Modeling of a Steam Heated Rotating Cylinder—A Grey-Box Approach

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Abstract— Drying is an important process in paper manufacturing, where steam heated cylinders are used to dry paper. Control of the moisture content is accomplished by adjusting the steam pressure in the cylinders. In previous work it has been shown that the dynamics from steam flow to steam pressure in the cylinder can be described by a linear second order black box model. This paper presents a first principles model which has a structure similar to the black box model.

I. INTRODUCTION

Moisture content is an important quality variable in paper manufacturing. There is a considerable economic incentive in keeping moisture well regulated [1]. A modern paper machine makes around 1000 tons of paper per day. With a paper price of $900 per ton, a reduction of moisture variations by 0.1% corresponds to a saving of more than $300,000 per year.

A good model of the dynamics of drying is essential for good moisture control. A key element is the dynamics that relates steam flow to pressure in the drying cylinders. A black box model is presented in [2]. This model structure has also been discussed in [3]. The model has an integrator, a pole, and a zero, and is called the IPZ-model. A black-box model is adequate for controller tuning purposes but it does not tell anything about the physics behind the dynamic behavior and it cannot be used for gain scheduling. In this paper we will present a first principles model, based on mass and energy balances. The primary model is a nonlinear differential-algebraic equation whose linearized version has the same structure as the IPZ-model. It is convenient to choose steam pressure and cylinder shell temperature as state variables, since both variables can be measured.

A main purpose of this grey-box model is to gain insight into how the physical properties influence the parameters of the IPZ model. A similar approach for a drum boiler can be found in [4]. The core of this work is presented in [5].

II. THE MODEL

Let \( q_s \) be the mass flow rate of steam into the cylinder, \( q_c \) be the condensation rate, and \( q_w \) be the siphon flow rate. Also, let \( V_s \) and \( V_w \) be the volume of steam and water in the cylinder, and let \( \rho_s \) and \( \rho_w \) be the densities of steam and water. The mass balances for water and steam are then

\[
\frac{d}{dt}(\rho_s V_s) = q_s - q_c \\
\frac{d}{dt}(\rho_w V_w) = q_c - q_w
\]

where no blow-through steam is assumed. Sometimes the blow-through steam is modeled as a fraction of \( q_s \) [6]. It then does not affect the dynamics of the system, only the steady state gain. The energy balances for steam, water and metal are

\[
\frac{d}{dt}(\rho_s u_s V_s) = q_s h_s - q_c h_s \\
\frac{d}{dt}(\rho_w u_w V_w) = q_c h_s - q_w h_w - Q_m \\
\frac{d}{dt}(m C_p T) = Q_m - Q_p
\]

where \( Q_m \) is the power supplied from the water to the metal, \( Q_p \) is the power supplied from the metal to the paper, \( h_s \) is the steam enthalpy, \( h_w \) is the water enthalpy, \( m \) the mass of the cylinder shell, \( C_p \) the specific heat capacity of the shell, \( T \) the mean temperature of the metal, \( u_s \) and \( u_w \) are the specific internal energies of steam and water. The power flow to the metal is given by

\[
Q_m = \alpha A_s (T_s - T)
\]
where \( D \) is the heat transfer coefficient from the steam-condensate interface to the centre of the cylinder shell, \( A_{cy} \) is the inner cylinder area, and \( T_s \) the steam temperature. The outer surface area of the cylinder is assumed to be equal to its inside area. The error is negligible (less than 5%), because the thickness of the cylinder shell is much smaller than the outer cylinder diameter. For simplicity, all steam within the cylinder cavity is assumed to be homogeneous with the same pressure and temperature. From (3) we make the assumption of a temperature gradient in the condensate layer and cylinder shell, as illustrated in Figure 1. The properties of the paper web are not included in the model and consequently there is no assumption on the paper temperature. The picture has taken inspiration from [9].

