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ML Identification of the Workload - Heart Rate Dynamics in Man using PRBS

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Abstract

Maximum Likelihood identification of the work load - heart rate dynamics is performed using PRBS-input. Second order models with one short time constant and one long time constant are obtained for three subjects. The individual variations are considerable. Some difficulties arise, especially in the noise estimation, because of the long time constant.

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1. Introduction:

The objective of the present work is to find the dynamics, when the human organism adapts to muscular excercise.

The main task of physiological systems is to create a "milieu interior" as good as possible. In order to describe their function the system dynamics has to be understood. One such system is the circulation system, and an interesting variable is the heart rate, which under some circumstances also reflects the blood flow through the heart.

The component properties of the system are not known quantatively. A useful tool is therefore identification, i.e. mathematical models are derived from experimental input output data.

One well suited method is the Maximum Likelihood (ML) identification [1]. An interactive program written by Ivar Gustavsson is available at the PDP-15 of the Division of Automatic Control, LTH.

At the Departments of Aviation and Naval Medicin, Karolinska Institutet, Stockholm, several experiments have been conducted, where the heart rate is registered for different changes in work load. The system has been analyzed with different methods for step, ramp and time inputs [2, 3]. Now three experiments are performed with a PRBS-input. Thus the load varies stepwise between two levels (chosen with respect to the subject) at "Pseudo Random" instants.

One main purpose is to examine the usefulness of the PRBS-input when dealing with physiological systems, where one severe restriction often is that the time for an experiment is limited.

2. Experimental method

An electrically braked cycle ergometer is used for the work load. The variable loads are obtained by resistors and controlled by a PRBS-generator. The heart rate is calculated from the ECG by a heart rate meter giving a continuous signal[4]. The heart rate, the work level and the time are recorded in analog form on magnetic tape. Before the registration begins, the subject is warming up, first with a rather high load and then on the lower level from which the experiment start. After the experiment the signals are filtered to avoid aliasing and then sampled with 10 sec. interval. The data is punched on paper tape in a suitable code for transfer to the PDP-15 in Lund.

3. Model and Data

The ML-identification as well as most other identification methods produces a linear model with time constant coefficients representing the dynamics of changes around a steady state value.

Therefore it has to be assumed that the system is linear within the load range. This is in reasonable agreement with steady state experiments for different work load[2]. Differences in time constants when increasing or decreasing the load cannot be clearly distinguished in the heart rate data.

The time constancy is probably more critical. It is one of the factors that limit the length of the experiment. In the experiment PRBSO2 (fig. 2) the heart rate increased more than expected after 30 min. The subject got tired.

In order to represent the data as changes around a steady state value both the heart rate and the work load are subtracted by the first measurement of the heart rate, y(0), and the work load, u(0), respectively. Thus it is assumed that the subject initially were in steady state with the heart rate y(0) for the work load u(0) (= the lower level of the PRBS). This assumption is not contradicted by the data.

A <u>visual</u> inspection of the data (figs 2-4) shows that the response might consist of one fast and one slow term. The fast mode seems to have a time constant of a fraction of a minute and a gain round 0.06 beats/kpm. The slow term induces an extra rise of 10 - 15 beats/min after 40 minutes for a mean load of 350 kpm/min. The time constant could from this inspection be anything between 5 minutes and infinity.

Three series are recorded, called PRBS02, PRBS03 and PRBS04. The sampling interval is 10 s. The minimum pulse length of the PRBS is 80 s and an experiment consists of two periods of length 15*80 s = 40 min. The sort of the input is kpm/min and of the output beats/min.

4. ML-Identification

Result: Identification with the ML-program is performed for all three series to the model

$$A\ (q^{-1})y(t) = B\ (q^{-1})u(t) + \lambda C\ (q^{-1})e(t) \qquad (1)$$

where q is the forward shift operator

$$q\ y(t) = y(t+h)$$

A, B and C are polynomials of order n, and e is a sequence of independent N(0,1) variables.

For a second order system (n=2) (1) thus means

$$y(t) + a_1y(t-1) + a_2y(t-2) = b_1u(t-1) + b_2u(t-2) + \lambda\{e(t) + c_1e(t-1) +$$

$$+ c_2e(t-2)\}$$

The result of the ML-identification is shown in Table 1 - 3.

Table 1: ML-Identification of PRBS02

	n=1	n=2	n=3
a_1	-0.853	-1.546	-1.827
a_2		0.563	0.993
a_3			-0.156
b_1	0.01134	0.01663	0.01644
b_2		-0.01529	-0.01956
b_3			0.00405
c_1	0.799	0.038	-0.187
c_2		-0.481	-0.492
c_3			0.003
λ	1.732	1.491	1.479
V	360.1	266.7	262.6
F	$F_{6,3} = 25$	$F_{9,6} = 1$	
	u(0) = 299	y(0) = 107.12	$u_{ampl} = 650$

Table 2: ML-Identification of PRBS03

	n=1	n=2	n=3
a_1	-0.807	-1.242	-0.733
a_2		0.303	-0.948
a_3			0.686
b_1	0.01345	0.02147	0.02556
b_2		-0.01688	-0.02082
b_3			-0.00427
c_1	0.618	-0.017	0.376
c_2		-0.333	-0.924
c_3			-0.435
λ	2.015	1.807	1.550
v	487.5	392.0	288.2
F	$F_{6,3}=20$	$F_{9,6}=29$	
	u(0) = 499	y(0)=91.64	$u_{ampl}= 700$

Table 3: ML-Identification of PRBS04

	n=1	n=2	n=3 (no conv)
a_1	-0.810	-1.735	-1.211
a_2		0.736	-1.503
a_3			0.365
b_1	0.01399	0.01561	0.01533
b_2		-0.01548	-0.00675
b_3			-0.00822
c_1	0.481	-0.624	0.008
c_2		-0.364	-0.561
c_3			-0.350
λ	1.526	1.342	1.35
V	279.4	216.0	218.6
F	$F_{6,3}=23$	$F_{9,6}=$ -	
	$u(0) = 499$	$y(0)=105.45$	$u_{ampl}=700$

When using the second order models the corresponding time constants of the contiuous system can be calculated.

