Self-tuning Regulators : Design Principles and Applications

Åström, Karl Johan

1979

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

LUTFD2/(TFRT-7177)/0-068/(1979)

SELF-TUNING REGULATORS -
DESIGN PRINCIPLES AND APPLICATIONS

K J ÅSTRÖM

INSTITUTIONEN FÖR REGLERTEKNIK
LUNDS TEKNISKA HÖGSKOLA
1979

Dokumentutgivare

Lund Institute of Technology
Dept of Automatic Control

Handläggare

Karl Johan Åström

Författare

Karl Johan Åström

Dokumentnamn

REPORT

Dokumentbeteckning

LUTFD2/(TFRT-7177)/1-068/(1979)

Utgivningsdatum

September 1979

Ärendebeteckning

06T6

10T4

Dokumenttitel och undertitel

Self-Tuning Regulators - Design Principles and Applications

Referat (sammandrag)

The basic ideas behind the self-tuning regulators are discussed. The regulators are motivated from the viewpoint of nonlinear stochastic control theory. Self-tuning regulators based on pole-zero assignment, minimum variance control and linear quadratic gaussian control theory as described in a common framework. Applications to a number of different industrial processes are described. Practical and theoretical issues relevant to design of self-tuning regulators are also discussed.

Referat skrivet av

author

Förslag till ytterligare nyckelord

44T0

Klassifikationssystem och -klass(er)

50T0

Indextermer (ange källa)

52T0

Omfång

68 pages

Övriga bibliografiska uppgifter

56T2

Språk

English

Sekretessuppgifter

60T0

ISSN

60T4

ISBN

60T6

Dokumentet kan erhållas från

Department of Automatic Control
Lund Institute of Technology
P O Box 725, S-220 07 LUND 7, Sweden

Mottagarens uppgifter

62T4

Pris

66T0

SELF-TUNING REGULATORS - DESIGN PRINCIPLES AND APPLICATIONS[1]

K. J. Åström

Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

The basic principles of self-tuning regulators are discussed. The regulators are motivated from the viewpoint of nonlinear stochastic control theory. Self-tuning regulators based on pole-zero placement, minimum variance control and linear quadratic gaussian control are described in a common framework. Relations between self-tuning regulators and model reference adaptive control are discussed. Applications to different industrial process control problems are described. Practical and theoretical issues of relevance to design of self-tuning regulators are also treated.

[1] This research was supported by the Swedish Board of Technical Development (STU).

1. INTRODUCTION

The tuning problem is one reason for using adaptive control. It is a well-known fact that many processes can be regulated satisfactorily with PI or PID regulators. It is fairly easy to tune a PI regulator which only has two parameters to adjust. However, for an installation which has several hundred regulators it is a substantial task to keep all the regulators well tuned. A PID regulator which has three or four parameters is not always easy to tune, particularly if the process dynamics is slow. The derivative action is, therefore, frequently switched off in industrial controllers.

Since many control loops are not critical three term controllers will undoubtedly be used extensively in the future too. With an increasing demand for efficiency in the use of energy and raw material there are, however, an increasing number of control problems where it is reasonable to use regulators which are more complicated than PID regulators. Such regulators, which may include feedforward, state feedback, and observers, can often have more than 10 adjustable parameters. It is not possible to adjust so many parameters without a systematic procedure. The lack of a suitable tuning procedure is one reason why modern control theory has not been used more extensively.

One possibility to tune a regulator is to develop a mathematical model for the process, and its disturbances, and to derive the regulator parameters from some control design procedure. The appropriate mathematical models can be obtained from physical modeling or from system identification. The drawback with such a procedure is that it may be fairly time consuming, and that it requires personnel with skills in modeling, system identification, and control design. The self-tuning regulator can be regarded as a convenient way to combine system identification and control design. Its name does, in fact, derive from such applications.

One reason for using adaptive control is thus to avoid the tuning problem. Another motivation for using adaptive control is that the characteristics of the process and its disturbances may change with time. If the changes are not too rapid a properly designed self-tuning regulator may be used for continuous tuning to obtain close to optimal performance.

Adaptive control has been a challenge to control engineers for a long time [1]. Many different schemes have been proposed [2-4]. In spite of this, progress in the field has been comparatively slow. One reason for this is that adaptive systems are difficult to understand because they are inherently nonlinear. The field is also fairly immature as a scientific discipline. Many ideas that are basically the same are derived and presented using very different approaches. There is a wide divergence in notations. A fundamental conceptual framework is also lacking. Recently there has been an increased interest in adaptive control. One reason for this is the availability of microprocessors which make it possible to implement adaptive controllers conveniently and cheaply [5-7]. Another reason is the success of adaptive control in pilot installations in industry [8-16]. A third reason is that some progress has recently been made in the theory of adaptive control [17-23].

The purpose of this paper is to give an overview of self-tuning regulators, which is <u>one</u> approach to adaptive control. The focus will be on concepts, theory, and applications. The paper is organized as follows.

A brief review of nonlinear stochastic control theory is given in Section 2. This theory gives a conceptual framework and a general structure of an adaptive regulator with many interesting features. The regulator obtained from nonlinear stochastic control theory is, however, so complicated that it can only be computed numerically in almost trivial cases. To obtain something useful it is thus necessary to make approximations. Self-tuning regulators are one approximation. The principles for design of self-tuning regulators are also discussed in Section 2. The basic idea can be described as follows. Start with a design method that will give adequate results if the parameters of models for the dynamics of the process and its environment are known. When the parameters are unknown, they are replaced by estimates obtained from a recursive parameter estimator. Since there are many available methods for designing control systems there are at least as many ways to design self-tuning regulators.

The general principles are illustrated in Section 3 by giving some details for self-tuning regulators based on assignment of poles and zeros. In this way, it is also possible to bring up the notion of algorithms with implicit and explicit identification. Self-tuning regulators based on minimum variance control and on linear quadratic control theory are briefly discussed in Section 4. In Section 5 self-tuning regulators are compared with model reference adaptive systems, which is another approach to adaptive control. It is shown that the two approaches are closely related. Applications of self-tuning regulators to real processes are discussed in Section 6. This includes control of papermachines, heat exchangers, ore crushers, and autopilots for ship steering. Some aspects on the available theory and practice of self-tuning regulators are given in Section 7. The paper ends with conclusions and references.

2. DESIGN PRINCIPLES

The principles for designing self-tuning regulators (STR) are presented in this section. A brief review of nonlinear stochastic control theory is first given. This provides a convenient conceptual framework. Self-tuning regulators are then introduced as approximations to the general nonlinear stochastic problem. It is also shown that the self-tuning regulators can be formulated for many problems which are not stochastic control problems.

Nonlinear Stochastic Control

In discrete time nonlinear stochastic control theory, the model of the system and its environment are specified by giving the conditional probability distribution of the state x(t+1) at time t + 1 given the state x(t) and the control u(t) at time t. The observation process is similarly specified by giving the conditional probability distribution of the observation y(t) at time t given the state x(t) and the control u(t) at time t. The criterion is formulated as to minimize the expected value of a loss function which is a scalar function of states and controls.

The problem of finding a control which minimizes the expected loss function is difficult. Useful explicit conditions for existence are not known, in general. Under the assumption that a solution exists a functional equation can, however, be derived using standard dynamic programming [24]. This functional equation, which is called the Bellman equation, can be solved numerically only in very simple cases. The approach is, nevertheless, of interest because it gives insight into the structure of the optimal controller. This structure is shown in Fig. 1. The controller can be thought of as composed of two parts, an estimator and a feedback regulator. The estimator generates the conditional probability distribution of the state of the process given all past data. This distribution is sometimes called the hyperstate of the problem. The feedback regulator is simply a nonlinear static function which maps the hyperstate into the space of controls.

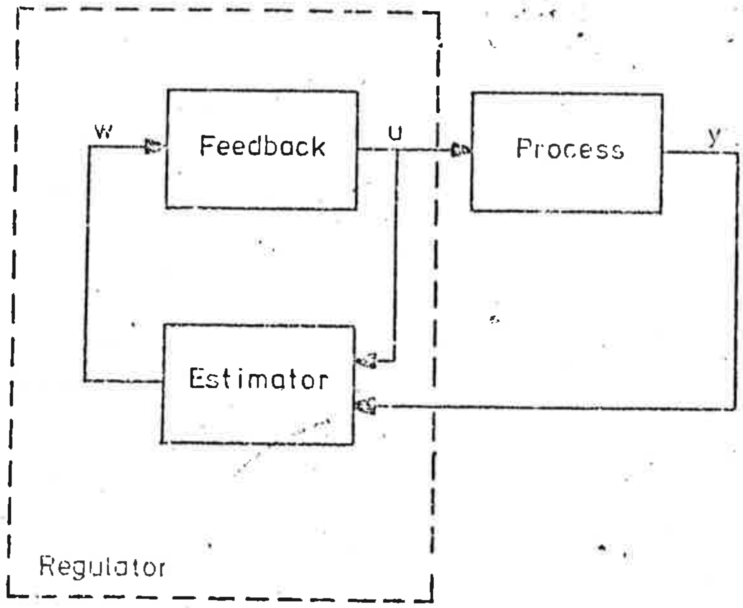

FIGURE 1. Structure of the regulator obtained from nonlinear stochastic control theory. The hyperstate w is generated from a dynamical system using u and y as inputs. The regulator is a static nonlinearity which gives the control variable u(t) as a function of the hyperstate w(t).

To solve a nonlinear stochastic control problem it is necessary to determine the estimator, which updates the hyperstate, and to solve the functional equation which gives the feedback regulator. The structural simplicity of the solution is obtained at the price of introducing the hyperstate which is a quantity of very high dimension. For example, a problem whose state space is R^4 has a hyperstate which is a distribution over R^4.

Nonlinear stochastic control is of substantial interest for adaptive control, because many of the aspects of adaptive control are captured by the nonlinear stochastic control formulation. In a problem with unknown but constant parameters the parameters are simply introduced as auxiliary state variables. Let a be a constant parameter, the associated state equation is then

$$a(t+1) = a(t). \tag{2.1}$$

If it can be assumed that the parameter is drifting like a random walk the following is an alternative to (2.1):

$$a(t+1) = a(t) + v(t), \tag{2.2}$$

where $\{v(t)\}$ is a sequence of uncorrelated random variables. For a system with constant but unknown parameters or drifting parameters it is natural to separate the state variables into two groups, the original state variables and the state variables associated with the parameters of the process. It is often assumed that the 'parameters' are changing at much slower rates than the other state variables. It is, however, not necessary to make such a distinction. Nonlinear stochastic control theory thus makes it possible conceptually to deal with rapid parameter variations.

Approximations

Since it is difficult to solve the Bellman equation, approximate solutions are of considerable interest. A simple example will be used to illustrate some common approximations. Consider a process described by

$$y(t+1) = y(t) + bu(t) + e(t) \tag{2.3}$$

where u is the control, y the output, e white noise and b a constant parameter. Equation (2.3) is a sampled data model of an integrator with unknown gain. Let the criterion be to minimize

$$\lim_{N\to\infty} E \frac{1}{N} \sum_{1}^{N} y^2(t). \tag{2.4}$$

If the parameter b is known, the control law which minimizes (2.4) is given by

$$u(t) = -\frac{1}{b}\,y(t). \tag{2.5}$$

If the parameter b has a gaussian prior distribution, it follows that the conditional distribution of b, given inputs u, and outputs y up to time t is normal with mean $\hat{b}(t)$ and covariance P(t). The hyperstate of the problem can then be characterized by the triple $[y(t),\hat{b}(t),P(t)]$. In this simple case the Bellman equation can be solved numerically [25-28]. The control law obtained can not be characterized in a simple way. It has, however, a very interesting property. The control signal will not only attempt to bring the output close to zero. When the parameters are uncertain the regulator will also inject signals into the system to reduce the uncertainty of the parameter estimates. This is referred to as <u>probing</u> [29]. The optimal control law will give the right balance between the tasks of keeping the control errors and the estimation errors small. This is called <u>dual control</u> [30].

The following control law

$$u(t) = -\frac{1}{\hat{b}(t)}\,y(t) \tag{2.6}$$

is an approximation which is called the <u>certainty equivalence control</u>. It is obtained simply by solving the control problem in the case of known parameters and substituting the known parameters with their estimates.

In the control law (2.6) the control variable u(t) is obtained by dividing the measured variable y(t) by the estimate $\hat{b}(t)$. This implies that there will be difficulties if the estimate $\hat{b}(t)$ is zero or very small. When using a certainty equivalence control like (2.6) it is therefore necessary to limit the controller gain. One simple way to do this, which works well in practice, is simply to put a hard bound on u(t) or its increment u(t) - u(t-1).

The difficulty associated with division by $\hat{b}$ in (2.6) is avoided in the control law

$$u(t) = - \frac{\hat{b}(t)}{\hat{b}^2(t) + P(t)} y(t) = - \frac{1}{\hat{b}(t)} \frac{\hat{b}^2(t)}{\hat{b}^2(t) + P(t)} y(t) \qquad (2.7)$$

which is called <u>cautious control</u> because it hedges and uses lower gain when the estimates are uncertain. Notice that the cautious control law minimizes the criterion

$$E[y^2(t)\,|\,y(t-1),y(t-2),\ldots] \qquad (2.8)$$

where $E[\cdot|\underline{D}]$ denotes the conditional expectation with respect to $\underline{D}$. The control law (2.7) does, however, not minimize (2.5). The minimization of (2.8) can be taken as the definition of cautious control in the general case.