Equation (1), (2), and (3) is a crude nonlinear model for the steam and condensate system in the cylinder cavity. To obtain a linear second-order model, we make a few simplifications. We assume that the steam in the cylinder is saturated. This means that the state of the steam can be characterized by one variable only and that it is sufficient to use either the mass balance or energy balance. Therefore we leave out the energy balance. In addition, the thermal dynamics of the water is very fast, so we replace it by a static model. Observing that the volumes are constrained by \( V_s + V_w = V \), where \( V \) is the total cylinder volume, the second mass balance in (1) can be eliminated. Since the water volume is small we also have \( V \approx V_s \). Summarizing, we find that the system can be described by the equations

\[
\frac{d}{dt}(\rho_s V) = q_s - q_c
\]

\[
\frac{d}{dt}(mC_p T) = Q_m - Q_p
\]

\[
0 = q_c h_s - q_w h_w - Q_m
\]

\[
Q_m = \alpha A_{cy} (T_s - T)
\]

which is a mass balance for the steam, an energy balance for the metal, a static energy balance for the water, and an algebraic equation for the energy flow. Eliminating the variables \( q_c \) and \( Q_m \), the model becomes

\[
\frac{d}{dt}(\rho_s V) = q_s h_s - q_w h_w - \alpha A_{cy} (T_s - T)
\]

\[
\frac{d}{dt}(mC_p T) = \alpha A_{cy} (T_s - T) - Q_p
\]

Assuming that the steam in the cylinder is saturated, enthalpies \( h_s, h_w \), density \( \rho \), and the temperature \( T \), are all functions of the pressure \( p \). The model can thus be written as

\[
\frac{d}{dt}(\rho_s V) = q_s h_s - q_w h_w - \alpha A_{cy} (T_s - T)
\]

\[
\frac{d}{dt}(mC_p T) = \alpha A_{cy} (T_s - T) - Q_p
\]

where the states are pressure \( p \) and mean metal temperature \( T \). The steam inlet flow, \( q_s \), is the input. The equilibrium is

\[
0 = q_s^0 h_s^0 - q_w^0 h_w^0 - \alpha A_{cy} (T_s^0 - T^0)
\]

\[
0 = \alpha A_{cy} (T_s^0 - T^0) - Q_p^0
\]

Hence

\[
Q_p^0 = \alpha A_{cy} (T_s^0 - T^0) = q_s^0 h_s^0 - q_w^0 h_w^0
\]

Linearizing around the equilibrium gives

\[
\left. \frac{d}{dt} \right|_{p=p^0} \frac{d\rho_s}{dp} = q_s^0 \frac{d}{dp} h_s^0 - q_w^0 \frac{d}{dp} h_w^0 - \alpha A_{cy} \frac{d}{dp} T_s^0 - \alpha A_{cy} \frac{d}{dp} T^0 \frac{d\rho_s}{dp}
\]

\[
+ \alpha A_{cy} \delta T + h_s^0 \frac{d}{dp} T_s^0
\]

\[
mC_p \left. \frac{d\delta T}{dt} \right|_{p=p^0} = \alpha A_{cy} \frac{d}{dp} T_s^0 \frac{d}{dp} - \alpha A_{cy} \delta T
\]

Fig. 1. A piece of the cross-section of a drying cylinder, visualizing the assumption on the temperature profile and the energy flow to the metal, from (3). The paper web is not included in the model and consequently there is no assumption on the paper temperature. The picture has taken inspiration from [9].
where the states are expressed in terms of deviations. Assuming that
\[
\begin{align*}
\frac{q_v}{\varphi} - q_w(p_0) \frac{\partial q_w}{\partial p} - h_w(p_0) \frac{\partial h_w}{\partial p} \ll \alpha A_c \frac{\partial T_s}{\partial p}
\end{align*}
\]  \tag{10}

the model becomes
\[
\begin{align*}
&h_s(p_0) V \frac{\partial \rho_s}{\partial p} \left. \frac{d \varphi}{dt} \right|_{p=p_0} = -\alpha A_c \frac{\partial T_s}{\partial p} \left. \varphi \right|_{p=p_0} + \\
&+ \alpha A_c \delta T + h_s(p_0) \delta q_s
\end{align*}
\]  \tag{11}

\[
mC_p \frac{d \delta T}{dt} = \alpha A_c \frac{\partial T_s}{\partial p} \left. \varphi \right|_{p=p_0} - \alpha A_c \delta T
\]

