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SOME COMPUTATIONAL RESULTS OBTAINED BY
PANUŠKA'S METHOD OF STOCHASTIC APPROXI-
MATIONS FOR IDENTIFICATION OF DISCRETE
TIME SYSTEMS

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DIVISION OF AUTOMATIC CONTROL

SOME COMPUTATIONAL RESULTS OBTAINED BY PANUŠKA'S METHOD OF STOCHASTIC
APPROXIMATIONS FOR IDENTIFICATION OF DISCRETE TIME SYSTEMS. [†]

J. Valis and I. Gustavsson

ABSTRACT

In this paper different variants of Panuška's method of stochastic approximations for system identification are used to estimate the discrete transfer function coefficients from simulated second order system input/output data sequences of finite length. Using Monte Carlo simulation mean values and variances of the estimates are computed and compared with those obtained using the method of maximum likelihood. Results confirm the superiority of M.L. estimates in this case.

[†] This work has been supported by the Swedish Board for Technical Development under Contract 68-336-f

1. A BRIEF OUTLINE OF PANUŠKA'S METHOD

In his papers [2], [3] V Panuška proposes a very simple method (perhaps the most simple method which could ever be used) for numerical identification of linear dynamic discrete time system of finite order with constant coefficients from normal operating records. Description of such a system can always be reduced to Åström's canonical form

$$y(k) + \sum_{i=1}^n a_i y(k-i) = \sum_{i=0}^n b_i u(k-i) + \lambda(e(k) + \sum_{i=1}^n c_i e(k-i)) \quad (1)$$

where $\{y(k)\}$ is the sequence of values of output signal
 $\{u(k)\}$ is the sequence of values of input signal
 $\{e(k)\}$ is a sequence of normal independent random variables with zero mean and variance 1 ($N(0,1)$)
 n is the order of the system

Panuška supposes, that finite length records of sequences

$$\{u(t), y(t); t = 1, 2, \dots, N\}$$

are given and that the order of the system n is known. Then he uses the following formulas for stochastic approximation of estimates of \hat{a}_i , \hat{b}_i , \hat{c}_i and $\hat{\lambda}$.

$$k = 1 + (t - 1) \bmod N \quad (2a)$$

$$\hat{\epsilon}(k) = y(k) + \sum_{i=1}^n \hat{a}_i(k) y(k-i) - \sum_{i=0}^n \hat{b}_i(k) u(k-i) - \sum_{i=1}^n \hat{c}_i(k) \hat{\epsilon}(k-i) \quad (2b)$$

$$G(t) = \alpha/t; \alpha \text{ suitable positive constant} \quad (2c)$$

$$\hat{a}_r(t+1) = \hat{a}_r(t) - G(t) \cdot \hat{\epsilon}(k) \cdot y(k-r) \quad r = 1, \dots, n \quad (2d)$$

$$\hat{b}_r(t+1) = \hat{b}_r(t) + G(t) \cdot \hat{\epsilon}(k) \cdot u(k-r) \quad r = 0, 1, \dots, n \quad (2e)$$

$$\hat{c}_r(t+1) = \hat{c}_r(t) + G(t) \cdot \hat{\epsilon}(k) \cdot \hat{\epsilon}(k-r) \quad r = 1, 2, \dots, n \quad (2f)$$

$$\hat{\lambda}^2 = \frac{1}{N} \sum_{k=1}^N \hat{\epsilon}^2(k) \quad (2g)$$

In [2] Panuška states without complete proof that the sequences $\hat{a}_r(t)$, $\hat{b}_r(t)$ and $\hat{c}_r(t)$ converge in the mean square sense to the true values of a_r , b_r and c_r respectively. He also gives some results of identification of artificially simulated data. Unfortunately, the paper does not contain complete answers to some important questions concerning variance, speed of convergence, stability of the computation, etc.

2. DISCUSSION OF THE ALGORITHM AND VARIANTS

Panuška's algorithm may have the following interpretation [Fig. 1].

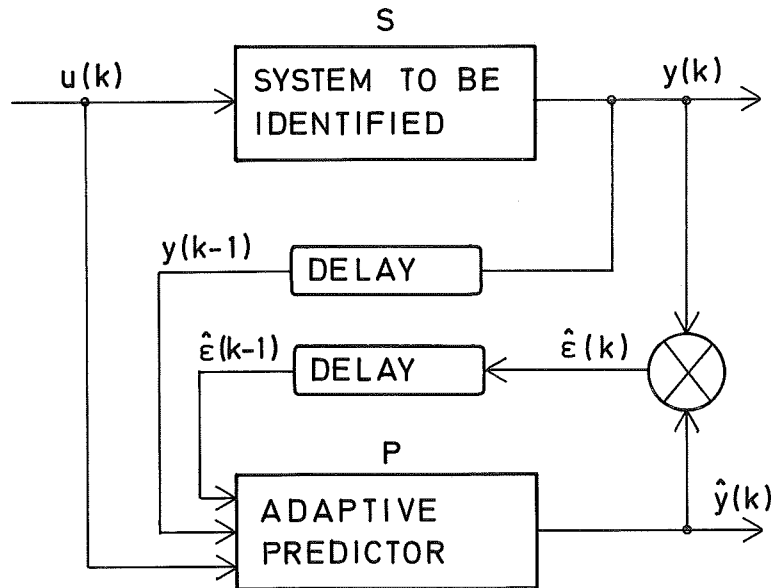


Fig. 1. Interpretation of Panuška's algorithm.

Let us assume, that the system S may be described by eq. (1). We want to adjust the coefficients of the adaptive predictor P , which predicts the value of output $\hat{y}(k)$ based on past values of input $u(k)$, $u(k-1)$, $u(k-2)$, ..., $u(k-n)$, past values of output $y(k-1)$, $y(k-2)$, ..., $y(k-n)$, and past prediction errors $\hat{\epsilon}(k-1)$, $\hat{\epsilon}(k-2)$, ..., $\hat{\epsilon}(k-n)$ in such a way, that the prediction errors $\hat{\epsilon}(k)$, defined by

$$\hat{y}(k) = \sum_{i=0}^n \hat{b}_i(k) u(k-i) - \sum_{i=1}^n \hat{a}_i(k) y(k-i) + \sum_{i=1}^n \hat{c}_i(k) \hat{\epsilon}(k-i) \quad (3)$$

$$\hat{\epsilon}(k) = y(k) - \hat{y}(k) \quad (4)$$

are „as small as possible“.

Exactly, it means that the $E\hat{\epsilon}^2(k)$ should be minimal and the $\{\hat{\epsilon}(k)\}$ sequence should be white.

Our „adjust strategy“ will be simply to minimize at each time instant k the square of instant prediction error $\hat{\epsilon}(k)$ using gradient method (neglecting the whole past history). It means

$$\begin{aligned} \hat{\epsilon}(k) = y(k) - \hat{y}(k) = y(k) + \sum_{i=1}^n \hat{a}_i(k) y(k-i) - \sum_{i=0}^n \hat{b}_i(k) u(k-i) - \\ - \sum_{i=1}^n \hat{c}_i(k) \hat{\epsilon}(k-i) \end{aligned} \quad (5)$$

and

$$\frac{\partial \hat{\epsilon}^2(k)}{\partial \hat{a}_r(k)} = 2 \hat{\epsilon}(k) \cdot y(k-r) \quad r = 1, 2, \dots, n \quad (6a)$$

$$\frac{\partial \hat{\epsilon}^2(k)}{\partial \hat{b}_r(k)} = -2 \hat{\epsilon}(k) \cdot u(k-r) \quad r = 0, 1, \dots, n \quad (6b)$$

$$\frac{\partial \hat{\epsilon}^2(k)}{\partial \hat{c}_r(k)} = -2 \hat{\epsilon}(k) \hat{\epsilon}(k-r) \quad r = 1, 2, \dots, n \quad (6c)$$

Putting (5) and (6a, b, c) into the general formula for the gradient minimization method, we get (2b - g).