Table 4:

PRBS 02	19 s	250 s
03	9 s	100 s
04	30 s	2000 s

Time constants of second order models.

The accuracy of especially the long time constant is poor, which is natural from the short experiment time (2 400 seconds).

The model output, that is the deterministic part of the output generated by the input, is shown in fig. 5 - 7 for the three series. A general comment is that the peaks are underestimated. Too much of the output is assumed to be generated by the noise. One reason for this is that the long time constant enters the noise-filter, there giving a rather large gain.

Test of model order:

When the system can be described by the model (1), the coefficients have nice statistical properties asymptotically. It is therefore possible to perform statistical tests on the number of parameters needed or on the model order [1,5].

As long as the test quantities, $F_{3n+3,3n}$, are greater than 2.5, it can be concluded at the risk level of 5 % that the correct model order is greater than or equal to n.

The F-test thus indicates second order models for PRBS02 and PRBS04 but third order model for PRBS03. The parameters of the third order model however have large estimated variance, and there is no continuous correspondence of the same order to that model because of poles on the negative real exis.

Test on the residuals:

The residuals, $\varepsilon(t)$, calculated from

$$C\ (q^{-1})\ (t) = A\ (q^{-1})y(t) - B\ (q^{-1})u(t) \qquad (2)$$

should be white if the system could be described by the model (1) of the same order as in (2). The auto correlation function of the residuals is therefore important for the decision of model order. In fig. 8 - 10 are shown the auto correlation functions normalized with the variance for the three series.

The one σ limit for lags > 0 is also shown. The residulas are almost uncorrelated for the second order models but correlated for the first order models as shown for PRBS04 in fig 11. The peak usually appearing at 8 lags emmanates from the minimum pulse length of the PRBS.

Both the F-tests and the residual tests thus make second order models reasonable.

5. Discussion

The ML-identification of the PRBS experiments verifies the results obtained by other methods [2,3]. The PRBS is an easily generated signal giving much information of systems also with rather high noise intensity. The ML-method has here some difficulties, especially with the noise estimation, because of the long time constant, long compared with the sampling interval and the experiment length. The convergence was slow for some cases, and convergence to local minima or solutions with indefinite second derivative has occurred. Different starting points sometimes gave somewhat different solutions.

Steady state:

Calculations are performed to find out from the data whether the system was in steady state at the start of an experiment or not. The starting values of the residual difference equation (2), normally assumed to be zero, were let free and estimated by the ML-method. No significant lossreduction was obtained. No initial deviation from steady state was thus detected in the data.

Bias:

Because of the output noise it might well happen that the steady state output corresponding to u(0) is somewhat different from y(0). This would introduce bias in the output signal used in the identification, and especially the estimation of the noise filter C/A might be influenced. Calculations on the data, however, showed no signigicant loss reduction for changes in the output bias.

Computer Program:

The ML-identifications are all performed on the PDP-15-computer with the interactive program written by I Gustavsson. The use of this program is extremely easy and comfortable.

Although the computer has no floating point arithetic hardware the computing time is reasonable, typically 80 sec for a second order model. All results are printed on line printer but also shown on a

Tectronix graphic display at the user's console. Changing model order, fixing parameters and including initial values is done by simple commands. The autocovariance function of the residuals can be calculated and shown on the display as well as input, output, model output, model error and residuals. All the diagrams in this report are hard copies of the display output.

The work was very much facilitated by this program.

