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Frantzich, Håkan

1998

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*Citation for published version (APA):*

Frantzich, H. (1998). *Uncertainty and Risk Analysis in Fire Safety Engineering*. [Doctoral Thesis (monograph), Division of Fire Safety Engineering]. Department of Fire Safety Engineering and Systems Safety, Lund University.

*Total number of authors:*

1

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INSTITUTE OF TECHNOLOGY  
DEPARTMENT OF FIRE SAFETY ENGINEERING  
REPORT LUTVDG/(TVBB-1016)

# Uncertainty and Risk Analysis in Fire Safety Engineering



Håkan Frantzich  
Lund 1998

*Uncertainty and Risk Analysis in Fire Safety Engineering*

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**Uncertainty and Risk Analysis in Fire Safety  
Engineering**

**Håkan Frantzich**

**ISSN 1102-8246**

**ISRN LUTVDG/TVBB--1016--SE**

**Keywords:** Risk analysis, uncertainty analysis, Monte Carlo simulation, FOSM, reliability index, fire engineering design, event tree, response surface.

**Abstract:** Two Quantitative Risk Analysis (QRA) methods are presented which can be used to quantify the risk to occupants in, for example, a building in which a fire has broken out. The extended QRA considers the inherent uncertainty in the variables explicitly. The standard QRA does not consider the uncertainties in the variables and must be complemented by a sensitivity analysis or an uncertainty analysis. Both methods provide risk measures, such as individual risk and FN curves. In the extended QRA these are presented in terms of statistical distributions. The standard QRA is more simple to perform and has been used extensively in many engineering fields. Both QRA methods have been applied to an example, structured with the event tree technique, to determine the risk to patients on a hospital ward.

In addition to the two risk analysis methods, separate uncertainty analysis methods are also presented. Both stochastic uncertainty and knowledge uncertainty are considered in the analysis, separately and combined. The importance of the variables is also investigated.

As both QRA methods are rather complex to use, a more simple method using design values in deterministic equations would be preferable for fire safety design purposes. A method of deriving these design values, based on quantified risk, is presented and complemented with an example which provides design values for a class of buildings. When these design values are known, so-called partial coefficients can be derived.

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Front page photo: Typical societal risks (hospital, shopping centre and transportation of hazardous goods). S-I Granemark and H Frantzich.

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## Nomenclature

$a_i^*$	vector of direction cosines
$A_r$	room area, m <sup>2</sup>
$c_i$	subscenario consequences
$e$	regression error
$F_s$	specific flow rate through a doorway, persons/(m+s)
$H_r$	room height, m
$M$	escape time margin, s
$M_S$	model uncertainty correction factor
$N_i$	number of fatalities
$N_o$	occupant density, persons/m <sup>2</sup>
$NoPat$	number of patients on the ward
$NoStaff$	number of nurses on the ward
$PatInRm$	number of patients in a patient room
$p_{day}$	fraction of fires occurring during the day
$p_{detection}$	operation probability for detection system
$p_{door}$	probability that door to the fire room is open
$p_{ET,i}$	event tree branch probability
$p_{fire}$	fire occurrence rate, fires/year
$p_{f_j}$	probability of failure for single failure mode
$p_{flaming}$	probability of flaming fire
$p_{help}$	fraction that require help during evacuation
$p_i$	subscenario probability
$p_{initial}$	scenario probability
$p_{sleeping}$	probability of sleeping patients
$p_{sprinkler}$	operation probability for sprinkler system
$p_{suppressed}$	probability that the fire will be suppressed by a member of staff or is selfextinguished
$p_{target}$	specified target probability of failure
$p_{u,i}$	subscenario probability of failure
$R^2$	coefficient of determination
$r$	correlation coefficient
$s_i$	subscenario description
$StaffInRm$	number of nurses in patient room

$t_{care}$	time taken to prepare a patient, s
$t_{det}$	time taken to detect the fire, s
$T_g$	temperature in hot smoke layer, °C
$t_{move}$	time required to move to a safe location, s
$t_{move}^{corr}$	movement time from corridor, s
$t_{move}^{room}$	movement time from patient room, s
$t_{patM}$	movement time for a patient, s
$t_{resp}$	response and behaviour time, s
$t_{resp,d}$	response and behaviour time (design value), s
$t_{resp}^{staff}$	member of staff response and behaviour time, s
$t_{staffM}$	movement time for a member of staff, s
$t_u$	time taken to reach untenable conditions, s
$t_u^{corr}$	time taken to reach untenable conditions in corridor, s
$t_u^{room}$	time taken to reach untenable conditions in fire room, s
$W$	door width, m
$x_i^*$	vector of "design point" in original space
$x_i^*$	vector of "design point" in standardised space
$x_{i,ch}$	characteristic value
$x_{i,d}$	design value
$z$	smoke layer height, m
$\alpha_f$	fire growth rate, kW/s <sup>2</sup>
$\alpha_{f,d}$	fire growth rate (design value), kW/s <sup>2</sup>
$\beta$	reliability index
$\beta_C$	reliability index according to Cornell
$\beta_{HL}$	reliability index according to Hasofer and Lind
$\gamma_i$	partial coefficient
$\delta$	regression coefficient
$\varepsilon$	experimental data
$\Theta$	general random variable
$\lambda$	regression coefficient
$\mu$	mean value
$\rho$	correlation coefficient
$\sigma$	standard deviation

## **1 Introduction**

The research work presented in this thesis is a partial fulfilment of the Doctor of Philosophy in Engineering (PhD Eng.) requirement at the department of Fire Safety Engineering at Lund University.

### **1.1 Background**

Traditionally, fire safety design has been highly reliant on prescriptive rules in building codes, NR (1988). This is particularly the situation for occupant safety in the case of fire. Regulations usually state in detail what measures should be taken in order to accomplish a minimum occupant fire safety level.

Detailed or prescriptive building regulations have one major advantage: they are easy to use. The architect merely has to consult the building code to find, for example, what the minimum width of the exits should be or what the maximum allowed walking distance to the nearest emergency exit is. In applying these fire safety regulations to building design, the architect does not have to consider what is actually safe. The safety is already implicitly embodied in the prescribed values.

There are, however, some deficiencies associated with this type of regulations. They are, for example, rather inflexible if not applied to a standard type building. Prescriptive regulations are suited to buildings of a certain type, for which they were initially derived. If the building in question does not fit into any standard type of building, the regulations may force the architect to incorporate too many, or inappropriate, fire safety measures. It could also happen that the safety level may be too low in some buildings. These regulations also vary from country to country. Hagiwara et al. (1994) give a review of how prescriptive regulations vary with respect to different countries.

It can therefore be stated, that despite the relatively easy implementation of prescriptive regulations, they are inflexible and

may lead to unnecessarily expensive buildings. As a consequence of these disadvantages, so-called performance-based building regulations have been developed in several countries during the last two decades, BBR94 (1994). Performance-based regulations define the objective for a certain regulation, but do not say how the objective should be accomplished.

It is a widespread misunderstanding that calculational procedures must be used to design fire safety measures that fulfil performance-based regulations. This is not true, as the method used to satisfy the regulation actually has nothing to do with the actual requirement. Performance-based regulations do not recommend any particular design method above another. Calculation methods have, however, become more frequent in the verification of the requirements stipulated. Engineering methods have become important with the development of tools to assist in the design procedure.

It is now relatively simple to achieve a fire safety design, since computer tools such as CFAST (Peacock et al., 1994) for smoke transport calculations, have become available. The architect just specifies a likely fire growth process and adds the relevant geometrical parameters, i.e. the design values, to obtain an answer. The program can calculate the time available for escape, which can then be compared with the assumed (calculated) evacuation time. When the available time exceeds the escape time, the safety of the occupants is ensured. This is the performance requirement: safe evacuation of the occupants. The problem is now not the rigid regulations but the rather easy process of obtaining a solution for fire safety, whether it be good or bad.

The relevant fire safety design values shall, according to published handbooks (for example, FEG, 1996) be chosen so as to cover the credible worst case scenario. But, it is still the architect who makes the choice from a range of credible values available. There are no commonly accepted design values of for example the response and

behaviour time for people escaping from a fire. The design values can also be decided upon in groups consisting of representatives from the authorities, the architect, the contractor and other specialists. Whether or not this procedure is adopted probably depends on the initiative taker, the building owner.

This procedure, using not commonly agreed design values, has become a problem, at least in Sweden. In addition, building procedures have been deregulated and the official local authority representative has been more or less removed from the design process. The responsibility now lies solely on the owner to ensure that the building is designed according to the fire safety requirements. The local building authority shall only determine whether or not the contractor can follow his own quality plan, and is able to ensure that fire safety is according to what is prescribed. The problem lies in that no acceptable design values have been derived, and there is no standard practice in the area of fire safety engineering. This is mainly due to the fast growth of knowledge in fire safety science and the inherent lack of engineering tradition. This is a phase that fire safety engineering has to go through in order to make fire safety engineering an accepted engineering science.

As the fire safety engineering tradition is less developed than some other engineering disciplines, there are no methods available which will reveal whether certain design strategies are sufficiently safe. In a situation in which the overall safety depends on, for example, just one fire hazard reducing system and no sensitivity analysis is performed, there will be an uncertainty as to whether the regulations are fulfilled or not. Trading, for example, multiple escape routes for an automatic fire alarm without examining the consequences in detail is evidence of this lack of engineering tradition.

There is a current tendency to use many technical systems without thorough analysis of the consequences. It should, however, be

stated that this was also common when prescriptive regulations were in force. It is therefore not legitimate to take the current fire safety engineering practice as an argument for discarding the performance-based regulations. Such regulations are vital for the rational development of building tradition, but this development must be guided in order to be efficient. In addition to the guidelines, it must be possible to quantify the required objectives in the regulation.

Safety can be ensured either by comparing the proposed design with accepted solutions, or with tolerable levels of risk, or by using design values in the calculations which are based on a specified level of risk. The first method, using accepted solutions, is more or less equivalent to the prescriptive regulation method. It has normally very little to do with optimising a solution for a specified risk level.

The other two methods are based on specified levels of risk. In the design process, the proposed design can be evaluated by applying risk analysis methods. This can also be done after the building has been completed, to check its fire safety. To be able to make full use of the advantages of performance-based regulations, the design should be based on risk analysis methods. This thesis describes methods that can be used in the design process. One such method is called the standard Quantitative Risk Analysis (QRA) method.

As many variables are associated with uncertainty, the risk analysis should be complemented by an uncertainty analysis. Applying uncertainty analysis to the standard QRA method will lead to the extended QRA method. Both methods can be used in the risk management process described later. The standard QRA method has been applied to fire safety problems but on a very limited scale only. The extended QRA method has, however, never been applied to fire safety problems. The standard QRA has, however, extensively been applied to other engineering fields such as for hazard analysis in chemical process plants.

As both QRA methods can be rather complex to use, a more simple method using design values in deterministic equations would be preferable for fire safety design purposes. Design values for input variables should be derived for the most common types of buildings and building use.

These design values, based on quantified risk, should not be confused with values estimated by architects based on experience. The latter values are the ones used today, as design values based on risk do not yet exist in the area of fire safety engineering. In other fields of engineering, e.g. in structural engineering, design values based on risk have been developed and are now in use (Thoft-Christensen et al., 1982). A procedure for deriving design values based on risk is presented in this thesis (Chapter 9) and the method is illustrated with an example. The method assumes that the risk is explicitly defined.

### **1.2 Purpose of the study**

The main purpose of this work is to present quantitative methods which can be used to determine risk levels to which occupants are exposed in buildings in the case of fire. The risk levels form the basis for making decisions regarding design strategy and measures to reduce the risk. Two Quantitative Risk Analysis (QRA) methods are presented. The extended QRA method explicitly considers the inherent uncertainty as it is part of the procedure. The standard QRA method does not take variable uncertainty into account. The standard QRA method should be complemented by a sensitivity or uncertainty analysis.

Another aim of this work is to introduce a method through which so-called design values can be obtained. This method is linked to an uncertainty analysis method, the analytical First Order Second Moment (FOSM) method and is used to derive design values assuming a specified risk level.

### **1.3 Risk management**

Risk management can be seen as the complete methodology in which the qualitative and quantitative analysis methods are contained. Risk management covers the complete analysis:

- definition of the goals,
- identification of the hazards,
- determination of the measures of risk
- evaluation of the tolerability of the risk
- definition of risk reduction measures
- implementation of the result into practice including monitoring.

Figure 1.1 shows the different processes in risk management procedure. This definition of the risk management process has been adopted from the International Electrotechnical Commission (IEC, 1995) and is that used in this thesis. There are other relationships between the definitions of risk analysis, risk assessment and risk management, e.g. Covello et al. (1993), where risk assessment is part of the risk analysis.

As can be seen from the figure, risk analysis is only one part of the risk management process. Risk analysis can be further separated in at least three levels, depending on how detailed the analysis is to be and the labour resources available

- qualitative methods
- semi-quantitative methods
- quantitative methods.

During risk analysis, all three levels can be used in sequence. The first methods are used to determine which scenarios are relevant to continue with in the quantitative risk analysis.



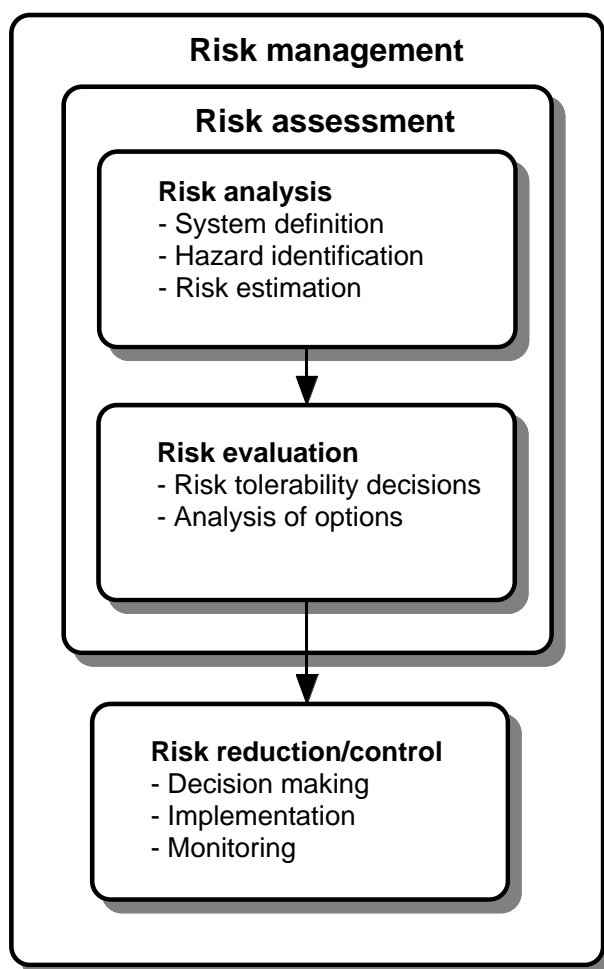


Figure 1.1. The activities in the risk management process.

### 1.3.1 Qualitative methods

Qualitative methods are used to identify the most hazardous events. The events are not ranked according to degree of hazard. For the chemical process industry, methods have been developed such as HazOp, What-if and different check-lists (CPQRA, 1989). Qualitative methods may be used as screening methods in the preliminary risk analysis.

### 1.3.2 Semi-quantitative methods

Semi-quantitative methods are used to determine the relative hazards associated with unwanted events. The methods are normally called index methods, point scheme methods, numerical grading, etc., where the hazards are ranked according to a scoring system. Both frequency and consequences can be considered, and different design strategies can be compared by comparing the resulting scores.

Various point scheme methods have been developed for fire safety analysis, for example, the Gretener system (BVD, 1980), and the NFPA 101M Fire Safety Evaluation System (Nelson et al., 1980 and NFPA 101M, 1987). The Gretener system has been developed by an insurance company and is mainly intended for property protection. It has, however, been widely used and is rather extensive in terms of describing the risk.

The major drawback of point scheme methods is that they contain old data. New technologies are included rather slowly. Influences on the methods from the authors background and from, for example, building traditions, are also unavoidable. The NFPA method favours North American building traditions and may not be applicable in Europe. On the other hand the simplicity of the methods is an advantage. Usually, only basic skills are required. A review of different risk ranking methods for fire safety is presented by Watts (1995).

Another semi-quantitative method which is used in this area focuses on risk classification (SRV, 1989). The hazards are judged in terms of the frequency and expected consequences. The frequency and consequences are selected from a list consisting of five levels. By combining the frequency class and consequence class, a measure of risk is obtained. This measure can be used to compare hazards. This analysis is usually performed on the societal level and is not applicable to a single building or industry.

Other industry-specific index methods are available, for example, for the chemical process industry (CPQRA, 1989). Typical index methods are the Equivalent Social Cost Index and Fatal Accident Rate.

The Equivalent Social Cost Index (ESCI) is an alternative expression of the average societal risk. The difference compared with the usual form of average societal risk is that a risk aversion factor,  $p$ , is included. Usually  $p$  is chosen to a value higher than 1.0 to consider the unwillingness for a large number of fatalities as the relationship then becomes non-linear. The Equivalent Social Cost Index can be expressed as

$$ESCI = \sum_{i=1}^n p_i N_i^p \quad [1.1]$$

Suitable values for the risk aversion factor  $p$  have been suggested to be between 1.2 and 2 (Covello et al., 1993).  $N_i$  is the number of fatalities per year in subscenario  $i$ . The term subscenario is defined in Chapter 3. The ESCI is a pure index for comparison of engineering measures. The relation to monetary units is not meaningful as the factor  $p$  is more or less based on judgement without any connection to tolerable risk levels.

The Fatal Accident Rate (FAR) is used in worker accident assessment. FAR expresses the number of deaths per  $10^8$  hours (approximately 1000 worker lifetimes). It is a measure that combines risk contributions from many sources. It is closely linked to an average individual risk measure used in the chemical process industry cf. Section 6.3.1.

### 1.3.3 Quantitative methods

The final level of analysis is the most extensive in terms of quantifying the risk. It is also the most labour intense. On this level, a distinction can be made between a deterministic analysis and a probabilistic analysis. The deterministic analysis focuses on

describing the hazards in terms of the consequences. No consideration is taken of the frequency of the occurrence. A typical example is the determination of the worst case scenario expressed as a risk distance. The deterministic approach has been used in estimating design equivalency for evacuation safety by Shields et al. (1992).

The probabilistic approach determines the quantified risk based on both frequency and consequences. The Quantitative Risk Analysis method uses information regarding the questions:

- what can go wrong?
- how often will it happen?
- what are the consequences if it happens?

This approach has been used in fire spread calculations in buildings and on ships (Fitzgerald, 1985). One of the more extensive fire risk programmes was developed in the USA during the 1990s (Bukowski et al., 1990). The methodology is used to derive the expected number of fatalities per year in buildings. The main objective was to study the influence on the risk of different types of building construction materials.

A quantitative probabilistic method has also been used to evaluate risk in health care facilities in the UK (Charters, 1996). This analysis uses an event tree approach similar to the one presented in this thesis. The consequences estimated are rather crude. It is, however, one of the first attempts to quantify the risk to patients and staff in a hospital.

The probabilistic approach has also been adopted in the proposed international standard for fire safety engineering as a procedure for identifying fire scenarios for design purposes (ISO/CD 13388).

For situations in which the risk management process is used at the design stage, the Australian Fire Engineering Guide (FEG, 1996),

proposes a rational structure of quantitative methods. Different levels are to be used depending on the relative benefits which are possible to obtain.

Three levels of quantitative analysis are identified:

- component and subsystem equivalence evaluation
- system performance evaluation
- system risk evaluation

The first level is basically used for comparative studies to evaluate equivalency between different design alternatives on the component level. Different alarm systems can be compared and evaluated in terms of equivalency with respect to a prescribed standard.

The second level considers the relation between two or more subsystems. The difference between the design alternatives is higher than in the first level. Evaluation aspects may include fire growth, smoke development and occupant evacuation.

The last level can be seen as a standard QRA where the whole building design is considered and measures of risk are derived.

#### **1.4 Uncertainty**

In many engineering situations, most variables used in the analysis will be associated with uncertainty. In performing a Quantitative Risk Analysis it is important to identify how these uncertainties affect the result. Therefore, an uncertainty analysis should complement the risk analysis. This is, however, seldom the case. It is believed, that the biggest benefit of uncertainty analysis would be to illuminate the fact that uncertainties exist.

#### **1.5 Overview of this thesis**

This thesis focuses on risk analysis and uncertainty analysis, employed separately or together.

The second chapter provides a brief introduction to different failure sources, i.e. what can go wrong and how? A major distinction is made between failure due to gross errors and failure due to random variability. Variation in variable outcome, due to random variability, can result either from stochastic uncertainty or from knowledge uncertainty.

Chapter 3 introduces the event tree approach on which the risk analysis is based. This chapter presents the idea of Kaplan-Garrick triplets and describes how the probability of an individual subscenario is derived.

Another vital part of the risk analysis concerns the description of undesired consequences. Chapter 4 describes how the consequences, in terms of occupants not being able to escape safely, are derived. The response surface technique is introduced.

The variables used in the analysis can be described as deterministic values or as probability distributions. The latter form is used when uncertainty is explicitly considered. Chapter 5 discusses the two methods by which the variables are assigned values.

The different risk analysis methods are presented in Chapter 6. The chapter is divided into three main sections, Standard QRA, Uncertainty Analysis and Extended QRA. The uncertainty analysis section is both an introduction to the extended QRA and a separate part. The uncertainty analysis can be performed separately from the risk analysis.

In Chapter 7 the risk analysis methods are applied to the case of a hospital ward. The purpose is to illustrate the methodologies and to discuss some practical problems regarding the analysis. The risk analysis will result in information about both the individual risk and in information on the risk to society.

Chapter 8 introduces the term tolerable risk. Any risk assessment must compare the risk analysis result with tolerable risk levels accepted by society. The chapter gives a brief introduction to this topic.

When an architect designs a building, consideration must be taken to ensure occupant safety in the case of fire. It is possible to perform a risk analysis to compare the actual risk with those in similar, acceptable building designs. This is, however, too complex a task to be performed for every single new building design. Therefore, for relatively simple buildings, so-called design values should be defined. Using these in the design process will lead to a safety level which satisfies the safety specification. In Chapter 9, a procedure is presented which can be used to derive the design values for some types of buildings.

This thesis has two appendices. The first presents the Matlab m-files which are used to sort the risk analysis data. These are described in detail and can be used directly in risk analysis using the computer software Matlab. The second appendix presents the general assumptions for the example risk analysis presented in Chapter 7.

### **1.6 Limitations**

This work is subjected to limitations and should be seen as a step towards introducing risk analysis methods into fire safety engineering. Definitions used in other engineering fields have often been used but this has not always been possible. This is due to the discrepancies between standardised definitions within the risk analysis society and due to traditions in the fire safety community.

The purpose of the methodology presented here is to provide methods by which occupant safety in the case of fire is ensured. No consideration has been taken of loss of property, damage to the environment or loss of vital societal systems. The latter can be

exemplified by the reluctance to lose a whole hospital or a school in a fire. The occupants will perhaps be rescued and the buildings may be insured, but the loss of the function may not be tolerable. The methods which are presented are, however, general and additional loss criteria can be included in the analysis.

The question of human error has been treated on a rather limited scale. This is a matter that must be dealt with, but the form for such work is different from the quantitative methods presented in this thesis. The frequency of human error is normally reduced by introducing routines and check-lists. This type of error can still be incorporated into the QRA methodology, in a formal manner (see Chapters 2 and 3).



## **2 Sources of failure**

### **2.1 Error classification**

When a risk analysis is to be performed, one must ask the questions "what can go wrong?", "how likely is it?" and "what are the consequences?". This is one of the most fundamental steps in the process and results in a list of possible outcomes, some of which result in people not being able to evacuate safely, i.e. the system fails.

Looking at the list of failures, it is possible to distinguish a pattern of similarities among the sources of failure. At least two types of failure can be identified,

- failure due to gross error and
- failure due to random variability.

When examining the evacuation from a building, which has taken place, it is probably rather easy to identify the reason why the occupants were not able to escape safely. But when performing a risk analysis for a future event, or executing an engineering design, sources of failure in the first category are very difficult to identify. This is because of the nature of gross errors. They originate from errors during the design process or from the risk analysis procedures. The division between the two types of failure is made because the methods with which the two types of failure are handled are different.

There are many other ways to categorise different sources failure, many of which are specific to a specific area of engineering (Blockley, 1980). The categorisation of failures into those caused by gross error and those caused by random variability is only one example, but a rational one. Types of failure can be distinguished by for example the nature of the error, the type of failure associated with the error, the consequences of the failure arising from the

error, those responsible for causing or for not detecting the error, etc.

## **2.2 Gross errors**

Gross error can be defined as fundamental errors which, in some aspect of the process of planning, design, analysis, construction, use or maintenance of the premises, have the potential to cause failure (Thoft-Christensen et al., 1982).

A risk analysis or a design can be performed on condition that the models and basic background information are correct and that procedures for design, analysis, maintenance, etc. are carried out according to relevant state-of-the-art standards. If this is not the case, changes must be made. Either other standards or control measures must be used or the conceptual model must be changed. A typical example of a gross error in fire engineering is neglecting to maintain vital functions such as the emergency lighting system or alarm system. When maintenance is neglected, the reliability of such systems can deviate from that which is specified.

Another example of gross errors is when changes are made on the construction site which are not examined or approved in the design phase of a project. Changing to different products which might seem harmless to the builder can lead to significant safety problems when the specific protection product is needed in a real hazardous situation.

### **2.2.1 Human error**

Many gross errors originate from human errors. Underlying causes may be, for example, lack of experience, education or formal qualification. But such errors can also occur due to incompetence and negligence.

During evacuation, many actions are taken by people which afterwards, may seem irrational or inefficient. The behaviour of people under the psychological stress can result in actions which

may not be the most rational. Actions such as investigating the unknown fire cue, alerting others and helping others are common. Even actions such as ignoring the threat have also been observed in fire investigations. Some of these actions can be considered irrational and will not bring the person into a safer position. These may be called human errors.

However, this type of human error should not be considered gross errors as it is part of the random uncertainty in people's reaction and behaviour. Reaction and behaviour, is one of the variables in the state function describing the evacuation process, see Chapter 4. It must, however, be noted that all human actions, described by the response and behaviour variable, will sooner or later lead to the decision to evacuate. This will also be the case for individuals ignoring the threat, but they may realise this too late to be able to escape safely. The choice of alternative design solutions may be able to help also such people.

An overview of the area of human error has been presented by Reason (1990), who also presents some rational measures to minimise the influence of gross error.

### ***2.3 Random variability***

The other type of failure is caused by the inevitable randomness in nature. This randomness results in a variability of the variables describing the system which might cause an error. Variables describing the system are not always known to a degree making it possible to assign the variable to a constant. Uncertainty is always present in the variables and this is one of the reasons why risk analysis is performed.

Failure occurs when the variable values are unfavourable for the system. If, for example, the fire growth in a room is extremely rapid and at the same time the occupant load is also very high, this may lead to the result that not all the people in the room can escape. The fire might result in a positive outcome, i.e. no

fatalities, if the occupant load was not that high, but the combined effect of the rapid growth and the high number of occupants, results in the accident.

The event can be seen as a random combination due to unfortunate circumstances. These failures are acceptable as long as their probabilities are independent and below that which is tolerable. The important matter is that uncertainties in the variables describing the system can, for some combinations, cause the system to fail. The uncertainty due to random variability can be further divided into the subclasses stochastic variability and knowledge uncertainty. The difference between these is described in the following Section, 2.3.1.

Variables which are subject to uncertainty are usually described by probability distributions, see Chapter 5, and randomness can assign a value to the variable which might be very high or very low, i.e. an unfavourable value. These values can occur due to circumstances which are unlikely to happen, but still which are possible. A very high fire growth rate can occur in a building, even if it might be unlikely. By using probability distributions, very unlikely events can also be considered.

### 2.3.1 Uncertainty caused by randomness

There are at least two types of uncertainty which must be distinguished as they originate from different conditions. Stochastic uncertainty or variability is the inevitable variation inherent in a process which is caused by the randomness in nature. This type of uncertainty can be reduced by exhaustive studies and by stratifying the variable into more nearly homogeneous subpopulations.

Knowledge uncertainty represents the variation due to a lack of knowledge of the process. This type of uncertainty can be reduced by further analysis of the problem and experiments, but it still originates from randomness.

Both types of uncertainty are described by the same measure, i.e. the probability distribution of the variable. But, they are otherwise fundamentally different as they describe different phenomena.

Normally, in uncertainty analysis, stochastic and knowledge uncertainties are treated without distinction, both contributing to the overall uncertainty. There are, however, situations where there is an interest in separating stochastic uncertainty from knowledge uncertainty. By doing this, it is possible to see the influence of the two types on the overall uncertainty, to determine which area requires further knowledge.

In model validation, it is also practicable to separate variability from knowledge uncertainty. The latter is then in the form of model uncertainty. One of the first attempts at using the approach of stochastic and knowledge uncertainties in an assessment in the area of fire safety engineering was presented by Magnusson et al. (1995) and Magnusson et al. (1997). The analysis was performed on calculations of evacuation reliability from an assembly room.

Stochastic uncertainty and knowledge uncertainty have also been referred to as, Type A uncertainty associated with "stochastic variability with respect to the reference unit of the assessment question", and Type B uncertainty "due to lack of knowledge about items that are invariant with respect to the reference unit in the assessment question" (IAEA, 1989). Examples of parameters that are coupled to the two types of uncertainty are given below.

- Variability, Type A: wind direction, temperature, fire growth rate in a particular class of buildings and occupant response times
- Knowledge uncertainty, Type B: model uncertainty, plume flow coefficient, acceptable heat dose to people and most reliability data for systems.

It should be mentioned that several variables may be affected by both kinds of uncertainty, and there is usually no clear separation between the two.

#### **2.4 Handling gross errors**

One cannot treat gross errors in the same way as random errors, regarding them as extreme values of a probability distribution. Gross errors alter the probability of failure by changing the complete model describing the system. Gross errors are reduced by measures such as training, internal or external control, proper organisation, maintenance of equipment, attitude of the staff, etc.

As a consequence of this, gross errors have not been considered by choosing probability distributions with tails which are infinite. Gross errors are normally considered in qualitative review processes. The rest of this thesis will be devoted to risk analysis with and without the influence of uncertainties. It is, of course, clear that the complete risk management process, must also consider potential gross errors.

#### **2.5 Systematic errors**

Systematic errors can belong to both categories of failure, gross error or error due to random variability, depending on whether the systematic error is known in advance or not. A systematic error is defined as the difference between the true value and the measured or predicted value. A systematic error can arise from biases in, for example, model prediction or expert judgements.

A known systematic error, such as a model uncertainty, can be treated as an uncertain variable or a constant correction. Unknown systematic errors, on the other hand, are more difficult to foresee, and must be considered as potential gross errors. Efforts must be made to minimise the influence of systematic errors. In some cases, they can be reduced by performing more experiments or by using different evaluation methods for expert judgement predictions. Using models outside the area for which they are

validated will contribute to the unknown systematic error. This must therefore be avoided. It is, however, usually not possible to reduce all systematic errors and some will remain and be unknown.

### **2.6 Uncertainty in subscenarios**

Another possible division of the uncertainty variables can be made to distinguish between uncertainty in the subscenario probability and uncertainty in the consequence description for each subscenario. This division is most relevant when the risk analysis covers the whole system (building), i.e. when performing a QRA.

The probabilities of the subscenarios are usually also random variables and subject to uncertainty. The reliability of, for example, a sprinkler system and an automatic fire alarm system will, to some extent, be a random variable and the outcome probability of the subscenarios will therefore, also be subject to uncertainty.

The uncertainty for each subscenario can be treated as a stochastic uncertainty, but this does not mean that the uncertainty in the consequence description will be a knowledge uncertainty. Both types of uncertainty are included in the description of the consequences. An extended QRA can, therefore, usually not distinguish between stochastic and knowledge uncertainties, see the discussion in Chapter 6.

## **3 Logical systems**

### **3.1 Event tree**

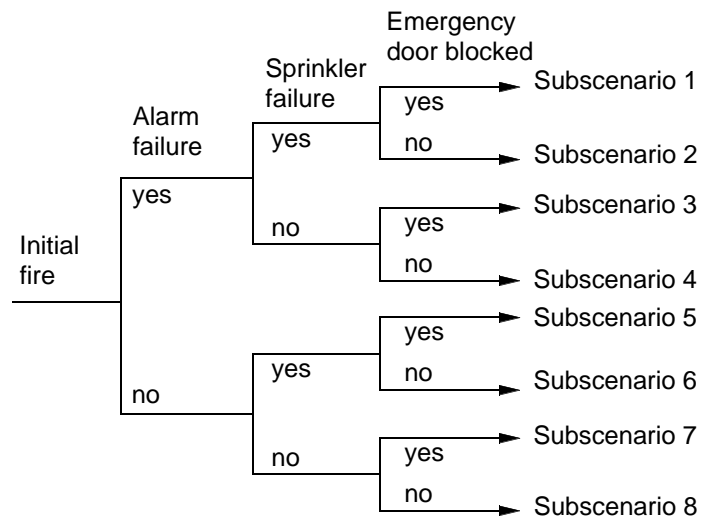
In the case of an accident, the final outcome is not known in advance. Different outcomes can occur depending on the initial conditions of the event. The circumstances of the scenario at the time of the accident will decide the final outcome. In the risk analysis procedure it is often necessary to examine a large number of scenarios with different chains of events. Each final event, outcome or subscenario can be assigned a probability of occurrence as a consequence of the uncertainty in which event will actually occur. In order to structure the possible event sequences arising from an initial event, the event tree approach may be used. An event tree provides a logic graphical description of the possible final events and is therefore a rational method for quantitative risk analysis.

The final events or outcomes in the event tree are denoted subscenarios. The scenario is an aggregation of all subscenarios. Different terminology exists regarding the extent of the scenario. In some literature the scenarios can be defined as the outcomes in the event tree. This is not the case in this thesis. An example of an event tree is presented in Figure 3.1.

The event tree describing evacuation in the case of a fire starts with an initiating event, the initial fire. Different installations or circumstances which will have an affect on the outcome can be treated as branch events.

At each branch point, different alternatives may occur. For example, an installation such as an automatic fire alarm system will either operate or fail. The alternatives at the branch point affects the following parts of the tree. Each event tree outcome is evidence of the chain of events leading to the final event.





*Figure 3.1. An event tree for a simple fire risk analysis.*

The event tree structures the scenario so that the relevant questions for the analysis can be identified:

- what can happen?
- what is the probability of each subscenario?
- what are the consequences of each subscenario?

