# Some Extensions of Poincaré-Bendixson Theory Applied to a Resonant Converter.

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# A LIMIT CYCLE OF A RESONANT CONVERTER

**Melin J. and Hultgren A.**

*Department of Technology, University of Kalmar, Sweden*

Abstract: *We study a piecewise linear system approximating the behavior of a switched DC/DC-converter. We give conditions for a unique limit cycle. The stability of a state space trajectory controlled converter is addressed. The discontinuity curves have been carefully examined with respect to uniqueness of intersecting solutions.* ***Copyright © 2003 IFAC***

Keywords: Piecewise linear system, Limit cycle, Switched system, Resonant converters.

## 1. INTRODUCTION

A group of physical system can be characterized by abrupt changes in its dynamics. Modeling that group of systems as a switched system has gained increased attention during the last years. See for instance (Schaft, *et al.*, 2000). The switched model enables more accurate analysis and more advanced control of the physical system compared to modeling with averaging methods or ignoring part of the dynamics. The resulting control algorithms are often calculation intensive but the development in real time capacity makes them possible to implement.

General analysis and control of switched systems is difficult and this report focus on the principal behavior of a certain controlled switched system, a switched electrical power converter. Applying switched control of a switched system, like an electrical power converter, can give rise to mathematically interesting solutions of the switched differential equations describing the controlled converter. Global analysis of the switched system is of course crucial when designing switched control of the physical system

The use of switched electrical power converters is widely spread due to the extensive use of electronic equipment. The demand of high bandwidth performance motivate modeling and control of the switched converters as switched systems. The analysed converter is of resonant type, having many advantages such as low switching losses at higher switching frequencies, and easier electromagnetic interference (EMI) filtering. However the control will be more complex compared to pulse width modulation, PWM, controlled converters, (Kazmierczuk and Czarkowski, 1995). The output of the converters is the current on the secondary side of a transformer, $i_0(t)$, see Fig. 1. The resonant converter is applied in a high voltage equipment with a capacitive load property. The capacitive load and the high transformer ratio, $n$, will give a comparatively large load capacitance, making the load voltage, $v_0$, only slowly varying. The converter is controlled by the supply voltage, $E$, and the four transistors $Z_1 - Z_4$ determining the voltage across the resonant circuit, $U_{AB}$, between junctions A and B. The control of the converter is performed with non linear state space feedback, determining the voltage $U_{AB}$ to be $E, -E$ or $0$. The transistors are modeled as ideal switches.

Different control methods have been proposed for the control of series resonant converters. A rather general non linear control method is chosen in this report implying state region feedback similar to the controllers suggested in (Oruganti and Lee, 1984), and

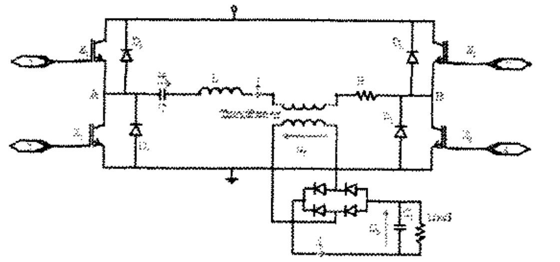

Fig. 1. The DC/DC converter circuit.

later by many authors i.e. (Rossetto, 1998). The closed system can in these cases be characterized as a switched system, in fact it is piece-wise linear. Resent works in modeling and analysis of switched converters as a switched system can be found in (Escobar *et al*,. 1999) and (Hultgren *et al.*, 2002). Analysis of piece-wise linear systems can for instance be found in (Johansson, 1999) and (Pettersson, 1999).

