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Åström, Karl Johan

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LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

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KARL JOHAN ÅSTRÖM

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY

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LUND INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AUTOMATIC CONTROL
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ANALYSIS OF ROHRS COUNTEREXAMPLES TO ADAPTIVE CONTROL

K.J. Åström

Department of Automatic Control

Lund Institute of Technology, Lund, S-22007 Sweden

ABSTRACT

Simulation results by Rohrs indicate potential difficulties with adaptive control. Analysis which explains the nature of the difficulties and ways to avoid them are presented in this paper.

1. INTRODUCTION

There has been a modest but noticeable progress in adaptive control over the past 30 years. See e.g. the review Åström (1981,1983). Stability proofs for certain simple adaptive algorithms have recently been proven. See Egardt (1979a,1980) Goodwin et al (1980), Morse (1980), Narendra et al (1980). Commercial adaptive regulators have also been announced by several manufacturers, Accuray, ASEA, Kockumation, Leeds and Northrup and others.

Rohrs et al (1980,1981,1982) and Rohrs (1983) have presented simulation studies of a simple model reference algorithm. They have shown that a very simple model reference adaptive system may become unstable if the strong assumptions made in the stability proofs are violated. The issues focus on unmodeled high frequency dynamics and high frequency excitation either from command signals or from disturbances.

Based on these studies and some analysis it is claimed that "...the adaptive algorithms considered cannot be used for practical adaptive control...".

The purpose of this paper is to analyse Rohrs examples to explain the origin of the difficulties and to propose remedies. The different instability mechanisms are discussed in Section 2. A discussion of each of them is then given in Sections 3, 4 and 5. The reasons for the difficulties are explained and remedies are given. The explanations given are different from the arguments given by Rohrs. General methods for avoiding the difficulties are given in Section 6 and the conclusions are given in Section 7.

2. THE INSTABILITY MECHANISMS

In his simulation studies Rohrs investigated a simple model reference adaptive control system with two adjustable parameters. The adaptive scheme works well and results in a stable closed loop system when applied to a first order system without disturbances. When adding unmodeled high frequency dynamics Rohrs showed that there may be unstable solutions to the equations describing closed loop system. Several instability mechanisms can be found. Instability may be caused by unmodeled high frequency dynamics in combination with:

- large command signals
- command signals with high frequency
- measurement noise.

All these mechanisms are assumed away in current stability proofs. The different instability mechanisms will be discussed in the following sections.

3. HIGH ADAPTATION GAIN

The simple model reference adaptive system for a single-input-single-output continuous time system can be described by the equations

$$y = G(p)u$$

$$u = \theta^T\varphi$$

$$\frac{d\theta}{dt} = -k\varphi e \qquad (3.1)$$

$$e = y - y_m$$

where u is the process input, y the process output, y_m the desired model output, e the error and θ a vector of adjustable parameters. The transfer function of the process is G and p = d/dt denotes the differential operator. The components of the vector φ are functions of the command signal. In Rohrs example the vector φ is given by

$$\varphi = [r \ \ -y]^T \qquad (3.2)$$

where r is the reference signal. It follows from (3.1) that

$$\frac{d\theta}{dt} + k\varphi[G(p)\varphi^T\theta] = k\varphi y_m \qquad (3.3)$$

This equation can be used for a heuristic discussion which will give us insight into the behaviour of the system. For this discussion it will be assumed that the process transfer function is of low pass character.

<u>Slow adaptation</u>

Assume first that the adaptation loop is much slower than the process dynamics. The term $G(p)\varphi^T\theta$ in (3.3) can then be approximated by

$$G(p)\varphi^T\theta \approx [G(p)\varphi^T]\theta$$

and the following approximation to (3.3) is obtained

$$\frac{d\theta}{dt} + k\varphi[G(p)\varphi^T]\theta \approx k\varphi y_m \qquad (3.4)$$

This is the normal situation because the MRAS algorithm is derived under this assumption. With the algorithm (3.1) it is however not easy to guarantee that the parameters change slowly as will be seen in the following.