Each final outcome, or subscenario, in the event tree has its own set of answers, called the Kaplan and Garrick triplet (Kaplan et al., 1981). A triplet is composed of the three variables,  $(s_i, p_i, c_i)$ , where  $i = 1$  to  $n$  with  $n$  equal to the number of subscenarios, i.e. the number of branches in the event tree.

The term  $s_i$  is the event description and  $p_i$  and  $c_i$  describe the probability and consequence of the subscenario. The term  $c_i$  can, in some applications, be a vector containing information on different consequences, for example, consequences for the environment, humans or economic loss. Different decision criteria cannot be mixed in one and the same assessment, cf. Chapter 6.

The consequences can be in the form of number of injuries, fatalities or people having their escape routes blocked. The formal derivation of the consequence measure will be further addressed in Chapter 4.

The total risk is the set of all triplets  $R = \{(s_i, p_i, c_i)\}$  for the scenario. In this definition of risk, all information regarding the calculated risk is included. Each subscenario is defined by its probability and its consequence. The set of triplets can be stored as three vectors, one for each component in the triplet.

At each branching point, the possible outcome probabilities for a two-way branch can be described as  $p_{failure}$  and  $p_{success} = 1.0 - p_{failure}$ . The probability of the final subscenario,  $p_i$ , for each branch, is simply the product of the branch probabilities leading to that subscenario. The probability of the initial event,  $p_{initial}$ , should also be included in  $p_i$ .

It is sometimes convenient to separate the probability of the initiating event and the probabilities of the events described by the event tree. The probability of each subscenario without, consideration of the initial event probability, can be denoted  $p_{ET,i}$ . The total subscenario probability can then be written

$$p_i = p_{initial} \cdot p_{ET,i} \quad [3.1]$$

The probability  $p_{initial}$  can be omitted when comparative studies are performed, when this probability is the same for all cases being investigated. The only differences then originate from different scenario descriptions, i.e. different event trees. The probability  $p_i$  in the triplet is replaced by  $p_{ET,i}$  for comparative studies. The sum of the  $p_{ET,i}$  can be written

$$\sum_n p_{ET,i} = 1.0 \quad [3.2]$$

As a consequence of this, the sum of the  $p_i$  can be written

$$\sum_n p_i = P_{initial} \quad [3.3]$$

Further refinements of the quantity  $p_i$  can be made to include, for example, variable uncertainty. This will be further described in Chapter 6.

The idea of triplets can also be used for situations where variables are subject to uncertainty. Inclusion of variable uncertainty makes it possible to answer the question "How certain is the calculated risk?"

Usually, both the outcome probability of the subscenario,  $p_i$ , and the description of the consequences,  $c_i$ , are subject to some uncertainty. Information concerning the state of knowledge of the variables must be included in both  $p_i$  and  $c_i$ . The set of triplets can then be written  $R = \{(s_i, p_i(\phi_i), \zeta_i(c_i))\}$  using the notation of Kaplan and Garrick. The state of knowledge in the probability of each subscenario is expressed by assuming that it follows a probability density function,  $p_i(\phi_i)$ , instead of being a single value. In the same way, the consequences can be subject to uncertainty which is expressed by the function  $\zeta_i(c_i)$ .

### **3.2 Fault tree**

The branch probabilities for the event tree can be determined in a number of ways e.g., based on historical records or by estimation, see Chapter 5. For complex technical systems, reliability data can be on a very low component level. To obtain the reliability or probability of failure for the whole system, fault tree analysis can be a helpful tool.

The fault tree technique is a method which can be used to estimate the probability of failure for a system, such as a sprinkler system, by tracing the events that result in the top event, the failure. The

top event can be reached in a number of ways, each starting with more basic events, safety systems and human reliability. A known reliability on a lower level can be used to determine the reliability of a higher level system, for example, the complete sprinkler system.

The basic events are combined by logical AND and OR gates, which finally lead to the unwanted event. At an AND gate both conditions must be fulfilled to trigger the next step but for an OR gate, one of the conditions is sufficient. Figure 3.2 shows a simple fault tree for an emergency lighting device where the top event is failure of the lamps to light. The fault tree shall only be seen as an illustration of the technique and is not complete.

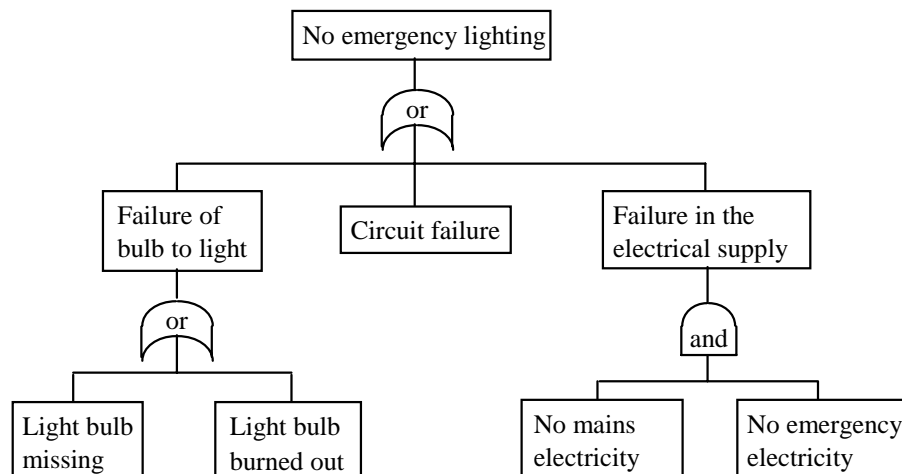


Figure 3.2. Example of a fault tree.

The fault tree technique is used quite frequently in the process industry to obtain the failure probability of complex systems for which the reliability is unknown. Regarding fire safety in buildings, this technique can also be used to obtain accurate

information for systems important to the people safety. In this thesis the failure probabilities are obtained based on historical records combined with estimations.

### **3.3 Problems with logical trees**

Apart from not being able to sufficiently estimate probabilities in the event or fault trees and having improper knowledge of the system, at least three other factors, able to create problems, are of importance, CPQRA (1989):

- common cause failures (CCF),
- human errors and
- external events.

A common cause failure is characterised by an error that will affect several points in the event tree. A typical example is a fault in the electrical supply. This will affect alarm systems, lighting conditions, etc., which are assumed to be operational in many of the subscenarios resulting from the event tree.

This example of a failure is introduced as a basic event in a number of fault trees used in determining the branch probabilities. The method of dealing with CCFs is to identify them, quantify them and finally formulate a defence against them to minimise the effects of the failure. One way to eliminate a common cause failure is to assume a correlation between variables.

Human-induced errors are an important factor in risk assessment. The Human Risk Assessment (HRA) (Reason, 1990) is basically designed to identify possible human errors, and to quantify them in order to incorporate them into the event tree analysis, see also Chapter 2. Knowing about the existence of possible human errors can lead to routines to minimise their effects. However, they should still be included as a possible source of failure in the event tree.

Typical examples of human error in the area of personal safety are misinterpretation of fire cues, unfortunate choices during evacuation and neglecting information transfer during maintenance of vital equipment. It should be emphasised that in overall risk management, human reliability must be addressed as the majority of errors that occur can be traced back to a erroneous human action (Murphy et al., 1996). HRA is, however, a completely separate task and will not be further addressed here.

External events can be classified into two classes (CPQRA, 1989):

- natural hazards: earthquakes, lightning, etc.
- man-induced events: aircraft crashes, sabotage, etc.

These events can be incorporated into the risk assessment if they are judged to be of importance. A separate analysis can also be performed to determine the consequences of these events. Further information concerning external events can be found in the PRA Procedures Guide, NUREG (1983).

## **4 The unwanted consequences**

### **4.1 Model for consequence calculation**

#### **4.1.1 The limit state function**

In the QRA of a system, the consequence in each subscenario must be quantified. The consequence is expressed, for example, in terms of the number of injured people or the amount of toxic gas released to the atmosphere.

The consequence can be formulated in terms of a performance function or a state function for each subscenario in the event tree. The state function describes one way or mode, in which the system can fail. The problem can generally be expressed as a matter of supply versus demand. The state function is the formal expression of the relationship between these two parameters. The simplest expression of a state function is basically the difference

$$G = X - Y \quad [4.1]$$

where  $X$  is the supply capacity and  $Y$  the demand requirement. The purpose of any reliability study or design is to ensure the condition  $X > Y$  through out the lifetime of the system, to a specified level indicated by  $P(X \leq Y) \leq p_{target}$ .

Failure is defined as when the state function  $G$  is less than or equal to zero. When the transition occurs, i.e. when  $G = 0$ , the state function is denoted the limit state function in order to emphasize that it defines the distinction between failure and success. The limit state function is used in risk analysis and design to determine the maximum consequences of a failure. In this thesis, it is understood that when the values of the consequences are derived it is done using the limit state function, i.e. for the condition that supply capacity equals the demand requirement.

In the evacuation scenario, the state function is composed of two time expressions, time available for evacuation and the time taken for evacuation. The variable  $G$  can be seen as the escape time margin. If the escape time margin is positive, all the people in the room will be able to leave before untenable conditions occur. On the other hand, if the margin is negative for a subscenario, some of the people cannot leave without being exposed to the fire hazard. The number of people subjected to this condition will depend on the magnitude of the time margin, the distance to the escape route, the initial occupant density, the occupant characteristics, etc. The components in the state function can be functions of other variables. There is no restriction on the number of functions or variables in the state function.

In the analysis in this thesis, the state function has the following general appearance

$$G = t_u - t_{det} - t_{resp} - t_{move} \quad [4.2]$$

where

$t_u$  = time taken to reach untenable conditions, i.e. the available escape time

$t_{det}$  = time taken to detect the fire

$t_{resp}$  = response and behaviour time of the occupants

$t_{move}$  = time required to move to a safe location.

The four time variables are, in turn, functions of other basic variables and constants. A basic variable is one which is subject to uncertainty. Variables compensating for model error can also be included in these functions. Additional variables can be introduced for specific subscenarios to better describe the actual situation.

It is possible to express the risk in terms of lack of escape time instead of number of people. It is, however, customary to express the risk by the number of people not being able to escape safely. In



the risk analysis, the escape time margin is reformulated in terms of the number of people not being able to evacuate within the available time, i.e. expressed by the limit state function. This is not necessarily equivalent to the number of fatalities. The available time is determined by the level set for untenable conditions.

#### 4.1.2 Untenable conditions

For evacuation analysis, the occurrence of the untenable conditions determines the available safe escape time. In most engineering risk analyses, the desired consequence should be expressed in terms of the number of fatalities, i.e. the number of people dying from the exposure. For evacuation analysis, this can be obtained by setting lethal exposure levels to what is considered untenable. Levels other than lethal, can be chosen.

In this thesis, two different definitions of untenable conditions were used. In the design process in fire safety engineering, untenable conditions are normally defined as escape routes becoming filled with smoke to a certain height above floor level. This criterion is often used in combination with other criteria such as the smoke temperature and toxic gas concentration.

The levels set do not imply that people become fatal victims of the fire, but they will have some difficulties in escaping through smoke and toxic gases created by the fire. These untenable conditions are usually assumed to define the time when the escape route is no longer available as a safe passage. The levels of exposure are chosen on the safe side to allow most occupants to be able to withstand them for a period of time. The risk measure using this definition of untenable conditions cannot be comparable to other risk measures in society, but can be used for comparative studies between different design solutions or buildings. Later in this thesis this level will be denoted the *critical* level of untenable conditions.

The other level of untenable conditions assumes that people will probably become fatal victims of the fire due to high temperature and smoke exposure. The exposure level is higher than for the critical level of untenable conditions. Using this definition, the risk analysis can be compared with similar analysis from other engineering fields. This level is denoted the *lethal* level of untenable conditions.

The problem with this definition lies in determining the lethal conditions. People are not equally sensitive to fire conditions and factors such as age, sex, physical and psychological health status play important roles. Limits on what can be regarded as lethal conditions must be determined, deterministically or be described as probability distributions. The latter will, however, result in an enormous work load if traditional engineering methods of predicting the consequences are used. In a purely statistical approach, this method of determining the tolerable human exposure could be used.

Both definitions of untenable conditions are based on what humans can stand in terms of heat and smoke exposure. The critical level can be related to the acute exposure to high temperature in combination with irritating smoke. But prolonged exposure can also be harmful, even at a lower level of exposure.

The cumulative exposure dose can cause the occurrence of what is considered untenable levels. Work by Purser (1995) has resulted in an extensive knowledge base in terms of upper limit exposure rates of humans to, for example, heat, radiation and toxic gases leading to incapacitation or death. The levels can be expressed as the instantaneous exposure rate or the dose. The dose expression is most common for the effects on humans of narcotic gases, but can also be used for thermal exposure responses. It should be mentioned that most of this type of work are performed on animals and not on humans. Questions may be raised to whether or not these data can be used to determine the tolerable exposure levels

on humans. These data are, however, the only existing and therefore those used.

A method of deriving the total exposure effect from different exposure sources is the Fractional Effective Dose (FED) method, introduced by Hartzell et al. (1985). The FED method sums the contributions from the various sources to give one variable value. When the FED has attained the value of 1.0, the occupant is defined as being incapacitated or dead, depending on the expressions used.

The problem in using this information is that the production term for narcotic gases in a fire is very uncertain and depends greatly on the fire scenario. Therefore, more simple deterministic values are used to express the occurrence of untenable conditions. The most commonly used variable is acute temperature exposure in conjunction with a limit on the smoke layer height. Conditions are defined as being untenable as soon as the conditions are fulfilled, and it is assumed that the escape route is instantaneously blocked.

The use of toxicological data in combination with temperature and radiation exposure, could in the future be used as a better prediction of untenable levels and for consideration of the inherent variation. This may be possible when better models, capable of predicting toxic gas concentrations in the vicinity of a fire room, become available. Toxicological data for determining untenable conditions is used in other areas of engineering, for example, in the prediction of the effects of toxic gas release to the atmosphere.

When determining the consequences of a release of toxic gas to the atmosphere, the Probit function is normally used (Finney, 1971). This is a measure that considers the exposure concentration, the exposure time and also the toxicological effect on the human body. Different exposure effects can be studied, from the smell of the gas to acute death. Different gases and exposure effects generate different values of the variables, which are used in the Probit

function. These are based on the estimated human tolerability to the gases. If the gas concentration at a specified location and exposure time is known, the number of victims, or people being subjected to its effects, can be estimated.

#### 4.1.3 The values of variables

A state expression may contain functions of random variables as well as independent basic random variables and constants. The response and behaviour time is, for example, usually determined as a single deterministic value or a distribution. There are no calculation models available to determine this time.

The values used to calculate both the probabilities and the consequences should be chosen carefully. This is the most critical part of the analysis, regardless of whether the task is to design the escape system for a building, to perform a standard QRA or to perform a complete uncertainty analysis, an extended QRA.

Many values are not easily determined and may be subject to uncertainty. For design purposes, values should be chosen to represent the "credible worst case" (ISO 13388, 1997). Taking the mean value of, for example, fire growth rate for a scenario does not necessarily represent credible scenarios sufficiently well. An upper percentile value could be chosen for the fire growth rate. Other values determined by physical properties of, for example, the building layout, are easier to determine. A distance can be measured with a high degree of certainty.

In building design and standard QRA, single values are used to determine the consequences and, if applicable, also the probabilities. In an explicit uncertainty analysis, the variables are defined by their respective distributions. The full information regarding the variation in the variables is then included in the distribution, and the credible worst case is normally considered as being within the distribution limits. Uncertainty analysis is further described in Chapter 6.

The values for the standard QRA can be chosen in two ways. Either the values are chosen to represent the best estimate for the variables or they can be chosen as conservative estimates, similar to those used for design purposes. Using the best estimate values results in a measure of risk that is also a best estimate. However, as there are uncertainties involved in the variables the best estimate measure of risk can be associated with large uncertainty. As the nature of a best estimate is to represent the average situation many situations, approximately half, will be worse than the estimated measure of risk. Performing an extended QRA leads to a quantification of this uncertainty.

The values for the standard QRA can also be chosen as conservative estimates. Using these in the analysis leads to a measure of risk that is on the safe side. How safe the measure is cannot be quantified without performing an extended QRA, but the measure of risk is not underestimated compared with the estimated average risk measure. One problem that can occur using these values is that the choices can be too conservative. Performing an uncertainty analysis or extended QRA can help solving this problem.

In the example risk analysis in this thesis, the latter method of choosing values for the standard QRA was used. The average measures of risk were also implicitly derived as they can be obtained from the extended QRA as the average values, for example, as the average risk profile.

Using values which are slightly conservative, similar to those used for design, in the standard QRA can be interpreted as performing a risk analysis on the design conditions. The implications of using either the best estimate values or the conservative values is discussed in Section 7.6.

In order to evaluate the influences from uncertainties, the standard QRA or the fire engineering design process should be complemented by a sensitivity analysis. It result in information concerning the relative importance between variables.

#### **4.2 Sensitivity analysis**

The purpose of the sensitivity analysis is to identify important variables, i.e. those controlling the result to a high degree.

Work has been done to determine what should be included in a sensitivity analysis (NKB, 1997) and Fire Engineering Guidelines (FEG, 1996). Factors that should be investigated with respect to the impact on the final result are

- variations in input data
- dependence on degree of simplification of the problem
- dependence on description of scenario, i.e. how the event tree is created
- reliability of technical and human systems

The variables identified as important should perhaps be chosen somewhat more conservatively than others. If the safety is highly dependent on just one function, redundancy should be considered. The analysis should identify variables of importance and what measures should be taken to eliminate or reduce the consequences of failure.

Sensitivity analysis only gives an indication of the importance of the variables involved in the analysis of a planned or existing building. If a more detailed investigation is necessary a complete uncertainty analysis should be performed. All information regarding the uncertainty in variables is then considered. Kleijnen (1995) provides a general description of sensitivity and uncertainty analysis.

### **4.3 System analysis**

The most simple situation occurs when the subscenario problem can be formulated as one single equation. The single limit state function contains all the information needed to describe the consequences of the subscenario. In some cases this is not sufficient as more than one failure mode can exist, i.e. the safety of the occupants can be jeopardised in more than one way.

When this is the case, the situation must be described by more than one equation. If these equations are correlated, they must be linked together to form a system which describes the expected consequences of the subsystem, cf. Section 6.6.2.

In evacuation analysis, the failure or unsuccessful evacuation is determined by the occurrence of the first failure mode. The evacuation safety of the subscenario is expressed as a series system, as only one failure mode is required. If one failure mode is fulfilled then at least one occupant is exposed to untenable conditions at any of the locations described by the subscenario.

In the area of structural reliability series systems, parallel systems and combinations of series and parallel systems can be identified. In fire safety engineering, the interest is purely on series system as occupants are prevented from further evacuation as soon as untenable conditions have arisen at least at one location.

The series system can be illustrated by a chain. The strength of the chain is dependent on the strength of the weakest link. The links can be expressed as limit state functions for the different locations, for example, fire room and corridor, for one subscenario. If one system fails, the whole system fails.

When numerical analysis methods are used to solve series problems, the limit state function can be expressed in terms of a number of separate equations. The consequences are derived from

sample calculations that are repeated. This may require several iterations before the subscenario consequences can be determined.

For analytical methods such as the First Order Second Moment (FOSM) method (Thoft-Christensen et al., 1982), the problem must be treated a little differently. Correlated single equations have to be treated simultaneously to derive the probability of interest. The probability of failure can, in most cases, not be determined in terms of a single value, but as an interval. Different methods are available to describe the bounds of the interval. This discussion will be further elaborated on in Section 6.5.1.

#### **4.4 Response surface method**

Usually, in a risk analysis the expressions in the limit state functions are derived by the use of computer programs. That is independent on whether it is a standard QRA or the complete uncertainty analysis that is the objective. In some cases, more than one computer program must be used to predict the consequence for every branch.

If only one consequence value, such as the number of people not being able to escape safely, is calculated for each event tree outcome, the use of the computer tools is normally rather straightforward. This is the situation in building design or in a standard QRA. The computer output results can be used directly, as input, in the risk analysis.

When considering uncertainties, as in the extended QRA or in uncertainty analysis, the computer programs must be used differently. This is because uncertainty analysis requires that the problem be formulated in a certain manner. The uncertainty analysis can either be performed as a numerical sampling procedure or as an analytical procedure.

When a numerical simulation procedure, such as a Monte Carlo method is used, a large number, usually more than 1000, of



calculations must be performed for each event tree outcome. It is rather inefficient to calculate the result directly for each subscenario 1000 times. If the computer program is specially designed to enable this iterative procedure it may be an integrated part of the analysis, see Iman et al. (1988) and Helton (1994) for reviews of the area. As this feature is rather uncommon in commercial programs, other approaches must be considered. One approach first approximates the computer output with an analytical expression, a response surface, which then, in the second step, easily can be applied in the Monte Carlo simulation, i.e. the uncertainty analysis. The arguments for using response surface equations are also valid if the uncertainty analysis is performed by analytical methods, such as the FOSM method.

The response surface, or meta model, is used to estimate the values from computer calculations or experiments on the basis of only a very few input variables. A response surface is normally created by using regression analysis. The term response surface is used to indicate that, when using several variables to represent the data, a surface is created in n-dimensional space, where n is the number of variables. In a two-variable case the response surface will become a line which is usually referred to as a regression line. Having more than two variables, the regression result will be a plane, linear or nonlinear, depending on the relationship between the variables. In this thesis the general term surface will be used even for the two-dimensional situation.

The response surface equation should represent the data as accurately as possible, at least in the region of interest. In the examples in this thesis, the interest is mainly in creating an analytical expression that will represent the output from one or two computer models as well as possible. The intent is not to carry out a detailed investigation, in order to find the most optimum expression for an uncertainty analysis.

There are other advantages with this method, apart from time saving, which are worth mentioning. As the output is derived from an equation, it is rather obvious which variables determine the result. The analysis is very transparent and easy to verify and reproduce. The results will not be determined by a black-box computer program. It is also rather easy to determine the quality of the output as only one or a few equations must be considered in a sensitivity analysis or uncertainty analysis. This is further explained in a following section.

The drawback of using the response surface technique is that a new uncertainty variable is introduced. The magnitude of this new uncertainty is usually small and its influence normally not very significant. To gain an understanding of how well the response surface equation predicts the computer output, the coefficient of determination,  $R^2$ , can be analysed.

The uncertainty resulting from the regression analysis can, however, be included in the total uncertainty analysis. This is elaborated on later on in this chapter. With good approximation methods this uncertainty will be small, and the benefit of having a fast calculation model outweighs this uncertainty.

#### ***4.5 Creating the response surface equation***

Creating the response surface equation for computer model outputs requires information on both the input values and the computer output results. Regression analysis is used to obtain the analytical relationship between the input parameters and their corresponding output (Ang et al., 1975).

Several methods are available to create this analytical equation, such as the method of least squares and the method of maximum likelihood. The response surfaces used in this thesis were derived using the method of least squares.

#### 4.5.1 The linear two-dimensional case

The simplest case of curve fitting is to derive an equation that represents data by a straight line, linear regression analysis. The task is to estimate  $\lambda$  and  $\delta$  in the expression

$$\hat{y} = \lambda + \delta x + e \quad [4.3]$$

giving the estimate of the real variable  $y$ , Figure 4.1. The equation can also be interpreted as providing the conditional estimate  $E(y|x)$ . The factor  $e$  represents the uncertainty in  $\hat{y}$ . The regression equation does not have to be restricted to two variables. Multiple variable regression analysis is similar, but the theoretical evidence will not be presented here.

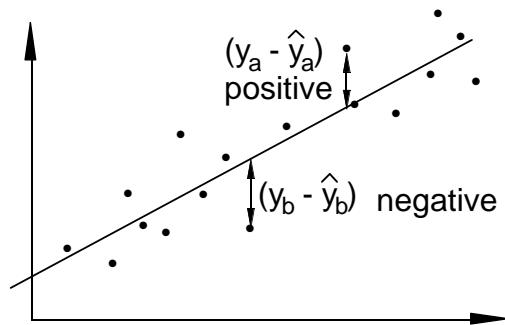


Figure 4.1. Simple linear regression.

The regression analysis introduces new uncertainties into the parameters  $\lambda$  and  $\delta$  as they only can be estimated and will therefore be associated with uncertainty, e.g. described by a mean and a standard deviation. This mean that  $\lambda$ ,  $\delta$  and  $e$  are subject to uncertainty as a result of the regression analysis.

The method of least squares works with any curve characteristics as the only objective is to minimise the difference between the sample data and the predicted surface. The important issue is to find a relation that describes the output in the best way and with as small a deviation from the data as possible.

The vertical differences between the data and the regression line, the residuals, will be evenly distributed on both sides of the regression line. This is a result of the method as it minimises the sum of the squares of the residuals. This means that the sum of the residuals is equal to 0.

The residual variance,  $s_e^2$ , is a measure of how well the regression line fits to the data. It shows the variation around the regression line. The variable  $e$  in Equation [4.3] is usually estimated by a normal distribution  $(0, s_e)$ .

Figure 4.2 shows the residuals from one of the sample calculations which will be presented in Chapter 7. The regression equation estimates the time before lethal conditions arise in the corridor of a hospital ward. All data points should preferably be located close to the solid horizontal line which represents the regression line. The vertical distances between the data points and the line shows the deviation between the computer results of this variable and the expression which was used in the uncertainty analysis. The depending variable, the fire growth rate,  $\alpha_f$ , is shown on the horizontal axis and the residuals of the time on the vertical axis.

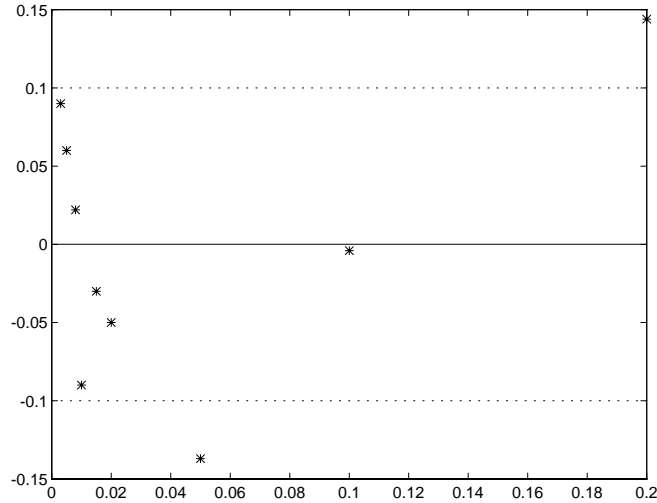


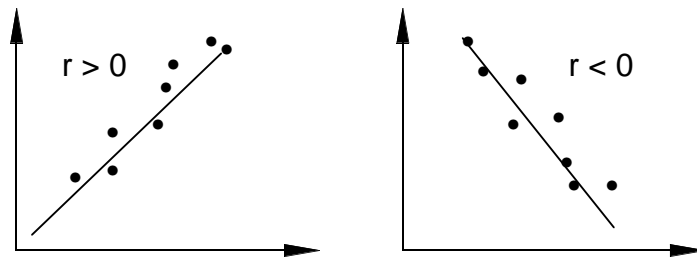
Figure 4.2. Residuals from the regression analysis of time before lethal conditions arise in a health care ward corridor as a function of the fire growth rate  $\alpha_f$  ( $\text{kW/s}^2$ ). No sprinklers are activated. The dotted lines indicate  $\pm$  one  $s_e$ .

The values on the vertical axis are logarithmic ( $\ln(t_u^{corr})$ ) due to reasons which are explained in Section 4.5.2.

The residuals are in the same units as the variable  $y$ , which means that the values from different regression analyses cannot be compared directly determining whether or not the regression shows good agreement. A normalised measure of the deviation is the correlation coefficient. The correlation coefficient,  $r$ , is a measure of how close the data are to a linear relationship, and is defined as

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} \quad [4.4]$$

The correlation coefficient can vary between -1 and +1, and values close to the outer limits of this interval represent good agreement. The sign indicates whether the correlation is positive or negative, see Figure 4.3.



*Figure 4.3. Correlation coefficients for a sample.*

In multiple linear regression analysis, the coefficient of determination,  $R^2$ , is used instead of the correlation coefficient. For the linear case with only one dependent variable  $r^2 = R^2$ .

$$R^2 = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad [4.5]$$

The coefficient of determination is a measure of how much of the residuals are explained by the regression model. The value should be as close as possible to 1. It is clear that the uncertainty in the prediction of  $y$  will depend on the sample size,  $n$ . Increasing the sample size decreases the overall uncertainty. The coefficient of determination,  $R^2$ , for the analysis presented in Figure 4.2 above, is 0.97, which indicates good agreement between the regression line and the data.

One of the problems that may occur when using a response surface instead of the actual computer output, is that the residuals may increase as the value of one or more variable is increased. If this

happens, the uncertainty introduced by the regression analysis may have to be considered important.

As the regression analysis is used together with other variables that are subjected to uncertainty, the uncertainty variables from the regression analysis must be compared to the other uncertainties. For most cases these new introduced uncertainties can be omitted as their contribution to the overall uncertainty can be considered small.

#### 4.5.2 Nonlinear problems

Linear problems are rare in most engineering disciplines. Most models result in nonlinear solutions and the traditional linear regression gives a poor representation of the data. There are two ways of solving this problem; optimising a nonlinear expression or transforming the model into a form that is linear, at least locally in the area of interest.

Most nonlinear solutions are based on approximating the data to a polynomial in various degrees, for example a 2nd order polynomial. The curve-fitting technique is more or less the same as that described above. This approach is normally considered rather laborious and other means are preferable if they are available.

The second technique transforms the data into a form in which the transformed variables are linear. One such transformation is to use the logarithmic values of the data. Other transformations such as squares or exponentials can also be considered. If the transformed values appear to be linearly dependent, linear regression analysis can be performed. The coefficient of determination can be used to determine the agreement between the data and the response surface for both the nonlinear and the transformed solutions. There are two good reasons for using the logarithmic values in some engineering applications.

1. In some cases the variation in the input variables is several orders of magnitude. The values located close to the upper limit of the response surface output, will then influence the parameters in the equation more than others.
2. For some parameter combinations, a polynomial relationship can result in negative responses which are physically impossible. This must definitely be avoided.

It appears that the linear approximation of the logarithmic data in determining the response surfaces is an appropriate choice for the cases considered in this thesis. The coefficient of determination,  $R^2$ , is generally very high in all equations. The large difference in magnitude of the variables will be drastically reduced and no negative responses will be derived using this approach. The response surface will have the following general appearance:

$$y = \exp(\lambda) \prod_{i=1}^n (x_i)^{\delta_i} \quad [4.6]$$

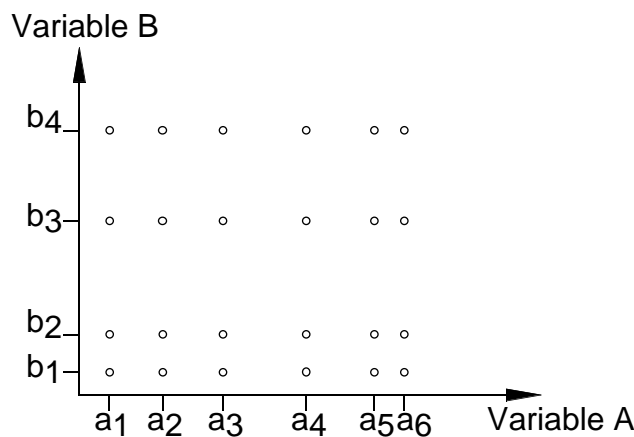
where  $n$  is the number of variables, and  $\lambda$  and  $\delta_i$  are the linear regression parameters. A problem arises when the uncertainties in  $\lambda$ ,  $\delta$  and  $e$  are to be transformed. If a numerical procedure is used for the uncertainty analysis this will normally not be a problem. For an analytical method using hand calculations, these new uncertainties become a problem which might cause exclusion of the method. An approximate solution can be used, i.e. excluding these uncertainties, or special software capable of considering regression parameter uncertainty can be used.

In the risk analysis presented here, both the standard QRA and the extended QRA, these uncertainties are omitted, as they are small in comparison with the other variable uncertainties. To be able to draw this conclusion, the single subscenario uncertainty analysis was performed both with and without the uncertainty information in  $\lambda$ ,  $\delta$  and  $e$ .



### 4.5.3 Design of experiments

Creating a response surface to represent the output from a computer program requires a set of outputs from the program, together with the corresponding input data. Several sampling methods are available which describe how the calculation procedure should be designed in order to minimise the total number of computer runs. The most extensive sampling method is the factorial method, which requires an output for every combination of input variables, Figure 4.4. The figure illustrates how computer outputs are calculated for every combination of the input variable values. The input data for the two variables are represented by  $a_1$  to  $a_6$  and  $b_1$  to  $b_4$ .



*Figure 4.4. A 4 by 6 level factorial sample. A circle indicates that an output is calculated.*

If the number of input variables and the different levels for each variable are large, methods are available which can reduce the number of computer runs by selecting certain combinations of the variables and corresponding levels. In these cases, methods such as the fractional factorial and Latin square methods (Vardeman, 1993) should be used. The term level is used here to define the number of values each variable will have in the calculation process

defining the various outputs. Using these methods will inevitably lead to some loss of information, but that has to be weighed against the time gained through the smaller number of computer simulations.

In this thesis, both the number of variables and levels are considered low and complete factorial studies have been performed. Usually, only one or a few variables are used as uncertainty variables in calculating the response. In the computer programs, several other input variables are needed but these are treated as deterministic constants without any uncertainty.

This is, of course, a simplification but it can be explained by the choice of subscenarios. Variables with great influence on the results, apart from the fire growth rate  $\alpha_f$ , are the dimensions of the building. As the example risk analysis calculations have been performed on a standardised building with fixed dimensions, this simplification is justified.

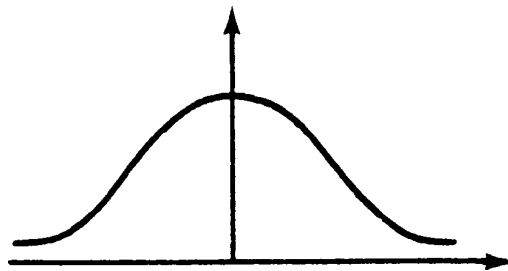
## 5 Describing random variables

### 5.1 Statistical distributions

When the uncertainty is to be explicitly included in the analysis, some of the variables must be defined as random variables. This is independent of whether the uncertainty is a stochastic or a knowledge uncertainty, cf. Chapter 2. One way of describing the variables is to use the probability density function (PDF) or frequency distribution for the variable,  $f_X$ .

A random variable can be represented by values within a specified interval, described by the frequency function. The distribution shows the probability that a specific value will be assigned. The distribution interval can either be limited by outer bounds, minimum and maximum values or be open, having no outer limits, Figure 5.1.

An example of a limited frequency function is the uniform distribution, having the same frequency for all values within the interval and defined by the minimum and maximum values. The normal distribution is an example of an open distribution.