This report is devoted to a mathematical analysis of the rich kind of solutions of the switched differential equations that can occur in a controlled switched system of the presented type. Our switched model is a planar, linear system with discontinuous right hand side, also called a Filippov system (Filippov, 1988). We have two discontinuity lines in the model, one circle centered in origin and the $x_1$-axis (Alexander and Seidman, 1998). We will prove regularity in most parts of the $x_1x_2$-plane (Grimshaw, 1993), (Yan-Qian *et al.*, 1986) and that we have unique limit cycles for some of the parameter values. At some parts on the discontinuity lines we have irregularity, so called sliding modes (Filippov, 1988), (Utkin, 1992), (Imura and Schaft, 2000), Boukal and Krivan, 1999, which is treated according to Filippov. We will prove that the model is dissipative, (Savelév, 1991). On the sliding mode part of the $x_1$-axis, we have numerical problems due to the simulation, so called "chattering approximation" (Alexander and Seidman, 1998), (Leine *et al.*, 2000). In chapter 2 we will introduce the model of the system, prove symmetry, dissipativity , and discuss some parameter values. In chapter 3 we will look into the discontinuity curves according to Filippov, look into sliding mode phenomena and prove existence and uniqueness of a limit cycle. In all simulations we use the parameters: $R = 0.2\Omega$, $L = 31\cdot 10^{-6}\text{H}$ and $C = 2\cdot 10^{-6}F$.

## 2. THE MODEL

The model for the resonant converter with the suggested feedback in case of constant load voltage $U_0$, with $u_C=x_1$ and $i_L=x_2$ is given by:

$$\begin{cases} \dot{x}_1 = \dfrac{x_2}{C} \\ \dot{x}_2 = -\dfrac{x_1 + R\cdot x_2 - u_i}{L} \end{cases} \qquad (1)$$

where the control parameter $u_i$ takes the values of $u_1 = E - U_0$, $u_2 = U_0 - E$, $u_3 = -U_0$ and $u_4 = U_0$ in their respectively regions $\Omega_i$, $i \in \{1,2,3,4\}$. The regions are defined by:

$$\begin{cases} \Omega_1: \ x_2 > 0,\ x_1^2 + x_2^2 < i_r^2 \\ \Omega_2: \ x_2 < 0,\ x_1^2 + x_2^2 < i_r^2 \\ \Omega_3: \ x_2 > 0,\ x_1^2 + x_2^2 > i_r^2 \\ \Omega_4: \ x_2 < 0,\ x_1^2 + x_2^2 > i_r^2 \end{cases}$$

Where $i_r > 0$ is the reference current.

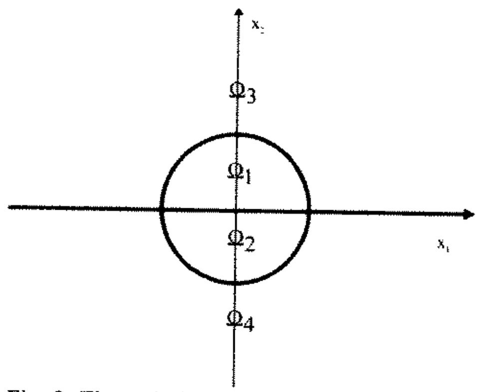

Fig. 2. The switching regions of the system.

In the circuit topology the load voltage $U_0$ can not exceed the supplied voltage $E$, so we have $E \geq U_0 > 0$. The fixed points are $(x_1, x_2) = (u_i, 0)$, the location and numbers of the fixed points depends on the parameters $E, U_0$ and $i_r$. The eigenvalues are $\lambda = -a \pm i\cdot b$, where $a = \dfrac{R}{2L}$ and $b = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$.

The circuit is supposed to have resonant property, so let $R > 0$, $L > 0$, $C > 0$ and $L/C > R^2/4$. This means that $a$ and $b$ are positive real numbers and the fixed points are stable foci. We shall assume the above conditions for the parameters through out the paper and our simulation parameters satisfy these criteria. We shall need the following symmetry property later on.

**Lemma 1:** *The vector field of the system (1) is symmetric with respect to any straight line through the origin. So called reflection symmetry in the origin or center symmetry.*

**Proof**: The differential equation for the trajectories is: $\dfrac{dx_2}{dx_1} = -\dfrac{L}{C}\cdot\dfrac{x_2}{x_1 + R\cdot x_2 - u_i}$.

