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Wittenmark, Björn; Bar-Shalom, Yaakov

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BJÖRN WITTENMARK

YAAKOV BAR-SHALOM

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY

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Author(s)
Björn Wittenmark

Yaakov Bar-Shalom

Supervisor

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Björn Wittenmark

Department of Automatic Control, Lund Institute of Technology, Lund, Sweden

Yaakov Bar-Shalom

Department of Electrical Engineering, University of Connecticut, Storrs, CT, USA

Abstract. One reason to make identification experiments is to obtain models which can be used to design controllers. Structures, for instance model orders, can be determined using statistical tests on the error between the model output and the output of the system. However, these tests do not determine how well the system can be controlled. One interesting problem is to predict the performance of the closed loop system, when a controller has been designed based on the identified model. The purpose of this paper is to show a way to test the sensitivity of the identified models. The uncertainties of the parameter estimates are used to give a prediction of the closed loop system performance. This is done without making any further experiments on the system. An algorithm is described which evaluates the increase in the expected cost due to parameter uncertainties. The new test is compared with other tests and numerical results from simulated data are given. The paper shows that the new test can be a useful complement for model validation.

Keywords. Modelling; identification; stochastic systems; structure testing; model validation.

1. INTRODUCTION

The area of identification contains many different and interesting problems. One important problem is model validation, i.e. how to choose the most appropriate structure of the model. Most identification methods are based on a given structure. The user has to test different structures and determine which one fits the data best. There are many tests described in the literature, see e.g. Åström (1967), Akaike (1974), Woodside (1971), and Bohlin (1978). Comparisons of different tests can be found in Gustavsson (1972), Unbehauen and Göhring (1974), van den Boom and van den Enden (1974), and Söderström (1977).

Many identification methods are so called prediction error methods. The prediction errors, the residuals, are often used to make statistical tests of different kinds, e.g. tests of whiteness and normality and F-tests. The results of these tests are, at least for real data, quite often inconsistent. The choice of the model structure will thus be subjective. The choice of model and model structure should be related to the ultimate use of the model. For instance one-step-ahead prediction error methods might give a model which is suitable for one-step-ahead predictions. The model may, however, be less suited for control purposes where it is necessary to have a good estimate of the static gain. In such cases it might be better to use a multi-step-ahead prediction method, see Åström and Källström (1979).

One goal for the identification of an unknown process can be to design a controller. However, most tests for model validation do not say anything about how well the process can be controlled. Further when making structure tests generally the parameter estimates only are used and not the uncertainties of the estimates. It is thus interesting to evaluate the performance of the closed loop system when the estimated model is used to design a controller. The straightforward way to test the derived controllers is to make experiments on the process. These experiments may be quite time consuming and only a limited number of controllers can be tested. Another way to test the performance of the closed loop system is through simulations. The controller can be based on the estimated model. The controller can then be tested by changing the "process" in the simulation. The process models can be obtained by generating different sets of parameters with the same mean values and covariance matrix as the estimated model. This is, however, a rather expensive simulation approach since many trials must be done.

The purpose of this paper is to show an approximate way to predict the behaviour of the closed loop system. This can be done without making any additional experiments on the real process or by making extensive simulations. The performance and the influence of

the parameter estimates are estimated using results from Bar-Shalom (1978). The different structures are evaluated with respect to the predicted loss for the closed loop system. The underlying idea is that one should choose a model structure which gives a desired behavior and which is also as insensitive as possible to errors in the parameters.

The paper is organized as follows: The problem is formally stated in Section 2. Section 3 contains a brief review of the theory. Section 4 discusses how the results can be used to determine structures of identified models. Numerical examples using simulated data are given in Section 5. Section 6 contains conclusions.

2. STATEMENT OF THE PROBLEM

The class of processes discussed in this paper consists of single input single output systems described by

$$y(t) + a_1 y(t-1) + \ldots + a_{na}\, y(t-na) =$$

$$= b_1 u(t-k-1) + \ldots + b_{nb} u(t-k-nb) + e(t). \quad (2.1)$$

The noise, e(t), is assumed to be white with zero mean and the standard deviation σ. The generalization to more general noise structures is discussed in Section 6.