Remark 1:

The computation of $\hat{\epsilon}(k)$ using formula (2b) may easily become unstable if the polynomial

$$\hat{C}_t(z^{-1}) = z^n (1 + \sum_{i=1}^n z^{-i} \hat{c}_i(k)) \quad (7)$$

has zeroes outside of the unit circle. Panuška avoids this difficulty by putting certain bounds on values of estimates $\hat{a}_r(k)$, $\hat{b}_r(k)$, $\hat{c}_r(k)$ and errors $\hat{\epsilon}(k)$. That means (q is suitable positive constant) for a -coefficients

$$\hat{a}_r(t+1) = \begin{cases} +q & \text{if } |\tilde{a}_r(t+1)| \geq q \\ \tilde{a}_r(t+1) & \text{if } |\tilde{a}_r(t+1)| < q \\ -q & \text{if otherwise} \end{cases} \quad (8)$$

where $\tilde{a}_r(t+1) = \hat{a}_r(t) - G(t) \cdot \hat{\epsilon}(t) y(t-r)$ and similar for $\hat{b}_r(t)$, $\hat{c}_r(t)$ and $\hat{\epsilon}(t)$.

Another way to assure the stability of the computation of $\hat{\epsilon}(k)$ is to change the values of $\hat{c}_r(t)$ so that polynomial $\hat{C}_t(z^{-1})$ remains stable. Then we have to check the stability of the polynomial (7) after each iteration step. This is relatively easy to do in first and second order system cases but for $n > 2$ we must use a suitable numerical method for stability check because of the large number of iteration steps.

Remark 2:

To obtain better estimates, Panuška uses several runs through the input-output records so that the values of estimates after the first pass are used as the starting values for the second pass and so forth. This is a very good idea, but in our opinion still better accuracy can be achieved when using

$$G(t) = \frac{\alpha}{t} \quad (9a)$$

or

$$G(t) = \frac{\alpha}{\beta + k}, \quad \beta \gg 1 \quad (9b)$$

instead of (2c) as a gain factor.

3. EXPERIMENTAL INVESTIGATION OF PANUŠKA'S METHOD

The papers [2], [3] contain results obtained by means of described method on identification of a 2nd order system

$$\begin{aligned} y(k) - 1.5y(k-1) + 0.7y(k-2) &= u(k-1) + 0.5u(k-2) + \varepsilon(k) + \\ &+ \varepsilon(k-1) + 0.2\varepsilon(k-2) \end{aligned}$$

where $\{u(k)\}$ and $\{\varepsilon(k)\}$ are generated as independent pseudorandom sequences $N(0,1)$. The results obtained from 500 input/output data pairs are very promising, but unfortunately the variance of these estimates cannot be found from only very few realisations.

To obtain better insight into properties of this method, we first wrote simple programs for simulation of a linear discrete-time system driven by pseudorandom normal signal and its identification from input/output records. We tried both proposed ways to assure the stability of computation of $\hat{\varepsilon}(k)$:

- 1) Original Panuška's with values of estimates of coefficients \hat{a} , \hat{b} , \hat{c} bounded to ± 5.0 and errors $\hat{\varepsilon}(k)$ bounded to ± 10.0 . This algorithm was called ORIGPAN.
- 2) Checking the stability of $\hat{C}_t(z^{-1})$ polynomial at each iteration step - at unstable iterations we did not change the estimates. This algorithm was called STABPAN.

In both algorithms different gain sequences $G(t)$ were used. In each example we computed mean values and variance of estimates by Monte Carlo method (from 20 different samples). Starting values for estimates and initial conditions were zero.

Numerical results are scheduled in Tab. 1. Surprising are large variances and bad estimates of c-coefficients (particularly c_2) in all cases when compared with result obtained by Åström's method of maximum likelihood (which can be considered as the most powerful method for identification at present time [1], [4]).

Not surprising is the (relative) superiority of ORIGPAN when compared with STABPAN because even if the \hat{C} -polynomial is stable the values of $\hat{\epsilon}$ may become very large and cause completely wrong changes of estimates.

For further comparison we chose the variant, which gave the most reasonable results (ORIGPAN, $\alpha = 5$, 5 passes denoted by f in Tab. 1) and tried it on two different samples of our 2nd order standard data. In this case the input signal was pseudorandom normal (0,1), length of sequences $N = 1000$ data pairs. Results, summarized in Tab. 2, seem not to be bad in comparison with M.L.E., but unfortunately the estimates obtained after 5th pass through the data cannot be considered as final because they show distinct trend to change further when we increase the number of passes as it is shown in Tab. 3.

The reason for this is quite obvious:

According to (2d - 2f) the changes of estimates of coefficients shall continue while

$$\frac{1}{t} \cdot \hat{\epsilon}(k) \cdot \hat{\epsilon}(k - r) \neq 0 \quad (r = 1, 2, \dots, n)$$

by other words the changes shall continue until the sequence $\{\hat{\epsilon}(k)\}$ becomes such that the first n-values of its autocovariance function

$$r_{\hat{\epsilon}\hat{\epsilon}}(\tau) = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{i=1}^t \hat{\epsilon}(i) \hat{\epsilon}(i - \tau) \quad \tau = 1, 2, \dots, n$$

are zero. In the case of input/output data of infinite length the sequences of estimates $\{\hat{a}(k)\}, \{\hat{b}(k)\}, \{\hat{c}(k)\}$ would therefore converge to their true values. Apparently this may not be the case if we perform iterations by repeated passes through the same data of finite length N because this is equivalent to the case when the original sequence of (generated) errors $\{\epsilon(k)\}$ is periodical with period N and the values of its autocovariance function

$$r_{\epsilon\epsilon}(\tau) = \frac{1}{N} \sum_{t=1}^N \epsilon(t) \epsilon(t - \tau)$$

may not be zero.

Actually, in our example of 2nd order standard data this sequence was not „very white" as it is shown in Tab. 4 and Fig. 2, where the estimates and covariances of the respective residuals are scheduled for different numbers of passes.

The algorithm ORIGPAN tries to make the values of $\hat{r}_{\epsilon\epsilon}(1)$ and $\hat{r}_{\epsilon\epsilon}(2)$ as small as possible (even if the original ones are not) without taking other values of $\hat{r}_{\epsilon\epsilon}$ into account while the autocovariance function of residuals of M.L. estimates has almost the same shape as the covariance of original noise.

To illustrate this we designed two more examples with different noise sequences and plotted the results in Fig. 3, 4 resp. Tab. 5. The „more white" sequence S_2 gave better estimate than the S_1 with relatively large values of $\hat{r}_{\epsilon\epsilon}(1)$ and $\hat{r}_{\epsilon\epsilon}(2)$.

4. CONCLUSIONS

It follows from the previous investigation that Panuška's method of stochastic approximations can be considered as interesting with the following advantages:

1. simplicity - at present time no more simple method is known
2. easy programmable - a computer subroutine contains a few statements only
3. suitable for real-time identification (see Appendix)

Unfortunately, the disadvantages are rather serious:

1. suitable gain sequence must be found experimentally
2. large numbers of passes through the data are needed to obtain more accurate results
3. the variances of estimates are greater than those obtained by Åström's M.L.E.
4. there is no measure of accuracy of estimates available.