The inequality in (10) will be commented and examined later in the simulations. Writing the system in standard state-space form, we find that
\[
\begin{align*}
\dot{x} &= Ax + Bq_s, \\
y &= Cx + Dq_s
\end{align*}
\]  \tag{12}

where \(x = [\varphi \ T]^{T}\) and
\[
A = \begin{bmatrix}
-\alpha A_c & \frac{A_c}{h_s V} \\
\frac{h_s V}{\rho_s} & \frac{C_p}{h_s V}
\end{bmatrix}, \\
B = \begin{bmatrix}
\frac{1}{V} \frac{\partial \rho_s}{\partial p} \\
0
\end{bmatrix}, \\
C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
D = 0
\]  \tag{13}

The steam properties are here assumed to be given in their equilibrium values. The matrix \(A\) has an eigenvalue at the origin and one eigenvalue on the negative real axis. The transfer function from steam flow to pressure is
\[
G(s) = b \frac{s + z}{s(s + \lambda)} = b \frac{s - a_{22}}{s(s - a_{11} - a_{22})} = \\
= \frac{b}{s} \left( \frac{z + \lambda - s}{s + \lambda} \right)
\]  \tag{14}

where
\[
b = \frac{1}{V} \frac{\partial \rho_s}{\partial p}, \\
\lambda = -a_{11} - a_{22} = \alpha A_c
\]  \tag{15}

The essential parameters of the model are
- Cylinder volume \(V\)
- Cylinder mass \(m\)
- Specific heat of metal \(C_p\)
- Area of the cylinder surface \(A_c\)
- Steam properties \(h_s, \frac{\partial \rho_s}{\partial p}, \frac{\partial T_s}{\partial p}\)
- Heat transfer coefficient \(\alpha\)

All parameters, except the heat transfer coefficient, \(\alpha\), are known beforehand, either by machine specifications or from a standard chemical handbook. The heat transfer coefficient is used to fit the model to the measured data. Note that it is only the last two items in the parameter list that depend on the operating point.

The following assumptions have been made in the development of the model
- No blow-through steam
- The steam in the cylinder is saturated
- Energy flow to paper is constant
- The thermal dynamics of the condensate is fast compared to the cylinder shell
- The condition (10)

The pole \(\lambda\) and the zero \(z\) are both proportional to the heat transfer coefficient \(\alpha\). For large \(s\) the transfer function (14) is approximated by
\[
G(s) \approx \frac{b}{s} \quad \text{where} \quad b \text{ does not depend on } \alpha.
\]  \tag{16}

For small \(s\) the transfer function can be approximated by
\[
G(s) \approx \frac{bz}{\lambda s} \quad \text{where} \quad \frac{bz}{\lambda} = \frac{h_s}{mC_p \frac{\partial T_s}{\partial p} + Vh_s \frac{\partial \rho_s}{\partial p}}
\]  \tag{17}
does not depend on \( \alpha \). Therefore neither the initial part of a step response nor the slope of the asymptote depend on the heat transfer coefficient.

As can be noticed, the grey-box model does not explain the time delay often seen in the black-box model. It is important to remember that we are dealing with sampled measurements and it has, in practice, been observed that the time delay of the system often is close to the sampling time. Moreover, we have seen from the derivation of the grey-box model that there are neglected dynamics in the IPZ-structure, which may possibly give rise to an estimation of the dead-time that is larger than the true value.

III. FREQUENCY RESPONSE

To investigate the dynamic behavior of the linearized model, given by (12), we will look at its frequency response. The machine dependent parameters are taken from a steam group of a fluting machine, running at an operating point with a steam pressure of 90 kPa (gauge pressure), a nominal speed of 450 – 600 m/min and a basis weight between 110 – 200 g/m². Simulation values are

- Cylinder volume: \( V = 12.6 \) m³
- Cylinder mass: \( m = 7610 \) kg
- Cylinder area: \( A_{cy} = 37.2 \) m²
- Heat capacity for cast iron: \( C_p = 500 \) J/(kg·°C)
- Steam properties for the given operating point

The nominal steam mass flow rate to each cylinder is approximately 0.25 kg/s. This value is obtained from a steam flow gauge, measuring the total machine steam consumption. Figure 2 shows the Bode plot of the system, where the gain is normalized by 400 kPa.