It is assumed that a set of input-output values are given. These data points are used to estimate the parameters a_i, b_i, and σ in (2.1) for different values of na, nb, and k. It is assumed that the identification method used gives estimates of the parameters together with the parameter uncertainties. The problem is now to determine the model which is most suitable for control. It is assumed that the purpose of the identification is to design a controller which minimizes the loss function

$$J = \frac{1}{2} E \Big\{ (y(N)-y_r(N))^2 + \sum_{t=0}^{N-1} \left[(y(t)-y_r(t))^2 + r(t)(u(t)-u_r(t))^2 \right] \Big\}. \quad (2.2)$$

The variables $y_r(t)$ and $u_r(t)$ are the desired trajectories of the output and the control respectively and r(t) is a weighting factor. The problem that will be discussed is to evaluate the loss function (2.2). An approximate calculation can be done for different estimated models. Also we will determine how the loss function is affected by the uncertainties of the parameter estimates in each model. The model validation will then be done by considering the estimated closed loop performance and the sensitivity of the closed loop performance with respect to the uncertainties of the parameter estimates.

3. THE EFFECT OF RANDOM PARAMETERS ON THE CONTROL PERFORMANCE

This section treats the problem of evaluating a loss function when the system contains random parameters.

Consider the system

$$x(t+1) = A(\theta)x(t) + B(\theta)u(t) + De(t+1) \quad (3.1)$$

where $A(\theta)$ and $B(\theta)$ are linear in unknown elements, θ, modelled as random parameters, independent from period to period but with known mean and variance. The process noise, e(t) is white with zero mean and variance σ. This is called the "white" parameter case. (In this section the system may have several inputs.) Consider the loss function

$$J = \frac{1}{2} E \Big\{ \sum_{t=0}^{N} [x(t)-x_r(t)]^T Q(t)[x(t)-x_r(t)] + \sum_{t=0}^{N-1} [u(t)-u_r(t)]^T R(t)[u(t) - u_r(t)] \Big\} \quad (3.2)$$

where $x_r(t)$ and $u_r(t)$ are the desired trajectories of the state and the input respectively. The minimization of (3.2) leads to the following optimal stochastic control, see e.g. Aoki (1967),

$$u^*(t) = L(t)x(t) + m(t) \quad (3.3)$$

where

$$L(t) = -\left[R(t) + \overline{B^T K(t+1)\, B}\right]^{-1} \cdot \cdot \overline{B^T K(t+1)\, A} \quad (3.4)$$

$$m(t) = -\left[R(t) + \overline{B^T K(t+1)\, B}\right]^{-1} \cdot \cdot \left[\bar{B}^T p(t+1) - R(t)u_r(t)\right] \quad (3.5)$$

and the bar over A and B denotes expectations over the unknown parameters, e.g.

$$\overline{B^T K(t+1) B} = E\left[B^T K(t+1) B\right]. \quad (3.6)$$

The matrix K, the vector p, and the scalar g are given by the following backward recursions

$$K(t) = Q(t) + \overline{A^T K(t+1) A} + \overline{A^T K(t+1) B}\, L(t) \quad (3.7)$$

$$p(t) = -Q(t)x_r(t) + \bar{A}^T p(t+1) + \overline{A^T K(t+1) B}\, m(t) \quad (3.8)$$

$$g(t) = \frac{1}{2} x_r^T(t)Q(t)x_r(t) + \frac{1}{2} u_r^T(t)R(t)u_r(t) + \frac{1}{2}\left[\bar{B}^T p(t+1) - R(t)u_r(t)\right]^T m(t) + g(t+1) \quad (3.9)$$

with the terminal conditions

$$K(N) = Q(N) \quad (3.10)$$

$$p(N) = -Q(N)x_r(N) \quad (3.11)$$

$$g(N) = \frac{1}{2} x_r(N)Q(N)x_r(N). \quad (3.12)$$

Note that the expectation of the type (3.6) leads to an expression containing the means and covariances of the random parameters θ. The resulting control is optimal for the case of "white" parameters and suboptimal otherwise. It is known under the name of open-loop feedback (OLF), see e.g. Bar-Shalom (1978) for further discussion.

