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THE ADAPTIVE NONLINEAR MODELER

K. J. Åström

Abstract

This paper describes a new systems component which is capable of generating a static nonlinearity adaptively. The device which may be viewed as a building block in the control engineers toolbox can be used for many different purposes e.g. to provide automatic calibration of nonlinear sensors, feedforward compensation, compensation for nonlinear valve characteristics, automatic gain scheduling etc.

Contents

1. Introduction

Much of the work on adaptive control has focused on linear system. In this paper we discuss a simple adaptive problem namely automatic modeling of static nonlinearities. There is a wide class of problems where the essential nonlinearity can be captured by a nonlinear static model. Generically this is the case when the nonlinearity appears either at the output or the input of the system. A nonlinear valve or a nonlinear sensor are typical examples. Nonlinearities of this type are often used in the design of practical control systems. In this paper we propose a device which can model static nonlinearities adaptively. Such a device can be used in many different ways some of which are outlined in the paper. This includes automatic linearization of sensors and actuators, compensation for static process nonlinearities and nonlinear feedforward compensation. It is also worth noticing that tables of process characteristics is an important part of expert control systems. The adaptive nonlinear modeler is one possibility to describe such functions with algorithms instead of rules.

The paper is organized as follows. Different ways to characterize a static nonlinearity are described in Section 2. Adaptive techniques to generate nonlinearities automatically are described in Section 3. It is shown that the ideas can be encapsulated in a system component called the Adaptive Nonlinear Modeler or ANM for short. Different ways to use this system component are discussed in Section 4. The ideas are summarized in the conclusions where various sophistications are also presented.

2. Static Nonlinearities

A static nonlinearity is simply a system whose input-output relation can be described as a nonlinear function

$$y = f(x) \tag{2.1}$$

where x is the input and y is the output. In this paper it will for simplicity be assumed that the domain and the range of f are subsets of the real numbers. There are many different ways to characterize general nonlinear functions. One possibility which is convenient for our purposes is to combine a table with an interpolation function. It is assumed that the value of the function is specified for discrete arguments $x_1, x_2, \ldots, x_n$ and that the values at intermediate points are specified by an interpolation formula. For simplicity we consider linear interpolation. The function is thus defined as

$$f(x) = \frac{x_{k+1} - x}{x_{k+1} - x_k} f\left(x_k\right) + \frac{x - x_k}{x_{k+1} - x_k} f\left(x_{k+1}\right) \tag{2.2}$$

Together with the table $\{x_k, f(x_k)\}$ the function f is then uniquely specified in the interval $[x_1, x_n]$.

3. Adaptation

An adaptive function generation problem will now be formulated. Assume that the points $x_1, x_2, \ldots, x_n$ are given and that there is a device which generates function values possibly with some errors. The problem is to find a mechanism which will generate the table $\{x_k, f_k\}$. When $x_k \le x \le x_{k+1}$ the following updating rule can be used

$$\begin{cases} \dfrac{df_i}{dt} = 0 \qquad i \neq k,\ k+1 \\[2ex] \dfrac{df_k}{dt} = \dfrac{1}{T} \dfrac{x_{k+1} - x}{x_{k+1} - x_k} \left[f(x) - y(x)\right] \\[2ex] \dfrac{df_{k+1}}{dt} = \dfrac{1}{T} \dfrac{x - x_k}{x_{k+1} - x_k} \left[f(x) - y(x)\right] \end{cases} \tag{3.1}$$

where

$$y(x) = \frac{x_{k+1} - x}{x_{k+1} - x_k} f_k + \frac{x - x_k}{x_{k+1} - x_k} f_{k+1} \tag{3.2}$$

To explain that the system has the desired property assume first that x remains at x_k for a long time. Then f_k converges to $f(x_k)$, because if $x = x_k$ we get

$$\frac{df_k}{dt} = \frac{1}{T} \left[f(x_k) - f_k\right] \tag{3.3}$$

This differential equation is asymptotically stable with the solution

$$f_k = f(x_k)$$

The situation is a little more complicated when x is not a grid point. Introduce

$$e = f(x) - y(x) = f(x) - \alpha f_k - \beta f_{k+1} \tag{3.4}$$

where

$$\begin{cases} \alpha = \dfrac{x_{k+1} - x}{x_{k+1} - x_k} \\ \beta = 1 - \alpha = \dfrac{x - x_k}{x_{k+1} - x_k} \end{cases} \tag{3.5}$$