The method of averaging

If the closed loop system is stable and if the parameters θ change much slower than φ then the equation (3.4) can be approximated by

$$\frac{d\theta}{dt} + \mathrm{avg}(k\varphi[G(p)\varphi^T])\,\theta = k\,\mathrm{avg}(\varphi y_m) \quad (3.5)$$

where the function avg is defined as the average calculated under the assumption that the parameters θ are constant. This approach is called the method of averaging. It is useful in order to determine the possible equilibrium values for the parameters and their stability.

Equation (3.5) is stable if $k\varphi[G(p)\varphi^T]$ is positive. This is true if G is SPR and if the input signal is persistently exciting. See Åström and Bohlin (1965), Åström and Eykhoff (1971) and Morgan and Narendra (1977).

Fast adaptation

The approximation (3.4) is based on the assumption that the parameters θ change much slower than the other system variables. It is unfortunately not possible to guarantee this by choosing the parameter k in (3.1) sufficiently small. This can be seen as follows.

If the parameters θ change more rapidly than φ then (3.3) can be approximated by

$$\frac{d\theta}{dt} + k\varphi\varphi^T G(p)\theta \approx k\varphi y_m \quad (3.6)$$

A linearization for constant φ_0 shows that the stability is governed by the algebraic equation

$$\det[sI+k\varphi_0\varphi_0^T G(s)]=s^{n-1}[s+KG(s)]=0 \quad (3.7)$$

where I is the identity matrix and K is given by

$$K = k\varphi_0^T\varphi_0 \quad (3.8)$$

is the equivalent adaptive loop gain. The stability can then be determined by a simple root-locus argument. For sufficiently large $k\varphi_0^T\varphi_0$ the system will always be unstable if the pole-excess of G(s) is larger than or equal to 2. Also notice that the equivalent gain K is proportional to $\varphi_0^T\varphi_0$. The equivalent gain can thus be made arbitrarily large by choosing the command signal large enough. It thus seems intuitively clear that the adaptive system can be made unstable by making the command signal large enough.

Once the source of the difficulty is recognized it is easy to find a remedy. Since the equivalent gain K in the adaptive loop is too large because of its signal dependence, one possibility is simply to modify the parameter updating law to

$$\frac{d\theta}{dt} = -\frac{k}{1+\varphi^T\varphi}\varphi e \quad (3.9)$$

Equation (3.6) then holds with

$$K = k\frac{\varphi\varphi^T}{1+\varphi^T\varphi} \quad (3.10)$$

With the modification (3.10) the equivalent gain in the adaptation loop will be small and the parameters θ will change arbitrarily slow for all signal levels. The actual value of the k can be chosen based on a simple root-locus argument for (3.7).

Equation (3.6) was originally derived by Shackcloth and Butchart (1966). It was also used by Parks (1966) under the name "adaptive step response". It was also used by Rohrs et al (1981) who used the term "d*-root locus". The modification (3.9) of the parameter updating law has been used by many authors e.g. Narendra and Lin (1980). It is also worthwhile to note that a law of this type is obtained automatically when adaptive laws are derived from recursive estimation. See Åström (1983b). The high gain instability mechanism is the same as the one discussed in Cyr et al (1983).

4. HIGH FREQUENCY COMMAND SIGNALS

This instability mechanism is due to an interplay between unmodeled high frequency dynamics and high frequency excitation. The phenomenon is easy to explain by using the equivalence between MRAS and STR. The model reference adaptive system can thus be interpreted as a system where process parameters are estimated and the regulator parameters are determined from some design principle.

The origin of the difficulty

The key problem is to understand what happens when a low order model is fitted to a process having high order dynamics. In such a case the low order model obtained will depend critically on the properties of the input signal. A simple example illustrates what happens.