The optimized cost of the problem with stochastic parameters is

$$J^S = J_1^S + J_2^S \tag{3.13}$$

where, with K, p and g obtained from (3.7) - (3.12),

$$J_1^S = \frac{1}{2} x^T(0)K(0)x(0) + p^T(0)x(0) + g(0) \tag{3.14}$$

is the cost without the process noise. The part due to the noise is

$$J_2^S = \frac{1}{2} \sum_{t=1}^{N} \mathrm{tr}\left(K(t)\, D\sigma^2 D^T\right). \tag{3.15}$$

The optimized cost for the same problem with known (deterministic) parameters will be denoted as

$$J^D = J_1^D + J_2^D \tag{3.16}$$

and its two components are given by (3.14) and (3.15) respectively with the following change: iterations (3.7) - (3.12) have no expectations over the parameters.

The sensitivity to parameter uncertainties of a system with uncertain parameters can be judged by comparing the costs of (3.13) and (3.16). Another concept that can be used in assessing the goodness of a model for control purpose is the trajectory confidence tube. This is a sequence of confidence regions about the predicted values of a component of the state vector in which the corresponding true value will lie with a certain probability.

If one uses the feedback law

$$u^0(t) = L^0(t)x(t) + m^0(t) \tag{3.17}$$

then the predicted trajectory for the closed loop system is

$$x^0(t+1) = \left(\overline{A} + \overline{B}\, L^0(t)\right) x^0(t) + \overline{B}\, m^0(t) \tag{3.18}$$

$$t = 0, \ldots, N-1; \quad x^0(0) = x(0).$$

The mean square value of the deviation of the ℓ-th komponent of $x(N)$ from the predicted value is

$$\rho_\ell^2(N) = \frac{1}{2} E\left\{\left(x(N)-x^0(N)\right)^T Q^0(N,\ell) \cdot \left(x(N)-x^0(N)\right)\right\} \tag{3.19}$$

where the elements of the matrix in the above quadratic form are all zero except

$$Q^0_{\ell\ell}(N,\ell) = 1. \tag{3.20}$$

The 1/2 in (3.19) was used for the convenience of programming. The result will be that the standard deviation of the ℓ-th component of the state at time N is $\sqrt{2}\,\rho_\ell(N)$. This result is exact for the case of white parameters, otherwise it is the OLF approximation which is quite attractive due to its simplicity of implementation.

The evaluation of (3.19) is done with recursions obtained similarly to the OLF control presented above. For details, see Bar-Shalom (1978). The result is

$$\rho_\ell^2(N) = \frac{1}{2} x^0(0)^T K^0(0)x^0(0) + p^0(0)^T x(0) + g^0(0) \tag{3.21}$$

where K^0, p^0, and g^0 (which for simplicity are not indexed by ℓ and N) are obtained from the following linear backward recursions:

$$K^0(t) = \overline{A^T K^0(t+1) A} + L^0(t)^T \overline{B^T K^0(t+1) B}\, L^0(t) + \overline{A^T K^0(t+1) B}\, L^0(t) + L^0(t)^T \overline{B^T K^0(t+1) A} \tag{3.22}$$

$$p^0(t) = \overline{A^T K^0(t+1) B}\, m^0(t) + L^0(t)^T \overline{B^T K^0(t+1) B}\, m^0(t) + L^0(t)^T \overline{B^T}\, p^0(t+1) + \overline{A}^T p^0(t+1) \tag{3.23}$$

$$g^0(t) = \frac{1}{2} m^0(t)^T \overline{B^T K^0(t+1) B}\, m^0(t) + \frac{1}{2} \mathrm{tr}\left(K^0(t+1)\, D\sigma^2 D^T\right) + p^0(t+1)^T \overline{B}\, m^0(t) + g^0(t+1) \tag{3.24}$$

$$t = N-1, N-2, \ldots, 0$$

with terminal conditions

$$K^0(N) = Q^0(N,\ell) \tag{3.25}$$

$$p^0(N) = -Q^0(N,\ell)\, x^0(N) \tag{3.26}$$

$$g^0(N) = \frac{1}{2} x^0(N)^T Q^0(N,\ell)\, x^0(N). \tag{3.27}$$

4. MODEL VALIDATION USING ESTIMATED CLOSED LOOP PERFORMANCE

In the previous section it was shown how the parameter uncertainties will affect the closed loop performance of a system with random parameters. In this section we will discuss how these results can be used for model validation.