There is empirical evidence that the cautious controller hedges too much. It has been observed in many simulations that the control signal will occasionally be zero over certain periods when the cautious controller is used. The mechanism can intuitively be explained as follows. If the estimate $\hat{b}$ is uncertain, i.e. $\hat{b}^2 << P$, then it follows from (2.7) that the control signal will be small. The uncertainty P may then increase and the control signal will decrease further until it finally becomes zero. When applied to stable systems the control signal will be zero for a period and normal operation will then be resumed. This phenomenon is called <u>turn off</u>. When the cautious controller is applied to an unstable system the output may become so large that the system can no longer be brought back into normal operation. The phenomenon is then called <u>escape</u>. The possibilities of turn off and escape makes the cautious control law less useful.

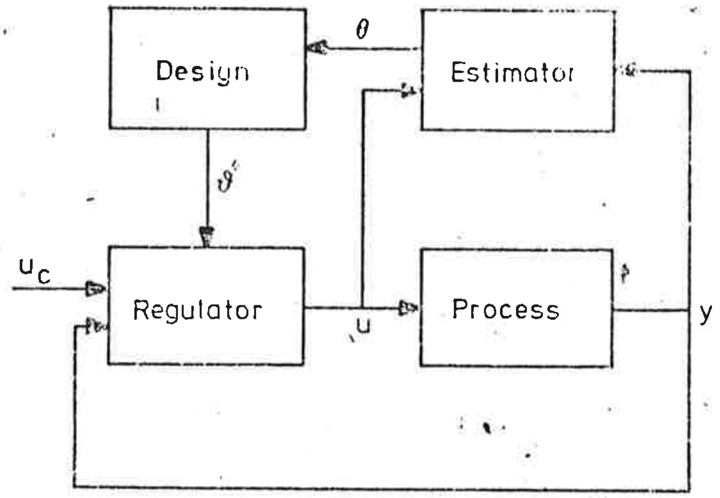

FIGURE 2. Schematic diagram of a self-tuning regulator.

Self-Tuning Regulators

A block diagram of a self-tuning regulator is shown in Fig. 2. The regulator can be thought of as composed of three parts, a recursive parameter estimator, a design calculator, and a regulator with adjustable parameters. The design method is chosen so that it gives the desired results when the parameters characterizing the process and its environment are known. When the parameters are unknown they are simply replaced by estimates. From the viewpoint of nonlinear stochastic control a self-tuning regulator is clearly a certainty equivalence control. Notice that the state is separated into two parts corresponding to the original process states and the parameters and that there are different signal paths for the two parts. Also notice that although the STR can be viewed as a certainty equivalence control the principle of STR can be applied to many problems which can not be formulated as stochastic control problems.

Since there are many different ways to do control design and to estimate parameters, there are many varieties of self-tuning regulators. Because the self-tuning regulators are certainty equivalence controls, it is also clear that they could be modified by introducing approximations of the effects of caution and probing if needed.

Disregarding the special structure of the regulator in Fig. 2 it can clearly be regarded as a nonlinear regulator which accepts an input y and generates a control u. The special structure is, however, very useful because it helps to understand how the regulator works and how it should be designed.

3. SELF-TUNERS BASED ON POLE-ZERO ASSIGNMENT

Design of self-tuning regulators based on pole-zero assignment will now be worked out in detail. A pole-zero assignment method for systems with known parameters is first described. It is then shown how this method is used to design self-tuners. Only discrete time systems are discussed.

Formulation

Consider a process characterized by

$$Ay = Bu \tag{3.1}$$

where A and B are polynomials in the forward shift operator. It is assumed that A and B are coprime and that

$\deg B < \deg A,$

where deg A denotes the degree of the polynomial A. It is desired to find a controller such that the closed loop is stable and that the transfer function from the command input u_c to the output is given by

$$G_M = \frac{Q}{P} \tag{3.2}$$

where P and Q are coprime and

$$\deg P - \deg Q >= \deg A - \deg B. \tag{3.3}$$

Condition (3.3) is necessary for the problem to be solved using a causal regulator.

Design Procedure

A general linear two-degree-of-freedom [31] regulator can be described by

$$Ru = Tu_c - Sy. \tag{3.4}$$

The closed loop transfer function relating y to u_c is given by

$$\frac{TB}{AR + BS} = \frac{Q}{P}, \tag{3.5}$$

where the right hand side is the desired closed loop transfer function G_M given by (3.2). The design problem is thus equivalent to the algebraic problem of finding polynomials R,S, and T such that (3.5) holds. It follows from (3.5) that factors of B, which are not also factors of Q, must divide R. Since factors of B correspond to open loop zeros it means that open loop zeros, which are not desired closed loop zeros, must be canceled. Factor B as

$$B = B^{+}B^{-} \tag{3.6}$$

where all the zeros of B^{+} are in a region $\underline{Z}$ of the complex plane and all zeros of B^{-} outside $\underline{Z}$. The region $\underline{Z}$ corresponds to modes that are sufficiently well damped. This means that all zeros of B^{+} correspond to well damped modes and all zeros of B^{-} correspond to unstable or poorly damped modes.

A necessary condition for solvability of the servo problem is that the specifications are such that

$$Q = Q_1 B^{-}. \tag{3.7}$$

Since deg P is normally less than deg (AR + BS) it is clear that there are factors in (3.5) which cancel. In state space theory it can be shown that the regulator (3.4) corresponds to a combination of an observer and a state feedback. See [32]. It is natural to assume that the observer is designed in such a way that changes in command signals do not generate errors in the observer. This means that the factor which cancels in the right hand side of (3.5) can be interpreted as the observer polynomial T_1.

The design procedure can be formulated as follows.

Data: Given the desired response specified by the polynomials P and Q, subject to the conditions (3.3) and (3.7), and the desired observer polynomials T_1. It is assumed that P and T_1 have all their zeros in $\underline{Z}$.

Step 1: Solve the equation

$$AR_1 + B^{-}S = PT_1 \tag{3.8}$$

with respect to R_1 and S.

Step 2: The regulator which gives the desired closed loop response is given by (3.4) with

$$R = R_1 B^+ \tag{3.9}$$

and

$$T = T_1 Q_1 . \tag{3.10}$$

□

The equation (3.8) can always be solved because it was assumed that A and B were coprime. This implies that A and B^- are also coprime. Equation (3.8) has infinitely many solutions. If R_1^0 and S^0 is one solution then another solution is given by

$$R_1 = R_1^0 + B^- F$$

$$S = S^0 - AF$$

where F is an arbitrary polynomial. All solutions give closed loop systems with the same transfer function (3.2) from command signal u_c to the output y. The response of the closed loop system to disturbances will, however, depend on the particular solution which is chosen.

The lack of uniqueness can, for example, be used to ensure that the regulator has integral action, that it is causal and that it is insensitive to measurement noise. Since the following discussion will not depend on the particular solution which is chosen, it is unnecessary to go into further details here. Let it suffice to mention [33] that there will always be a causal regulator if

$$\deg T_1 = \deg A - \deg B^+ - 1.$$

It is often reasonable to choose a solution such that

$$\deg R = \deg S = \deg T,$$

which means that there will be direct terms both in the feedback and feedforward paths.

Least Squares Parameter Estimation

A recursive parameter estimator is an important part of a self-tuning regulator. See Fig. 2. There are many different methods which could be used for parameter estimation, for example stochastic approximation, least squares, extended least squares, generalized least squares, multistage least squares, instrumental variables, and maximum likelihood. A review of recursive estimation methods is given in [34] where also many references are given. There is unfortunately no recursive parameter estimator which is uniformly best. Consequently there are many different possibilities. For simplicity only recursive least squares estimation is used in this paper. Least squares is one of the simplest recursive estimation procedures. The method unfortunately gives biased estimates if the disturbances are correlated.

In recursive least squares estimation the criterion

$$V = \sum_{k=1}^{t} \lambda^{t-k} \varepsilon^2(k) \tag{3.11}$$

where

$$\varepsilon(t + \deg A) = Ay(t) - Bu(t) \tag{3.12}$$

is minimized. The parameter λ is a weighting factor which gives lower weight to old measurements. To describe the algorithm the model (3.1) is written explicitly as

$$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) =$$

$$= b_0 u(t-k) + \ldots + b_m u(t-m-k). \tag{3.13}$$

Introduce a vector of parameter estimates

$$\theta = [\hat{a}_1 \dots \hat{a}_n \quad \hat{b}_0 \dots \hat{b}_m]^T \tag{3.14}$$

and a vector of regressors

$$\phi(t) = [-y(t-1) \dots -y(t-n) \quad u(t-k) \dots u(t-m-k)]^T. \tag{3.15}$$

The recursive least squares estimate is then given by

$$\theta(t+1) = \theta(t) + P(t+1)\phi(t+1)\varepsilon(t+1), \tag{3.16}$$

where

$$\varepsilon(t+1) = y(t+1) - \theta^T(t)\phi(t+1), \tag{3.17}$$

and

$$P(t+1) = [P(t) - P(t)\phi(t)R(t)\phi^T(t)P(t)]/\lambda, \tag{3.18}$$

where

$$R(t) = [\lambda + \phi^T(t)P(t)\phi(t)]^{-1}.$$

There are other ways to perform the least square calculations. Square root algorithms, [35-38], are useful if the problem is poorly conditioned. Fast algorithms, [39-41], can be used if many parameters have to be estimated and if computing time is crucial.

Interpretations

Equation (3.16) can be interpreted as a quasi-Newton iteration for minimizing ε^2. It follows from (3.17) that

$$\phi = - \operatorname{grad}_\theta \varepsilon. \tag{3.19}$$

The term $\phi\varepsilon$ in (3.16) can thus be interpreted as the gradient of $\varepsilon^2/2$. The matrix P in (3.16) modifies the gradient direction and determines the step length. The matrix P can thus be interpreted as a <u>gain factor</u> which determines the rate of change of the estimate. The matrix P satisfies the equation

$$P^{-1}(t+1) = \lambda P^{-1}(t) + \phi(t+1)\phi^T(t+1). \tag{3.20}$$

When λ is equal to one, the matrix P will thus decrease monotonically. This means that the gain of the estimator will decrease to zero. Recall (3.11) which shows that λ can be interpreted as a factor which discounts past measurements. When $\lambda < 1$ the matrix P will not go to zero as time increases. The magnitude of the gain factor will depend on λ. The gain P will be larger if λ is smaller. The gain of the estimator will thus depend on the rate in which past data is discounted. There are also other possibilities to obtain an estimator with nondecreasing gain. If it is assumed that the parameters are drifting as in (2.2) and $\lambda = 1$ Equation (3.18) is replaced by

$$P(t+1) = P(t) + P(t)\Phi(t)R(t)\phi^T(t)P(t) + R_1, \tag{3.21}$$

where

$$R_1 = \operatorname{cov}[v(t), v(t)].$$

Under stationary conditions when R_1 is different from zero the gain factor P will settle on a level which is determined by R_1. The larger R_1 is, i.e. the larger the assumed drift rate is, the larger will the gain factor be. The model (3.21) is more flexible than (3.18) because the rate of change of the parameters can be set individually. Different drift rates for different parameters can, however, also be incorporated in (3.18) by replacing λ in (3.18) by a diagonal matrix.

There are proposals to adjust the forgetting factor automatically [42]. The principle is to choose λ as

$$\lambda = 1 - \alpha\varepsilon^2/\overline{\varepsilon^2} \tag{3.22}$$

where $\overline{\varepsilon^2}$ is the mean value of ε^2 over a certain period. It has been found empirically [43] that a forgetting factor λ between 0.95 and 0.99 works well if there are continuous disturbances, provided that the problem is not overparametrized. If there are too many parameters the matrix

$$\Sigma\phi(t)\phi^T(t)$$

will be poorly conditioned. With a forgetting factor the matrix P can then be very large in certain directions.

In a servo problem with few disturbances the major excitation comes from the changes in the command signal. Such changes may be irregular and it has been found that there may be bursts in the process output if Eq. (3.18) is used with λ less than one. The presence of bursts can be understood intuitively as follows. The negative term in (3.18) represents the reduction in parameter uncertainty due to the last measurement. When there are no changes in the set point the vector $P(t)\phi(t)$ will be zero. There will not be any changes in the parameter estimate and the negative term in the right hand side of (3.18) will be zero. The equation (3.18) then reduces to

$$P(t+1) = \frac{1}{\lambda} P(t)$$

and the matrix P will thus grow exponentially if $\lambda < 1$. If there are no changes for a long time the matrix P may thus become very large. A change in the command signal may then lead to large changes in the parameter estimates and in the process output. The large values of the matrix P may also lead to numerical problems. Examples which illustrate this behaviour are found e.g. in [42] and [44]. These results are partly due to bad numerics.