If a transfer function B(s)/A(s) is determined by least squares the criterion is

$$V_T(\theta) = \frac{1}{T}\int_0^T [A(p)F(p)y(t)-B(p)F(p)u(t)]^2 dt \quad (4.1)$$

where u is the process input, y the process output, and the components of the vector θ are the coefficients of the polynomials A(s) and B(s), and F(s) is the transfer function of a low pass filter (a state variable filter). If the input-output data is generated by a process with the transfer function G(p) the least squares criterion (4.1) can be written as

$$V_T(\theta) = \frac{1}{T}\int_0^T \{[A(p)G(p)-B(p)][F(p)u(t)]\}^2 dt \quad (4.2)$$

If the input u is a sinusoid with frequency ω_k it follows that the steady state loss function V_∞ is zero if there are model parameters such that

$$\frac{B(i\omega_k)}{A(i\omega_k)} = G(i\omega_k) \quad (4.3)$$

The transfer function of the estimated model is thus exactly equal to transfer function of the process at the frequency ω_k.

If the input signal is a sum of sinusoids with frequencies $\omega_1, \omega_2, \ldots, \omega_N$ and if the polynomials A(s) and B(s) are of sufficiently high degree the model obtained will agree exactly with the transfer function of the process at those frequencies. More general results are given by Mannerfelt (1981).

The observations in Rohrs simulations will now be explained. The MRAS used by Rohrs can be interpreted as an STR where a model with the transfer function

$$G_1(s) = \frac{b}{s+a} \tag{4.4}$$

is fitted to the process transfer function. With the control law

$$u(t) = k_r r(t) - k_y y(t) \tag{4.5}$$

and the reference model

$$G_m(s) = \frac{a_m}{s+a_m} \tag{4.6}$$

the parameters are related as follows

$$a = a_m(1-k_y/k_r)$$
$$b = a_m/k_r \tag{4.7}$$

The parameters k_r and k_y are those that are updated in the MRAS algorithm.

In Rohrs example the process transfer function is

$$G(s) = \frac{458}{(s+1)(s^2+30s+229)} \tag{4.8}$$

It is now clear what happens if the model (4.4) is fitted to (4.8) when the input is a sine wave. The condition (4.3) gives

$$a = \frac{229-31\omega^2}{259-\omega^2}$$
$$b = \frac{458}{259-\omega^2} \tag{4.9}$$

where ω is the frequency of the reference signal. Numerical values of k_r, k_y, a and b for different ω are given in Table 1. The difficulties in fitting the first order model (4.4) to the third order model (4.8) using sinusoidal data can be appreciated from Table 1. The correct low frequency model corresponds to a=1 and b=2. Compare with (4.8). The models obtained for low frequencies are quite reasonable. At the frequency ω = 2.71 (4.8) has a phaseshift of $\pi/2$. The only possibility to get a model (4.4) with this property is to have a = 0. For higher frequencies the least squares estimation gives an unstable first order model (4.4). The process (4.8) has a phaseshift of π at the frequency $\omega = \sqrt{259} \approx 16.09$ rad/s. The only possibility to get this property from the first order model (4.4) is to have the degenerate case when the parameters a and b are infinitely large.

Table 1 - Steady state values of estimated parameters for different frequencies of the command signal.

ω	a	b	k_r	k_y
0	0.88	1.77	1.70	1.20
1	0.77	1.78	1.69	1.26
2	0.41	1.80	1.67	1.44
2.71	0.00	1.82	1.65	1.65
3	-0.20	1.83	1.64	1.75
4	-1.10	1.88	1.59	2.17
5	-2.33	1.96	1.53	2.72
10	-18.06	2.88	1.04	7.31
16.09	$-\infty$	∞	0	17.03
20	86.32	-3.25	-0.93	25.65

Equilibrium values for the regulator parameters

Additional insight is obtained by computing the equilibrium values of the regulator parameters k_r and k_y. The MRAS used by Rohrs is described by the equations

$$\begin{cases} u = k_r r - k_y y \\ \frac{dk_r}{dt} = -kre \\ \frac{dk_y}{dt} = kye \\ e = y - y_m \\ y_m = G_m(p) r \end{cases} \tag{4.10}$$

