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Eklund, Karl

1968

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

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K. EKLUND

REPORT 6809 NOVEMBER 1968
LUND INSTITUTE OF TECHNOLOGY
DIVISION OF AUTOMATIC CONTROL

LINEAR MATHEMATICAL MODELS OF THE DRUM-DOWNCOMER-RISER LOOP OF A DRUM BOILER †

K. Eklund

ABSTRACT

This report presents linearized mathematical models of the drum-downcomer-riser loop of a drum boiler. The set of equations corresponding to the basic assumptions gives a 6:th order model. Further simplifications give a 5:th and a 4:th order model. The dynamic properties of the models are compared. The models have the form of FORTRAN programs. The program input data is boiler construction parameters e.g. the drum size and thermal data determined by the drum pressure. The program output is the linearized model on standard form. The report also presents an investigation of the influence of some model parameters e.g. the drum size and the feedwater temperature, on the model dynamics.

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†This work has been supported by the Swedish Board for Technical Development under Contract 68-336-f.

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LIST OF SYMBOLS

The capital letters following the symbol is the notation used in the programs.

L_d, ALD	total length of downcomer tubes [m]
L_{d1}, ALD1	the length of one downcomer tube (gravitational head) [m]
L_r, ALR	total length of riser tubes [m]
L_{r1}, ALR1	the length of one riser tube (gravitational head) [m]
A_d, AD	total flow area of downcomer tubes [m^2]
A_r, AR	total flow area of riser tubes [m^2]
D_d, DD	total diameter of downcomer tubes [m]
D_r, DR	total diameter of riser tubes [m]
M_r, RMASS	mass of riser tubes [kg]
M_w, WMASS	mass of drum liquid [kg]
V_s, VS	volume of vapor phase in drum [m^3]
A, ADR	area of drum liquid surface [m^2]
Q_g	heat flow from gases to riser walls [kJ/s]
Q_r, QR	heat flow from riser walls to steam-water mixture in the riser tubes [kJ/s]
ρ_w, DEW	density of drum liquid [kg/m^3]
ρ_s, DESS	density of saturated vapor [kg/m^3]
ρ_m, DEM	mean density of steam-water mixture in the riser tubes [kg/m^3]
ρ_o, DE	density of the steam-water mixture at the outlet of the risers [kg/m^3]
T_w, TW	the drum liquid temperature [^oC]
T_r, TR	the temperature of the riser tubes [^oC]
T_s, TS	the saturation temperature [^oC]
h_w, HW	the drum liquid enthalpy [kJ/kg]
h_{ws}, HWS	enthalpy of saturated liquid [kJ/kg]
h_{ss}, HSS	enthalpy of saturated vapor [kJ/kg]
h_e, HE	enthalpy of evaporation [kJ/kg]
h_{fw}, HFW	enthalpy of feedwater [kJ/kg]
h_m, HMM	mean enthalpy of steam-water mixture in the risers [kJ/kg]
h_o, HM	enthalpy of steam-water mixture at the outlet of the risers [kJ/kg]

w_w, WW	downcomer mass flow [kg/s]
w_o, W	mass flow at the outlet of the risers [kg/s]
w_s, WS	outlet steam flow [kg/s]
w_e, WE	evaporation mass flow [kg/s]
w_{fw}, WFW	feedwater flow [kg/s]
p_d	drum pressure [bar]
p_{md}	mud drum pressure [bar]
c_w	downcomer flow velocity [m/s]
c_o	riser flow velocity at the outlet of the risers [m/s]
y	drum liquid level [m]
z	length coordinate
x_o	steam quality of the outlet of the risers
x_m, XM, XROOT(1)	mean value of steam quality of the steam-water mixture in the risers

Constants

g, GA	acceleration due to gravity [m/s^2]
k_t, PK1	temperature-pressure proportional constant at saturation state
k_d, PK2	density-pressure proportional constant at saturation state
k_h, PK3	water enthalpy-pressure proportional constant at saturation state
k_e, PK4	evaporation constant
k_r, PK5	heat transfer coefficient from riser tube walls to steam-water mixture [kJ/s $^oC^3$]
c_{pw}, PK6	heat capacitance for drum liquid [kJ/kg ^oC]
c_r, PK8	heat capacitance for the riser tubes [kJ/kg ^oC]
k_s, PK9	vapor enthalpy-pressure proportional constant at saturation state
ζ_i	energy loss coefficient for entrance and exit losses
f_d, FD	energy loss coefficient for downcomer flow
f_r, FR	energy loss coefficient for riser flow
ξ	loss factor

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d	drum or downcomer
md	mud drum
o	riser exit
w	water
s	steam or saturated
r	riser
fw	feedwater

1. INTRODUCTION

This report presents linearized mathematical models of 4:th, 5:th and 6:th order of the drum-downcomer-riser loop of a drum boiler. The reasons for a model is to design control laws for steady-state control and to illustrate the application of the systematic modelling technique described in {4}. Of particular interest is to investigate the qualitative influence of model complexity and model parameters on the dynamic behaviour of the model.

Computation of steady-state solutions, linearization and reduction to standard form S(A,B,C,D) of the equations for a process of some complexity is a very extensive work. For this reason FORTRAN programs for the models were developed. Program input data are boiler construction parameters such as drum size, tube length etc. and thermal data determined by the drum pressure. Two temperatures, viz. the drum liquid temperature and the temperature of the risers have to be guessed. A complete list of input data is found in the numerical examples in Appendix C. The output of the program is the linearized model on standard form. This gives us a very useful tool to investigate the influence of model parameter changes and various physical approximations on the model dynamics. The essential difficulty with this approach is to assign the smallest number of state variables to the linearized set of equations in order to reduce this set to standard form. The report presents a solution to this problem which applies to the 4:th and 5:th order models. A generel method which solves the special problems concerning the 6:th order model was also developed. This method as well as a systematic modelbuilding technique is described in {4}.

The order of the model depends on which simplifying assumptions we make. The basic assumptions in this report gives a 6:th order model. It is now interesting to investigate the influence of further simplifications on the dynamic behaviour of the model. Neglecting time derivatives of the velocity of the fluid in the momentum equations for the downcomers and risers, the order of the model is reduced to five. If we consider the fluid in the downcomers and the risers as a rigid body with no heat capacity, the order of the model is reduced to four. The step responses of the state variables of these three models to a step change in the input variables viz.

the heat flow to the risers, the feedwater flow and the steam outlet flow are shown in Fig. 7, 8 and 9. The state variables correspond to the following physical quantities

x_1 drum pressure
x_2 drum level
x_3 drum liquid temperature
x_4 riser tube temperature
x_5 steam quality
x_6 a linear combination of the downcomer and the riser mass flow

Comparing the responses of the models there seems to be a large difference in the gain between the 4:th order model and the other models. However, this difference is mostly due to a distinction in the unstable mode of the models. The unstable mode is a consequence of the assumption that the steam outlet flow is a process input variable. The drum level response of the 5:th and 6:th order models to a step change in the steam outlet flow is a typical non-minimum phase response (Fig. 9). The 4:th order model does not show this behaviour. This was expected since the 4:th order model does not include the steam quality variable as a state variable. The steam quality variable takes at least qualitatively the effect of bubble formation in the risers and the drum into account. The bubble formation is the physical reason for the non-minimum phase behaviour of the real process. Between the 5:th and 6:th order models there is practically no difference at all. Then we can conclude that there is no reason to choose the 6:th order model. If the non-minimum phase characteristics for the drum level is essential we should prefer the 5:th order model.

In section 2 we give a process description and define the process input and output variables. In section 3 we state the most essential assumptions and discuss their physical motivation. The derivation of the equations is given in section 4. The equations are linearized and given in a compact form. In section 5 we derive the linearized equations for the reduced order models. A method to reduce a set of linear equations to standard form is presented in section 6. The method has been applied to the 4:th and 5:th order models. The assignment of state variables to the linearized system is discussed. The digital computer programs for the models are described in section 7. In section 8 we give some numerical examples. We investigate the influence of parameter changes such as the size of the drum, the feedwater temperature and the two guessed temperatures on the model dynamics. It is interesting to notice that it is possible to choose a feedwater temperature which gives zero gain between feedwater flow and drum pressure. In Appendix A we give the coefficients of the linearized equations in terms of the steady-state values of the used variables. Appendix B contains the program listings and Appendix C gives the numerical results of the examples in section 8.

2. DESCRIPTION OF THE PROCESS

The process configuration is shown in Fig. 1. Into the drum enter the feedwater flow and the steam-water mixture from the risers. The flows which leave the drum are the output steam flow and the water flow into the downcomers. A heat flow is supplied to the risers. The circulation is natural and thus due to the density difference between the fluid in the downcomers and the risers. The process input variables are

- feedwater flow
- feedwater enthalpy
- heat flow to risers
- steam outlet flow

and the process output variables are

- drum pressure
- drum liquid level

Notice that the fuel flow is not the heat input variable. If the fuel is oil there is approximately a static relation between the fuel flow and the heat flow to the risers. The heat may be transfered by radiation or convection or both. The proportions in which the heat is transfered are different in different boilers. The fuel-heat flow relation is then also different.

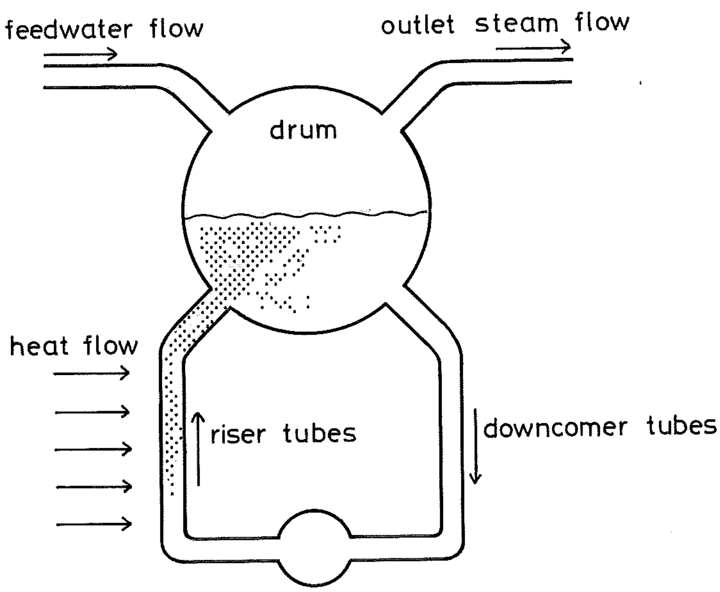


Fig. 1 - The considered drum-downcomer-riser loop of a drum boiler

We have taken the steam flow as an input to the process. This can be interpreted as a control loop which keeps the steam flow constant. The flow changes only if a change of the "set point" of this control loop is ordered.

The process includes some very intricate physical phenomena. The risers contain a boiling liquid and we have both the vapor phase and the liquid phase. The amount of each phase changes along the risers. The two phases have different densities and move with different velocities. There is a heat exchange between the vapor and the liquid. The temperature of the risers is not the same everywhere. The heat flow to the risers and the refrigeration supplied by the fluid are different in different parts of the riser banks.

The temperature of the feedwater is normally less than the saturation temperature. The temperature difference causes vapor to condense and effect the bubble formation in the drum. Thus both the pressure and the liquid level are influenced.

An accurate description of the dynamic behaviour of such processes becomes very complex. The process is a distributed system. Some of the phenomena are difficult to express in mathematical terms. If the ambition is to derive a model of low order some simplifying assumptions must be applied.