There are many ways to eliminate bursts. Perturbation signals may be added to ensure that the process is properly excited. The estimation algorithm may be modified. One possibility is to stop the updating of the matrix P(t) when the signal $P(t)\phi(t)$ is smaller than a given value. Another possibility is to replace P(t) by

$$[\alpha I + \sum_{k=1}^{t} \lambda^{t-k} \phi(k)\phi(k)]^{-1}$$

where α is a small number. This ensures that the matrix P stays bounded.

An Explicit Self-Tuning Regulator

A self-tuning controller based on pole-placement design and least squares estimation can be obtained by implementing the scheme of Fig. 2 directly. The following algorithm is obtained.

ALGORITHM E1. (Basic explicit algorithm)

Data: The polynomials P, T_1, and Q_1 are given.

Step 1: Estimate the parameters of the model

$$Ay(t) = Bu(t) \tag{3.1}$$

by least squares.

Step 2: Factor the estimated polynomial $\hat{B}$ into $\hat{B}^+$ and $\hat{B}^-$ where $\hat{B}^+$ has all its zeros in the region $\underline{Z}$ and $\hat{B}^-$ has all its zeros outside $\underline{Z}$.

Step 3: Solve the linear equation

$$\hat{A}R_1 + \hat{B}^- S = PT_1. \tag{3.8}$$

Step 4: Calculate the control variable u from

$$Ru = Tu_c - Sy \tag{3.4}$$

where

$$R = R_1\hat{B}^+$$

and

$$T = T_1Q_1.$$

(Notice that there are many solutions and that a choice has to be made [50].)

The steps 1,2,3, and 4 are repeated at each sampling period. □

This algorithm is called an algorithm based on <u>estimation of process</u> parameters or an algorithm with <u>explicit identification</u>, because the parameters of the process model (3.1) in the standard form are estimated. Using the terminology of model reference adaptive systems, the algorithm is also called <u>indirect</u> [45], because the parameters of the regulator are updated indirectly via estimation of the process parameters (Step 1) and the design calculations (Steps 2,3, and 4).

If the parameter estimates converge the closed loop system obtained will have the transfer function

$$G = \frac{Q_1 \hat{B}^-}{P}$$

from the command signal to the output. The polynomial $\hat{B}^-$ is the polynomial that corresponds to the unstable or poorly damped process zeros. Notice that the closed loop response will change if $\hat{B}^-$ changes.

There are two difficulties with Algorithm E1. The factorization may be difficult and time consuming. Equation (3.8) is poorly conditioned if the polynomials $\hat{A}$ and $\hat{B}$ have factors which are close. It is, therefore, of interest to consider cases where the factorization problem can be avoided. Such schemes are discussed in [46],[47] and [50]. When the polynomial $\hat{A}$ has zeros which are close to the zeros of the polynomial $\hat{B}$ it is reasonable to cancel such factors before Step 3. An example of this is discussed in [48].

A third difficulty with the algorithm E1 is that the least squares parameter estimation method will give biased estimates if there are correlated disturbances. One possibility to avoid this is to use another estimation method. The first step in the algorithm can also be replaced by:

Step 1*: Estimate the parameters of the model

$$A\bar{y}(t) = B\bar{u}(t),$$

where

$$\bar{y} = \frac{1}{T_1}\, y, \quad \bar{u} = \frac{1}{T_1}\, u$$

by least squares.

This requires, of course, that the polynomial T_1 is known. If the polynomial T_1 is not known the following algorithm can be used.

ALGORITHM ESR. (Explicit algorithm for combined servo and regulation problem)

Data: The polynomials P and Q_1 are given.

Step 1: Estimate the parameters of the model

$$Ay(t) = Bu(t) + Ce(t)$$

by recursive maximum likelihood.

Step 2: Factor the estimated polynomial $\hat{B}$ into $\hat{B}^+$ and $\hat{B}^-$ where $\hat{B}^+$ has all its zeros in the region $\underline{Z}$ and $\hat{B}^-$ has all its zeros outside $\underline{Z}$.

Step 3: Solve the linear equation

$$\hat{A}R_1 + \hat{B}^- S = PT_1,$$

where $T_1 = \hat{C}$.

Step 4: Calculate the control variable u from

$$Ru = Tu_c - Sy,$$

where

$$R = R_1\hat{B}^+$$

and

$$T = T_1Q_1.$$

The steps 1,2,3, and 4 are repeated at each sampling period. □

An Implicit Self-Tuning Regulator

The design calculations required for the explicit algorithms may be time consuming. It is possible to obtain different algorithms where the design calculations are simplified considerably. The basic self-tuning regulator [59] is a prototype for algorithms of this type. The basic idea is to rewrite the process model in such a way that the design step is trivial. By a proper choice of model structure the regulator parameters are updated directly and the design calculations are thus eliminated. Algorithms of this type are called algorithms based on implicit identification of a process model. In the terminology of model reference adaptive systems the corresponding algorithms are also called direct methods [45] because the parameters of the regulator are updated directly. An example of an explicit algorithm will now be given.

Consider a process described by (3.1) with $B^- = 1$, which means that all zeros are well damped. Assume that it is desired to find a feedback such that the transfer function from the reference value to the output is

$$\frac{z^{\deg B}}{P} .$$

Equation (3.5) gives

$$\frac{TB}{AR + BS} = \frac{z^{\deg B}}{P} .$$

The polynomial B thus divides R. Introduce

$$R = R_1 B.$$

Then

$$PT = z^{\deg B}(AR_1 + S). \tag{3.23}$$

Hence

$$PTy = z^{\deg B}(AR_1 y + Sy) = z^{\deg B}(R_1 Bu + Sy) =$$

$$= z^{\deg B}(R_1 Bu + Sy) = z^{\deg B}(Ru + Sy), \tag{3.24}$$

where the second equality is obtained from (3.1). The process can thus be represented either by (3.1) or by (3.23). The representation (3.23) has the advantage that the polynomials R and S required by the polynomial design occur explicitly in the model. If the model (3.23) is available the pole-placement design is thus trivial because the regulator (3.4) is obtained from the model (3.23) by inspection. The following self-tuning control algorithm can now be obtained.

ALGORITHM I2. (Implicit algorithm with all process zeros cancelled)

Data: Given the polynomials P and T, where P is normalized such that $P(1) = 1$.

Step 1: Estimate the parameters of the polynomials R and S in the model

$$PTy = z^{\deg B}(Ru + Sy) \tag{3.25}$$

by least squares.

Step 2: Calculate the control signal using

$$\hat{R}u = Tu_c - \hat{S}y, \tag{3.26}$$

where $\hat{R}$ and $\hat{S}$ are the polynomials estimated in Step 1. The steps 1 and 2 are repeated at each sampling period. □

This algorithm was originally proposed in [49]. Since the specifications require that all process zeros are cancelled, they must be sufficiently well damped for the algorithm to function. The algorithm will thus not work for nonminimum-phase systems.

To write the equation (3.25) it is necessary to know the pole excess

$$k = \deg A - \deg B$$

of the sampled system. The number k can often be estimated based on physical knowledge. There are, however, cases where it can be difficult to determine k apriori. It has been found empirically that it is not dangerous to overestimate k. If the pole excess of the sampled model is underestimated the leading coefficient r_0 of the polynomial R will be zero. The control law (3.26) is then not causal. If the estimate of r_0 is small the gain of the controller will be very large. Compare with the discussion of Equation (2.6).

Since the least squares method gives biased estimates if the disturbances are correlated, the following variation of the algorithm I2 is useful.

ALGORITHM I3.

Data: Given the polynomials P and T where P is normalized such that $P(1) = 1$.

Step 1: Estimate the parameters of the polynomials

$$Py = z^{\deg B}(R\bar{u} + S\bar{y}),$$

where

$$\bar{u} = \frac{1}{T}\, y,\ \bar{y} = \frac{1}{T}\, y$$

by least squares.

Step 2: Calculate the control variable using

$$\hat{R}u = Tu_c - \hat{S}y,$$

where $\hat{R}$ and $\hat{S}$ are the polynomials estimated in Step 1. The steps 1 and 2 are repeated at each sampling period. □

Further discussions on self-tuners based on pole-zero assignment are given in [46], [50-55], where many additional references are given.

4. SELF-TUNERS BASED ON STOCHASTIC CONTROL THEORY

Self-tuners based on minimum variance control and on linear quadratic gaussian control theory (LQG) will be discussed in this section. The discussion is limited to single input single output systems. It is assumed that the process to be controlled is governed by the model

$$Ay(t) = Bu(t) + Ce(t) \tag{4.1}$$

where u is the input, e white noise and y the output. It is assumed that the reader is familiar with the design problem for systems with known parameters [56],[57].

Minimum Variance Control

In this case the criterion is to minimize

$$E[y^2(t)] \tag{4.2}$$

in steady state. A typical application is minimization of fluctuations in quality variables in process control. If the parameters of the model (4.1) are known the control strategy which minimizes (4.2) is obtained by solving (3.23) with

$$P = z^{\deg A}$$

$$T = C.$$

The equation (3.23) then becomes

$$z^{\deg A - \deg B} C = AR_1 + S.$$

The particular solution such that

$$\deg R_1 < \deg A - \deg B = k$$

$$\deg S < \deg A$$

is chosen. The minimum variance control law is then given by (3.4) with $R = R_1 B$, S and $T = C$. Notice that B divides R, which means that all process zeros are cancelled. This means that the controller can not be expected to work well unless all process zeros are well damped. A minimum variance controller for non-minimum phase systems is given in [57]. The minimum variance controller attempts to bring the predicted output equal to the desired output after a time interval of length

$$h\ (\deg A - \deg B) = hk$$

where h is the sampling period. This means that the sampling period is the major design variable which determines the closed loop bandwidth.

For the case $C = z^{\deg A}$ it is easy to obtain a minimum variance self-tuner. The algorithm is a special case of the implicit pole placement Algorithm I2. It is easy to derive it directly. Equation (4.3) gives

$$
\begin{aligned}
y(t + 2 \deg A - \deg B) &= AR_1 y(t) + Sy(t) = \\
&= R_1 Bu(t) + Sy(t) + R_1 e(t + \deg A) = \\
&= Ru(t) + Sy(t) + R_1 e(t + \deg A),
\end{aligned}
\tag{4.4}
$$

where Equation (4.1) was used to obtain the second equality. The transformed model (4.4) is a convenient starting point for an implicit self-tuner because it contains the controller parameters explicitly. A self-tuning minimum variance regulator is now obtained as follows.

ALGORITHM IMV. (Implicit minimum variance controller)

Step 1: Estimate the parameters in the model

$$y(t + k + \deg R) = Ru(t) + Sy(t) \tag{4.5}$$

$$k = \deg A - \deg B$$

by least squares.

Step 2: Calculate the control variable from

$$\hat{R}u(t) = y_r(t + \deg \hat{R}) - \hat{S}y(t) \tag{4.6}$$

where the coefficients of the polynomials $\hat{R}$ and $\hat{S}$ are the estimates obtained in Step 1.

The steps 1 and 2 are repeated in each sampling period. □

This is the basic self-tuner discussed in [58-60]. It is natural that the algorithm will work well when applied to a process (4.1) such that $C = z^{\deg A}$. In this case, the parameter estimates will converge to the corresponding minimum variance regulator. It is more surprising that the regulator will work well also when applied to a model (4.1) with $C \neq z^{\deg A}$. In this case, the least squares parameter estimates will be biased. In spite of this, the regulator will, however, converge to the minimum variance controller if it converges at all [58],[59]. Notice, however, that the parameter estimates may not necessarily converge [17], [85].

When using minimum variance control the variable y is often introduced as the control error. With this choice of variables the reference value y_r is zero. The control law (4.6) then has one redundant parameter. The redundant parameter can be eliminated by reparametrizing the estimation model (4.5) as

$$y(t+k) = r_0[u(t) + r_1'u(t-1) +\ldots+ r_{n_R} u(t-n_R)] +$$

$$+ s_0y(t) + s_1y(t-1) +\ldots+ s_{n_S} y(t-n_S). \tag{4.7}$$

The control law (4.6) then becomes

$$u(t) = -\frac{1}{\hat{r}_0} [\hat{s}_0y(t) +\ldots+ \hat{s}_{n_S} y(t-n_S)] -$$

$$- \hat{r}_1'u(t-1)-\ldots- \hat{r}_{n_R}' u(t-n_R). \tag{4.8}$$

It is shown in [59] that the estimate $\hat{r}_0$ can be fixed apriori provided that

$$0.5 \leqq \hat{r}_0/r_0 < \infty. \tag{4.9}$$

If the algorithm converges with $\hat{r}_0 = r_0$ it will still converge if (4.9) holds. The convergence rate of the parameters is, however, influenced by $\hat{r}_0$. The fastest convergence is obtained for $r_0 = \hat{r}_0$.