The model (2.1) can be written in the state space form (3.1) with

$$A(\theta) = \begin{pmatrix} -a_1 & & \\ \vdots & & \\ -a_{na} & & I \\ 0 & & \\ \vdots & & \\ 0 & & \\ \hline 0 & 0 & \dots 0 \end{pmatrix}, \quad B(\theta) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ b_1 \\ \vdots \\ b_{nb} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{matrix} k$$

$$\text{and} \quad D^T = \begin{pmatrix} 1 & 0 \dots 0 \end{pmatrix}, \tag{4.1}$$

and where $y(t) = x_1(t)$. The dimension of the state is $n = \max(na, nb+k)$. All the states can be expressed in old inputs and outputs, i.e. all the states can be regarded as directly measurable. The results from Section 3 can now be used to determine the loss function (2.2). The loss functions J^S and J^D can be evaluated for different identified models. The loss $J^D = J_1^D + J_2^D$ can be compared for the different models. J_1^D is due to the deviation in the trajectory and J_2^D is due to the additive noise, $e(t)$. A model with good fit to the data should give a low loss compared with a model which badly fits the data. There is one problem by only comparing J^D for different structures. The problem is that the controllers are tested on different models, i.e. the control law derived for one structure is tested on the same structure and the cross coupling between the different losses comes only from the fact that the same data set has been used in the identification. This might have the consequence that a too simple model can give a low loss since this model might be easy to control. This problem can be avoided by making the model validation in two steps. In the first step the models that are obviously too simple are eliminated using a conventional statistical test, for instance the F-test, Åström (1967), or AIC, Akaike (1974). In the second step one can use the estimated closed loop performance for the final decision.

When comparing different models it is also interesting to look at the sensitivity with respect to the parameter uncertainties. The standard deviations of the parameter estimates are usually not used in the standard statistical tests. The standard deviations are sometimes qualitatively compared for different model structures. A too complex structure usually gives larger standard deviations. The difference $\Delta J = J^S - J^D$ between the stochastic loss and the deterministic loss is a measure of the increase in the expected loss due to the parameter uncertainties. It is thus interesting to look at both J^D and ΔJ. Also the confidence tubes can be used to test the sensitivity of the models. The uncertainty radius around the predicted trajectory is another measure of the influence of the parameter uncertainties. An illustrative example with two econometric models is given in Bar-Shalom (1978).

The derivation in Section 3 is done for the extreme so called "white" parameter case. The unknown parameters are at each step of time drawn from a given distribution. The other extreme case is when the parameters of the process are assumed to be fixed during one realization, but for each realization drawn from the same distribution. In practice the situation is probably in between. The white parameter case is an "optimistic" evaluation of the effect of the parameter errors. When computing the lossfunction (3.2) we have to evaluate expressions of the type $E(\theta_i\theta_j)^n$. From Jensen's inequality for convex functions it follows that

$$E(\theta_i\theta_j)^n \geq (E(\theta_i\theta_j))^n.$$

The right hand side of the inequality corresponds to the white parameter case and the left hand side to the single realization case.

To summarize the advantages of the new test are:

- It takes the parameter uncertainties into account.
- It calculates an approximate expected loss of the closed loop performance. (The calculations are exact in the white parameter case.)
- Time consuming on-line tests are avoided.
- Extensive Monte Carlo simulations are avoided.

The drawbacks are:

- The solution is exact only for the white parameter case.
- The computations are nontrivial but not too time consuming. (Equations (3.7) - (3.9) have to be iterated and expectations of the type (3.6) have to be evaluated.)

In the evaluation of different models one should not use *one* test but weight different tests against each other. The new test is then a good complement to standard tests as it gives an estimate of the *closed loop* performance.

5. NUMERICAL EXAMPLES

Two numerical examples will be given which show how the expected closed loop performance can be used for model validation.