Change $x_1$ to $-x_1$ and $x_2$ to $-x_2$, then we have $\dfrac{dx_2}{dx_1} = -\dfrac{L}{C}\cdot\dfrac{x_2}{x_1 + R\cdot x_2 + u_i}$, but $u_2 = -u_1$ and $u_4 = -u_3$. This completes the proof.

**Remark:** *Lemma 1 implies that any closed curve* $h(x_1, x_2) = 0$, *which is a solution to the differential equation above satisfies* $h(-x_1, -x_2) = 0$. *A consequence is that limit cycles to the system (1) posses that property.*

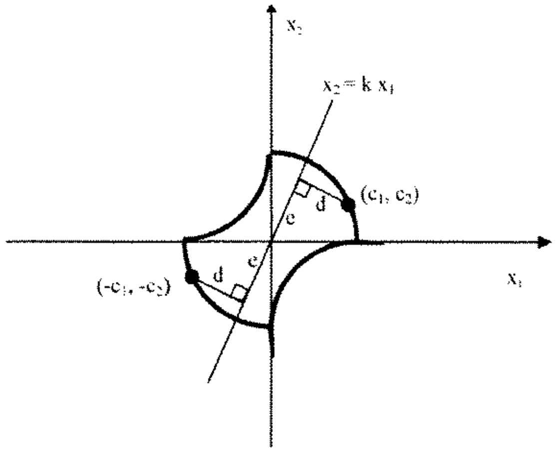


Fig. 3. Centre symmetry of the system.

We now prove that all solutions enter a bounded region in the phase plane.

**Theorem 2:** *The system (1) is dissipative.*

**Proof**: Choose a starting point $(r_0,0)$ with $r_0 > i_r$ and $r_0 > U_0$. The solution then enters $\Omega_4$ and calculations give eq. (2):

$$\begin{cases} x_1(t) = U_0 + (r_0 - U_0)\cdot e^{-at}\cdot\left(\cos(bt) + \dfrac{a}{b}\cdot\sin(bt)\right) \\ x_2(t) = -\dfrac{r_0 - U_0}{bL}\cdot e^{-at}\cdot\sin(bt) \end{cases}$$

If $r_0$ is large enough, the solution is in $\Omega_4$ when $0 < t < t_1$ without intersecting the circle. Where $t_1$ is the smallest $t > 0$ such that $x_2(t) = 0$, that is $t_1 = \pi/b$. The solution intersects the negative $x_1$-axis at $(r_1,0)$, where $r_1 = x_1(t_1)$, this implies $r_1 = U_0 - (r_0 - U_0)\cdot e^{-\frac{a\pi}{b}}$ ( 3 ). We have $r_1 = U_0 - r_0\cdot e^{-\frac{a\pi}{b}} + U_0\cdot e^{-\frac{a\pi}{b}} > -r_0\cdot e^{-\frac{a\pi}{b}} > -r_0$. Lemma 1 implies dissipativity. This completes the proof, see Fig. 4.

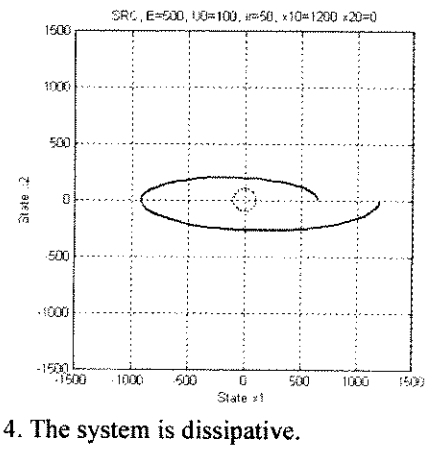


Fig. 4. The system is dissipative.