Then

$$\frac{d}{dt} 2e^2 = 2e \frac{de}{dt} = 2e \left[-\alpha \frac{df_k}{dt} - \beta \frac{df_{k+1}}{dt} \right]$$

$$= -2 \frac{\alpha^2 + \beta^2}{T} e^2$$

The error e will thus go to zero. Notice, however, that the values f_k and f_{k+1} will not converge to their correct values because the matrix

$$-\frac{1}{T} \begin{bmatrix} \alpha & \beta \\ \alpha & \beta \end{bmatrix}$$

has a zero eigenvalue. Also notice that there are other weightings between the derivatives of f_k and f_{k+1} which will give the desired result. The weighting in (3.1) is, however, quite reasonable.

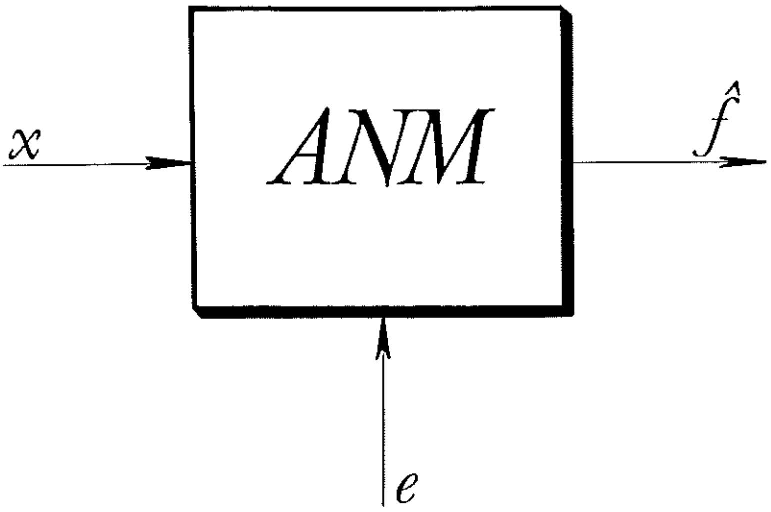

Figure 1. The adaptive nonlinear modeller.

A System Component

It is thus straightforward to obtain the desired adjustment mechanism. A system component which is a useful building block will now be described. Such a component is shown in the blockdiagram of Figure 1. The block has x as its primary input and f as its primary output. The states of the system are the table entries $f_1, f_2, \dots, f_n$ which are updated as

$$\begin{cases} \dfrac{df_i}{dt} = 0 & i \neq k,\ k+1 \\[2ex] \dfrac{df_k}{dt} = \dfrac{1}{T} \dfrac{x_{k+1} - x}{x_{k+1} - x_k}\, e & \\[2ex] \dfrac{df_{k+1}}{dt} = \dfrac{1}{T} \dfrac{x - x_k}{x_{k+1} - x_k}\, e & \end{cases} \tag{3.6}$$

where e is an auxiliary input. The output of the system is defined as

$$y = \frac{x_{k+1} - x}{x_{k+1} - x_k} f_k + \frac{x - x_k}{x_{k+1} - x_k} f_{k+1} \tag{3.7}$$