Linear Quadratic Control

The self-tuning regulators based on minimum variance control suffer the same drawback as minimum variance control for systems with known parameters, namely that all process zeros are cancelled in the design. This is of little importance if the zeros are well inside the unit disc. The cancellation is, however, disastrous if the system is not minimum-phase. Another drawback with minimum variance control is that the control signals may be excessively large. This can be overcome by a suitable choice of the sampling period. Another possibility is, however, to base the design on linear quadratic gaussian control theory [56]. For single input single output systems the model (4.1) is thus used and the criterion is

$$\lim_{N\to\infty} E \frac{1}{N} \sum_{t=1}^{N} [y^2(t) + \rho u^2(t)]. \qquad (4.10)$$

For the explicit algorithm it is necessary to find the steady state solution of a Riccati-equation [32] or to solve a spectral factorization problem [33]. It is thus necessary to perform a reasonable amount of calculations at each step. With the increasing computing power of micro-processors such calculations are, however, feasible in many situations. Further discussions of explicit self-tuners based on stochastic control theory are given in [60-62], where many additional references are given. It is an interesting problem to find the corresponding implicit algorithms.

5. RELATIONS TO MODEL REFERENCE ADAPTIVE SYSTEMS

Model reference adaptive systems (MRAS) is another approach to adaptive control. The original MRAS concept was developed by Prof. H. P. Whitaker at MIT [63] in connection with work on adaptive flight control systems. The main design goal was to obtain an adaptive servo for a continuous time problem. Superficially MRAS appears to be very different from the STR which was originally developed for stochastic regulation of a discrete time system. Disregarding the types of problems to which STR and MRAS were first developed, it turns out that the algorithms are very similar. The purpose of this section is to explore the connections between STR and MRAS in some detail. The basic principles of MRAS are first presented. A servo design problem is then considered. For systems with known parameters it is shown that the system obtained by applying the pole-placement design can be interpreted as model following. It is then shown that a self-tuning regulator based on implicit identification is equivalent to a model reference system.

Model Reference Adaptive Systems

In MRAS the specifications are given in terms of a reference model which tells how the process output should respond to the command signal. A schematic diagram of an MRAS is shown in Fig. 3. Notice that the reference model is part of the control system. The regulator can be thought of as having two loops. The inner loop is an ordinary control loop composed of the process and a regulator. The parameters of the regulator are adjusted in an outer loop in such a way that the error e between the process

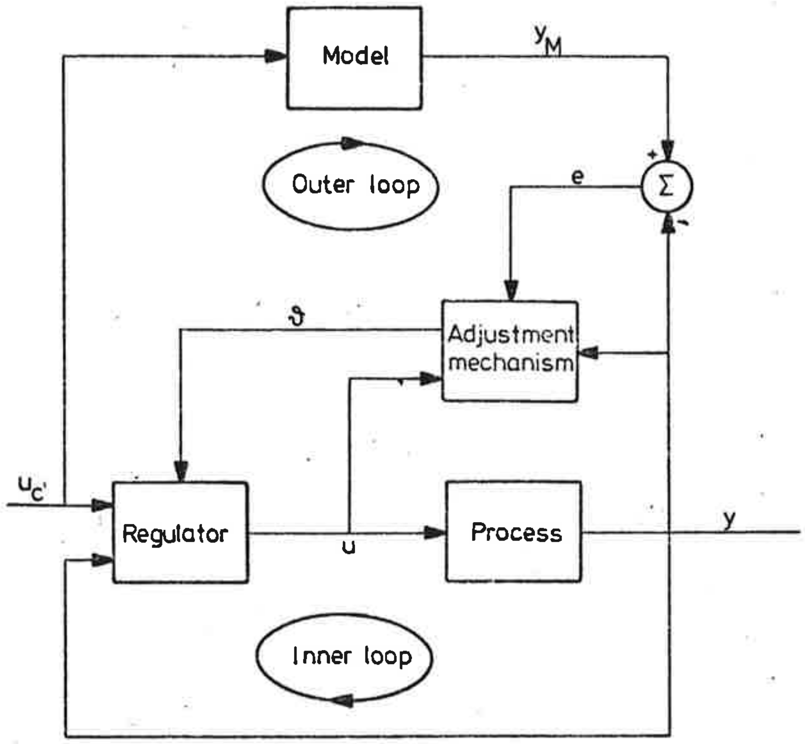

FIGURE 3. Block diagram of model reference adaptive system (MRAS).

output y and the model output y_M becomes small. The key problem is to determine the adjustment mechanism so that a stable system, where the error goes to zero, is obtained.

The following parameter adjustment mechanism, called the 'MIT-rule', was used in the original MRAS [63].

$$\frac{d\nu_i}{dt} = -k\frac{\partial e}{\partial \nu_i}e, \qquad i = 1,\ldots,n. \tag{5.1}$$

The variables $\nu_1,\ldots,\nu_n$ are the adjustable regulator parameters. The quantity e denotes the model error

$$e = y - y_M \tag{5.2}$$

and $\partial e/\partial \nu_i$, $i = 1,\ldots,n$, are the sensitivity derivatives. The constant k is a parameter which determines the adjustment rate.

The MIT rule can be interpreted as an algorithm for minimizing e^2. It was found that the adjustment rule could easily give unstable closed loop systems. A major step forward in the design of MRAS was when Parks [64] showed that Lyapunov stability theory could be used to obtain a modified adjustment rule which guaranteed stability. The modification consisted in replacing the model error and the sensitivity derivative in (5.1) by filtered signals. When the pole excess of the system is greater than one, the filtering proposed by Parks did, however, amount to taking derivatives of the error. The number of derivatives increased with the pole excess. Much work was devoted to the search for modified adjustment rules. The major tools were Lyapunov stability theory [65], [66] and Popov's hyperstability theory [67], [68]. A major innovation was made by Monopoli [69]. He showed that the derivatives that appeared when filtering the error could be eliminated by augmenting the model error e by terms which were different from zero only when the parameter estimates changed. In the adjustment rule proposed by Monopoli the error e in (5.1) was thus replaced by an <u>augmented error</u>. Both the error and the associated sensitivity derivatives were also filtered. The adjustment rules based on the augmented error have been generalized to the discrete time case [70],[71]. It is, however, only recently that correct stability results have been obtained [19],[20],[22],[23]. These stability results are dealt with in detail in [72].

Interpretation of Pole Placement as Model Following

The case of systems with known parameters will first be considered. It was shown in [33] that the regulator obtained by the pole-placement design method discussed in Section 3 can be interpreted as model following control. This is natural because the regulator aims at making the transfer function from the command signal u_c to the output equal to the model (3.2). To see this explicitly the feedback law (3.4) will be rewritten. It follows from (3.8), (3.9), and (3.10) that

$$\frac{T}{R} = \frac{T_1Q_1}{B^+R_1} = \frac{(AR_1 + B^-S)Q_1}{PB^+R_1} = \frac{AQ_1}{B^+P} + \frac{SB^-Q_1}{B^+R_1P} = \frac{A}{B}\frac{Q}{P} + \frac{S}{R}\frac{Q}{P} .$$

The feedback law (3.4) can thus be rewritten as

$$u = \frac{A}{B} y_M + \frac{S}{R} (y_M - y), \tag{5.3}$$

where

$$y_M = \frac{Q}{P} u_c . \tag{5.4}$$

Notice that the signal y_M can be interpreted as the model output. It follows from (4.2) that the control law can be thought of as composed of two parts. There is one feedforward term

$$\frac{A}{B} y_M = \frac{AQ}{BP} u_c \tag{5.5}$$

and one feedback term

$$\frac{S}{R} (y_M - y). \tag{5.6}$$

The feedforward term is generated by feeding the command signal through a cascade combination of the desired model $G_M = Q/P$ and an inverse A/B of the process model. Notice that the transfer function of the inverse process model A/B is not causal. The combination AQ/(BP) is, however, causal because of (3.3). The feedback term (5.6) is obtained by feeding the error e through a system with the transfer function S/R.

Implicit STR and MRAS

It will now be shown that the implicit self-tuning pole placement algorithm I1 is equivalent to an MRAS. For this purpose it is necessary to go into some details of the algorithm.

Introduce

$$\phi(t+1) = [y(t+n_s)\ldots y(t)u(t+n_R)\ldots u(t)]^T, \tag{5.7}$$

where

$$n_s = \deg S$$

$$n_R = \deg R.$$

Since the regulator is causal it follows that $n_R \geqq n_S$. In the implicit algorithm the estimated parameters are equal to the regulator parameters. Hence

$$\theta = \nu = [s_0\ldots s_{n_S}\ r_0\ldots r_{n_R}]. \tag{5.8}$$

The residual ε can then be written as

$$\varepsilon(t+1) = PT_1y(t) - Ru(t) - Sy(t) = PT_1 - \phi^T(t+1)\nu \tag{5.9}$$

where

$$l = \deg PT_1.$$

If recursive least squares parameter estimation is used the formula for updating the parameter estimates is given by (3.16). Since $\nu = \theta$ it follows that

$$\nu(t+1) = \nu(t) + P(t+1)\phi(t+1)\varepsilon(t+1) \tag{5.10}$$

where ϕ is given by (5.7), ε by (5.0), and P by (3.18). Since the estimated parameters θ equals the regulator parameters ν the equation (5.10) can clearly be interpreted as an adjustment rule for the regulator parameters. Notice that it follows from (5.9) that

$$\phi(t) = - \operatorname{grad}_{\nu} \varepsilon(t). \tag{5.11}$$

The vector ϕ can thus be interpreted as a sensitivity derivative. A comparison with (5.1) now shows that (5.12) is a discrete time version of the MIT rule. The main difference is that the model error $e = y - y_M$ in the MIT rule is replaced by the residual ε in the recursive least squares given by (5.9). Another difference is that the parameter k in (5.1) is replaced by the matrix P which is given by (3.18). The matrix P serves two purposes: it modifies the gradient direction and it gives an appropriate step length. While (5.1) can be viewed as a gradient algorithm to minimize e^2 (5.10) can be considered as a quasi-Newton method for minimizing ε^2. To explore the relations between MRAS and STR further it is of interest to find the relations between the model error e and the residual ε.

An Interpretation of the Residual ε

Equation (5.9) gives

$$\varepsilon = PT_1y - Ru - Sy.$$

It follows from (5.9) that the prediction of y(t+1) based on data available at time $t + n_R$ is given by

$$PT_1\hat{y} = \hat{R}u + \hat{S}y.$$

It thus follows that

$$\varepsilon = PT_1(y-\hat{y}) = PT_1[(y-y_n) - (\hat{y}-y_M)]$$

or

$$\varepsilon = PT_1[e - \hat{e}] \tag{5.12}$$

where

$$e = y - y_M$$

is the model error.

It thus follows that the residual ε is obtained by forming the difference $e - \hat{e}$, where $\hat{e}$ is the prediction of e, and filtering the difference with PT_1.

The residual ε is identical to the augmented error introduced by Monopoli [69]. This equivalence was shown in [73-75]. Monopoli was led to introducing the augmented error by very different arguments. It is interesting to see that the augmented error falls out automatically from the STR formulation. Notice that the error estimate $\hat{e}$ can be zero in certain cases. If $\theta(t+1)$ goes to $\theta(t)$ then $\hat{e}$ will be zero. Some control laws are designed so that $\hat{y} = y_M$ which also implies $\hat{e} = 0$. In these cases the residual is simply a filtered model error.

Another Interpretation of the Residual ε

It is also possible to give still another interpretation of the residual ε. Equation (5.9) gives

$$\varepsilon(t+l) = PT_1 y(t) - \hat{R}_{t+l-1} u(t) - \hat{S}_{t+l-1} y(t), \tag{5.13}$$

where the subscripts on R and S have been introduced to denote that the coefficients of the polynomials are based on estimates obtained at time $t + l - 1$. The control signal u is however generated from

$$\hat{R}_{t+n_R} u(t) = Tu_c(t) - \hat{S}_{t+n_R} y(t).$$

It follows from (3.10) that

$$Tu_c = T_1 Q_1 u_c(t) = T_1 P y_M(t).$$

Hence

$$\hat{R}_{t+n_R} u(t) = T_1 P y_M(t) - \hat{S}_{t+n_R} y(t).$$

Subtracting this equation from Equation (5.13) gives

$$\begin{aligned}\varepsilon(t+l) &= PT_1[y(t) - y_M(t)] + [\hat{R}_{t+n_R} - \hat{R}_{t+l-1}]u(t) + \\ &\quad + [\hat{S}_{t+n_R} - \hat{S}_{t+l-1}]y(t) = \\ &= PT_1 e(t) + [\theta(t+n_R) - \theta(t+l-1)]\phi(t+n_R).\end{aligned} \tag{5.14}$$

The residual ε can thus be generated by filtering the model error $e = y - y_M$ and adding a correction term which is proportional to the differences between parameter estimates at times which differ by deg P + deg T_1 - deg R - 1. The correction term will vanish if the parameter estimates converge. The error form (5.14) was used by Landau [71], [86]. The correction term was introduced in order to apply the hyperstability theorem. The notions of 'apriori' and 'aposteriori' reference models were introduced to give an interpretation of the correction term. Again it is interesting to observe that the correction term is obtained directly from the STR formulation.

6. APPLICATIONS

This section gives an overview of different applications of self-tuning regulators.

Control of Basis Weight and Moisture Content on Paper Machines

A schematic drawing of a paper machine is shown in Fig. 4. Basis weight and moisture content are important variables which characterize the quality of the finished product. Control of these quality variables can be conveniently described by stochastic control theory. Disturbances arise from many different sources.