3. EXISTENCE AND UNIQUENESS OF A LIMIT CYCLE

We shall begin this section by introducing our main result. There are two bifurcation parameters the load voltage, $U_0$, and the difference between the input voltage and the load voltage, $E - U_0$.

**Theorem 3:** *Let* $U_0 \le z\cdot i_r$ *and* $E - U_0 \le z\cdot i_r$, *where* $z = \sqrt{(L/C - 1)^2 + R^2}$, *if* $E - U_0 > i_r$ *and* $U_0 < i_r$, *then the system (1) has*

*i) no fixed points and*

*ii) a unique limit cycle, if* $R = 0$.

**Remark:** *We conjecture that ii) in theorem 3 holds for* $R > 0$, *but it seems difficult to construct a proof of this fact. Our simulations have so far not contradicted this. We will later on in this paper divide the proof of the theorem into two cases and we will show case 2 only when* $R = 0$.

By proving Theorem 3 we shall study our two discontinuity curves. Filippov defines three types of sliding modes: transversal, repulsion and attracting sliding mode. Following Filippov we let $f^+$ and $f^-$ denote the two fields at different sides of the discontinuity curve $\gamma$, given by the equation $\varphi(x_1, x_2) = 0$. Define $f^0 = \alpha\cdot f^+ + (1-\alpha)\cdot f^-$, $0 \le \alpha \le 1$, where $\alpha = \dfrac{\langle grad\varphi, f^-\rangle}{\langle grad\varphi, f^- - f^+\rangle}$. According to Filippov we now have $\dot{x} = f^0(x)$, $x \in \gamma$,

**Remark:** *We introduce the definition of regularity. With regularity we will mean uniqueness of solutions. We have that property every where in the phase plane outside the discontinuity curves, and at the discontinuity curves where there is transversal sliding mode.*

*3.1 Sliding mode at the* $x_1$ *– axis.*

Transversal sliding mode, at the $x_1$-axis, occurs in the intervals $]-\infty, -i_r[$ and $]i_r, \infty[$. Repulsion sliding mode occurs in the interval $]-i_r, i_r[$. Attracting sliding mode will never occur at the $x_1$-axis. See Fig. 5.

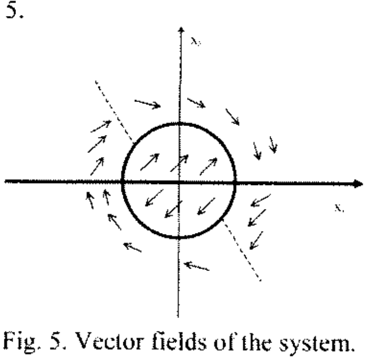


Fig. 5. Vector fields of the system.

*3.2 Sliding mode at the circle.*

Transversal sliding mode will appear where there is no attracting sliding mode. Repulsion sliding mode will never appear on the circle. Attracting sliding mode will appear between the points A and B, described below. Lemma 1 implies that we only have to consider the lower half plane. Let $A$ be the point where the attracting sliding mode starts and $B$ where it stops. See Fig. 6.

**Theorem 4:** *1) If* $E-U_0 \le z\cdot i_r$ *then* $A:\left(\bar{x},-\sqrt{i_r^2-\bar{x}^2}\right)$ *where*

$$\bar{x}=\frac{\left(\frac{L}{C}-1\right)\cdot(E-U_0)-R\cdot\sqrt{z^2\cdot i_r^2-(E-U_0)^2}}{z^2}$$

*2) If* $U_0 \le z\cdot i_r$ *then* $B:\left(\hat{x},-\sqrt{i_r^2-\hat{x}^2}\right)$ *where*

$$\hat{x}=-\frac{\left(\frac{L}{C}-1\right)\cdot U_0+R\cdot\sqrt{z^2\cdot i_r^2-U_0^2}}{z^2}$$

**Proof:**

We have $\begin{cases} f^+(x_1,x_2)=\left(\frac{x_2}{C},-\frac{x_1+R\cdot x_2+E-U_0}{L}\right) \\ f^-(x_1,x_2)=\left(\frac{x_2}{C},-\frac{x_1+R\cdot x_2-U_0}{L}\right)\end{cases}$

in respectively region and $\varphi(x_1,x_2)=x_1^2+x_2^2$. But $B$ is determined by $\langle grad\varphi, f^+\rangle=0$,this gives us $\bar{x}$. $A$ is determined by $\langle grad\varphi, f^-\rangle=0$, replace $E-U_0$ by $-U_0$ in $\bar{x}$. This gives us $\hat{x}$ and completes the proof.