Example 5.1

The system is described by

$$y(t) - 1.5y(t-1) + 0.7y(t-2) = \\ = u(t-1) + 0.7u(t-2) + e(t) \tag{5.1}$$

The variance of $e(t)$ is $\sigma^2 = 1$. The data were generated by letting $u(t)$ be a PRBS-signal

with period 127 and the amplitude ±0.14. The basic period of the PRBS signal was 1, i.e. the shortest time between changes in the signal is 1 timeunit. This gives a signal to noise ratio of about 1/25 (in power). The parameters were identified using the method of least squares with n = na = nb and k = 0. Table 5.1 shows the lossfunction

$$V = \sum_{1}^{N_0} \hat{e}(t)^2$$

for different number of data points, N_0, and different orders n. The variable $\hat{e}(t)$ is the difference between the output from the system and the model output.

TABLE 5.1 The lossfunction from the least squares identification for different numbers of data points, N_0, and different orders of the model, n, for the system (5.1).

N_0 \ n	Lossfunction 1	2	3
100	168.63	87.82	87.26
500	994.29	486.40	485.12
1000	1952.78	981.28	976.33

There is a significant decrease in the lossfunction when the order of the model is increased from 1 to 2 or 3. The F-test when comparing model orders 2 and 3 gives the following test quantities:

N_0	Test quantity
100	0.30
500	0.67
1000	2.52

The test quantity for the F-test at the 5 % level is about 3. It is only for $N_0 = 1000$ that one would consider that the process might be of third order.

The lossfunctions J^S and J^D were calculated for different orders and different number of data points. The results are summarized in Table 5.2. It is assumed that the reference trajectory for the output is

$$y_r(t) = \begin{cases} t & 0 \leq t \leq 5 \\ 5 & 5 \leq t \leq 15 \end{cases}$$

and that $u_r(t) = 0$ for $0 \leq t \leq 15$. Further $r(t) = 0.001$. For $N_0 = 100$ it is seen that the stochastic loss J^S is smallest for n = 1, but this model order has already been eliminated by the F-test. The deterministic loss is smallest for n = 2 for $N_0 = 100$ and 500. The second order model gives a smaller increase than for n = 3 in the loss when the parameter uncertainties are taken into account, i.e. $\Delta J = J^S - J^D$ is smaller. The radius of the confidence tube for the output at the final point is also given in Table 5.2. The radius is about the same for n = 2 and 3 when N = 500 or 1000. This indicates that the model is of second order.

Example 5.2

The following nonminimum phase system was simulated

$$y(t) - 0.9y(t-1) = u(t-2) + 2u(t-3) + e(t)$$

with $\sigma^2 = 1$. The input signal was a PRBS signal with period 127 and a basic period of 10 samples. The amplitude of the input was ±0.34. The signal to noise ratio is about 1.

The system was identified using the least squares method ($N_0 = 1000$) using the model

$$y(t) + a_1y(t-1) + a_2y(t-2) = b_1u(t-1) + b_2u(t-2) + b_2u(t-3) + e(t) \quad (5.2)$$

with the parameters identified according to the structures shown in Table 5.3.

TABLE 5.2

N_0	n	Stochastic cost J_1^S	J_2^S	$\rho_y(15)$	Deterministic cost J_1^D	J_2^D	$\rho_y(15)$
100	1	2.9268	15.6328	1.0615	0.5013	12.9082	-
	2	32.1177	24.4080	2.1503	0.5097	6.8728	0.6763
	3	16.0822	25.8780	1.7199	0.5032	6.9784	0.6813
500	1	1.4179	15.4216	1.0298	0.5039	14.2269	-
	2	1.0555	8.6166	0.7833	0.5030	7.3670	0.7002
	3	1.1564	8.7490	0.7958	0.5027	7.3775	0.7007
1000	1	0.8599	15.1908	1.0076	0.5031	14.6800	-
	2	0.8017	8.2512	0.7440	0.5042	7.4064	0.7018
	3	0.8522	8.3688	0.7508	0.5036	7.3852	0.7008

TABLE 5.3 The table shows which parameters in (5.2) that were identified for the different models. The other parameters were fixed to 0.