*Figure 5.1. Illustration of a probability density function (PDF) for a symmetrical open distribution.*

Other possible representations of a random variable are the cumulative distribution function (CDF) and the complementary cumulative distribution function (CCDF). The three types of representation, PDF, CDF and CCDF, contain the same information expressed in three different ways. The latter two present the cumulative frequency from the PDF.

The CDF describes the probability  $P(X \leq x)$  for the random variable  $X$  at any given  $x$  defined in the interval  $-\infty < x < \infty$ . It is important to distinguish between  $x$  and  $X$ . The lower case  $x$  is the argument of the function  $F_X$  describing the cumulative distribution function.

The mathematical relationship between the PDF and the CDF is defined as

$$F_X(x) = \int_{-\infty}^x f_X(t) dt \quad [5.1]$$

It is further assumed that the random variable  $X$  is continuous in the interval of interest.

The complementary cumulative distribution function (CCDF) is closely linked to the cumulative distribution function and is defined as  $1 - F_X(x)$ . In risk analysis, the use of the CCDF is quite common as it answers the question "How likely is it that the consequences are worse than a specified value?". In mathematical terms this can be expressed as

$$1 - F_X(x) = P(X > x) = 1 - F_X = \int_x^{\infty} f_X(t) dt \quad [5.2]$$

The probability density function (PDF) is the most common representation of a random continuous variable in quantitative risk analysis.

Similarly, if the variable is represented by a discrete function it can be described by its probability mass function (PMF) in analogy with its continuous relative.

Each random variable is represented by one or more parameters. The parameters can, for example, be minimum and maximum values or the mean value and the standard deviation. The normal distribution is, for example, represented by the mean value and the standard deviation.

### **5.2 Correlated variables**

The random variables may be linked together by a dependence relationship, i.e. they are correlated. The correlation between variables is important in risk calculations. The correlation can be either positive or negative. A positive correlation will tend to make the distributions deviate in the same direction, a high value of the variable  $X$  is likely to follow a high value of  $Y$ .

The correlation can be described by the covariance,  $C(X, Y)$  or by the correlation coefficient,  $\rho_{XY}$ . The correlation coefficient can be seen as a normalised covariance. If  $X$  and  $Y$  are statistically independent,  $C(X, Y) = 0$ , which also means that they are not correlated. Noncorrelated variables can, however, not be defined as statistically independent.

The correlation coefficient is the most frequent measure of correlation and it is always between -1 and +1. Note the similarity to the correlation coefficient,  $r$ , defined for a sample in Chapter 4.

### **5.3 The choice of distribution**

One task is to determine the most appropriate type of distribution for each variable and the corresponding parameter values. The data forming the basis for the choice of a specific distribution are usually limited. This leads to the question, "How should the distribution be selected in order to represent the variable as accurately as possible?".

Firstly, as pointed out by Haimes et al. (1994), the distribution is not formally selected. The distribution is evidence of, and a result of, the underlying data. In many cases the distribution type is determined by what is previously known about the variable. For example, a strength variable cannot have negative values, which eliminates some distributions. Two categories can be defined depending on the amount of data available separated:

- if the amount of data is large
- if the amount of data is small or irrelevant.

This implies that there are two methods available for the selection of a distribution and the corresponding parameters. The probability distribution of the event can be estimated either according to the classical approach, or according to the subjective approach, also known as the Bayesian approach, after the English mathematician Thomas Bayes (1702-1761).

### 5.3.1 The classical approach

If the data base is large, the distribution can be easily determined by fitting procedures. The parameters of the distribution can be derived by standard statistical methods. This is normally referred to as the classical approach.

The classical approach defines the probability on the basis of the frequency with which different outcome values occur in a long sequence of trials. This means that the parameters, describing the variable, are assigned based on past experiments. There is no judgement involved in this estimation. It is based purely on experimental data.

Additional trials will only enhance the credibility of the estimate by decreasing the variability. The errors of the estimate are usually expressed in terms of confidence limits.

An example of the frequency defined according to the classical approach is illustrated by the calculation of the probability that throwing a dice will result in a 'four'. The conditions of the experiment are well defined. Throwing the dice a thousand times will lead to the probability of  $1/6$  that the result will be a 'four'. The probability will not be exactly  $1/6$  but close to it. Increasing the number of trials will improve the probability.

### 5.3.2 The Bayesian approach

If only a small amount of data is available, this data together with expert judgement can be used to form the basis for the choice of distribution, which has the highest degree of belief. The choice will thus be partially subjective. By applying the method of Bayes, the subjective distribution can be updated in a formal manner, as soon as new data become available.

Bayes' method assumes that the parameters of the random variables are also random variables and can therefore be combined with the variability of the basic random variable in a formal statistical way by using conditional probabilities. This assumption will reflect the probable uncertainty inherent in the variable.

The estimate of a parameter which is based on subjective judgement is improved by including observation data in the estimate. The new estimate is a probability, on condition that experiments or other observations have been performed, and that these results are known. The method can be used for both discrete probability mass functions and continuous probability density functions.

Applying the dice example to this approach means that the person, conducting the experiment, does not have to throw the dice at all. He knows from past experience and assumptions that the probability will be  $1/6$  if the dice is correctly constructed. He makes this estimate by judgement. If the dice is corrupt and favours the outcome 'two' this will only be seen in the experiment

conducted according to the classical approach. The subjective estimate will, therefore, be false prediction of the true probability of the outcome 'four'. However, he can make a few throws to see if the dice is correctly balanced or not. Based on the outcome of this new experience, he can update his earlier estimate of the true probability, using Bayes' theorem. If subsequent new trials are performed and the probability continuously updated, subjective method will converge towards the classical estimate of the probability.

### 5.3.3 Bayes' theorem

In the following, a brief formal description of Bayes' theorem will be presented. A more detailed description can be found in, for example, Ang et al. (1975).

Each variable can be assigned a PDF which the engineer thinks represents the true distribution reasonably well. This first assumption is denoted the prior density function. The improved distribution, achieved by including new data, is denoted the posterior density function.

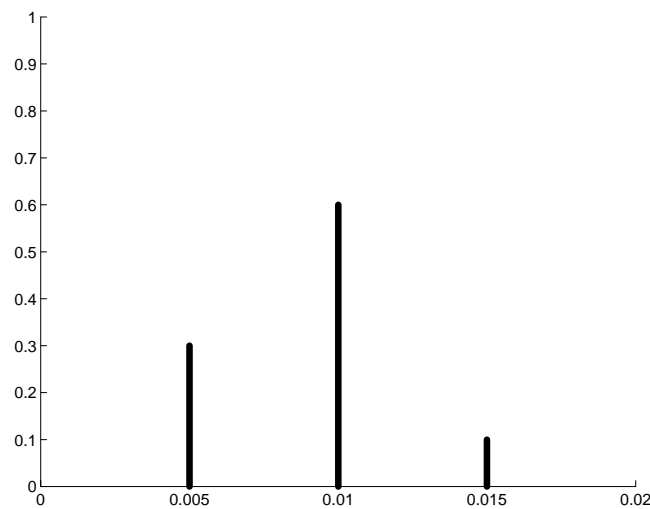
For a discrete variable, Bayes' theorem can be formulated as

$$P(\Theta = \theta_i | \varepsilon) = \frac{P(\varepsilon | \Theta = \theta_i) P(\Theta = \theta_i)}{\sum_{i=1}^n P(\varepsilon | \Theta = \theta_i) P(\Theta = \theta_i)} \quad [5.3]$$

describing the posterior probability mass function for the random variable  $\Theta$  expressed by  $i = n$  possible values. The posterior probability is the result of considering new experimental values,  $\varepsilon$ , in combination with the prior probability  $P(\Theta = \theta_i)$ . The term  $P(\varepsilon | \Theta = \theta_i)$  is defined as the conditional probability that  $\varepsilon$  will occur, assuming that the value of the variable is  $\theta_i$ . A short example will be used to illustrate the method.

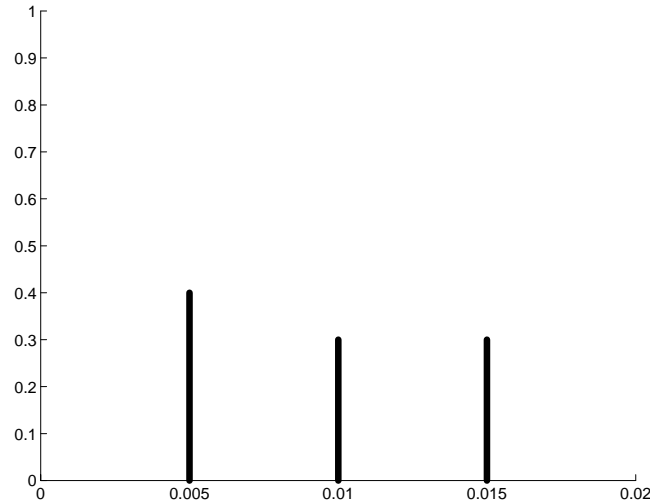


Assume that the probability of a fire occurring which can be described by the fire growth rate,  $\alpha_f$ , can be expressed as the discrete function illustrated in Figure 5.2. The figure illustrates the probability (vertical axis) as a function of the fire growth rate,  $\alpha_f$  (horizontal axis). The value  $\bar{\alpha}_f$  can be calculated giving  $0.009 \text{ kW/s}^2$ , as can be expected from the figure.



*Figure 5.2. Prior probability mass function of  $\alpha_f$ .*

After carrying out an extensive post-fire investigation on similar fire scenarios, the investigators' results indicate a slightly different probability function, as illustrated in Figure 5.3. This new information will be used to update existing information in terms of the prior probability information. It is evident that the new data are more uniformly distributed.



*Figure 5.3. New data on the variable  $\alpha_f$  after the post-fire investigation.*

The posterior probabilities for  $\alpha_f = 0.005, 0.01$  and  $0.015 \text{ kW/s}^2$  can now be evaluated.

$$P(\alpha_f = 0.005) = \frac{0.3 \cdot 0.4}{0.3 \cdot 0.4 + 0.6 \cdot 0.3 + 0.1 \cdot 0.3} = 0.36$$

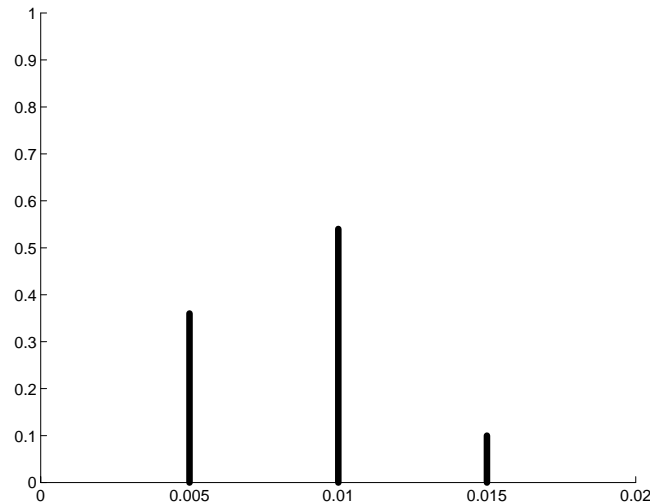
The other two probabilities can be derived in the same manner

$$P(\alpha_f = 0.01) = 0.54$$

$$P(\alpha_f = 0.015) = 0.10$$

and are illustrated in Figure 5.4. The new value of  $\bar{\alpha}_f$  can be derived based on the posterior probability function

$$\bar{\alpha}_f = 0.36 \cdot 0.005 + 0.54 \cdot 0.01 + 0.10 \cdot 0.015 = 0.0087 \text{ kW / s}^2$$



*Figure 5.4. Posterior probability mass function of  $\alpha_f$ .*

The theorem can also be used for continuous functions and the appearance is similar to that in the discrete situation. The solution usually requires numerical integration procedures.

$$f''(\theta) = \frac{P(\varepsilon|\theta)f'(\theta)}{\int_{-\infty}^{\infty} P(\varepsilon|\theta)f'(\theta)d\theta} \quad [5.4]$$

where  $f''(\theta)$  is the posterior PDF and  $f'(\theta)$  is the prior PDF for the variable  $\Theta$ .

#### **5.4 Fire safety engineering data**

In the area of fire safety engineering, it is usually difficult to obtain the data forming the new knowledge  $\varepsilon$ . It may not even be possible to estimate a new posterior function, and the prior function must be relied upon.

The reason for this practice is that much data can only come from post-fire investigations. Information on human responses and actions in actual situations can only come from this type of investigation. Performing experiments may not provide an alternative for ethical reasons. Some physical variables can, however, be measured without any involvement of humans and data can be collected and used together with Bayes' theorem.

Another problem arises when new data are to be used to improve the prior function. When the new data are very limited, perhaps only one or two data points, they may be considered not very representative of the variable. Applying these new values will thus result in a posterior function which is less realistic. Bayes' theorem does not consider the number of observations in the new data.

These problems are not unique to fire safety engineering. Lack of data is a problem in many engineering fields. Care must thus be taken to use as accurate data as possible, and to not use small samples to update the priori data. In fire safety engineering, many of the parameters still have to be subjectively estimated with little statistical support.

### ***5.5 Distributions used in fire safety engineering***

How should a type of prior distribution and its corresponding parameters be chosen? A number of researchers have tried to establish rules governing the choice of distribution based on, for example, the amount of data present. According to Haines et al. (1994), for small samples the mean value should be calculated and combined with a subjective estimate of the upper and lower bounds and the shape of the distribution. If large uncertainties are suspected, log-transformed distributions should be used instead of uniform, triangular or normal distributions.

For fire safety risk analysis, the first step is to establish the minimum and maximum limits for each variable. The next task is to estimate the mean values and the standard deviation, or other

parameters, for each of the basic variables. The final step is to choose a distribution type for the variables, based on which has the highest degree of credibility. This must be done for each random variable in the system, such as for the response time of the occupants, and also for variables such as reliability or availability of an automatic fire detection system.

For most variables, such as fire growth rate, there is a more or less extensive data base, which provides a credible range (minimum values to maximum values) for the specific parameter. The data are not systematically assembled, but the information exists and must be sought after in a number of sources. Collecting and systematically organising the relevant data is a task which must be given high priority in future work.

The type of distribution most frequently used in this thesis, is the normal distribution. This has been used for variables such as time spent for investigation after the alarm signal and occupant movement time. It is believed to represent the variables in a suitable way. A lognormal distribution has been chosen for the fire growth rate as it gives no negative values and is believed to represent the variable in the best possible way.

## 6 Quantitative methods

### 6.1 Introduction

The Quantitative Risk Analysis (QRA) is focused on the combined effect of frequency and consequences of a possible accident. The frequency is usually derived using event tree techniques sometimes combined with fault tree analysis, see Chapter 3. For each branch in the event tree, denoted a subscenario, the consequences will be determined. The consequence expresses the value of the unwanted event. The frequency and consequences are formally combined in the QRA. The quantitative risk analysis process can be illustrated as in Figure 6.1.

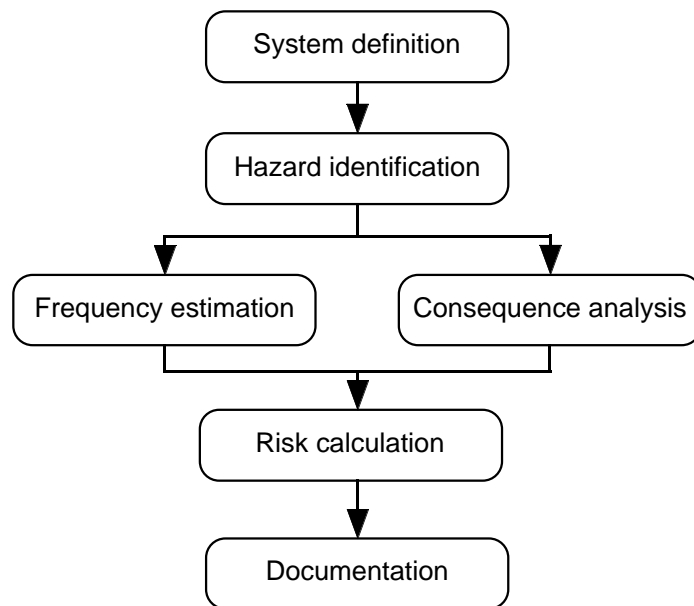


Figure 6.1. Risk analysis process.

The first step, before starting to quantify the risk, is related to defining and describing the system. The system is defined in terms of one or more scenarios. In the risk analysis the system must also

be defined in terms of physical limitations, i.e. which physical area should be considered in the analysis? In Chapter 7, where a sample risk analysis of a hospital ward is presented, the definition of the system boundary is further discussed.

After the system has been described, the hazards are identified and quantified, the next step in the process, according to Figure 6.1, is to evaluate the risk, i.e. perform the quantitative risk analysis. The results of the analysis are, for example, the individual risk and the societal risk, see Section 6.3.

Different criteria can be used in determining the consequences. It is not necessarily the hazard to humans that governs the analysis. The objective of the analysis could be to minimise the maximum allowed release of gas or to optimise safety measures restricted by constraints such as authority regulations or maximum cost levels. The analysis method presented in this thesis uses the decision criterion that occupants in a building shall be prevented from being exposed to harmful conditions if a fire starts on the premises. The occupants of the premises have a right, determined by societal regulations, to a certain degree of protection in the case of fire. This type of criterion is classified as a rights-based criterion according to the classification system of Morgan et al. (1990). The risk as defined in this thesis is then a measurement of not being able to satisfy this criterion. The risk is defined in terms of the complete set of triplets  $R = \{(s_i, p_i, c_i)\}$ , giving the standard QRA triplets.

Other decision criteria that may be used are utility-based criteria and technology-based criteria. Utility-based criteria are often based on a comparison between cost and benefit. The objective of the analysis can therefore be to maximise the utility. In order to choose an optimum solution, both the cost and the benefit must be expressed in the same unit, usually in a monetary unit. An overview of decision making can be found in Gärdenfors et al. (1986) and in Klein et al. (1993). Decision making, as a general

topic, will be briefly mentioned in Chapter 8 when tolerable risk is discussed.

Normally, a QRA is a rather complex task. It is difficult to perform the analysis as it is labour intensive and the degree of detail is high. It is also very difficult to evaluate a QRA as many of the assumptions are not well documented. In some cases, the only person able to reproduce the analysis is the one who carried out the analysis in the first place. It is therefore advisable to follow some golden rules for risk analysis. Morgan et al. (1990) defined a set of "ten commandments" for risk analysis which can be summarised as:

- perform the analysis in an open and transparent manner
- document all relevant assumptions and decisions taken throughout the process
- describe the uncertainties involved even if no explicit uncertainty analysis is performed
- expose the document to peer review.

Before continuing, it should be clearly stated that the term risk is not well defined. At the 1996 Annual Meeting of the Society for Risk Analysis Stan Kaplan said:

"The words risk analysis have been, and continue to be a problem. Many of you here remember that when our Society for Risk Analysis was brand new, one of the first things it did was to establish a committee to define the word 'risk'. This committee labored for 4 years and then gave up, saying in its final report, that maybe it's better not to define risk. Let each author define it in his own way, only please each should explain clearly what way that is."(Kaplan, 1997)

In this thesis, risk is defined as: the quantitative measure of the condition that people are not able to escape safely before the untenable conditions have occurred on the premises. The risk is



expressed both to individuals and as the societal risk considering multiple fatalities. See also the sample risk analysis in Chapter 7.

## **6.2 Performing a QRA**

In order to perform a fully quantitative risk analysis, a number of questions regarding, for example, the extent of the analysis must first be answered. The choice of system boundaries and system level will have a fundamental influence on the choice of analysis approach and methodology. The optimal choice of assessment method will be dependent on factors such as:

- whether the calculation tool is a computer program or an analytical expression
- to what extent variable uncertainty is explicitly considered
- whether the analysis is concerned with a single subscenario or the whole event tree.

Different approaches are available for quantitative risk analysis, which can be organised according to the illustration in Figure 6.2.

The first factor is related to how computer results are used in the uncertainty analysis. Computer program output can be used either directly in the analysis as an integrated part of the methodology or indirectly providing results which are used to create analytical response surface equations, see Chapter 4.

The second factor concerns the extent of the analysis in terms of explicitly considering variable uncertainty. If no uncertainties are considered in the definition of the variables, a standard quantitative risk analysis can be performed. In a standard QRA, the events will be described in terms of deterministic point estimates. The subsequent risk results, both individual risk and the societal risk, are also presented as deterministic values without any information on the inherent uncertainty. Simple deterministic calculations can be performed by hand, but computer calculation is normally the most rational.

If a more thorough analysis of the scenario is the objective, the impact of uncertainty in the variables defining the subscenarios should be examined. Usually, most variables are associated with uncertainty and the risk measure can be further improved by considering such uncertainties. The work load associated with the analysis will, however, be drastically increased.

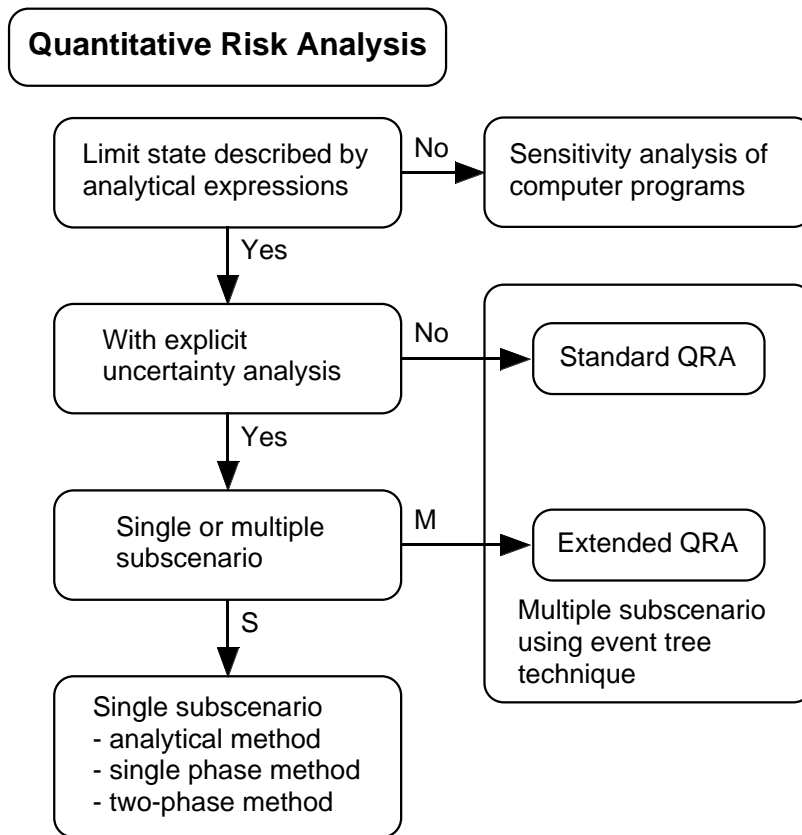


Figure 6.2. Risk analysis procedures.

The third factor is concerned with the level of analysis when considering uncertainty explicitly. Two different approaches can be taken regarding uncertainty analysis, depending on the level of examination. Only one subscenario at a time can be considered, or

the whole event tree can be regarded as a system, see Figure 6.2. The uncertainty analysis determines how uncertainties in outcome probability and consequences are propagated. This results in a more detailed description of the scenario.

For the analysis of one single subscenario, there are at least three methods available: one analytical method and two numerical simulation methods. The analytical first order reliability method is called analytical because it is possible to derive the resulting risk measure, the reliability index  $\beta$ , analytically for simple cases.

The two numerical methods, the single phase and the two-phase methods, are based on Monte Carlo simulations in which the variable distributions are estimated by sampling procedures. The two-phase simulation method makes it possible to separate two types of uncertainty, i.e. stochastic uncertainty and knowledge uncertainty, cf. Chapter 2. The first numerical method is more direct as it does not distinguish between different types of uncertainty.

The results of all three methods are, for the simplest case, the probability of failure of the subsystem,  $p_{u,i}$ , assuming that the subscenario has occurred. The probability of failure can, together with the probability  $p_i$ , be used to produce a better estimate of the risk contribution from subscenario  $i$ .

Considering variable uncertainty on the system level, i.e. performing a QRA, leads to the extended QRA.

### **6.3 Risk measures**

Before the different risk analysis methods are presented, it is appropriate to introduce the various measures by which the risk can be expressed. A more detailed explanation is given as the risk analysis methods are described.

It is possible to identify at least two types of risk measures

- individual risk, IR and
- societal risk, SR.

Those two are the most frequent risk measures in current risk analyses. But, comparing risk measures from different risk analyses is a difficult task, as the measures must be based on similar assumptions and be defined in the same manner. The purpose of this thesis is to illustrate a basic methodology for risk analysis in building fires. Simple treatment of the term risk is therefore emphasised.

### 6.3.1 Individual risk

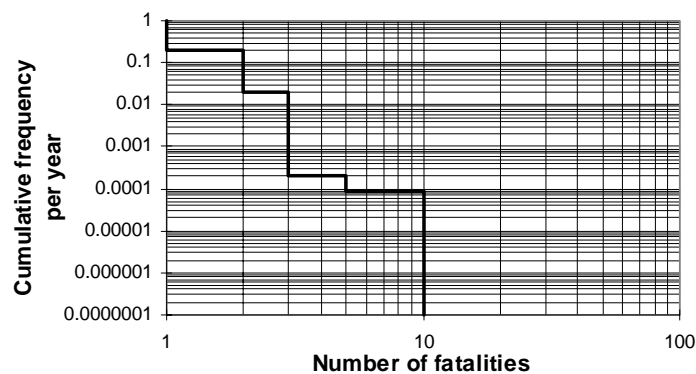
The individual risk is defined as the risk to which any particular occupant is subjected at on the location defined by the scenario. If an occupant is inside a building, he or she will be subjected to a risk in terms of the hazard frequency. The individual risk is usually expressed in terms of a probability per year of being subjected to an undesired event, i.e. the hazard, considering all subscenarios.

### 6.3.2 Societal risk

The societal risk is concerned with the risk of multiple fatalities. In this case, not only the probability that the subscenario leads to the unwanted event is considered, but also the number of people subjected to the hazard. People are treated as a group with no consideration given to individuals within the group, and the risk is defined from the societal point of view.

The societal risk is often described by the exceedance curve of the probability of the event and the consequences of that event in terms of the number of deaths. This curve is known as the FN curve (Frequency Number curve) or risk profile, see Figure 6.3. The curve shows the probability (cumulative frequency) of consequences being worse than a specified value on the horizontal axis.

This measure of risk is of particular interest as official authorities do not usually accept serious consequences, even with low probabilities.



*Figure 6.3. Example of an FN curve.*

Another form in which the societal risk can be expressed is as the average societal risk measure, which is an aggregated form of the FN curve. The average risk is expressed in terms of the expected number of fatalities per year.

#### **6.4 Standard quantitative risk analysis**

A quantitative risk analysis in the area of fire safety engineering should preferably be based on an event tree description of the scenarios. The problem can then be analysed in a structured manner. Consideration can be taken of, for example, the reliability of different installations. The standard QRA is most frequently used in describing risk in the process industries and in infrastructure applications. It has also been applied in the area of fire safety engineering, but as part of a more comprehensive risk assessment of a system, for example safety in railway tunnels.

The standard QRA is based on a high number of deterministic subscenario outcome estimates, but the method is still considered

probabilistic. When a large number of subscenarios are treated, each with its individual probability, this will lead to a probabilistic measure of the risk. The FN curve can, therefore, be seen as the empirical CCDF for the whole event tree.

In the standard QRA, the consequences and probabilities of the scenarios can be examined individually or together, as a system, depending on the objective of the analysis. The idea of triplets is used to give the procedure a rational structure, see Chapter 4. Both individual risk and societal risk can be calculated using this method.

The most frequent type of risk analysis (standard QRA) does not explicitly include any uncertainty analysis. To study the influence of uncertainties in branch probabilities or variables, an extended QRA must be performed, see Section 6.7.

#### 6.4.1 Societal risk

Various methods are available to express the societal risk when the triplets have been derived. The most common method is to express the risk in terms of an FN curve or a risk profile in a log-log diagram. An FN curve answers the question "How likely is it to be worse than this?", i.e. the frequency of exceedance. The number in the FN curve is usually equivalent to the number of fatalities in the risk analysis. This means that the risk is not constant in terms of number of deaths and the probability of those deaths, as the cumulative probability always decreases as the consequences increase.

The FN curve from a standard QRA can be used to compare different design solutions or to determine whether or not the design complies with tolerable risk levels. Tolerable risk levels have been developed for some large infrastructures in a number of countries. The tolerable risk can be defined as a limit line in the FN diagram, usually together with a grey zone in which the risk is tolerable but

should preferably be decreased. Different levels of tolerable risk will be further discussed in Chapter 8.

An unwanted event is defined in this thesis as that when people are unable to escape the threat of the fire within the available escape time. The available escape time is defined by the time taken to reach critical or lethal levels of untenable conditions. Therefore, the more general term risk profile will be used to represent the societal risk instead of the term FN curve. The risk profile is equivalent to an FN curve when the unwanted consequences are derived from the lethal levels of untenable conditions.

To create a risk profile, the triplets must be ordered in increasing order of consequence, i.e. so that  $c_i < c_{i+1}$ , see Table 6.1. The risk profile can be plotted as a step function as in Figure 6.4. The probabilities,  $p_i$ , in this figure are the event tree probabilities,  $p_{ET,i}$ . The maximum value on the vertical axis is therefore equal to 1.0, as

$$P = \sum_{i=1}^n p_{ET,i} = 1.0. \quad [3.2]$$

*Table 6.1. Triplets sorted in order of increasing consequence.*

$s_i$	$p_i$	$c_i$	Cumulative $p_i$
$s_1$	$p_1$	$c_1$	$1 - p_1$
$s_2$	$p_2$	$c_2$	$1 - \sum_{i=1}^2 p_i$
$s_3$	$p_3$	$c_3$	$1 - \sum_{i=1}^3 p_i$
-----	-----	-----	---
$s_{n-1}$	$p_{n-1}$	$c_{n-1}$	$1 - \sum_{i=1}^{n-1} p_i$
$s_n$	$p_n$	$c_n$	0

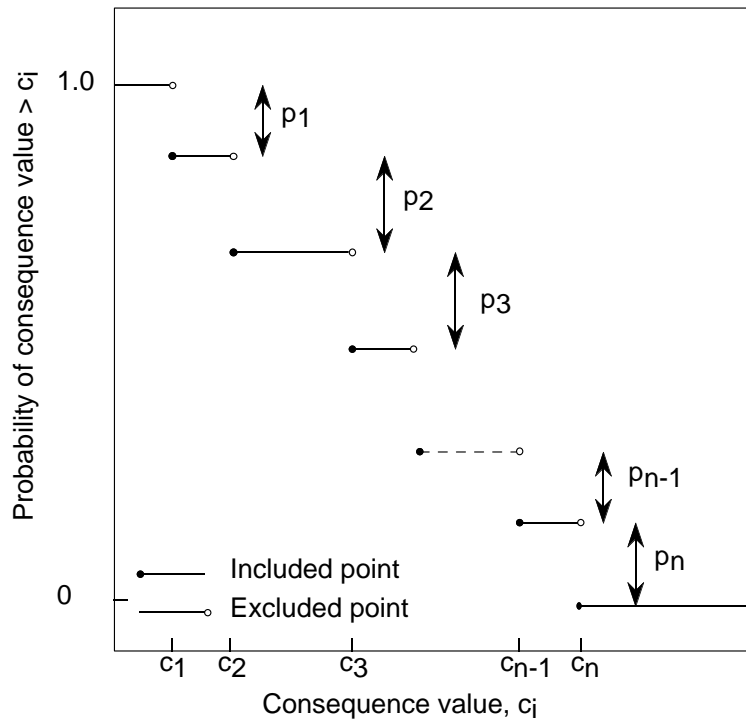


Figure 6.4. Construction of a risk profile in a QRA.

Small event trees can be evaluated by hand calculations, but if the event tree is large, commercial software is recommended. In addition, a number of Matlab m-files have been developed which can be used to sort the data and draw the diagrams. They are presented in Appendix A, and can also be used for the extended QRA presented later.

The profile displays the information contained in the probabilities  $p_i$  and the consequences  $c_i$  for all scenarios, fire locations and hazard targets. A scenario is more safe the closer it is to the lower left corner of the diagram. The simple illustration in Figure 6.4



does not contain any information about the real quantitative risk as it only concerns the different values of  $p_{ET,i}$ . To complete the analysis, the initial scenario probability must also be entered and multiplied by the individual  $p_{ET,i}$ .

In some risk analysis the curve is not presented as a step function, but as a continuous function. If the number of subscenarios is high, the step function will tend to become a continuous line. But if this is not the case and the line connects the filled circles in the figure, the risk profile will give an erroneous result. The risk will be underestimated.

Another measure with which to present the societal risk is to condense the information in the risk profile into one number, the average societal risk. The average risk makes it possible to compare different design alternatives in a simple way. The average risk is basically the sum of the probabilities and consequences in all subscenarios, and can be expressed:

$$ASR = \sum_{i=1}^n p_i c_i \quad [6.1]$$

giving the average expected number of fatalities per year. It can be used as a tool to further analyse the risk in a building or at a specific location.

#### 6.4.2 Individual risk

The total individual risk for any particular occupant can be derived for each subscenario in the event tree and then summarised for all events to give a total individual risk,  $IR$ . Generally, the risk measure should be a summation over all considered fire scenarios, fire locations and hazard targets. The total individual risk can be used to compare different risk situations with each other e.g. a fire threat with a threat from a chemical plant. However, the appropriateness of the comparison should be considered.

The individual risk can be seen as the conditional risk in being at the location. When inside a building the momentary risk to which the occupants are individually subjected is the individual risk. The risk measure does not consider the time during which a person is in the building, for example, every year.

In the chemical process industry, other types of individual risk measures can be used in risk analysis. The maximum individual risk is the risk to which the most exposed person is subjected near a hazard as the hazardous zone, for example, when originating from a toxic gas release, is a function of distance from the source. Different distances imply different risks, and the number of exposed occupants may vary depending on the distance. The fraction of occupants at a specific location is used to weight the location-specific individual risk. Summing all these contributions results in the average individual risk. This means that selecting a sufficiently large area and a sufficiently high number of occupants may lead to a very small average individual risk.

As a fire in a building can be considered to be confined within a fire compartment, the risk is constant within the compartment. The individual risk to any occupant in the fire compartment will be the same, i.e. the maximum individual risk when defined as in CPQRA (1989). If the fire spreads to other compartments it might be possible to estimate an average risk measure. Because of the different definitions of individual risk, care should be taken when comparing individual risk measures especially weighted measures.