7. References

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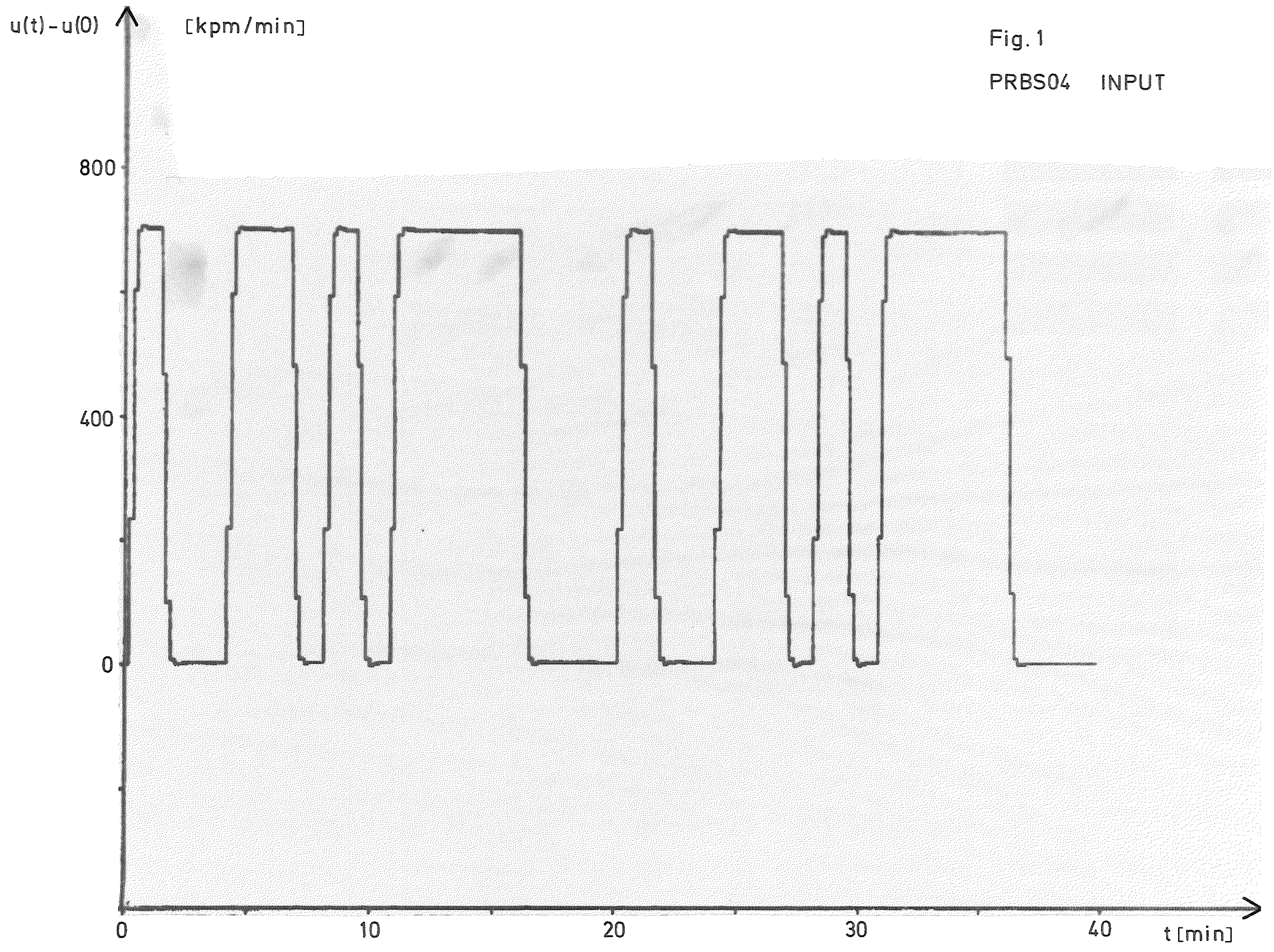
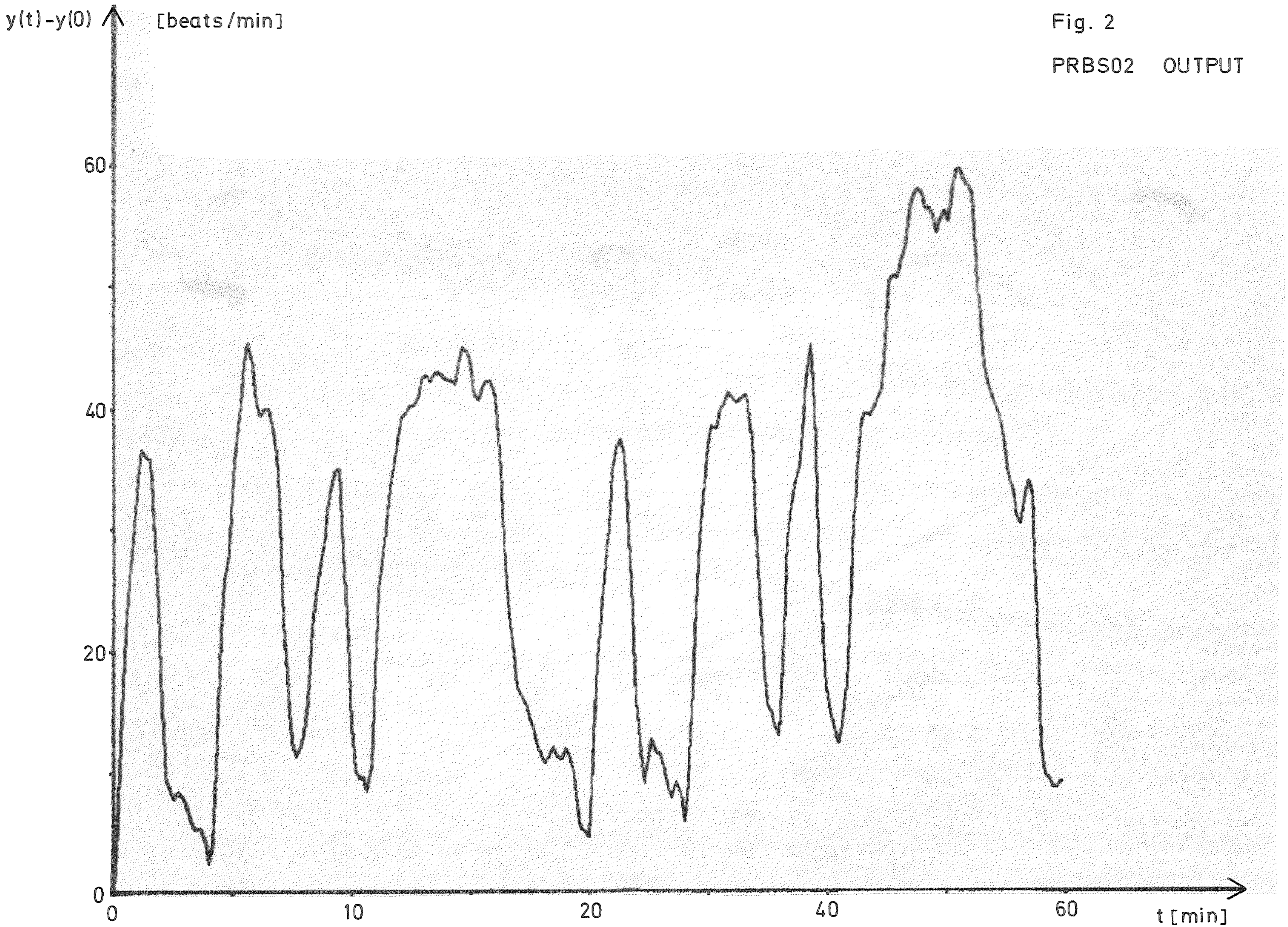


