$ = mass flow rate of the steam leaving the last section of the low-pressure turbine, (WLS5), [kg/s].

State Variable

$p_{cc2}$ = saturation pressure of condensate leaving the condensor, (PCC2), [Pa].

Variables

$h_{cc2p}$ = derivative of $h_{cc2}$ with respect to pressure, (HCC2P), [(kJ/kg)/Pa],

$h_{cw2}$ = enthalpy of the cooling water leaving the condensor, (HCW2), [kJ/kg],

$\rho_{cc2}$ = density of the condensate leaving the condensor, (RCC2), [$kg/m^3$],

$\rho_{cw2}$ = density of the cooling water leaving the condensor, (RCW2), [$kg/m^3$],

$T_{cc2}$ = temperature of the condensate at the outlet of the condensor, (TCC2), [$^oC$],

$T_{cc2p}$ = derivative of the temperature at the outlet of the condensor with respect to the pressure, (TCC2P), [$^oC/Pa$],

$T_{cw1}$ = temperature of the cooling water at the inlet of the condensor, (TCW1), [$^oC$], and

$T_{cw2}$ = temperature of the cooling water at the outlet of the condensor, (TCW2), [$^oC$].

Output

$h_{cc2}$ = enthalpy of the condensate leaving the condensor, (HCC2), [kJ/kg].

Parameters

$c_{cm2}$ = specific heat of the metal of the condensor, (CCM2), [kJ/(kg $^{o}$C)],

$h_{cc3}$ = difference between the enthalpy of the condensate and the cooling water at the outlet of the condensor, (HCC3), [kJ/kg],

$m_{cm2}$ = mass of the metal of the condensor, (MCM2), [kg],

$V_{cc2}$ = volume of the condensate in the condensor, (VCC2), [m$^3$], and

$V_{cw2}$ = volume of the cooling water in the condensor, (VCW2), [m$^3$].

Basic Physical Equations

The energy balance for the whole condensor becomes:

$$\frac{d}{dt}(V_{cc2}\rho_{cc2}h_{cc2} + m_{cm2}c_{cm2}T_{cm2} + V_{cw2}\rho_{cw2}h_{cw2}) =$$

$$= w_{\ell s5}h_{\ell s6} + w_{cw1}h_{cw1} - w_{cw2}h_{cw2} - w_{fw1}h_{cc2} \qquad (19.1)$$

According to assumption (A19.1) we have

$$h_{cw2} = h_{cc2} - h_{cw3} \qquad (19.2)$$

and according to assumption to (A19.2) we have

$$T_{cm2t} = T_{cc2t} \qquad (19.3)$$

The combination of equations (19.1), (19.2) and (19.3) gives:

$$(V_{cc2}\rho_{cc2}h_{cc2p} + m_{cm2}c_{cm2}T_{cc2p} + V_{cw2}\rho_{cw2}h_{cc2p})\frac{d}{dt}(p_{cc2}) =$$

$$= w_{\ell s5}h_{\ell s6} + w_{cw1}h_{cw1} - w_{cw2}h_{cw2} - w_{fw1}h_{cc2}$$

Algebraic Equations

The equations are coded on lines 335-38 in subroutine REGU, Appendix A56, on lines 90-91, 105-06, 121, 134, 147, 162-163, and 188-92 in subroutine COND, Appendix A47-49.

The enthalpy of the cooling water at the outlet of the condensor is given by

$$h_{cw2} = h_{cc2} - h_{cc3}$$

Thermodynamic Equations

The enthalpy of the condensate at the outlet of the condensor is given by:

$$h_{cc2} = HWP(p_{cc2})$$

The density of the condensate is given by:

$$\rho_{cc2} = RWP(p_{cc2})$$

The density of the cooling water is given by:

$$\rho_{cw2} = RHP(h_{cw2},p_{cw1})$$

The temperature of the condensate is given by:

$$T_{cc2} = TLP(p_{cc2})$$

The temperature of the cooling water at the inlet is given by:

$$T_{cw1} = THP(h_{cw1}, p_{cw1})$$

The temperature of the cooling water at the outlet is given by:

$$T_{cw2} = THP(h_{cw2}, p_{cw1})$$

The derivative of the enthalpy of condensate with respect to pressure is given by:

$$h_{cc2p} = HWPP(p_{cc2})$$

The derivative of the temperature of the condensate with respect to pressure is given by:

$$T_{cc2p} = TLPP(p_{cc2})$$

Differential Equations

The derivative of the pressure of the steam in the condensor with respect to time is given by:

$$\frac{d}{dt}(p_{cc2}) = \Big[w_{\ell s5}h_{\ell s6} + w_{fc1}h_{fc1} - w_{fw1}h_{cc2} - w_{cw1}(h_{cw2}-h_{cw1})\Big]/\tau_{cc2}$$

where

$$\tau_{cc2} = (V_{cc2}\rho_{cc2}h_{cc2p} + m_{cm2}c_{cm2}T_{cc2p} + V_{cw2}\rho_{cw2}h_{cc2p})$$

## 20. THE LOW-PRESSURE FEEDWATER PREHEATER

The low-pressure preheater consists of three heat-exchangers (stages). The feedwater (condensate) from the condensor enters the first stage. The temperature of the feedwater in the first stage is rised by the steam extracted after the second section of the low-pressure turbine and by condensate leaving the second stage of the feedwater preheater. The condensate is passed on into the condensor. The feedwater, which has been heated in the first stage of the feedwater preheater, enters the second stage. The temperature of the feedwater in the second stage is rised by steam extracted after the first section of the low-pressure turbine and by condensate leaving the third stage of the feedwater preheater. The condensate is passed on into the first stage. The feedwater, which has been heated in the second stage of the feedwater preheater, enters the third stage. The temperature of the feedwater in the third stage of the feedwater preheater is rised by a fraction of the steam extracted after the fourth section of the intermediate-pressure turbine.

### Properties

The temperature profile of the feedwater preheater is shown in fig. 20.1. The heat transfer coefficients are large and the temperature differences are small compared to temperature increases. If the stages should have been modelled with two (three) independent energy storing elements; the feedwater, the metal, (and the condensate), one (two) very fast mode(s) should have been obtained. The fast mode(s) describe(s) the equilibration of the three media in the stage of the feedwater preheater.

The mass flow rates of extraction steam depend on the difference between the pressures of the steam in the turbine and the pressures of the steam in the feedwater preheater. The temperature