The gain is independent of \( \alpha \), both at high and low frequencies, as shown in (16) and (17). The heat transfer coefficient has a considerable influence on both the gain and phase in the mid-frequency range. For the purpose of designing a PI-controller, this difference in gain and phase influences the controller parameters. It can also be seen that, a higher heat transfer coefficient yields a lower steam pressure gain, since there is a larger heat transfer through the cylinder and a higher condensation rate. This has also been pointed out in [2].

In (10) an inequality, that depends on the operating point and cylinder dimensions, was utilized to make a significant simplification. Using values from this example, we can examine its justification. Apart from the steam properties, we also need an expression for the derivative of siphon flow rate, \( q_s \), with respect to the cylinder pressure. Some experimental values are given in [7] and using those we find that the right hand side of (10) is 10 to 20 times larger than the left hand side. The next section shows that the model has a good fit to experimental data.

IV. COMPARISONS WITH PLANT DATA

To evaluate the accuracy of the grey-box model it has been calibrated and validated against measurements from a steam- and condensate system. The experiments have been carried out on a paper board machine and signals have been measured with a sampling time of 1 s. The cylinder data is

- Cylinder volume: \( V = 18.4 \) m³
- Cylinder mass: \( m = 8300 \) kg
- Cylinder area: \( A_{cy} = 45.5 \) m²

Since the input signal (steam input flow) is not manipulated directly, neither measured, a model for a steam valve has to be added to (12). A simple approach is to assume a linear relationship between the controller signal, \( u \), and the steam input flow, \( q_s \), namely

\[
q_s = du
\]

where \( d \) is a valve constant which will be a second calibration parameter together with the heat transfer coefficient, \( \alpha \). Using this valve description we keep the linearity and IPZ-structure in the model, given in (12).

In order to calibrate the model, the functions idgrey.m and pem.m in System Identification Toolbox for Matlab, were used to find the optimal calibration parameters. The optimization method is based on minimizing the prediction error, see [8].

Figure 3 shows an open loop step response together with the calibrated model, where the model output is bias corrected. The calibration parameters obtained are

\[
\alpha = 1820 \text{ W/(m}^2\cdot\text{K)}; \quad d = 0.00308 \text{ kg/(s}\%\text{)}
\]
In literature, we need a heat transfer coefficient through only the condensate film, \( \alpha_c \). From [9] the relationship

\[
\alpha = \frac{1}{\frac{1}{T_c} + \frac{T_y}{\lambda_{cy}}}
\]  \hspace{1cm} (21)

is given, where \( T_y \) is the distance into the cylinder where the temperature is equal to the mean cylinder temperature, and \( \lambda_{cy} \) is the thermal conductivity of the cylinder shell. In this example, the cylinder thickness is 25 mm (the mean temperature, \( T \), occurs in the middle of the cylinder shell), and the thermal conductivity is 50 W/(m·K). The heat transfer coefficient through the condensate is then

\[
\alpha_c = 3340 \text{ W/(m}^2\text{·K)}
\]

In [9], typical values of the heat transfer coefficient, \( \alpha_c \), are given. They depend strongly on the condensate thickness and activity, and can vary between 500 and 4000 W/(m²·K), where 2000 is a nominal value. Nevertheless, the fact that our estimated value from the model is within that range, gives support for the legitimacy of the model.

The total mass flow rate of steam, during the experiment, to the drying section is 85 ton/h. The machine has 93 cylinders, so the average steam flow per cylinder is 0.254 kg/s. The parameter \( d \), in (20), and the average valve opening gives the steam flow to the particular cylinder in the model, namely 0.154 kg/s. The steam flow is likely to vary a great deal between different drying groups but by comparing the two values we know that also the second calibration parameter is realistic.