Fig. 1

PRBS04 INPUT

Fig. 2

PRBS02 OUTPUT



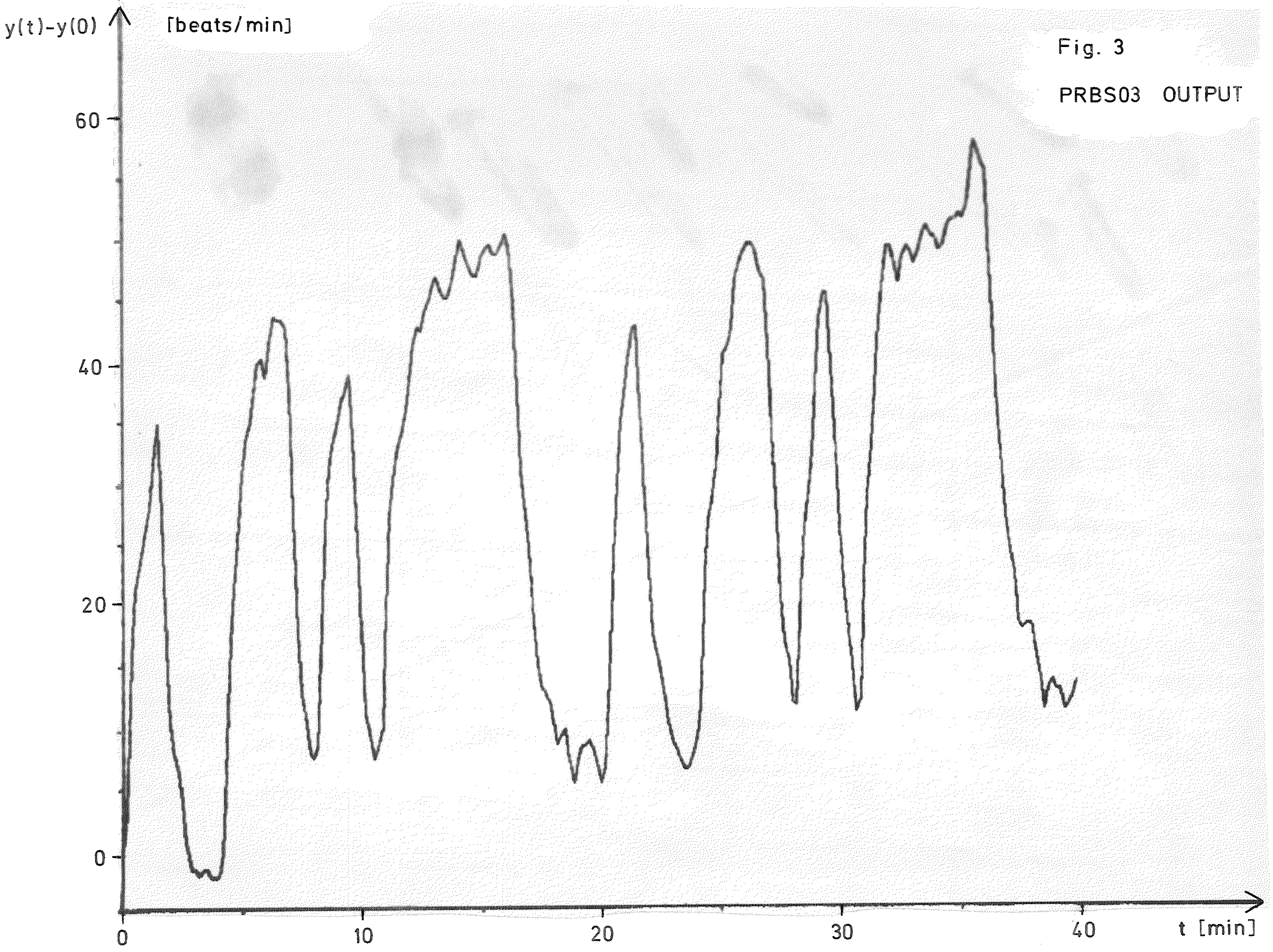
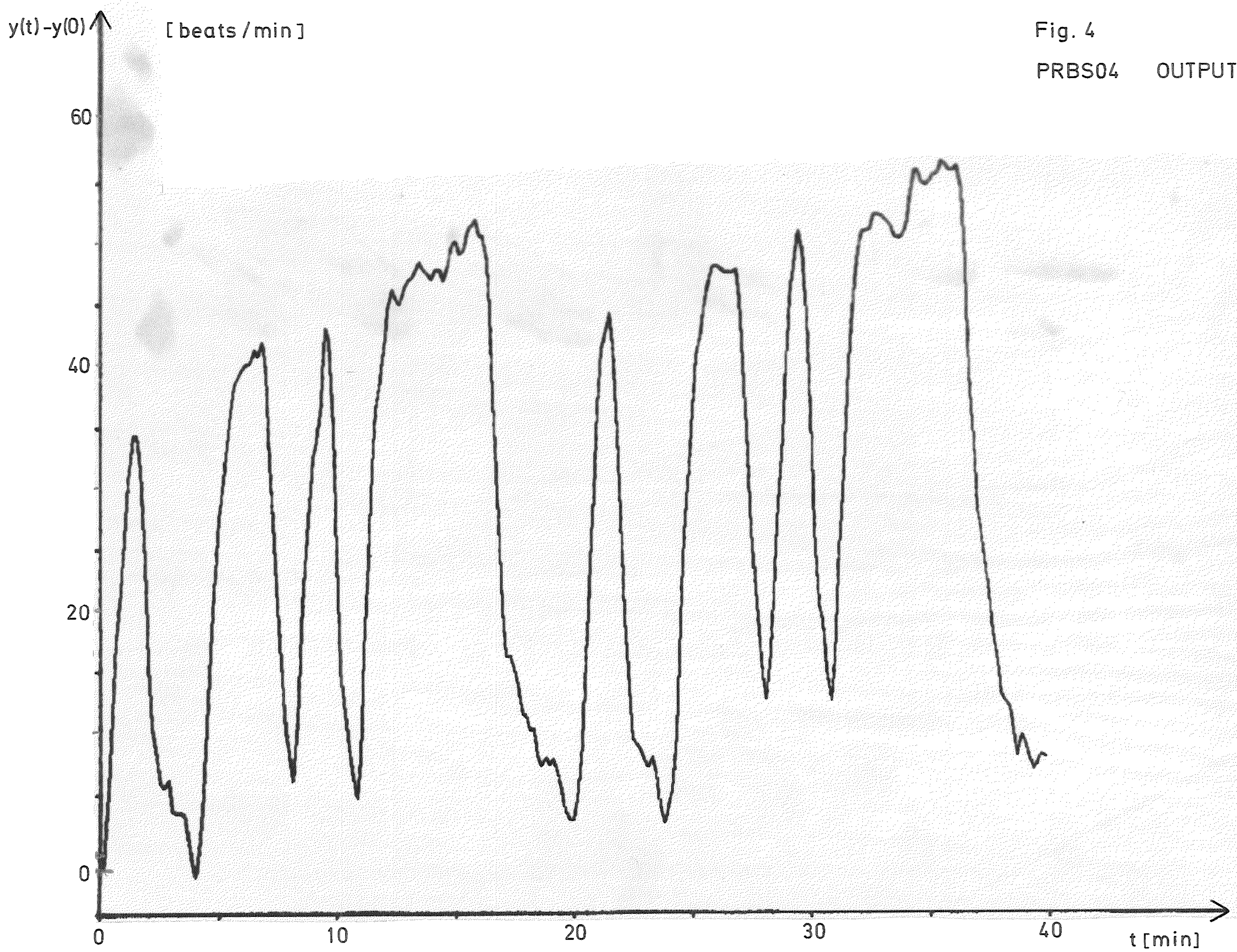