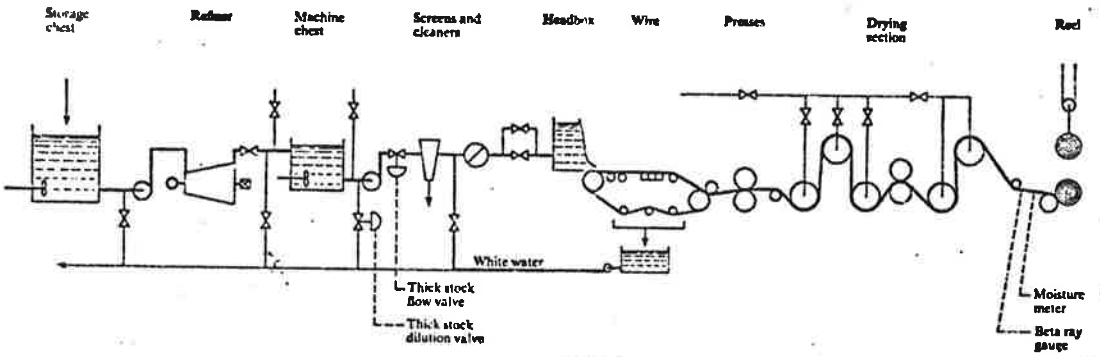

FIGURE 4. Schematic drawing of a paper machine. From [56].

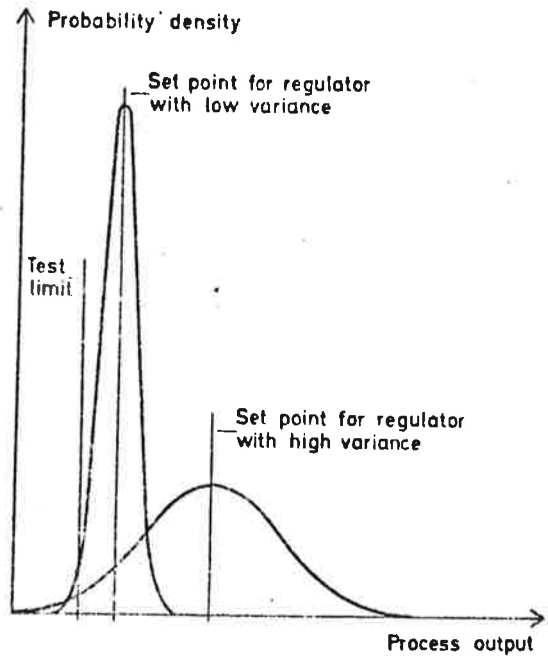

FIGURE 5. Benefits of minimum variance control. By reducing the variance of the output signal, the mean value can be moved closer to the target. From [56].

Their net effect on the output can be modeled as stochastic processes. The purpose of the control is to reduce the fluctuations in the quality variables. This goal is well described mathematically as to minimize the variances of the process outputs. See Fig. 5. There is no natural way to introduce any penalty on the control variables. The thick stock flow valve is the control variable for the basis weight loop and the steam pressure is the control variable for the moisture loop. The process dynamics of both loops are characterized by time delays and low order dynamics. There are couplings in the sense that changes in the thick stock flow valve will influence both basis weight and moisture content. Changes in the steam flow to the drying section influences the moisture content only. There are also interactions in the measuring devices because the beta ray gauge measures both basis weight and moisture content. These couplings can, however, easily be eliminated. Simple processing of the measured signals gives dry basis weight and moisture content. The basis weight is then controlled by a feedback from the dry basis weight signal to the thick stock flow valve and the moisture content is controlled by a feedback from the moisture content signal to the steam flow to the drying section. A feedforward signal e.g. from the thick stock flow measurement to the steam flow to the dryers, is introduced to compensate for the interaction in the process. A more detailed description of the control problem is given in [56], [76], and [77].

It has been verified by experiments on many plants that the dynamics of the plant and the disturbances can be adequately described by an ARMAX process of low order. See [56]. By identification of process dynamics and disturbance characteristics based on plant experiments it was demonstrated that substantial improvements over PID control could be achieved by using a basis weight regulator having the form

$$u(t) = -\frac{s_0 + s_1 z^{-1}}{1 + r_1 z^{-1} + r_2 z^{-2} + r_3 z^{-3}}\, y(t). \tag{6.1}$$

The parameters of a regulator like (5.1) can not be conveniently tuned by hand. Instead the parameters were determined by system identification based on plant experiments and control design as described in [56]. Such a procedure is comparatively costly because it requires appropriate identification software and skilled personnel. To obtain a reasonably good model it is necessary to experiment on the plant for at least 2 hours. It was, therefore, attempted to tune the parameters with a self-tuning regulator. Since the criterion was well defined and since the sampling period and the regulator structure could be chosen based on prior experience it was very straightforward to apply the self-tuning regulator.

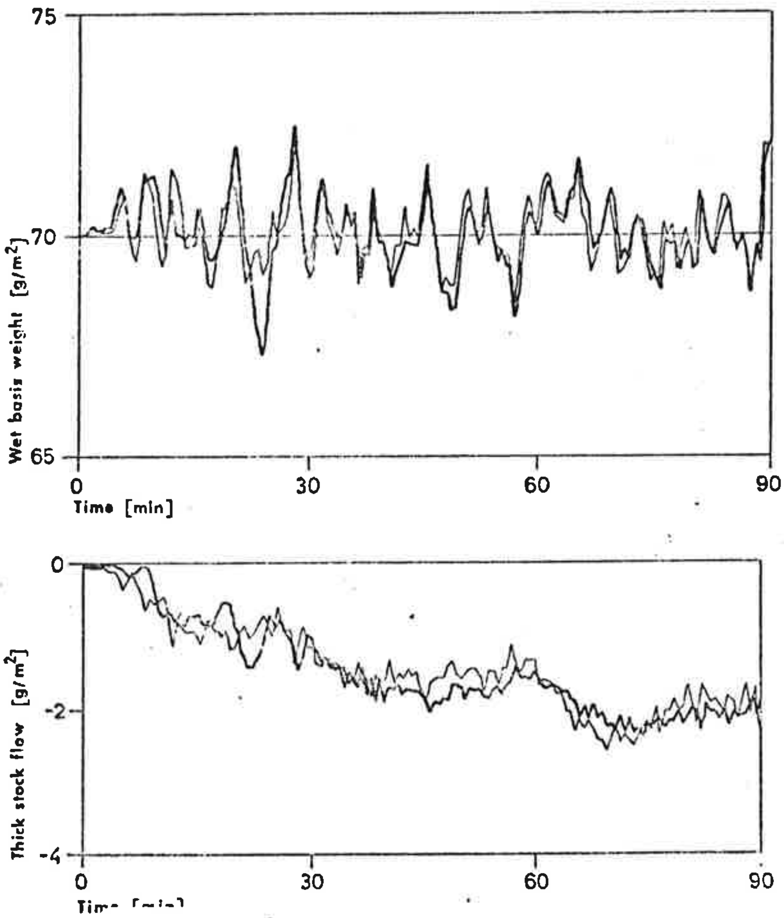

FIGURE 6. Simulation of minimum variance control (thin lines) and self-tuning control (thick lines) of basis weight of a paper machine. From [59].

Process control computers were available at the plant. The code for the simple self-tuning regulator based on least squares identification and minimum variance control (Algorithm IMV) was simply introduced as a special control algorithm, and experiments were started. A typical result is illustrated by the simulation shown in Fig. 6. This simulation is based on measured plant disturbances and models for the process dynamics estimated from plant experiments. In Fig. 6 the self-tuning regulator was initialized with all estimates equal to zero. It is seen from Fig. 6 that the fluctuations in basis weight obtained from the self-tuning regulator are larger than those obtained from the minimum variance regulator for the first 30 minutes. After 30 minutes there are, however, very small differences between the outputs of the two regulators. It is perhaps even more instructive to look at the control signals generated by the different regulators. It is seen from Fig. 6 that the self-tuner is fairly sluggish in the initial period. After 30 minutes there are, however, only minor differences in the control signal. Results similar to those shown in Fig. 6 were obtained when controlling the actual plants. On typical basis weight and moisture control loops the self-tuner will give close to optimal performance after a tuning period of 15 minutes to 2 hours.

In the example shown in Fig. 6 no apriori information about the parameters was assumed. In many applications in the paper mill it is possible to start the algorithm with reasonable estimates. The convergence time will then be shorter. The tuning time should be compared with the time required to make a good identification experiment. This time is between 2 and 5 hours. In the paper machine applications there have not been any problems with phenomena like 'turn off' or 'covariance blow up'. One reason is that the disturbances are persistent and fairly stationary.

In the paper mill the self-tuning regulator was used as a tuner, since the chief instrument engineer did not like the parameters of important control loops to be changed in his absence. The simple self-tuning regulator has been applied to many simple flow and level loops, to basis weight and moisture loops [78] on several different paper machines and also to recovery boilers in the paper mill.

It is much easier to implement a simple self-tuning regulator than to go through the procedure of process experiments, system identification and control design. It is thus clear that the self-tuning regulator gives a considerable saving of engineering work compared to previously used methods.

Control of a Heat Exchanger

This application is described in [79]. A schematic drawing of the heat exchanger is shown in Fig. 7. This type of heat exchanger is a common component in heating and ventilation systems. The warm air leaving a room gives away some of its enthalpy to the fresh air supplied to the room. The warm and cold air streams pass through the rotor which has axial channels. The rotor is made of a material which can absorb heat and moisture. The enthalpy

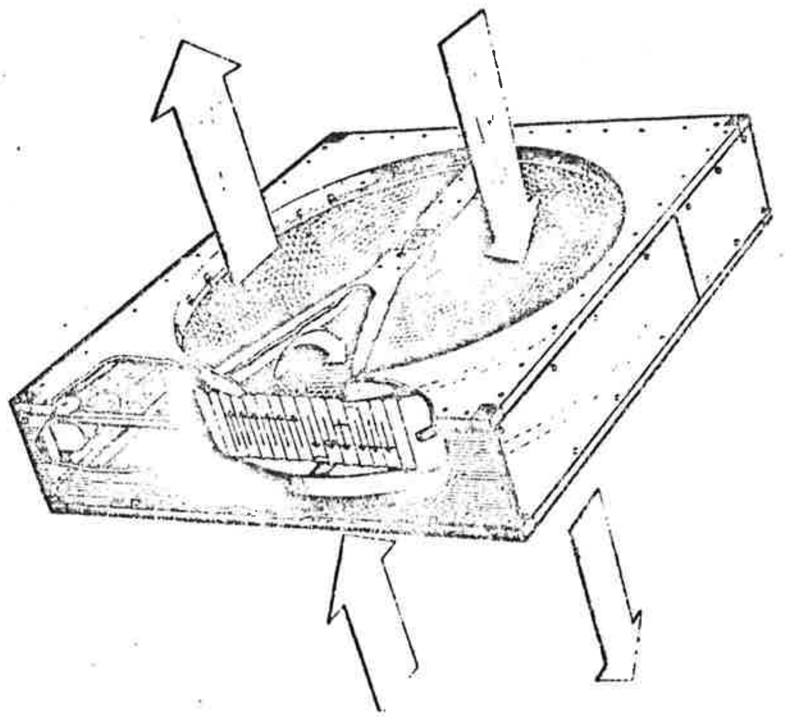

FIGURE 7. Schematic drawing of a heat exchanger.

is exchanged because the rotor segments are alternating between the warm and cold streams when the rotor rotates. The control problem is to adjust the angular velocity of the rotor in such a way that the room temperature is constant. The disturbances are due to sunshine, heat generated from people and other heat sources. In the particular case there are also sensor noise because the sensors are thermistors in the air streams. It is desirable to keep temperature fluctuations small but in contrast with the paper machine example there is no natural loss function. For a fixed operating condition it is easy to obtain good control simply by using integrating feedback from room temperature to rotor speed. The major difficulty is that the gain of the process changes drastically with operating conditions. It was not possible to find an auxiliary variable to schedule the controller gain since the gain depends on moisture temperature and rotor speed. A particular regulator structure was chosen. This structure was derived based on knowledge of the physics of the process.

The primary controlled variable is chosen as the thermal efficiency defined by

$$v = \frac{T_{cout} - T_{cin}}{T_{win} - T_{wout}} \tag{6.2}$$

where the subscript c stands for cold and w for warm air. The thermal efficiency v is first computed based on measurements of T_{cin}, T_{win}, T_{wout}, and knowledge of the desired T_{cout}. The advantage of choosing this variable as the controlled variable rather than the cold outlet temperature is that the effect on the control loop of some of the process nonlinearities are eliminated at the price of three extra thermistors. The thermal efficiency is then controlled by a feedback from the computed thermal efficiency to commanded rotor rotation speed.