An example illustrates how the adaptive nonlinear modeler can be used

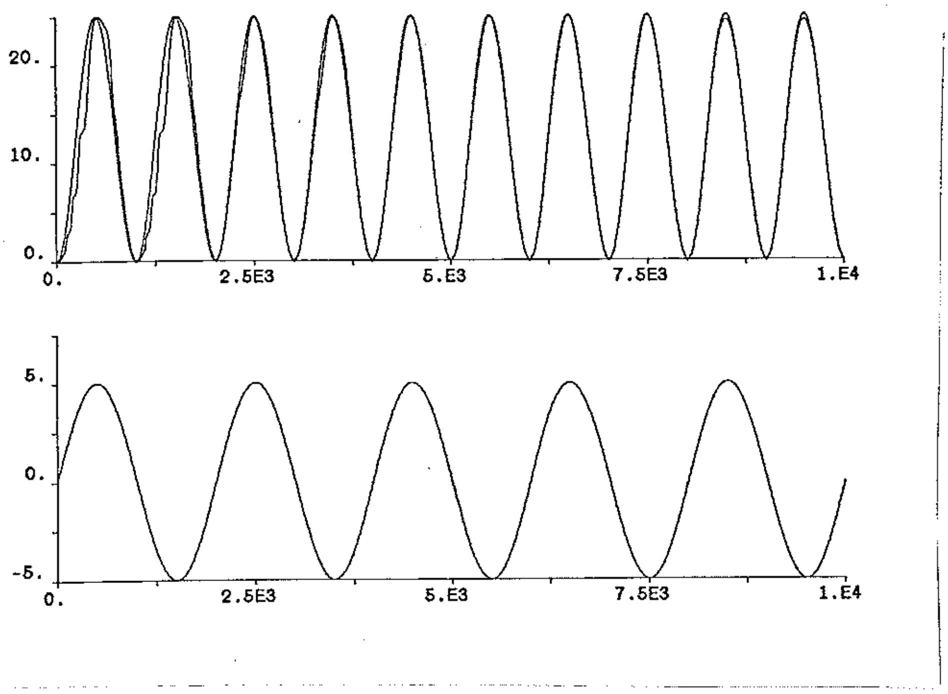

Figure 2. Simulation of the ANM.

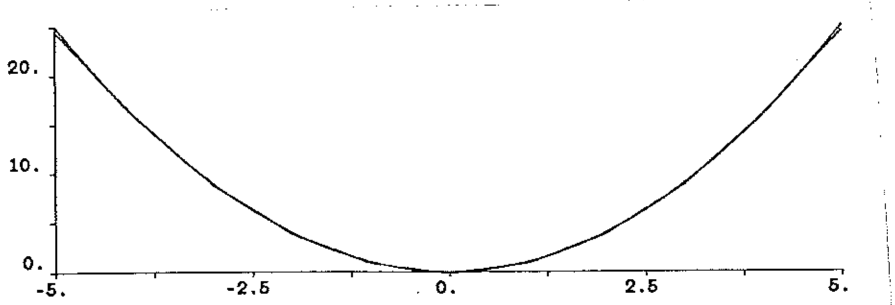

Figure 3. The nonlinearity $y = x^2$ and its estimate obtained from the ANM.

Example 3.1
Suppose that the ANM is used to find the nonlinearity $y = x^2$. Let the input signal be $x = 5 \sin 0.00314t$ and let the table entries used in the ANM be $x_1 = -6$, $x_2 = -5, \ldots, x_{11} = 6$. Figure 2 shows the input signal and the outputs of the nonlinearity and the ANM. The adaptation is switched off at time $t = 8000$ and the ANM is run as an interpolator for a period of the input signal. Figure 3 shows the actual nonlinearity and its estimate obtained from the ANM.

□

Derivatives

It is sometimes useful to also have the derivative of the function available. This can be generated numerically as follows

$$f'(x_k) = \frac{f_{k+1} - f_{k-1}}{x_{k+1} - x_{k-1}} \tag{3.8}$$

Inverse Functions

It is also possible to generate many other functions of the function f. If f is monotone we can e.g. be interested in having the inverse function. This is obtained simply by reversing the roles of x and f in the table entries.

4. Applications

A number of examples which illustrate how the mechanism may be used will now be presented.

Automatic Sensor Linearization

Sensors with nonlinear characteristics can conveniently be linearized using the ANM. For this purpose it is assumed that an accurate sensor which can be used for calibration is available. A system which performs the automatic linearization is shown in Figure 4. An input signal u is sent to the sensor and the reference sensor from some kind of test rig. When the signal is swept over the range the ANM will automatically adjust so that the combination of the sensor and the ANM will give the same result as the reference sensor.