The process is divided into three components which are treated separatly

- the downcomers
- the risers
- the drum

The input variables to one component are process input variables and/or output variables from the other components. The output variables are process output variables and/or input variables to other components.

3. ASSUMPTIONS

Physical phenomena which should be described with partial differential equations are described with one ordinary differential equation. Thus all parameters are lumped. The thermal state equations used are linearized relations where the coefficients for certain steady-state conditions may be found in e.g. steam tables. The heat transfer rates are obtained from empirical relations found in literature {14}. The heat transfer coefficients are calculated from the actual steady-state values. Below the additional assumptions which apply to the different components are listed.

The downcomers

(a) No boiling occur in the downcomers

(b) The liquid temperature is the same as the drum liquid temperature

Comments

If the drum pressure is decreasing fast enough, boiling will occur in the downcomers. The circulation flow decreases and the refrigeration of the riser tubes may become insufficient. The result is a process failure. If we want to take this phenomena into account we have to describe the behaviour of the downcomers quite accurately {11}. No attempt to include constraints on the time derivative of the drum pressure in the model has been made. The assumptions made allow us to apply only a momentum equation.

The risers

(a) There is no velocity difference between the vapor and the liquid phases in the risers.

(b) The steam quality is linear distributed along the risers.

(c) There is always saturation state in the risers.

Comments

The first two assumptions are coupled to each other. In the real process there is a velocity difference between the vapor and liquid phase. That is the amount of heat transfered to a unit mass of the steam-water mixture becomes less in the model than in the real process. If we assume constant steam quality along the riser it will be difficult to reach the true value of the steam quality at the outlet of the risers with a reasonable circulation rate in the downcomer-riser loop. The second assumption will supply a solution to this problem and is physically motivated. If a mean value of the steam quality is used in the calculations, we will get two times this value at the outlet of the risers. If we want to take the velocity difference explicity into account we have to introduce the so called slip factor. This is the standard technique in the theory of two-phase flow and is found in e.g. {2}, {5}, {8}, {17}.

The drum

(a) There is no temperature gradient in the liquid and the vapor phase in the drum.

(b) The vapor phase is always of saturation state.

(c) The feedwater is so entered into the drum that the liquid phase becomes slightly undercooled.

(d) The condensation or evaporation rate in the drum is proportional to the difference between liquid temperature and the saturation temperature.

(e) The feedwater temperature does not vary.

Comments

Assumptions (a) and (b) are physically well-grounded since the turbulence in the drum is considerable. Assumptions (c) and (d) allow us to describe the mass transportation phenomena caused by the subcooled feedwater. This assumptions are due to Ergin and collaborators {3}. The liquid level variations due to the bubble formation is taken into account through the steam quality variable This will introduce a non minimum phase characteristic for the drum level dynamics.

The feedwater temperature variations due to small load disturbances are in most drum boilers neglectable. This process input is thus set equal to a constant.

Other assumptions made are stated and commented when the equations are derived or follow from the equations used.

4. DERIVATION OF EQUATIONS

The physical equations used are essentially the laws of conservation of energy, mass and momentum. In the momentum equations correction terms are included which take friction, entrance and exit losses into account. Moreover, as mentioned in Section 3 static relations describing heat transmission and thermal relations are used.

4.1 Downcomers

The component is shown in Fig. 2 and the variables used are indicated.

Since the fluid in the downcomers is water, it can be considered incompressible and since there is no heat exchange with the environment we only have to apply a momentum equation. Notice that the water in the downcomers instantaneously assume the temperature of the water in the drum. The downcomers terminate in a mud drum where a certain amount of the kinetic energy is dissipated due to turbulence. The downcomers are serveral parallell coupled tubes and may be treated as one tube. The Bernoulli equations for instationary flow without friction yields

$$\int_{p_d}^{p_{md}} \frac{dp}{\rho_\omega} + \int_{L_{d1}}^{o} g dz + \int_{o}^{c_\omega} c\, dc + \int_{o}^{L_{d1}} \frac{dc}{dt} dz = 0 \qquad (4.1.1)$$

The integration shall be performed along a stream-line A with the co-ordinate z. The integration is possible since the variables ρ_ω, c and dc/dt can be considered as constant or zero throughout the stream-line A. Integrating and adding correction terms of friction we get

$$(p_d - p_{md}) \cdot \frac{1}{g} = \{f_d \frac{L_d}{D_d} + \zeta_{d1} + \zeta_{d2} + 1\} \cdot \frac{\omega_\omega^{\ 2}}{2g\ A_d^{\ 2}\ \rho_\omega} - \rho_\omega\ L_{d1} + \frac{L_{d1}}{gA_d} \frac{d\omega_\omega}{dt} \qquad (4.1.2)$$

The velocity c_ω is replaced with the mass flow ω_ω. The proportional constant f_d also takes energy losses in tube bends into account.

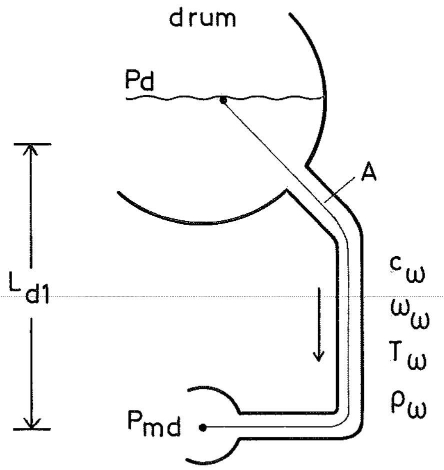


Fig. 2 - Simplified diagram of the downcomer

4.2 Risers

In Fig. 3 the component under consideration is shown and variables used are indicated. Again the parallell coupled tubes are treated as one tube. We have assumed that the steam quality is distributed along the riser. Hence the velocity, the density and the enthalpy of the steam-water mixture are dependent of the length co-ordinate z. In the equations, according to the assumptions, we must use some mean values of these variables.

Fig. 4 shows the simplified riser and the steam distribution. Let the steam distribution be

$$x = f(z) \qquad (4.2.1)$$

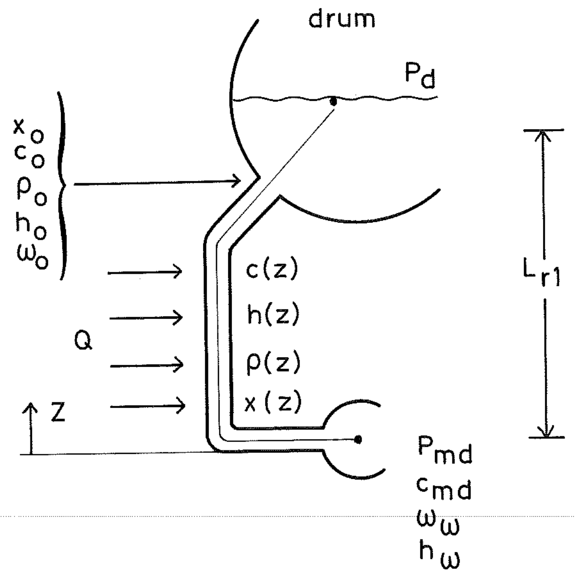


Fig. 3 - Simplified diagram of the risers

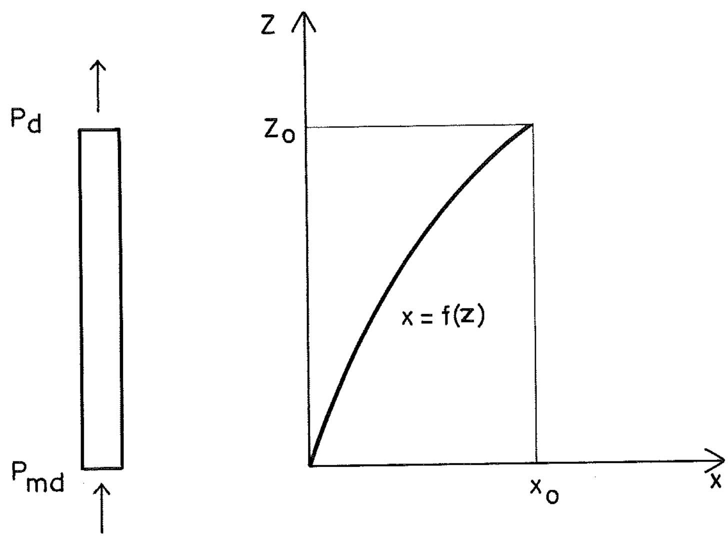


Fig. 4 - The steam quality distribution along the riser

Then the mean value x_m of the steam quality is defined as

$$x_m = \frac{1}{z_o} \int_o^{z_o} f(z)\, dz \qquad (4.2.2)$$

In every section of the riser we have

$$\frac{1}{\rho} = \frac{x}{\rho_s} + \frac{1-x}{\rho_\omega} \qquad (4.2.3)$$

If we define the mean value of the density of the steam-water mixture as the mean value of the steam quality above and use equations (4.2.1) and (4.2.3) we get

$$\rho_m = \frac{1}{z_o} \int_o^{z_o} \frac{\rho_s \cdot \rho_\omega}{\rho_s + f(z)(\rho_\omega - \rho_s)}\, dz \qquad (4.2.4)$$

Since we assumed that there always is saturation state in the risers, the densities ρ_ω and ρ_s are independent of the co-ordinate z.

Further we have in every section of the riser

$$h = h_{\omega s} + (h_{ss} - h_{\omega s})x \qquad (4.2.5)$$

and thus the mean value is

$$h_m = h_{\omega s} + \frac{1}{z_o} \int_o^{z_o} (h_{ss} - h_{\omega s})\, f(z)\, dz \qquad (4.2.6)$$

The enthalpies $h_{\omega s}$ and h_{ss} are independent of the co-ordinate z according to the assumptions.

Assume a linear distribution. Hence

$$f(z) = K \cdot z$$

and

$$x_m = \frac{K\, z_o}{2} \quad ; \quad x_o = 2x_m \qquad (4.2.7)$$

$$\rho_m = \frac{\rho_\omega\, \rho_s}{2x_m(\rho_\omega - \rho_s)} \ln\left(1 + 2\,\frac{\rho_\omega - \rho_s}{\rho_s}\, x_m\right) \qquad (4.2.8)$$

$$h_m = h_{\omega s} + (h_{ss} - h_{\omega s})\, x_m \qquad (4.2.9)$$

The momentum equation yields

$$\int_{P_{md}}^{P_d} \frac{dp}{\rho(z)} + \int_{o}^{L_{rl}} g dz + \int_{\sqrt{\xi}\, c_\omega}^{c_o} c dc + \int_{o}^{L_{rl}} \frac{dc(z)}{dt} dz = 0 \qquad (4.2.10)$$

The integration shall be performed along the stream-line B in Fig. 3. As indicated in the equation (4.2.10) the density ρ and the acceleration dc/dt are dependent of the variable z. This means that equation (4.2.10) cannot be directly integrated. However, using the mean value theorem for integrals we get

$$\int_{P_{md}}^{P_d} \frac{dp}{\rho(z)} = \frac{1}{\rho(z_1)} \int_{P_{md}}^{P_d} dp \quad ; \quad 0 \leqslant z_1 \leqslant L_{rl} \qquad (4.2.11)$$

$$\int_{o}^{L_{rl}} \frac{dc(z)}{dt} dz = \frac{dc(z_2)}{dt} \int_{o}^{L_{rl}} dz \quad ; \quad 0 \leqslant z_2 \leqslant L_{rl} \qquad (4.2.12)$$