In the standard QRA, the individual risk is derived as a point estimate without any account of uncertainties being taken in the variables in the limit state functions. The individual risk is usually expressed as a probability of being affected by the unwanted consequence, per year. The individual risk for each scenario is obtained from

$$IR = \sum p_i \text{ for all } i \text{ in which } c_i > 0 \quad [6.2]$$

where  $p_i$  is the probability of subscenario  $i$  occurring. The  $p_i$  included in the individual risk measure are those for which  $c_i > 0$ , i.e. at least one person is not able to escape safely from the location. If the consequences  $c_i$  are 0 or less, there is no risk. This definition of individual risk is also adopted in the committee document ISO/CD 13387 (1997).

Because of the often comparative nature of the work in fire safety engineering,  $p_{initial}$  may sometimes be disregarded. The only probability that will enter into the analysis will then be  $p_{ET,i}$ . The differences originate from using different event trees. This can be a rational procedure as most other probabilities can be treated as equal in different design strategies. If, however, for example, different staff training procedures are assumed, differences might be found in the probability of the initiating event,  $p_{initial}$ . This simplification, i.e. excluding  $p_{initial}$ , can then not be made if a true risk analysis is the objective.

#### 6.4.3 Limitations

One problem with the standard QRA, when it is used in, for example, the chemical industry, is the way it handles actions taken by people at the time of the accident. If people are exposed to a hazardous condition they will most likely try to leave the area of danger.

This is normally not addressed in the traditional procedures for a QRA. The traditional standard QRA, does not assume that people try to evacuate. This means that subscenarios in which people have evacuated before untenable conditions have occurred, are also accounted for.

The individual and societal risk in fire safety engineering should not include subscenarios in which people have evacuated before untenable conditions have occurred, even if these conditions arise later in the fire development. This condition is a consequence of the limit state function approach. It means that the fire safety engineering risk measures will be a better prediction of the true risk as they consider people escaping the fire.

This, however, introduces a restriction in the risk measures compared with traditional QRA. The fire safety engineering risk measures are based on the condition of having a certain number of people present in the building when the fire starts. For a small number of people being at the location, they may be able to leave before untenable conditions arise, and this subscenario will not add to the risk. But if a higher number of people were at the same location, some of them may not be able to leave in time. This situation will therefore increase the risk. The risk measure is therefore dependent on the number of people present at start.

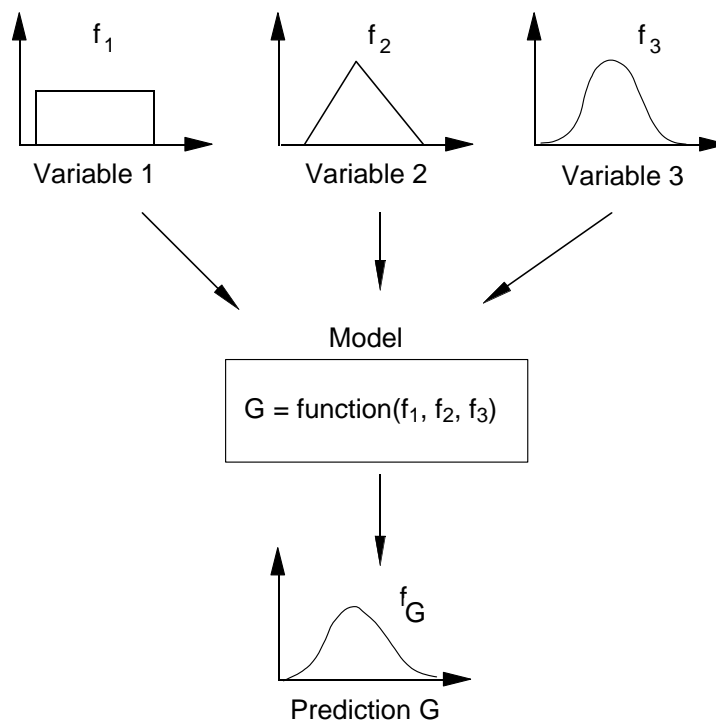
### **6.5 Uncertainty analysis**

In every risk analysis there are a number of variables which are of random character. This means that when single deterministic values are used, as in the standard QRA or in the routine design process, there is no way knowing how reliable the results are.

In many engineering fields, historically accepted or calculated design values have been derived to consider the inherent uncertainty. Using these values result in a design with a specified risk level. In the area of fire safety engineering, no such values are yet available and much engineering design is based on subjective judgement and decisions made by the architect. Values are then sometimes chosen on the conservative side and sensitivity analysis is performed to identify important variables.

A better way of illuminating the uncertainty in the results, is to carry out an uncertainty analysis in which the variables are

described by distributions instead of single values. The variation or uncertainty in a variable is described by its probability density function (PDF). The methods presented in this section are used to propagate the uncertainties of each variable through the limit state functions to result in an estimate of the joint probability density function. The result will be expressed as distributions of the limit state function  $G(X)$  or as confidence limits of the risk profile. Figure 6.5 shows schematically the propagation of uncertainties in the variables  $X_1$ ,  $X_2$  and  $X_3$ , through a model  $G(X)$ . The results of the uncertainty analysis can be used to improve the estimated risk measures, individual risk and societal risk.



*Figure 6.5. Propagation of uncertainty through a model.*

Depending on the level of uncertainty analysis, see Figure 6.2, there is a distinction between how the methods can be used and which are suitable for a specific task. Analysis can be performed on two levels:

- on a single subscenario described by one or more equations or
- on multiple subscenarios described by an event tree.

The difference is, in principle, whether analysis is carried out on one of the subscenarios in the event tree or if it considers the whole event tree. In the single subscenario analysis, three types of methods can be used:

- the analytical FOSM method,
- a numerical sampling method without distinction between the types of uncertainty, or
- a numerical sampling method distinguishing between two types of uncertainty; stochastic uncertainty and knowledge uncertainty.

The multiple scenario analysis is more straightforward and is basically an extension of the standard QRA procedure, but the uncertainty in the variables is explicitly considered. It is achieved by numerical sampling procedures.

For both levels, the description of the consequences employs limit state functions in which both deterministic and random variables can be included.

### ***6.6 The single subscenario***

In this case, the consequence limit state is described by one or more analytical equations in random variables. The methods determine how uncertainties in variables are propagated through the limit state functions. Usually, only one equation is used to represent the consequences.

The methods result in the probability,  $p_{u,i}$ , which can be seen as the probability of failure of the system described by the analytical equation. Probability of failure is the common term in structural reliability analysis. The term failure is usually equivalent to the case when the load is higher than the strength, i.e. generally expressed as  $G(X) < 0$ , see Section 4.1.1, where  $G(X)$  represents the limit state function.

For evacuation scenarios, this is equivalent to the escape time exceeding the available time. If numerical sampling methods are used detailed information is provided on the shape of the resulting distribution. This means that probabilities other than  $P(G(X) < 0)$  can be obtained. This information can be used to estimate the risk of multiple fatalities for this single subscenario. This is, however, not common procedure as the multiple fatality risk is usually derived for the whole scenario using the standard or the extended QRA technique.

The analytical method does not provide information about the PDF but has other advantages. Apart from the probability of failure, it provides information on the location of the so-called design point. The design point is defined by a set of variable values which, when combined in the limit state function, results in the highest probability of failure. The analytical method can, therefore, be used to derive design values based on a specified probability of failure, see Chapter 9.

#### 6.6.1 The analytical reliability index $\beta$ method

The reliability index  $\beta$  method has been used extensively in the area of structural engineering. It has also been applied to other scientific fields such as in public health assessment (Hamed, 1997) and in fire safety assessment (Magnusson et al. 1994 and Magnusson et al., 1995; 1997). It is a supply-versus-demand-based model and it provides information about the reliability of the system described by the limit state function. The term reliability is

here defined as the probabilistic measure of performance and expressed in terms of the reliability index  $\beta$ .

As both the supply and the demand sides of the problem are subject to uncertainty, some situations may occur in which the demand exceeds the supply capacity. This introduction will be limited to treating single limit state function representations of a single subscenario.

When multiple failure modes are present, a similar slightly modified methodology can be used, see Section 6.6.2. The failure mode describes one manner in which the system can fail, i.e. when at least one occupant in a building is unable to evacuate. In each subscenario the failure modes are described by the limit state functions.

Let the random variables be defined by:

$$\begin{aligned} R &= \text{supply capacity} \\ S &= \text{demand requirement} \end{aligned}$$

and the safety margin,  $M$ , by

$$G = M = R - S \quad [6.3]$$

The objective of the analysis is to determine the reliability of the event  $R < S$  in terms of the probability  $P(R < S) = p_{u,i}$ . If the probability density functions of  $R$  and  $S$  are known and if  $R$  and  $S$  are statistically independent, the probability of failure of the system may be derived as

$$p_{u,i} = \int_0^{\infty} F_R(s) f_S(s) ds \quad [6.4]$$



where  $F$  denotes the cumulative distribution function and  $f$  the probability density function.

If the variables  $R$  and  $S$  are correlated, the probability of failure can be derived from the joint probability density function  $f_{R,S}(r, s)$ . There are, however, only a few cases in which the joint probability density function can be derived analytically. In other cases it can be derived by numerical integration methods or with Monte Carlo sampling technique. There is still a need for a simple method to estimate the reliability of systems described by one limit state function.

One such method is the First Order Second Moment (FOSM) method. The limit state equation is approximated by a first order Taylor expansion and the method uses the two first moments, i.e. the mean and the standard deviation.

For design purposes, the FOSM method can be used on different levels, depending on the amount of information available. In the literature concerning the reliability of structural safety, four levels can be identified directly or indirectly linked to FOSM methods, Thoft-Christensen et al. (1982).

- Level 1. Deterministic method. The probability of failure is not derived directly but the reliability is expressed in terms of one characteristic value and safety factors or partial coefficients. This is normally the level at which design is carried out.
- Level 2. The probability of failure can be approximated by describing the random variables with two parameters, usually the mean value and the standard deviation. No consideration is taken of the type of distribution. This level is used when information regarding the statistical data is limited and the knowledge regarding the distribution type is lacking (FOSM method).

- Level 3. Analysis on this level considers the type of random variable distribution. The "true" probability of failure can be derived by numerical methods. If the variables are normally or lognormally distributed, noncorrelated and the limit state function is linear, exact analytical methods are available. Otherwise, the probability of failure will be approximated.
- Level 4. On this level, economical aspects are also considered in the analysis in terms of a cost-benefit analysis.

The analysis in this thesis will be executed on levels 2 and 3. Higher order levels can be used to validate lower level methods. In Chapter 9, a method will be presented which derives design values on level 1 based on uncertainty analysis carried out on level 2, valid for certain classes of buildings.

The following condensed presentation of the FOSM method will be limited to a level 2 analysis using a nonlinear limit state equation, for noncorrelated variables. Correlated variables and higher order analysis levels can be treated similarly and the reader is referred to more detailed references (Thoft-Christensen et al., 1982), (Ang et al., 1984) and (Madsen et al., 1986).

The reliability or measure of safety is defined by the reliability index  $\beta$ . This contains information regarding both the mean value and the standard deviation of the safety margin. There are different definitions of the reliability index  $\beta$ . The first was introduced by Cornell in the late 1960s (Cornell, 1969).

The mean and the standard deviation of the margin can be derived as

$$\mu_M = \mu_R - \mu_S \text{ and} \quad [6.5]$$

$$\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2} \quad [6.6]$$

when recalling that  $G(X) = M = R - S$ , Eq. [6.3].

The reliability index is defined by Cornell as

$$\beta_C = \mu_M / \sigma_M \quad [6.7]$$

If the parameters  $R$  and  $S$  are normally distributed, the margin  $M$  is also normally distributed. The parameter  $(M - \mu_M)/\sigma_M$  is then  $N(0,1)$  and the probability of failure can be derived as

$$p_{u,i} = 1 - \Phi(\beta_C) \quad [6.8]$$

where  $\Phi$  is the normal cumulative distribution function. Using this definition the reliability index is a measure of reliability that can be transformed into a probability of failure of the limit state equation. It can also be used to better predict the individual risk.

There have been several objections to using  $\beta_C$  as it is not consistent with some definitions of the limit state function (Thoft-Christensen et al., 1982). A better measure of reliability is the Hasofer and Lind reliability index  $\beta_{HL}$  (Hasofer et al., 1974). This reliability index is defined as the shortest distance between the failure surface, defined by  $G(X) = 0$  and the standardised origin, see Figure 6.6. The standardised variables are expressed as

$$X' = \frac{X - \mu_X}{\sigma_X} \quad [6.9]$$

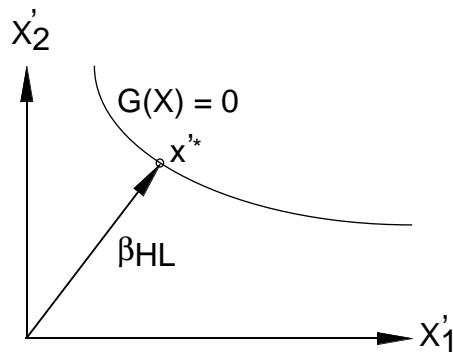


Figure 6.6. Reliability index  $\beta$  and the limit state function in the standardised space.

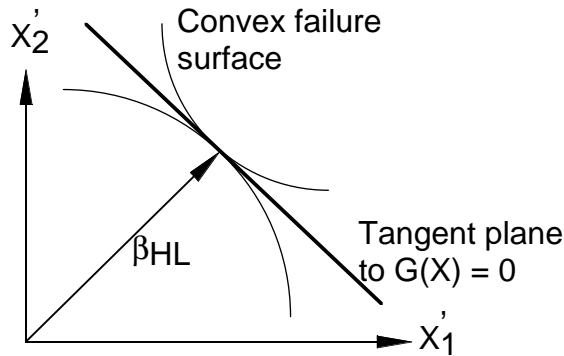
The point  $x'^*$  on the failure surface is of considerable importance. It is the so-called design point. If a design is carried out using values from the design point this will result in the highest probability of failure, thereof the name.

The method can be further extended to derive design values valid for a certain class of buildings. Design values on level 1 may be based on a specified risk level, expressed in terms of a target reliability, cf. the method presented in Chapter 9.

If the limit state function is linear and the variables are noncorrelated, the shortest distance, i.e.  $\beta_{HL}$ , can be found directly using basic algebra. However, this is seldom the case. When the limit state function is nonlinear, approximate methods must be considered, for example, geometrical optimisation of the problem which can be solved numerically or analytically.

The analytical methods are usually based on a first order Taylor approximation of the limit state function. The approximation may either be on the safe or unsafe side, depending on the curvature of the failure surface, see Figure 6.7. If the failure surface is concave

the result will be an unsafe estimate. There are methods to determine the extent of this uncertainty (Augusti et al., 1984).



*Figure 6.7. Tangent plane approximation to  $G(X) = 0$ .*

An iterative method that can be used to find  $\beta_{HL}$ ,  $x_i^*$  and an estimate of the probability of failure is the Rackwitz algorithm (Ang et al., 1984). In the form in which it is presented here, noncorrelated variables are assumed and that it is possible to approximate the non linear limit state function by a first order Taylor expansion. The variables are expressed by the two first moments, the mean and the standard deviation, i.e. the analysis is on level 2.

The design point can be expressed in scalar form as

$$x_i^* = -a_i^* \beta_{HL} \quad [6.10]$$

where  $a_i^*$  are the direction cosines (the unit vector) in the  $x_i'$  direction

$$a_i^* = \frac{\left( \frac{\partial g}{\partial X_i} \right)_*}{\sqrt{\sum_i \left( \frac{\partial g}{\partial X_i} \right)_*^2}} \quad [6.11]$$

The procedure employed in the Rackwitz algorithm is as follows:

1. Assume initial values for  $x_i^*$  for  $i = 1$  to  $n$
2. Calculate  $x_i^{**} = \frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}}$
3. Evaluate  $\left( \frac{\partial g}{\partial X_i} \right)_*$  and  $a_i^*$  at  $x_i^*$
4. Calculate  $x_i^* = \mu_{X_i} - a_i^* \sigma_{X_i} \beta$
5. Substitute  $x_i^*$  in  $g(x_1, x_2 \dots x_m) = 0$  and solve the equation system for  $\beta$
6. Use  $\beta$  to improve the values of  $x_i^{**} = -a_i \beta$
7. Repeat steps 3 to 6 until convergence in  $\beta$  is obtained.

The probability of failure  $p_{u,i}$  can be estimated by Eq. [6.8] replacing  $\beta_C$  by  $\beta_{HL}$ , assuming a normal distribution of the state function.

It is possible to consider the type of distribution explicitly by using the method on a level 3 approach by transforming non-normal variables to equivalent normal variables. Procedures are also available to consider correlated variables and a second order approximation of the limit state equation. In such cases, hand

calculations are not recommended as they become rather complex. Commercial computer programs are available that can handle these situations, for example, STRUREL from RCP in Germany (STRUREL, 1994) and Proban from Det Norske Veritas in Norway (Tvedt, 1989). STRUREL was used for the FOSM calculations in this thesis.

The advantage of the reliability index  $\beta$  method is that it is simple to use, and it has been widely used in structural and off-shore reliability studies. It can be used to derive sensitivity measures showing the importance of the variables, see Section 6.6.4. Another advantage is that it provides the design point at which the probability of failure is highest. It can therefore be used to derive design values for a level 1 design.

The most obvious disadvantage is that the method does not provide any information regarding the distribution of the limit state equation. The only measure that can be used for subsequent risk analysis is the probability of failure which on a level 2 analysis is an estimate of the true probability of failure. Methods on levels 3 and 4 can be used if the variable distributions are known. A better estimate of the probability of failure can be obtained by integrating the relationship in Eq. [6.4].

The program STRUREL employs an additional sampling procedure of the distributions in order to derive a better prediction of the the probability of failure than when the normal distribution is chosen as on level 2.

### 6.6.2 Reliability index $\beta$ method with multiple failure modes

Multiple failure modes can exist in a subscenario if the consequences are determined at more than one location or if failure can result from different sources. An example of the former when a floor consisting of a corridor and adjacent rooms, is treated as one system. The resulting consequences are determined on the floor as

one unit but the consequences in the corridor and in each of the rooms are derived from separate limit state functions.

The interest is to determine the uncertainty for the floor as such, i.e. from a system point of view. Therefore, both failure modes, expressed in terms of the limit state functions for the corridor and for the room, must be considered jointly. This is the situation in the example in Chapter 7.

Another situation where more than one limit state function can be used is when the consequences can be related to different types of fire-related criteria. Different functions can be used to determine the unwanted consequences depending on whether the limit state is reached due to heavy smoke, high temperature level, radiation, etc.

All these limit states generate separate state functions and must be treated as separate failure modes. In fire safety engineering, it is, however, convenient to treat them as one limit which is determined by the condition first reached. Traditionally, this is the procedure by which fire safety design is carried out.

Therefore, the system can be treated as a series system, which fails when at least one mode fails. Failure is then defined as "at least one occupant is unable to evacuate". The FOSM problem can be described as an extension of Figure 6.7, where each failure mode is defined by a separate failure surface, see Figure 6.8.

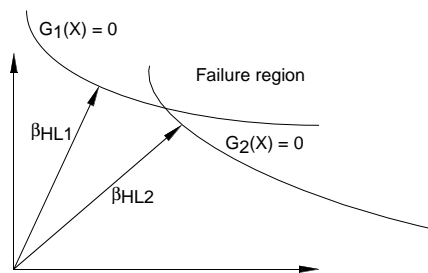


Figure 6.8. Subscenario with two failure modes.



The general probability of failure can be determined by the integral

$$p_{u,i} = \int \dots \int_{x_1, x_2, \dots, x_n} f(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad [6.12]$$

The integral should be calculated over all failure regions. This equation is normally very difficult to solve exactly, but upper and lower bounds for the probability of failure can be determined.

If the different failure modes are independent, i.e. the limit state functions are not correlated, the joint probability of failure can easily be determined. A probability of failure,  $p_{f_j}$ , is linked to each mode,  $j = 1, 2, \dots, n$ . The joint probability of failure can then be determined as

$$p_{u,i} = 1 - \prod_{j=1}^n (1 - p_{f_j}) \quad [6.13]$$

where  $j$  is the number of possible failure modes. This assumption, of independence is definitely not evident in the cases considered in this thesis. When the failure modes are related to the same fire hazard, they are correlated. The correlation in this case is positive as the fire will result in similar consequences in all failure modes, i.e. at all locations. A rapidly growing fire results in short times available for evacuation at all locations on the fire-affected floor.

If the correlation between the failure modes is known, methods are available to determine the exact probability of failure. In other cases, it is usually sufficient to be able to determine an interval in which the exact probability of failure is located. If the modes are completely correlated, i.e.  $\rho = 1$ , the probability of failure is determined by the mode which has the highest individual probability of failure.

The interval for  $p_{u,i}$  can now be estimated as

$$\max_j p_{f_j} \leq p_{u,i} \leq 1 - \prod_{j=1}^n (1 - p_{f_j}) \quad [6.14]$$

where the bounds are determined by the degree of correlation. The equation is only valid for positively correlated functions and when the modes are in series. Negative correlation can also be determined by the same procedures see, for example, Ang et al. (1984).

The interval described above is relatively broad and a narrower interval is sometimes preferable. Other bounds have been developed, for example, the Ditlevsen bounds, which are applicable for series systems and give the best result when  $\rho < 0.6$ . In the examples given in Chapter 7, the Ditlevsen bounds are derived together with the interval as defined above. The theoretical basis of the Ditlevsen bounds is described in Ang et al. (1984) and Thoft-Cristensen et al. (1982).

### 6.6.3 Numerical sampling methods

With today's computer power numerical simulation of the uncertainty propagation process in an equation can be efficiently carried out by sampling methods. Complex limit state functions including iteration procedures can be efficiently handled. The method is based on samples which are drawn from each variable distribution,  $f_1, f_2 \dots f_n$ .

Each set of samples is used in the limit state equation to calculate one value of the variable  $G$ , see Figure 6.3. Repeating this, for example 2000 times, results in 2000 values of the variable  $G$ . The number of repeated calculations, or iterations, necessary depends on the convergence criteria chosen. The simulation can be stopped when the relative change in the mean or standard deviation of the distribution of  $G$  is sufficiently small.

The result is an approximation of the joint PDF for the variable  $G$ . As mentioned previously, two types of numerical methods can be identified. In the first method both types of random variability, stochastic uncertainty and knowledge uncertainty, are treated without distinction. This results in a single PDF including both types of uncertainty. This method is denoted single phase method.

The second method, the two-phase method, distinguishes between stochastic uncertainty and knowledge uncertainty. The effects of these two types of uncertainty can therefore be observed separately.

In the single phase method, the uncertainties in the variables,  $f_i$ , are represented by the uncertainty of  $G$ . It is then possible to state the degree of confidence in the parameter  $G$  based on the uncertainty. The result is a single PDF, CDF or CCDF. The probability of failure is normally defined as the probability that  $G < 0$ ,  
 $P(G < 0) = p_{u,i}$ .

This probability can be used to derive a better estimate of the individual risk measure, see Section 6.7.2. The probability of failure represents the probability that the unwanted event will occur given the fact that the subscenario has occurred. It defines the probability of failure due to the inherent uncertainties in the subscenario. The samples can be treated with traditional statistical methods to derive the mean value, standard deviations and confidence limits, etc. Commercial computer programs are available, for example, @Risk (@Risk, 1994).

Two different types of sampling procedures have been used, Simple Random Sampling (SRS) and Latin Hypercube Sampling (LHS). The SRS method is what is normally called a random sampling method. The LHS method is usually more efficient and provides better agreement between the sample distribution and the theoretical "true" distribution. The better agreement is also seen in the representation of extreme values. The reason behind this, is that the samples are stratified within the distribution bounds. The LHS

method is also better to predict very high or very low values. The method is described in IAEA (1989).

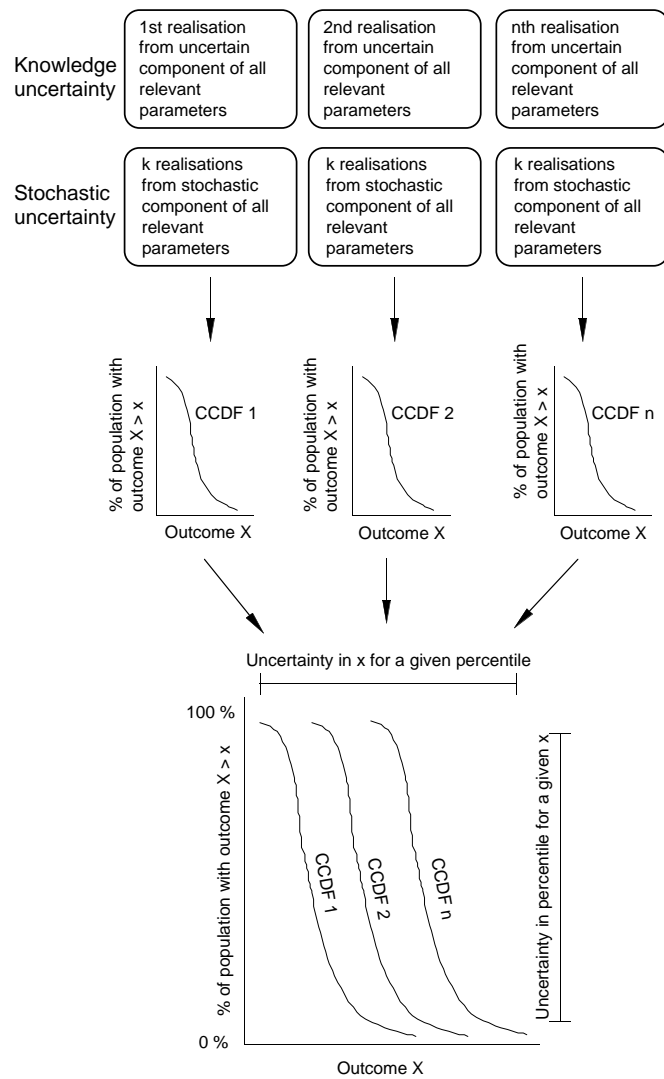
The disadvantage with LHS is that there are no methods available to determine confidence limits for fractiles in the sample. This is, however, not a serious drawback for risk analysis but might be important in model testing, IAEA (1989).

In cases where both uncertainty types, stochastic uncertainty and knowledge uncertainty, are present, the two-phase method should be used. Using this method, it is possible to distinguish the influence of the two types of uncertainty on the overall uncertainty. This may indicate a possibility of reducing the knowledge uncertainty, leaving the stochastic uncertainty, which cannot easily be reduced.

Briefly, the procedure is as following: Let  $\mathbf{X}_s$  denote the vector of variables with stochastic uncertainty and  $\mathbf{X}_k$  the vector of variables characterised by knowledge uncertainty. Single random elements of the vectors are called  $x_{s,i}$  and  $x_{k,i}$ , respectively. Sample vectors are called  $\mathbf{x}_s$  and  $\mathbf{x}_k$ , respectively. See Figure 6.9 which is adopted from MacIntosh et al. (1994).

First, single values,  $x_{k,i}$ , are randomly sampled, using SRS, from distributions representing knowledge uncertainty forming a vector  $\mathbf{x}_k$ . When this vector is determined, random samples are drawn, using LHS, from each of the stochastically varying parameters,  $X_{s,i}$ , giving a vector  $\mathbf{x}_s$ .

Keeping the  $\mathbf{x}_k$  vector constant, the last step is repeated, for example  $k = 2000$  times, resulting in a single CCDF curve for the assessment outcome variable,  $G$ . New values are sampled from  $\mathbf{X}_k$ , i.e. the next values of  $\mathbf{x}_k$ . The procedure is repeated  $n$  times, producing  $n$  CCDF curves for the assessment outcome, in this case the number of people not able to escape safely.



*Figure 6.9. Two-phase simulation of uncertainty in one scenario.*

The parameter  $n$  is here the number of simulations resulting in the representation of knowledge uncertainty. The value of  $n$  depends on the level of accuracy of the confidence limits of the CCDF curves. The number of times each of the variability iterations is repeated,  $k$ , is chosen according to the desired level of accuracy of the results in each CCDF.

In the sample calculation in Chapter 7, using this method, a value of  $n$  equal to 59 was used. The reason for choosing  $n = 59$  is explained below, and is based on the description of the derivation of distribution-free statistical limits from a simple random sample (IAEA, 1989):

"Upper ( $u\%$ ,  $v\%$ ) statistical tolerance limits are upper  $v\%$  confidence limits for the desired  $u\%$  fractile. Therefore one may be  $v\%$  confident that they are not underestimates of the desired  $u\%$  fractile. The smallest value  $n$  that satisfies the requirement

$$1 - (\text{fractile percentage } u/100)^n \geq \text{confidence level percentage } v/100$$

is the size of a simple random sample such that the maximum prediction value in the sample is an upper ( $u\%$ ,  $v\%$ ) statistical tolerance limit. For  $u = v = 95$  one obtains a sample size of  $n = 59$ .

Thus computation of the prediction value for only 59 m-tuples of parameter values from a simple random sample suffices for the maximum prediction value in the sample to be an upper 95% confidence limit of the desired 95% fractile of the subjective probability distribution of the model prediction.

It is not necessary to assume a particular type of distribution of the model prediction in the derivation of these limits. For this reason they are called 'distribution free' tolerance limits. It should be noted that the sample size required to obtain a

distribution free ( $u\%$ ,  $v\%$ ) statistical tolerance limit is independent of the number  $m$  of uncertain parameters and is determined by  $u$  and  $v$  only."

Practically this means that the left- and right-hand extreme curves in a diagram with 59 CCDF curves are the 5th and 95th percentile, respectively, with a confidence degree of 95%. In this way, it is possible to obtain a confidence interval for the safety margin.

The CCDF curves from the 59 different calculations can be interpreted in terms of uncertainty. The uncertainty due to stochastic uncertainty can be observed as differences in the slope of the CCDF curves or, more precisely, the difference between the highest and lowest value of a single CCDF. Large differences or low slopes indicate a high variability.

The uncertainty coupled to knowledge uncertainty can be seen as the deviation between the far left and far right CCDF in the diagram. A large distance between these two curves indicates a large knowledge uncertainty, see Figure 6.10.

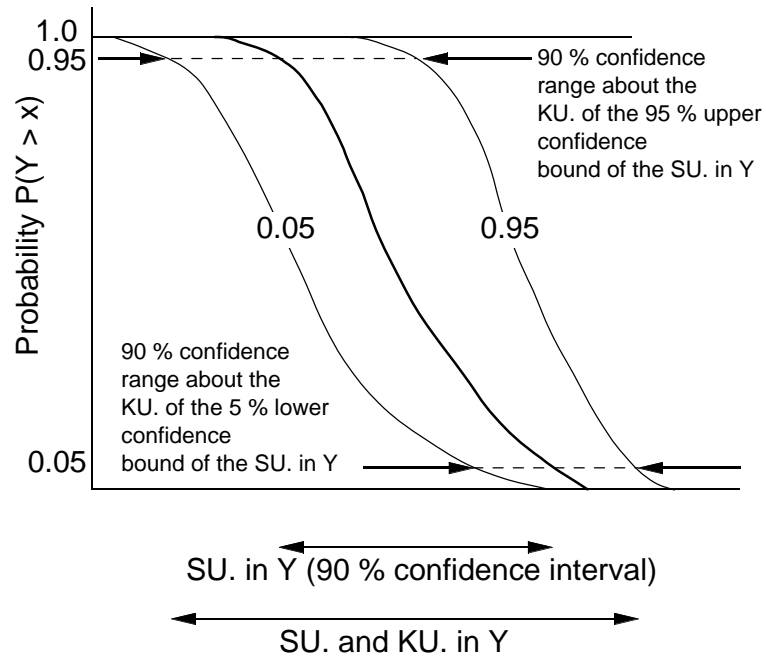


Figure 6.10. Interpretation of the results from a two-phase simulation, McCone (1994). SU = stochastic uncertainty and KU = knowledge uncertainty.

#### 6.6.4 Importance of variables

In addition to an uncertainty analysis it is also important to determine the contribution to the total uncertainty from each variable or function of variables. Different ranking methods have been developed, both analytical and numerical. Correlated variables may make the ranking difficult and measures must be taken to eliminate the problem (IAEA, 1989).

##### *Analytical method*

The most simple method of determining the relative importance of variables with respect to their uncertainties is to compare the



variances. The variance of the joint distribution,  $G$ , is composed of the variances of each of the variables. These are propagated through the limit state functions. For linear problems, the variance of the limit state function can easily be determined as

$$\begin{aligned} \text{Var}(G) &= \text{Var}(b_1 X_1 + b_2 X_2 + \dots + b_n X_n) = \\ & b_1^2 \text{Var}(X_1) + b_2^2 \text{Var}(X_2) + \dots + b_n^2 \text{Var}(X_n) \\ & + 2 \sum_{i=1}^n \sum_{j=i+1}^n b_i b_j C(X_i, X_j) \end{aligned} \quad [6.15]$$

The fractions  $\frac{b_i^2 \text{Var}(X_i)}{\text{Var}(G)}$  can be used to determine the order in which the variable uncertainties affect the total uncertainty.

In engineering problems, however, such linear relationships are not very frequent. Most problems are nonlinear and other methods must be used.

One simple method is based on a first order approximation of the limit state function following the same methodology as the reliability index  $\beta$  method presented in Section 6.6.1. An estimate of the variance of  $G$  can be derived from the first order Taylor expansion of the limit state function

$$\text{Var}(G) \cong \sum_{i=1}^n \sigma_{X_i}^2 \left( \frac{\partial G}{\partial X_i} \right)_*^2 = \sum_{i=1}^n \left( \frac{\partial G}{\partial X_i} \right)_*^2 \quad [6.16]$$

The variance should be evaluated at the most probable failure point, indicated in the equation by the asterisk. The equation is only valid for noncorrelated variables. Each variable component of the total variance in the standardised space is equal to

$$\text{Var}(X_i) = \left( \frac{\partial G}{\partial X_i} \right)_*^2 \quad [6.17]$$

In a similar way as for the linear problem, the contribution of the variance of each variable to the total variance can be derived as

$$a_i^* = \frac{\left( \frac{\partial g}{\partial X_i} \right)_*}{\sqrt{\sum_i \left( \frac{\partial g}{\partial X_i} \right)_*^2}} \quad [6.11]$$

which is the same as the direction cosines in the FOSM method of deriving the reliability index  $\beta$  and the probability of failure. The direction cosines, in the vector  $\mathbf{a}$ , can therefore be used as a measure of the relative importance of the variables. Better estimates of the importance measures can be obtained by using the second order Taylor expansion of the limit state function. Correlated variables can be treated with the same technique, but the correlation between the variables must be considered (see Ang et al., 1975).