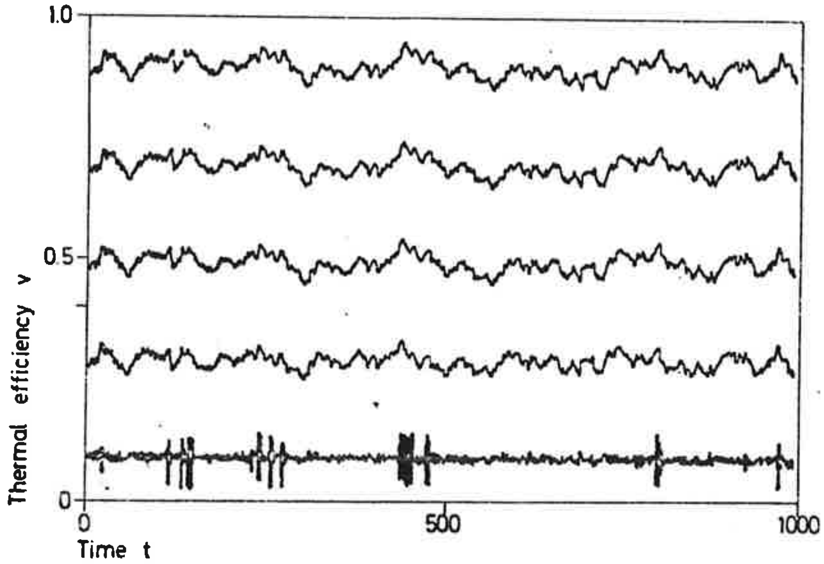

FIGURE 8. Thermal efficiency of a heat exchanger controlled by an integrating controller having fixed gain.

The dynamics relating thermal efficiency to rotor speed is characterized by a time delay and low order dynamics. The major difficulty in controlling the heat exchanger is that the static gain varies considerably. In steady state the relation between rotor speed u and thermal efficiency v is approximately given by

$$v = f(u) = \frac{au}{1 + au} . \tag{6.3}$$

If the units are chosen in such a way that $0 =< u =< 1$ then a may be 20 in a typical case. The static gain thus varies between 1 and 20.

The heat exchanger can be controlled by an integrating controller. Due to the gain variations there are, however, difficulties when a regulator with constant gain is used. This is illustrated by the simulation in Fig. 8. It is clearly seen from this figure that the loop gain is too high at low levels and too low at high levels of efficiency. If the relation (6.3) was accurate and did not change with time then Equation (6.3) could be used to schedule the gain of the controller as a function of the thermal efficiency. This is unfortunately not possible because the parameter a in (6.3) changes with time, temperature, and moisture content. A simple self-tuning regulator was therefore used to eliminate the gain variations. The self-tuning regulator is based on estimation of the parameter b in the model

$$v(t+1) - v(t) = b[u(t) - u(t-1)] = b\nabla u(t) \tag{6.4}$$

by least squares. In this case the estimate is particularly simple since only one parameter is estimated. The estimate is given by the following equations:

$$\hat{b}(t+1) = \hat{b}(t) + P(t+1)\nabla u(t)\varepsilon(t+1)$$

$$\varepsilon(t+1) = \nabla v(t) - \hat{b}(t)\nabla u(t)$$

$$P(t+1) = \frac{P(t)}{\{\lambda + P(t)[\nabla u(t)]^2\}}.$$

The variable v denotes the thermal efficiency and u denotes the angular velocity of the rotor. Having obtained the estimate $\hat{b}$ of b the following control law is then used:

$$u(t) = u(t-1) - (k_0/\hat{b})v(t) \tag{6.5}$$

where k_0 is an empirical constant. The 'cautious' control law

$$u(t) = u(t-1) - \frac{k_0\hat{b}}{\hat{b}^2 + P}v(t) \tag{6.6}$$

was also tried but there was little difference in performance compared to the certainty equivalence control (6.5).

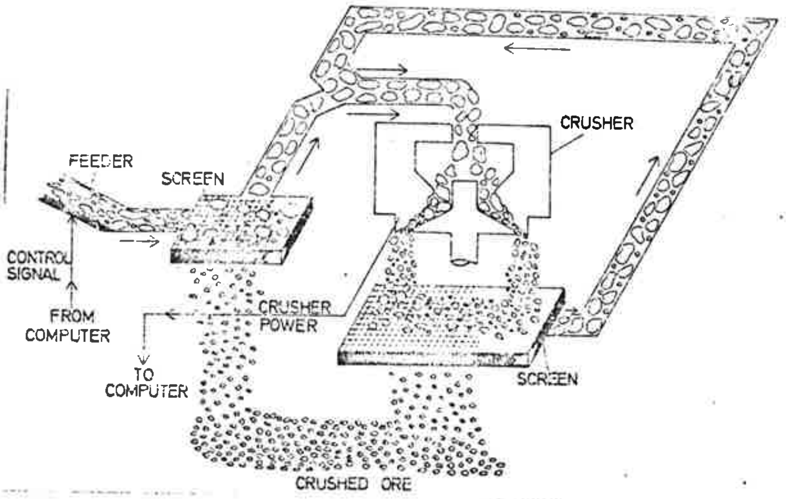

FIGURE 10. Schematic drawing of an ore crushing plant. From [80].

small ore lumps are separated from the large lumps. The larger ore lumps are transported to the crusher where the lumps are crushed. After the crusher there is another separation screen. Lumps with a diameter larger than 2.5 cm are recirculated to the crusher. The crusher is driven by an electric motor via a slip clutch which releases the motor and stops the line if the torque is too high. The control variable is the amount of ore fed into the line and the controlled variable is the power of the crusher motor. The goal of the control is to keep production as high as possible while avoiding overloading. This can be formulated approximately as to reduce the variance in the crusher power. By the usual argument, illustrated by Fig. 5, the set point of the crusher power can be moved closer to the target and the average production is increased as a consequence. The trade-off between high production and risk for overload is reflected in the choice of set point. The disturbances are due to variations in lumpsize, crushability, and variations of the crusher characteristics due to wear. The plant dynamics is characterized by a time delay of

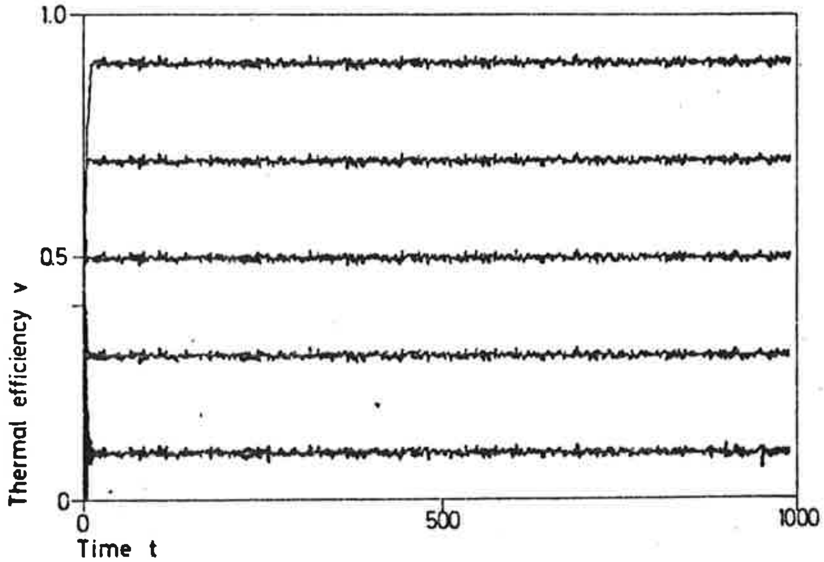

FIGURE 9. Thermal efficiency of the heat exchanger with a self-tuning integrating regulator.

The performance of the self-tuning regulator is illustrated by the simulation results shown in Fig. 9. The models and the disturbances were the same as when generating the results shown in Fig. 8. The Figures 8 and 9 are thus directly comparable.

It is clear that the self-tuning regulator handles the gain variations very well. The behaviour of the self-tuning regulator on the actual plant is similar to that shown in the simulations. See [79].

Control of an Ore Crushing Plant

This example is described in detail in [80]. A schematic drawing of the process is shown in Fig. 10. The plant consists of an ore bin, a feeder, two screens, an ore crusher, and conveyor belts. The ore is transported from the bin to a screen, where the

70-80 s in the recycle loop and time constants of 10-20 s in the crusher itself.

When the experiments were started there was very little apriori knowledge of the characteristics of the process and its environment. Some step responses were therefore determined initially. An interesting aspect of the experiments was that they were performed using teleprocessing between a plant in Kiruna in northern Sweden and a computer in Lund in southern Sweden. The distance between the two places is about 1800 km.

It was decided to try the simple self-tuning regulator based on minimum variance control (Algorithm IMV). Based on the time delays of the process it was decided that a sampling period of 20 s was reasonable. Since the time delay of the process was 40-50 s the parameter k in the self-tuner should have the value 3 or 4.

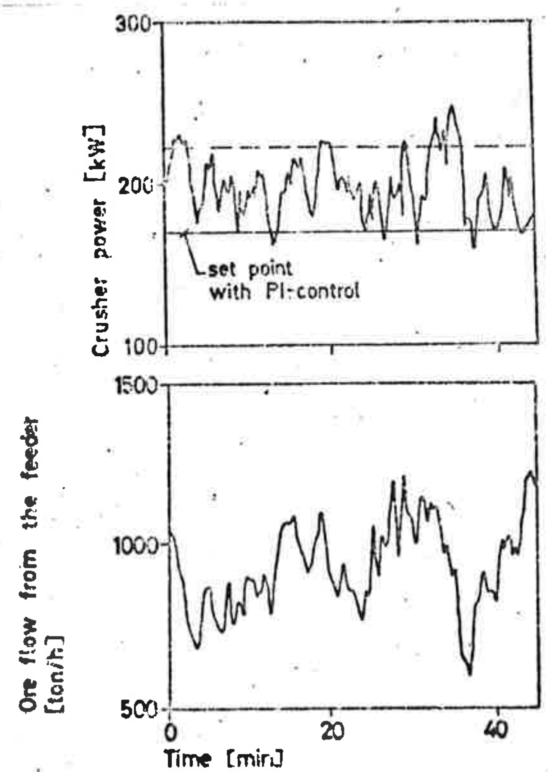

FIGURE 11. Results of an ore crusher experiment [79].

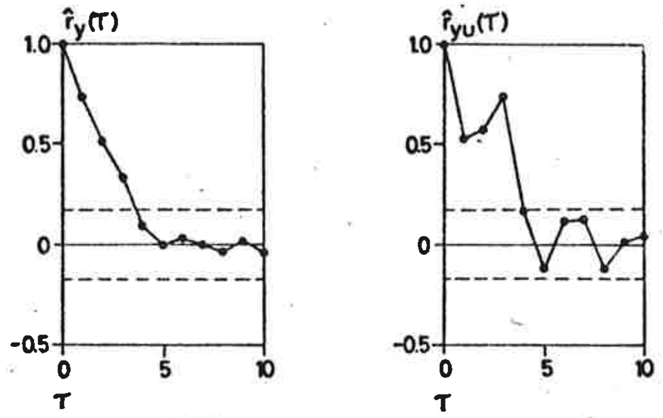

FIGURE 12. Covariance functions. From [80].

It was found experimentally that k = 4 worked better than k = 3. The complexity of the regulator was determined experimentally by controlling the plant with regulators having different complexity. The sample covariance function and the cross covariance between the output and the control variable were determined. The complexity of the regulator was increased until the conditions

$$\begin{aligned} r_y(\tau) &= 0, \quad \tau > k \\ r_{yu}(\tau) &= 0, \quad \tau > k \end{aligned} \tag{6.7}$$

which hold for the minimum variance controller, were fulfilled. See [56]. It was found that a simple self-tuning regulator with deg R = 3 and deg S = 4 was performing well. The forgetting factor λ was also determined empirically. The value $\lambda = 0.99$ was chosen after some experimentation. The value $\lambda = 0.95$ was found to be slightly better during start-up and during periods with a high variability in the ore properties. The results of one experiment are illustrated in Figs. 11, 12, and 13.

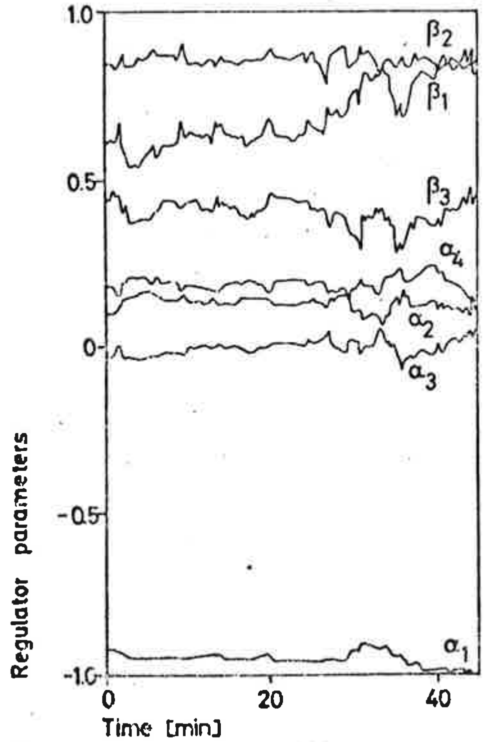

FIGURE 13. Parameter estimates. The parameter r_0 is fixed. From [79].