The points z_1 and z_2 cannot be determined considering the assumptions made and will vary with changing operating conditions of the boiler. Then we have to guess the values of these variables. A simplifying assumption is

$$\frac{1}{\rho(z)} = \frac{1}{\rho_m} \qquad (4.2.13)$$

and

$$\frac{dc(z_2)}{dt} = \frac{dc_o}{dt} \qquad (4.2.14)$$

The preserved fraction of the kinetic energy in the mud drum is set equal to ξ. ξ is less than one and greater than zero. Integrating and adding correction terms for energy losses we get

$$(P_{md} - P_d) \cdot \frac{1}{g} = (f_r \frac{L_r}{D_r} + 1) \frac{\omega_o^2 \cdot \rho_m}{2A_r^2\, g\, \rho_o^2} + \zeta_3 \frac{\omega_\omega^2}{2A_r^2\, g\, \rho_\omega}$$

$$+ \zeta_4 \frac{\omega_o^2}{2A_r^2\, g\, \rho_o} - \xi \frac{\omega_\omega^2 \cdot \rho_m}{2A_d^2 \rho_\omega^2} + \rho_m \cdot L_{rl} + \frac{\rho_m L_{rl}}{A_r \cdot g} \frac{d}{dt} (\frac{\omega_o}{\rho_o})$$

(4.2.15)

The velocities c_{md}, c_ω and c_o are replaced with the corresponding mass flows. The entrance and exit energy losses are set dependent of the entrance and exit velocities, respectively. The continuity equation becomes

$$\omega_\omega - \omega_o = A_r \cdot L_{rl} \frac{d}{dt} \rho_m \tag{4.2.16}$$

and the energy equation

$$Q_r + \omega_\omega \cdot h_\omega - x_o \omega_o h_{ss} - (1-x_o)\omega_o h_{\omega s} = A_r L_{rl} \frac{d}{dt} (\rho h)_m \tag{4.2.17}$$

Terms due to the rate of change of the kinetic energy are neglected since they are small compared to the heat energy. It is very difficult to compute the mean value d/dt $(\rho \cdot h)$ along the riser, because ρ depends on pressure, flow etc. We use the approximation

$$\frac{d}{dt} (\rho h)_m = \frac{d}{dt} \rho_m h_m$$

where ρ_m and h_m is the mean value of the density and the enthalpy respectively. The linearized equation then contains the time derivative of ρ_m which is eliminated using the mass balance across the riser.

As mentioned in Section 2 the heat flow to the risers Q_g, is taken as a process input variable. The energy equation applied to the riser gives

$$Q_g - Q_r = M_r \cdot C_r \cdot \frac{d}{dt} T_r \tag{4.2.18}$$

where T_r is the mean temperature of the riser tubes. In literature often the following empirical formula is used to describe the heat transportation from the riser tube walls to the steam water mixture {1}, {5}.

$$Q_r = k_r (T_r - T_s)^3 \tag{4.2.19}$$

The proportional constant k_r is calculated from steady state values.

4.3 The drum

Fig. 5 gives the simplified diagram of the drum. The two phases, the vapor phase and the liquid phase, in the drum are treated separately. The energy and continuity equations are applied to the liquid phase and the continuity equation to the vapor phase. The energy equation of the vapor phase is then automatically satisfied. Thus we get the following equations using variable notations indicated in Fig. 5.

$$(1-2x_m)h_{\omega s}\cdot\omega_o+h_{f\omega}\omega_{f\omega}-h_\omega\cdot\omega_\omega-h_{ss}\omega_e = \frac{d}{dt}(M_\omega h_\omega) \qquad (4.3.1)$$

$$(1-2x_m)\omega_o+\omega_{f\omega}-\omega_\omega-\omega_e = \frac{d}{dt}\cdot M_\omega \qquad (4.3.2)$$

and

$$2x_m\omega_o + \omega_e - \omega_s = \frac{d}{dt}(V_s\rho_s) \qquad (4.3.3)$$

According to the assumptions the evaporation or condensation rate in the drum is

$$\omega_e = k_e(T_\omega - T_s) \qquad (4.3.4)$$

In equations (4.3.1) and (4.3.3) two new variables, the mass of the liquid phase in the drum M_ω and the volume of the vapor phase in the drum V_s, are introduced. Under small disturbances, these variables are approximately proportional to the process output variable, the drum level. The proportional constants are easily derived. This is used when the linearized set of equations are derived. However in general we have

$$V_s = V_s(y) \qquad (4.3.5)$$

$$M_\omega = M_\omega(y) \qquad (4.3.6)$$

The functions are nonlinear due to the bending contour of the drum.

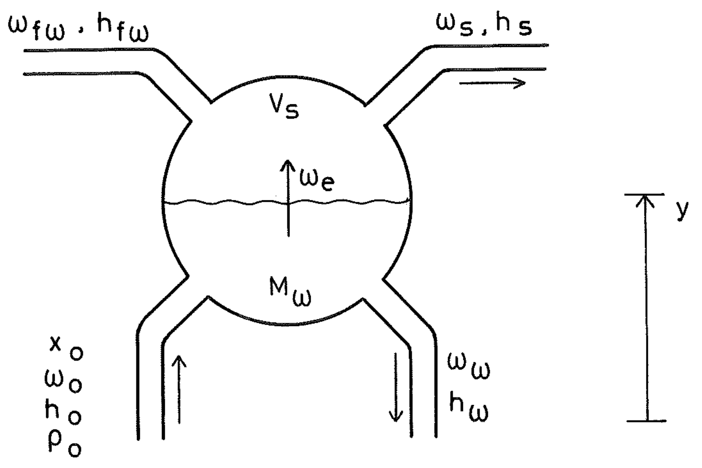


Fig. 5 - The simplified drum diagram

4.4 Thermal state equations

The thermal state of the steam-water mixture in the riser and of the vapor phase in the drum is the saturation state. Then there is only one independent thermal variable. Choose the drum pressure as the independent variable. Hence

$$T_s = T_s(P_d) \tag{4.4.1}$$

$$\rho_s = \rho_s(P_d) \tag{4.4.2}$$

$$h_{ss} = h_{ss}(P_d) \tag{4.4.3}$$

$$h_{\omega s} = h_{\omega s}(P_d) \tag{4.4.4}$$

The functions are nonlinear. The liquid phase in the drum is not of saturation state but is slightly undercooled. The dependence of the pressure of the enthalpy of the liquid phase is neglectable and thus

$$h_\omega = h_\omega(T_\omega) \tag{4.4.5}$$

where $h_\omega(T_\omega)$ is a nonlinear function.

4.5 Steady state values

The set of equations which describe the boiler dynamic behaviour is given by the equations (4.1.2), (4.2.3), (4.2.7), (4.2.8), (4.2.9), (4.2.15), (4.2.16), (4.2.17), (4.2.18), (4.2.19), (4.3.1), (4.3.2), (4.3.3), (4.3.4), (4.3.5), (4.3.6), (4.4.1), (4.4.2), (4.4.3), (4.4.4) and (4.4.5). If we include the process input variables in the n-vector u and the process output variables in the k-vector y and all the internal variables in the ℓ-vector v, a compact notation is

$$f(\dot{v}, v, u) = 0 \tag{4.5.1a}$$

$$g(y, v, u) = 0 \tag{4.5.1b}$$

where f is a ℓ-vector and g a k-vector whose components are nonlinear functions of the indicated variables. The set of equations is consistent if the dimensions of the vectors given above are the true dimensions. If all time derivatives are set equal to zero we get

$$f(0, v, u) = 0 \tag{4.5.2a}$$

$$g(y, v, u) = 0 \tag{4.5.2b}$$

If a solution to this set of nonlinear equations exists, this solution yields the steady state values of the variables v and y. Computer programs which solve set of nonlinear equations, are available. However, in this study no such general approch to this problem have been used since the structure of the equations is quite simple.

Except for construction parameters such as the size of the drum, the length of tubes and the empirical correction constants, the steady state values of the following variables are known

- the heat input rate $\overline{Q}_g$
- the feedwater enthalpy $\overline{h_{f\omega}}$
- the drum pressure $\overline{P_d}$
- the temperature of the riser tubes $\overline{T_r}$
- the temperature of the liquid phase in the drum $\overline{T_\omega}$

The last two variables are not process input variables or otherwise given from environment conditions. Thus they have to be guessed or measured on the actual physical process. However, usually it is possible to get a fairly good estimate of these variables.

The drum pressure determines all other variables associated with the fluid in saturation state. Hence we know the steady state values of

- the temperature $\overline{T}_s$
- the density of the vapor phase $\overline{\rho_s}$
- the density of the liquid phase $\overline{\rho_\omega}$
- the enthalpy of the vapor phase $\overline{h_{ss}}$
- the enthalpy of the liquid phase $\overline{h_{\omega s}}$

Equation (4.2.8) can be approximated with a straight line

$$\rho_m = c_1 x_m + c_2 \tag{4.5.3}$$

The line is adjusted to fit as good as possible in the interval in which the mean value of the steam quality is supposed to vary. This approximation is used when the steady-state value of the mean value of the steam quality is computed. Manually the equations (4.1.2), (4.2.3), (4.2.15), (4.2.16), (4.2.17) and (4.5.3) with time derivatives set equal to zero are reduced to a third order polynomial in the mean value of the steam quality. Using the real positive root of the polynomial the original set of equations is solved for the other variables. Hence we know the steady state values of the variables

- the mean value of the steam quality $\overline{x_m}$
- the mean value of the density of the steam-water mixture $\overline{\rho_m}$
- the outlet density of the steam-water mixture $\overline{\rho_o}$
- the mean value of the enthalpy of the steam-water mixture $\overline{h_m}$
- the circulation mass flow $\overline{\omega_o}$

Equation (4.2.19) gives the heat transfer coefficient

$$k_r = \frac{\overline{Q}_r}{(\overline{T}_r - \overline{T}_s)^3} \tag{4.5.4}$$

The equations (4.3.1), (4.3.2), (4.3.3) and (4.3.4) with time derivatives set equal to zero give the evaporation constant

$$k_e = \frac{\overline{\omega}_o}{h_{ss} - h_{f\omega}} \left[2\overline{x}_m \frac{\overline{h}_{\omega s} - \overline{h}_{f\omega}}{\cdot \overline{T}_s - \overline{T}_\omega} - \frac{\overline{h}_{\omega s} - \overline{h}_\omega}{\overline{T}_s - \overline{T}_\omega} \right] \tag{4.5.5}$$

Now equations (4.3.4), (4.3.3) and (4.3.2) determines the steady state values of

- the evaporation mass flow $\overline{\omega}_e$
- the outlet steam flow $\overline{\omega}_s$
- the feedwater flow $\overline{\omega}_{f\omega}$

The details of the computation of the steady state values are found in the program and the program output.