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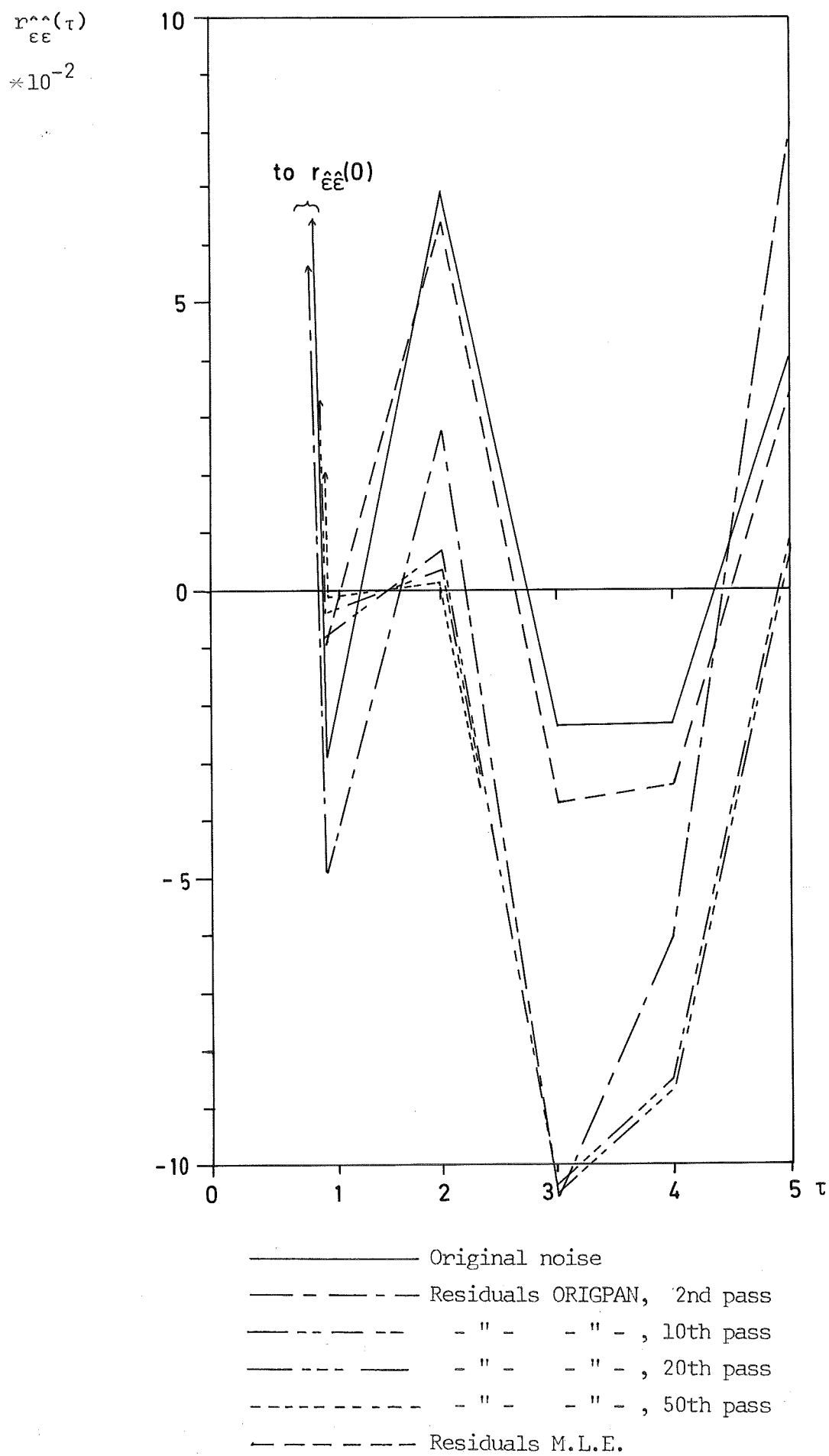


Fig. 2 Covariance of Original Noise and Residuals
2nd order standard data

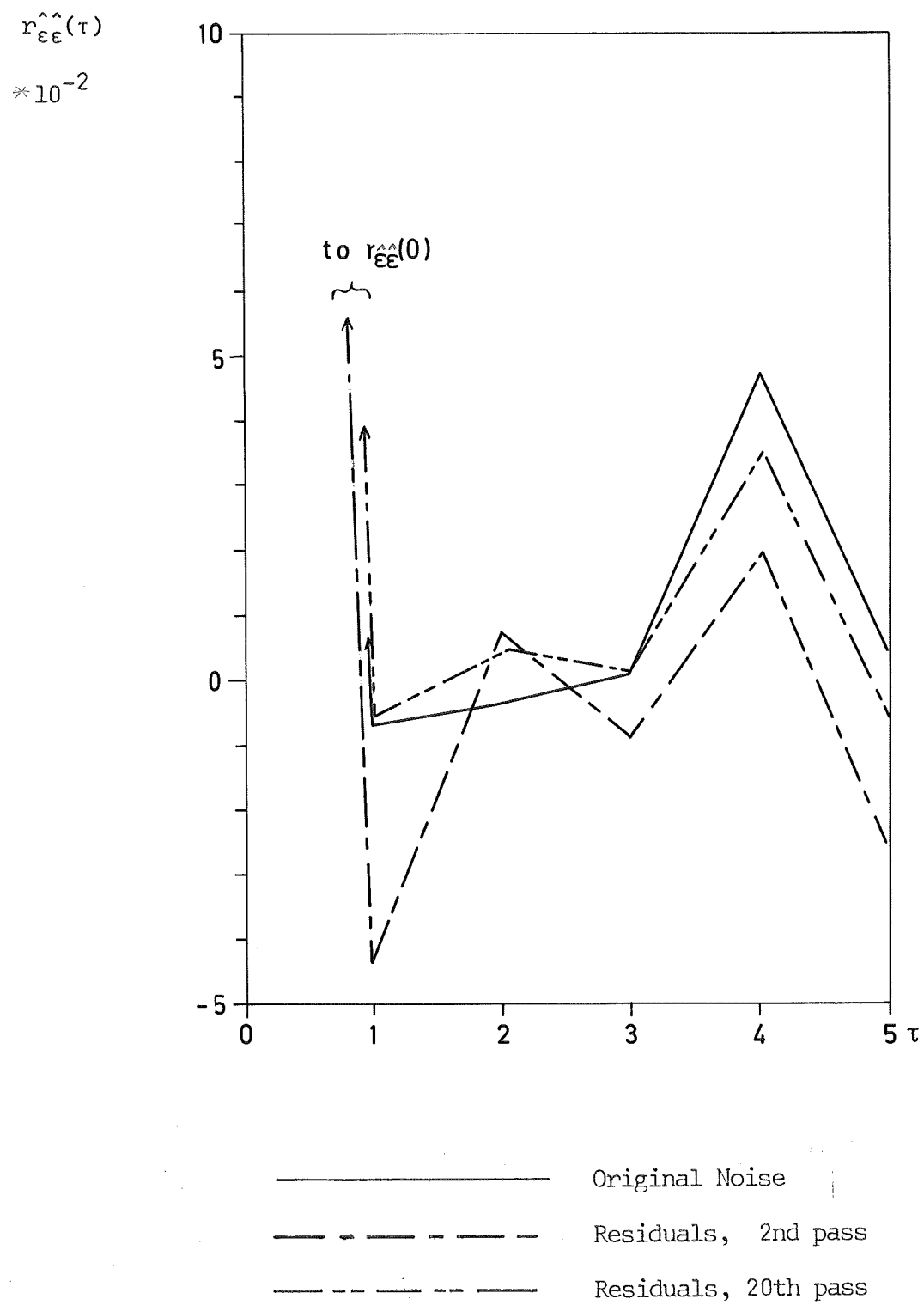


Fig.3 2nd Order Standard System Sample S_2
 Covariance of Original Noise and Residuals