The following lemma describes the location of $A$ and $B$,and can be proved by elementary calculations.

**Lemma 5:** *1) If* $E-U_0 \le z\cdot i_r$ *then* $\bar{x}\in\left[-\frac{R\cdot i_r}{z},\frac{(L/C-1)\cdot i_r}{z}\right]$.

*2) If* $U_0 \le z\cdot i_r$ *then* $\hat{x}\in\left[-i_r,-\frac{R\cdot i_r}{z}\right]$.

*3) If* $E-U_0\le z\cdot i_r$ *then* $\bar{x}>0$ *if and only if* $E-U_0>R\cdot i_r$.

*4) The attracting sliding mode part of the circle can be reduced to only one point* $\bar{x}=\hat{x}=-\frac{R\cdot i_r}{z}$ *if and only if* $E=U_0=0$.

For later use we calculate some points, $r_A$, $r_B$ and $r'_B$, connected to the previous introduced points on the circle, A and B.

**Lemma 6:** *1) Let* $(r_A,0)$ *be the starting point at the positive* $x_1$*-axis outside the circle for the solution which intersects the circle at* $A$, *then*

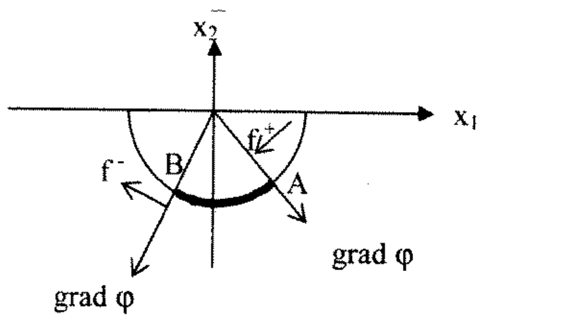


Fig. 6. Attracting sliding mode at the circle.

$$r_A=U_0+\sqrt{b^2L^2(i_r^2-\bar{x}^2)+\left(aL\sqrt{i_r^2-\bar{x}^2}+U_0-\bar{x}\right)^2}\cdot e^{a\cdot t_A}$$

*where* $t_A=\frac{1}{b}\cdot\left(\pi-\arctan\left(\frac{bL\sqrt{i_r^2-\bar{x}^2}}{aL\sqrt{i_r^2-\bar{x}^2}+U_0-\bar{x}}\right)\right)$

*2) Let* $(r_B,0)$ *be a starting point at the positive* $x_1$*-axis outside the circle for the solution which intersects the circle at* $B$, *then*

$$r_B=U_0+\sqrt{b^2L^2(i_r^2-\hat{x}^2)+\left(aL\sqrt{i_r^2-\hat{x}^2}+U_0-\hat{x}\right)^2}\cdot e^{a\cdot t_B}$$

*where* $t_B=\frac{1}{b}\cdot\left(\pi-\arctan\left(\frac{bL\sqrt{i_r^2-\hat{x}^2}}{aL\sqrt{i_r^2-\hat{x}^2}+U_0-\hat{x}}\right)\right)$

*3) Let* $(r'_B,0)$ *be the point at the negative* $x_1$*-axis where the solution from* $B$ *intersects the* $x_1$*-axis, then* $r'_B=U_0-(r_B-U_0)\cdot e^{-\frac{a\cdot\pi}{b}}$. See Fig. 7.