4.6 The linearized set of equations

The perturbed variables that is the difference between the actual value and the steady state value of the variables, are denoted as the variables themselves. The linearized set of thermal state equations become

$$T_s = k_T P_d \tag{4.6.1}$$

$$\rho_s = k_d P_d \tag{4.6.2}$$

$$h_{ss} = k_s P_d \tag{4.6.3}$$

$$h_{\omega s} = k_h P_d \tag{4.6.4}$$

where the proportional constants are obtained from e.g. a steam table. The linearized variants of equation (4.3.5) and (4.3.6) are

$$V_s = - A y \tag{4.6.5}$$

$$M_\omega = A \rho_\omega y \tag{4.6.6}$$

Hence the bending of the drum contour is neglected. Using the above equations the left hand variables are eliminated from the other equations. It is further relativly simple to eliminate the outlet density ρ_o and one of the variables ρ_m and x_m using the linearized variants of the equations (4.2.3) and (4.2.8). From a computational point of view the variables ρ_m and x_m are equivalent. However, the most informative variable is the mean value of the steam quality and we choose to keep this variable. The pressure difference in the two momentum equations (4.1.2) and (4.2.15) is also eliminated. The linearized system is then

Eq. used

$$a_1\dot{P}_d + a_2\dot{x}_m + a_3\omega_o + a_4\omega_\omega = 0 \qquad (4.2.16\ell)\ (4.2.8\ell)$$

$$a_5\dot{\omega}_o + a_6\dot{\omega}_\omega + a_7P_d + a_8x_m + a_9\omega_o + a_{10}\omega_\omega = 0 \qquad (4.1.2\ell)\ (4.2.15\ell)$$

$$a_{11}\dot{P}_d + a_{12}\dot{x}_m + a_{13}P_d + a_{14}T_\omega + a_{15}x_m + a_{16}\omega_o + a_{17}\omega_\omega + a_{18}Q_r = 0 \qquad (4.2.16\ell)\ (4.2.17\ell)$$

$$a_{19}P_d + a_{20}T_r + a_{21}Q_r = 0 \qquad (4.2.19\ell)$$

$$a_{22}\dot{T}_r + a_{23}Q_g + a_{24}Q_r = 0 \qquad (4.2.18\ell)$$

$$a_{25}\dot{y} + a_{26}\dot{T}_\omega + a_{27}P_d + a_{28}T_\omega + a_{29}x_m + a_{30}\omega_{f\omega} + a_{31}\omega_o + a_{32}\omega_\omega + a_{33}\omega_e = 0 \qquad (4.3.1\ell)$$

$$a_{34}\dot{y} + a_{35}x_m + a_{36}\omega_{f\omega} + a_{37}\omega_o + a_{38}\omega_\omega + a_{39}\omega_e = 0 \qquad (4.3.2\ell)$$

$$a_{40}\dot{P}_d + a_{41}\dot{y} + a_{42}x_m + a_{43}\omega_s + a_{44}\omega_o + a_{45}\omega_e = 0 \qquad (4.3.3\ell)$$

$$a_{46}P_d + a_{47}T_\omega + a_{48}\omega_e = 0 \qquad (4.3.4\ell)$$

The ℓ after the number of the equation indicates that the equation has been linearized before used. The coefficients are found in Appendix 1 expressed as functions of the steady state values of the variables and of the boiler construction parameters. Before the momentum equation for the riser is linearized the acceleration term is simplified by setting $d/dt(\omega_o/\rho) = {}^1/\rho \frac{d}{dt}\omega_o$

Introduce the notation

$$x_1 = P_d$$
$$x_2 = y$$
$$x_3 = T_\omega$$
$$x_4 = T_r$$
$$x_5 = x_m$$
$$x_6 = \omega_o$$
$$x_7 = \omega_\omega$$

The process input variables are

$$u_1 = Q_g$$
$$u_2 = \omega_{f\omega}$$
$$u_3 = \omega_s$$

and the internal variables are

$$v_3 = \omega_e$$
$$v_4 = Q_r$$

The rewritten linearized system then becomes

$$\left[\begin{array}{ccccccc|ccccccc|ccc|cc}
a_1 & & & & a_2 & & & & & & & & a_3 & a_4 & & & & & \\
 & & & & & a_5 & a_6 & a_7 & & & & a_8 & a_9 & a_{10} & & & & & \\
a_{11} & & & & a_{12} & & & a_{13} & & a_{14} & & a_{15} & a_{16} & a_{17} & & & & & a_{18} \\
 & & & & & & & a_{19} & & & a_{20} & & & & & & & & a_{21} \\
 & & & a_{22} & & & & & & & & & & & a_{23} & & & & a_{24} \\
 & a_{25} & a_{26} & & & & & a_{27} & & a_{28} & & a_{29} & a_{31} & a_{32} & & a_{30} & & a_{33} & \\
 & a_{34} & & & & & & & & & & a_{35} & a_{37} & a_{38} & & a_{36} & & a_{39} & \\
a_4 & a_{41} & & & & & & & & & & a_{42} & a_{44} & & & & a_{43} & a_{45} & \\
 & & & & & & & a_{46} & & a_{47} & & & & & & & & a_{48} &
\end{array}\right]
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_7 \\ x_1 \\ x_2 \\ \vdots \\ x_7 \\ u_1 \\ u_2 \\ u_3 \\ v_3 \\ v_4 \end{bmatrix} = 0 \qquad (4.6.7)$$

The reduction of this system to standard form requires that the order of the dynamic system is determined and that the internal variables are eliminated. The rank of the submatrix corresponding to the time derivatives of the variables x determines the maximum order of the dynamic system. This is motivated in Section 6. Here the sixth and seventh column vectors are linear dependent and thus the order of this dynamic system is at most six.

5. REDUCTION OF THE ORDER OF THE SYSTEM BY APPROXIMATIONS

The originally derived dynamic system is maximally of the 6:th order. It is desirable that the model which describes the dynamics of the system is of as low order as possible. The fastest dynamic lapses are those of the downcomer and the riser. The dynamics of the fluid of these components give two differential equations. We will reduce the maximum order of the dynamic system to five and four respectively by

(i) neglecting the acceleration terms in the Bernoulli equations of the downcomer and the riser

(ii) neglecting all dynamics of the downcomer-riser loop

The step responses of the state variables of the reduced and original order models to a step change in the input variables are found in Fig. 7, 8 and 9.

5.1 The reduced system equations (i)

The assumption (i) only causes the time derivatives of the variables ω_o and ω to vanish. The variables ω_o and ω are now internal variables. Put

$$v_1 = \omega_o$$

$$v_2 = \omega_\omega$$

The linearized system equations are then

$$\left[\begin{array}{ccccc|cccc|cccc|cccc}
a_1 & & & & a_2 & & & & & & & & & a_3 & a_4 & & \\
 & & & & & a_7 & & & & a_8 & & & & a_9 & a_{10} & & \\
a_{11} & & & & a_{12} & a_{13} & & a_{14} & & a_{15} & & & & a_{16} & a_{17} & & a_{18} \\
 & & & & & a_{19} & & & a_{20} & & & & & & & & a_{21} \\
 & & & a_{22} & & & & & & & a_{23} & & & & & & a_{24} \\
 & a_{25} & a_{26} & & & a_{27} & & a_{28} & & a_{29} & & a_{30} & & a_{31} & a_{32} & a_{33} & \\
 & a_{34} & & & & & & & & a_{35} & & a_{36} & & a_{37} & a_{38} & a_{39} & \\
a_{40} & a_{41} & & & & & & & & a_{42} & & & a_{43} & a_{44} & & a_{45} & \\
 & & & & & a_{46} & & a_{47} & & & & & & & & a_{48} &
\end{array}\right]
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_5 \\ x_1 \\ x_2 \\ \vdots \\ x_5 \\ u_1 \\ u_2 \\ u_3 \\ v_1 \\ \vdots \\ v_4 \end{bmatrix} = 0 \tag{5.1.1}$$

The rank of the submatrix formed by the first five column vectors is five and thus the order of the dynamic system is maximally five.

5.2 The reduced system equations (ii)

The assumption (ii) means that we neglect the acceleration terms of the momentum equations of the risers and the downcomers, that the fluid of the risers is incompressible and that the heat capacity of the fluid of the risers is zero. The equations (4.2.16) (4.2.17) are then replaced by

$$\omega_\omega = \omega_o \tag{5.2.1}$$

$$Q_r + h_\omega \omega_\omega - x_o \cdot \omega_o \cdot h_{ss} - (1 - x_o) h_{\omega s} \cdot \omega_o = 0 \tag{5.2.2}$$

The set of equations which now describes the process, does not include the time derivative of the mean value of the steam quality x_m. If we put

$$v_1 = \omega_o = \omega_\omega$$

$$v_2 = x_m$$

we get

$$\left[\begin{array}{cccc|ccccccc|cccc}
 & & & & \alpha_1 & & & & & & & \alpha_2 & \alpha_3 & & \\
 & & & & \alpha_4 & & \alpha_5 & & & & & \alpha_6 & \alpha_7 & & \alpha_8 \\
 & & & & \alpha_9 & & & \alpha_{10} & & & & & & & \alpha_{11} \\
 & & & \alpha_{12} & & & & & \alpha_{13} & & & & & & \alpha_{14} \\
 & \alpha_{15} & \alpha_{16} & & \alpha_{17} & & \alpha_{18} & & & \alpha_{19} & & \alpha_{20} & \alpha_{21} & \alpha_{22} & \\
 & \alpha_{23} & & & & & & & & \alpha_{24} & & \alpha_{25} & \alpha_{26} & \alpha_{27} & \\
\alpha_{28} & \alpha_{29} & & & & & & & & & \alpha_{30} & \alpha_{31} & \alpha_{32} & \alpha_{33} & \\
 & & & & \alpha_{34} & & \alpha_{35} & & & & & & & \alpha_{36} &
\end{array}\right]
\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_4 \\ x_1 \\ \vdots \\ x_4 \\ u_1 \\ u_2 \\ u_3 \\ v_1 \\ \vdots \\ v_4 \end{bmatrix} = 0$$

(5.2.3)

The coefficients are found in Appendix 1. The last six and the first equations are obtained from the last six and the second equations of equation (5.1.1) if equation (5.2.1) is used. The second equation is the linearized variant of equation (5.2.2). In the same manner as before it is found that the order of the system is maximally four.

6. REDUCTION TO STANDARD FORM

Two different methods have been used to solve the reduction problem. In the general case the number of time-derivatives of physical variables can exceed the number of state variables. The problem is then to assign a proper number of state variables. In the set of equations which represent the 4:th and 5:th order models, the number of state variables equal the number of time-derivatives of physical variables. The reduction problem in this case is quite simple and the state variables will equal the physical variables of which time-derivatives appear in the original set of equations. The used method is described below.

In the original set of equations eq (4.6.7) time-derivatives of seven of the physical variables appears. However, the order of the corresponding dynamic system on standard form is maximally six. This gives a reduction problem of a general character. A method which solves this general reduction problem has been derived and is presented in a separate report {4} . This report also includes a FORTRAN-program for the method. The 6:th order model presented here has been generated with this program. Since the method is documented in {4} no details are given in this rep t.

The reduction method

The problem is: given a set of equations

$$AA\ y = 0 \tag{6.1}$$

where AA is a constant matrix of order $\ell \times (2r + m + nv)$. The vector y is

$$y = [\dot{z}_1 \ \dots \ \dot{z}_r \ z_1 \ \dots \ z_r \ u_1 \ \dots \ u_m \ v_1 \ \dots \ v_{nv}]^T$$

and

$$\ell = r + nv$$

Determine the standard form of equation (6.1)

$$\dot{x} = Ax + Bu \tag{6.2}$$

where A and B are constant matrices of order n x n and n x m respectively. The state variables are a linear combination of the variables z

$$x = L \cdot z$$

where L is of order n x r, $n \leq r$.