Model No.	Identified parameters				
	a_1	a_2	b_1	b_2	b_3
1	x		x		
2	x			x	
3	x				x
4	x			x	x
5	x	x		x	x
6	x	x	x	x	x

TABLE 5.4 Lossfunctions and test quantities when going from model structure m_1 to model structure m_2.

Model No. m_1	$V(m_1)$	Test quantity from m_1 to m_2		
		m_2 4	5	6
1	1264.60	-	-	67.87
2	1142.60	86.95	43.46	29.32
3	1067.60	15.80	7.92	5.62
4	1050.94	-	0.06	0.54
5	1050.88	-	-	1.02
6	1049.80	-	-	-

Model 4 is the model with correct structure. Models 5 and 6 are more complex structures. Model 3 can be regarded as an approximation where the nonminimum phase part is approximated with a pure time delay.

Table 5.4 shows the lossfunctions and the F-test quantities for different changes in the model. The test quantity should be greater than about 3 if the more complex model should be preferred on a 5 % confidence level.

Table 5.5 shows J_1^D, J_2^D, ΔJ, and $\rho(15)$ when the reference trajectory is

$$y_r(t) = \begin{cases} 0 & 1 \leq t \leq 6 \\ t-6 & 6 \leq t \leq 10 \\ 5 & 11 \leq t \leq 15 \end{cases}$$

$$u_r(t) = 0 \qquad 1 \leq t \leq 15$$

and with $r(t) = 0.001$.

The models can be divided into two groups, 1 - 3 and 4 - 6 respectively. In the first group ΔJ is small but we can eliminate these models by using the result of the F-test shown in Table 5.4. The models 4 and 5 have about the same performance and it can be preferable to use the simpler model, i.e.

TABLE 5.5 Deterministic loss, increase in the loss due to parameter uncertainties and radius of the confidence tube for the system in Example 5.2.

Model	J_1^D	J_2^D	ΔJ	$\rho(15)$
1	0.000678	9.486	0.0309	0.7952
2	0.000557	15.383	0.0210	1.0286
3	0.000498	18.623	0.0168	1.1476
4	0.000597	15.375	0.4135	0.9768
5	0.000594	15.271	0.4815	0.9735
6	0.000552	13.610	1.2173	0.7257

model 4. Comparing models 4 and 6 we see that model 6 has lower deterministic loss but on the other hand the increase in the loss due to the parameter uncertainties are 2-3 times larger. This in combination with Table 5.4 indicate that model 4 should be preferable.

6. CONCLUSIONS

A new test for model validation has been discussed. The test is based on an approximate closed loop performance, where the controller is based on the estimated model. The test has the advantage that it considers the parameter estimates as well as the parameter uncertainties.

The theory described in this paper is done only for the model structure (2.1), i.e. the least squares structure. If we have a more general noise structure with

$$e(t) = \varepsilon(t) + c_1\varepsilon(t-1) + \ldots + c_{nc}\varepsilon(t-nc)$$
$$\varepsilon(t) \in N(0,\sigma)$$

some modifications have to be done. The state space model will then also contain uncertain parameters in the D-vector in (3.1) i.e.

$$D^T = \begin{pmatrix} 1 & c_1 \ldots c_{nc} & 0 \ldots 0 \end{pmatrix}. \qquad (6.1)$$

In Section 3 this only affects the loss J_2^S if we still assume that all the states are measured. The optimal control law will still be the same linear feedback from all the states. If the system has A and B matrices according to (4.1) and a D-vector as (6.1), then we can not directly write the state as a linear combination of a finite set of the inputs and outputs. Further approximations must be done. It could be assumed that the states are reconstructed using a Kalman filter. The reconstruction errors will then increase the loss functions J^S and J^D. The state estimator has to be based on the mean values of the parameter estimates. If we consider the white parameter case there will

then be an additional increase in the loss due to the mismatch between the process and the model. If we disregard these extra losses due to the state estimation we will still quite easily be able to compute the estimated closed loop performance according to the equations given in Section 3. Only smaller changes are needed. To summarize, the presented test makes it possible to estimate the performance of the closed loop system. This can be done without further experiments or Monte Carlo simulations. The new test is a good complement to standard tests for model validation.

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