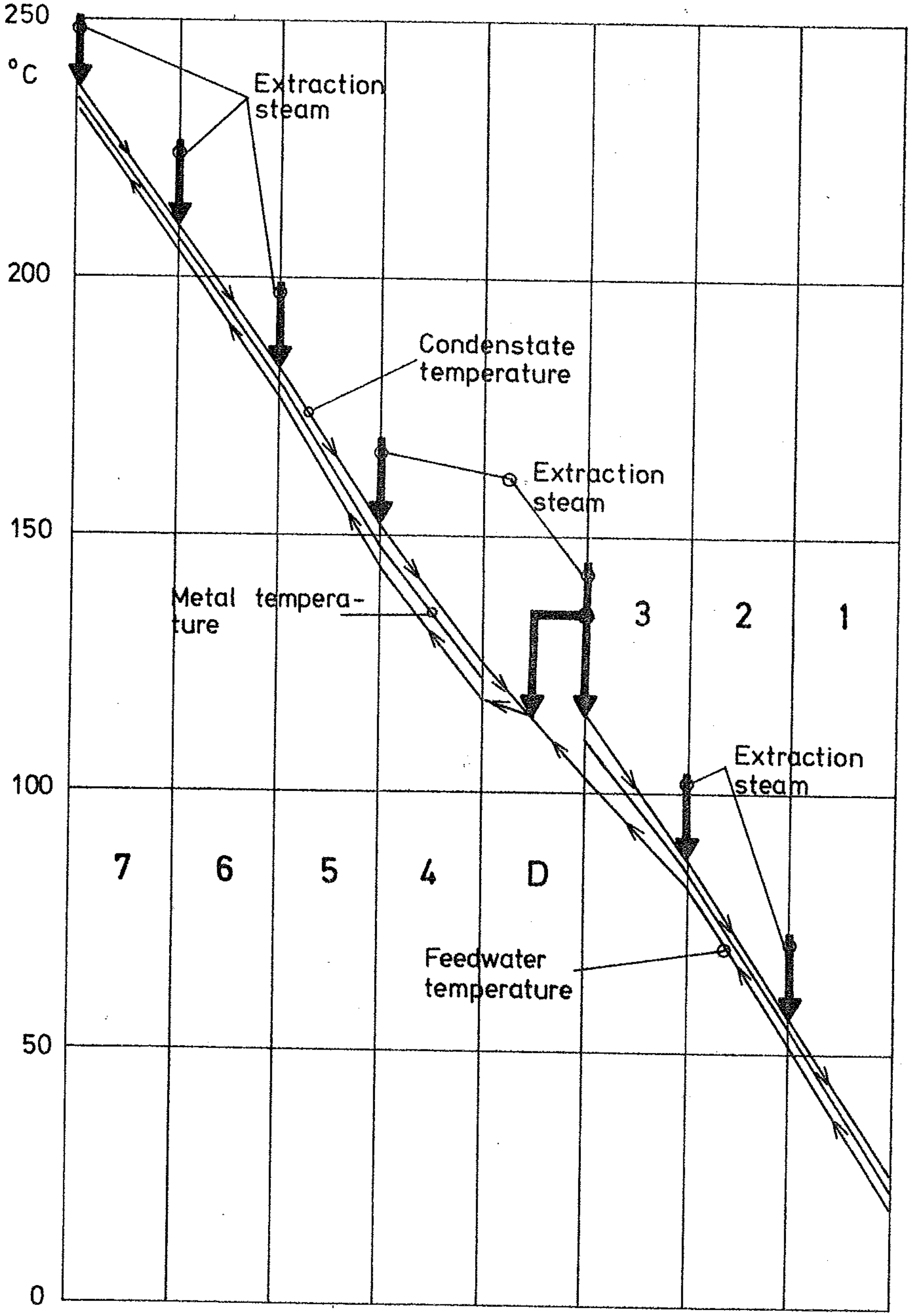


Fig. 20.1 - Temperature profile of the feedwater preheater and the deaerator.

of the feedwater is self-regulated, which can be explained in the following way: Assume that the temperature of the feedwater is lower than the equilibrium value. The saturation pressure of the condensate in the feedwater preheater is then reduced. The mass flow rate of extraction steam is increased due to the increased pressure difference. The heat flow to the feedwater preheater is increased and the temperature of the feedwater increases until the equilibrium value is reached. This means that the mass flow rates of extraction steam and the temperatures of the feedwater are determined by an involved interaction between the very rapid dynamics of the turbines and the slow dynamics of the feedwater preheater.

The model will also be used to study the effects of rapid variations of the mass flow rates of the extraction steam. The possibilities of rapid changes of output power without introducing severe conditions for the drum system and the superheaters will be investigated.

In order to simulate the varying equilibrium temperatures of the feedwater and the effects of rapid variations of the mass flow rate of the extraction steam it is important that the steam pressures are represented in the model. Therefore it was decided to model each stage of the feedwater preheater with a single lumped system with the saturation pressure of the steam as state variable.

Assumptions

The individual stages of the low-pressure feedwater preheater can be treated as single lumped systems with the saturation pressure of the steam as state variables (A20.1).

The difference between the enthalpy of the condensate and the enthalpy of the feedwater is constant (A20.2).

The difference between the temperature of the metal and the temperature of the condensate is constant (A20.3).

The enthalpy of the condensate, which enters a stage of the low-pressure preheater, is equal to the enthalpy of saturated water at the pressure of the steam in the corresponding stage of the low-pressure preheater (A20.4).

The difference between the saturation pressure of the steam in the condensor and the saturation pressure of the condensate leaving the first stage of the feedwater preheater is constant (A20.5).

Comments

Assumption (A20.1) is crude if it is desired to describe the internal phenomena of the preheater stages. The purpose of the model is to describe the temperature of the feedwater leaving the seventh stage of the feedwater preheater. The purpose of the model is also to make it possible to study the variation of the output power due to variations of the mass flow rates of the extraction steam. Eklund assumed in [1] that the temperature of the feedwater leaving the economizer was constant but his experimental results showed later on that his assumption was too crude. The inclusion of a model of the economizer dynamics as well as the feedwater preheater dynamics is a significant improvement over [1]. McDonald et.al. did not include any model of the feedwater preheater dynamics. The possibility of investigating the effects of the manipulation of the mass flow rate of extraction steam is a considerable extension of the purpose of the model compared to purposes of the models proposed in the literature.

Assumptions (A20.2 to A20.5) are very reasonable considering fig. 20.1.

Inputs

$h_{cc2}$ = enthalpy of the condensate at the outlet of the condensor, (HCC2), [kJ/kg],

$h_{is8}$ = enthalpy of the steam after the fourth section of the intermediate pressure turbine, (HIS8), [kJ/kg],

$h_{\ell s2}$ = enthalpy of the steam after the first section of the low-pressure turbine, (HLS2), [kJ/kg],

$h_{\ell s4}$ = enthalpy of the steam after the second section of the low-pressure turbine, (HLS4), [kJ/kg],

$p_{as2}$ = saturation pressure of the steam in the deaerator, (PAS2), [Pa],

$p_{cc2}$ = saturation pressure of the steam and the condensate in the condensor, (PCC2), [Pa],

$w_{is8}$ = mass flow rate of extraction steam after the fourth section of the intermediate pressure turbine, (WIS8), [kg/s],

$w_{\ell s2}$ = mass flow rate of extraction steam after the first section of the low-pressure turbine, (WLS2), [kg/s],

$w_{\ell s4}$ = mass flow rate of extraction steam after the second section of the low-pressure turbine, (WLS4), [kg/s], and

$w_{\ell s5}$ = mass flow rate of steam into the third section of the low-pressure turbine, (WLS5), [kg/s]

States

$p_{fs1}$ = saturation pressure of the steam in the first stage of the feedwater preheater, (PFS1), [Pa],

$p_{fs2}$ = saturation pressure of the steam in the second stage of the feedwater preheater, (PFS2), [Pa], and

$p_{fs3}$ = saturation pressure of the steam in the third stage of the feedwater preheater, (PFS3), [Pa].

Variables

$h_{f\ell 1}$ = saturation enthalpy of water corresponding to the steam pressure in the first stage of the feedwater preheater, (HFL1), [kJ/kg],

$h_{f\ell 2}$ = saturation enthalpy of water corresponding to the steam pressure in the second stage of the feedwater preheater, (HFL2), [kJ/kg],

$h_{f\ell 3}$ = saturation enthalpy of water corresponding to the steam pressure in the third stage of the feedwater preheater, (HFL3), [kJ/kg],

$h_{f\ell 1p}$ = derivative of $h_{f\ell 1}$ with respect to pressure, (HFL1P), [(kJ/kg)/Pa],

$h_{f\ell 2p}$ = derivative of $h_{f\ell 2}$ with respect to pressure, (HFL2P), [(kJ/kg)/Pa],

$h_{f\ell 3p}$ = derivative of $h_{f\ell 3}$ with respect to pressure, (HFL3P), [(kJ/kg)/Pa],

$h_{fw1}$ = enthalpy of feedwater after the first stage of the feedwater preheater, (HFW1), [kJ/kg],

$h_{fw2}$ = enthalpy of feedwater after the second stage of the feedwater preheater, (HFW2), [kJ/kg],

$p_{fc1}$ = saturation pressure corresponding to the enthalpy of the condensate after the first stage of the feedwater preheater, (PFC1), [Pa],

$\rho_{f\ell 1}$ = density of the condensate in the first stage of the feedwater preheater, (RFL1), [kg/m$^3$],

$\rho_{f\ell 2}$ = density of the condensate in the second stage of the feedwater preheater, (RFL2), [kg/m$^3$],