Fig. 3

PRBS03 OUTPUT

Fig. 4

PRBS04 OUTPUT



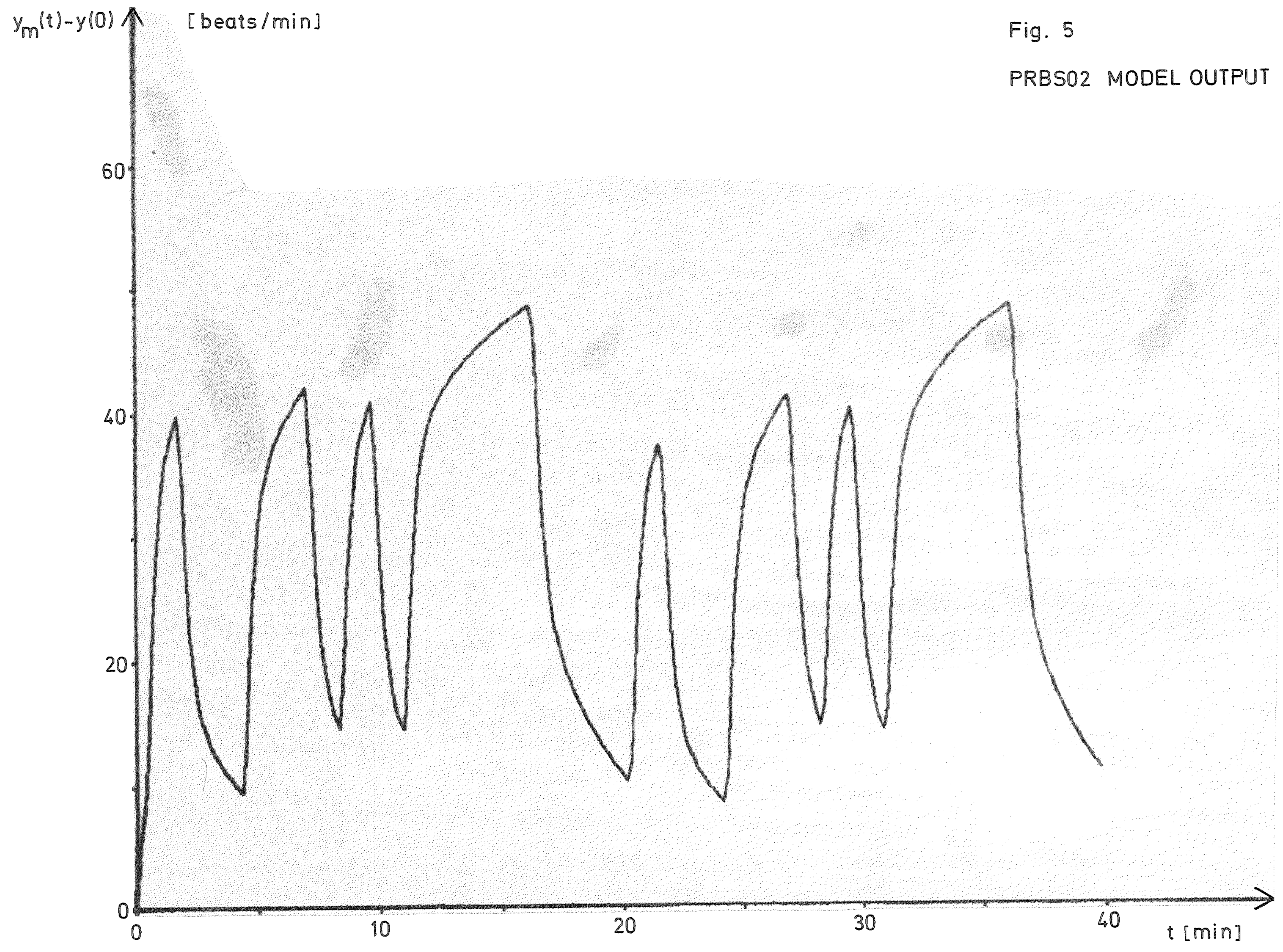


Fig. 5

PRBS02 MODEL OUTPUT

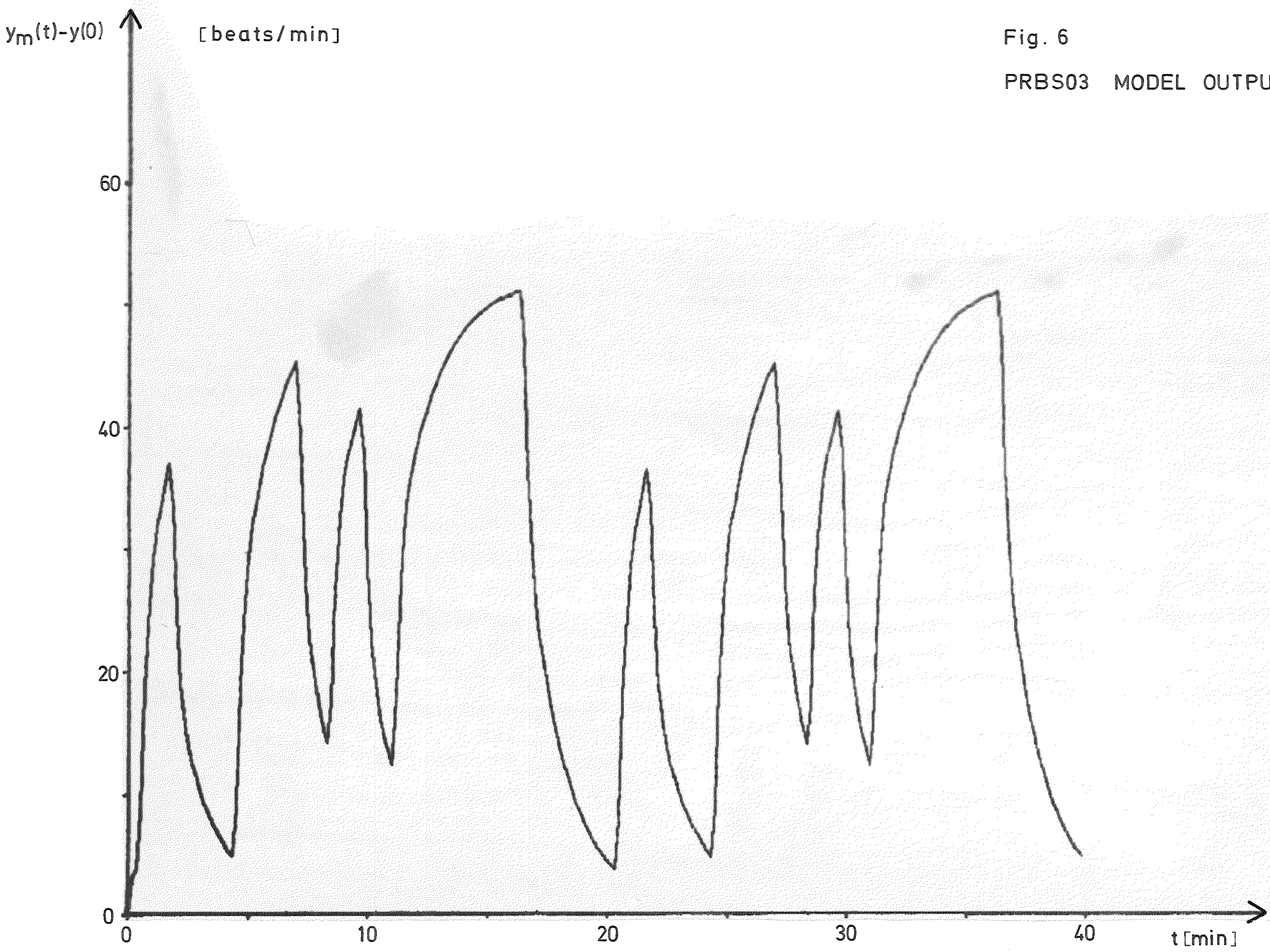