#### *Numerical methods*

A number of different ranking methods are available when numerical sampling procedures are used to determine the joint distribution of the limit state function (IAEA, 1989):

- correlation coefficients
- partial correlation coefficients
- standardised partial regression coefficients
- rank correlation coefficients

Most of these determine the degree of correlation between variables and the limit state function  $G$  in terms of the linear or nonlinear relationship between  $G$  and  $X_i$ . Most coefficients are calculated with the commercially available computer packages which are used for the sampling procedures.

### **6.7 Extended quantitative risk analysis**

#### **6.7.1 Societal risk**

The standard QRA is performed without explicitly considering the uncertainty which is inevitably present in each variable. Instead, the variables are assigned values which, for example, are on the safe side, i.e. conservative estimates which will cover the credible worst cases. Other possible values that can be used in the standard QRA are the most likely values.

The results from such an analysis are usually presented as a risk profile, Section 6.3.1, at least for the societal risk measure, but such profiles do not contain any information on the uncertainty. If one wishes to know how certain the calculated risk profiles are the uncertainties in the variables involved must also be considered. To obtain this information, risk analysis, according to the standard QRA method, should be combined with an uncertainty analysis. Formalising this methodology results in the extended QRA.

The extended QRA can be used to express the degree of credibility in the resulting median risk profile by complementing the profile with confidence bounds. Similarly, it is possible to state the degree of accomplishment of defined goals, for example, expressed in terms of tolerable risk levels defined by society. Figure 6.11 shows schematically the median risk profile with its upper and lower confidence bounds. A cut curve can be drawn for a value on the horizontal axis showing the risk profile probability for a given consequence value. This cut curve results in, for example, the frequency of the 5th and the 95th percentiles for the risk profiles on condition of a specified value of the consequences.

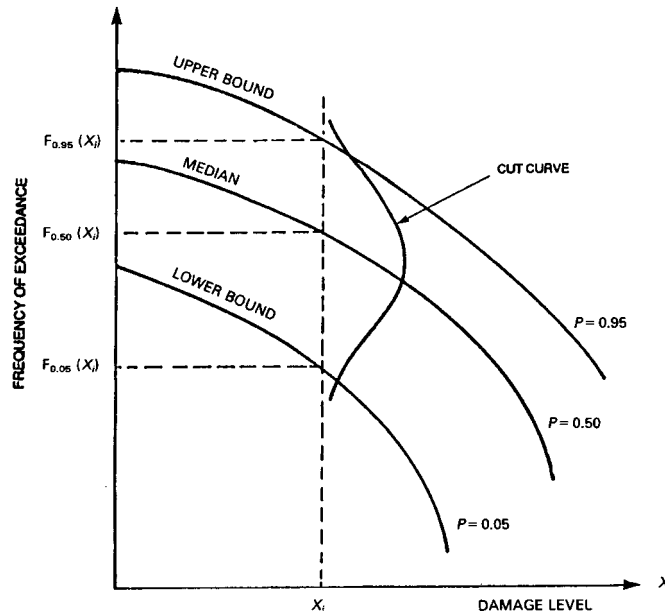
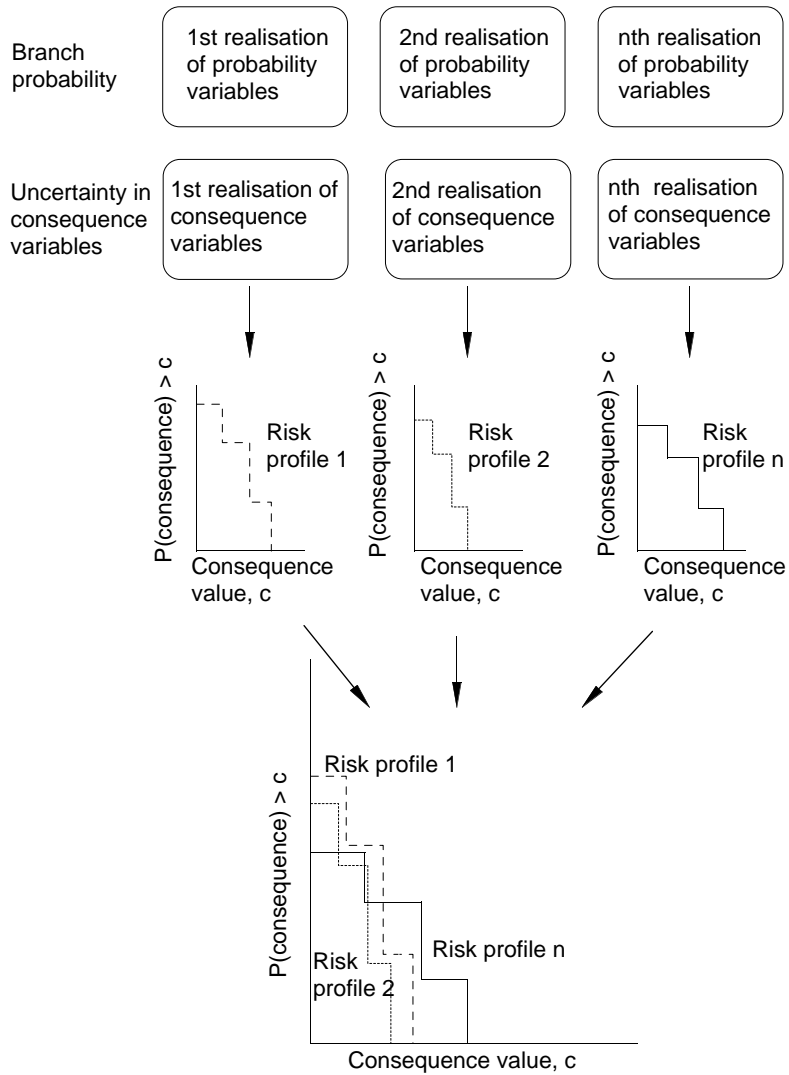


Figure 6.11. Uncertainty in risk profiles (CPQRA, 1989).

The procedure for performing an extended QRA is similar to that for the standard QRA. As the variables are not constant but are expressed in terms of frequency distributions, the propagation of uncertainty must be modelled for all subscenarios simultaneously. Simply, the process can be seen as a standard QRA which is repeated a large number of times. For each new iteration, the variables are assigned new values according to the frequency distribution. This results in a new risk profile for each iteration, providing a family of risk profiles. The family of risk profiles can be used to describe the uncertainty inherent in the resulting risk measure.

The technique employing triplets can also be used for the extended QRA. The information concerning the state of knowledge of the variables must be included in the representation of both  $p_i$  and  $c_i$ ,

i.e. both the branch probability and the consequence are subject to uncertainty. Figure 6.12 shows the process schematically.



*Figure 6.12. Uncertainty analysis of a QRA.*

The societal risk resulting from the extended QRA can be expressed in terms of a family of risk profiles such as those shown in Figure 6.13. It is clear that the information is very extensive. Therefore, alternative presentation methods may have to be used in order to be able to interpret the information. A better method is to present the societal risk profiles in terms of the median or mean risk profile and to complement these with relevant confidence bounds. The confidence interval can, for example, be the 80% interval. Doing this for the example in Figure 6.13 results in Figure 6.14.

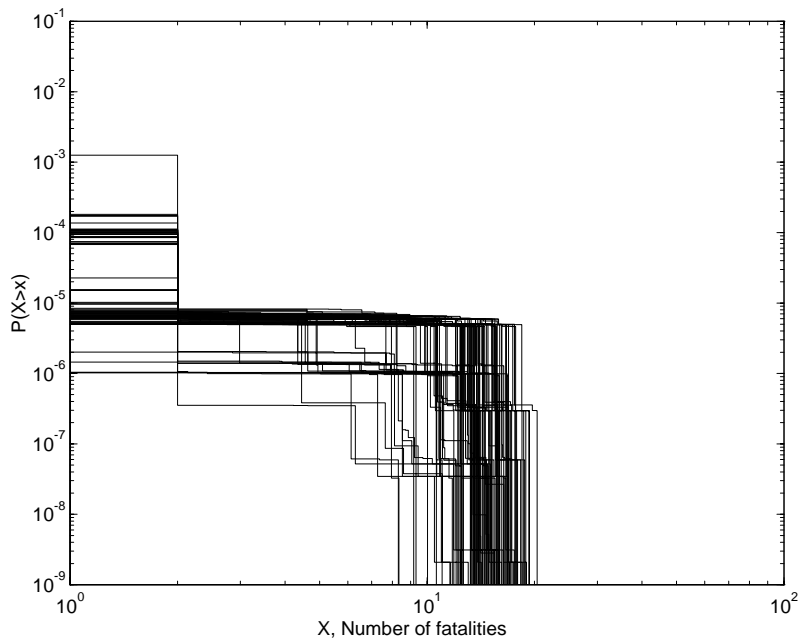
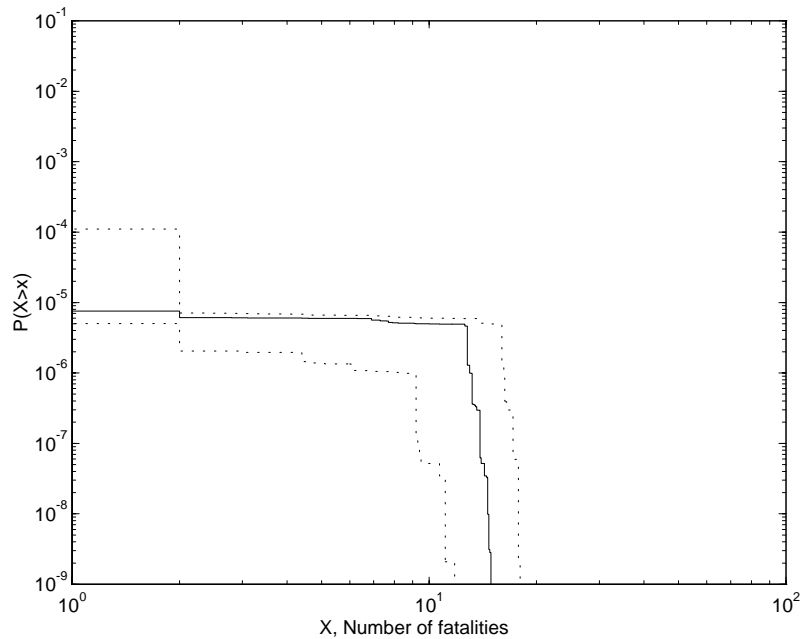


Figure 6.13. Family of risk profiles from an extended QRA.



*Figure 6.14. Simple presentation technique for the extended QRA method. Lines indicate the median risk profile together with the 10th percentile and 90th percentile confidence bounds.*

The confidence limits are constructed from the family of risk profiles in the following manner. For each point on the horizontal axis, an imaginary vertical line is drawn. This line crosses each of the individual risk profiles once. The probability values, i.e. the values on the vertical axis, for these points of interception can be used to determine the mean value, the median value and the desired confidence level values, for each imaginary line drawn from the horizontal axis.

These new values, for example the median values, for each horizontal axis value, can be plotted in a diagram. The values are derived in the vertical direction indicating the uncertainty in this direction. This means, for example, that the confidence limits are

derived on condition of the value on the horizontal axis, cf. Figure 6.11.

Uncertainty is normally associated with both the subscenario outcome probability and with the description of the subscenario consequence. In defining the bounds of the analysis some of these uncertainty variables may be considered less important. Situations can occur where the subscenario probability,  $p_i$ , can be treated as a deterministic value. This can be done if these probabilities are known to a high degree of confidence. As a consequence of this the extended analysis can be divided in two subcategories depending on which variables are assumed to be random.

The first subcategory only considers the uncertainty in the description of the consequences and treats the branch probabilities as deterministic values. When the branch probabilities are fixed between each iteration, the triplet representation can be written  $R = \{(s_i, p_i, \zeta_i(c_i))\}$ , i.e. the probabilities do not change, cf. Chapter 3. In this case, Figure 6.12 can be simplified as the branch probabilities are the same for each iteration.

The second subcategory considers the uncertainties in both branch probability and consequence. The complete set of triplets can now formally be written  $R = \{(s_i, p_i(\phi_i), \zeta_i(c_i))\}$ . The total uncertainty in the risk profile will be increased by adding the branch probability. Both subcategories can, however, be seen as extended QRA procedures.

In the same manner as for the standard QRA, the average risk can be calculated. But as the variables are subject to uncertainty the average risk will also be subject to uncertainty and will consequently be presented as a distribution. Each iteration will generate one sample of the average risk derived according to Eq. [6.1]. These average risk values will form the distribution of the average risk.



### 6.7.2 Individual risk

When a risk analysis is combined with an uncertainty analysis, it is possible to consider the uncertainty in the individual risk measure. Some combinations of the variables used to derive the consequence in a subscenario will lead to conditions resulting in fatalities or blocked escape routes. Similarly, due to randomness, some subscenarios will not always contribute to the individual risk measure. Therefore, there will be a degree of uncertainty in the individual risk originating from variable uncertainty.

The individual risk resulting from the extended QRA,  $IR_u$ , can be expressed in terms of a distribution, for example a CDF, instead of just one single value. The CDF can be condensed into a more easily comparable single value, still including information regarding the uncertainty. The distribution shows, however, the uncertainty in individual risk in a more complete manner. The condensed single value  $IR_u$  can be obtained as the mean value from the distribution of individual risk.

This deterministic value can also be obtained as

$$IR_u = \sum p_i p_{u,i} \quad [6.18]$$

where  $p_{u,i}$  is the probability that the subscenario leads to the unwanted event.

The probability,  $p_{u,i}$ , can preferably be estimated from the probability that  $c_i > 0$ , which is a result of the uncertainty analysis, cf. Sections 6.5 and 6.6.

This more general expression of the individual risk, Eq. [6.18], can naturally also be used in the standard QRA, without explicitly performing the uncertainty analysis. The probability,  $p_{u,i}$ , must then be obtained as a qualitative estimate. In this way a full uncertainty analysis need not be performed, but a possibly better single value estimate of the individual risk can be obtained. However, this

approach should be used with the greatest care as it is always difficult to make estimates of such a quantity. In some cases it might, however, be the only alternative.

The extended QRA is performed as a numerical procedure which involves Monte Carlo sampling techniques. Today, no commercial software exists that can handle both subscenario variable uncertainty and branch probabilities. Therefore, a set of Matlab routines has been developed to perform the sorting of data and plotting of curves. The input to these Matlab m-files is in the form of vectors or matrices composing the set of triplets for the event tree. The m-files are described in Appendix A and are ready to use.

### 6.7.3 Application of the method

The extended QRA has not been widely used, and standard methods for its application have not been developed, cf. the standard QRA. Some analysis has been performed on the problem of isolating radioactive waste in long-term storage facilities (Helton, 1994). This method of analysis has not yet been applied to fire safety engineering.

After having studied the single subscenario uncertainty analysis, a justified question is: "Is it possible to determine the influences from the two types of uncertainty, stochastic and knowledge uncertainty?". The answer to that question is both yes and no.

Yes, it is possible to determine the stochastic uncertainty and distinguish it from the knowledge uncertainty. The requirement for this is that it is possible to assign one type of uncertainty giving rise to the single CCDF and the other uncertainty type giving rise to the distribution of CCDFs. The single CCDF occurs because each of the subscenarios occurs with a particular probability.

According to (Helton et al., 1997), this is equivalent to the condition that the subscenarios take place randomly and the uncertainty is referred to as stochastic. The knowledge uncertainty

is the type of uncertainty that generates the uncertainty in the outcome of the subscenarios. The description of the subscenario will, therefore, be defined by variables which are subject to knowledge uncertainty, which can be reduced if better information is available. These variables, defining the subscenario consequences, have to be assigned parameters according to the best knowledge and they are not random due to arbitrariness in the nature.

In this sense, stochastic uncertainty and knowledge uncertainty can be differentiated. But, what happens if there is also a variability in the description of the outcome consequences? There are then stochastic uncertainty contributions to both the description of the single CCDF and to the distribution of the CCDFs.

This is the present situation regarding the application of this approach to fire safety engineering. There is stochastic uncertainty, actually to a large extent, also in the description of the fire hazard. There is currently no information available that can be used to predict the fire development that can be expected in an arbitrary room, in for example a hospital ward or an apartment. Uncertainty concerning the contents of the room and the source of ignition and fire development makes it impossible to determine these variables based on past experience and models.

Also, factors such as; is the window open? and will there be occupants present able to extinguish the fire?, are impossible to determine in advance. These factors have to be defined as random by nature. The stochastic uncertainty will, therefore, be part of the description of the outcome consequences.

It is, therefore, not relevant to use the terminology "distinction between stochastic uncertainty and knowledge uncertainty" in fire engineering risk analysis. It is more important to describe what the extended QRA risk profiles really mean, i.e. a description of the overall uncertainty in societal risk.

## **7 Sample risk analysis of a hospital ward**

### **7.1 Introduction**

To illustrate the methodology presented in the previous chapter a sample case will be analysed. The analysis is based on a fictive hospital ward, in a large hospital in a Swedish city. The risk measures will be derived using both the standard QRA and the extended QRA, considering the uncertainty in the variables. In addition to these, a separate uncertainty analysis will be performed on some of the subscenarios in the scenario.

Two levels of untenable conditions will be used in the analysis to determine the available escape time, both *critical* and *lethal*, cf. Section 4.1.2, as quantified in Appendix B. The first of these levels is related to defined tolerable levels which can be used for design purposes in Sweden. Comparing the results obtained from the two analyses, differing only in terms of defined untenable levels, will reveal the relative safety obtained using the critical level of untenable conditions. Using the critical conditions, does not imply that patients on the ward become fatal victims during the fire. They will, however, be restricted in their use of the escape routes as they will be filled with smoke. The staff are excluded from the risk measure and are only considered as an aid to patient evacuation. It is assumed that the staff are able to escape before they become fatal victims.

A more correct risk analysis is based on lethal levels of exposure. There is, however, considerable difficulty in defining the conditions that result in death of the patient. Patients are sensitive to different degrees to the conditions resulting from the fire. When the conditions in this example are defined as lethal it is understood that the conditions in the ward then have reached the definition of lethal conditions. Whether or not the patients become fatal victims as a result of being exposed to these conditions can, of course, be debated. The conditions are chosen to represent a rather serious

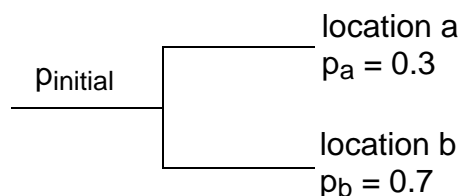
situation and it is assumed that these conditions represent fatal conditions.

### **7.2 Definition of the system**

The problem is structured according to the event tree methodology. To limit the size of the event tree some limitations are introduced. First, the analysis is restricted to only one ward in the hospital. Second, only one fire source is used for the illustration. Third, the only target specified in this analysis are the patients on the ward. A more complete analysis should cover all possible fire locations, sources and targets.

More fire sources and fire locations may not necessarily increase the reliability of the risk measures. The chosen fire source and location are representative of all sources and locations on the ward. Separation of the chosen fire location into more locations could be performed within the analysis, see Figure 7.1.

When further subdivisions in the event tree are made, the result will only affect the resolution of the societal risk profile. It is then assumed that the initial part of the event tree is unaffected.



*Figure 7.1. Multiple fire scenario analysis.*

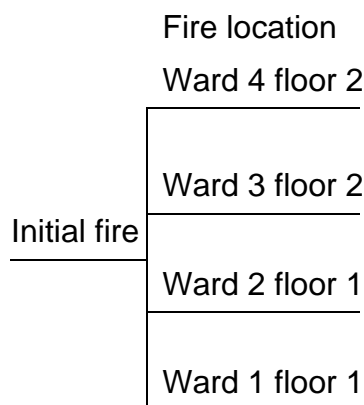
The current analysis, using only one representative fire source, i.e. fire scenario, will result only in a slightly less detailed risk profile.

Multiple fire sources are implicitly considered in the extended QRA as the development of the fire is subject to uncertainty. This uncertainty can be seen as representing different fire sources or fire

growth development. In the standard QRA, a representative value is used for all fires chosen according to principles for this type of analysis.

The magnitude of the societal risk is dependent only on the risk analysis perspective, i.e. whether the analysis covers one ward or the whole hospital. Therefore, the physical boundaries of the analysis must be made appropriately.

The risk analysis could be extended to consider, for example, the whole hospital. If a larger part of the hospital is considered in the analysis, the initial fire probability will also be higher because of the higher number of possible fire locations. If this more global analysis is chosen, the event tree and initial probability must be chosen accordingly. This can be illustrated as in Figure 7.2.



*Figure 7.2. Example of the initial part of a complete event tree for the risk analysis of a hospital.*

The probability of the fire starting in any of the wards in the hospital, will not differ due to the more global analysis perspective, but the total initial probability of fire occurring will be higher for the whole hospital. This results in a displacement of the risk profile

higher up in the diagram, indicating a higher risk than when only a single ward is considered.

If identical situations are assumed for the four wards in Figure 7.2 and the likelihood of fire occurrence is the same for all the wards, the societal risk profile for the whole hospital is increased by a factor of four compared with the single ward risk profile. The consequences are not changed, if the fires are independent events and fire spread between wards is neglected, but the probabilities are increased. In the real situation it might not be appropriate to assume similar likelihood of fire in the wards. Different activities on the wards and differences in types of installations may affect the likelihood of fire breaking out.

The true risk level for the hospital is, of course, dependent on the actual situation on each floor, but the fire probability at a specific location is not changed by changes in the analysis perspective. Therefore, the risk measures will depend on the limitations set out for the analysis. If acceptable risk measures are available, it might be possible to determine the maximum allowed size of, for example, a building, as the risk increases with building size.

It is also necessary to be aware of how society is defined. From a societal point of view it may make no difference, with respect to fire hazards, whether we have one hospital with four wards or four hospitals with one ward each. The risk to society is the same but the societal risk for each facility is different. This is based on the assumption that the fire does not spread from one ward to another in the four-ward hospital. The individual risk is not affected by the analysis perspective.

Some of the limitations of this approach can be reduced by performing an uncertainty analysis of the safety on the ward, in which the deterministic values are replaced by variables defined by a distribution, the extended QRA. For example, different fire sources are considered during this analysis.

The purpose of this example is merely to demonstrate the methodology. It is further assumed that the quantitative risk analysis has been preceded by qualitative screening methods to determine the quantitative scenario.

The event tree used for this analysis is presented in Appendix B, in Figures B1, B2 and B3, together with the basic assumptions for the calculations.

### **7.3 Standard QRA**

#### **7.3.1 Societal risk**

Based on the information in Appendix B, a standard QRA of the hospital ward can be performed. The analysis results in 100 subscenarios. The first 96 subscenarios represent the final outcomes of the event trees in Figures B2 and B3; 48 for daytime conditions and 48 for night-time conditions. The last four subscenarios can be found in the initial part of the event tree (Figure B1) but do not result in any unwanted consequences as the fire is either extinguished or will not continue to grow.

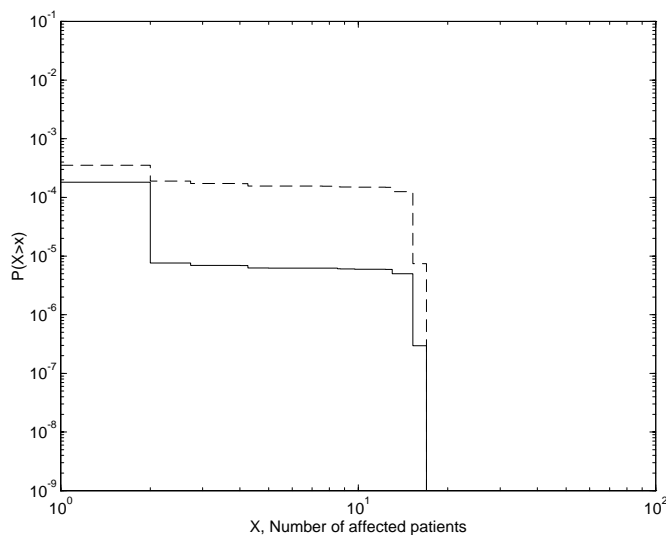
The Kaplan-Garrick triplets are derived and collected in two vectors, containing subscenario probabilities and consequences. The consequences are expressed in terms of the number of patients not being able to escape within the available time defined by the occurrence of untenable conditions. The consequences have been derived using values representing the credible worst conditions. No specific distribution fractile has been used in estimating these values, see Chapter 4. The triplets are sorted according to the procedure described in Chapter 6 and the resulting risk profile is shown in Figure 7.3 as the solid line.

The initial probability of fire,  $p_{initial}$ , has been chosen to be 0.07 fires per year per ward based on the reasons presented in Appendix B. The vertical axis in the diagram for societal risk will then express the probability of the occurrence per year per ward.



An alternative design strategy, with no sprinkler system, was also examined using this method. The dashed line shows this design risk profile. This risk profile indicates a higher risk, which is rational and understandable as sprinkler systems are assumed to decrease the hazard of fires.

The profiles in Figure 7.3 were derived using the critical conditions for untenable environment. The same information, using lethal conditions, can be found in Figure 7.4.



*Figure 7.3. Risk profile for the standard quantitative risk analysis using critical untenable conditions. Dashed line = risk profile for design alternative without sprinkler system.*

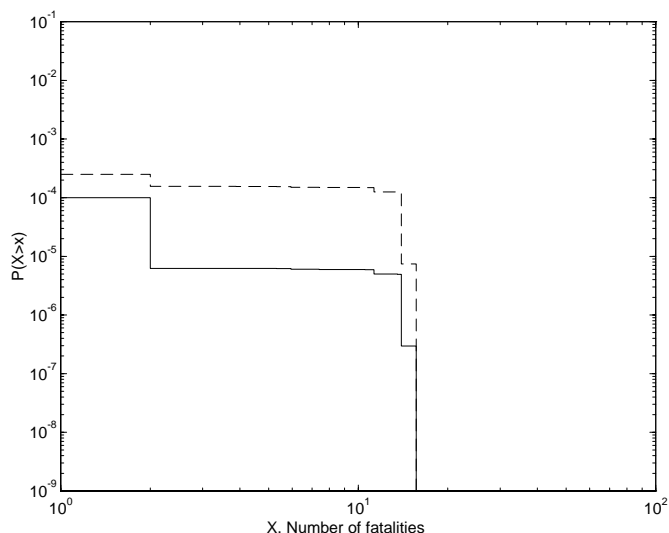


Figure 7.4. Risk profile for the standard quantitative risk analysis using lethal untenable conditions. Dashed line = risk profile without sprinkler system.

There is only a small difference between the corresponding curves in the two figures, indicating a small difference in available escape time between the two conditions critical and lethal. The risk profile for critical conditions is, of course, located slightly higher than that for lethal conditions. This difference should not be interpreted as a difference in risk level, but indicates the difference in the definition of hazardous environment. Using the more severe lethal conditions result in a longer available escape time.

The result is that a higher number of patients can be evacuated within the available time. The maximum number of patients unable to be evacuated is also different: 17 patients versus 15.5 patients for the critical and lethal conditions, respectively. The conclusion that can be drawn from this is that there is only a small time margin between the two levels of untenable conditions for growing fires. One should bear in mind that the fires are assumed to grow continuously as a function of time. Other kinds of fire development

might result in a different situation. It is, however, believed that the trend would be similar, i.e. a small difference in available time between critical conditions and lethal conditions.

Based on the results, the average societal risk can also be obtained. The products of the probability and the consequences for each subscenario are summed to give the average societal risk, according to Eq. [6.1]. For this scenario the average risk using critical conditions is  $4.4 \cdot 10^{-4}$  persons per year per ward. This means that in an average year  $4.4 \cdot 10^{-4}$  persons will become exposed to untenable conditions and prevented from further evacuation due to fire.

This can be expressed in another way. A fire resulting in at least one victim, i.e. a patient being exposed to the defined untenable conditions, can be expected to occur once every 227 years. The measure can be used to compare different design solutions and to make statements on relative safety.

The average societal risk of suffering multiple fatalities in the ward can also be calculated using the same procedure but different criteria for determining the available time. When the risk measure is derived assuming the lethal conditions, the value found is  $2.7 \cdot 10^{-4}$  persons per year per ward.

The two values of average societal risk given above were derived for a situation in which sprinklers are operating with a conditional probability of 0.96. The alternative design solution without the sprinkler system has also been analysed in terms of the average societal risk. The corresponding values for the average societal risk are  $2.7 \cdot 10^{-3}$  patients per year per ward, assuming critical conditions and  $2.2 \cdot 10^{-3}$  patients per year per ward assuming lethal conditions. There is thus, a difference in risk of a factor of approximately 6 - 10 between the scenarios with and without sprinklers.

This comparison between a ward with and without sprinklers was presented to illustrate the capability of the method. Similar results can be obtained by comparing situations with and without an automatic fire detection device, etc. The risk profiles and average risk measures will be different, but it is possible to illustrate the benefit of devices which increase safety in a quantitative manner.

The question is whether the ward without the sprinkler system is acceptable or not. The sprinkler-equipped ward may result in an "oversafe" and too expensive situation. On the other hand, with a sprinkler system, a higher number of patients could be housed on the ward with the same risk level as the ward without the sprinkler system.

### 7.3.2 Individual risk

The individual risk has also been derived for the two levels of untenable conditions. The individual risk is defined here as the probability per year of the escape routes being blocked by the fire or patients being killed by the fire, depending on the choice of untenable conditions.

In the critical conditions case, the individual risk was equal to  $1.8 \cdot 10^{-4}$  per year and  $1 \cdot 10^{-4}$  per year in the lethal conditions case. The measure of risk is the sum of the subscenario outcome probabilities leading to the unwanted event, i.e. that the escape routes are blocked or that a patient is killed, cf. Eq. [6.2]. These values can also be found from the risk profiles as the points at which the curves cross the vertical axis.

## **7.4 Extended QRA**

### 7.4.1 Societal risk

To obtain information on the reliability of the risk profile, an uncertainty analysis of the fire in the hospital ward can be performed. The difference compared with the previous section is that most variables are now associated with uncertainty.

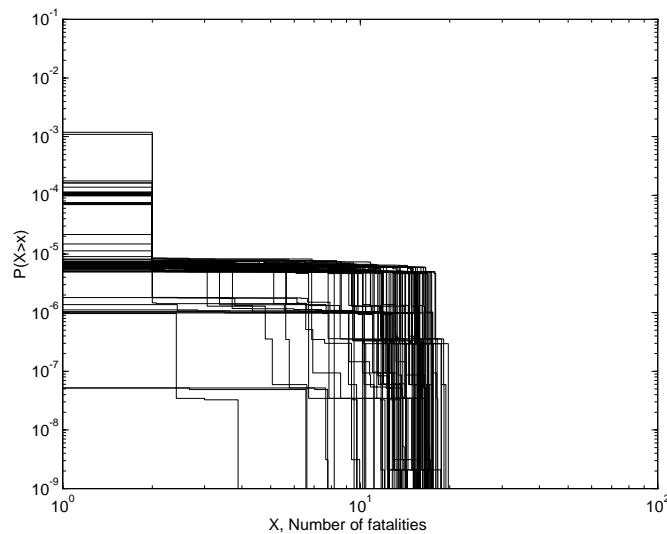
The branch probabilities in the event tree are initially treated as deterministic values, i.e. without uncertainty. Two of these, the initial fire probability and the reliability of the automatic fire alarm, will later be subject to uncertainty to determine the influence on the total uncertainty.

The consequences, in terms of the number of patients not being able to escape safely, are derived by response surface equations. This method is adopted because the calculations must be executed a large number of times in order to reflect the uncertainty. The variables derived from the regression analysis, i.e. the regression coefficients, are subject to uncertainty. This is a result of the regression analysis as the response surface only approximates the results of the computer output. The regression coefficients will, however, be treated as deterministic values due to their low importance in the final measures of risk. The influence of the uncertainty of the response surface variables will be studied in the single subscenario uncertainty analysis in Section 7.4.

The main source of uncertainty is the uncertainty in the variables defining the consequences. In the extended QRA only results from calculations using lethal levels of untenable conditions are presented. This is not a major limitation as the results from calculations using critical levels only will be slightly different. The sampling technique used for the extended QRA is Latin Hypercube Sampling (LHS). The number of samples was 100. Increasing this number will improve the accuracy of the risk measures, but also increase the amount of data.

In the first case, which the subscenario probabilities are treated as deterministic values, these values are collected in a single vector. The consequence values,  $c_i$ , are collected in a matrix where each row defines one iteration or sample of the derived consequences. The magnitude of the values of  $c_i$  will vary between 0 and 22 patients. The maximum theoretical value of  $c_i$  was, however, never achieved.

Each subscenario in the event tree is equivalent to one column in the matrix. The matrix consists of 100 columns and 100 rows. The Kaplan-Garrick triplets,  $(s_i, p_i, c_i)$  are treated according to the method presented in Chapter 6. The resulting risk profiles are illustrated in Figure 7.5.

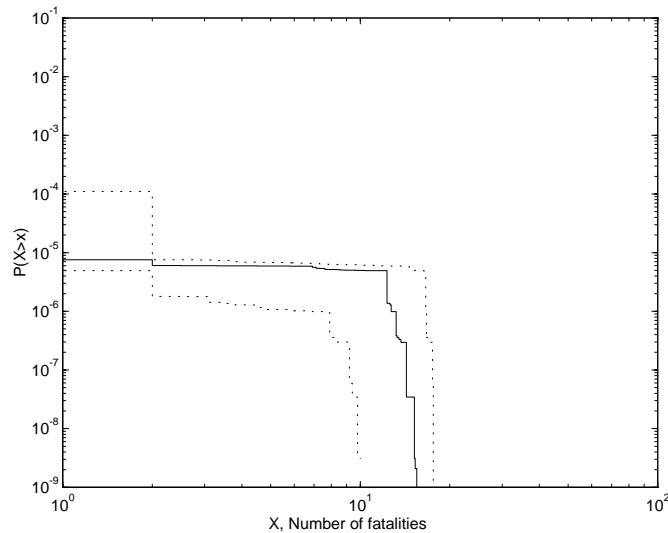


*Figure 7.5. Family of risk profiles for the extended quantitative risk analysis using lethal untenable conditions.*

As can be seen, in this form, the large number of curves makes them impractical to use as a basis for determining the uncertainty in the risk. Therefore, curves representing the median curve together with percentile curves should be used. The data were processed using the Matlab file `mymat.m` which reads the data shown in Figure 7.5 and sorts it to enable another Matlab file, `drawfrac.m`, to plot the curves of the percentiles of interest. The Matlab files are presented in Appendix A.

The resulting 10th, 50th and 90th percentiles of the data in Figure 7.5 are shown in Figure 7.6. The band between the dotted lines represents 80% of the data, which also means that 10% falls

outside each dotted line. The curves represent the results of the complete extended QRA including sprinkler subscenarios.