$\rho_{f\ell 3}$ = density of the condensate in the third stage of the feedwater preheater, (RFL3), [kg/m$^3$],

$\rho_{fw1}$ = density of the feedwater in the first stage of the feedwater preheater, (RFW1), [kg/m$^3$],

$\rho_{fw2}$ = density of the feedwater in the second stage of the feedwater preheater, (RFW2), [kg/m$^3$],

$\rho_{fw3}$ = density of the feedwater in the third stage of the feedwater preheater, (RFW3), [kg/m$^3$],

$T_{fc1}$ = saturation temperature corresponding to $h_{fc1}$, (TFC1), [$^o$C],

$T_{f\ell 1}$ = saturation temperature corresponding to $p_{fs1}$, (TFL1), [$^o$C],

$T_{f\ell 2}$ = saturation temperature corresponding to $p_{fs2}$, (TFL2), [$^o$C],

$T_{f\ell 3}$ = saturation temperature corresponding to $p_{fs3}$, (TFL3), [$^o$C],

$T_{f\ell 1p}$ = derivative of $T_{f\ell 1}$ with respect to pressure, (TFL1P), [$^{o}$C/Pa],

$T_{f\ell 2p}$ = derivative of $T_{f\ell 2}$ with respect to pressure, (TFL2P), [$^{o}$C/Pa],

$T_{f\ell 3p}$ = derivative of $T_{f\ell 3}$ with respect to pressure, (TFL3P), [$^{o}$C/Pa],

$T_{fw1}$ = temperature of the feedwater after the first stage of the feedwater preheater, (TFW1), [$^{o}$C],

$T_{fw2}$ = temperature of the feedwater after the second stage of the feedwater preheater, (TFW2), [$^{o}$C],

$T_{fw3}$ = temperature of the feedwater after the third stage of the feedwater preheater, (TFW3), [$^{o}$C],

$w_{f\ell 1}$ = mass flow rate of condensate into the first stage of the feedwater preheater, (WFL1), [kg/s], and

$w_{f\ell 2}$ = mass flow rate of condensate into the second stage of the feedwater preheater, (WFL2), [kg/s].

Outputs

$h_{fc1}$ = enthalpy of the condensate leaving the first stage of the feedwater preheater, (HFC1), [kJ/kg],

$h_{fw3}$ = enthalpy of the feedwater leaving the third stage of the feedwater preheater, (HFW3), [kJ/kg],

$w_{fc1}$ = mass flow rate of condensate leaving the first stage of the feedwater preheater, (WFC1), [kg/s], and

$w_{fw1}$ = mass flow rate of feedwater entering the first stage of the feedwater preheater, (WFW1), [kg/s].

Parameters

$c_{fm1}$ = specific heat of the metal of the first stage of the feedwater preheater, (CFM1), [kJ/(kg $^{o}$C)],

$c_{fm2}$ = specific heat of the metal of the second stage of the feedwater preheater, (CFM2), [kJ/(kg $^{o}$C)],

$c_{fm3}$ = specific heat of the metal of the third stage of the feedwater preheater, (CFM3), [kJ/(kg $^{o}$C)],

$h_{f\ell 8}$ = enthalpy difference between the condensing steam and the feedwater in the feedwater preheater, (HFL8), [kJ/kg],

$m_{fm1}$ = mass of the metal of the first stage of the feedwater preheater, (MFM1), [kg],

$m_{fm2}$ = mass of the metal of the second stage of the feedwater preheater, (MFM2), [kg],

$m_{fm3}$ = mass of the metal of the third stage of the feedwater preheater, (MFM3), [kg],

$p_{cc3}$ = difference between the saturation pressure of the steam in the condensor and the saturation pressure of the condensate leaving the first stage of the feedwater preheater, (PCC3), [Pa],

$V_{fc1}$ = volume of the condensate in the first stage of the feedwater preheater, (VFC1), [m$^{3}$],

$V_{fc2}$ = volume of the condensate in the second stage of the feedwater preheater, (VFC2), [$m^3$],

$V_{fc3}$ = volume of the condensate in the third stage of the feedwater preheater, (VFC3), [$m^3$],

$V_{fw1}$ = volume of the feedwater in the first stage of the feedwater preheater, (VFW1), [$m^3$],

$V_{fw2}$ = volume of the feedwater in the second stage of the feedwater preheater, (VFW2), [$m^3$], and

$V_{fw3}$ = volume of the feedwater in the third stage of the feedwater preheater, (VFW3), [$m^3$].

Basic Physical Equations

The energy balances of the three stages of the low-pressure preheater become:

$$\frac{d}{dt}(V_{fc1}\rho_{f\ell 1}h_{f\ell 1} + m_{fm1}c_{fm1}T_{fm1} + V_{fw1}\rho_{fw1}h_{fw1}) =$$

$$= w_{\ell s4}h_{\ell s4} + w_{f\ell 1}h_{f\ell 1} + w_{fw1}h_{cc2} - w_{fw1}h_{fw1} - w_{fc1}h_{fc1}$$

$$\frac{d}{dt}(V_{fc2}\rho_{f\ell 2}h_{f\ell 2} + m_{fm2}c_{fm2}T_{fm2} + V_{fw2}\rho_{fw2}h_{fw2}) =$$

$$= w_{\ell s2}h_{\ell s2} + w_{f\ell 2}h_{f\ell 2} + w_{fw1}h_{fw1} - w_{fw1}h_{fw2} - w_{f\ell 1}h_{f\ell 1}$$

$$\frac{d}{dt}(V_{fc3}\rho_{f\ell 3}h_{f\ell 3} + m_{fm3}c_{fm3}T_{fm3} + V_{fw3}\rho_{fw3}h_{fw3}) =$$

$$= 0.7w_{is8}h_{is8} + w_{fw1}h_{fw2} - w_{fw1}h_{fw3} - w_{f\ell 2}h_{f\ell 2}$$

Assumption (A20.2) gives:

$$h_{fw1} = h_{f\ell 1} - h_{f\ell 8}$$

$$h_{fw2} = h_{f\ell 2} - h_{f\ell 8}$$

$$h_{fw3} = h_{f\ell 3} - h_{f\ell 8}$$

Assumption (A20.3) gives:

$$\frac{d}{dt}(T_{fm1}) = \frac{d}{dt}(T_{f\ell 1})$$

$$\frac{d}{dt}(T_{fm2}) = \frac{d}{dt}(T_{f\ell 2})$$

$$\frac{d}{dt}(T_{fm3}) = \frac{d}{dt}(T_{f\ell 3})$$

The energy balances of the three stages of the low-pressure feedwater preheater now become:

$$(V_{fc1}\rho_{f\ell 1}h_{f\ell 1p} + m_{fm1}c_{fm1}T_{f\ell 1p} + V_{fw1}\rho_{fw1}h_{f\ell 1p})\frac{d}{dt}(p_{fs1}) =$$

$$= w_{\ell s4}h_{\ell s4} + w_{f\ell 1}h_{f\ell 1} - w_{fw1}(h_{fw1}-h_{cc2}) - w_{fc1}h_{fc1}$$

$$(V_{fc2}\rho_{f\ell 2}h_{f\ell 2p} + m_{fm2}c_{fm2}T_{f\ell 2p} + V_{fw2}\rho_{fw2}h_{f\ell 2p})\frac{d}{dt}(p_{fs2}) =$$

$$= w_{\ell s2}h_{\ell s2} + w_{f\ell 2}h_{fc2} - w_{fw1}(h_{fw2}-h_{fw1}) - w_{f\ell 1}h_{f\ell 1}$$

$$(V_{fc3}\rho_{f\ell 3}h_{f\ell 3p} + m_{fm3}c_{fm3}T_{f\ell 3p} + V_{fw3}\rho_{fw3}h_{f\ell 3p})\frac{d}{dt}(p_{fs3}) =$$

$$= 0.7w_{is8}h_{is8} - w_{fw1}(h_{fw3}-h_{fw2}) - w_{f\ell 2}h_{f\ell 2}$$

Algebraic Equations

The equations of the low-pressure feedwater preheater are coded on lines 86, 92-96, 107-110, 122-125, 135-138, 148-152, 164-167, 180-182, and 193-197 in subroutine COND, Appendix A47-A49.