In (22) the grey-box model is compared with the corresponding black-box model, adjusted on the same data set. The velocity gain in the models is normalized with the measuring range of the pressure gauge. The black-box model is not shown in the figures but it gives a slightly better fit to the data, since it has more degrees of freedom (three parameters to adjust instead of two). There is also a difference in the parameters of the transfer functions. The time delay comes from the identification procedure and is equal to the sampling time.

\[
G_{\text{grey}}(s) = \frac{0.00243(S)}{20.4s + 1} e^{-s}
\]

\[
G_{\text{black}}(s) = \frac{0.00176(S)}{21.3s + 1} e^{-s}
\]  \hspace{1cm} (22)

The grey-box model has also been validated graphically by using the measured control signal values to simulate an output. The model output is then compared with the measured steam pressure. Figure 4 shows such an evaluation. The excitation in the control signal is generated by a series of steps in the set point (closed loop), which is not shown in the figure to keep it clear.

V. A MODIFIED MODEL

It has previously been observed that in some occasional cases the IPZ-structure is not sufficient to describe the pressure dynamics in a steam cylinder. This can be resolved by changing the integrator to a real pole [5]. In the grey-box model this can be accomplished by changing the assumption that the energy flow \( Q_p \) is constant to

\[
Q_p = \alpha_p A_c \eta (T - T_p)
\]  \hspace{1cm} (23)

where \( \alpha_p \) is the heat transfer coefficient from the center of the cylinder to the paper-cylinder interface, \( \eta \) is the fraction of dryer surface covered by the paper web, and \( T_p \) is the paper temperature. By letting \( T_p \) be constant, (13) becomes
and the other matrices are unchanged. By examining data sets where the IPZ-structure is sufficient with cases where is not, an explanation to the modeling problem is found. If $\alpha >> \alpha_p$ then model (13) is adequate and $\alpha \approx \alpha_p$ means that (24) is a better structure. This has also been verified by simulation of the primary DAE system (1)–(3) in Modelica. An brief overview of Modelica is found in [10]. Closer examination of (24) shows that $\alpha >> \alpha_p$ gives a system with one fast pole and one close to the origin. When $\alpha_p$ is increased, the slow pole moves along the real negative axis towards the other pole, and it can then no longer be regarded as an integrator.

It can be shown, knowing that all factors in the elements of (24) are positive, that the eigenvalues of $A'$ are real and negative. The characteristic equation of (24) is

$$s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21} = 0$$  \hspace{1cm} (25)

Identification of the parameters gives

$$a_{11} + a_{22} = -\frac{\alpha A_{cy}}{h_j V} \frac{\partial p_s}{\partial p} - \frac{\alpha A_{cy} + \alpha_p A_p}{mC_p} < 0$$

and Routh’s criterion shows stability. The roots of (25) are

$$s_{1,2} = \frac{a_{11} + a_{22}}{2} \pm \sqrt{\left(\frac{a_{11} - a_{22}}{2}\right)^2 + a_{12}a_{21}}$$  \hspace{1cm} (26)

and since both $a_{12}$ and $a_{21}$ are positive, the solution has no complex parts. The relation between the position of the poles and the physical parameters is a bit more complicated than in (15). It can also be shown that the initial dynamics of the modified model in (24) are equal to (16). The low frequency properties will be different for the two models though since (12) contains an integrator and has no steady state gain. For small $s$ the modified model becomes

$$G(s) \approx \frac{h_j (\alpha A_{cy} + \alpha_p A_p)}{\alpha A_{cy} \alpha_p A_p \frac{\partial T_s}{\partial p}} \frac{\partial T_s}{\partial p}$$  \hspace{1cm} (28)

VI. CONCLUSIONS

This paper has presented a grey-box model for the pressure dynamics in a rotating drying cylinder. The model was derived from first principles. It has been evaluated with real plant data and compared with a corresponding black-box model. The main purpose of the grey-box model is to gain insight into the physical laws behind the black-box model. This can have effect on the mechanical design of the dryer section, such as the siphon shape and form, dryer bars, cylinder dimensions etc. It is also useful for the controller design.

There is also a potential to make a recursive identification of the heat transfer coefficient for fault detection with respect to condensate evacuation. It would then be beneficial to have a separate pressure meter and mass flow meter installed at the drying cylinder of interest, to acquire an accurate estimate. An important thing to remember here is that the model is an approximation of the real process. Model errors will therefore be included in the calibration parameter and its absolute value might be uncertain. The relative value is a useful parameter though.

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