Fig. 6

PRBS03 MODEL OUTPUT

Fig. 7

PRBS04 MODEL OUTPUT

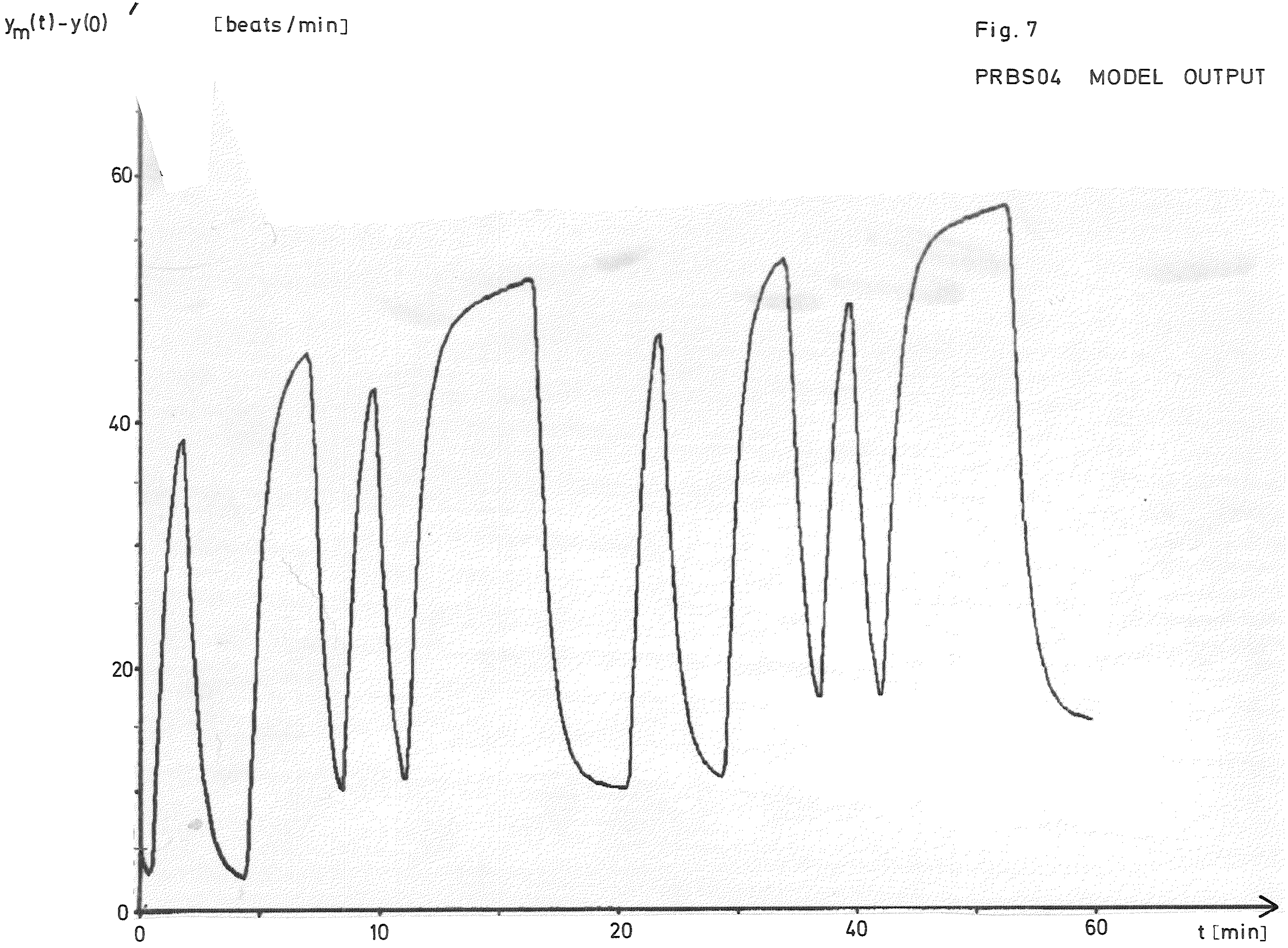


Fig. 8