The saturation pressure of the condensate leaving the first stage is given by:

$$p_{fc1} = p_{cc2} + p_{cc3}$$

The enthalpies of the feedwater leaving the three stages are given by:

$$h_{fw1} = h_{f\ell 1} - h_{f\ell 8}$$

$$h_{fw2} = h_{f\ell 2} - h_{f\ell 8}$$

$$h_{fw3} = h_{f\ell 3} - h_{f\ell 8}$$

The mass flow rates of condensate leaving the three stages are given by:

$$w_{fc1} = w_{f\ell 1} + w_{\ell s4}$$

$$w_{f\ell 1} = w_{f\ell 2} + w_{\ell s2}$$

$$w_{f\ell 2} = 0.7w_{is8}$$

The mass flow rate of feedwater entering the first stage is given by:

$$w_{fw1} = w_{\ell s5} + w_{fc1}$$

Thermodynamic Equations

The enthalpy of the condensate leaving the first stage is given by:

$$h_{fc1} = HWP(p_{fc1})$$

The enthalpies of the condensate in the three stages are given by:

$$h_{f\ell 1} = HWP(p_{fs1})$$

$$h_{f\ell 2} = HWP(p_{fs2})$$

$$h_{f\ell 3} = HWP(p_{fs3})$$

The derivatives of the enthalpies of the condensate with respect to the pressures of the steam in the three stages are given by:

$$h_{f\ell 1p} = HWPP(p_{fs1})$$

$$h_{f\ell 2p} = HWPP(p_{fs2})$$

$$h_{f\ell 3p} = HWPP(p_{fs3})$$

The densities of the condensate in the three stages are given by:

$$\rho_{f\ell 1} = RWP(p_{fs1})$$

$$\rho_{f\ell 2} = RWP(p_{fs2})$$

$$\rho_{f\ell 3} = RWP(p_{fs3})$$

The densities of the feedwater leaving the three stages are given by:

$$T_{fc1} = TLP(p_{fc1})$$

The temperatures of the condensate in the three stages are given by:

$$T_{f\ell 1} = TLP(p_{fs1})$$

$$T_{f\ell 2} = TLP(p_{fs2})$$

$$T_{f\ell 3} = TLP(p_{fs3})$$

The derivatives of the temperatures of the condensate with respect to the pressures of the steam in the three stages are given by:

$$T_{f\ell 1p} = TLPP(p_{fs1})$$

$$T_{f\ell 2p} = TLPP(p_{fs2})$$

$$T_{f\ell 3p} = TLPP(p_{fs3})$$

The temperatures of the feedwater leaving the three stages are given by:

$$T_{fw1} = THP(h_{fw1}, p_{as2})$$

$$T_{fw2} = THP(h_{fw2}, p_{as2})$$

$$T_{fw3} = THP(h_{fw3}, p_{as2})$$

Differential Equations

The derivatives of the pressures of the steam in the three stages are given by:

$$\frac{d}{dt}(p_{fs1}) = \big(w_{\ell s4}h_{\ell s4} + w_{f\ell 1}h_{f\ell 1} - w_{fc1}h_{fc1} - $$

$$- w_{fw1}(h_{fw1}-h_{cc2})\big)/\tau_{fs1}$$

$$\frac{d}{dt}(p_{fs2}) = \big(w_{\ell s2}h_{\ell s2} + w_{f\ell 2}h_{f\ell 2} - w_{f\ell 1}h_{f\ell 1} - $$

$$- w_{fw1}(h_{fw2}-h_{fw1})\big)/\tau_{fs2}$$

$$\frac{d}{dt}(p_{fs3}) = \big(0.7w_{is8}h_{is8} - w_{f\ell 2}h_{f\ell 2} - $$

$$- w_{fw1}(h_{fw3}-h_{fw2})\big)/\tau_{fs3}$$

where

$$\tau_{fs1} = (V_{fc1}\rho_{f\ell 1}h_{f\ell 1p} + m_{fm1}c_{fm1}T_{f\ell 1p} + V_{fw1}\rho_{fw1}h_{f\ell 1p})$$

$$\tau_{fs2} = (V_{fc2}\rho_{f\ell 2}h_{f\ell 2p} + m_{fm2}c_{fm2}T_{f\ell 2p} + V_{fw2}\rho_{fw2}h_{f\ell 2p})$$

$$\tau_{fs3} = (V_{fc3}\rho_{f\ell 3}h_{f\ell 3p} + m_{fm3}c_{fm3}T_{f\ell 3p} + V_{fw3}\rho_{fw3}h_{f\ell 3p})$$

## 21. THE DEAERATOR

The feedwater, which has been heated in the third stage of the feedwater preheater enters the deaerator. The feedwater reaches the saturation temperature in the deaerator.

### Properties

The water-steam surface in the deaerator is very large. The feedwater is heated to the boiling state by steam extracted after the intermediate-pressure turbine.

### Assumptions

The deaerator can be treated as a single lumped system with the saturation pressure of the steam as state variable (A21.1).

The mass and energy storage in the steam volume of the deaerator can be neglected (A21.2).

The difference between the temperature of the feedwater and the temperature of the metal in the deaerator is constant (A21.3).

### Comments

Assumption (A21.1) is very reasonable as long as the purpose is not to study internal phenomena of the deaerator.

Assumption (A21.2) is very reasonable as long as the purpose is not to study pressure control of the deaerator.

Assumption (A21.3) is physically well-founded, due to the high heat transfer coefficient.

Inputs

$h_{fc4}$ = enthalpy of the condensate leaving the fourth stage of the feedwater preheater, (HFC4), [kJ/kg],

$h_{fw3}$ = enthalpy of the feedwater leaving the third stage of the feedwater preheater, (HFW3), [kJ/kg],

$h_{is8}$ = enthalpy of the steam extracted after the intermediate-pressure turbine, (HIS8), [kJ/kg],

$w_{fc4}$ = mass flow rate of the condensate leaving the fourth stage of the feedwater preheater, (WFC4), [kg/s],

$w_{fw5}$ = mass flow rate of the feedwater leaving the deaerator, (WFW5), [kg/s], and

$w_{is8}$ = mass flow rate of the steam extracted after the intermediate-pressure turbine, (WIS8), [kg/s].

State

$p_{as2}$ = saturation temperature of the steam in the deaerator, (PAS2), [Pa].

Variables

$h_{aw1}$ = enthalpy of the feedwater needed for the control of the level in the deaerator, (HAW1), [kJ/kg],

$h_{aw2p}$ = derivative of $h_{aw2}$ with respect to the pressure in the deaerator, (HAW2P), [(kJ/kg)/Pa],

$\rho_{aw2}$ = density of the feedwater in the deaerator, (RAW2), $[kg/m^3]$,

$T_{aw2}$ = temperature of the feedwater leaving the deaerator, (TAW2), $[^{o}C]$, and

$T_{aw2p}$ = derivative of $T_{aw2}$ with respect to the pressure in the deaerator, (TAW2P), $[^{o}C/Pa]$.

Output

$h_{aw2}$ = enthalpy of the feedwater leaving the deaerator, (HAW2), [kJ/kg].

Parameters

$c_{am2}$ = specific heat of the metal of the deaerator, (CAM2), $[kJ/(kg\ ^{o}C)]$,

$h_{aw3}$ = enthalpy of the feedwater in the storage tank, (HAW3), [kJ/kg],

$m_{am2}$ = mass of the metal of the deaerator, (MAM2), [kg], and

$V_{aw2}$ = volume of the feedwater in the deaerator, (VAW2), $[m^3]$.