*Figure 7.6. Percentiles from the extended quantitative risk analysis using lethal untenable conditions. The lines represent the 10th, 90th and 50th (or median) percentiles.*

It is interesting to see when the standard QRA analysis and the extended analysis coincide. Which percentile in the extended QRA is represented by the standard analysis? To answer this question the results of the standard QRA and the percentiles obtained from the extended QRA using lethal conditions can be plotted in the same diagram. Figure 7.7 shows the standard QRA results and the 90th percentile from the extended QRA.

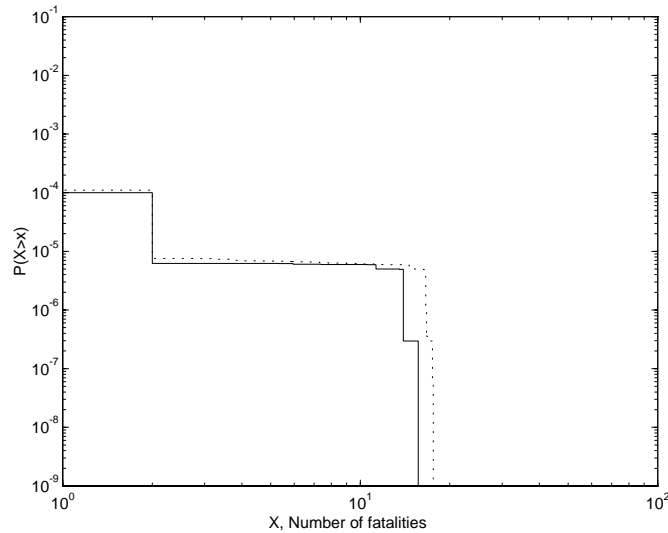


Figure 7.7. The dotted line is the 90th percentile from the extended analysis and the solid line is the standard QRA result.

Comparing the 90th percentile in the extended QRA with the result of the standard QRA it shows that the two lines are rather close for small consequences, but deviate above about 10 victims. This means that using design values on the safe side of each variable distribution results in a level of safety that can be said to be certain to a specified level of confidence, in this case approximately 90%.

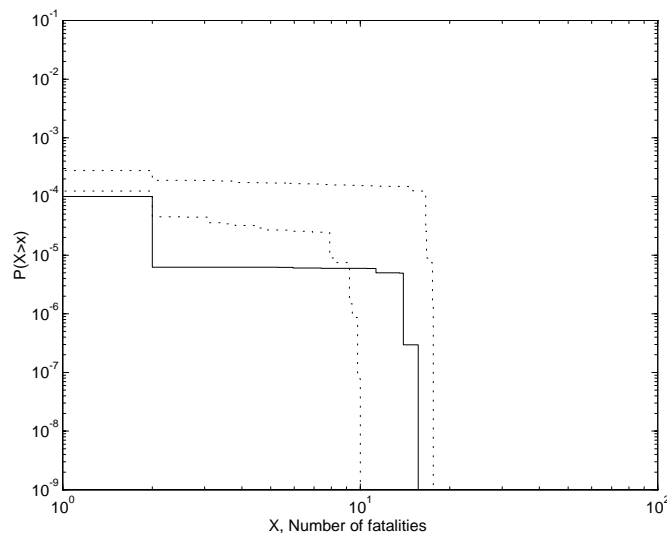
The method can also, as for the standard QRA, be used to evaluate the benefits of different design strategies in efforts to obtain an optimal solution. The QRA methods can be used both in the design process and in safety checking of existing buildings and plants. The methods are not a traditional design method which is provided with design variables leading to a specified risk level, but can be used to check design solutions.

As the variation in risk profiles originates from the uncertainty in the actual scenario if it occurs, it can be used to determine if additional safety measures, afford any measurable increase in



safety. If the new design risk profile falls between the accepted confidence limits, there is no statistically significant change in safety, on the specified confidence level, due to the additional safety measures. The risk profile resulting from a new design might not be different from the inherent uncertainty of the scenario.

As an example of this the following situation will be examined. The baseline scenario is defined as the scenario without the sprinkler system. The 80% confidence interval (10% - 90%) is derived for this situation. The question is "Will a sprinkler system increase the level of safety to a level that is statistically significant?". The result can be seen in Figure 7.8.



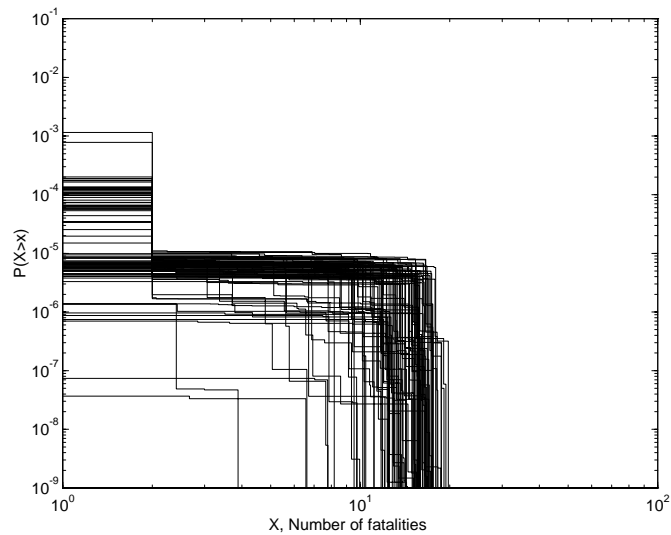
*Figure 7.8. Confidence limits at the 10th and 90th levels for the baseline scenario (dotted lines) and standard QRA risk profile for scenario with the sprinkler system (solid line).*

When the sprinkler system is installed, the risk is decreased to a degree that is statistically significant for low number of fatalities. As the number of fatalities increases, the sprinkler system has a

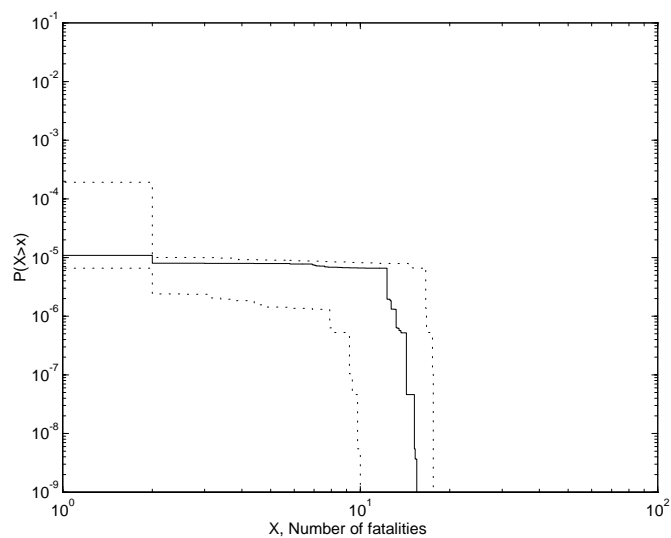
lesser effect on the measure of risk. The overall probability for such occurrences is, however, low. A high number of fatalities occurs when the sprinkler system is not able to operate.

In all the risk profiles presented so far the probability for each of the subscenarios has been treated as a deterministic value. To address this simplification, the analysis will be extended to consider uncertainty in the initial fire probability,  $p_{initial}$ , and in the reliability measure for the automatic fire detection system. In the previous analysis these variables were assigned the values 0.07 fires/year and 0.06 for the probability of failure per demand for the detection system.

In the following analysis the initial probability will be uniformly distributed between 0.04 and 0.1 fires per year. The reliability of the automatic fire detection system, i.e. the probability that it will work, will be uniformly distributed between 0.9 and 0.98 per demand. The results of these calculations are presented in Figures 7.9 and 7.10. The sampling technique used was the LHS method.



*Figure 7.9. Risk profiles for the scenario where  $p_{initial}$  and detection system reliability are subject to uncertainty.*



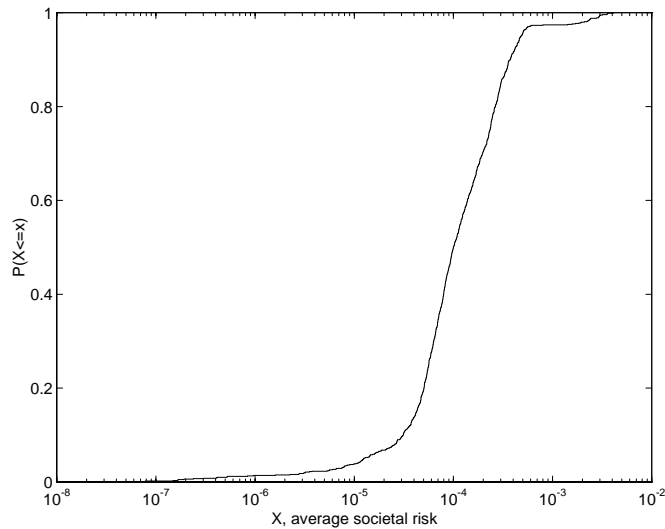
*Figure 7.10. Confidence limits for the scenario where  $p_{initial}$  and detection system reliability are subject to uncertainty. The lines represent the 10th, 90th and 50th (or median) percentiles.*

The above two figures probably show the most complete picture of the risk measure for the scenario. It is obvious that the overall uncertainty increases as the number of variables subject to uncertainty is increased.

The difference between these results and the result from the scenario where the uncertainties in  $p_{initial}$  and detection system reliability are not considered, is significant and should not be disregarded. One problem that might arise is in defining the number of variables that should be considered with their uncertainties. Adding another variable subject to uncertainty will increase the overall uncertainty.

The answer must be to include all variables, including the uncertainty information, in the analysis that significantly affect the total uncertainty. This could present a serious problem if a high number of variables are subject to high uncertainty. Variables which are not so important in the final uncertainty can be neglected. Studying one subscenario at the time the importance of the variables can be identified by using the methods presented in Section 7.5. Defining the limitations of the problem, expressed in terms of the scenario, could also help to solve this problem.

In the same way as for the standard QRA, the average societal risk can be determined but now in the form of a distribution instead of a single value. This distribution can be used to derive confidence limits. The average societal risk distribution for the sample case is shown in Figure 7.11.



*Figure 7.11. Cumulative distribution of the average risk measure.*

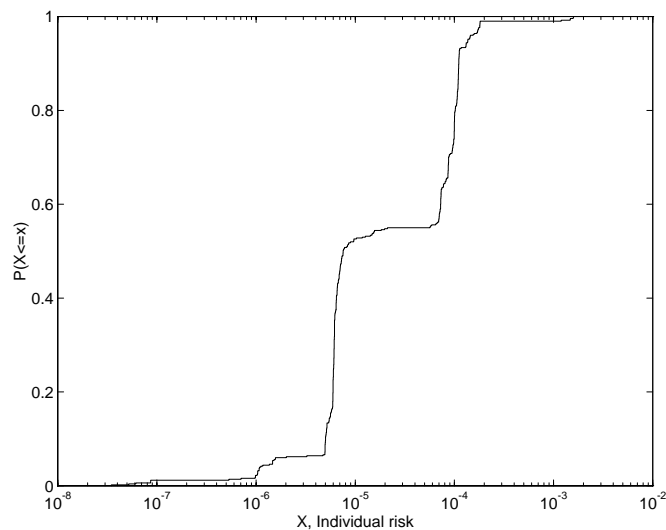
The information in Figure 7.11 can be condensed into one single deterministic value which can be used compared with the risk measure derived from the standard QRA. This aggregated value can be described as the "average" average societal risk.

Inevitably, some information regarding the uncertainty in the average societal risk will be lost, but that may be compensated for by easier comparison. The average value obtained from the extended QRA defined by the result in Figure 7.11 is  $2.1 \cdot 10^{-4}$  persons per year per ward. This value should be compared with the result obtained from the standard QRA of  $2.7 \cdot 10^{-4}$  persons per year per ward. Considering the uncertainty in the variables will decrease the average value as some subscenarios, in the extended QRA, in some iterations will result in less fatalities than in the standard QRA. The contribution to the total risk measure will therefore be less.

### 7.4.2 Individual risk

The individual risk can also be determined in the extended analysis. As each subscenario is subject to uncertainty in terms of the consequences, it is possible to determine the probability that at least one person will be subjected to untenable conditions due to uncertainties in the subscenario descriptions.

In each subscenario there will be situations in which no unwanted consequences will arise, for example if the fire develops slowly and the response of the staff is very rapid. This randomness can also be seen in the individual risk when expressed in terms of a distribution, in this case a CDF showing the cumulative probability that the risk measure is less than the specified value on the horizontal axis, Figure 7.12.



*Figure 7.12. Distribution of individual risk in terms of probability per year, from the extended QRA.*

In many situations it is not necessary to use all the information contained in the CDF. The information can, as for the average societal risk measure, be aggregated into a single value, the mean

individual risk. This individual risk measure,  $IR_u$ , is the mean value of the distribution and considers the uncertainty in the variables defining the consequences.

This measure can be obtained either by deriving the mean value of the distribution or according to the procedure resulting in Eq. [6.18]. This equation defines how the subscenario probabilities  $p_i$ , derived in the standard QRA, should be adjusted to take into consideration the uncertainty in each subscenario,  $p_{u,i}$ . Both procedures result in the same value,  $IR_u$ .

The single value of the individual risk of becoming a fatal victim when the variable uncertainties are also considered was found to be  $6.3 \cdot 10^{-5}$  per year. This value should be compared to  $1 \cdot 10^{-4}$  per year when no consideration is taken of the uncertainties in the variables. The individual risk is decreased and is actually a better prediction of the risk as it is based on a broader base of information about the scenario. A more detailed investigation concerning the probability  $p_{u,i}$  is presented in Section 7.5.

### **7.5 Uncertainty analysis of individual subscenarios**

In the previous section of this chapter the whole event tree was evaluated in terms of uncertainty in the resulting risk. The method was used to determine the uncertainty propagation from a global perspective.

The influence of single variable uncertainties is not easily determined with this approach. The individual variable uncertainty is not very important in relation to the combined effect and the global view of the uncertainty.

To be able to evaluate the effect of uncertainties in single variables on the total uncertainty, each subscenario must be evaluated separately. By doing this, the importance of each variable in each subscenario can be determined and ranked. The variables which

are considered important can then be observed more carefully and their uncertainty can perhaps be reduced.

The most important result of the uncertainty analysis on the subscenario level is, however, the probability that the unwanted event will occur due to the inherent uncertainty in the variables,  $p_{u,i}$ . In this case, it is equal to the probability that the evacuation of the patients are obstructed. Different methods are available for calculating this, as described in Chapter 6. In this illustration all three methods described in Chapter 6 have been used, but on a limited number of subscenarios.

The subscenarios on which the analysis has been performed are 1, 3, 13 and 49, cf. Figures B2 and B3 in Appendix B. These subscenarios reflect different degrees of help required by a patient, the influence of the automatic fire alarm and whether the fire occurs during the day or night.

The subscenario uncertainty analysis is limited to the study of cases where the untenable conditions are defined as lethal. In this analysis, most variables involved are subject to uncertainty, i.e. also the parameters used in the regression equations. As it is understood that the subscenarios have occurred in performing this analysis, i.e. implicitly assuming that  $p_i$  is equal to 1, the subscenario occurrence probabilities are not considered.

#### 7.5.1 The analytical reliability index $\beta$ method

The most important result obtained from the First Order Second Moment (FOSM) method is the reliability index  $\beta$ . This index is a measure of performance and is used to express safety in terms of patient safety on the ward. A high value of  $\beta$  represents high safety.

The reliability index can also be transformed into a probability of failure, i.e. the probability that at least one patient is prevented from evacuation, cf. Eq. [6.8]. The reliability index used in this



calculation was first presented by Hasofer et al. (1974), and is denoted  $\beta_{HL}$ .

The probability of failure is theoretically derived by integrating Eq. [6.4]. But this is not usually possible in practice. Approximations of the probability of failure can instead be evaluated where the accuracy depends on the choice of evaluation level. The sample case calculation was performed on level 3, cf. Section 6.6.1, using the computer software STRUREL (STRUREL, 1995). The results obtained are the reliability index  $\beta_{HL}$ , and the probability of failure.

The most obvious problem in applying this method to the sample case is that the consequences or safety are expressed as more than one single limit state function. The reliability cannot be exactly determined using the method presented in Chapter 6 as it is designed for only one limit state function at a time.

The method can, however, be used to derive the interval bounds between which the probability of failure is located. Each failure mode is analysed and results in a probability of failure. These are used to estimate the simple bounds for the exact probability of failure for the multiple failure mode system.

The calculations were performed with the software STRUREL as it is applicable to a situation with multiple failure modes. The program gives the so-called Ditlevsen bounds for the probability of failure, which are more narrow than the simple bounds, cf. Section 6.6.2. Therefore, a better estimate of the probability of failure can be calculated. The calculated Ditlevsen bounds are very narrow and in Table 7.1 are the bounds presented as the mean values of  $\beta_{HL}$  and probability of failure.

*Table 7.1. Reliability index  $\beta_{HL}$  and probability of failure for uncertainty analysis of subscenarios 1, 3, 13 and 49.*

Sub-scenario	$p_{u,i}$ %	$\beta_{HL}$
1	57	-0.18
3	5	1.6
13	56	-0.16
49	92	-1.4

The wider and more simple bounds based on basic algebra have also been derived in order to show that these are outside the Ditlevsen bounds. This has only been done for subscenario 3. The resulting probability of failure for the two modes in subscenario 3, i.e. in the room and in the corridor, are 4.5% and 0.8%, respectively. The separate failure mode probabilities are used to derive the simple bounds based on Eq. [6.14]. The resulting probability of failure interval according to this equation is  $4.5\% \leq p_{u,3} \leq 5.3\%$ .

It is clear from studying the results in Table 7.1, that variable uncertainty has a significant effect on the reliability of the different subscenarios. There can be many explanations as to why the reliability index  $\beta_{HL}$  shows such a large deviation, from -1.4 to 1.6.

First, if subscenario 1, 13 or 49 occurs, it is likely that some patients will be unable to escape safely. Subscenario 3 has a high likelihood of a positive outcome, corresponding to a low probability of failure.

The difference between subscenarios 1 and 3 is related to the physical capabilities of the patients on the ward. In the first case, the time required to prepare and move each patient from the room to a safe location is much longer than in the second. The average difference for each patient is 20 seconds for the preparation time and 30 seconds for the movement time. Bearing this in mind and

the fact that there are 22 patients in total on the ward, the difference in safety can be understood.

It is clear that patient capability has an influence on the reliability and that actions could be taken to reduce the probability of failure, for example, by increasing the number of staff. The small difference in safety between subscenarios 1 and 13 is related to the small difference in detection time for manual detection and automatic detection. If the manual detection time is increased, the probability of failure is increased. The difference between subscenarios 1 and 49 reflects whether the fire occurs during the day or during the night.

The independent subscenarios can be compared with each other by simply regarding their individual reliabilities. However, this gives a false picture of the total safety of the system, in this case the ward. Different subscenarios have different probabilities of occurrence and by comparing the subscenarios as described above implies that each subscenario has equal importance. This is obviously not correct and consideration should be taken of the underlying probability of subscenario occurrence.

Another important matter in the analytical uncertainty method is the degree of importance of the individual variables in the overall uncertainty of the subscenario. If each variable importance is identified and perhaps reduced the overall safety can be increased.

The variable importance values,  $a_i^*$ , cf. Eq. [6.11], can also be derived for situations of multiple failure modes. The parameter  $a_i$  represents the relative importance of each variable but the geometrical interpretation is not so simple for this case.

The value and direction of  $\beta_{HL}$ , the latter indicated by the vector  $\mathbf{a}^*$ , is determined for an equivalent failure plane in which both the failure planes of both modes are included. The equivalent failure

plane is used as a representation of the total limit state, i.e. all failure modes, of the system.

The most important variables for the subscenarios are presented in Table 7.2. Some variables have been left out as their contribution to the subscenario uncertainty is small. Therefore, the sum of squares of  $a_i^*$  will be less than 1.0. A value close to -1 or 1 represents a high degree of importance.

*Table 7.2. Relative importance of variables in the total uncertainty in subscenarios expressed in terms of  $a_i^*$ .*

Variable	1	3	13	49
$M_S$	0.24	0.27	0.23	0.21
$\alpha_f$	-0.52	-0.62	-0.38	-0.57
$t_{det}$	-0.40	-0.66	N.A.	-0.69
$t_{care}$	-0.25	<-0.1	-0.28	-0.18
$t_{patM}$	-0.59	<-0.1	-0.78	-0.21
$\lambda_{room}$	<-0.1	0.17	<-0.1	0.2
$\delta_{room}$	<-0.1	0.17	<-0.1	0.2
$\lambda_{corr}$	0.20	0.12	0.26	<-0.1
$\delta_{corr}$	-0.19	-0.1	-0.24	<-0.1

N.A. Not applicable. Detection time determined by automatic alarm.

Generally, high importance in the overall uncertainty is related to the variables  $\alpha_f$  and  $t_{det}$ , which are the fire growth rate and manual detection time, respectively. Also,  $t_{patM}$  is highly significant in subscenarios 1 and 13. Decreasing the uncertainties in these variables result in a higher level of safety, as is the aim of this type of analysis. The parameters resulting from the regression analysis forming the response equations for the available escape time in the room,  $\lambda_{room}$  and  $\delta_{room}$ , and in the corridor  $\lambda_{corr}$  and  $\delta_{corr}$ , are also significant but to a small degree.

A further analysis was performed on subscenario 3, treating the regression parameters as deterministic values. The analysis showed that if these parameters are random variables, the reliability in terms of  $\beta_{HL}$ , will decrease from 1.8 to 1.6. This is equal to an increase in the probability of failure from 4% to 5%, i.e. a small difference.

The regression coefficients can be classified as belonging to knowledge uncertainty, which can be reduced by further studies. The benefit of such a task might be questioned as the most important variables can be characterised as belonging to stochastic uncertainty. However, stochastic uncertainty cannot be reduced without further subdivision of the subscenarios into more homogeneous subgroups.

#### 7.5.2 Numerical sampling methods

Numerical sampling methods are more straightforward to use in the determination of the uncertainty propagation for each subscenario. The samples are derived by one or more limit state functions and are not sensitive to the number of failure modes. An increased computational time can, however, be expected.

The subscenario problem, i.e. the limit state function, is normally defined as a series of calculations which can be set up as a few lines of computer code or macro lines in a spreadsheet program. The equations are calculated for every iteration, resulting in a single output, in this case the number of fatalities. This is repeated a large number of times, usually more than 1000, to obtain sufficiently accurate information about the distribution of the consequences.

As in the analytical method, importance parameters, cf. Section 6.6.4, can be obtained which determine the relative importance of the variables in the limit state function(s). As with the analytical method, the uncertainty propagation has been performed for the same four subscenarios.

Both numerical procedures, the single phase and the two-phase methods in Figure 6.2, have been used to analyse the results. First, the method which does not distinguish between stochastic and knowledge uncertainty will be discussed.

*Single phase sampling method*

The primary result will be the probability of failure of the individual subscenario. The term failure is defined as the escape time margin being less than zero, i.e. the available time is less than the required escape time. This situation will occur as a result of the randomness in the variables in the limit state function. The uncertainty inherent in the variable descriptions will result in some situations in which the unwanted event occurs, and the escape time margin is less than zero. The probability of failure is a measure of how frequent this situation is.

The subscenario frequency,  $p_i$ , does not have any influence on the results of this analysis. It is assumed that the subscenario has occurred. The resulting probability of failure can, however, be used to improve the risk measure, see Section 7.3, as it is a measure of the probability  $p_{u,i}$ .

Another result of this analysis is the relative importance of the variables in the limit state function. Different measures of the relative importance are available, see Chapter 6. In this analysis, one of the more simple measures, the correlation coefficients, was used to rank the variables in terms of their importance. The resulting probability of failure is presented in Table 7.3. The number of iterations was 2000 and the samples were obtained with Latin hypercube sampling, LHS.

*Table 7.3. Resulting probability of failure.*

Sub-scenario	$P_{u,i}$ %
1	55.7
3	4.9
13	53.9
49	91.2

The results can be compared with the results of the analytical method presented in the previous section. The comparison shows good agreement between the methods.

The relative importance of the variables resulting from this sample case, using the correlation coefficient, is presented in Table 7.4. Only the most important variables are given in the table.

*Table 7.4. Relative importance of variables on the total uncertainty in subscenarios 1, 3, 13 and 49.*

Variable	1	3	13	49
$\alpha_f$	0.4	0.26	0.36	0.41
$t_{det}$	0.25	0.28	N.A	0.30
$t_{patM}$	0.65	-	0.70	0.66
$M_s$	-0.18	-0.1	-0.23	-0.17
$t_{care}$	0.20	-	0.22	0.24
$\gamma_{room}$	-	-0.12	-	-
$\delta_{room}$	-	0.1	-	-
$\gamma_{corr}$	-0.21	-	-0.29	-0.22
$\delta_{corr}$	0.20	-	0.20	0.19

N.A. Not applicable. Detection time determined by automatic alarm.

The most important variables are, in general, the fire growth rate factor,  $\alpha_f$ , the manual detection time and the patient-related variables, preparation time and movement time. This is what

would be expected from observing the uncertainty in each of these variables. The uncertainty in these variables is rather large. The results correspond, in principal, to the ranking obtained with the analytical method. What is somewhat surprising is the relatively large importance of the fire growth rate as it is raised to the power of  $-0.3$  -  $-0.5$  in the limit state functions. However, the fire growth rate is involved in more than one expression in the limit state function, which may be one explanation. General conclusions can not, however, be drawn at this stage, due to the limited number of subscenarios examined.

It is important in risk management to identify significant variables and to try to reduce them as far as is practicable. Therefore, the above method can be used in addition to the standard QRA. It must be emphasised that the number of examined subscenarios must then be higher in order to gain a more complete picture of the underlying uncertainties. An alternative solution may be to perform a sensitivity analysis, but this results in a lower amount of information. The procedure involved in the sensitivity analysis is, however, probably less extensive.

#### *Two-phase sampling method*

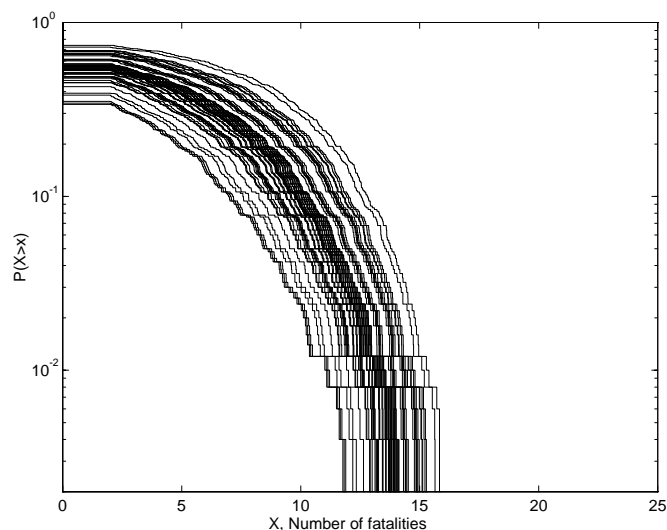
In the above so-called single phase simulation, no distinction is made between stochastic and knowledge uncertainty. But, according to theory there is a fundamental difference between them, cf. Chapter 2. Some variables will be random by nature and cannot easily be reduced. Others, for example model uncertainty, can be reduced if better models are used or if more extensive model tests are carried out.

To determine the influence on the overall uncertainty due to the two uncertainty types, a so-called two-phase sampling procedure can be used. To illustrate the method a sample calculation has been performed on subscenario 13, separating stochastic uncertainty from knowledge uncertainty. The procedure is described in Section 6.6.3.



Most variables in the sample case will be categorised as inevitable or inherent variation. The only variable that will be categorised as knowledge uncertainty is the model uncertainty,  $M_S$ . This can be further reduced as more information becomes available.

If the model uncertainty is separated from the other variables, the results of the two-phase simulation can be illustrated as in Figures 7.13 and 7.14. The number of individual CCDF curves is 59, due to reasons presented in Chapter 6. The outermost two curves represent the 5 percentile and the 95 percentile which are known with a degree of confidence of 95%. Better estimates can be obtained if a higher number of CCDF curves are calculated.



*Figure 7.13. Result from the two-phase uncertainty analysis of subscenario 13.*

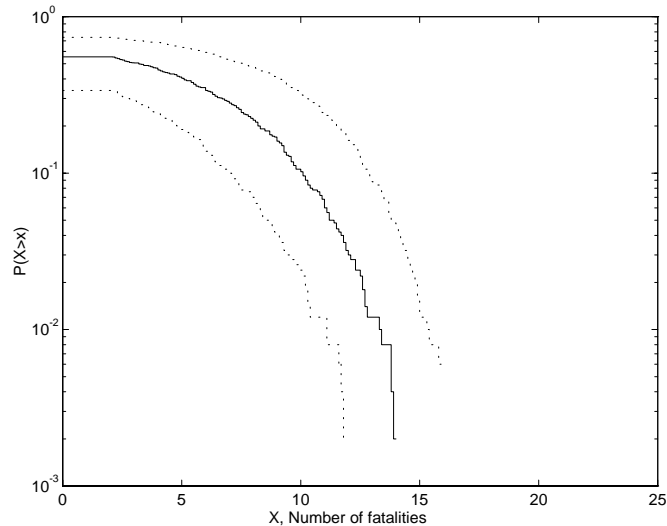


Figure 7.14. The 90% confidence interval for the number of fatalities in subscenario 13 using the two-phase method.

The differences between the individual CCDF curves represent the influence on the total uncertainty of the model uncertainty. If a better model is used, or if better information regarding the prediction ability of the model is available, the horizontal spacing between the curves will decrease.

Each of the curves represents stochastic uncertainty and the family of CCDFs represents the knowledge uncertainty. The correct single probability distribution is unknown due to this type of uncertainty, i.e. due to the model uncertainty. The type of information available can be in the following form (see also Figure 6.10).

The probability that more than 10 patients will be able to escape safely can be identified as 32% with a 95 % degree of confidence. Similarly, at a confidence level of 50%, the probability of less than 10 patients being able to escape is approximately 10%. The uncertainty in predicting  $P(c_{13} > 10)$  is a result of the uncertainty in the model prediction defined as  $M_5$ . Due to the separation of

stochastic variability and knowledge uncertainty it is possible to state the degree of confidence of the results obtained from the single-phase simulation.

The probability  $P(C_{13} > 10)$  varies between 2.3% and 32% with a degree of confidence of 90%. This is a variation of approximately one order of magnitude. To improve the total risk analysis result, reduction of the knowledge uncertainty, in this case the model uncertainty must be given high priority. This conclusion is, however, based on an analysis of only one subscenario. Such a general statement cannot be made without examining more subscenarios.

### **7.6 Concluding remarks**

The quantitative methods presented in this chapter can be helpful to the architect in designing new buildings. Using these methods, it is possible to derive risk measures both with and without explicitly considering the inherent uncertainty in the system. These risk measures can be used either to compare different design solutions or to compare a solution with accepted risk levels. The former use is the most likely as acceptable risk levels have not been defined for most architectural work. Only in very special cases have tolerable risk levels been used in the design process, e.g. in large infrastructures. The risk analysis methods can be used for comparison with traditionally accepted design solutions.

The values of the variables which were used to derive, for example, the consequences in the standard QRA were chosen to represent slightly conservative cases. It is possible to choose the most likely values instead, and the resulting risk profile will, in such a case, represent an estimate of the most likely risk profile. The problem with using best estimates is that there is a good chance that the true risk profile, or any other risk measure, will be exceeded. The average risk profile, for the hospital ward example, can be found as the average profile in an extended QRA and it is

obvious that this risk profile can be exceeded due to the randomness in the variable outcome.

This is the reason for choosing values on the conservative side in the standard QRA. The resulting risk profile will indicate a higher risk but it is a safe estimate of the risk. In addition, the best estimate risk profile will probably not represent the true best estimate, even if the average values are used, as all sources of hazards are not encountered in the analysis. Performing a risk analysis usually involves a screening procedure which is used to identify the hazards with which the analysis is concerned.

This thesis also presents four subscenarios in detail. The most important result of these analyses is the probability of failure, i.e. the probability that at least one person is prevented from being evacuated within the time available for evacuation. This result arises from the inherent uncertainty in the variables defining the consequences in the subscenarios. Other relevant results can be obtained, for example, measures of the relative importance of the variables in the limit state functions by performing the analysis for a given subscenario.

Having determined those important variables, it can be seen that the uncertainty in some variables can be neglected. Neglecting the uncertainty in a variable, i.e. treating it as a deterministic value, results in a lower risk. The loss of information must, however, be compared with the possible saving in work load due to the simplification of the problem. Many of the variables have been subjectively estimated in terms of the uncertainty parameters, and it is, therefore, reasonable to only consider the most important variables in an uncertainty analysis. For example, it might be sufficient to consider the uncertainty in variables with direction cosines,  $a_i^*$ , greater than, say 0.2, cf. Section 7.5.1. Further analysis is needed to determine the consequences of disregarding the uncertainty information in the variables with direction cosines smaller than 0.2.

## **8 Risk evaluation**

In order to apply risk analysis methods in practice, criteria for tolerable risk levels must be defined. Tolerable levels should be determined based on the choice of decision criteria used for the analysis. Utility-based decision criteria, cf. Chapter 6, are used to estimate the costs of different alternatives and to optimise the cost versus the benefit.

Rights-based decision criteria are used in risk analysis to determine safety according to what is acceptable from the point of view of society. The risk measures, individual risk and societal risk, are then compared with levels that society is willing to accept in terms of, for example, human losses. The performance of a system will be constrained to meeting the specified level or generally, not exceed the specified level.

The term "tolerable risk level" is used instead of the more commonly used phrase "acceptable risk". The reason for this is that it may not be possible to determine the acceptable level as there may not be a generally accepted risk level (Davidsson et al., 1997).

The following description provides a brief introduction to the area of tolerable risk levels. This area is outside the scope of this thesis, but is of general interest as it links risk analysis to the design of new buildings. The design values, presented in Chapter 9, are derived from information regarding tolerated risk. An overview of the area of risk evaluation can be found in Davidsson et al. (1997) and in HSE (1987), the latter, however, is restricted to individual risk.

### **8.1 Tolerable risk**

There are no generally accepted tolerable risk levels. There are risks involved in many technological systems in use in society today, but the risk level is normally not explicitly defined. Quantified risk levels have only been used for a few, large

construction projects, e.g. relating transportation of hazardous goods. Risk criteria must be based on what society is willing to accept. In Davidsson et al. (1997) four main principles have been defined for evaluating risk.

1. The risk shall be avoided if possible by reasonable means. Costs and technological means usually define the constraints.
2. The risk shall not be unproportionally large compared to the benefits.
3. The risk shall be evenly distributed in society.
4. Disasters shall be avoided. Accidents are preferred.