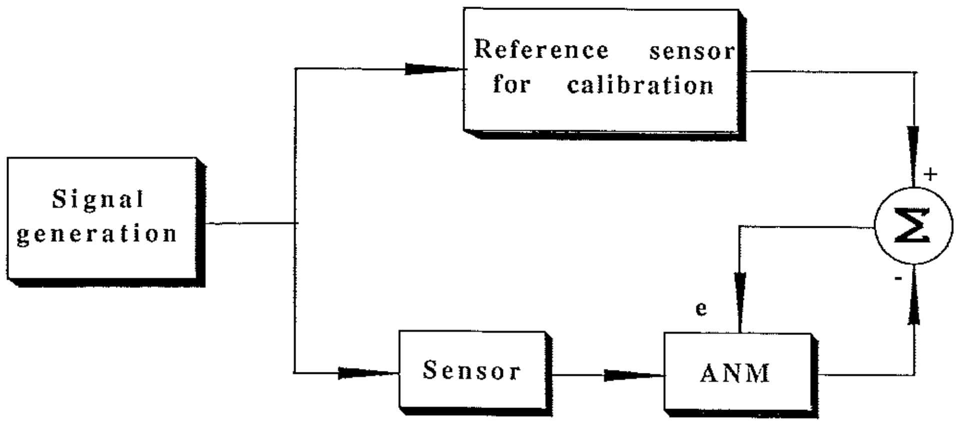

Figure 4. Automatic sensor linearization using the ANM.

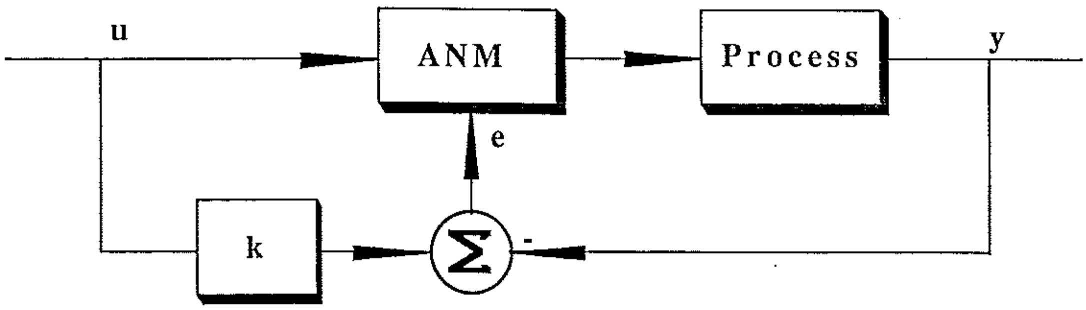

Figure 5. Using the ANM to linearize process dynamics.

Automatic Linearization of Process Dynamics

Consider a process where the major linearity is a static nonlinearity at the process input. A typical example is a flow loop with a nonlinear valve. An adaptive nonlinear modeler can be used to linearize the system as is shown in Figure 5. The ANM is connected in series with the process. Its error signal is formed as the difference between the nominal process output y_m and the actual process output y. The parameter k is the nominal process gain. The time constant in the adaptation loop should be chosen much larger than the time constants of the process. This selection is discussed further in Section 5.