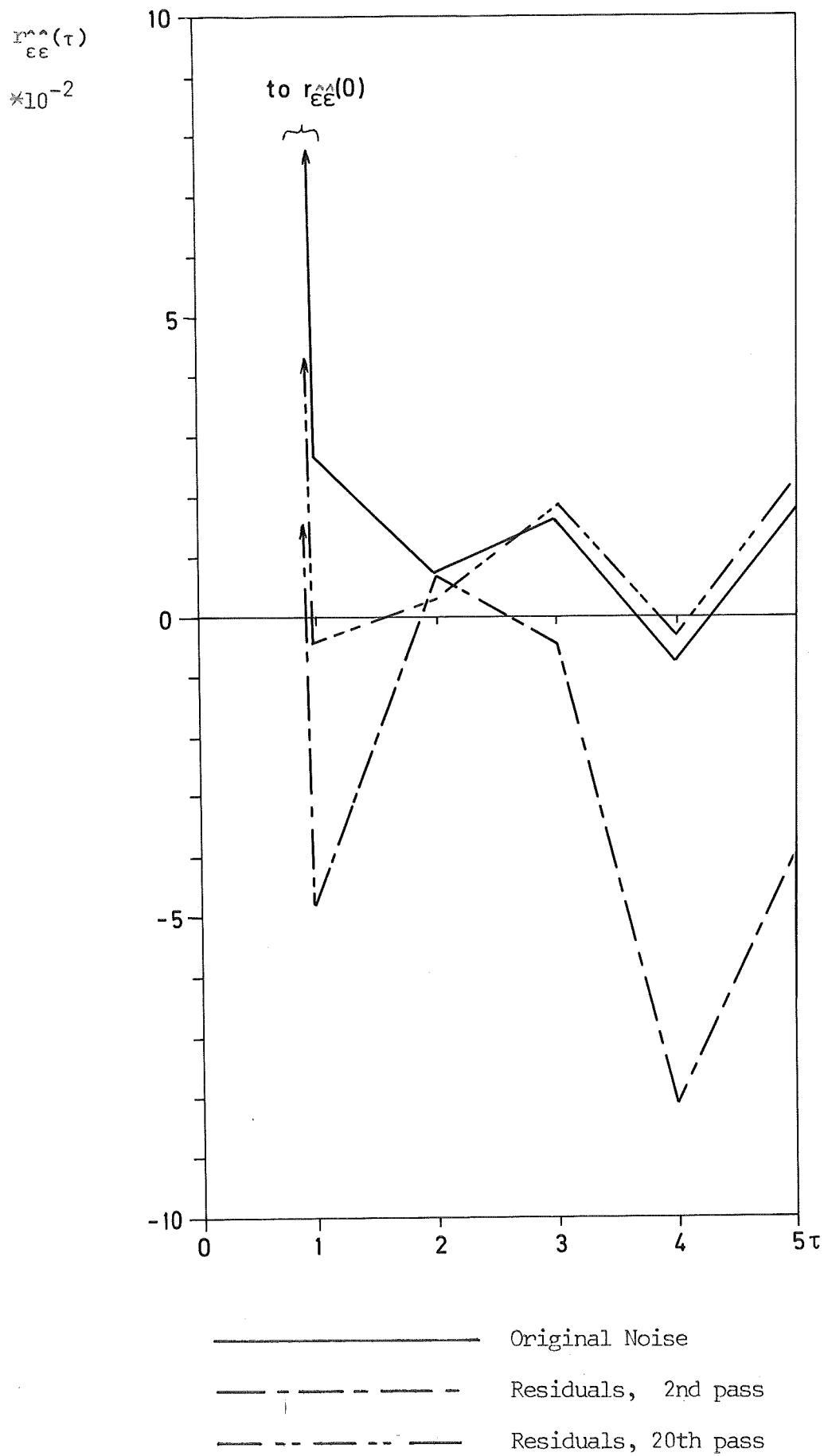


Fig. 4. 2nd Order Standard System, Sample S_1
 Covariance of Original Noise and Residuals

Tab. 1.		Input Signal Pseudorandom Gaussian, (0,1) generated by RANSS with starting value 1 Noise: Pseudorandom Gaussian, generated by RANSS with starting value 3 Length of record N = 500 Bounds: Coeff ± 5.0 Errors ±10.0																
Example mark	Vari- ant of algo- rithm	Gain seq.			Results - all scheduled values are sample means from 20 different realisations.													
		ORIGPAN	STABPAN	Number of passes	b ₁		b ₂		a ₁		a ₂		c ₁		c ₂			
					\hat{b}_1	±δ	\hat{b}_2	±δ	\hat{a}_1	±δ	\hat{a}_2	±δ	\hat{c}_1	±δ	\hat{c}_2	±δ		
a	X	X		2.5	1	0.737	.19	1.116	.30	-1.422	.08	0.643	.078	-0.543	.12	-0.034	.07	
b	X	X		2.5	5	0.668	.23	1.205	.27	-1.407	.08	0.626	.086	-0.533	.14	-0.156	.10	
c	X	X		5.0	1	0.631	.39	1.197	.32	-1.258	.30	0.653	.35	-0.408	.25	0.035	.18	
d	X	X		5.0	5	0.650	.49	1.175	.41	-1.252	.22	0.655	.40	-0.336	1.26	0.004	.20	
e	X			2.5	5	0.895	.05	1.069	.075	-1.487	.02	0.682	.018	-0.825	.045	0.062	.06	
f	X			5.0	5	0.873	.04	1.060	.074	-1.481	.024	0.667	.02	-0.785	.043	0.040	.06	
g		X		0.5	5	0.725	.37	1.305	.37	-1.736	1.7	0.258	1.35	-0.164	.49	0.119	.21	
h		X		1.0	5	0.735	.62	1.421	.98	-1.384	1.22	0.615	.29	-0.220	.67	0.119	.31	
i		X		2.5	5	7*10 ³	3*10 ⁴	1.005	1.8	-5.5*10 ³	2*10 ⁴	0.425	3.8	-0.619	.47	0.163	.30	
j		X		5.0	5	N	O N S E	N S E N S E										
k			X	2.5	5	1.036	.65	1.026	.22	-1.474	.05	0.674	.04	-0.797	.08	0.048	.05	
l			X	5.0	5	1.947	1.34	0.935	.43	-2.268	3.5	0.569	0.4	-0.785	.2	0.053	.16	
m	X		$G_t = \frac{a}{2000+K}$	5.0	5	0.860	.04	1.073	0.05	-1.479	.02	0.668	.006	-0.776	.012	0.035	.02	

Tab. 2

Input Signal: Pseudorandom Gaussian (0,1) generated by RANSS										Sample q on cards					
Noise: Pseudorandom Gaussian (0,1) generated by RANSS										Sample r on tape 18					
Length of record N = 1000															
Sample	Used method	b_1		b_2		a_1		a_2		c_1		c_2		Computing time in S	
		1.000	$\pm \delta$	0.500	$\pm \delta$	-1.500	$\pm \delta$	0.700	$\pm \delta$	-1.000	$\pm \delta$	0.200	$\pm \delta$		
		\hat{b}_1	\hat{b}_2	\hat{a}_1	\hat{a}_2	\hat{c}_1	\hat{c}_2	\hat{c}_1	\hat{c}_2	\hat{c}_1	\hat{c}_2				
q	ORIGPAN 1 pass	1.043		0.644		-1.296		0.591		-0.593		0.075		-	
	ORIGPAN 5 pass	1.064		0.498		-1.459		0.678		-0.948		0.226		11	
	M.L.E.	1.018	.032	0.476	.042	-1.508	.009	0.708	.008	-1.022	.031	0.233	.03	≈30	
r	ORIGPAN 1 pass	1.002		0.646		-1.494		0.673		-0.729		0.032		-	
	ORIGPAN 5 pass	0.999		0.534		-1.491		0.704		-0.945		0.219		17	
	M.L.E.	1.008	.03	0.518	.039	-1.503	.009	0.703	.008	-1.006	.031	0.231	.031	≈30	

Tab. 3

Original Panuška with bounds N = 1000 2nd order standard data sample q BE = 10.00 BC = 5.00 ALFA = 5.00					
PASS NO. 1 COEF 1.0431+000	6.4419-001	-1.2961+000	5.9090-001	-5.9337-001	7.5014-002
PASS NO. 2 COEF 1.0670+000	5.5026-001	-1.3950+000	6.4703-001	-8.0823-001	1.5418-001
PASS NO. 3 COEF 1.0691+000	5.1778-001	-1.4285+000	6.6109-001	-8.8998-001	1.8526-001
PASS NO. 4 COEF 1.0671+000	5.0428-001	-1.4468+000	6.7027-001	-9.2766-001	2.0888-001
PASS NO. 5 COEF 1.0639+000	4.9775-001	-1.4586+000	6.7771-001	-9.4832-001	2.2567-001
TRUE: $b_1 = 1.000$	$b_2 = 0.500$	$a_1 = -1.500$	$a_2 = 0.700$	$c_1 = -1.000$	$c_2 = 0.200$