### **8.2 Risk measures**

Tolerable risk levels can be expressed in different ways, using a deterministic point estimate approach or a probabilistic approach. Deterministic estimates may be in the form of a required minimum distance or a safety zone. Probabilistic measures considers both the probability and the consequences of a hazard.

The probabilistic approach is preferred above the deterministic. The reason is that the latter is difficult to use when risk reduction measures are taken. Disagreement may arise concerning the value of risk reducing measures. Probabilistic risk analysis may provide a better formal solution to these disagreements.

The traditional way of presenting the results of a probabilistic risk approach is to use FN curves and measures of the individual risk, see Chapters 6 and 7. Society defines the constraints of the analysis. Usually, this is in the form of a limit line in the FN diagram below which the results of the analysis must lie. The average societal risk may also be used as a measure of tolerable risk. The average risk expresses the number of expected fatalities per year. The individual risk also indicates the maximum tolerable

risk and is usually expressed as the probability per year. Figure 8.1 shows schematically how the tolerable risk criteria can be used on an FN curve. The sloping line indicates the maximum tolerable level, and it is clear that the results of the analysis do not meet the safety goals as part of the curve exceeds the limit line.

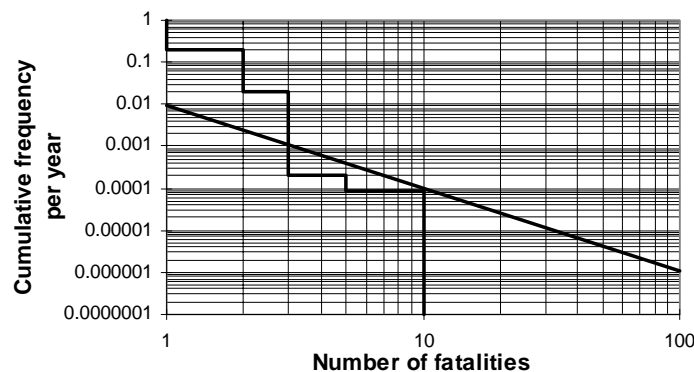


Figure 8.1. Example of tolerable risk criteria (sloping line).

Different countries use different limit lines in terms of starting point and slope of the line. The slope is expressed as the value of the exponent of the curve. The curve above has a slope of -2 as the frequency decreases by a factor of  $10^{-2}$  when the consequence is increased by a factor of  $10^1$ .

Slope coefficients of -1 or -2 are common. A slope coefficient  $< -1$  indicates the tendency of increased risk aversion. High number consequences are less willingly accepted than low number consequences, as the average tolerable risk measure decreases with increasing consequence. With a slope coefficient -1, the average tolerable risk is constant.

In some countries, two limit lines have been adopted, a maximum tolerable risk line and another below which the risk can be neglected. If the FN curve is located between these two lines the

hazard should be reduced if possible. Whether this is possible or not can, for example, be determined by utility-based analysis considering both cost and benefit of the actions proposed to reduce the risk. This intermediate region is then usually denoted ALARP (As Low As Reasonably Practicable). The exact meaning of this has been interpreted by the British HSE (1997) for different situations.

In many cases, the individual risk is specified as a single value of  $10^{-6}$  per year. This value is based on the Dutch risk criteria (Vrijling et al., 1995) which states that the increase in risk of dying due to exposure to a hazard should not be higher than 0.01 times the lowest death rate. The minimum risk of dying due to normal causes in The Netherlands is  $10^{-4}$  for young people.

The degree of voluntary exposure to risk is not considered in the above mentioned risk values or references. Vrijling et al. (1995) proposed that whether or not the risk is accepted voluntarily should also be considered when deciding the tolerable risk. This leads directly to the area of risk perception, i.e. how do people perceive a threat? This is an area which falls outside the scope of this thesis but Sjöberg et al. (1994) have given an overview of the topic.

### **8.3 Uncertainty in QRA**

Performing an extended QRA provides information on the degree of credibility of, for example, the resulting risk profile or FN curve. The tolerability can then be expressed in terms of a level which should be met by a specified resulting fractile of the FN curve.

This approach has been implicitly adopted in assessing the safety of releasing radionuclides according to the US Environmental Protection Agency standards for geological disposal of radioactive waste (US EPA, 1985; 1993). The standard defines a limit curve criterion for the allowed release of a number of radioactive products in terms of a curve in an FN diagram. The number in the FN diagram is related to a normalised radionuclide release.



In addition to the limit curve, the standard also states that the risk profile must lie below the limit with "reasonable expectation". This statement implies that uncertainties in the system must also be considered. Helton et al. (1997) have shown how this regulation can be complied with in different waste location assessments. The 90 percentile of the risk profile was used to determine the agreement with regulations.

This form of criteria is still rather uncommon. The calculation procedures necessary to meet the criteria are also complex and time consuming. It is a major step from using simple societal risk criteria in terms of the traditional limit in the FN diagram to the approach used in the US EPA standard. However, it is believed that performing a risk analysis without considering the inherent uncertainties leads to an uncertain result. Otherwise, the benefit of performing the analysis may be more of a technical nature than identification of some real risk.

## **9 Design values based on risk**

### **9.1 Introduction**

The current procedure in the fire safety design of a building depends, to a high degree on the competence of the architect. This is especially true when so-called engineering design methods are used. The architect must specify input parameters, decide which scenarios are the most appropriate and decide whether the proposed measures are adequate or not. This puts pressure on the architect to define a solution which is acceptable to both to the building owner and the authorities.

If prescriptive solutions are chosen, the result will be more strictly according to traditionally accepted solutions and rather easy to verify. The engineering solution is more difficult to verify according to acceptable risk levels, but gives more flexibility, cf. Chapter 1. The two methods or design strategies are denoted

- the prescriptive method and
- the engineering method.

The first relies on historically accepted solutions and will, in some cases, result in a design that is not very efficient in terms of costs involved and safety. The design cost will, however, be low as the time spent on design is low. The engineering method costs more during the design stage, but will hopefully result in a more cost effective solution. However, the task depends on the integrity of the architect to provide an acceptable solution. The problem is that there are no generally accepted engineering methods available in fire safety engineering nor are any tolerable risk levels available.

In other engineering fields, such as in structural engineering, so-called design values have been derived which can be used together with accepted calculation methods. This results in an accepted engineering solution to the problem. The design values in, for

example, structural engineering lead to a predefined safety level as they are derived from a specified target risk. This risk is usually described as the target reliability index  $\beta$ . More advanced methods exist in parallel with commonly used engineering methods, cf. level 3 and 4 methods in Section 6.6.1. The area of fire safety science is young and has not yet developed standardised engineering methods.

There are procedures which can be used in fire safety design but design values based on risk measures have not been developed (ISO/CD 13387, 1997; NKB, 1997; BSI, 1997 and Fire Engineering Guidelines, 1996). The procedures rely heavily on the use of expert teams, credible worst case scenarios and sensitivity studies.

This should not be considered nonrational, but there is a need for an engineering method which is based on the use of predefined design values derived from an accepted risk level. The remainder of this chapter will describe a method with which the so-called design values can be derived. The engineering procedures in the above mentioned references can be used for the more complex buildings where it is impossible to derive design values due to the small number of buildings of that type.

## **9.2 Theory of the method**

The method by which the design values are derived is based on the FOSM analytical reliability index  $\beta$  method presented in Chapter 6. More specifically, the method employs the Hasofer and Lind reliability index  $\beta_{HL}$  (Hasofer et al., 1974). The method provides information on the design point. Using the values defining the design point leads to a result with a reliability equal to the reliability index  $\beta_{HL}$ . This index can be used to estimate the probability that the system will fail, in this case equivalent to the situation where at least one person is unable to escape safely.

The method has been used in the area of structural engineering to derive design values and partial coefficients (Thoft-Christensen et al., 1982 and Sørensen et al., 1994). The principle has been demonstrated for fire safety in assembly rooms where both the design values and partial coefficients were derived (Frantzich et al., 1997). This example is presented later in this chapter.

The general procedure for deriving design values has been outlined by Thoft-Christensen et al. (1982)

- set limits on the range of scenarios or subscenarios for which the deterministic equation shall be valid
- identify the main uncertainty contributors
- select the desired safety format, i.e. the number of partial coefficients and their position in the design equation
- select the appropriate characteristic values to be used as fixed deterministic quantiles
- determine the partial coefficients, to be used together with the corresponding characteristic values, or design values to achieve the required reliability or level of safety.

The last of these points will be analysed using an optimisation procedure.

### **9.3 The design problem**

The design problem can be formulated in terms of a limit state function

$$G(X_1, X_2, \dots, X_n) = 0 \quad [9.1]$$

This description is similar to that presented in Chapter 6. The previous use of the reliability index  $\beta_{HL}$  method was simple; specify the input data in terms of random variables and constants and derive the reliability index. Now, however, the objective is to find a solution that satisfies the condition

$$P(G < 0) < p_{target} \quad [9.2]$$

The procedure is to specify the target reliability index  $\beta_{HL}$  and the variables, both random and constants, and to vary the design parameter until the target reliability index  $\beta_{HL}$  is obtained. The design parameter may, for example, be the escape door width, which is the parameter the designer wants as the result using the deterministic design equation.

The method provides the vector of design values,  $x_{i,d}$  (the design point) which is used to define the partial coefficients,  $\gamma_i$ , based on the characteristic values,  $x_{i,ch}$

$$x_{i,d} = \gamma_i \cdot x_{i,ch} \quad [9.3]$$

The characteristic value is defined as a chosen percentile from each relevant distribution, e.g. the 50th, 80th or other percentile. The partial coefficients,  $\gamma_i$ , result from this choice.

One problem in using the method as described in Chapter 6, is that the result will only be valid for the specified subscenario and building described by the limit state function  $G(X)$ . A design guide must be valid for more than one specified subscenario in one single building. It must be valid for a class of buildings with, for example, the same occupancy type, using the same safety concept. The safety concept may be defined in terms of the same type of installations, for example, sprinklers and escape alarm systems.

Also, the design guide cannot refer the architect to a level 2 or 3 reliability index method. The method should be deterministic and on level 1, cf. Chapter 6. Each class of buildings would yield a separate deterministic design equation or set of design values. The solution to the problem is to derive the design point vector, using an optimisation procedure that fulfils two conditions

- keep the average safety level constant for the class of buildings
- minimise the difference in the required and obtained safety level, taken over all individual buildings.

Two criteria must be met in order to use this method:

- it must be possible to define the limit state function  $G(X)$
- statistical information on the variables  $X_i$  must exist.

These two criteria are rather obvious. The limit state function can be defined as expressing the number of people not able to escape within the available time.

As mentioned in Chapter 5, the available statistical database in fire safety engineering are extensive but not easily accessible. Only a few attempts have been made to organise the available data. Therefore, much statistical information must be assumed.

The use of characteristic values and partial coefficients may seem confusing as the result of the procedure and what the architect needs are the design values. The partial coefficients are derived from the design values and a choice of the characteristic values. This safety format has been proven rational for structural engineering, and it may be applied also to fire safety engineering.

There is, however, no evidence that this is the most adequate procedure but there is an advantage in using this safety format. The same characteristic values can be used for a large group of buildings, more than one building type class, and the differences in design can be identified by using different partial coefficients. The coefficients can be seen as a form of safety factors which are based on a specific risk.

#### **9.4 Method**

A design guide should cover many types of buildings under various conditions in order to be a useful tool. Design equations

and relevant design values must be available for the different conditions specified in the guide. Therefore, the guide should clearly define under what conditions the values can be used. The values must not be used outside the bounds for which they have been derived.

Examples of conditions that may vary when using a specific equation are room area, room height and type of occupancy. The result given by the equation is a design variable which may be, for example, the required door width, the number of staff or the maximum occupancy load. Using the design values in the design equation will yield a value of the desired design variable that satisfies the required safety conditions defined in Section 9.3.

There will be a variation in safety within the class bounds, but it is the purpose of the method in deriving the design values to minimise this variation. If large variations in safety are found, the overall uncertainty must be decreased or the class must be further divided into smaller classes.

There is a conflict between the number of classes required by the safety standard, in order to cover all variations, and the number with which the architect wishes to work. The architect wants as small a number of classes as possible. On the other hand, if equations are derived for classes which cover a large proportion of the population of buildings, there will probably be difficulties in obtaining the equations and satisfying the general conditions. If the conditions for the building types in one class are too diverse, convergence problems will arise in the optimisation procedure. The class must cover similar types of buildings in order to ensure small deviations in the safety within the class.

To illustrate the procedure of deriving the design values and the corresponding partial coefficients, an example will be given (Frantzich et al., 1997). This is an example of the limit state function for the evacuation time margin from a large assembly

building where the objective is to derive a design equation for the required escape door width,  $W$ . The general limit state function for the subscenario is

$$G = 1.67\alpha_f^{-0.26} H_r^{0.44} A_r^{0.54} M_S - 5.36\alpha_f^{-0.48} H_r^{0.7} - t_{resp} - \frac{N_o A_r}{WF_s} \quad [9.4]$$

The condition for this subscenario is that an automatic fire alarm system is installed and working. There is no sprinkler system and all the occupants are able to evacuate by themselves. The expressions in the limit state function are derived using the response surface technique. The values of the variables in the function are based on subjective judgement.

Let the class of buildings, for which the design equation is to be applicable, be defined by the floor area and room height intervals

$$\begin{aligned} 3 \text{ m} < \text{room height, } H_r < 8 \text{ m} \\ 1000 \text{ m}^2 < \text{floor area, } A_r < 1600 \text{ m}^2 \end{aligned}$$

The objective is to derive design values for the fire growth rate,  $\alpha_{f,d}$ , and occupant response time,  $t_{resp,d}$ , for the building class so that the variation in safety level is minimised over the calculated examples. The other variables subject to uncertainty, the occupant density,  $N_o$ , and model uncertainty,  $M_S$ , and the specific flow rate through the doorway,  $F_s$ , are treated as deterministic variables in order to provide a transparent solution that can be illustrated in a two-dimensional space.

The class is assumed to be represented by six combinations of floor area and room height; (3, 1000), (5, 1000), (8, 1000), (3, 1600), (5, 1600) and (8, 1600). The procedure involves finding a limit state function for each of the six combinations, such that the respective  $\beta_{HL}$  values are as close to the specified  $\beta_{target}$  as



possible. The different limit state functions are derived by choosing different values for the design variable, i.e. the escape door width, considering the uncertainty variables  $\alpha_f$  and  $t_{resp}$ . The expression for the required escape door width in terms of the design values is

$$W = \frac{\frac{N_o A_r}{F_s}}{1.67\alpha_{f,d}^{-0.26} H_r^{0.44} A_r^{0.54} M_S - 5.36\alpha_{f,d}^{-0.48} H_r^{0.7} - t_{resp,d}} \quad [9.5]$$

Each of the six representations will have a limit state function

$$G_j(\alpha_f, t_{resp}) = 1.67H_{r,j}^{0.44} A_{r,j}^{0.54} M_S (\alpha_f^{-0.26} - \alpha_{f,d}^{-0.26}) - 5.36H_{r,j}^{0.7} (\alpha_f^{-0.48} - \alpha_{f,d}^{-0.48}) - (t_{resp} - t_{resp,d}) \quad [9.6]$$

from substituting Eq. [9.5] in Eq. [9.4] where  $H_{r,j}$  and  $A_{r,j}$  are the room height and floor area values for each representation. The method is to find a vector of the design values,  $\alpha_{f,d}$  and  $t_{resp,d}$  that minimises the object function

$$\sum_{j=1}^6 (\beta_j(\alpha_{f,d}, t_{resp,d}) - \beta_{target})^2 \quad [9.7]$$

where

$$\beta_j(\alpha_{f,d}, t_{resp,d}) = \min_A \sqrt{\left(\frac{\alpha_f - \mu_{\alpha_f}}{\sigma_{\alpha_f}}\right)^2 + \left(\frac{t_{resp} - \mu_{t_{resp}}}{\sigma_{t_{resp}}}\right)^2} \quad [9.8]$$

and  $A = \{(\alpha_f, t_{resp}); G_j(\alpha_f, t_{resp}) \leq 0\}$

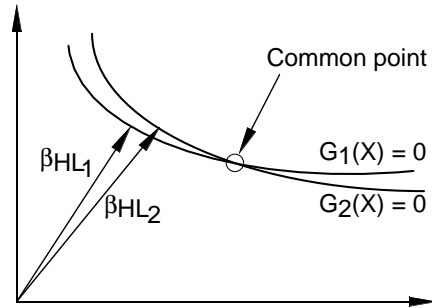
The algorithm used to derive the design value vector can be described by

1. Assume initial values for the design values.
2. Solve the six design variable values, i.e. the six values of  $W$ .
3. Derive the six separate reliability indices  $\beta_{HL,j}(X_{i,d})$  using, for example, the FOSM method (see Chapter 6).
4. Calculate the sum of the squares according to Eq. [9.7].
5. Use a numerical optimisation procedure that calculates the vector of the design values that minimises Eq. [9.7].
6. When the vector of the design values has been derived, the corresponding partial coefficients can be obtained.

As this optimisation procedure uses different limit state functions, one for each building condition, there is a formal error in using the expression design value vector, for the result. In Chapter 6, the design values were obtained as the variable values at the design point on the surface  $G(X) = 0$  having the shortest distance to the standard origin. The design point obtained from the optimisation procedure is not compatible with this definition.

The procedure does not result in one explicitly defined point which simultaneously coincides with each of the six individual correctly defined design points. The design point resulting from the procedure will, however, hopefully be very close to each of the individual limit state function design points.

Figure 9.1 illustrates the difference in design points between the individual limit state functions and the common point obtained from the optimisation procedure. The term "common point" may be the most appropriate to distinguish it from the well defined "design point". But, as the common point is used to derive the design values, the term design point is still used despite this slight inconsistency.



*Figure 9.1. Identification of the common design point and two individual limit state function design points with corresponding  $\beta_{HL}$  vectors.*

When defining the convergence criteria for the optimisation procedure, the maximum distance from the common design point to each of the individual design points can be defined. This is done in order to avoid the common design point becoming too remote from each of the individual limit state function design points. In the procedure, no such criterion was employed. The only objective was to minimise the sum of the squares according to Eq. [9.7]. This can result in an individual value of  $\beta_{HL,j}$  which is less than the target  $\beta_{HL}$  value, as other values can be higher.

To avoid numerical solutions that result in  $\beta_{HL,j}$  values smaller than the target value of the reliability index, the following object function may be considered:

$$\min_{j \in \{1, \dots, 6\}} (\beta_j(\alpha_{f,d}, t_{resp,d}) - \beta_{target})_{\vee} \quad [9.9]$$

where  $(x)_{\vee} = x$  if  $x \geq 0$  but  $\infty$  if  $x < 0$ . This expression will only consider solutions that are safer than the lower limit defined by  $\beta_{target}$ .

Thoft-Christensen et al. (1982) suggested that other object functions can be used to consider the different degree of importance of the building conditions representing the class. This can be performed, for example, by adding different weights to the object function:

$$\sum_{j=1}^6 w_j (\beta_j(\alpha_{f,d}, t_{resp,d}) - \beta_{target})^2 \quad [9.10]$$

where  $w_j$  are the weights for the representations.

### **9.5 Design values**

In order to derive the design value vector, the target reliability,  $\beta_{target}$ , must be specified. The value for this example was chosen to be 1.4 based on subjective assessment. This corresponds to an approximate probability of failure of 8% assuming that the subscenario has occurred, i.e. the fire has started and has not been extinguished. The two variables,  $\alpha_f$  and  $t_{resp}$ , are both subject to uncertainty and must therefore be specified as random variables. The procedure is on level 2 according to the definitions in Chapter 6, and  $\alpha_f$  and  $t_{resp}$  are defined by the mean value and the standard deviation, see Table 9.1.

*Table 9.1. Random variables*

Variable	$\mu$	$\sigma$	
$\alpha_f$	0.05	0.01	kW/s <sup>2</sup>
$t_{resp}$	100	80	s

The other variables in the expression are treated as constants. Values are given in Table 9.2.

Table 9.2. Constant variables

Variable		
$M_S$	1.35	-
$N_o$	0.7	persons/m <sup>2</sup>
$F_s$	1.0	persons/(m s)

The values are chosen based on assessment and are perhaps slightly too small considering the occupancy type in the building. They should, therefore, be seen only as sample values for the purpose of illustration.

The iteration procedure was performed with standard Matlab functions in combination with an optimisation toolbox. The results of the calculation indicate that the design values should be  $(\alpha_{f,d}, t_{resp,d}) = (0.053, 210)$ . Using these values, the corresponding design equation can be obtained as a result of Eq. [9.5]:

$$W = \frac{0.7A_r}{4.8H_r^{0.44} A_r^{0.54} - 21.9H_r^{0.7} - 210} \quad [9.11]$$

As the problem is solved numerically there may be more than one solution that fulfils the conditions, i.e. there may be local minima. In order to avoid false results, several starting points for the initial design value were used. The resulting limit state curves are shown in Figures 9.2 and 9.3. The starting point is illustrated by an asterisk. The standard origin is defined by the o symbol. The line originating from the asterisk is the search path for the procedure. The vertical axis shows the occupant response time,  $t_{resp}$  and the horizontal axis, the fire growth rate,  $\alpha_f$ .

It should be emphasised that the curves showing the limit state functions are not fixed in the diagram but are moved for each iteration in the optimisation procedure. The figures show the final positions of the curves. The required escape door widths for the six cases are presented on the right of each figure. Due to the small difference between the common design point and the individual

design points for the six limit state functions, they cannot be observed directly.

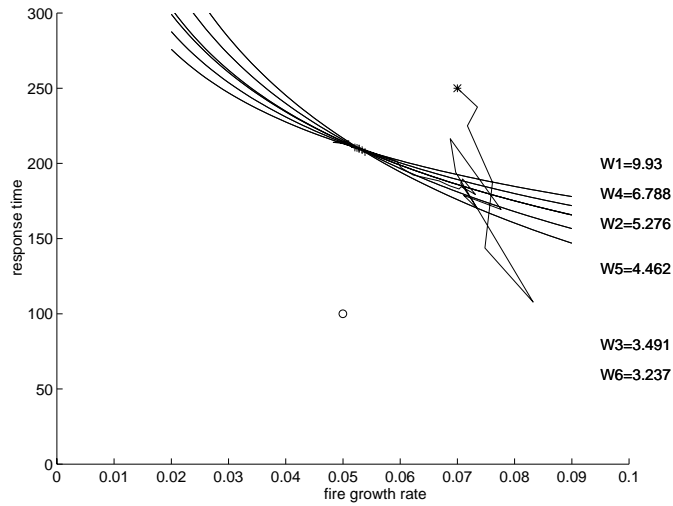


Figure 9.2. Illustration of the iterations and the final limit state functions. Starting point (0.07, 250).

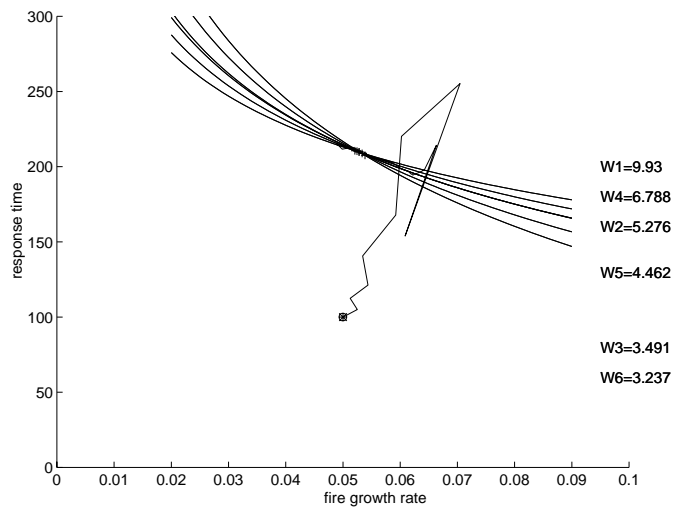


Figure 9.3. Illustration of the iterations and the final limit state functions. Starting point (0.05, 100).

Having determined the design values, corresponding partial coefficients can be obtained. The design equation will then have a different appearance:

$$W = \frac{0.7 A_r}{2.25 \alpha_{f, ch}^{-0.26} \gamma_{\alpha_f}^{-0.26} H_r^{0.44} A_r^{0.54} - 5.36 \alpha_{f, ch}^{-0.48} \gamma_{\alpha_f}^{-0.48} H_r^{0.7} - 210} \quad [9.12]$$

To be able to derive the partial coefficients the characteristic values must be determined. Characteristic values should normally be chosen such that load variables, such as the occupant response time, are seldom exceeded. The characteristic value for strength variables, such as the available escape time, should normally be exceeded. However, the values chosen should not be so large or so small that they are never observed. The choice is greatly influenced by expert judgement.

It is also clear that the safety format, with characteristic values and partial coefficients, is suitable for structural engineering, but not necessarily for fire safety engineering. The uncertainty in the variables is only of interest conditional that the subscenario has occurred. There is thus a probability  $p_i$  which must also be considered, and which can perhaps be included in the target reliability value. A structural element, on the other hand, is always subject to the load/strength environment. There are no conditional forces, except those resulting from accidents. Other design values are used to consider the likelihood of a specific accident, and are expressed in terms of another target value for the structural system reliability.

### **9.6 Problems with the method**

It is evident that there are some problems associated with this method in addition to the general limitations explained at the beginning of this chapter. The numerical solutions may encounter convergence problems if too broad building classes are chosen. The class should incorporate similar building types and occupancies and the class bounds must be evaluated and examined

more than once to find an optimum interval. In order to ensure reasonably large classes, the variable uncertainty must be low and the subscenario must be described as accurately as possible.

This can, for example, be achieved by considering the correlation between variables defining the representative buildings in the class during the evaluation. In the example in this chapter, a correlation between the occupant response time and the room area should perhaps have been included in order to give a better description of the limit state. If this correlation had been included in the limit state function, the building type class could have been larger.

Another problem is encountered in cases where more than two variables are subject to uncertainty. It may be difficult to find one well-defined design point as the elementary problem in  $\mathbf{R}^3$  results in a line of design points as the limit state functions are defined as planes in the space. The solution may be to force the common design point towards the individual limit state function design points. The choice of object function is then very important.



## **10 Summary, conclusions and future work**

Fire safety design has traditionally been reliant upon prescriptive regulations and detailed design solutions. As building codes now allow engineering solutions to the design objectives these solutions have become more frequent. One particular problem in fire safety engineering design is the lack of accepted design values, which forces the architect to choose these values more or less on his or her own judgement. Occupant safety will then be determined by the experience and the skill of the architect. As the values used for design will be subjectively chosen, the resulting risk level will be unknown. This thesis presents two Quantitative Risk Analysis (QRA) methods which can be used to quantify the risk to occupants in, for example, a building in which a fire breaks out.

The two methods, standard QRA and extended QRA, are similar. The extended QRA, however, considers the inherent uncertainty in the variables explicitly. The standard QRA does not allow this, and must be complemented by a sensitivity analysis or an uncertainty analysis. The standard QRA is more simple to perform and has been used extensively in many engineering fields. Both QRA methods have been applied to an example to determine the risk to patients on a hospital ward in which a fire breaks out. The fire scenario is structured with the event tree technique, resulting in 100 event tree outcomes or subscenarios.

Both risk analysis methods results in risk measures such as the individual risk and the societal risk. The societal risk is expressed both in terms of the FN curve and the average societal risk. The FN curve provides the probability that the consequences are worse than a specified consequence value. The average societal risk measure defines the average number of fatalities per year. The term FN curve implies that the consequences express the number of fatalities. As other descriptions of the consequences are also used the more general term risk profile is used instead of the term FN curve. The extended QRA results in risk measures which are

described as statistical distributions instead of single values. The confidence in the risk measures can therefore be explicitly defined. It is also shown that using the extended QRA results in risk measures which are better predictions of the risk compared to those resulting from the standard QRA. The reason for this is that the scenario can be described in a more accurate way by including the uncertainty of the variables in the analysis.

In order to assist in handling the data resulting from the risk analysis procedures a number of Matlab routines have been developed. After the risk analysis data have been calculated, these data have to be processed to enable a rational presentation of the risk measures. The routines are designed to sort the data and draw the most important diagrams, such as FN diagrams.

In addition to the two risk analysis methods, uncertainty analysis methods are also presented. These determine the probability that a subscenario will result in an undesired event on condition that the subscenario has occurred. Uncertainties in the variables are considered in this analysis. Both stochastic uncertainty and knowledge uncertainty are considered in the analysis either separately or combined. The extended QRA is a method in which the uncertainty analysis and the risk analysis are formally combined.

The response surface technique has been proven to be rational as a substitute for computer simulations for the extended QRA or the uncertainty analysis. The response surface is used to recreate the computer outputs based on only a few input variables. The computer programs used in fire safety engineering are normally based on a large number of input variables. The execution times are also rather long and the response surface is therefore a good alternative to directly linking the computer program to the procedure for the uncertainty analysis.

One major result of the uncertainty analysis is the relative importance of the variables. Being able to rank the variables in terms of their importance can be useful identifying those variables which are most important in the overall safety. Controlling these or reducing their uncertainties should be a result of the uncertainty analysis.

As both QRA methods are rather complex to use, a more simple method using design values in deterministic equations would be preferable for fire safety design purposes. It is not possible for an architect to perform a complete risk analysis for every new design solution and simpler methods must be used. The design values should be based on quantified risks in order to obtain similar risk levels in different buildings or building designs. This thesis presents a method of deriving these design values and it is complemented with an example which provides design values for a class of buildings. When these design values are known, so-called partial coefficients can be derived. Deriving these design values and defining the deterministic equations must be one of the tasks that is performed in the near future. Fire safety design can then be performed in a way that eliminates some of the subjectivity in today's design procedure.

To be able to execute such a task, the uncertainties in the variables needed for design must be determined. For most variables, such as the fire growth rate, there are more or less extensive databases, which provide a credible range (minimum values to maximum values) for the specific parameter. The data are not systematically assembled, but the information exists and must be sought after in many sources. Collecting and systematically organising the relevant data is a task which must be given high priority in future work. The next step should then be to establishing what is the de facto risk due to fires in buildings. With this information available, it may be possible to determine the tolerable risk level forming the basis for deriving the design values.

Finally, to sum up, the thesis contains methods and procedures which will hopefully contribute to an extended use of approaches based on quantitative risk in fire safety engineering design. The methods are applied on calculation examples for illustration purposes.

## **Acknowledgements**

I would like to express my sincere gratitude to my supervisor, Professor Sven Erik Magnusson, for his patience and encouragement during the process which resulted in this thesis.

I would also like to specially thank Dr Kazunori Harada, at the Building Research Institute (BRI) in Tsukuba in Japan, for his help when we started our work at the Department on uncertainty analysis. Also, Prof. Björn Holmquist and Mr Jesper Rydén, at the Department of Mathematical Statistics, deserve special thanks for their assistance in the mathematical work.

Another person to whom I am indebted to is Dr Yuji Hasemi, now at the Waseda University in Tokyo. At the beginning of my research career he invited me to stay at the BRI, to learn about the research being carried out there. This was most appreciated.

Many thanks are also directed to Mr Robert Jönsson and all my colleagues at the Department of Fire Safety Engineering.

Most of this work was supported by the Development Fund of the Swedish Construction Industry (SBUF) and by the Swedish Fire Research Board (Brandforsk), which are gratefully acknowledged.

The IEC Central Office in Geneva and the Center for Chemical Process Safety of the American Institute of Chemical Engineers are acknowledged for permitting me to use their figures (Figures 1.1 and 6.11).

Finally, my wife Annika and daughter Matilda deserve my special thanks for their continuous support and belief that I would finally, one day, complete my thesis. This is it.

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## **Appendix A. Description of Matlab files**

Matlab files were used to sort data and draw risk profiles. They can be seen as short script files written in a sort of Matlab-compatible computer source code. Each file can be used separately but should be used in a sequence to eliminate problems when, for example, a figure is to be drawn. Certain diagram formats cannot handle zero values and these must therefore be removed.

The files presented here are:

*noneg.m*  
*ccdfs.m*  
*mymat.m*  
*drawfrac.m*

It is assumed that the consequences, in terms of the number of people not being able to escape, are located in the matrix "cons". The probability of each subscenario must be located in the matrix "prob". Each row in *cons* and *prob* represents one sample iteration and each column represents one subscenario.

For the standard QRA this means that *cons* and *prob* are two line vectors as there is only one row containing the data for the analysis. In the extended QRA the data will be located in matrices. The uncertainty in the unwanted consequences for each subscenario is represented by the variation in each column in the matrix *cons*. In the same way, the uncertainty in subscenario probability be found as the variation in each column in the matrix *prob*.

### ***noneg.m***

The file *noneg.m* removes all zero values from the matrix *cons*. The reason for using this file is to make it possible to draw all risk profile lines when the diagram axes are logarithmic. If zero values are present it is impossible to draw the line. The zero values are replaced by the value 0.01.

```
%File start
[ROW,COLUMN]=size(cons);
for R=1:ROW
    for K=1:COLUMN
        if cons(R,K)<=0
            cons(R,K)=0.01;
        end
    end
end
%Last line in noneg.m
```

Care should be taken if, for example, mean consequences are derived after having used *noneg.m*. There will be a slight error in some parameters as a consequence of the 0.01 replacing zero values.

### ***ccdfs.m***

The file *ccdfs.m* sorts the data for each iteration in the extended QRA or in the resulting vector from the standard QRA. The data are sorted in increasing order, as described in Chapter 6. The probability values are increased by adding  $10^{-12}$  in order to enable drawing of the lines in a log-log-diagram. The data contained in *cons* and *prob* are not overwritten after being processed in *ccdfs.m*.

```
%File start
%Consequence data in matrix cons. Probability data in matrix
%prob.
%All prob values are increased 1e-12 to make it possible to draw
%correct figure lines.
```

```
figmin=1; %Minimum value of x for
           %the diagram.
figmax=100; %Maximum value of x for
            %the diagram.
```



```
row=size(cons,1);           % Checks number of
                             % iterations.
for N=1:row
    sumprob=sum(prob(N,:));  % Exchange N to 1 if
                             % standard QRA.
    x=cons(N,:);           % Consequence vector for
                             % Nth iteration.
    y=prob(N,:);           % Probability vector for Nth
                             % iteration.
                             % Exchange N to 1 if
                             % standard QRA.

    [z,i]=sort(x);
    index=i+1;
    indvekt=[1 index];     % First position in probability
                             % vector gets index 1.

    mincons=z(1);
    nyz=[mincons z];
    nyy=[0 y];
    [co,pr]=stairs(nyz,sumprob-cumsum(nyy(indvekt))+1e-12);
                             % Vectors containing data for
                             % the risk profile line.
    loglog(co,pr)          % Draws line in loglog
                             % diagram.

        for N=1
            hold on
        end
    axis([figmin figmax 1e-9 1e-1])
end

xlabel('X, Number of fatalities')
ylabel('P(X>x)')
title('Risk profiles, extended QRA')
% Last line in ccdfs.m
```

### ***mymat.m***

In order to create the fractile curves from the family of risk profiles created with *ccdfs.m*, the data must be processed by *mymat.m*. The information for creation of the fractiles is obtained in the following manner. For each point on the horizontal axis in the diagram produced by *ccdfs.m*, an imaginary vertical line is drawn. The interval between the values on the horizontal axis is specified by the user. The vertical line crosses each risk profile once. The file *mymat.m* determines the interceptions of the vertical line and the risk profiles for every value on the horizontal axis.