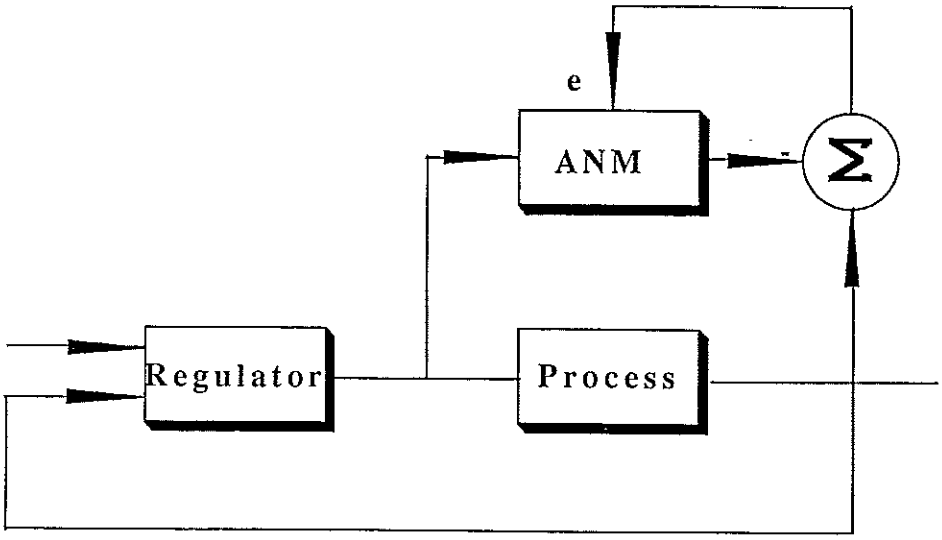

Figure 6. Use of the ANM to find a nonlinear process characteristics.

Determination of a Static Process Model

A natural use of the ANM is to generate a nonlinear static model of a process. This can be done as shown in Figure 6. To avoid stability problems the time constant of the ANM should be chosen sufficiently large. As a rule of thumb it should be larger than the dominant time constants of the process.

Automatic Generation of a Gain Schedule

The system shown in Figure 7 can be modified to provide automatic gain scheduling. The output of the ANM which gives the derivative of the function is then used to modify the process gain. Such a scheme will of course only work well for a process where the major scheme of parameter variations is due to changes in the static gain of the process.

Nonlinear Feedforward Compensation

Consider a situation where there are major upsets due to a disturbance which can be measured. One possibility to make a static compensation is to feed the disturbance through a nonlinearity which gives the control signal necessary to compensate statically for the disturbance. Figure 8 shows how the ANM can be

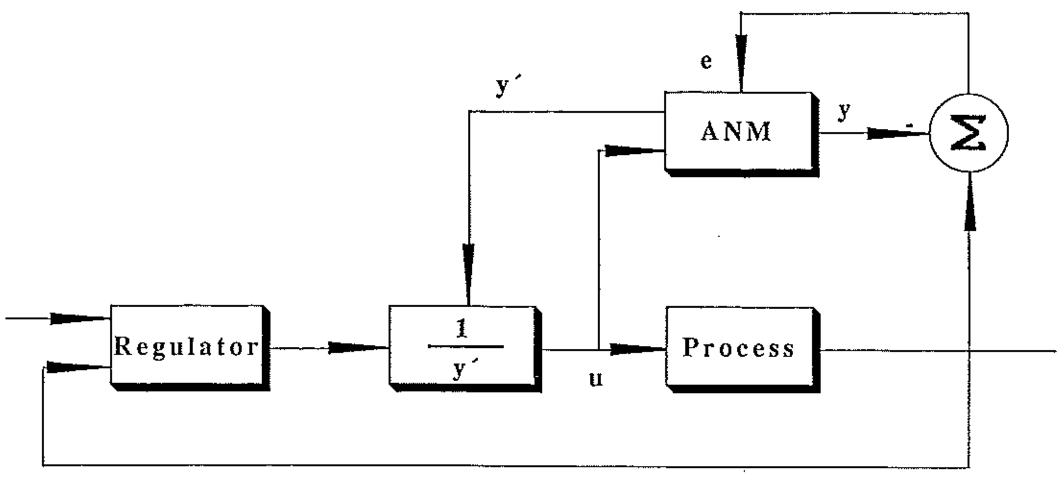

Figure 7. Use of the ANM for automatic generation of a gain schedule.