Partition the matrix AA

$$AA\ y = \left[\begin{array}{c|c|c} AA_{11} & AA_{12} & AA_{13} \\ \hline AA_{21} & AA_{22} & AA_{23} \end{array}\right] \left[\begin{array}{c} \dot{z} \\ \hline z \\ u \\ \hline v \end{array}\right] = 0 \tag{6.3}$$

where

AA_{11}	r x r	AA_{21}	nv x r
AA_{12}	r x (r + m)	AA_{22}	nv x (r + m)
AA_{13}	r x nv	AA_{23}	nv x nv

Evaluate equation (6.3)

$$AA_{11}\ \dot{z} + AA_{12} \begin{bmatrix} z \\ u \end{bmatrix} + AA_{13}\ v = 0 \tag{6.4a}$$

$$AA_{21}\ \dot{z} + AA_{22} \begin{bmatrix} z \\ u \end{bmatrix} + AA_{23}\ v = 0 \tag{6.4b}$$

If the inverse of the matrix AA_{23} exists, then

$$v = -\ AA_{23}^{-1}\ AA_{21}\ \dot{z} - AA_{23}^{-1}\ AA_{22} \begin{bmatrix} z \\ u \end{bmatrix} \tag{6.5}$$

Combining the equations (6.4a) and (6.5) we get

$$(AA_{11} - AA_{13}\ AA_{23}^{-1}\ AA_{21})\dot{z} + (AA_{12} - AA_{13}\ AA_{23}^{-1}\ AA_{22}) \begin{bmatrix} z \\ u \end{bmatrix} = 0$$

or

$$C\ \dot{z} + D \begin{bmatrix} z \\ u \end{bmatrix} = 0$$

The solution is

$$\dot{z} = -\ (AA_{11} - AA_{13}\ AA_{23}^{-1}\ AA_{21})^{-1}\ (AA_{12} - AA_{13}\ AA_{23}^{-1}\ AA_{22}) \begin{bmatrix} z \\ u \end{bmatrix}$$

or

$$\dot{z} = \left[\begin{array}{c|c} A & B \end{array}\right] \begin{bmatrix} z \\ u \end{bmatrix} \tag{6.6}$$

if the inverse exists.

The serious drawback of this method is that it fails when the number of time-derivatives $\dot{z}$ exceeds the number of state variables. We must have n = r. The state variables will have a very simple physical interpretation. Equations (6.2) and (6.6) yield

$$x = z \tag{6.7}$$

Before the matrix AA is partitioned, it is rearranged. Consider the matrix

$$AA3 = \begin{bmatrix} AA_{13} \\ AA_{23} \end{bmatrix}$$

If the rank of AA3 equals nv, the matrix AA is arranged so that the rank of AA_{23} equals nv. If the rank of AA3 does not equal nv, there is no invertable submatrix AA_{23}.

Consider the matrix

$$C = (AA_{11} - AA_{13}\, AA_{23}^{-1}\, AA_{21})$$

If the computation of the inverse of C fails, this indicates that the rank of the matrix AA_{11} is less than r. On the other hand if the rank of AA_{11} is r the existence of the inverse of C is usually assured.

The reduction of equation (6.1) to standard form with this method is thus possible if

- the rank of the matrix AA3 equals nv
- the rank of the matrix C equals r

7. A DESCRIPTION OF THE COMPUTER PROGRAMS

The program which generates a 5:th order model is called DR5M (DRum 5:th order Model) and that which generates a 4:th order model DR4M. The differences are the linearized system matrix AA and dimensions of matrices. The programs are written in FORTRAN. The necessary subroutines are POLRT, MIART and EIGUNS. POLRT finds the roots of polynomials. MIART determines the rank of matrices, inverts matrices and solves linear equations. EIGUNS computes the eigenvalues of nonsymmetric matrices. The program lists are found in Appendix B. In order to avoid too cumbersome program listings, auxilary subroutines concerned with standard mathematical operations are omitted. The program variables are given in List of symbols. The matrix notations used are the same as elsewhere in the report. The program output is found in Appendix C. Program input is listed under INPUT DATA.

The input data is dimension parameters of the boiler and state equation proportional constants. We also need the coefficients c_1 and c_2 of the linear approximation of equation (4.2.4). c_1 and c_2 are manually computed. We have

$$x_o = \beta\, x_m$$

where β equals two. If we want to change the steam distribution we have to change β. As before ξ is the fraction of the kinetic energy which is not lost in the mud drum.

The computation of steady state values requires that we solve a set of nonlinear equations. Using equations (4.1.2), (4.2.15), (4.5.3), (4.2.3), (4.2.16) and (4.2.17) with time derivatives set equal to zero we get

$$-\Delta p = d_1\, \omega_\omega^{\,2} + d_2$$

$$\Delta p = d_3 \frac{\omega_o^{\,2}\, \rho_m}{\rho_o^{\,2}} + d_4\, \omega_\omega^{\,2} + d_5 \frac{\omega_o^{\,2}}{\rho_o} + d_6\, \omega_\omega^{\,2} \cdot \rho_m + d_7\, \rho_m$$

$$\rho_m = c_1\, x_m + c_2$$

$$\frac{1}{\rho_o} = d_8\, x_m + d_9$$

$$\omega_o = \omega$$

$$d_{10}\ \omega_o + d_{11}\ \omega_\omega + d_{12}\ x_m + d_{13} = 0$$

where

$$\Delta p = (p_{md} - p_d)/g$$

We have used equation (4.2.7) to replace x_o by x_m. By hand computation the set of equations is reduced to a 3:rd order polynomial in x_m

$$\alpha_4\ x_m^3 + \alpha_3\ x_m^2 + \alpha_2\ x_m + \alpha_1 = 0$$

The coefficients and roots of the polynomial are computed in the program. If there is more than one positive real root the program execution is stopped. If there is only one positive real root the steady state values of all other variables are computed and printed. The elements of matrix AA are computed and printed.

Now, if possible a invertable submatrix AA23 is established. The rank of matrix AA3 is computed with MIART. The rank and the vectors IBETA and JBETA are printed. The elements of the vectors IBETA and JBETA which not equal zero indicate linear independent rows and columns respectively. If the rank of the rectangular matrix AA3 is less than the order of the quadratic submatrix AA23, the program execution is stopped. If the rank equals this order a reduction to standard form is still possible. Using IBETA the matrix AA is arranged so that the indicated linear independent rows become the last rows of AA. Now, the inverse of the matrix AA23 exists. Subroutine MIART allow us to compute the inverse of AA23 at the same time as the rank of AA3. The rearranged matrix AA is partitioned and the matrices C and D are computed. Using MIART the linear equation is solved. The determinant of C and the content of the vectors IBETA and JBETA are printed. If MIART fails to solve the linear equation the program execution is stopped. Otherwise the reduction has succeeded and the system matrices A and B are printed. The eigenvalues of the matrix A are computed with EIGUNS.

The transfer functions are computed with program LAPLACE which needs the subroutines POLRT and TRANS. Subroutine TRANS computes the coefficients of the characteristic polynomial and the coefficients of the numerator polynomials of the transfer functions.

8. NUMERICAL EXAMPLES

The boiler data used is taken from a conventional power station boiler. The maximum steam flow is 350 t/h and the drum pressure is 140 bar. The energy loss coefficients of flow are calculated from empirical equations given in literature. The friction coefficients f_r and f_d can be used to adjust the steam quality at the outlet of the risers. The feedwater temperature is $\sim 230\ ^{o}C$. The heat flow to the risers is calculated from a heat balance in steady state. Two temperatures have to be guessed, the drum liquid temperature and the temperature of riser tubes. The riser temperature is often approximately known. A straight line is manually fitted to equation (4.2.8) in the interval $0.07 \leqslant x_m \leqslant 0.09$. The thermal state data and proportional constants are taken from steam tables. If the pressure is changed we thus have to change thermal state data, proportional constants and the coefficients of the straight line fit.

In the examples the influence of

- the two guessed temperatures
- the factor ξ
- the size of the drum
- the feedwater temperature

on significant parameters of the 4:th and 5:th order models is investigated. Open loop step responses of the state variables of the 4:th, 5:th and 6:th order models are compared.

8.1 The 4:th order model

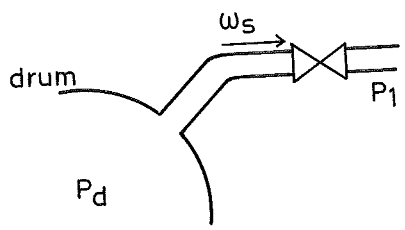
The results are given in Appendix C. The printouts of one run are packed on one page. The state variables of the 4:th order model are

x_1 the drum pressure
x_2 the drum liquid level
x_3 the drum liquid temperature
x_4 the mean temperature of riser tubes

<u>Example 1</u>

The drum liquid temperature has been set to 326 $^{\circ}C$ which is about 10 $^{\circ}C$ below the saturation temperature. The riser temperature is 480 $^{\circ}C$ and the factor ξ is 0.5. The friction coefficients are taken to 0.1 and 0.2 which is reasonable. All other data are boiler construction parameters or determined of the drum pressure. The page marked Example 1 Appendix C gives the printouts of the program.

The eigenvalues of the matrix A give the dynamic behaviour of the system except for non-minimum phase characteristics. The pure integration is associated with the drum liquid level. A simple physical interpretation of the other eigenvalues is difficult to state because of the interaction of the system. The cause of the unstable mode of the system is the assumption of constant outlet steam flow. Simulate a superheater with a pure restriction and take the pressure p_1 as the new input variable. See Fig. 6.



<u>Fig. 6</u> - The modified outlet of the drum

The flow is then determined by

$$\omega_s = k \sqrt{p_d - p_1} \tag{8.1.1}$$

or linearized

$$\Delta\omega_s = \frac{k}{2\sqrt{\bar{p}_d - \bar{p}_1}} (\Delta p_d - \Delta p_1) \tag{8.1.2}$$

where Δ denotes the perturbed variables and a bar indicates the steady state values of the variables. A proper choice of p_1 gives

$$\Delta\omega_s = 10(\Delta p_d - \Delta p_1) \tag{8.1.3}$$

The steam flow ω_s does only appear in the mass balance of the vapor phase in the drum. Replacing the steam flow by the pressure Δp_1 as the input variable simply means to add the third column of the matrix B to the first column of the matrix A. The third column of matrix B now becomes $[10 \quad 0 \quad 0 \quad 0]^T$. The eigenvalues of the modified matrix A become

0.0000 + 000
-6.6458 - 002
-9.1828 - 002
-1.0064 + 000

The integration is still there but all other eigenvalues are affected and have negative real parts.

The mean value of the steam quality is 7.64% which gives 15.3% at the outlet of the risers. This is a little bit too small. The true value is approximately 20%.

<u>Example 2</u>

The riser wall temperature is decreased to 460 $^{\circ}$C. The slight change of the eigenvalues indicates that the guessed value of the mean temperature of riser tubes is not critical.

<u>Example 3</u>

The example includes two runs. The drum liquid temperature is varied and set to 330 $^{\circ}$C and 335.6 $^{\circ}$C which is 6.6 $^{\circ}$C and 1.0 $^{\circ}$C below the saturation temperature respectively. The results should be compared with the result in Example 2. The increase of the temperature to 330 $^{\circ}$C gives a slightly faster system. The second guess makes the last eigenvalue of the table approximately ten times larger. Thus if the temperature difference saturation temperature-drum liquid temperature is not made too small the system eigenvalues are neglectable disturbed. The mean value of the steam quality increases with increased drum liquid temperature. Also the evaporation mass flow increases. Using program LAPLACE the transfer functions of run 330 $^{\circ}$C are computed.

Example 4

The example includes two runs with the factor ξ set equal to 1 and 0. The resulting eigenvalues are practically the same. The influence of this factor can be neglected.

Example 5

The size of the drum is reduced with a factor 5. Compare the result with the result in Example 3, run 330 ^{O}C. The system becomes faster. The absolute value of two eigenvalues increases approximately three times and one ten times. This result agrees with the expected one.