Basic Physical Equations

The energy balance of the deaerator becomes:

$$\frac{d}{dt}(V_{aw2}\rho_{aw2}h_{aw2} + m_{am2}c_{am2}T_{am2}) =$$

$$= 0.3w_{is8}h_{is8} + w_{fw1}h_{fw3} + w_{aw1}h_{aw1} + w_{fc4}h_{fc4} - w_{fw5}h_{aw2}$$

Assumption (A21.3) gives:

$$\frac{d}{dt}(T_{am2}) = \frac{d}{dt}(T_{aw2})$$

The energy balance now becomes:

$$(V_{aw2}\rho_{aw2}h_{aw2p} + m_{am2}c_{am2}T_{aw2p})\frac{d}{dt}(p_{aw2}) =$$

$$= 0.3w_{is8}h_{is8} + w_{fw1}h_{fw3} + w_{aw1}h_{aw1} + w_{fc4}h_{fc4} - w_{fw5}h_{aw2}$$

## Algebraic Equations

The equations of the deaerator are coded on lines 97-98, 126--127, 153-154, 186-187, and 204-212 of subroutine COND, Appendix A47-A49. The mass flow rate of the feedwater needed for level control in the deaerator is given by:

$$w_{aw1} = w_{fw5} - w_{fw1} - w_{fc4} - 0.3w_{is8}$$

The enthalpy of the feedwater needed for level control in the deaerator depends on the direction of the flow:

$$h_{aw1} = \begin{cases} h_{aw2} & \text{if} \quad w_{aw1} \leq 0 \\ h_{aw3} & \text{if} \quad w_{aw1} > 0 \end{cases}$$

## Thermodynamic Equations

The enthalpy of the feedwater leaving the deaerator is given by:

$$h_{aw2} = HWP(p_{as2})$$

The derivative of $h_{aw2}$ with respect to the pressure of the steam in the deaerator is given by:

$$h_{aw2p} = HWPP(p_{as2})$$

The density of the feedwater leaving deaerator is given by:

$$\rho_{aw2} = RWP(p_{as2})$$

The temperature of the feedwater leaving the deaerator is given by:

$$T_{aw2} = TLP(p_{as2})$$

The derivative of $T_{aw2}$ with respect to the pressure of the steam in the deaerator is given by:

$$T_{aw2p} = TLPP(p_{as2})$$

Differential Equation

The derivative of the pressure of the steam in the deaerator with respect to time is given by:

$$\frac{d}{dt}(p_{as2}) = (0.3w_{is8}h_{is8} + w_{fw1}h_{fw3} + w_{aw1}h_{aw1} + $$

$$+ w_{fc4}h_{fc4} - w_{fw5}h_{aw2})/\tau_{as2}$$

where

$$\tau_{as2} = (V_{aw2}\rho_{aw2}h_{aw2p} + m_{am2}c_{am2}T_{aw2p})$$

## 22. THE HIGH-PRESSURE FEEDWATER PREHEATER

The high-pressure preheater consists of four heat-exchangers (stages). The feedwater pump sucks water from the deaerator and pumps it through the fourth, fifth, sixth and seventh stages of the feedwater preheater. The steam extracted after the third section of the intermediate-pressure turbine is fed to the fourth stage. The condensate leaving the fifth stage also contributes to the heat flow of the fourth stage. The fifth stage is heated by steam extracted after the second section of the intermediate-pressure turbine and by condensate from the sixth stage. Steam extracted after the first section of the intermediate-pressure turbine is used to rise the temperature of the feedwater in the sixth stage. The condensate leaving the seventh stage does also contribute to the heat flow in the sixth stage. The seventh stage is heated by steam extracted after the high-pressure turbine.

### Properties

With the motivation given in Section 20 it was decided to model each stage of the high-pressure feedwater preheater with a single lumped system with the saturation pressure of the steam as state variable.

### Assumptions

The individual stages of the high-pressure feedwater preheater can be treated as single lumped systems with the saturation pressure of the steam as state variables (A22.1).

The difference between the enthalpy of the condensate and the enthalpy of the feedwater is constant (A22.2).

The difference between the temperature of the metal and the temperature of the condensate is constant (A22.3).

The enthalpy of the condensate, which enters a stage of the high-pressure preheater, is equal to the enthalpy of saturated water at the pressure of the steam in the corresponding stage of the low-pressure preheater (A22.4).

The difference between the saturation pressure of the steam in the deaerator and the saturation pressure of the condensate leaving the fourth stage of the feedwater preheater is constant (A22.5).

Comments

Assumptions (A22.1 to A22.5) are similar to assumptions (A20.1 to A20.5) and the same comments are applicable.

Inputs

$h_{aw2}$ = enthalpy of the feedwater leaving the deaerator, (HAW2), [kJ/kg],

$h_{hs2}$ = enthalpy of the steam leaving the high-pressure turbine, (HHS2), [kJ/kg],

$h_{is2}$ = enthalpy of the steam leaving the first section of the intermediate-pressure turbine, (HIS2), [kJ/kg],

$h_{is4}$ = enthalpy of the steam leaving the second section of the intermediate-pressure turbine, (HIS4), [kJ/kg],

$h_{is6}$ = enthalpy of the steam leaving the third section of the intermediate-pressure turbine, (HIS6), [kJ/kg],

$p_{as2}$ = saturation pressure of the steam in the deaerator, (PAS2), [Pa],

$p_{fw6}$ = pressure of the feedwater leaving the feedwater pump, (PFW6), [Pa],

$p_{fw8}$ = pressure of the steam leaving the high-pressure feedwater preheater, (PFW8), [Pa],

$w_{fw5}$ = mass flow rate of feedwater leaving the feedwater pump, (WFW5), [kg/s],

$w_{hs2}$ = mass flow rate of extraction steam leaving the high-pressure turbine, (WHS2), [kg/s],

$w_{is2}$ = mass flow rate of extraction steam leaving the first section of the intermediate-pressure turbine, (WIS2), [kg/s],

$w_{is4}$ = mass flow rate of extraction steam leaving the second section of the intermediate-pressure turbine, (WIS4), [kg/s], and

$w_{is6}$ = mass flow rate of extraction steam leaving the third section of the intermediate-pressure turbine, (WIS6), [kg/s].

States

$p_{fs4}$ = saturation pressure of the steam in the fourth stage of the feedwater preheater, (PFS4), [Pa],

$p_{fs5}$ = saturation pressure of the steam in the fifth stage of the feedwater preheater, (PFS5), [Pa],

$p_{fs6}$ = saturation pressure of the steam in the sixth stage of the feedwater preheater, (PFS6), [Pa], and

$p_{fs7}$ = saturation pressure of the steam in the seventh stage of the feedwater preheater, (PFS7), [Pa].

Variables

$h_{f\ell 4}$ = saturation enthalpy of water corresponding to the steam pressure in the fourth stage of the feedwater preheater, (HFL4), [kJ/kg],

$h_{f\ell 5}$ = saturation enthalpy of water corresponding to the steam pressure in the fifth stage of the feedwater preheater, (HFL5), [kJ/kg],

$h_{f\ell 6}$ = saturation enthalpy of water corresponding to the steam pressure in the sixth stage of the feedwater preheater, (HFL6), [kJ/kg],

$h_{f\ell 7}$ = saturation enthalpy of water corresponding to the steam pressure in the seventh stage of the feedwater preheater, (HFL7), [kJ/kg],

$h_{f\ell 4p}$ = derivative of $h_{f\ell 4}$ with respect to pressure, (HFL4P), [(kJ/kg)/Pa],

$h_{f\ell 5p}$ = derivative of $h_{f\ell 5}$ with respect to pressure, (HFL5P), [(kJ/kg)/Pa],

$h_{f\ell 6p}$ = derivative of $h_{f\ell 6}$ with respect to pressure, (HFL6P), [(kJ/kg)/Pa],

$h_{f\ell 7p}$ = derivative of $h_{f\ell 7}$ with respect to pressure, (HFL7P), [(kJ/kg)/Pa],

$h_{fw4}$ = enthalpy of the feedwater leaving the fourth stage of the feedwater preheater, (HFW4), [kJ/kg],

$h_{fw5}$ = enthalpy of the feedwater leaving the fifth stage of the feedwater preheater, (HFW5), [kJ/kg],