**Proof**: We shall start with 2), a solution in $\Omega_4$ with starting point $(r_B,0)$ is according to eq. (2)

$$\begin{cases} x_1(t)=U_0+(r_B-U_0)\cdot e^{-at}\cdot\left(\cos(bt)+\frac{a}{b}\cdot\sin(bt)\right) \\ x_2(t)=-\frac{r_B-U_0}{b\cdot L}\cdot e^{-at}\cdot\sin(bt)\end{cases}.$$

If this solution intersects $B$ we have $\begin{cases} x_1(t_B)=\hat{x} \\ x_2(t_B)=-\sqrt{i_r^2-\hat{x}^2}\end{cases}$. If we solve this system with respect to $t_B$ and $r_B$ the result follows. 3) follows

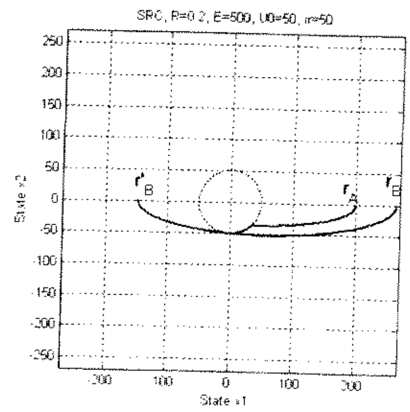


Fig. 7. Sliding mode parameters.

from formula (3). 1) follows by replacing $\hat{x}$ by $\bar{x}$ in $t_B$ and $r_B$. The proof is complete.

**Lemma 7:** *If a solution intersects the circle between* $A$ and $B$*, the solution slides along the circle to the left and leaves it at B.*

**Proof**: From the proof of theorem 4 we have $f^+$ and $f^-$, calculations give $f^0(x_1,x_2)=(x_2/C,-x_1/C)$ this implies $\begin{cases}\dot{x}_1=x_2/C\\ \dot{x}_2=-x_1/C\end{cases}$. Let the solution intersect the circle at $\left(a,-\sqrt{i_r^{\,2}-a^2}\right)$ between $A$ and $B$, then we have

$$\begin{cases}x_1(t)=a\cdot\cos(t/C)-\sqrt{i_r^{\,2}-a^2}\cdot\sin(t/C)\\ x_2(t)=-a\cdot\sin(t/C)-\sqrt{i_r^{\,2}-a^2}\cdot\cos(t/C)\end{cases}.$$

This solution slides along the circle to the left and leaves at $B$ enters $\Omega_4$ and intersects the negative $x_1$ -axis at $(r'_B,0)$. This completes the proof.

**Remark:** *Lemma 1 implies that there is a corresponding attracting sliding mode part of the circle in the upper half plane.*

**Lemma 8:** *The system (1) has no limit cycles entirely inside any of the regions* $\Omega_i, i=1,2,3,4$.

**Proof:** Let $f$ be the vector field of the system (1), then the divergence of $f$ is given by: $divf(x_1,x_2)=\dfrac{\partial}{\partial x_1}\left(\dfrac{x_2}{C}\right)+\dfrac{\partial}{\partial x_2}\left(-\dfrac{x_1+R\cdot x_2-u_i}{L}\right)=$ $=-R/L\neq 0$. This completes the proof.

**Remark:** *Theorem 2 implies that the system (1) has no limit cycles entirely outside the circle.*

***Lemma 9:*** *There exists a trajectory* $\gamma_C$ *with starting point* $(r_C,0)$, $r_C>i_r$ *such that* $\gamma_C$ *enters the circle in the fourth quadrant at* $C$, *where* $C$ *is located to the right of* $A$. *Furthermore then* $\gamma_C$ *intersects the circle at* $B$, *enters* $\Omega_4$ *and intersects the negative* $x_1$*-axis at* $(r'_B,0)$.