PRBS02 RESIDUAL AUTOCOVARIANCE

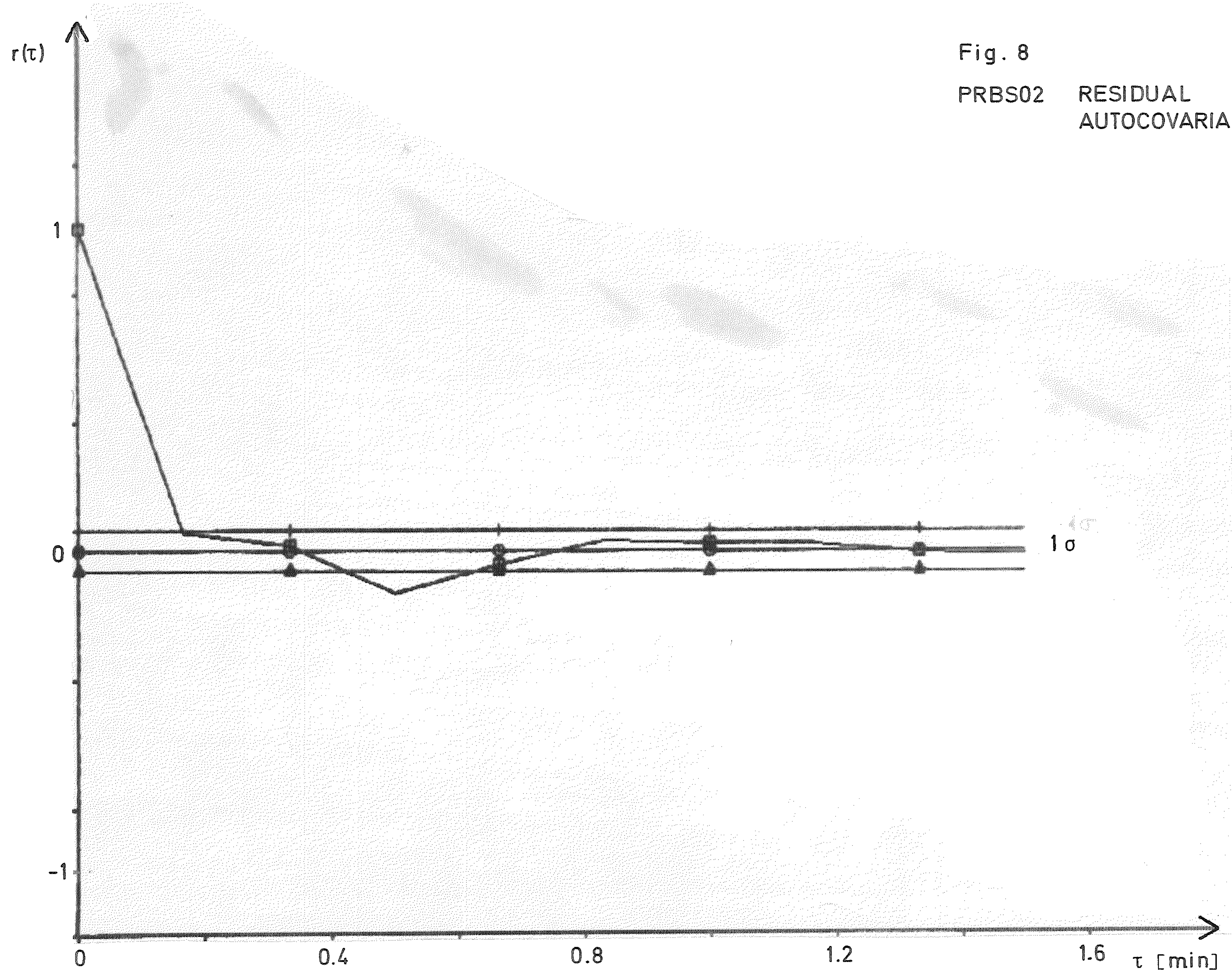


Fig. 9

PRBS03 RESIDUAL AUTOCOVARIANCE

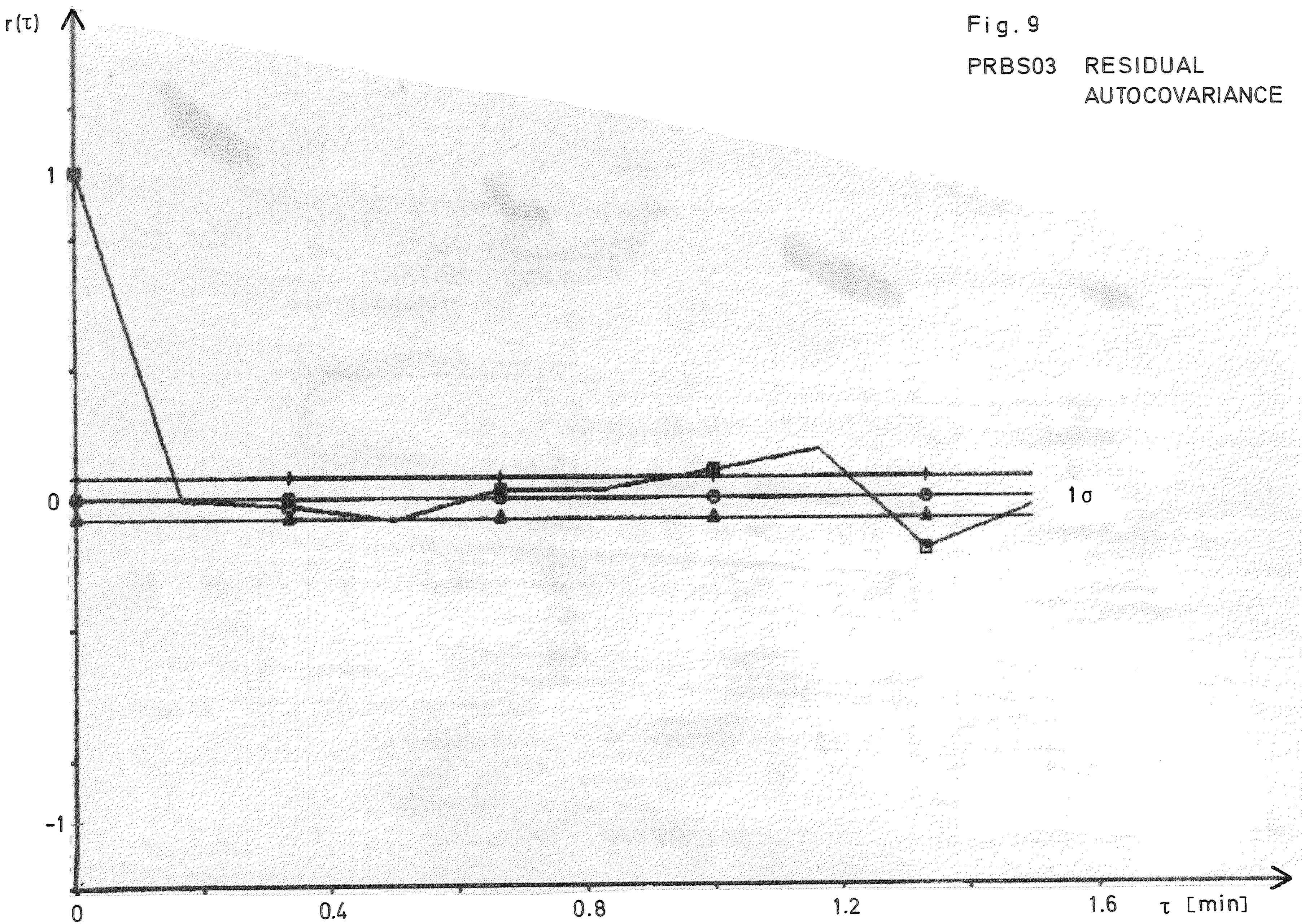