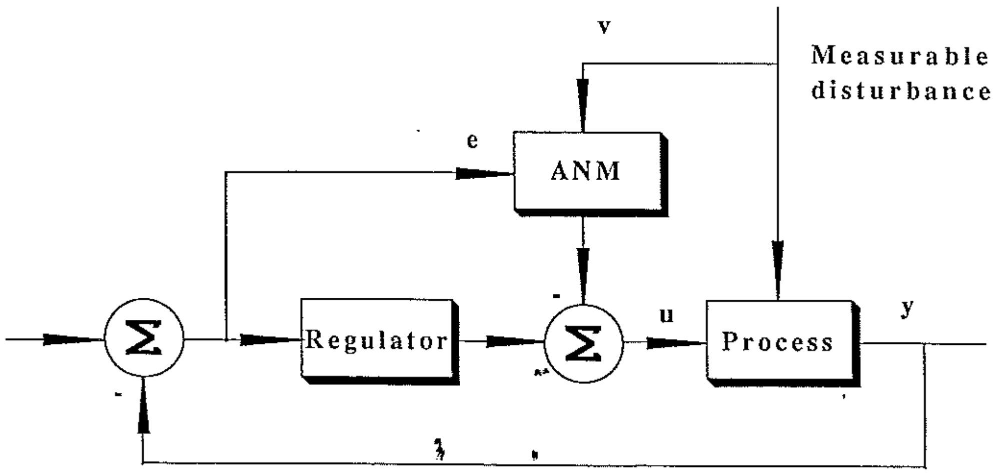

Figure 8. Use of the ANM for automatic generation of a nonlinear feedforward.

used to generate this nonlinearity automatically. Notice that the regulator should not have integral action when the ANM tunes.

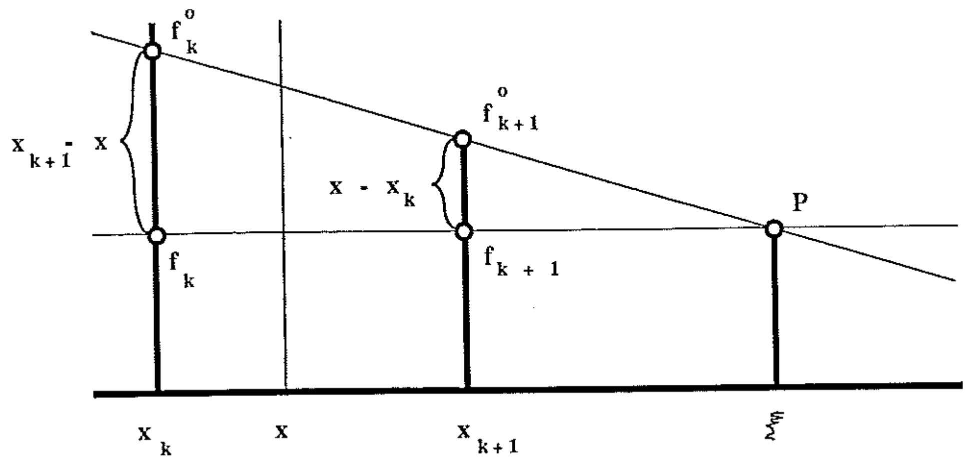

Figure 9. Graphical illustration of the behavior of the updating algorithm.

The behavior of the system can be understood as follows. The disturbance v will generate a control error under proportional control. The ANM will change its parameters as long as there is an error. The coefficients of the nonlinearity will thus be adjusted until the error vanishes.

5. Analysis

In many cases the action of the ANM can be understood simply from linear control theory. Consider for example the automatic linearization case shown in Figure 2. It suffices to analyse what happens when the input x is between two grid points. The equations for updating the table entries are then given by equation (3.1). When x is kept constant the table entries f_k and f_{k+1} will thus change in such a way that they have a constant ratio. Let the initial values be f^o_k and f^o_{k+1}. It follows from (3.1) that

$$(x-x_k)\,\frac{df_k}{dt} = (x_{k+1} - x)\,\frac{df_{k+1}}{dt}$$

If x is constant the changes in f_k and f_{k+1} thus have a constant ratio. This is illustrated in Figure 9. The equilibrium solution to (3.1) can thus be constructed