The procedure uses the data from the matrices *cons* and *prob* which must be of equal size. The procedure generates two new matrices, *Resmat* and *Sortmat*, and a vector, *myx*. The vector *myx* contains the values on the horizontal axis for which the probability values in *Resmat* and *Sortmat* are derived. *Resmat* contains the probability values for each risk profile in the diagram generated by *ccdfs.m*. Connecting these points will result in a risk profile similar to that generated directly by *ccdfs.m*. A small deviation is inevitable as the information regarding the probability values is evaluated at the values on the horizontal axis, the *myx* values. If the risk profile steps do not coincide with the values in *myx*, and they will probably not, a slight decrease in accuracy will follow. The difference is, however, not detectable by eye.

The other matrix generated is *Sortmat*. This matrix contains the same results as *Resmat* but the columns are sorted in increasing order. Fractile curves can now be plotted as each row in *Sortmat* contains data for the specified fractiles. The information is used in the Matlab file *drawfrac.m*, described below.

```
%File start
```

```
clear Resmat  
clear Sortmat
```

```
%Clear old matrix.
```

```
% Define bounds and step length for the output matrices' horizontal
% axis.
mini=0; % Minimum value of x for
% the diagram.
maxi=20; % Maximum value of x for
% the diagram.
step=0.1; % Step length in myx. Hint:
% use steps shorter than 1.0.

myx=[mini:step:maxi];
nostep=length(myx);
[rad1,noscen]=size(cons); % Matrix size.

% Following lines sort data.
for N=1:rad1
    sumproba=sum(prob(1,:));
    x1=cons(N,:);
    y1=prob(1,:);
    [z1,i1]=sort(x1);
    index1=i1+1;
    indvekt1=[1 index1];
    nyy1=[0 y1];
    probvekt=sumproba-cumsum(nyy1(indvekt1));

% Search for probability interceptions.
    Pos=1;
    for XStep=1:nostep
        if myx(XStep)>=z1(noscen)
            Resmat(N,XStep)=abs(probvekt(noscen+1));
        else
            if myx(XStep)<z1(Pos)
                Resmat(N,XStep)=probvekt(Pos);
            else
                Pos=Pos+1;
                if myx(XStep)<z1(Pos)
                    Resmat(N,XStep)=probvekt(Pos);
                else
                    continue;
                end
            end
        end
    end
end
```

```
        while myx(XStep)>=z1(Pos)
            Pos=Pos+1;
        end
        Resmat(N,XStep)=probvekt(Pos);
    end
end
end
end
Sortmat=sort(Resmat);
%Last line in mymat.m
```

### ***drawfrac.m***

After the risk analysis results have been processed with *mymat.m*, the relevant fractile can be plotted in a diagram. The following file simply extracts the relevant data from the matrix *Sortmat* in order to draw the line, for a specified fractile. The file must be modified for each new fraction to be drawn. The variable *myfrac* defines which fractile is to be extracted from *Sortmat*.

```
%File start
%Uses data in Sortmat and myx created in mymat.m.
%Output is also stored in vector y_frac.
%SPECIFY THE DESIRED FRACTILE AS myfrac.
myfrac=10;

[row2,nocol]=size(Sortmat);
frac=row2*myfrac/100;
y_frac=Sortmat(frac,:);
[ax,ay]=stairs(myx,y_frac);
ax(1,1)=0.01;                                     %Hint: use this substitution
                                                    %to draw the line to the
                                                    %vertical axis if first value
                                                    %on horizontal axis is 0.

loglog(ax,ay,':')                                %Draws line in loglog
                                                    %diagram.
```

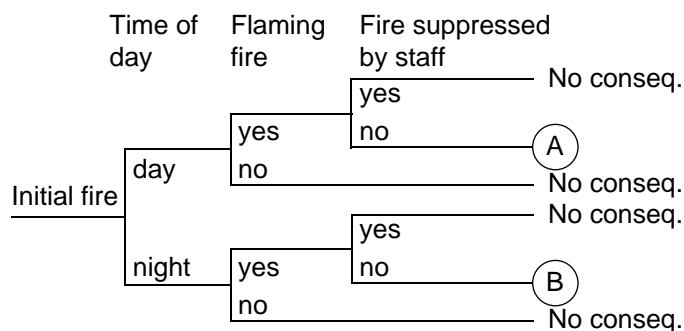
```
axis([1 100 1e-9 1e-1])  
xlabel('X, Number of fatalities')  
ylabel('P(X>x)')  
title('Confidence interval, extended QRA')  
%Last line in drawfrac.m
```

## **Appendix B. General assumptions for the sample scenario**

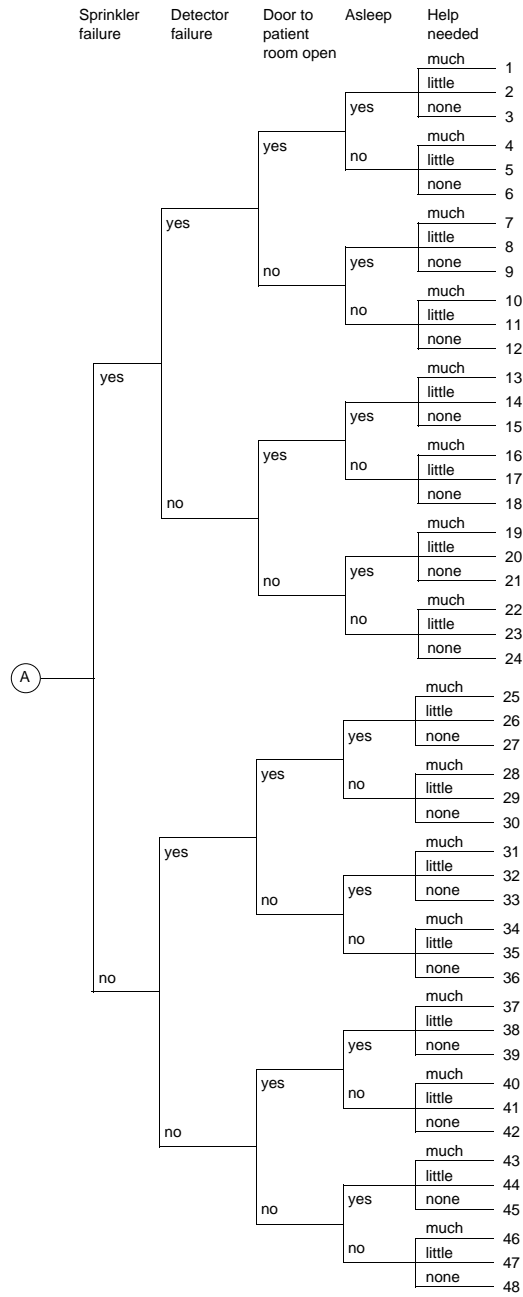
This appendix presents the background information necessary to perform both the standard and the extended QRA and the uncertainty analysis for subscenarios 1, 3, 13 and 49. If only the standard QRA is the objective, much of the information is not required.

### ***B1 Defining the scenario***

The sample calculation is defined by the scenario illustrated in Figures B1, B2 and B3. Figure B1 shows the initial part of the event tree leading to the two final parts, A and B. Part A defines subscenarios 1 to 48 and is shown in Figure B2. Part B, defining subscenarios 49 to 96, has the same general appearance, but differs in terms of when the fire starts, see Figure B3. In the initial part of the event tree, four subscenarios are identified which do not result in any unwanted consequences. In these, the fire may have been suppressed by the staff or will not grow. If these subscenarios occur, no evacuation will be necessary.

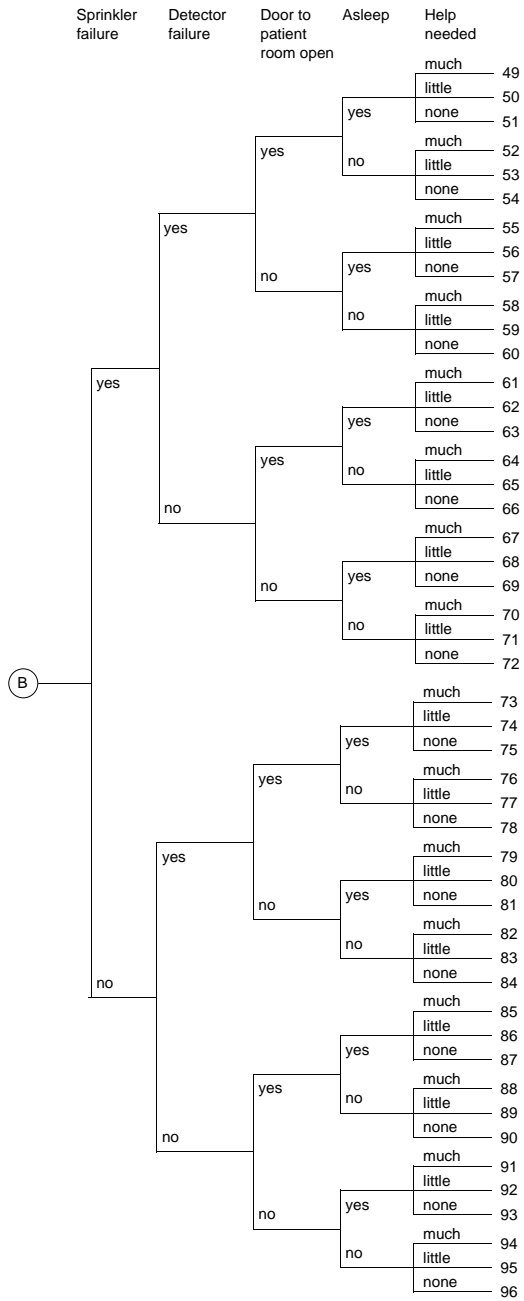


*Figure B1. Initial part of the event tree for fire on the hospital ward.*



*Figure B2. Continuation of the event tree for daytime conditions.*

*Appendix B. General assumptions for the sample scenario*



*Figure B3. Continuation of the event tree for night-time conditions.*



Each subscenario is defined by an individual limit state function which, considering the variables, reflects the current condition. The definitions of all variables and the derivation of the necessary equations are presented in this appendix.

### **B2 Initial fire probability**

In defining the risk to which patients in a hospital ward is exposed, it is necessary to know the fire occurrence rate, i.e. the probability that a fire will start. The statistics in this area are, unfortunately, rather limited. It is usually possible to predict the number of fires occurring in a town or country each year. Some information is given in Rutstein (1979) which relates the probability of fires occurring to the floor area, in m<sup>2</sup>, of the building. According to this reference the probability of having a fire in a hospital ward per year can be calculated as

$$p_{fire} = 0.0007 \cdot A^{0.75} \quad [B1]$$

This probability has been derived from reports from fire departments in the UK. The expression gives an average value of the probability and the deviation can be large. As the number of fires in hospitals is low, the reliability of the expression can be questioned. The value of the exponent has been arbitrarily assumed to be 0.75 due to low incident rate. It is, however, generally assumed that the probability increases with increasing building area. The probability increase can be assumed to be slower and therefore the exponent may be chosen to be less than one.

Using this expression for the hospital ward studied in this thesis gives a probability of a fire event of 0.077 fires per year. The floor area used for this calculation was 35 x 15 m<sup>2</sup>.

In the BSI Draft for development (1997), the overall probability of a fire event in a hospital is assumed to be 0.3 fires per year. This value is of course highly dependent on the size of the hospital. The probability derived using Eq. [B1] is valid only for a single ward in

a hospital and should be less than the probability of a fire starting at any place in a hospital.

Some preliminary Swedish data concerning fire occurrence rates are available from fire departments in the country. The data have been collected from the rescue reports following an emergency operation handled by the fire departments. Almost all Swedish health care facilities (including hospitals) are equipped with smoke detectors which are connected to the local fire department. This means that if a fire occurs in a hospital, it is very likely that the fire department will be notified of the fire. The fire department rescue reports are therefore a good estimate of the number of actual fires in a hospital.

The data are from 3 Swedish fire departments, located in different parts of the country. The incidents reported are those in which a fire has definitely started, and the false alarms have been removed. The number of reported fires is compared with the number of hospital wards in the area covered by the fire department.

The ward has been chosen as the dependent variable as the variation in size between wards is assumed to be low. This is a simplification as there are differences between wards, but the number of fires is small and other dependent variables, such as the number of fires per m<sup>2</sup>, would not necessarily increase the reliability in the prediction of fire frequency. The total number of fires reported was 59.

*Table B1. Fire frequencies in hospital wards per year in three towns in Sweden (Frantzich, 1996).*

Town	Fire frequency per year
Helsingborg	0.038
Lund	0.078
Solna	0.068

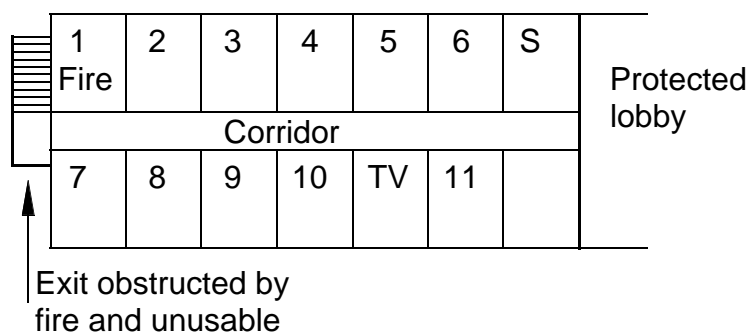
The initial fire probability,  $p_{initial}$ , has been set at 0.07 fires per year. The value for the wards in Helsingborg is half that of the others, but still of the same order of magnitude. This difference will be examined in the extended QRA, where  $p_{initial}$  is treated in some calculations as a random variable. The variable  $p_{initial}$  will then be assumed to belong to a uniform distribution [0.04, 0.1] fires per year. On the basis of the statistics from the fire departments, it is assumed that the probability of a fire occurring at night is 0.33 and during the day, 0.67.

The condition leading to evacuation of the ward, is that a fire is initiated and will continue to grow. This means that a smouldering fire will not lead to evacuation unless it develops into a flaming fire. It is assumed that a smouldering fire is harmless, at least on the time scale considered here. Calculations of the conditions in a room in which there is a smouldering fire have been performed using input parameters from Quintiere et al. (1982).

### ***B3 Building characteristics***

The calculations were performed on a hospital ward with fixed dimensions to reduce the number of calculation scenarios. The ward complies with the minimum recommendations for hospital wards set out in the former Swedish Building Code (NR, 1989). These recommendations state that the walking distance to the closest evacuation exit from any point on the ward should not exceed 30 m.

It is always assumed that the fire is located in a room close to one exit preventing it from being used. Figure B4 shows the assumed ward with 11 patient rooms, a TV room and a staff room. The exit to the right leads to a protected lobby which, in the other direction, is connected to a second ward. The patients and the staff are considered to be safe when they have reached the protected lobby.



*Figure B4. Ward layout. TV indicates a TV-room and S the staff room.*

All rooms in the ward are  $5 \times 6 \times 3.2 \text{ m}^3$  and the corridor is  $35 \times 3 \times 3 \text{ m}^3$ . All patient rooms are equipped with one window to the outside and a door leading to the corridor. The window is  $0.9 \times 0.9 \text{ m}^2$  and the window sill is located 1.2 m above floor level. It is assumed that the window is initially closed in the fire room and breaks when the fire in the room reaches a certain temperature. The door between the patient room and the corridor is  $1.2 \times 2.1 \text{ m}^2$ . It may be open or closed according to the subscenario definition.

The door between the corridor and the protected lobby is open only during evacuation. Otherwise it is closed, as it is equipped with a closing device. The door has a height of 2.1 m. The patient rooms are not separate fire compartments, but it is assumed that no smoke can leak directly from one patient room to another. The walls between the patient rooms and the corridor prevent smoke from leaking into the corridor.

The ceiling and walls are covered with gypsum plasterboard and the floor is concrete. These conditions are common for the whole ward.

The ward is equipped with a sprinkler system designed to extinguish a fire. The sprinkler system is designed according to the

Swedish regulation RUS 120:4 (1993). The sprinkler heads activate at a temperature of 68°C and are of quick-response type ( $RTI$  value  $35\sqrt{m \cdot s}$ ). The coverage area of each sprinkler head is 20 m<sup>2</sup>, which means two sprinkler heads per patient room.

The likelihood that a sprinkler system will work and be able to extinguish a fire is assumed to correspond to a probability of operation of 0.96. This value was chosen based on judgement combined with information in Bukowski (1997). This value has been used without any uncertainty.

An automatic fire alarm system is installed in the building. The alarm system is equipped with smoke detectors in every patient room and in common areas. The alarm system is monitored for errors and well maintained. The alarm system does not only indicate the presence of a fire, but gives also an alarm to the staff and patients in the ward. The sounding of the alarm informs the staff that there is a fire in the ward.

The likelihood that the automatic fire detection system will work and be able to detect a fire is assumed to correspond to a probability of operation of 0.94. This value was chosen based on judgement combined with information in Bukowski (1997). The reliability of this system is considered less well defined than that of the sprinkler system. It has therefore been subjected to uncertainty in some of the extended QRA calculations. The probability of operation will then follow a uniform distribution [0.9, 0.98]. The mean probability value will be the same with and without the uncertainty consideration.

#### ***B4 Staff and patients***

There are 22 patients on the ward, two in each patient room. The physical conditions of the patients may vary according to the subscenario. Three different physical conditions have been used to determine their need for help and their mobility. The number of patients in each of the three categories will depend on whether day

or night is considered, table B2. These proportions, used as branch probabilities in the event tree, are purely arbitrary and may vary between wards.

*Table B2. Proportions of patients in various groups according to need for help in evacuation.*

Need for help	Day	Day	Night	Night
	Sleeping	Awake	Sleeping	Awake
Much help needed	0.7	0.1	0.75	0.1
Little help needed	0.2	0.2	0.15	0.15
No help needed.	0.1	0.7	0.1	0.75

Help is needed to make the patient aware of the situation and to prepare the patient for evacuation. Different patient categories require different amount of time. The time period,  $t_{care}$ , is defined as the time spent by the staff in preparing a patient for movement and the physical movement time to the corridor. The values of  $t_{care}$  for the six different patient categories are given in Tables B3 and B4.

Rather low values have been chosen for  $t_{care}$ . This implies that patients requiring a great deal of help in preparation and movement have been excluded from this investigation.

The movement time along the corridor to the safe lobby is determined by  $t_{patM}$  and includes the time required by the staff to reach the next patient. In the extended QRA, both  $t_{care}$  and  $t_{patM}$  are normally distributed.

*Table B3. Duration of  $t_{care}$  and  $t_{patM}$  for the standard QRA. Values are in seconds.*

Awake or asleep	Need for help	$t_{care}$	$t_{patM}$
Awake	None	10	25
Awake	Little	15	40
Awake	Much	20	50
Asleep	None	13	25
Asleep	Little	25	50
Asleep	Much	40	60

*Table B4. Duration of  $t_{care}$  and  $t_{patM}$  for the extended QRA. Values are the mean and standard deviation in seconds.*

Awake or asleep	Need for help	$t_{care}$	$t_{patM}$
Awake	None	[5,5]	[20,5]
Awake	Little	[10,5]	[30,30]
Awake	Much	[15,5]	[40,40]
Asleep	None	[10,3]	[20,5]
Asleep	Little	[20,5]	[40,30]
Asleep	Much	[30,10]	[50,30]

The number of members of staff on the ward depends on if it is daytime or night-time. During the day, 4 nurses are on the ward and during the night 2. There are never more nurses than patients in one room.

After the fire has been detected by either the automatic fire alarm or manually, the staff spend some time reacting and interpreting the situation. As they are trained to respond to various kinds of signals, the response time,  $t_{resp}^{staff}$ , is rather short. The staff response time is assumed to be normally distributed [10,3] seconds in the extended QRA. In the standard QRA, the value is assumed to be 10 seconds.

If a fire occurs, the staff will most likely be able to put it out. Therefore, situations in which the ward must be evacuated have the following characteristics; the staff are not able to distinguish the

fire and it does not self-extinguish. This is a very infrequent event and its probability has been estimated on the basis of statistics and discussions with other fire professionals. The probability of successful extinction by the staff or self-extinguishment, has been set to 0.95.

If the staff do not tackle the fire, they will move towards the patients. This movement time,  $t_{staffM}$  is assumed to follow a normal distribution [15,5] seconds. In the standard QRA the value used is 20 seconds.

The evacuation of the ward must be completed before untenable conditions arise. The limits used to define untenable conditions are given in Table B5. The condition first reached determines the available time for evacuation. Two levels of untenable conditions have been used in the risk analysis, *critical* and *lethal*.

The critical conditions are similar to those recommended for fire safety design in Sweden. In addition to the critical conditions, lethal conditions were used to define the available escape time. The lethal conditions chosen were based on work by Purser (1995) and are assumed to be relevant for hospital environments. The temperature levels have been deliberately chosen to be slightly lower than Purser's suggestion due to the assumed lower lethal limits for hospital patients. He suggested exposure to 120°C for some minutes as the lethal limit assuming water-vapour-saturated smoke.

*Table B5. Untenable conditions.*

Type	Critical	Lethal
Radiation at floor level	2.5 kW/m <sup>2</sup>	2.5 kW/m <sup>2</sup>
Smoke layer height ( $z$ )	1.5 m if $T_g > 80^\circ\text{C}$	1.0 m if $T_g > 100^\circ\text{C}$
Temperature in layer ( $T_g$ )	80°C if $z < 1.5$ m	100°C if $z < 1.0$ m
Toxicity	FED = 0.5	FED = 1.0



Toxicity is measured in terms of the Fractional Effective Dose, FED, which considers the effect of a number of toxic gases, Bukowski et al. (1989).

### ***B5 Fire specifications***

The energy release rate from the fire is assumed to follow an  $\alpha_f t^2$  relationship. It is assumed that the fire always arises in a patient room and does not spread to a neighbouring room or corridor during the time of interest. The time available for escape depends on how fast the fire grows, i.e. the growth rate of the fire,  $\alpha_f$ .

It is reasonable to assume a low value of the growth rate. Tests on the fire behaviour of hospital beds, indicate a growth rate of approximately  $0.01 \text{ kW/s}^2$  (Holmstedt et al., 1983). The bed used for that test was a standard bed used in hospitals until a couple of years ago. Newer beds are especially designed to be difficult to ignite and fires in such beds are reported to have a substantially slower growth rate in initial fire development.

After the fire in the Hillhaven Nursing Home in Norfolk, Virginia, USA in 1989 it was determined that the fire in the bed ignited, had a growth rate of approximately  $0.01 \text{ kW/s}^2$  (Nelson et al., 1991). In simulations of patient room fires growth rates in the region of  $0.0001 - 0.00025 \text{ kW/s}^2$  have been used which are very low (Notarianni, 1993).

After examining similar fires it was decided to use a fire growth rate following a lognormal distribution  $[0.01, 0.005] \text{ kW/s}^2$ . This will result in untenable conditions in the fire room within a few minutes, which is in good agreement with experiments and post-fire investigations. The value used for the standard QRA was chosen to be  $0.007 \text{ kW/s}^2$  to include the very slow growing fires reported.

## **B6 Expressions for the limit state function**

### **B6.1 Available escape time**

The unwanted consequences are derived by comparing the available time and the escape time, cf. Eq. 4.2. For situations where the difference, i.e. the escape time margin, is negative, some people will not be able to evacuate in time. The unwanted consequences are expressed in the number of patients not being able to escape safely. The safety is expressed by the limit state function for each location of interest. In this case, two locations will be studied, the fire room and the corridor. The choice of the fire room is obvious, but the choice of the corridor may need some explanation. As all patients are moved out from their rooms, they all must pass along the corridor. The conditions in the corridor will then determine the time available for evacuation of the ward. Untenable conditions will only occur in the corridor if the door to the fire room is left open. Otherwise, untenable conditions will not arise at that location.

The program CFAST (Peacock et al., 1994) has been used to derive a response surface, describing the time taken to reach untenable conditions as a function of the growth rate of the fire,  $\alpha_f$ . CFAST calculates temperature, smoke layer height, radiation, etc., in every room in the scenario. The user defines the scenario by room structure and layout. The response surface was created by the method of least squares of the logarithmic values, as described in Chapter 4. All response surface equations in this sample risk analysis will have the same general appearance

$$t_u \text{ (or } t_{\text{det}}) = \exp(\lambda + \ln(\alpha_f) \delta) \quad [\text{B2}]$$

where  $\lambda$  and  $\delta$  are the regression coefficients from the linear analysis, see Tables B6 - B8. To be completely accurate, the uncertainty  $s_e$  in the prediction of the variable, should also be accounted for. This factor has only a small effect on the total

uncertainty in these cases and has been omitted here. If large deviations between the computer output and the regression equation are observed,  $s_e$  should be included.

The event tree results in a number of different fire situations, each with a new response surface equation describing the time available for evacuation. New expressions must be derived describing whether

- the door to patient room is open or closed after passage, and whether
- the sprinkler system operates or not.

*Door open/closed*

After patients have been removed, the door between the corridor and the patients' room can either be left open or it can be closed. If the door is closed after passage of both patients, the conditions in the corridor will never reach untenable levels. If the door is left open after the patients have been removed, untenable conditions will eventually arise in the corridor. Untenable conditions will always occur in the patient room in which the fire started, independent of whether the door is open or closed. For the CFAST calculations it is assumed that the door was opened after 90 seconds to let the patients escape. If it was closed after passage, it is assumed that the door will be completely closed after 150 seconds, otherwise, it will be kept open.

Equations have been derived for subscenarios both with and without the sprinkler system. The sprinkler activation times in the patient room were, for all subscenarios, much longer than the time taken to reach untenable conditions. The sprinkler operation will therefore not affect the time available for escape from the fire room. The sprinklers will, however, affect the conditions in the corridor if the door to the patient room is left open; thus increasing the overall safety. When sprinklers activate, they will for many

situations result in an infinite available escape time, i.e. the conditions will never reach untenable levels (critical and lethal).

The available escape time in the corridor is also dependent on when or whether the window breaks in the patient room. There are few data when windows break and how much of the glass that falls out. A sensitivity study has been performed to examine how the available escape time in the patient room and in the corridor depends on when the window breaks (Frantzich, 1996). Based on the results of this study, it was assumed here that the windows was 60% open when the fire gas temperature in the room had reached 250°C.

*Table B6. Regression coefficients for time taken to reach critical conditions, mean and standard deviation.*

Condition	$\lambda$	$\delta$	No. obs.	$R^2$
Fire room open door	[2.77, 0.03]	[-0.42, 0.01]	10	1.00
Fire room closed door	[2.71, 0.06]	[-0.43, 0.01]	10	0.99
Corridor sprinklers work	[4.60, 0.10]	[-0.13, 0.04]	3 *	0.90
Corridor sprinklers fail	[4.10, 0.11]	[-0.35, 0.02]	9	0.96

\* Untenable conditions only for fire growth rate  $\alpha_f > 0.05 \text{ kW/s}^2$ .

*Table B7. Regression coefficients for time taken to reach lethal conditions, mean and standard deviation.*

Condition	$\lambda$	$\delta$	No. obs.	$R^2$
Fire room open door	[2.95, 0.09]	[-0.48, 0.02]	10	0.99
Fire room closed door	[3.21, 0.05]	[-0.37, 0.01]	10	0.99
Corridor sprinklers work	- *	-	-	-
Corridor sprinklers fail	[4.28, 0.10]	[-0.34, 0.02]	9	0.97

\* Lethal conditions did not arise in corridor.

## B6.2 Detection time

The model Detact-t2 was used to calculate detection times for smoke detectors for different fire growth rates (Evans et al., 1985). The Detact-t2 model calculates the activation time for a given fire and detector configuration. The smoke detectors are assumed to behave like heat detectors but with a much faster response. The detectors have the following characteristics:  $RTI = 0.5 \sqrt{m \cdot s}$ , activation temperature = 25°C, i.e. 5°C above ambient temperature. A response surface equation was created for the detection time according to Eq. B2. The parameters are presented in Table B8. The prediction error,  $s_e$ , in the detection time was considered small.

*Table B8. Regression coefficients for smoke detector detection time.*

Condition	$\lambda$	$\delta$	No. obs.	$R^2$
All conditions	[3.02, 0.04]	[-0.31, 0.01]	5	0.99

If the smoke detectors fail to operate, someone on the ward must observe the fire and alert the staff. The manual detection time,  $t_{det}$ , is assumed to be a random variable, normally distributed with the parameters [90, 45] seconds for daytime and [120, 60] seconds for night-time. These two distributions are chosen purely based on

judgement. The night-time distribution results in longer detection times as there are fewer members of staff present than during the day. In the standard QRA, the mean values plus one standard deviation were chosen to represent the conditions.

### **B6.3 Model uncertainty**

There is no computer model that predicts reality without any error. Limitations and simplifications in the models inevitably result in deviations between the predicted values and those measured in a test or a real fire situation. There is no such thing as a perfect model. A correction factor must be used to compensate for some of the differences between experimental results and predictions.

Two computer models have been used in this study. CFAST was used for the prediction of the time available for evacuation or the time taken to reach untenable conditions. Detact-t2 was used to calculate the activation times for detectors and sprinkler heads. Based on the results of a few experiments, the difference in available escape time may be treated as a random variable,  $M_S$ , normally distributed [1.35, 0.1] (Magnusson et al., 1995). The model CFAST underestimates the time available for evacuation by a factor of 1.35 on average.

The uncertainty in the Detact-t2 model is unknown. The activation time in reality is highly dependent on the ceiling configuration and other obstructions in the upper part of the room. The variation in detection time may be significant. Better prediction can be obtained by Computational Fluid Dynamic (CFD) models which currently are under development (Andersson, 1997).

### **B6.4 Movement time**

Movement will take place from two locations:

- from the patient room to the corridor
- from the corridor to a safe place outside the ward.

First after the staff have responded to the alarm, do they move towards the patients during the time period,  $t_{staffM}$ . Then they start to prepare the patients for movement. The time required to move the patients from their room to the corridor can be derived using the following equation

$$t_{move}^{room} = t_{care} \cdot (PatInRm / StaffInRm). \quad [B3]$$

The variables  $PatInRm$  and  $StaffInRm$  indicate the number of patients and members of staff in the patient room during evacuation, i.e. 2 patients and 2 nurses.

After the patients have been evacuated from the room in which the fire started, the rest of the patients may also need to be evacuated. This will be the situation if the door between the corridor and the patient room is left open. If it is closed, there is no need to evacuate the other patients.

The expression for the evacuation time for the whole corridor is

$$t_{move}^{corr} = (t_{care} + t_{patM}) \cdot (NoPat / NoStaff). \quad [B4]$$

The time required to evacuate each patient is now the sum of the preparation time,  $t_{care}$ , and the movement time to the safe place,  $t_{patM}$ .  $NoPat$  and  $NoStaff$  are equal to the total number of patients and member of staff on the ward.

### B6.5 Limit state function

The problem can now be formulated in terms of the number of people that might not be able to be evacuated before untenable conditions arise. The limit state function is based on the escape time margin which expresses the time difference between the available time and the required escape time. The appearance of the limit state function depends on whether the door to the patient room where the fire started is open or not. The closed door subscenario means that only the patient room containing the fire

must be evacuated. Differences in variable values will result in different consequence values, defined by the variable  $c_i$ .

The maximum number of patients that may be trapped in the subscenarios in which the door is being closed is 2 as this is the maximum number of patients in a room. If the door is left open the number is increased to 22 as the other patients also have to be evacuated and might be subjected to the hazard. The two limit state functions can be formulated as escape time margins for the fire room and the corridor as:

$$\text{Room margin} = t_u^{\text{room}} M_S - t_{\text{det}} - t_{\text{resp}}^{\text{staff}} - t_{\text{staff}M} - t_{\text{move}}^{\text{room}} \quad [\text{B5}]$$

$$\text{Corridor margin} = t_u^{\text{corr}} M_S - t_{\text{det}} - t_{\text{resp}}^{\text{staff}} - t_{\text{staff}M} - t_{\text{move}}^{\text{corr}} \quad [\text{B6}]$$

These can be expressed, for each subscenario  $i$ , in terms of the number of patients:

- when the door is closed
  - $\text{RoomCons} = 2$ , if Room margin  $< 0$ .
  - $c_i = \text{RoomCons}$
- when the door is left open
  - $\text{CorrCons} = (\text{Corridor margin} / t_{\text{move}}^{\text{corr}}) \text{NoPat}$  if Corridor margin  $< 0$ .
  - $c_i = \text{RoomCons} + \text{CorrCons}$

If the escape time margin is positive, all patients have been evacuated before untenable conditions occur. A short summary of the values used in the risk analysis is presented in Table B9.



*Table B9. Values of variable used in the risk analysis.*

Variable	Standard QRA	Uncertainty analysis
$\alpha_f$	0.007 kW/s <sup>2</sup>	LN[0.01, 0.005] kW/s <sup>2</sup>
$M_S$	1.35	N[1.35, 0.1]
$t_{det}$ (day)	135 s	N[90, 45] s
$t_{det}$ (night)	180 s	N[120, 60] s
$t_{resp}^{staff}$	10 s	N[10,3] s
$t_{staffM}$	20 s	N[15,5] s
$t_{care}$	see Table B3	see Table B4
$t_{patM}$	see Table B3	see Table B4
$PatInRm$	2	2
$StaffInRm$	2	2
$NoPat$	22	22
$NoStaff$ (day)	4	4
$NoStaff$ (night)	2	2

*Table B10. Probability variables used in the risk analysis.*

Variable	Standard QRA	Uncertainty analysis
$P_{initial}$	0.07	Unif[0.04, 0.1]
$P_{day}$	0.67	0.67
$P_{flaming}$	0.5	0.5
$P_{suppressed}$	0.95	0.95
$P_{sprinkler}$	0.96	0.96
$P_{detection}$	0.94	Unif[0.9, 0.98]
$P_{door}$	0.9	0.9
$P_{sleeping}$ (day)	0.05	0.05
$P_{sleeping}$ (night)	0.95	0.95
$P_{help}$	see Table B2	see Table B2