Fig. 10

PRBS04 RESIDUAL AUTOCOVARIANCE

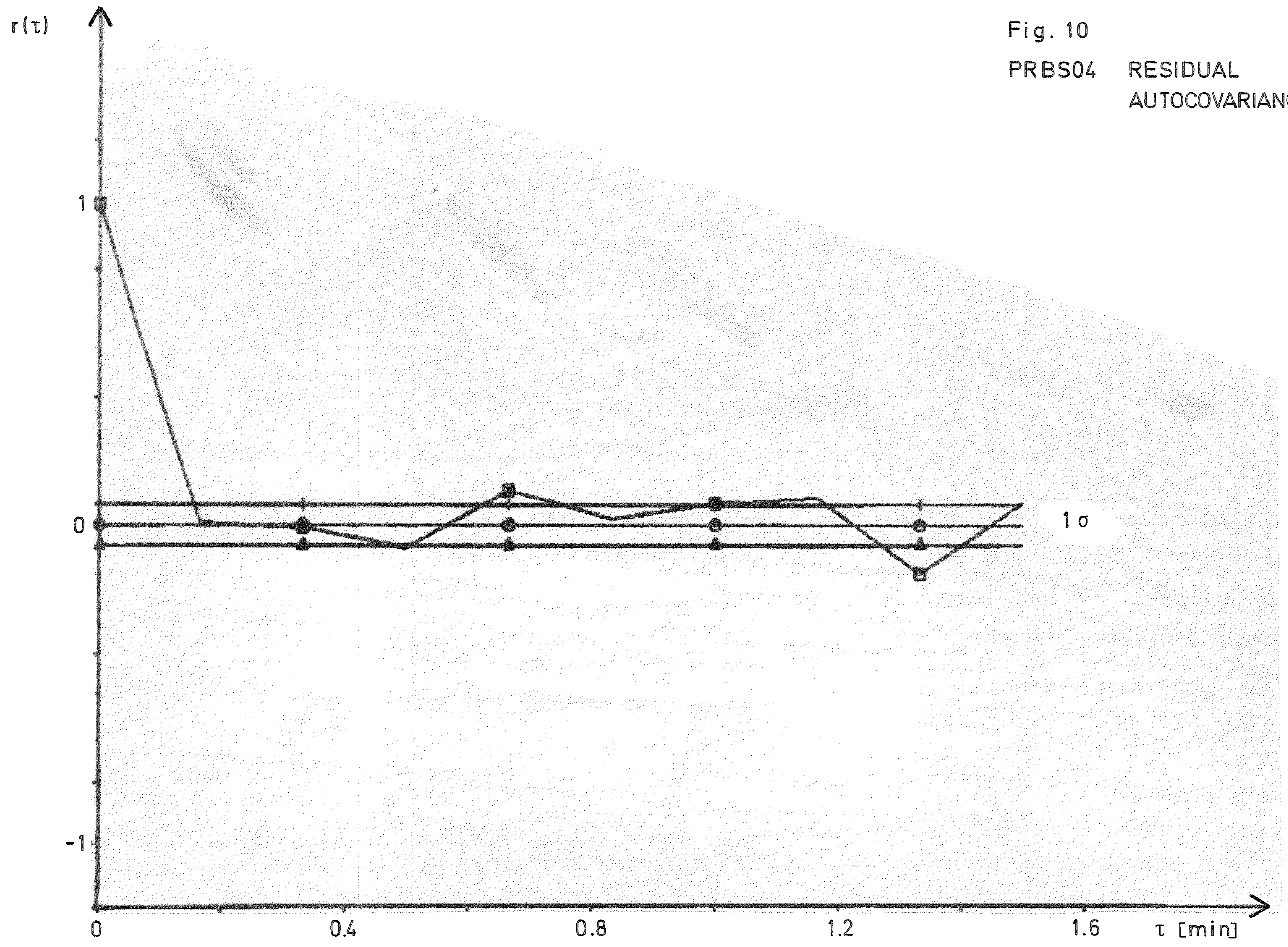


Fig. 11

PRBS04 RESIDUAL AUTOCOVARIANCE

1:st order model