Example 6

The feedwater temperature is increased to 310 ^{O}C. The heat input flow is changed so that the steam flow do not change. All other data is the same as in Example 3, run 330 ^{O}C. Only the eigenvalues corresponding to the two fastest modes of the system have changed. Using program LAPLACE the transfer functions are computed. The output variables are the drum pressure and the drum liquid level. The coefficient matrix B1 contains the coefficients of the n-1 power of the numerator polynomials. B2 the coefficients of the n-2 power and so forth. The transfer function from feedwater flow to drum pressure is

$$G_{12} = 0.01056 \frac{s(s + 0.0108)(s + 0.0726)}{s(s - 0.00392)(s + 0.048)(s + 0.191)}$$

and the transfer function given by Example 3, run 330 ^{O}C is

$$G_{12}' = 0.01056 \frac{s(s - 0.285)(s + 0.0956)}{s(s - 0.00395)(s + 0.112)(s + 0.465)}$$

or if only the dominating mode is kept

$$G_{12}(s) = 9.0 \cdot 10^{-4} \frac{1}{s - 0.00392}$$

$$G_{12}'(s) = - 5.5 \cdot 10^{-3} \frac{1}{s - 0.00392}$$

Thus the gain changes sign. The gain depends on two mutually counteractive factors (a) the compression caused by the liquid level changes (b) the cooling effect of the changed flow. The influence of (b) depends on the feedwater temperature. It is then possible to choose a feedwater temperature which gives approximately zero gain.

8.2 The 5:th order model

The results are given in Appendix C. The state variables of the 5:th order model are

x_1 the drum pressure
x_2 the drum liquid level
x_3 the drum liquid temperature
x_4 the mean temperature of riser tubes
x_5 the mean value of steam quality

The first four state variables are thus the same as in the 4:th order model.

The parameter changes made are the same as before. The comments made in the Examples 1, 2, 3 and 4 of the 4:th order models hold as well for the 5:th order model. In Example 5 the changes of the eigenvalues of the matrix A are less than the corresponding changes in the 4:th order case. Using the figures of Example 6 and Example 3 we find that the sign of the gain of the transfer-function from feedwater flow to drum pressure is dependent of the temperature of the feedwater. This result is also obtained in Example 6 of the 4:th order model. In Example 2 the full program output is shown.

8.3 The 6:th order model

The system matrices used in the analog simulation are shown in Appendix C. The input data is the data used in Example 1. The state variables are

x_1 the drum pressure
x_2 the drum liquid level

x_3 the drum liquid temperature
x_4 the mean temperature of riser tubes
x_5 the mean value of steam quality
x_6 $\omega_o + 2.656\omega_\omega$, thus a linear combination of the mass flow at the outlet of the risers and the mass flow of the downcomers.

If we compare the eigenvalues with the result in Example 1 of the 5:th order model, we find that the new state variable introduced corresponds to the eigenvalue -1.683. This mode is fast and should not contribute considerably to the dynamic behaviour of the system. The agreement of the other eigenvalues is very good.

No changes of input data have been made since this model is nearly the same as the 5:th order model.

8.4 Comparison of the 4:th, 5:th and 6:th order models

The open loop responses of state variables of the models to a step change in the heat input flow, the feedwater flow and the steam outlet flow are shown in Fig. 7, Fig. 8 and Fig. 9 respectively. The input data is the data of Example 1. There is an extremely good agreement between the 5:th and the 6:th order model. This means that the acceleration terms of the momentum equations of the downcomers and the risers can be neglected. The dynamic behaviour of the system is not affected by this approximation. The essential difference between the 4:th and the 5:th order model is that the non-minimum phase response of the drum liquid level does not appear in the 4:th order model. This non-minimum phase behaviour is a property of the real process. Thus the steam quality variable, which is a state variable in the 5:th order model takes at least qualitatively the effects of steam bubble formation in the riser and the drum into account. If we want to include this effect in the model the dynamic behaviour of the risers cannot be neglected. Notice this two essential results of the model comparison. There seems to be a quite large difference between the gains of the 4:th and the 5:th order model. The difference is however mostly due to the distinction between the unstable mode of the models.

9. ACKNOWLEDGEMENT

The author wish to express his gratitude to Mr. K. Mårtensson who has written the subroutines MIART, POLRT and EIGUNS which are used frequently in the model programs.

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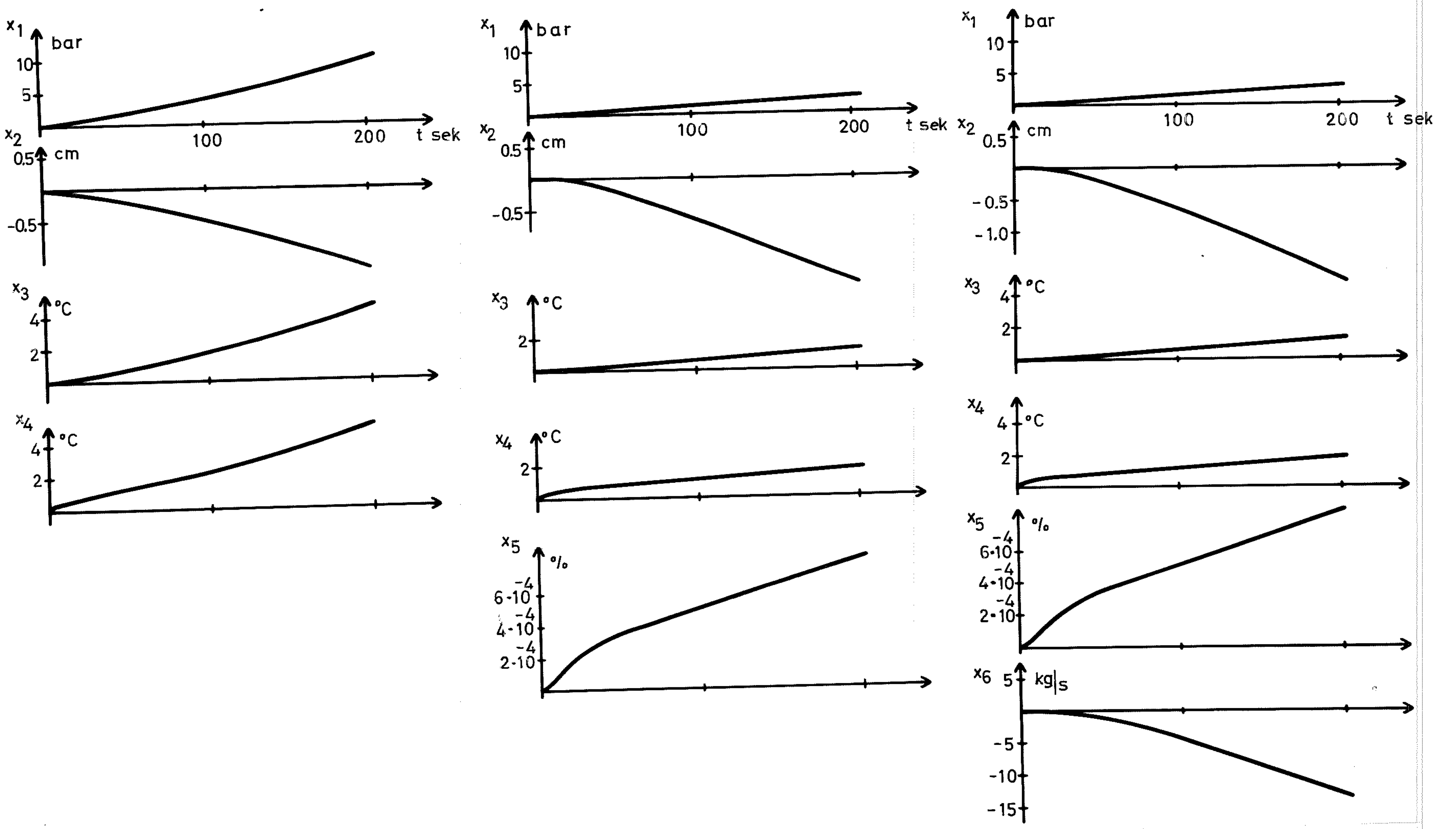


Fig. 7 - Open loop responses of the state variables of the 4:th, 5:th and 6:th order models to a step change in the heat flow to the risers of 2000 kJ/s (∼ 1%)