An Autopilot for Ship Steering

This application is described in detail in [12] and [81] where many additional references are given. An ordinary autopilot for a ship is based on feedback from measurements of heading (and possibly also the rate of change of the heading) to the rudder angle. A PID algorithm is commonly used. An autopilot has different functions. It should be able to maintain the ship at a constant course and it should be able to handle maneuvers. The dynamics of a ship will change with speed, trim, loading, and water depth. The characteristics of the disturbances will also change considerably with weather and wind. Although, it is in many cases possible to find constant settings of an ordinary autopilot which will guarantee stability over a wide range of operating conditions, there is a considerable advantage in having an adaptive autopilot. It is a common experience on tankers that ordinary autopilots do not work well in bad weather. The reason is partly that the PID algorithm is too simple to handle the requirements and partly that proper tuning to different weather conditions is required.

The design of an autopilot for straight course keeping can be formulated as a stochastic control problem. The ship dynamics can be described as a linear dynamical system and the disturbances can be characterized as random processes. Fortunately there is also a natural loss function which fits well into the stochastic control formulation. It can be shown by hydrodynamic theory that the average increase in drag due to yawing and rudder motion can be approximately described by

$$\frac{\Delta R}{R} = \mu(\overline{\psi}^2 + \zeta\overline{\delta}^2), \tag{6.8}$$

where R is the drag, ψ the heading deviation, δ the rudder angle, and $\overline{\psi}^2$ denotes the quadratic mean value. The values $\mu = 0.014$ deg^{-2} and $\zeta = 0.1$ are typical for a tanker. It is thus natural to use the criterion

$$V = \frac{1}{T}\int_0^T \{[\psi(t) - \psi_{ref}(t)]^2 + \zeta\delta^2(t)\}dt \tag{6.9}$$

as a basis for the design and evaluation of autopilots for steady state course keeping. One unit of the loss function would then correspond to an increase of 1.4% of the average drag or about the same increase in fuel consumption.

Since the design of an autopilot can be formulated as a linear quadratic stochastic control problem an adaptive autopilot can be designed using the corresponding self-tuning regulator. Several such designs have been made, simulated and field tested. The

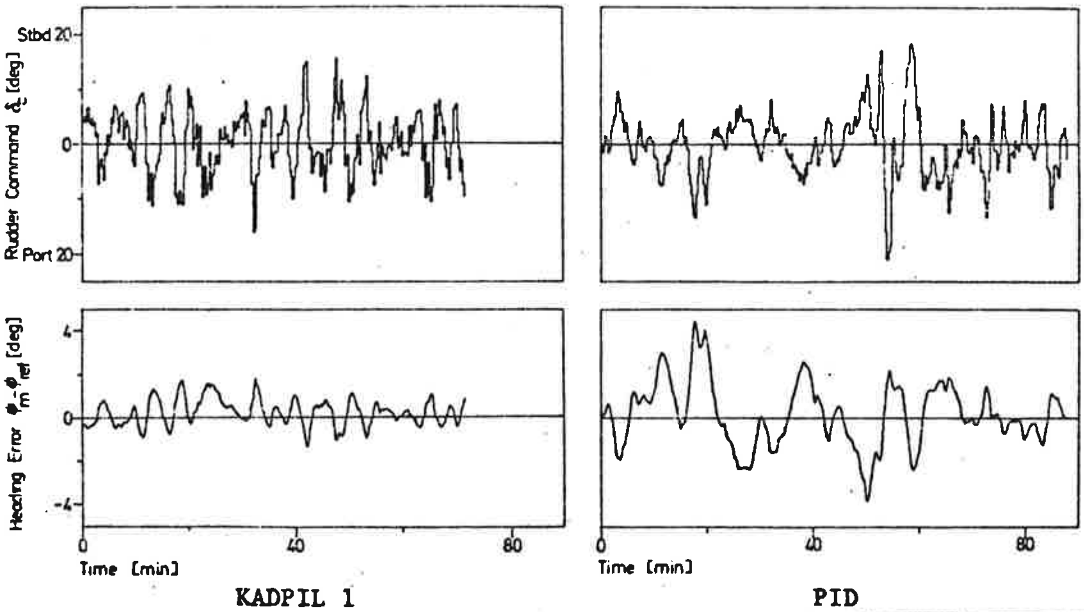

FIGURE 14. Results from an experiment with a 255,000 ton tanker comparing a well-tuned PID regulator with a self-tuning regulator. The values of the loss function (6.9) are 3.7 for the adaptive and 5.6 for the PID control. From [81].

experiments have been carried out on several large tankers. Available on board computers were used in the experiments. Due to memory constraints several simplifications were made. Since the LQG self-tuners require the solution of Riccati equations, which is space consuming, it was also attempted to use the simple self-tuner based on minimum variance control. This was reasonably successful provided that the prediction horizon was chosen appropriately. Extensive comparisons between different regulator structures including well-tuned PID-regulators were made. One comparison is illustrated in Fig. 14. It follows from (6.8) that the adaptive autopilot reduces the drag by 2.7% in comparison with an ordinary autopilot based on PID control. In these comparisons the ordinary autopilot was at all times retuned for best performance.

The magnitude of the improvements depends on the operating conditions. The largest improvements are found in bad weather when the ship is fully loaded. The self-tuning ship steering autopilot has been tried on several different tankers. On one ship it has been in continuous operation for several years.

7. CONCLUSIONS

A systematic way to design self-tuning regulators has been presented. The basic idea is very straightforward. A design procedure for a system with known parameters which fits the particular application is first chosen. When the parameters of the process are not known they are estimated recursively and the regulator parameters are recalculated at each step using the updated estimates. In some cases the model can be reparametrized so that the design calculations are avoided. Since there are many different design methods and many different parameter estimation methods, there are consequently a large variety of self-tuning regulators. So far only a small number of the available combinations have been explored. Some comments on the theory and practice of self-tuning regulators will now be given.

Theory

There are many interesting and important theoretical problems associated with self-tuning regulators. Stability, convergence, and performance are some of the key problems, but there are also many other questions like convergence rate and parametrization which are of considerable interest.

The stability problem is, of course, of major interest both theoretically and practically. There have recently been major advances in stability theory for self-tuners. Most of the results are, however, limited to the simple self-tuner based on least squares estimation and minimum variance control [59] or its direct multivariable generalization [82],[83]. The simple self-tuner will work well only for minimum-phase systems. Conditions for stability when there are no disturbances are given in [20]. The corresponding results for the case of bounded disturbances are found in [19]. Mean square stability for the stochastic case is investigated in [21]. Similar results for the continuous time problem are given in [22] and [23]. The theoretical results are also limited because of the assumptions made. The unpleasant assumption is that an upper bound of the order of the system must be known. This bound also determines the complexity of the self-tuner. In practice this is highly unrealistic because the process to be controlled will be of very high order (probably infinite dimensional) and the self-tuning regulator will be based on a simplified model. It is shown in [84] that regulators based on drastically simplified models can actually work very well.

The convergence problem has also been investigated for the simple self-tuners [58] based on least squares and minimum variance control. The surprising result that the correct regulator is the only possible equilibrium point, even if there is structural mismatch between the model and the process, is also given in [59]. Ordinary differential equations, which describe the mean paths of the parameter estimates, are derived in [17]. Necessary conditions for local stability around the equilibrium points are given in [85].

In summary there have recently been very interesting results in the theory of self-tuning regulators. It would be useful to eliminate some of the assumptions made. It would also be highly desirable to extend the analysis to other than the most simple self-tuning algorithms.

Practice

The basic algorithm for a PID regulator is very simple:

$$u = K[e + \frac{1}{T_I}\int^t e(s)\, ds + T_D \frac{de}{dt}].$$

An implementation of this algorithm in analog or digital hardware does, however, not necessarily give a good controller. In practice it is also necessary to consider operator interface, filtering of the signals, automatic/manual transfer, bumpless parameter changes, reset windup, nonlinear output, like gap, and saturation, etc. Notice that many of these functions involve nonlinearities which are not easily analyzed. Whether a PID regulator works well in an industrial environment depends very much upon how well the problems discussed above are handled. Similar things also apply to self-tuning regulators. The list is, however, longer because the basic algorithm is more complicated than the PID algorithm. For example, windup occurs in a PID regulator because the integrator in the algorithm could achieve large values if the control value saturates or if it is driven manually. In a self-tuner with a forgetting factor windup can also occur in the estimator. The self-tuning regulator can operate in many different modes like estimation only, tuning etc. The problem of operator interface is particularly important. A key problem is how the specifications are entered and how an operator should interact with the controller. There are many different possibilities ranging from the case where there are no buttons at all on the panel to fairly complicated operator interfaces. Certainly there are many interesting possibilities as is illustrated on self-tuning regulators which are already on the market or which are in the process of coming out.

Uses of Self-Tuners

Self-tuning regulators can be used in many different ways. Since the regulator becomes an ordinary constant gain feedback if the parameter estimates are kept constant, the self-tuner can be used as a tuner to adjust the parameters of a control loop. In such an application the self-tuner is connected to the process and run until satisfactory performance is obtained. The self-tuner is then disconnected and the system is left with the constant parameter regulator obtained. This mode of using the self-tuner is convenient to implement in a package for direct digital control (DDC-package). The DDC-package is simply provided with a tuning routine which can be connected to an arbitrary loop in the package.

The self-tuner can also be used to build up a gain schedule. In such a case the system is run at different operating points and the controller parameters obtained are stored. When the process has been run at a sufficient number of operating points a table for scheduling the controller parameters can be generated by interpolation and smoothing of the values obtained.

The self-tuner can, of course, also be used as a truly adaptive controller for systems with varying parameters. In cases where rapid adaptation over widely varying operating conditions are required combinations between gain-scheduling and self-tuning can also be considered.

Abuses of Self-Tuners

Compared with a three-term controller the self-tuner is a sophisticated controller. Such a controller can, of course, be misused. The self-tuner should certainly not be used if a simpler controller will do the job. Before considering a self-tuning regulator it is, therefore, useful to check if a constant parameter regulator or a regulator with gain scheduling will do the job. When designing a self-tuning regulator it is also useful to consider the particular application carefully and decide upon a design method which is suitable for the particular problem if a model for the process and its environment are known. A parameter estimation scheme which works wehl for the particular situation should also be chosen before the details of the design are considered.

Summary

The word self-tuning regulator may lead to the false conclusion that such regulators can be switched on and used blindly without any apriori considerations. This is definitely not true. The self-tuning regulator is a fairly complex control law. A proper design involves the choices of gross features like underlying design and estimation methods and decisions on details like initialization, selection of parameters, and safeguard methods. Proper choices require insight and knowledge. There are known cases where bad choices have been disastrous. All theory required is definitely not available. Based on experiences from a few applications I believe, however, that self-tuning regulators can and will be used profitably, even if some of their properties are not fully understood theoretically. It is thus my hope that this paper may inspire some of you to acquire the appropriate knowledge and try some schemes of your own. It is also my hope that some of you will tackle the important theoretical problems that remain.

ACKNOWLEDGMENTS

I have had the pleasure to work with adaptive and self-tuning regulators for a long time. Much insight and knowledge, which is reflected in this paper, was obtained when working with students who did their Ph.D. work in this area: B. Wittenmark, L. Ljung, T. Söderström, I. Gustavsson, U. Borisson, J. Sternby, L. Jensen, J. Holst, C. G. Källström, and B. Egardt. The benefit derived from this is gratefully acknowledged.

REFERENCES

1. Gregory, P. C. (editor), "Proceedings of the Self Adaptive Flight Control Systems Symposium," WADC Technical Report 59-49, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, March 1959.
2. Mishkin, E., and Braun, L., "Adaptive Control Systems," McGraw-Hill, 1961.
3. Unbehauen, H., and Schmid, Chr., "Status and Application of Adaptive Control Systems," Automatic Control Theory and Applications, 3 pp. 1-12, 1975.
4. Asher, R. B., Adrisani, D., and Dorato, P., "Bibliography on Adaptive Control Systems," Proc. IEEE, 64, pp. 1126-1240, 1976.
5. Clarke, D. W., and Gawthrop, P. J., "Implementation and Application of Microprocessor-Based Self-Tuners," Preprints 5th IFAC Symposium on Identification and System Parameter Estimation, Darmstadt, September 1979, pp. 197-208.
6. Neuman, C. P., and Morris, R. L., "Mini and Micro Computer Implementation of Adaptive Control Systems," Proc. First International Symposium on Mini- and Microcomputers in Control, San Diego, January 1979.

7. Neuman, C. P., and Baradello, C. S., "Microcomputer Implementation of Adaptive Controllers," Proc. First International Symposium on Mini- and Microcomputers in Control, San Diego, January 1979.

8. Åström, K. J., Borisson, U., Ljung, L., and Wittenmark, B., "Theory and Applications of Self-Tuning Regulators," Automatica, 13, pp. 457-476, 1977.

9. Dumont, G. A., and Bélanger, P. R., "Self-Tuning Control of a Titanium Dioxide Kiln," IEEE Trans., AC-23, pp. 532-537, 1978.

10. Kurz, H., Isermann, R., and Schumann, R., "Development, Comparison and Application of Various Parameter-Adaptive Digital Control Algorithms," Proc. 7th IFAC World Congress, Helsinki, 1978.