**Proof:** Let $R=0$, then $\bar{x}=\dfrac{E-U_0}{L/C-1}$ and $\hat{x}=-\dfrac{U_0}{L/C-1}$. In $\Omega_3$ we have the solution:

$$\begin{cases}x_1(t)=U_0-E+(r_C+E-U_0)\cdot\cos(bt)\\ x_2(t)=-\dfrac{r_C+E-U_0}{bL}\cdot\sin(bt)\end{cases} \quad (4).$$

If this solution intersects the circle at $B$ we have: $\begin{cases}x_1(t)=\hat{x}\\ x_2(t)=-\sqrt{i_r^{\,2}-\hat{x}^2}\end{cases}$, this implies

$$r_C=U_0+\sqrt{L/C\cdot i_r^2-\frac{4E^2-L/C\cdot U_0^2}{L/C-1}},$$

put this into (4) and use $x_1(t)^2+x_2(t)^2=i_r^{\,2}$ and then we have the coordinates of point

$$C:\left(\frac{2E-U_0}{L/C-1},\ -\sqrt{i_r^{\,2}-\left(\frac{2E-U_0}{L/C-1}\right)^2}\right).$$

This proves that $C$ is located to the right of $A$ when $R=0$. Now let $R>0$, but still small. Then $A,B$ moves to the left and $C$ to the right. This proves the lemma.

**Remark:** *Such a trajectory* $\gamma_C$ *never intersects the attracting sliding mode part of the circle, except at B, and we have:* $i_r<r_C<r_A<r_B$.

*3.3 Proof of Theorem 3*

We divide the proof into two different cases:

*Case 1,* $r_C\leq -r'_B<r_B$. A trajectory which starts in $(-r'_B,0)$ intersects the attracting sliding mode part of the circle and eventually intersects the negative $x_1$ -axis at $(r'_B,0)$. Lemma 1 implies that such a trajectory is a limit cycle, see Fig. 8.

*Case 2,* $i_r<-r'_B<r_C$. We will prove this when $R=0$. Let $\gamma_0$ be a trajectory with starting point $(r_0,0)$, $r_0>i_r$. $\gamma_0$ enters $\Omega_4$ and intersects the circle at $C'':(c''_1,c''_2)$ passes through the circle, leaves it at $C':(c'_1,c'_2)$ and reaches the negative $x_1-axis$ at $(r''_1,0)$. see Fig. 9.

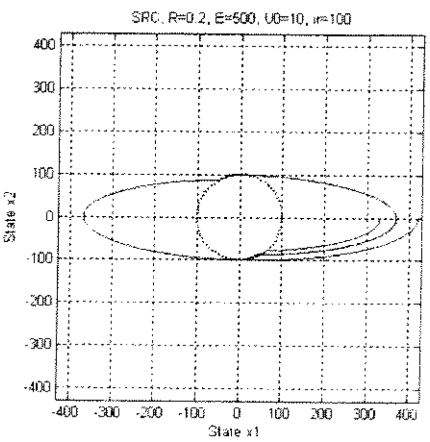


Fig. 8. Limit cycle of the system

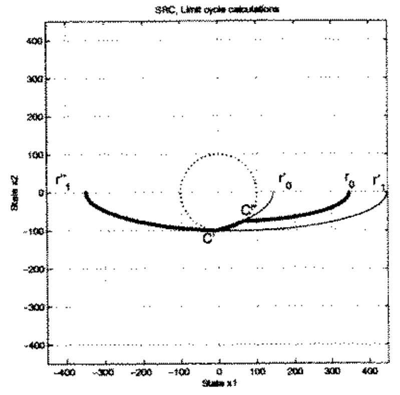


Fig. 9. Limit cycle calculations

Let $C^0:(c_1^0,c_2^0)$ be either one of $C'$ *or* $C''$, eq. *2* in $\Omega_3$ gives:

$$\begin{cases} U_0 - E + (r_0' + E - U_0)\cdot\cos(bt) = c_1^0 \\ -\dfrac{r_0' + E - U_0}{bL}\cdot\sin(bt) = c_2^0 \end{cases}$$

using $\sin^2(bt)+\cos^2(bt)=1$ gives

$$\left(\frac{c_1^0 + E - U_0}{r_0' + E - U_0}\right)^2 + \left(\frac{-bLc_2^0}{r_0' + E - U_0}\right)^2 = 1, \text{ and } (c_2^0)^2 = i_r^2 - (c_1^0)^2$$

implies

$$(r_0')^2 + 2(E-U_0)r_0' + (L/C-1)(c_1^0)^2 - 2(E-U_0)c_1^0 - L/C\cdot i_r^2 = 0,$$