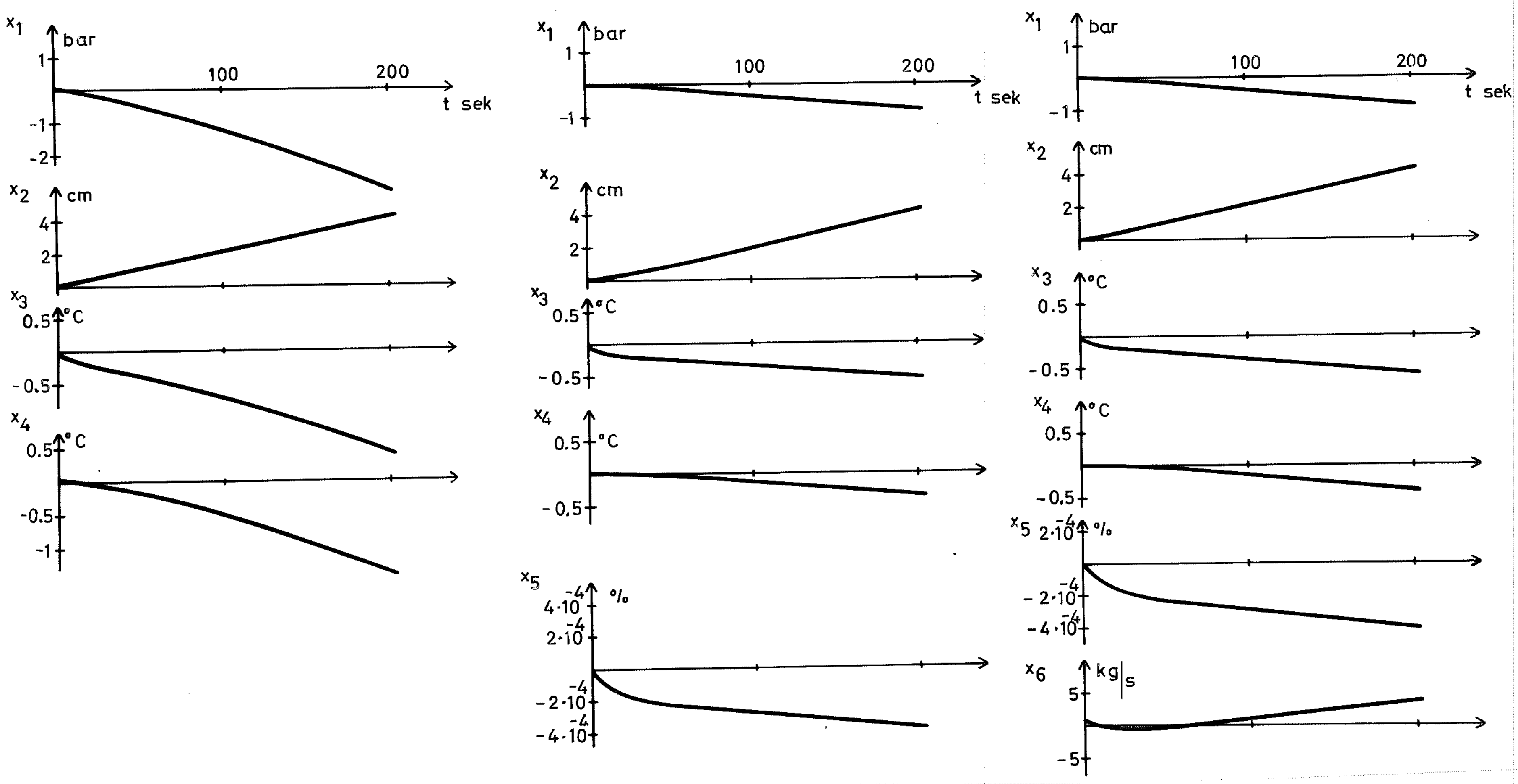
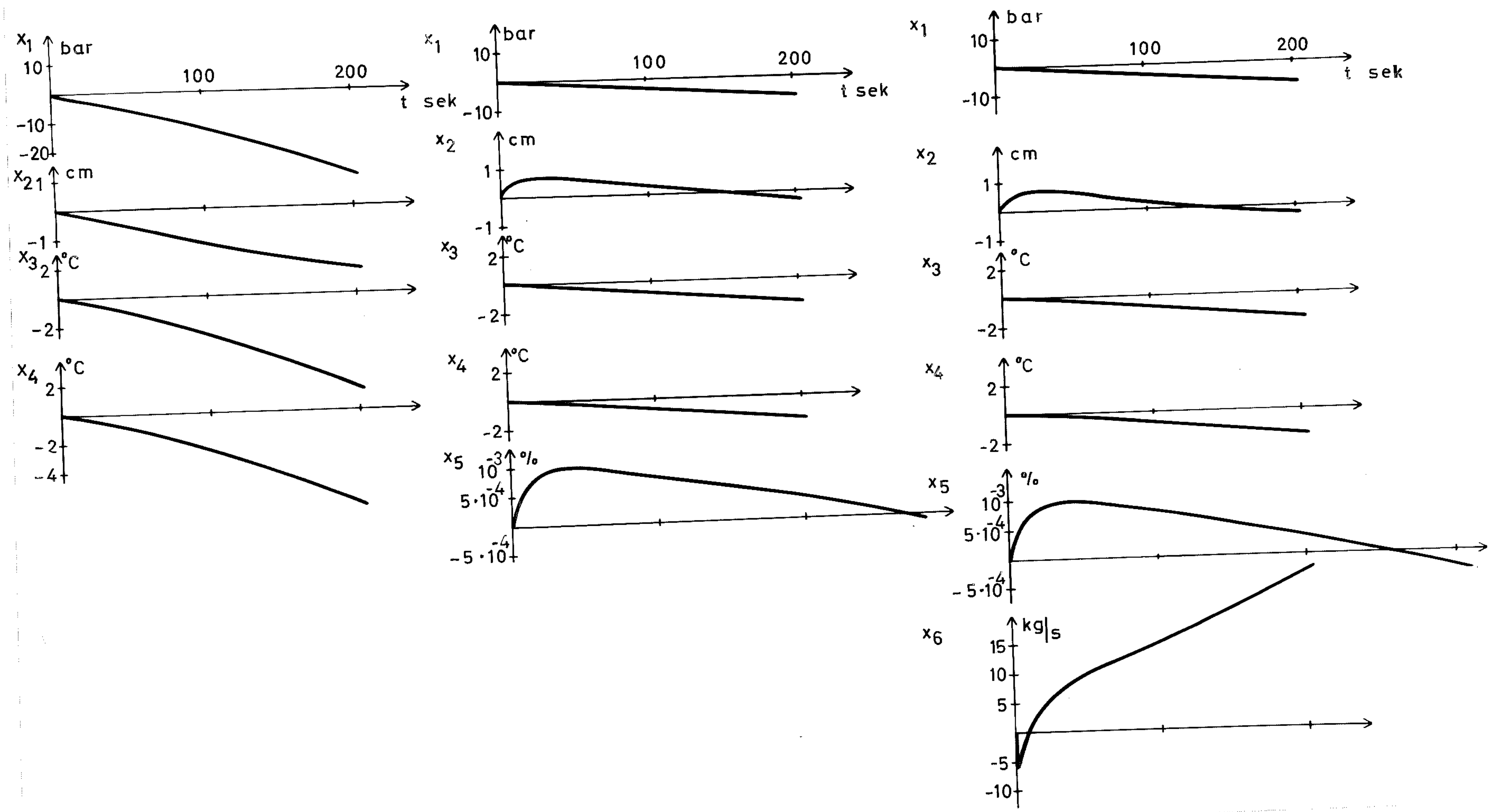


Fig. 8 - Open loop responses of the state variables of the 4:th, 5:th and 6:th order models to a step change in the feedwater flow of 2 kg/s (∼ 1%)



<u>Fig. 9</u> - Open loop responses of the state variables of the 4:th, 5:th and 6:th order models to a step change in the outlet steam flow of 2 kg/s (∿ 1%)

APPENDIX A

Reasonable numerical values of the constants $\zeta_1, \ldots \zeta_4$ are used in the expressions below. The values of the variables are the steady state values.

1.1 Coefficients of the equations (4.6.7) and (5.1.1)

$$a_1 = \left[\frac{\rho_\omega^2}{\beta x_m (\rho_\omega - \rho_s)^2} \ln\left(1 + \frac{\beta(\rho_\omega - \rho_s)}{\rho_s} x_m\right) - \frac{\rho_\omega^2}{\rho_s(\rho_\omega - \rho_s)} \cdot \frac{1}{1 + \frac{\beta(\rho_\omega - \rho_s)}{\rho_s} x_m} \right] k_d$$

$$a_2 = \frac{\rho_\omega}{x_m} \cdot \frac{1}{1 + \frac{\beta(\rho_\omega - \rho_s)}{\rho_s} x_m} - \frac{\rho_\omega \rho_s}{\beta x_m^2 (\rho_\omega - \rho_s)} \ln\left(1 + \frac{\beta(\rho_\omega - \rho_s)}{\rho_s} x_m\right)$$

$$a_3 = \frac{1}{A_r L_{r1}}$$

$$a_4 = - a_3$$

$$a_5 = \frac{\rho_m L_{r1}}{\rho A_r g}$$

$$a_6 = \frac{L_{d1}}{A_d g}$$

$$a_7 = (b_1 b_2 - b_3 b_4) k_d$$

where

$$b_1 = \left(f_r \frac{L_r}{D_r} + 1\right) \frac{\omega_o^2}{2g A_r^2 \rho_o^2} - \xi \frac{\omega^2}{2g A_d^2 \rho_\omega^2} + L_{r1}$$

$$b_2 = a_1 / k_d$$

$$b_3 = \left(f_r \frac{L_r}{D_r} + 1\right) \frac{\omega_o^2 \rho_m}{g A_r^2 \rho_o^3} + 0.1 \frac{\omega^2}{g A_r^2 \rho_o^2}$$

$$b_4 = \frac{\beta x_m \rho_o^2}{\rho_s^2}$$

$$a_8 = b_1 \cdot a_2 + b_3 b_5$$

where

$$b_5 = \beta\left(\frac{1}{\rho_s} - \frac{1}{\rho_\omega}\right)\rho^2$$

$$a_9 = \left(f_r \frac{L_r}{D_r} + 1\right)\frac{\omega_o \rho_m}{g A_r^2 \rho_o^2} + 0.2 \frac{\omega}{g A_r^2 \rho_o}$$

$$a_{10} = b_6 + b_7$$

where

$$b_6 = 1.8 \cdot \frac{\omega_o}{g A_r^2 \rho_\omega} - \xi \frac{\omega_o \rho_m}{g A_d^2 \rho_\omega^2}$$

$$b_7 = \left(f_d \frac{L_d}{D_d} + 3\right)\frac{\omega_o}{g A_d^2 \rho_\omega}$$