$h_{fw6}$ = enthalpy of the feedwater leaving the sixth stage of the feedwater preheater, (HFW6), [kJ/kg],

$p_{fc4}$ = saturation pressure corresponding to the enthalpy of the condensate leaving the fourth stage of the feedwater preheater, (PFC4), [Pa],

$\rho_{f\ell 4}$ = density of the condensate in the fourth stage of the feedwater preheater, (RFL4), [kg/m$^3$],

$\rho_{f\ell 5}$ = density of the condensate in the fifth stage of the feedwater preheater, (RFL5), [kg/m$^3$],

$\rho_{f\ell 7}$ = density of the condensate in the seventh stage of the feedwater preheater, (RFL7), [kg/m$^3$],

$\rho_{fw4}$ = density of the feedwater in the fourth stage of the feedwater preheater, (RFW4), [kg/m$^3$],

$\rho_{fw5}$ = density of the feedwater in the fifth stage of the feedwater preheater, (RFW5), [kg/m$^3$],

$\rho_{fw6}$ = density of the feedwater in the sixth stage of the feedwater preheater, (RFW6), [kg/m$^3$],

$\rho_{fw7}$ = density of the feedwater in the seventh stage of the feedwater preheater, (RFW7), [kg/m$^3$],

$T_{fc4}$ = saturation temperature corresponding to $h_{fc4}$, (TFC4), [$^o$C],

$T_{f\ell 4}$ = saturation temperature corresponding to $p_{fs4}$, (TFL4), [$^o$C],

$T_{f\ell 5}$ = saturation temperature corresponding to $p_{fs5}$, (TFL5), [$^o$C],

$T_{f\ell 6}$ = saturation temperature corresponding to $p_{fs6}$, (TFL6), [$^o$C]

$T_{f\ell 7}$ = saturation temperature corresponding to $p_{fs7}$, (TFL7), [$^{o}$C],

$T_{f\ell 4p}$ = derivative of $T_{f\ell 4}$ with respect to pressure, (TFL4P), [$^{o}$C/Pa],

$T_{f\ell 5p}$ = derivative of $T_{f\ell 5}$ with respect to pressure, (TFL5P), [$^{o}$C/Pa],

$T_{f\ell 6p}$ = derivative of $T_{f\ell 6}$ with respect to pressure, (TFL6P), [$^{o}$C/Pa],

$T_{f\ell 7p}$ = derivative of $T_{f\ell 7}$ with respect to pressure, (TFL7P), [$^{o}$C/Pa],

$T_{fw4}$ = temperature of the feedwater leaving the fourth stage of the feedwater preheater, (TFW4), [$^{o}$C],

$T_{fw5}$ = temperature of the feedwater leaving the fifth stage of the feedwater preheater, (TFW5), [$^{o}$C],

$T_{fw6}$ = temperature of the feedwater leaving the sixth stage of the feedwater preheater, (TFW6), [$^{o}$C],

$T_{fw7}$ = temperature of the feedwater leaving the seventh stage of the feedwater preheater, (TFW7), [$^{o}$C],

$w_{f\ell 4}$ = mass flow rate of the condensate entering the fourth stage of the feedwater preheater, (WFL4), [kg/s],

$w_{f\ell 5}$ = mass flow rate of the condensate entering the fifth stage of the feedwater preheater, (WFL5), [kg/s], and

$w_{f\ell 6}$ = mass flow rate of the condensate entering the sixth stage of the feedwater preheater, (WFL6), [kg/s].

Outputs

$h_{fc4}$ = enthalpy of the condensate leaving the fourth stage of the feedwater preheater, (HFC4), [kJ/kg],

$h_{fw7}$ = enthalpy of the feedwater leaving the seventh stage of the feedwater preheater, (HFW7), [kJ/kg], and

$w_{fc4}$ = mass flow rate of the condensate leaving the fourth stage of the feedwater preheater, (WFC4), [kg/s].

Parameters

$c_{fm4}$ = specific heat of the metal of the fourth stage of the feedwater preheater, (CFM4), [kJ/(kg $^{o}$C)],

$c_{fm5}$ = specific heat of the metal of the fifth stage of the feedwater preheater, (CFM5), [kJ/(kg $^{o}$C)],

$c_{fm6}$ = specific heat of the metal of the sixth stage of the feedwater preheater, (CFM6), [kJ/(kg $^{o}$C)],

$c_{fm7}$ = specific heat of the metal of the seventh stage of the feedwater preheater, (CFM7), [kJ/(kg $^{o}$C)],

$h_{f\ell 8}$ = enthalpy difference between the condensate and the feedwater in the feedwater preheater, (HFL8), [kJ/kg],

$m_{fm4}$ = mass of the metal of the fourth stage of the feedwater preheater, (MFM4), [kg],

$m_{fm5}$ = mass of the metal of the fifth stage of the feedwater preheater, (MFM5), [kg],

$m_{fm6}$ = mass of the metal of the sixth stage of the feedwater preheater, (MFM6), [kg],

$m_{fm7}$ = mass of the metal of the seventh stage of the feedwater preheater, (MFM7), [kg],

$p_{as3}$ = difference between the saturation pressure of the steam in the deaerator and the saturation pressure of the condensate leaving the fourth stage of the feedwater preheater, (PAS3), [Pa],

$V_{fc4}$ = volume of the condensate in the fourth stage of the feedwater preheater, (VFC4), [$m^3$],

$V_{fc5}$ = volume of the condensate in the fifth stage of the feedwater preheater, (VFC5), [$m^3$],

$V_{fc6}$ = volume of the condensate in the sixth stage of the feedwater preheater, (VFC6), [$m^3$],

$V_{fc7}$ = volume of the condensate in the seventh stage of the feedwater preheater, (VFC7), [$m^3$],

$V_{fw4}$ = volume of the feedwater in the fourth stage of the feedwater preheater, (VFW4), [$m^3$],

$V_{fw5}$ = volume of the feedwater in the fifth stage of the feedwater preheater, (VFW5), [$m^3$],

$V_{fw6}$ = volume of the feedwater in the sixth stage of the feedwater preheater, (VFW6), [$m^3$], and

$V_{fw7}$ = volume of the feedwater in the seventh stage of the feedwater preheater, (VFW7), [$m^3$].

Basic Physical Equations

The energy balances of the four stages of the high-pressure preheater become:

$$\frac{d}{dt}(V_{fc4}\rho_{f\ell 4}h_{f\ell 4} + m_{fm4}c_{fm4}T_{fm4} + V_{fw4}\rho_{fw4}h_{fw4}) =$$

$$= w_{is6}h_{is6} + w_{f\ell 4}h_{f\ell 4} + w_{fw5}h_{aw2} - w_{fw5}h_{fw4} - w_{fc4}h_{fc4}$$

$$\frac{d}{dt}(V_{fc5}\rho_{f\ell 5}h_{f\ell 5} + m_{fm5}c_{fm5}T_{fm5} + V_{fw5}\rho_{fw5}h_{fw5}) =$$

$$= w_{is4}h_{is4} + w_{f\ell 5}h_{f\ell 5} + w_{fw5}h_{fw4} - w_{fw5}h_{fw5} - w_{f\ell 4}h_{f\ell 4}$$

$$\frac{d}{dt}(V_{fc6}\rho_{f\ell 6}h_{f\ell 6} + m_{fm6}c_{fm6}T_{fm6} + V_{fw6}\rho_{fw6}h_{fw6}) =$$

$$= w_{is2}h_{is2} + w_{f\ell 6}h_{f\ell 6} + w_{fw5}h_{fw5} - w_{fw5}h_{fw6} - w_{f\ell 5}h_{f\ell 5}$$

$$\frac{d}{dt}(V_{fc7}\rho_{f\ell 7}h_{f\ell 7} + m_{fm7}c_{fm7}T_{fm7} + V_{fw7}\rho_{fw7}h_{fw7}) =$$