Tab. 4

2nd Order standard data, sample q , $N = 1000$												
	b_1	b_2	a_1	a_2	c_1	c_2	$r_{\epsilon\epsilon}(0)$	$r_{\epsilon\epsilon}(1)$	$r_{\epsilon\epsilon}(2)$	$r_{\epsilon\epsilon}(3)$	$r_{\epsilon\epsilon}(4)$	$r_{\epsilon\epsilon}(5)$
Original system	1.000	0.500	-1.500	0.700	-1.000	0.200	1.01320	-0.02981	0.06935	-0.02303	-0.02293	0.04137
ORIGPAN 2nd pass	1.067	0.550	-1.395	0.647	-0.808	0.154	1.49710	-0.04949	0.02798	-0.10575	-0.05912	0.08046
ORIGPAN 10th pass	1.051	0.490	-1.483	0.697	-0.986	0.262	1.0828	-0.00769	0.00701	-0.10308	-0.08477	0.00853
ORIGPAN 20th pass	1.040	0.489	-1.494	0.706	-1.004	0.280	1.0552	-0.00388	0.00354	-0.10464	-0.08607	0.00433
ORIGPAN 50th pass	1.032	0.489	-1.501	0.707	-1.015	0.293	1.0394	-0.00127	0.00164	-0.10480	-0.08493	0.00523
M.L.E.	1.018	0.476	-1.508	0.708	-1.022	0.233	1.02392	-0.01003	0.06479	-0.03748	-0.03399	0.03542

Tab. 5

Second order standard system												
Input: Pseudorandom Gaussian (0,1) generated by RANSS with starting value 3 (both samples)												
Noise: Pseudorandom Gaussian (0,1) generated by RANSS. Sample S_1 with starting value 5												
Sample S_2 with starting value 1												
Length: N = 1000												
	b_1	b_2	a_1	a_2	c_1	c_2	$r_{\varepsilon\varepsilon}(0)$	$r_{\varepsilon\varepsilon}(1)$	$r_{\varepsilon\varepsilon}(2)$	$r_{\varepsilon\varepsilon}(3)$	$r_{\varepsilon\varepsilon}(4)$	$r_{\varepsilon\varepsilon}(5)$
Original sample S_1	1.000	0.500	-1.500	0.700	-1.000	0.200	0.94431	0.02674	0.00639	0.01609	-0.00793	0.01812
ORIGPAN 2nd pass	1.024	0.503	-1.500	0.681	-0.838	0.063	1.31524	-0.04784	0.00659	-0.00461	-0.08108	-0.03867
ORIGPAN 20th pass	0.982	0.459	-1.519	0.722	-0.945	0.161	1.00728	-0.00402	0.00313	0.01936	-0.00305	0.02352
Original sample S_2	1.000	0.500	-1.500	0.700	-1.000	0.200	0.98573	-0.00786	-0.00325	0.00065	0.03824	0.00485
ORIGPAN 2nd pass	0.912	0.552	-1.491	0.664	-0.781	0.032	1.37403	-0.04381	0.00759	-0.00827	0.01986	-0.02558
ORIGPAN 20th pass	1.024	0.446	-1.509	0.699	-0.999	0.196	1.01185	-0.00588	0.00492	0.00120	0.03534	-0.00489

Appendix A.

Programs, Subroutines and Functions used for Computation

A 1.	RANSS	Subroutine
A 2.	BOUND	Real function
A 3.	ORIGPAN	Preliminary versions used in early trials (for results in Tab. 1)
A 4.	STABPAN	
A 5.	ORIGPAN 2	
A 6.	RTPAN with subroutine	
	RETORPA	Real time version of ORIGPAN
A 7.	GACF	Subroutine

A 1. RANSS (KK, RR)

used for generation of noise, resp. input signal sequences.

RANSS - 1 of 2
29 Jan 65

Title Random Floating Point Numbers in Normal Distribution (RANSS)

CO-OP Class/Index Code GS WISC RANSS

CO-OP Organ. Code WISC (adapted from CODA RNDEV)

Program Language CODAP-1/COMPASS

Computer 1604/3600

Programmer RNDEV by Wm Silverman, adapted by Richard G. Wolfe for the University of Wisconsin Computing Center

Contributing Organ. University of Wisconsin Computing Center (Formerly Numerical Analysis Laboratory)

Date February 1963

UWCC ID Code C0010-00/S0010-00

1.0 PURPOSE

To generate random floating point numbers distributed according to the Normal (Gauss) distribution with a mean of 0 and a variance of 1. The range of the generated distribution is between plus and minus 8.

2.0 USAGE

An odd integer between 0 and 2⁴³ is assigned by the User prior to the first call to RANSS, and this number determines the sequence of deviates that will be generated. This odd integer is changed by the subroutine to prepare for the next call and generation.

2.1 Calling Sequence.

2.1.1 FORTRAN-63 Calling Sequence: CALL RANSS (KK,RR)

2.2 Arguments Or Parameters.

KK is the odd integer assigned as above.

RR will be the generated deviate.

2.4 Space Required.

1604

27₈ locations

3600

41₈ locations

RANSS - 2 of 2
29 Jan 65

2.12 Timing.

1604

approximately

1400 microseconds

3600

approximately

380 microseconds

3.0 METHOD

The number KK is multiplied by 5^{13} and reduced modulo 2^{43} . The process is repeated 16 times and the results summed. The numbers obtained after each multiplication and reduction have many of the characteristics of uniformly distributed random variables which take their values in the odd integers between 0 and 2^{43} . If we call the i th such variable X_i and put $S_n = \sum_{i=1}^n X_i$, then the following result is implied by the Central Limit Theorem:

$$\lim_{n \rightarrow \infty} P \left\{ \frac{S_n - nM}{\sigma\sqrt{n}} \leq X \right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-t^2/2) dt$$

where M is the mean of the uniform variable X , and $M = 2^{42}$, and σ^2 is the variance of X , and $\sigma^2 = \frac{1}{3} (2^{42}-1)^2$ and the right hand term is the distribution function of a normally distributed variable with mean 0 and variance 1. Asymptotic considerations show that for $n \geq 16$ the approximation is sufficiently accurate for single precision arithmetic. So $n = 16$ is used with $16M = 2^{46}$ and $\sigma\sqrt{16} = \frac{4(2^{42}-1)}{\sqrt{3}}$.

A 2. BOUND (X, B)

This real function is used to limit the value of estimates of coefficients and residuals to certain value B.

The value of this function is equal to

$$\begin{aligned} &X \text{ when } |x| \leq B \\ &B \cdot \text{sign } X \text{ when } |x| > B \end{aligned}$$

FTN5.4B

18/02-89

```
REAL FUNCTION BOUND(X,B)
IF(ABS(X).LE.B) GO TO 1
IF(X) 2,1,3
1 BOUND=X S RETURN
2 BOUND=-B S RETURN
3 BOUND=B S RETURN
END
```

A 3. ORIGPAN - used in early trials for results in Tab. 1

Requires: Subroutine	RANSS
Real function	BOUND

18/07-68

4B

```

PROGRAM ORIGPAN
  DIMENSION U(510),Y(510),E(510),A(10),B(10),C(10),EA(10),EB(10),
1 EC(10),PC(10),BAC(20,9),EBAC(9),VBAC(9)
  REAL BOUND
  BE=10. $ BC=5.
  READ 100,NN
  PRINT111,NN
111 FORMAT(*SIMULATION*/ *LENGTH *I5)
  READ 101,N
  READ 102,(B(I),I=1,N)
  READ 102,(A(I),I=1,N)
  READ 102,(C(I),I=1,N)
  READ 101,NE
  PRINT 107,(B(I),I=1,N)
  PRINT109,(A(I),I=1,N)
  PRINT110,(C(I),I=1,N)
100 FORMAT(I4)
101 FORMAT(I3)
102 FORMAT(10F4,1)
103 FORMAT(F4,1)
  ICOUNT = 1
  ALFA = 2.5
  PRINT 112,ALFA
112 FORMAT(*ALFA=*F6.2)
  IT1=1$IT2=3
  DO 38 IREA=1,20
  DO 21 I=1,NN
  CALL RANSS(IT1,U(I+10))
  CALL RANSS(IT2,E(I+10))
  21 CONTINUE
  DO 22 I=1,10
  22 U(I)=Y(I)=E(I)=0.
  DO 23 K1=1,NN
  K=K1+10$EE=E(K)
  DO 24 I=1,N
  J=K-I
  24 EE=EE+B(I)*U(J)+C(I)*E(J)-A(I)*Y(J)
  Y(K)=EE
  23 CONTINUE
  PRINT 113,IREA
113 FORMAT(*IDENTIFICATION NO.*I4)
  DO 26 I=1,NE
  26 EB(I)=EA(I)=EC(I)=0.
  DO 45 KK=1,ICOUNT
  E(11)=0.
  DO 25 K1=2,NN
  K=K1+10 $ EE=Y(K)
  G=ALFA/((KK-1)*NN+K1)
  DO 27 I=1,NE
  J=K-1$ EE=EE+EA(I)*Y(J)-EB(I)*U(J)-EC(I)*E(J)
  27 CONTINUE
  E(K)=BOUND(EE,BE)
  G=G+E(K)
  DO 28 I=1,NE
  J=K-I
  EC(I)=BOUND(EC(I)+G+E(J),BC)