$$a_{11} = \frac{k_h - x_m(k_h - k_s)}{h_e}$$

$$a_{12} = 1$$

$$a_{13} = \frac{\omega_o(k_h - \beta x_m(k_h - k_s))}{A_r L_{rl} \rho_m h_e}$$

$$a_{14} = -\frac{\omega_o c_{p\omega}}{A_r L_{rl} \rho_m h_e}$$

$$a_{15} = \frac{\beta \omega_o}{A_r L_{rl} \rho_m}$$

$$a_{16} = -\frac{(1-\beta)x_m}{A_r L_{rl} \rho_m}$$

$$a_{17} = - \frac{h_\omega - h_m}{A_r L_{r1} \rho_m h_e}$$

$$a_{18} = - \frac{1}{A_r L_{r1} \rho_m h_e}$$

$$a_{19} = 3k_r (T_r - T_s)^2 \cdot k_T$$

$$a_{20} = - 3k_r (T_r - T_s)^2$$

$$a_{21} = 1$$

$$a_{22} = 1$$

$$a_{23} = - \frac{1}{M_r \ c_r}$$

$$a_{24} = \frac{1}{M_r \ c_r}$$

$$a_{25} = A \ \rho_\omega$$

$$a_{26} = \frac{M_\omega \ c_{p\omega}}{h_\omega}$$

$$a_{27} = - \frac{(1-\beta x_m)\omega_o \ k_h - \omega_e \ k_s}{h_\omega}$$

$$a_{28} = \frac{\omega_o \ c_{p\omega}}{h_\omega}$$

$$a_{29} = \frac{\omega_o \ h_{\omega s}\beta}{h_\omega}$$

$$a_{30} = - \frac{h_{f\omega}}{h_\omega}$$

$$a_{31} = - \frac{(1-\beta x_m) h_{\omega s}}{h_\omega}$$

$$a_{32} = 1$$

1.2 Coefficients of equation (5.2.3)

$\alpha_1 = a_7$

$\alpha_2 = a_9 + a_{10}$

$\alpha_3 = a_8$

$\alpha_4 = -(k_h - \beta x_m(k_h - k_s))\omega_o$

$\alpha_5 = c_{p\omega} \cdot \omega_o$

$\alpha_6 = (h_\omega - h_{\omega s} - \beta x_m h_e)$

$\alpha_7 = -\beta \cdot \omega_o \cdot h_e$

$\alpha_8 = 1$

$\alpha_9 = a_{19}$

$\alpha_{10} = a_{20}$

$\alpha_{11} = a_{21}$

$\alpha_{12} = a_{22}$

$\alpha_{13} = a_{23}$

$\alpha_{14} = \alpha_{24}$

$\alpha_{15} = \alpha_{25}$

$\alpha_{16} = a_{26}$

$\alpha_{17} = a_{27}$

$\alpha_{18} = a_{28}$

$\alpha_{19} = a_{30}$

$\alpha_{20} = a_{31} + a_{32}$

$\alpha_{21} = a_{29}$

$\alpha_{22} = a_{33}$

$\alpha_{23} = a_{34}$

$\alpha_{24} = a_{36}$

$\alpha_{25} = a_{37} + a_{38}$

$\alpha_{26} = a_{35}$

$\alpha_{27} = a_{39}$

$\alpha_{28} = a_{40}$

$\alpha_{29} = a_{41}$

$\alpha_{30} = a_{43}$

$\alpha_{31} = a_{44}$

$\alpha_{32} = a_{42}$

$\alpha_{33} = a_{45}$

$\alpha_{34} = a_{46}$

$\alpha_{35} = a_{47}$

$\alpha_{36} = a_{48}$

$$a_{33} = \frac{h_{ss}}{h_{\omega}}$$

$$a_{34} = a_{25}$$

$$a_{35} = \omega_o \beta$$

$$a_{36} = -1$$

$$a_{37} = -\ (1-\beta x_m)$$

$$a_{38} = 1$$

$$a_{39} = 1$$

$$a_{40} = 1$$

$$a_{41} = -\ \frac{\rho_s \cdot A}{V_s\ k_d}$$

$$a_{42} = -\ \frac{\beta\ \omega_o}{V_s\ k_d}$$

$$a_{43} = \frac{1}{V_s\ k_d}$$

$$a_{44} = -\ \frac{\beta x_m}{V_s\ k_d}$$

$$a_{45} = -\ \frac{1}{V_s\ k_d}$$

$$a_{46} = k_e \cdot k_T$$

$$a_{47} = -\ k_e$$

$$a_{48} = 1$$

```
      PROGRAM DR4M
C
C     COMPUTES THE STANDARD FORM S(A,B,C,D) OF A FOURTH ORDER BOILER
C     MODEL GIVEN THE ORIGINAL LINEARIZED EQUATION MATRIX AA,THE
C     INPUT DATA ARE MACRO BOILER DATA.
C
C     SUBROUTINE REQUIRED
C        FOLRT,MIART
C
      DIMENSION AA(20,20),AA11(5,5),AA12(10,10),AA13(5,5),AA21(5,5),AA22
     1(10,10),AA23(5,5),AA23I(5,5),AA3(10,10),BA(20,20),B(10,10),C(10,10
     1),D(10,10),A(10,10),CD(20,20),IBETA(10),JBETA(5),XROOT(5),ALFA(5),
     1ROOTR(10),ROOTI(10),XROOTR(5)
 1100 READ 1060,NR
 1060 FORMAT(I3)
      IF(NR-99) 1061,1061,998
 1061 READ 1101,ADE
 1101 FORMAT(E8.1)
      READ 1010,FD,ALD,ALD1,AD,DD,FR,ALR,ALR1,AR,DR,DEW,DESS,TW,TR,TS,PK
     11,PK2,PK3,PK6,PK8,PK9,HSS,HE,HFW,HWS,HW,VS,WMASS,RMASS,ADR,QR,BETA
     1,XSI,C1,C2
 1010 FORMAT(6F12.5)
C
C     PRINT INPUT DATA
C
      PRINT 1051
 1051 FORMAT(1H1)
      PRINT 1009
 1009 FORMAT(1H1,2(/),11H INPUT DATA,2(/))
      PRINT 1011
 1011 FORMAT(/,15H DOWNCOMER DATA,/)
      PRINT 1012,FD
 1012 FORMAT(21H FRICTION COEFFICIENT,14X,F11.5)
      PRINT 1013,ALD
 1013 FORMAT(22H TOTAL LENGTH OF TUBES,13X,F11.5)
      PRINT 1014,ALD1
 1014 FORMAT(19H LENGTH OF ONE TUBE,16X,F11.5)
      PRINT 1015,AD
 1015 FORMAT(16H TOTAL FLOW AREA,19X,F11.5)
      PRINT 1016,DD
 1016 FORMAT(20H TOTAL TUBE DIAMETER,15X,F11.5)
      PRINT 1017
 1017 FORMAT(2(/),11H RISER DATA,/)
      PRINT 1012,FR
      PRINT 1013,ALR
      PRINT 1014,ALR1
      PRINT 1015,AR
      PRINT 1016,DR
      PRINT 1018,DEW
 1018 FORMAT(2(/),28H DENSITY OF SATURATED LIQUID,7X,F11.5)
      PRINT 1019,DESS
 1019 FORMAT(27H DESITY OF SATURATED VAPOR,8X,F11.5)
      PRINT 1020,TW
 1020 FORMAT(24H DRUM LIQUID TEMPERATURE,11X,F11.5)
      PRINT 1021,TR
 1021 FORMAT(23H RISER TUBE TEMPERATURE,12X,F11.5)
```

```
      PRINT 1050,RMASS
 1050 FORMAT(20H MASS OF RISER TUBES,15X,F11.5)
      PRINT 1022,TS
 1022 FORMAT(23H SATURATION TEMPERATURE,12X,F11.5)
      PRINT 1023,HW
 1023 FORMAT(24H ENTHALPY OF DRUM LIQUID,11X,F11.5)
      PRINT 1024,HFW
 1024 FORMAT(22H ENTHALPY OF FEEDWATER,13X,F11.5)
      PRINT 1025,HWS
 1025 FORMAT(29H ENTHALPY OF SATURATED LIQLID,6X,F11.5)
      PRINT 1026,HSS
 1026 FORMAT(28H ENTHALPY OF SATURATED VAPCR,7X,F11.5)
      PRINT 1027,HE
 1027 FORMAT(24H ENTHALPY OF EVAPORATION,11X,F11.5)
      PRINT 1028
 1028 FORMAT(2(/),10H DRUM DATA,/)
      PRINT 1029,WMASS
 1029 FORMAT(20H MASS OF DRUM LIQUID,15X,F11.5)
      PRINT 1030,VS
 1030 FORMAT(24H VOLUME OF VAPOR IN DRUM,11X,F11.5)
      PRINT 1031,ADR
 1031 FORMAT(28H LIQUID SURFACE AREA IN DRLM,7X,F11.5)
      PRINT 1032,QR
 1032 FORMAT(/,31H HEAT INPUT RATE TO RISER TUBES,3X,F12.5)
      PRINT 1033
 1033 FORMAT(2(/),10H CONSTANTS,/)
      PRINT 1034,PK1
 1034 FORMAT(9H PK1 (KT),26X,F11.5)
      PRINT 1035,PK2
 1035 FORMAT(9H PK2 (KD),26X,F11.5)
      PRINT 1036,PK3
 1036 FORMAT(9H PK3 (KH),26X,F11.5)
      PRINT 1037,PK6
 1037 FORMAT(10H PK6 (CPW),25X,F11.5)
      PRINT 1039,PK8
 1039 FORMAT(9H PK8 (CR),26X,F11.5)
      PRINT 1040,PK9
 1040 FORMAT(9H PK9 (KS),26X,F11.5)
      PRINT 1070,BETA
 1070 FORMAT(5H BETA,30X,F11.5)
      PRINT 1071,XSI
 1071 FORMAT(4H XSI,31X,F11.5)
      PRINT 1072,C1
 1072 FORMAT(3H C1,32X,F11.5)
      PRINT 1073,C2
 1073 FORMAT(3H C2,32X,F11.5)
      GA=9.81
C
C     FACTORS OF COEFFICIENTS OF POLYNOMIAL IN STEAMQUALITY IS
C     CALCULATED
C
      AV1=GA*AD*AD
      AV2=GA*AR*AR
      AV3=QR*QR
      A1=(FD*ALD/DD+3.)/(2.*AV1*DEW)
      A2=ALD1*DEW
```

```
      B1=(FR*ALR/DR+1.)/(2.*AV2)
      B2=0.9/(AV2*DEW)
      B3=0.1/AV2
      B4=XSI/(2.*AV1*DEW*DEW)
      B5=ALR1
      D1=EETA*(1./DESS-1./DEW)
      D2=1./DEW
      E1=FW-HWS
      E2=EETA*HE
C
C     COEFFICIENTS OF POLYNOMIAL IN STEAMQLALITY IS CALOULATED
C
      ALFA(4)=C1*(AV3*B1*D2*D2+B5*E2*E2)
      ALFA(3)=AV3*B1*D1*(C2*D1+2.*C1*D2)+B5*E2*(C2*E2-2.*C1*E1)-A2*E2*E2
      ALFA(2)=AV3*(B1*D2*(C1*D2+2.*C2*D1)+E3*D1-B4*C1)+B5*E1*(C1*E1-2.*C
     12*E2)+2.*A2*E1*E2
      ALFA(1)=AV3*(B1*C2*D2*D2+B2+B3*D2-B4*C2*A1)+E1*E1*(B5*C2-A2)
      PRINT 10
   10 FORMAT(1H1,2(/))
C
C     ROOTS OF POLYNOMIAL IS CALCULATED
C
      CALL POLRT(ALFA,3,ROOTR,ROOTI,5,IER)
C
      KUF=IER+1
      GO TO(39,39,39,41,42),KUF
   41 PRINT 46
   46 FORMAT(66H UNABLE TO DETERMINE ROOT WITH 500 ITERATIONS ON 5 START
     1ING VALUES)
      GO TO 999
   42 PRINT 47
   47 FORMAT(32H HIGH ORDER COEFFICIENTS IS ZERO)
      GO TO 999
   39 PRINT 40
   40 FORMAT(36H ROOTS OF POLYNOMIAL IN STEAMQUALITY,2(/))
      PRINT 45
   45 FORMAT(14X,10H REAL PART,5X,15H IMAGINARY PART,/)
      DO 50 I=1,3
      PRINT 55,ROOTR(I),ROOTI(I)
   55 FORMAT(10X,E15.5,2X,E15.5)
   50 CONTINUE
C
C     TEST ON REAL POSITIVE ROOTS
C
      J=0
      DO 60 I=1,3
      IF(ABSF(ROOTI(I))-1.E-006) 65,60,60
   65 J=J+1
      XROOTR(J)=ROOTR(I)
   60 CONTINUE
      K=0
      DO 70 I=1,J
      IF(XROOTR(I)) 70,70,75
   75 K=K+1
      XROOT(K)=XROOTR(I)
   70 CONTINUE
```

```
      IF(K-1) 80,85,80
   80 PRINT 81
   81 FORMAT(48H FAILURE NO OR MORE THAN TWO POSITIVE REAL ROOTS)
      GO TO 999
   85 CONTINUE
C
C     COMPUTE AND PRINT STEADY STATE VALUES
C
      XM=XROOT(1)
      W=QR/(E2*XM-E1)
      DE=1./(D1*XM+D2)
      DEM=C1*XM+C2
      AV8=TS-TW
      PK4=W*(BETA*XM*(HWS-HFW)/AV8-(HWS-HW)/AV8)/(HSS-HFW)
      WE=-PK4*AV8
      WS=WE+BETA*XM*W
      WFW=WS
      PK5=QR/(TR-TS)**3
      HM=HWS+BETA*XM*HE
      HMM=HWS+XM*HE
      PRINT 86,XM
   86 FORMAT(//,28H MEAN VALUE OF STEAM QUALITY,E15.5,//)
      PRINT 90,W
   90 FORMAT(16H TOTAL MASS FLOW,15X,E15.5)
      PRINT 101,DE
  101 FORMAT(30H OUTLET DENSITY OF STEAM-WATER,/,8H MIXTURE,23X,E15.5)
      PRINT 102,DEM
  102 FORMAT(28H MEAN DENSITY OF STEAM-WATER,/,8H MIXTURE,23X,E15.5)
      PRINT 95,PK4
   95 FORMAT(24H CONSTANT OF EVAPORATION,7X,E15.5)
      PRINT 100,WE
  100 FORMAT(22H EVAPORATION MASS FLOW,9X,E15.5)
      PRINT 105,WS
  105 FORMAT(18H OUTLET STEAM FLOW,13X,E15.5)
      PRINT 91,PK5
   91 FORMAT(31H HEAT TRANSFER COEFFICIENT FROM,/,27H RISER TUBES TO STE
     1AM-WATER,/,8H MIXTURE,23X,E15.5)
      PRINT 92,HM
   92 FORMAT(27H OUTLET ENTHALPY OF MIXTURE,/,15H IN RISER TUBES,16X,E15
     1.5)
      PRINT 103,HMM
  103 FORMAT(25H MEAN ENTHALPY OF MIXTURE,/,15H IN RISER TUBES,16X,E15.5
     1)
C
C     ALL STEADY STATE VALUES KNOWN.
C
C
C     THE ELEMENTS OF MATRIX AA IS CALCULATED
C
      DO 200 I=1,8
      DO 200 J=1,16
  200 AA(I,J)=0.
      AV4=DEW-DESS
      AV5=1.+(BETA*AV4*XM)/DESS
      AV6=LOGF(AV5)
      AV9=DEW*DEW
```