$$= w_{hs2}h_{hs2} + w_{fw5}h_{fw6} - w_{fw5}h_{fw7} - w_{f\ell 6}h_{f\ell 6}$$

Assumption (A22.2) gives

$$h_{fw4} = h_{f\ell 4} - h_{f\ell 8}$$

$$h_{fw5} = h_{f\ell 5} - h_{f\ell 8}$$

$$h_{fw6} = h_{f\ell 6} - h_{f\ell 8}$$

$$h_{fw7} = h_{f\ell 7} - h_{f\ell 8}$$

Assumption (A22.3) gives:

$$\frac{d}{dt}(T_{fm4}) = \frac{d}{dt}(T_{f\ell 4})$$

$$\frac{d}{dt}(T_{fm5}) = \frac{d}{dt}(T_{f\ell 5})$$

$$\frac{d}{dt}(T_{fm6}) = \frac{d}{dt}(T_{f\ell 6})$$

$$\frac{d}{dt}(T_{fm7}) = \frac{d}{dt}(T_{f\ell 7})$$

Assumption (A22.5) gives:

$$p_{fc4} = p_{as2} + p_{as3}$$

The energy balances of the four stages of the high-pressure feedwater preheater now become:

$$(V_{fc4}\rho_{f\ell 4}h_{f\ell 4p} + m_{fm4}c_{fm4}T_{f\ell 4p} + V_{fw4}\rho_{fw4}h_{f\ell 4p})\frac{d}{dt}(p_{fs4}) =$$

$$= w_{is6}h_{is6} + w_{f\ell 4}h_{f\ell 4} - w_{fw5}(h_{fw4}-h_{aw2}) - w_{fc4}h_{fc4}$$

$$(V_{fc5}\rho_{f\ell 5}h_{f\ell 5p} + m_{fm5}c_{fm5}T_{f\ell 5p} + V_{fw5}\rho_{fw5}h_{f\ell 5p})\frac{d}{dt}(p_{fs5}) =$$

$$= w_{is4}h_{is4} + w_{f\ell 5}h_{f\ell 5} - w_{fw5}(h_{fw5}-h_{fw4}) - w_{f\ell 4}h_{f\ell 4}$$

$$(V_{fc6}\rho_{f\ell 6}h_{f\ell 6p} + m_{fm6}c_{fm6}T_{f\ell 6p} + V_{fw6}\rho_{fw6}h_{f\ell 6p})\frac{d}{dt}(p_{fs6}) =$$

$$= w_{is2}h_{is2} + w_{f\ell 6}h_{f\ell 6} - w_{fw5}(h_{fw6}-h_{fw5}) - w_{f\ell 5}h_{f\ell 5}$$

$$(V_{fc7}\rho_{f\ell 7}h_{f\ell 7p} + m_{fm7}c_{fm7}T_{f\ell 7p} + V_{fw7}\rho_{fw7}h_{f\ell 7p})\frac{d}{dt}(p_{fs7}) =$$

$$= w_{hs2}h_{hs2} - w_{fw5}(h_{fw7}-h_{fw6}) - w_{f\ell 6}h_{f\ell 6}$$

Algebraic Equations

The equations of the high-pressure feedwater preheater are coded on lines 87, 99-104, 111-115, 128-132, 139-143, 155--160, 168-179, and 213-226 in subroutine COND, Appendix A47--A49.

The saturation pressure of the condensate leaving the fourth stage is given by:

$$p_{fc4} = p_{as2} + p_{as3}$$

The enthalpies of the feedwater leaving the four stages are given by:

$$h_{fw4} = h_{f\ell 4} - h_{f\ell 8}$$

$$h_{fw5} = h_{f\ell 5} - h_{f\ell 8}$$

$$h_{fw6} = h_{f\ell 6} - h_{f\ell 8}$$

$$h_{fw7} = h_{f\ell 7} - h_{f\ell 8}$$

The mass flow rate of condensate leaving the fourth stage is given by:

$$w_{fc4} = w_{f\ell 4} + w_{is6}$$

The mass flow rates of condensate entering the high-pressure stages are given by:

$$w_{f\ell 4} = w_{f\ell 5} + w_{is4}$$

$$w_{f\ell 5} = w_{f\ell 6} + w_{is2}$$

$$w_{f\ell 6} = w_{hs2}$$

Thermodynamic Equations

The enthalpy of the condensate leaving the fourth stage is given by:

$$h_{fc4} = HWP(p_{fc4})$$

The enthalpy of the condensate in the four stages are given by:

$$h_{f\ell 4} = HWP(p_{fs4})$$

$$h_{f\ell 5} = HWP(p_{fs5})$$

$$h_{f\ell 6} = HWP(p_{fs6})$$

$$h_{f\ell 7} = HWP(p_{fs7})$$

The derivatives of the enthalpies of the condensate with respect to the pressures of the steam in the four stages are given by:

$$h_{f\ell 4p} = HWPP(p_{fs4})$$

$$h_{f\ell 5p} = HWPP(p_{fs5})$$

$$h_{f\ell 6p} = HWPP(p_{fs6})$$

$$h_{f\ell 7p} = HWPP(p_{fs7})$$

The densities of the condensate in the four stages are given by:

$$\rho_{f\ell 4} = RWP(p_{fs4})$$

$$\rho_{f\ell 5} = RWP(p_{fs5})$$

$$\rho_{f\ell 6} = RWP(p_{fs6})$$

$$\rho_{f\ell 7} = RWP(p_{fs7})$$

The densities of the feedwater leaving the four stages are given by:

$$\rho_{fw4} = RHP(h_{fw4}, p_{fw6})$$

$$\rho_{fw5} = RHP(h_{fw5}, p_{fw6})$$

$$\rho_{fw6} = RHP(h_{fw6}, p_{fw8})$$

$$\rho_{fw7} = RHP(h_{fw7}, p_{fw8})$$

The temperature of the condensate leaving the fourth stage is given by:

$$T_{fc4} = TLP(p_{fc4})$$

The temperatures of the condensate in the four stages are given by:

$$T_{f\ell 4} = TLP(p_{fs4})$$

$$T_{f\ell 5} = TLP(p_{fs5})$$

$$T_{f\ell 6} = TLP(p_{fs6})$$

$$T_{f\ell 7} = TLP(p_{fs7})$$

The derivatives of the temperatures of the condensate with respect to the pressures of the steam in the four stages are given by:

$$T_{f\ell 4p} = TLPP(p_{fs4})$$

$$T_{f\ell 5p} = TLPP(p_{fs5})$$

$$T_{f\ell 6p} = TLPP(p_{fs6})$$

$$T_{f\ell 7p} = TLPP(p_{fs7})$$

The temperatures of the feedwater leaving the four stages are given by:

$$T_{fw4} = THP(h_{fw4}, p_{fw6})$$

$$T_{fw5} = THP(h_{fw5}, p_{fw6})$$

$$T_{fw6} = THP(h_{fw6}, p_{fw8})$$

$$T_{fw7} = THP(h_{fw7}, p_{fw8})$$

Differential Equations

The derivatives of the pressures of the steam in the four stages are given by:

$$\frac{d}{dt}(p_{fs4}) = \big(w_{is6}h_{is6} + w_{f\ell 4}h_{f\ell 4} - w_{fw5}(h_{fw4}-h_{aw2}) - - w_{fc4}h_{fc4}\big)/\tau_{fs4}$$

$$\frac{d}{dt}(p_{fs5}) = \big(w_{is4}h_{is4} + w_{f\ell 5}h_{f\ell 5} - w_{fw5}(h_{fw5}-h_{fw4}) -$$

$$- w_{f\ell 4}h_{f\ell 4}\big)/\tau_{fs5}$$

$$\frac{d}{dt}(p_{fs6}) = \big(w_{is2}h_{is2} + w_{f\ell 6}h_{f\ell 6} - w_{fw5}(h_{fw6}-h_{fw5}) -$$

$$- w_{f\ell 5}h_{f\ell 5}\big)/\tau_{fs6}$$

$$\frac{d}{dt}(p_{fs7}) = \big(w_{hs2}h_{hs2} - w_{fw5}(h_{fw7}-h_{fw6}) -$$

$$- w_{f\ell 6}h_{f\ell 6}\big)/\tau_{fs7}$$

where

$$\tau_{fs4} = (V_{fc4}\rho_{f\ell 4}h_{f\ell 4p} + m_{fm4}c_{fm4}T_{f\ell 4p} + V_{fw4}\rho_{fw4}h_{f\ell 4p})$$

$$\tau_{fs5} = (V_{fc5}\rho_{f\ell 5}h_{f\ell 5p} + m_{fm5}c_{fm5}T_{f\ell 5p} + V_{fw5}\rho_{fw5}h_{f\ell 5p})$$

$$\tau_{fs6} = (V_{fc6}\rho_{f\ell 6}h_{f\ell 6p} + m_{fm6}c_{fm6}T_{f\ell 6p} + V_{fw6}\rho_{fw6}h_{f\ell 6p})$$

$$\tau_{fs7} = (V_{fc7}\rho_{f\ell 7}h_{f\ell 7p} + m_{fm7}c_{fm7}T_{f\ell 7p} + V_{fw7}\rho_{fw7}h_{f\ell 7p})$$