```

```
      EB(I)=BOUND(EB(I)+G*U(J),BC)
28  EA(I)=BOUND(EA(I)-G*Y(J),BC)
25  CONTINUE
30  PRINT 104, KK
      PRINT 1000, ((EB(I), I=1, NE), (EA(I), I=1, NE), (EC(I), I=1, NE))
1000 FORMAT(*COEF*9E12,4)
45  CONTINUE
      DO 39 I=1, NE
        I1=I+NE $ I2=I+2*NE
        BAC(IREA, I)=EB(I)
        BAC(IREA, I1)=EA(I)
39  BAC(IREA, I2)=EC(I)
38  CONTINUE
      ND=3*NE
      DO 41 I=1, ND
        SUM=0.
        DO 40 J=1, 20
40  SUM=SUM+BAC(J, I)
41  EBAC(I)=SUM/20.
      PRINT 1001, (EBAC(I), I=1, ND)
1001 FORMAT(/*EXP=*9E12,4)
      DO 42 I=1, ND
        DO 42 J=1, 20
42  BAC(J, I)=BAC(J, I)-EBAC(I)
      DO 43 I=1, ND
        SUM=0.
        DO 44 J=1, 20
44  SUM=SUM + BAC(J, I)*BAC(J, I)
43  VBAC(I)=SUM/20.
      PRINT 1002, (VBAC(I), I=1, ND)
1002 FORMAT(*VAR=*9E12,4)
104  FORMAT(*PASS NO,*I3)
107  FORMAT(*B=*(E12,5))
109  FORMAT(*A=*(E12,5))
110  FORMAT(*C=*(E12,5))
      CALL EXIT
      END
```

A 4. STABPAN - used in early trials for results in Tab. 1.

Requires: Subroutine RANSS

23/06-68

```

PROGRAM STABPAN
DIMENSION U(510),Y(510),E(510),A(10),B(10),C(10),EA(10),EB(10),
1EC(10),PC(10),BAC(20,9),LBAC(9),VBAC(9)
BE=10. $ BC=5.
READ 100,NN
PRINT 111,NN
111 FORMAT(*SIMULATION*/ *LENGTH *15)
READ 101,N
READ 102,(B(I),I=1,N)
READ 102,(A(I),I=1,N)
READ 102,(C(I),I=1,N)
READ 101,NE
PRINT 107,(B(I),I=1,N)
PRINT 109,(A(I),I=1,N)
PRINT 110,(C(I),I=1,N)
100 FORMAT(I4)
101 FORMAT(I3)
102 FORMAT(10F4.1)
103 FORMAT(F4.1)
ICOUNT=5
ALFA==.
PRINT 112,ALFA
112 FORMAT(*ALFA=*F6.2)
IT1=1,IT2=3
DO 38 IREA=1,20
DO 21 I=1,NN
CALL SANSS(IT1,U(I+10))
CALL SANSS(IT2,E(I+10))
21 CONTINUE
DO 22 I=1,10
22 U(I)=Y(I)=E(I)=0.
DO 23 K1=1,NN
K=K1+10 SEE=E(K)
DO 24 I=1,N
J=K-I
24 EE=EE+B(I)*U(J)+C(I)*E(J)-A(I)*Y(J)
Y(K)=EE
23 CONTINUE
PRINT 113,IREA
113 FORMAT(*IDENTIFICATION NO.*14)
DO 26 I=1,NE
26 EB(I)=EA(I)=EC(I)=0.
DO 45 KK=1,ICOUNT
E(11)=0.
DO 25 K1=2,NN
K=K1+10 SEE=Y(K)
G=ALFA/((KK-1)*NN+K1)
DO 27 I=1,NE
J=K-I EE=EE+EA(I)*Y(J)-EB(I)*U(J)-EC(I)*E(J)
27 CONTINUE
E(K)=EE
G=G+E(K)
DO 46 I=1,NE
J=K-I
46 PC(I)=EC(I)+G+E(J)
IF(ABS(PC(2)).LT.1..AND.ABS(PC(1)).LT.1.+PC(2)) 47,25

```

28/06-66

```

47 DO 28 I=1,NE
   J=K-I
   EC(I)=PC(I)
   EB(I)=EB(I)+G*U(J)
28 EA(I)=EA(I)-G*Y(J)
25 CONTINUE
30 PRINT 104, KK.
   PRINT 1000, ((EB(I), I=1, NE), (EA(I), I=1, NE), (EC(I), I=1, NE))
1000 FORMAT(*COEF*9E12.4)
45 CONTINUE
   DO 39 I=1, NE
     I1=I+NE 5 I2=I+2*NE
     BAC(I,EA,I)=EB(I)
     BAC(I,EA,I1)=EA(I)
39 BAC(I,EA,I2)=EC(I)
38 CONTINUE
   ND=3*NE
   DO 41 I=1, ND
     SUM=0.
     DO 40 J=1, 20
40 SUM=SUM+BAC(J,I)
41 EBAC(I)=SUM/20.
   PRINT 1001, (EBAC(I), I=1, ND)
1001 FORMAT(/*EXP*9E12.4)
   DO 42 I=1, ND
     DO 42 J=1, 20
42 BAC(J,I)=BAC(J,I)-EBAC(I)
     DO 43 I=1, ND
       SUM=0.
       DO 44 J=1, 20
44 SUM=SUM + BAC(J,I)*BAC(J,I)
43 VBAC(I)=SUM/20.
   PRINT 1002, (VBAC(I), I=1, ND)
1002 FORMAT(*VAR*9E12.4)
104 FORMAT(*PASS NO.*I3)
107 FORMAT(*B=*(E12.5))
109 FORMAT(*A=*(E12.5))
110 FORMAT(*C=*(E12.5))
   CALL EXIT
   END

```

ARY DECK***
 ,1500, , ,

A 5. ORIGPAN 2

Complete program used for computation of Tab. 2 and Tab. 3.

Requires Subroutine GACF¹⁾
 Real function BOUND

¹⁾Remark: The use of subroutine GACF for computation of
 covariance of residuals after each pass through
 the data is not vital and may be omitted by skipping
 the respective statements at the end of program,
 i.e.

```
              CALL GACF(...  
              PRINT 888,(...  
888 FORMAT(...
```


18/02-89

5.4B

PROGRAM ORIGPAN 2
 NUMERICAL IDENTIFICATION OF LINEAR DISCRETE TIME SYSTEM FROM NORMAL
 OPERATING RECORDS USING PANUSKA'S METHOD OF STOCHASTIC APPROXIMATIONS
 (V. PANUSKA A STOCHASTIC APPROXIMATION METHOD OF IDENTIFICATION OF
 LINEAR SYSTEMS USING ADAPTIVE FILTERING, DEPT. OF ENG. SC. OXFORD
 UNIVERSITY, OXFORD, ENGLAND SUBMITTED TO 1988 JACC CONFERENCE)

MODIFIED AND PROGRAMMED BY JAROSLAV VALIS 12.6.1988 IN INSTITUT FOR
 REGLERINGSTEKNIK IH LUND

BE BOUND FOR ERRORS
 BC BOUND FOR COEFFICIENTS
 NN NUMBER OF DATA PAIRS (MAX 1000)
 U INPUT SEQUENCE
 Y OUTPUT SEQUENCE
 NE ASSUMED ORDER OF THE SYSTEM
 ICOUNT NUMBER OF PASSES THROUGH THE DATA