11. Keviczky, L., Hetthéssy, J., Hilger, M., and Kolostori, J., "Self-Tuning Adaptive Control of Cement Raw Material Blending," Automatica, 14, pp. 525-532, 1978.

12. Källström, C. G., Åström, K. J., Thorell, N. J., Eriksson, J., and Sten, L., "Adaptive Autopilots for Large Tankers," Proc. 7th IFAC World Congress, Helsinki, 1978.

13. Irving, E., "Improving Power Network Stability and Unit Stress with Adaptive Generator Control," Automatica, 15, pp. 31-46, 1979.

14. Buchholt, F., Clement, K., and Bar Jorgensen, S., "Self-Tuning Control of a Fixed Bed Chemical Reactor," Preprints 5th IFAC Symposium on Identification and System Parameter Estimation, Darmstadt, September 1979, pp. 1213-1218.

15. Haber, R., Hetthéssy, J., Keviczky, L., Vajk, I., Fehér, A., Czeiner, N., Császar, Z., and Turi, A., "Identification and Adaptive Control of a Glass Furnace by a Portable Process Computer Laboratory," Preprints 5th IFAC Symposium on Identification and System Parameter Estimation, Darmstadt, September 1979, pp. 209-220.

16. Zinober, A.S.I., "The Self-Adaptive Control of Overhead Crane Operations," Preprints 5th IFAC Symposium on Identification and System Parameter Estimation, Darmstadt, September 1979, pp. 1161-1168.

17. Ljung, L., "Analysis of Recursive Stochastic Algorithms," IEEE Trans. AC-22, pp. 551-575, 1977.

18. Ljung, L., "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes," IEEE Trans. AC-22, pp. 539-551, 1977.

19. Egardt, B., "Stability of Model Reference Adaptive and Self-Tuning Regulators," Thesis, CODEN: LUTFD2/(TFRT-1017)/1-163/, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1978.

20. Goodwin, G. C., Ramadge, P. J., and Caines, P. E., "Discrete Time Multivariable Adaptive Control," Report Div. of Applied Science, Harvard University, Cambridge, Mass., U.S.A., November 1978.

21. Goodwin, G. C., Ramadge, P. J., and Caines, P. E., "Discrete Time Stochastic Adaptive Control," Report Div. of Applied Science, Harvard University, Cambridge, Mass., U.S.A., December 1978.

22. Narendra, K. S., Lin, Y.-H., and Valavani, L. S., "Stable Adaptive Controller Design - Part II: Proof of Stability," S&IS Report 7904, Yale University, April 1979.

23. Morse, A. S., "Global Stability of Parameter-Adaptive Control Systems," S&IS Report 7902R, Yale University, March 1979.

24. Bellman, R., "Adaptive Processes - A Guided Tour," Princeton University Press, 1961.

25. Bohlin, T., "Optimal Dual Control of a Simple Process with Unknown Gain," Report TP 18.196, IBM Nordic Laboratory, Lidingö, Sweden, 1969.

26. Patchell, J. W., "On the Structure of Stochastic Control Laws," Dr. Phil. Dissertation, University of Oxford, May 1971.

27. Åström, K. J., Wittenmark, B., "Problems of Identification and Control," J. Mathematical Analysis and Applications, 34, pp. 90-113, 1971.

28. Sternby, J., "Topics in Dual Control," Thesis, CODEN: LUTFD2/(TFRT-1012)/1-135/, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1977.

29. Florentin, J. J., "Optimal Probing, Adaptive Control of a Simple Bayesian System," Int. J. on Control, 13, pp. 165-177.

30. Feldbaum, A. A., "Dual Control Theory I - IV," Automation & Remote Control, 21, pp. 874-880, 21, pp. 1033-1039, 22, pp. 1-12, 22, pp. 109-121, 1960.

31. Horowitz, I. M., "Synthesis of Feedback Systems," Academic Press, New York, 1963.

32. Åström, K. J., "Reglerteori (Control Theory)," Almqvist & Wiksell, Stockholm 1967, 2nd edition 1976 (in Swedish).

33. Åström, K. J., "Algebraic System Theory as a Tool for Regulator Design," in Halme et al. (eds.), "Topics in System Theory," Publication in honour of Professor Hans Blomberg on the occasion of his sixtieth birthday on December 18th, 1979. Acta Polytechnica Scandinavica, Ma 31, Helsinki 1979. Also available as report CODEN: LUTFD2/(TFRT-7164)/1-023/, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1979.

34. Söderström, T., Ljung, L., and Gustavsson, I., "A Comparative Study of Recursive Identification Methods," Report TFRT-3085, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1974.

35. Lawson, C. L., and Hanson, R. J., "Solving Least Squares Problems," Prentice-Hall, Englewood Cliffs, New Jersey, 1974.

36. Peterka, V., "A Square-Root Filter for Real-Time Multivariable Regression," Kybernetica, 11, pp. 53-67, 1975.

37. Bierman, G. J., "Factorization Methods for Discrete Sequential Estimation," Academic Press, New York, 1977.

38. Thornton, C. L., and Bierman, G. J., "Filtering and Error Analysis via the UDU Covariance Factorization," IEEE Trans., AC-23, pp. 901-907, 1978.

39. Levinson, N., "The Wiener RMS (Root Mean Square) Error in Filter Design and Prediction," J. Math. Phys., 28, pp. 261-278, 1947.

40. Kailath, T., "Some New Algorithms for Recursive Estimation in Constant Systems," IEEE Trans., IT-19, pp. 750-760, 1973.

41. Lindquist, A., "Optimal Filtering of Continuous-Time Stationary Processes by Means of the Backward Innovation Process," Siam J. Control, 12, pp. 747-754, 1974.

42. Fortescue, T. R., Kershenbaum, L. S., and Yostie, B. E., "Implementation of Self Tuning Regulators with Variable Forgetting Factors," Report, Dept. of Chem. Eng. and Chem. Technology, Imperial College, London, 1978.

43. Wittenmark, B., "A Self-Tuning Regulator," Thesis, TFRT-1003, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1973.

44. Morris, A. J., Fenton, T. P., and Nazer, Y., "Application of Self-Tuning Regulator to the Control of Chemical Processes," in H. R. van Nauta Lemke and H. B. Verbruggen (eds.), "Digital Computer Applications to Process Control," preprints 5th IFAC/IFIP Int. Conf., The Hague, Netherlands, 1977.

45. Narendra, K. S. and Valavani, L. S., "Direct and Indirect Adaptive Control," Automatica, to appear.

46. Åström, K. J., Westerberg, B., and Wittenmark, B., "Self-Tuning Controllers Based on Pole-Placement Design," Report, CODEN: LUTFD2/(TFRT-3148)/1-052/, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1978.

47. Åström, K. J., "New Implicit Adaptive Pole-Zero Placement Algorithms for Nonminimum-Phase Systems," Report, CODEN: LUTFD2/(TFRT-7172)/1-021/, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1979.

48. Åström, K. J., "Simple Self-Tuners I," Report CODEN: LUTFD2/(TFRT-7184)/1- /, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1979.

49. Clarke, D. W., and Gawthrop, B. A., "Self-Tuning Controller," Proc. IEE 122, pp. 929-934, 1975.

50. Åström, K. J., and Wittenmark, B., "Self-Tuning Controllers Based on Pole-Zero Placement," Report, CODEN: LUTFD2/(TFRT-7180)/1-43/, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1979.

51. Clarke, D. W., and Gawthrop, P. J., "Self-Tuning Control," Proc. IEE 126, pp. 633-640, 1979.

52. Elliott, H., and Wolovich, W. A., "Parameter Adaptive Identification and Control," IEEE Trans., AC-24, pp. 592-599, 1979.

53. Wouters, W.R.E., "Adaptive Pole Placement for Linear Stochastic Systems with Unknown Parameters," Preprints IEEE Conference on Decision and Control," New Orleans, 1977.

54. Wellstead, P. E., Edmunds, J. M., Prager, D., and Zanker, P., "Self-Tuning Pole/Zero Assignment Regulators," Int. J. Control, 30, pp. 1-26, 1979.

55. Wellstead, P. E., Prager, D., and Zanker, P., "Pole Assignment Self-Tuning Regulator," Proc. IEE 126, pp. 781-787, 1979.

56. Åström, K. J., "Introduction to Stochastic Control Theory," Academic Press, New York, 1970.

57. Åström, K. J., "Stochastic Control Problems," in Coppel, W.A., (ed.), "Mathematical Control Theory. Lecture Notes in Mathematics," Springer-Verlag, Berlin, 1978.

58. Åström, K. J., and Wittenmark, B., "On the Control of Constant but Unknown Systems," Proc. 5th IFAC World Congress, Paris, 1972.

59. Åström, K. J., and Wittenmark, B., "On Self-Tuning Regulators," Automatica, 9, pp. 185-199, 1973.

60. Peterka, V., "Adaptive Digital Regulation of Noisy Systems," Proc. 2nd IFAC Symposium on Identification and Process Parameter Estimation, Prague, 1970.

61. Åström, K. J., "A Self-Tuning Regulator for Non-Minimum Phase Systems," Report TFRT-3113, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1974.

62. Åström, K. J., and Wittenmark, B., "Analysis of a Self-Tuning Regulator for Nonminimum Phase Systems," Preprints IFAC Symposium on Stochastic Control, Budapest, September 1974.

63. Whitaker, H. P., Yamron, J., and Kezer, A., "Design of Model-Reference Adaptive Control Systems for Aircraft," Report R-164, Instrumentation Laboratory, MIT, Cambridge, Mass., September 1958.

64. Parks, P. C., "Lyapunov Redesign of Model Reference Adaptive Control Systems," IEEE Trans., AC-11, pp. 362-367, 1966.

65. Monopoli, R. V., "The Kalman-Yakubovich Lemma in Adaptive Control System Design," IEEE Trans., AC-18, pp. 527-529, 1973.

66. Lindorff, D., and Carroll, R., "Survey of Adaptive Control Using Lyapunov Design," Int. J. Control, 18, pp. 897-914, 1973.

67. Popov, V. M., "Hyperstability of Control Systems," Springer-Verlag, 1973.

68. Landau, I. D., "A Survey of Model-Reference Adaptive Techniques: Theory and Applications," Automatica, 10, pp. 353-379, 1974.

69. Monopoli, R. V., "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Trans., AC-19, pp. 474-484, 1974.

70. Ionescu, T., and Monopoli, R. V., "Discrete Model Reference Adaptive Control with an Augmented Error Signal," Automatica, 13, pp. 507-517, 1977.

71. Landau, Y. D., "Adaptive Control - The Model Reference Approach," Marcel Dekker, New York, 1979.

72. Narendra, K. S., and Lin, Y.-H., "Design of Stable Model Reference Adaptive Controllers," Proc. of the Workshop on Applications of Adaptive Control, Yale University, Aug. 1979.

73. Egardt, B., "Unification of Some Continuous-Time Adaptive Control Schemes," Proc. IEEE AC-24, pp. 588-592, 1979.

74. Egardt, B., "Unification of Some Adaptive Control Schemes - Part II, Discrete Time," Proc. IEEE, AC, to appear.

75. Egardt, B., "Stability of Adaptive Controllers," Lecture Notes in Control and Information Sciences, Vol. 20, Springer-Verlag, Berlin, 1979.

76. Åström, K. J., "Computer Control of a Paper Machine - An Application of Linear Stochastic Control Theory," IBM J. Res. & Dev., 11, pp. 389-405, 1967.

77. Åström, K. J., "Applications to Paper Making," in Curriculum and Methods in Control and Systems Engineering, Vol. II, IBM Corp., 1970.

78. Borisson, U., and Wittenmark, B., "An Industrial Application of a Self-Tuning Regulator," Proc. IFAC Symp. on Digital Computer Applications to Process Control, Zürich, 1974.

79. Jensen, L., and Hänsel, R., "Computer Control of an Enthalpy Exchanger," Report TFRT-3081, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1974.

80. Borisson, U., and Syding, R., "Self-Tuning Control of an Ore Crusher," Automatica, 12, pp. 1-7, 1976.

81. Källström, C. G., Åström, K. J., Thorell, N. E., Eriksson, J., and Sten, L., "Adaptive Autopilots for Tankers," Automatica, 15, pp. 241-254, 1979.

82. Peterka, V., and Åström, K. J., "Control of Multivariable Systems with Unknown but Constant Parameters," Proc. 3rd IFAC Symposium on Identification and System Parameter Estimation, The Hague, June 1973.

83. Borisson, U., "Self-Tuning Regulators for a Class of Multivariable Systems," Automatica, 15, pp. 209-215, 1979.

84. Åström, K. J., "Self-Tuning Control of a Fixed Bed Chemical Reactor System," Report, CODEN: LUTFD2/(TFRT-3151)/1-066/, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1978.

85. Holst, J., "Local Convergence of Some Recursive Stochastic Algorithms," Preprints 5th IFAC Symposium on Identification and System Parameter Estimation, Darmstadt, September, 1979.

86. Landau, I. D., and Lozano, R., "Design and Evaluation of Discrete Time Explicit MRAC for Tracking and Regulation," Report LAG 79.14, Laboratoire d'Automatique de Grenoble, June, 1979.