$(1')$ $\Rightarrow$

$$r_0' = U_0 - E + \sqrt{(E-U_0)^2 + L/C\cdot i_r^2 + 2(E-U_0)c_1^0 - (L/C-1)(c_1^0)^2}$$

now replace $c_1^0$ with $c_1'$ *and* $c_1''$ respectively, thus:

$$\begin{cases} r_0' = U_0 - E + \sqrt{(E-U)^2 + L/C\cdot i_r^2 + 2(E-U_0)c_1' - (L/C-1)(c_1')^2} \\ r_0' = U_0 - E + \sqrt{(E-U_0)^2 + L/C\cdot i_r^2 + 2(E-U_0)c_1'' - (L/C-1)(c_2'')^2} \end{cases}$$

$\Rightarrow c_1'' = \dfrac{2(E-U_0)}{L/C-1} - c_1'$, $(2')$ consider the "virtual" trajectory with starting point $(r_1',0)$ through $C'$, see Fig. 9. according to $(1')$ we have:

$$(L/C-1)(c_1')^2 + 2U_0c_1' - L/C\cdot i_r^2 - 2U_0r_1' + (r_1')^2 = 0 \Rightarrow$$

$$c_1' = -\frac{U_0 + \sqrt{L/C(L/C-1)i_r^2 + 2(L/C-1)U_0^2 - (L/C-1)(r_1')^2 + U_0^2}}{L/C-1}$$

eq.3 implies $r_1'' = 2U_0 - r_1'$, let $r_1'' = -r_0$ $\Rightarrow$ $r_1' = 2U_0 + r_0$, put this into the formula for $c_1'$, then

$$c_1' = -\frac{U_0 + \sqrt{L/C(L/C-1)i_r^2 + U_0^2 - (L/C-1)r_0^2 - 2(L/C-1)r_0U_0}}{L/C-1}$$

and using $(2')$ implies:

$$c_1'' = \frac{2E - U_0 + \sqrt{L/C(L/C-1)i_r^2 + U_0^2 - (L/C-1)r_0^2 - 2(L/C-1)r_0U_0}}{L/C-1}$$

consider now instead the trajectory $\gamma_0$ in $(1')$, giving $r_0^2 - 2U_0r_0 + (L/C-1)(c_1'')^2 + 2U_0c_1'' - L/C\cdot i_r^2 = 0$, using the formula for $c_1''$ gives:

$$r_0 = E\cdot\sqrt{\frac{L/C(L/C-1)i_r^2 + U_0^2 - E^2}{(L/C-1)E^2 + (L/C-1)^2U_0^2}}, (3')$$

if $E^2 \le L/C(L/C-1)i_r^2 + U_0^2$.

We can also see that $C''$ is located to the right of $C$ ($C$ as in the proof of lemma 9 ) this means that $\gamma_0$ is above $\gamma_C$ ( $\gamma_C$ as in lemma 9 ), and therefore $\gamma_0$ never intersects the sliding mode part of the circle. We have shown that if we choose $r_0$ as in $(3')$, there is a unique limit cycle in this case.

## 4. CONCLUSION

In this paper we have proved the existence and uniqueness of a limit cycle of a piecewise linear system with respect to two bifurcation parameters: the load voltage and the difference between the input voltage and the load voltage. We have proved some properties of the system, such as symmetry and dissipativity. Our findings show that the dynamics of the system become much more complicated if the two bifurcation parameters values increase.

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