THIS PROGRAM REQUIRES THE REAL FUNCTION BOUND(X,B)

```

  DIMENSION U(1010),Y(1010),E(1010),EA(10),EB(10),EC(10)
  DIMENSION ACVF(21)
  REAL BOUND
  BE=10. $ BC=5.
  READ 100,NN
  PRINT 111,NN,BE,BC
111 FORMAT(*ORIGINAL PANUSKA WITH BOUNDS*/ *NN=*10,/* BE=*F5.2,
1* BC=*F5.2)
100 FORMAT(I4)
  ICOUNT = 20
  ALFA = 5.
  PRINT 112,ALFA
112 FORMAT(*ALFA=*F6.2)
  DO 22 I=1,10
  22 U(I)=Y(I)=E(I)=0.
  NNN=NN+10
  READ 999,(U(I),Y(I),I=11,NNN)
999 FORMAT(10F8.3)
  READ 102, NE
101 FORMAT(10F8.3)
102 FORMAT(I3)
  DO 26 I=1,NE
  26 EB(I)=EA(I)=EC(I)=0.
  DO 45 KK=1,ICOUNT
  DO 25 K1=1,NN
  K=K1+10 $ EE=Y(K)
  G=ALFA/((KK-1)*NN+K1)
  DO 27 I=1,NE
  J=K-1$ EE=EE+EA(I)*Y(J)-EB(I)*U(J)-EC(I)*E(J)
27 CONTINUE
  E(K)=BOUND(EE,BE)
  G=G*E(K)
  DO 28 I=1,NE
  J=K-I
  EC(I)=BOUND(EC(I)+G*E(J),BC)
  EB(I)=BOUND(EB(I)+G*U(J),BC)

```

10/02-04

TNS.48

```
28 EA(I)=BOUND(EA(I)-G*Y(J),BC)
25 CONTINUE
30 PRINT 104, KK
104 FORMAT(*PASS NO.*13)
PRINT 1000, ((EB(I), I=1, NE), (EA(I), I=1, NE), (EC(I), I=1, NE))
CALL GACF(E, 1010, 11, 1010, ACVF, 11, 21)
PRINT 888, (ACVF(I), I=1, 11)
888 FORMAT(/*AUTOCOVARANCE FUNCTION OF RESIDUALS*/10E12.4)
1000 FORMAT(*COEF*9E12.4)
45 CONTINUE
CALL EXIT
END
```

A 6. RTPAN with RETIORPA

Complete example of use and results (Tab. 5, sample S_1 , Fig. 4).

RTPAN is the program used for computation of results shown in Tab. 5, Fig. 3 and Fig. 4.

Requires:	Subroutine RETIORPA	}	All included in the listing of the program
	Real function BOUND		
	Subroutine GSIMDATA ¹⁾		
	Subroutine GACF		
	Subroutine CLEAN ²⁾		
	Subroutine RANSS		

Remark: ¹⁾ Subroutine GSIMDATA simulates run of a given linear stochastic system for given input sequence

²⁾ Subroutine CLEAN is used to set all elements of a desired vector array to zero.

RETIORPA is universal subroutine which performs one iteration of Panuška's algorithm on latest data pair and therefore can be used for real time identification. It also computes the covariance of residuals.

01/04-69

PROGRAM RTPAN

THIS IS AN EXAMPLE HOW TO USE SUBROUTINE RETIORPA FOR IDENTIFICATION
 OF A SIMULATED SYSTEM OF SECOND ORDER
 AUTHOR: JAROSLAV VALIS 07/03-69 LTH LUND SWEDEN

SUBROUTINE REQUIRED

RETIORPA

BOUND (REAL, FUNCT.)

RANSS (PSEUDORANDOM NORMAL NUMBERS (0,1) GENERATOR)

GSIMDATA (SYSTEM SIMULATOR)

GACF (COMPUTES AUTOCOVARIANCE)

CLEAN

DIMENSION U(1000),Y(1000),E(1000),A(2),B(2),C(2),REE(11),U1(2),
 Y1(2),E1(11)

A(1)=-1.5 \$ A(2)=.7 \$ B(1)=1. \$ B(2)=.5 \$ C(1)=-1. \$ C(2)=.2

N=2

NN=1000

IT1=3

IT2=5

GENERATE INPUT SEQUENCE

DO 1,K=1,NN

1 CALL RANSS(IT1,U(K))

SIMULATE RUN OF GIVEN SYSTEM

CALL GSIMDATA(N,A,B,C,1.,IT2,NN,U,Y,E)

COMPUTE COVARIANCE OF ORIGINAL NOISE

CALL GACF(E,NN,1,NN,REE,11,11)

PRINT 100,(REE(I),I=1, 1)

100 FORMAT(*00V. OF ORIG. NOISE*/12F9.5)

SET ZERO INITIAL CONDITIONS

CALL CLEAN(A,N)

CALL CLEAN(B,N)

CALL CLEAN(C,N)

CALL CLEAN(E1,11)

CALL CLEAN(U1,N)

CALL CLEAN(Y1,N)

CALL CLEAN(REE,11)

START IDENTIFICATION

ALFA=5

IPASS=20

REPEAT (IPASS) TIMES

DO 2 IC=1,IPASS

KK=(IC-1)*NN

DO 3 K=1,NN

G=ALFA/(KK+K)

3 CALL RETIORPA(U(K),Y(K),U1,Y1,N,A,B,C,5.,10.,G,E1,REE,11)

PRINT OUT THE RESULTS

PRINT 101,IC

101 FORMAT(*PASS NO. *I3)

```
PRINT 102,(A(I),I=1,N)
102 FORMAT(/AAA 1.0000*10F8.4)
PRINT 103,(B(I),I=1,N)
103 FORMAT(*BB *10F8.4)
PRINT 104,(C(I),I=1,N)
104 FORMAT(*CC 1.0000*10F8.4)
DO 4 I=1,11
  4 REE(I)=REE(I)/NN
PRINT 105,(REE(I),I=1,11)
105 FORMAT(*OOV,(F RES.,/12F9.5)
CALL CLEAN(REE,11)
  2 CALL CLEAN(E1,11)
CALL EXIT
END
```

01/04-09

SUBROUTINE RETIORPA(UK,YK,U,Y,NE,A,B,C,BC,BE,GAIN,E,REE,L)
 REAL TIME IDENTIFICATION OF LINEAR SYSTEM USING STOCHASTIC
 APPROXIMATIONS
 REFERENCE, PANUSKA, V., A STOCHASTIC APPROXIMATION METHOD FOR IDENTIFICATION
 OF LINEAR SYSTEMS USING ADAPTIVE FILTERING, JACC 1968
 AUTHOR, JAROSLAV VALIS, 07/03-69

UK - LATEST VALUE OF INPUT
 YK - LATEST VALUE OF OUTPUT
 U - VECTOR OF LENGTH (NE) CONTAINING LAST NE VALUES OF INPUT
 Y - VECTOR OF LENGTH (NE) CONTAINING LAST NE VALUES OF OUTPUT
 NE - ORDER OF THE ESTIMATED MODEL
 A - VECTOR OF LENGTH (NE) CONTAINING CURRENT ESTIMATES OF A-COEF.
 B - VECTOR OF LENGTH (NE) CONTAINING CURRENT ESTIMATES OF B-COEF.
 C - VECTOR OF LENGTH (NE) CONTAINING CURRENT ESTIMATES OF C-COEF.
 BC - BOUND VALUE FOR COEFFICIENTS A,B,C
 BE - BOUND VALUE FOR ERRORS E
 GAIN - GAIN CONSTANT
 E - VECTOR OF LENGTH (L) CONTAINING LAST L RESIDUALS
 REE - VECTOR OF LENGTH (L) CONTAINING CURRENT COVARIANCE OF RESIDUALS E
 L - LENGTH OF VECTORS E, REE

SUBROUTINE REQUIRED
 BOUND

DIMENSION A(NE),B(NE),C(NE),REE(L),E(L),U(NE),Y(NE)