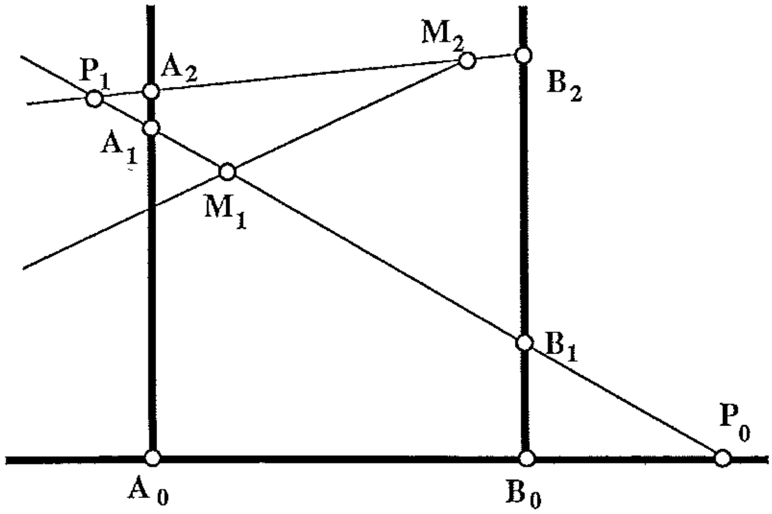

Figure 10. Graphical construction of the equilibrium solutions for two measurements M_1 and M_2.

as follows. Draw a line through (x_k,f_k^o) and (x_{k+1},f^o_{k+1}). Rotate this line through the point P such that the line goes through the measured value (x,f). The new equilibrium values of f_k and f_{k+1} are obtained as the intersections of the line with the lines $x = x_k$ and $x = x_{k+1}$. Let ξ be the x-coordinate of P. It follows that

$$\frac{\xi - x_{k+1}}{\xi - x_k} = \frac{x - x_k}{x_{k+1} - x}$$

Hence

$$\xi = x_{k+1} + (x_{k+1} - x_k)\,\frac{x - x_k}{x_k + x_{k+1} - 2x}$$

Notice that $\xi = x_{k+1}$ if $x = x_k$. It is thus easy to construct the equilibrium solution geometrically. Figure 10 shows what happens for two measurements.

Notice that if no weighting is introduced in (3.1) it follows that

$$\frac{df_k}{dt} = \frac{df_{k+1}}{dt}$$

The straight line will then be displaced parallel to its original position. This means that there will be no convergence if measurements are inside a given interval.

Improved Algorithms

It is clear from the previous discussion that several iterations are required for the estimates to converge. Algorithms which converge faster can be constructed at the expense of the complexity of the algorithm. One possibility is to construct a least squares algorithm for estimating the f_k and f_{k+1}. This would require storage of three additional parameters.

It is also possible to make an adaptive change of the table entries $x_1, x_2, \ldots, x_n$ so that the points are denser when the function f is steep.

Dynamic Properties

The dynamic properties of the adjustment loop is governed by the feedback from the output of the ANM to its error input. In many cases this feedback is simply -1. This is the case in Figure 2, Figure 6 and Figure 7. The adjustment loop is then simply a linear system with the characteristic equation

$$sT + 1 = 0$$

In other cases like in Figure 5 and Figure 8 there is dynamics in the feedback which must be taken into account. This is illustrated by an example.

Example 5.1 - Linearization of Process Dynamics
Consider the system in Figure 5. Let the process dynamics be described by the transfer function G(s). The dynamics of the adjustment loop is then described by the characteristic equation

$$1 + \frac{G(s)}{sT} = 0$$

To make sure that the adjustment loop is stable it is then necessary to choose T sufficiently large. A suitable value can be found by the root-locus method provided that G(s) is known.

□

In some cases the situation is even more complex. This is illustrated by an additional example.

Example 5.2 - Nonlinear Feedforward
Consider the system shown in Figure 6. Let the process dynamics be described by G(s). The characteristic equation of the adjustment loop is then

$$1 + \frac{G(s)}{sT} = 0$$

The analysis of stability and performance is similar to Example 5.1.

□

6. Conclusions

The idea presented in this report seems to be a useful component with many applications. It can be extended and generalized in many different ways. We have chosen to represent a function by a table of values and an interpolation polynomial. Many other representations can be used for example orthogonal polynomials, other interpolation polynomials etc. In this paper we have only considered functions $f \; ; R \rightarrow R$. It is straightforward to generalize the results to functions from R^n to R and a little bit harder to extend the results to functions from R^n to R^m. We have also used fixed grid points. They can be adjusted adaptively to have more points where the function changes rapidly.

The idea can also be extended to compensation of nonlinearities with a hysteresis characteristic. This can be done by representing the hysteresis by two functions and an internal state which tells which branch to choose.