## 23. THE THERMODYNAMIC EQUATIONS

The thermodynamic state of water in liquid, boiling and vapor phase can be characterized by e.g. enthalpy, entropy, pressure, density, or temperature. If two of these variables are known it is usually possible to compute the other variables. Pressure and temperature are possible to measure and they define the state except if the water is boiling. The knowledge of enthalpy or density makes it possible to determine the state completely. In the early stages of the modelling process it was decided to base the thermodynamic state on enthalpy and pressure. From these two variables it is always possible to compute the other variables. It was also decided to develope routines for the computation of density and temperature of water in liquid, boiling, and vapor phase. It was also decided to develope routines for the computation of enthalpy, density, and temperature of saturated steam and water. All routines are based on Schmidt [6].

### Density of Water in Liquid, Boiling, and Vapor Phase.

The density and its derivatives are nonlinear functions of the enthalpy and the pressure:

$$\rho = \mathrm{RHP}(h,p)$$

$$\left(\frac{\partial \rho}{\partial h}\right)_{p=\mathrm{const}} = \mathrm{RHPH}(h,p), \text{ and}$$

$$\left(\frac{\partial \rho}{\partial p}\right)_{h=\mathrm{const}} = \mathrm{RHPP}(h,p)$$

The functions RHP, RHPH, and RHPP are defined by the subroutine RHP, Appendix C1-C6. The subroutine is called by:

```
CALL RHP(H,P,R,RH,RP)
```

where

H = enthalpy, [kJ/kg],
P = pressure, [Pa],
R = density, [kg/m$^3$],
RH = derivative of the density with respect to enthalpy, [(kg/m$^3$)/(kJ/kg)], and
RP = derivative of the density with respect to pressure, [(kg/m$^3$)/Pa].

Temperature of Water in Liquid, Boiling and Vapor Phase

The temperature and its derivatives are nonlinear functions of the enthalpy and the pressure.

$$T = THP(h,p),$$

$$\left(\frac{\partial T}{\partial h}\right)_{p=const} = THPH(h,p), \text{ and}$$

$$\left(\frac{\partial T}{\partial p}\right)_{h=const} = THPP(h,p)$$

The functions THP, THPH, and THPP are defined by the subroutine, THP, Appendix C7-C12. The subroutine is called by:

```
CALL THP(H,P,T,TH,TP)
```

where

H = enthalpy, [kJ/kg]
P = pressure, [Pa],
T = temperature, [$^o$C],
TH = derivative of the temperature with respect to enthalpy, [$^o$C/(kJ/kg)], and

TP = derivative of the temperature with respect to pressure, [$^{o}$C/Pa].

Enthalpy of Condensing Steam

The enthalpy and its derivative are nonlinear functions of the pressure.

h = HSP(p) and

$$\frac{\partial h}{\partial p} = \text{HSPP}(p)$$

The functions HSP and HSPP are defined by the subroutine HSP, Appendix D1-D2. The subroutine is called by:

```
CALL HSP(P,H,HP)
```

where

P = pressure, [Pa],
H = enthalpy, [kJ/kg], and
HP = derivative of enthalpy with respect to pressure, [(kJ/kg)/Pa].

The result of the test program, Appendix D13-D16, is given in Appendix D19. The maximum absolute error is 9.2 kJ/kg and the maximum relative error is 0.4%.

Enthalpy of Boiling Water

The enthalpy and its derivative are nonlinear functions of the pressure

$h = HWP(p)$ and

$$\frac{\partial h}{\partial p} = HWPP(p)$$

The functions HWP and HWPP are defined by the subroutine HWP, Appendix D3-D4. The subroutine is called by:

```
CALL HWP(P,H,HP)
```

where

P = pressure, [Pa],
H = enthalpy, [kJ/kg], and
HP = derivative of the enthalpy with respect to pressure, [(kJ/kg)/Pa].

The results of the test program, Appendix D13-D16, is given in Appendix D2. The maximum absolute error is 8.5 kJ/kg and the maximum relative error is 3.3%.

Density of Condensing Steam

The density and its derivative are nonlinear functions of the pressure

$\rho = RSP(p)$, and

$$\frac{\partial \rho}{\partial p} = RSPP(p)$$

The functions RSP and RSPP are defined by the subroutine RSP, Appendix D5-D6. The subroutine is called by:

```
CALL RSP(P,R,RP)
```

where

P = pressure, [Pa],
R = density, [kg/m$^3$], and
RP = derivative of the density with respect to the pressure, [(kg/m$^3$)/Pa].

The result of the test program, Appendix D13-D16, is given in Appendix D17. The maximum absolute error is 5.7 kg/m$^3$ and the maximum relative error is 2.7%.

Density of Boiling Water

The density and its derivative are nonlinear functions of the pressure

$$\rho = RWP(p), \text{ and}$$

$$\frac{\partial \rho}{\partial p} = RWPP(p)$$

The functions RWP and RWPP are defined by the subroutine RWP, Appendix D7-D8. The subroutine is called by:

```
CALL RWP(P,R,RP)
```

where

P = pressure, [Pa],
R = density, [kg/m$^3$], and
RP = derivative of the density with respect to the pressure, [(kg/m$^3$)/Pa].

The result of the test program, Appendix D13-D16, is given in Appendix D18. The maximum absolute error is 7.6 kg/m$^3$ and the maximum relative error is 1.7%.

Temperature of Condensing Steam and of Boiling Water

The temperature and its derivative are nonlinear functions of the pressure

T = TLP(p), and

$$\frac{\partial T}{\partial p} = TLPP(p)$$

The functions TLP and TLPP are defined by the subroutine TLP, Appendix D9-D10. The subroutine is called by:

```
CALL TLP(P,T,TP)
```

where

P = pressure, [Pa],
T = temperature, [$^{o}$C], and
TP = derivative of the temperature with respect to the pressure, [$^{o}$C/Pa].

The result of the test program, Appendix D13-D16, is given in Appendix D21. The maximum absolute error is 14$^{o}$C and the maximum relative error is 3.3%.

Organization of the Subroutines

All subroutines are based on interpolation in tables of accurate values of the nonlinear functions. The range of the subroutines RHP and THP is $0.01 \cdot 10^5 \leq p \leq 500 \cdot 10^5$ [Pa], and $0.0 \leq h \leq 4000$ [kJ/kg]. The range of the subroutines HSP, HWP, RSP, RWP, and TLP is $0.01 \cdot 10^5 \leq p \leq 221.29\ 10^5$ [Pa]. The value of $\partial h/\partial p$ and $\partial \rho/\partial p$ tends to infinity when p tends to the critical point, $p = 221.29 \cdot 10^5$ Pa. Near the critical point these functions are approximated by the inverse of a quadratic function.

The subroutine SEARCH, Appendix D11, is used to find the interval of the independent variable to interpolate in. The subroutine INT13, Appendix D12, is used to interpolate the function, using a third order polynomial. The interpolation in the subroutines RHP and THP uses a two-dimensional linear approximation function. Special care has been exercised to ascertain continuity near the border to the saturated region.

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