COMPUTE THE LATEST RESIDUAL

EE=YK

DO 1 I=1,NE

1 EE = EE+A(I)*Y(I)-B(I)*U(I)-C(I)*E(I)

EE = BOUND(EE,BE)

UPDATE ESTIMATES

G = GAIN*EE

DO 2 I=1,NE

C(I)=BOUND(C(I)+G*E(I),BC)

B(I)=BOUND(B(I)+G*U(I),BC)

2 A(I)=BOUND(A(I)-G*Y(I),BC)

UPDATE I/O DATA

DO 3 I=2,NE

I1=NE-I+1

U(I1+1)=U(I1)

3 Y(I1+1)=Y(I1)

U(1)=UK

Y(1)=YK

DO 4 I=2,L

I1=L-I+1

4 E(I1+1)=E(I1)

E(1)=EE

UPDATE COVARIANCES

DO 5 I=1,L

5 REE(I)=REE(I)+EE*E(I)

RETURN

END

01/04-09

```

REAL FUNCTION BOUND(X,B)
IF(ABS(X).LE.B) GO TO 1
IF(X) 2,1,3
1 BOUND=X $ RETURN
2 BOUND=-B $ RETURN
3 BOUND=B $ RETURN
END

```

01/04-09

```

SUBROUTINE CLEAN(A,N)
DIMENSION A(N)
DO 1 I=1,N
1 A(I)=0.
RETURN
END

```

01/04-09

```

SUBROUTINE GSIMDATA(N,A,B,C,LAMBDA,IT2,NN,U,Y,E)
DIMENSION A(N),B(N),C(N),U(NN),Y(NN),E(NN)
REAL LAMBDA
DO 1 I=1,NN
CALL RANSS(IT2,EE)
E(I)=EE*LAMBDA
1 CONTINUE
DO 2 K=1,NN
EE=E(K)
DO 4 I=1,N
J=K-I
IF(J)2,2,4
4 EE=EE+B(I)*U(J)-A(I)*Y(J)+C(I)*E(J)
2 Y(K)=EE
C END OF SIMULATION
RETURN
END

```

01/04-09

```

SUBROUTINE GACF(X,NN,N1,N2,RXX,IT,N3)
DIMENSION X(NN),RXX(N3)
INTEGER TAU
NT=N2-N1+2
DO 1 TAU=1,IT
XX=0.
LB=N1+TAU-1
DO 2 K=LB,N2
J=K-TAU+1
2 XX=XX+X(K)*X(J)
1 RXX(TAU)=XX/(NT-TAU)
RETURN
END

```

RY DECK***
100, 1

ON STARTED AT 1507 -25

COV.OF ORIG.NOISE
 0.94431 0.02674 0.00630 0.01600 -0.00793 0.01812 -0.04917

PASS NO. 1

AA 1.0000 -1.3962 0.5021
 BB 1.0275 0.5256
 CC 1.0000 -0.6749 0.0731
 COV.OF RES.
 28.58597 -1.46674 -1.74870 3.04233 8.31074 -1.78609 3.15400

PASS NO. 2

AA 1.0000 -1.4997 0.6813
 BB 1.0236 0.5033
 CC 1.0000 -0.8384 0.0630
 COV.OF RES.
 1.31524 -0.04784 0.00659 -0.00461 -0.08108 -0.03867 -0.08520

PASS NO. 3

AA 1.0000 -1.5204 0.6907
 BB 1.0117 0.4852
 CC 1.0000 -0.8614 0.1000
 COV.OF RES.
 1.18973 -0.01580 0.01096 0.02072 -0.02930 0.00551 -0.05179

PASS NO. 4

AA 1.0000 -1.5248 0.7083
 BB 1.0040 0.4761
 CC 1.0000 -0.8806 0.1168
 COV.OF RES.
 1.12799 -0.01632 0.01337 0.02078 -0.02027 0.00999 -0.05020

PASS NO. 5

AA 1.0000 -1.5248 0.7129
 BB 0.9990 0.4700
 CC 1.0000 -0.8954 0.1258
 COV.OF RES.
 1.09370 -0.01561 0.00938 0.01924 -0.01701 0.01168 -0.04963

PASS NO. 6

AA 1.0000 -1.5237 0.7157
 BB 0.9956 0.4676
 CC 1.0000 -0.9061 0.1320
 COV.OF RES.
 1.07244 -0.01396 0.00774 0.01893 -0.01429 0.01367 -0.04797

PASS NO. 7

AA 1.0000 -1.5226 0.7177
 BB 0.9931 0.4654
 CC 1.0000 -0.9140 0.1369
 COV.OF RES.
 1.05804 -0.01228 0.00691 0.01906 -0.01198 0.01557 -0.04626

PASS NO. 8

AA 1.0000 -1.5217 0.7100
 BB 0.9911 0.4639
 CC 1.0000 -0.9201 0.1403
 COV.OF RES.
 1.04763 -0.01082 0.00534 0.01926 -0.01010 0.01720 -0.04470

PASS NO. 9

AA 1.0000 -1.5211 0.7100
 BB 0.9895 0.4628
 CC 1.0000 -0.9247 0.1441
 COV.OF RES.
 1.03977 -0.00962 0.00587 0.01943 -0.00862 0.01853 -0.04349

PASS NO. 10

AA 1.0000 -1.5206 0.7205
 BB 0.9882 0.4620
 CC 1.0000 -0.9284 0.1469
 COV.OF RES.
 1.03360 -0.00862 0.00547 0.01953 -0.00745 0.01959 -0.04247

PASS NO. 11

AA 1.0000 -1.5202 0.7210
 BB 0.9871 0.4614
 CC 1.0000 -0.9314 0.1423
 COV.OF RES.
 1.02863 -0.00780 0.00511 0.01960 -0.00653 0.02043 -0.04104

PASS NO. 12

AA 1.0000 -1.5199 0.7213
 BB 0.9861 0.4609
 CC 1.0000 -0.9339 0.1513
 COV.OF RES.
 1.02455 -0.00711 0.00479 0.01962 -0.00578 0.02111 -0.04097

PASS NO. 13

AA 1.0000 -1.5197 0.7215
 BB 0.9853 0.4605
 CC 1.0000 -0.9360 0.1531
 COV.OF RES.
 1.02114 -0.00652 0.00450 0.01962 -0.00518 0.02160 -0.04043

PASS NO. 14

AA 1.0000 -1.5195 0.7217
 BB 0.9846 0.4602
 CC 1.0000 -0.9378 0.1546
 COV.OF RES.
 1.01825 -0.00601 0.00424 0.01961 -0.00469 0.02210 -0.03996

PASS NO. 15

AA 1.0000 -1.5194 0.7218
 BB 0.9840 0.4600
 CC 1.0000 -0.9393 0.1560
 COV.OF RES.
 1.01577 -0.00557 0.00401 0.01958 -0.00428 0.02246 -0.03902

PASS NO. 16

AA	1.0000	-1.5193	0.7219			
BB		0.9835	0.4598			
CC	1.0000	-0.9406	0.1573			
COV. OF RES.						
	1.01362	-0.00518	0.00380	0.01954	-0.00395	0.02276 -0.03932

PASS NO. 17

AA	1.0000	-1.5192	0.7219			
BB		0.9831	0.4596			
CC	1.0000	-0.9418	0.1584			
COV. OF RES.						
	1.01173	-0.00484	0.00361	0.01950	-0.00366	0.02300 -0.03907

PASS NO. 18

AA	1.0000	-1.5191	0.7220			
BB		0.9826	0.4595			
CC	1.0000	-0.9428	0.1593			
COV. OF RES.						
	1.01098	-0.00454	0.00344	0.01945	-0.00343	0.02321 -0.03887

PASS NO. 19

AA	1.0000	-1.5191	0.7220			
BB		0.9823	0.4594			
CC	1.0000	-0.9437	0.1602			
COV. OF RES.						
	1.00860	-0.00427	0.00328	0.01941	-0.00322	0.02338 -0.03870

PASS NO. 20

AA	1.0000	-1.5190	0.7220			
BB		0.9820	0.4593			
CC	1.0000	-0.9445	0.1611			
COV. OF RES.						
	1.00728	-0.00402	0.00313	0.01936	-0.00305	0.02352 -0.03870