If the transfer function of the process is G(s) and if a constant regulator is used then the closed loop transfer function becomes

$$G_c = \frac{k_r G}{1+k_y G} \tag{4.11}$$

and the control error can be written as

$$e(t) = [G_c(p) - G_m(p)]r(t).$$

It follows from (4.10) that the regulator parameters are constant for a sinusoidal reference signal with frequency ω if

$$G_c(i\omega) = \frac{k_r G(i\omega)}{1+k_y G(i\omega)} = G_m(i\omega) \tag{4.12}$$

Using G given by (4.8) we get after some calculations

$$k_r = \frac{3(259-\omega^2)}{458}$$
$$k_y = \frac{2(137+7\omega^2)}{229} \tag{4.13}$$

Some numerical values of k_r and k_y are given in Table 1. The desired closed loop system for the nominal process model is obtained for k_r = 1.5 and k_y = 1.0. The models obtained for $0 \le \omega < 2$ give reasonable values of the parameters.

Simple calculations show that the closed loop system obtained by applying the feedback (4.5) to (4.8) is stable if

$$0 < k_y < \frac{3900}{229} \approx 17.03$$

This value of k_y corresponds to $\omega = \sqrt{259}$. See Table 1.

The equilibrium values for the regulator gains for sinusoidal reference values thus give a stable closed loop system only if the frequency is less than 16.09 rad/s. Notice that a and b are infinite for this frequency.

Remedy

Once the mechanisms which cause the difficulties are understood it is easy to find suitable remedies. The key problem when fitting a low order model to a complex process is that the low order model obtained will critically depend on the frequency content of the input signal. To achieve identifiability it is also necessary that the components of the input signal in the useful frequency range are persistently exciting. To ensure that a reasonable model is obtained it is therefore necessary to monitor the frequency content of the input signal and the conditions for excitation. If the input signal is not suitable then a proper perturbation signal must be added. It can be demonstrated by simulation that the oscillations arising for $\omega > 16.09$ can be quenched by adding a reference signal with sufficient low frequency content. Sampled systems also behave differently because the sample-and-hold will introduce additional frequencies. If it is not possible to add a perturbation signal the parameters should be updated only when there is proper excitation. Further details of a scheme of this type is given in Section 6. It also follows that the instability is generated by poor estimation and not by high gain and large phaseshift.

5. MEASUREMENT NOISE

When measurement noise was added Rohrs also found that the parameter k_y could drift until the stability limit is reached. To understand this mechanism consider the equations for updating the regulator gain

$$\frac{dk_r}{dt} = - kr(y - y_m) \tag{5.1}$$

$$\frac{dk_y}{dt} = ky(y-y_m) \tag{5.2}$$

It follows from equation (5.2) that any measurement error in y will result in a drift of k_y with positive rate because of the term y^2. This is a property of the particular MRAS algorithm used and not an intrinsic property of adaptive control. A simple remedy has been known for a long time by practitioners of MRAS. Equation (5.2) is replaced by

$$\frac{dk_y}{dt} = k\tilde{y}(y-y_m) \tag{5.2'}$$

where $\tilde{y}$ is a filtered version of y. Because of the dynamics of the filter high frequency disturbances will not cause drift. Also notice that there is a drastic difference between the continuous time case and the discrete time case in this respect. In the discrete time algorithms the term $y(y-y_m)$ is replaced by $y(t-d)[y(t) - y_m(t)]$, see Ljung and Söderström (1983). The filtering of the regression vectors and the modification of the error have in fact been two important themes in the development of model reference adaptive systems. See Landau (1979).

Another drawback with the updating formula (5.1) and (5.2) is that they can be interpreted as estimation algorithms with forgetting of old data. See Åström (1980a,b). Hägglund (1983) has proposed superior algorithms which only discount data in the directions where there is new information.

Rohrs results for a constant reference signal are also easy to explain. When the reference signal is a constant the input u to the process is also a constant. Since a constant input signal is persistently exciting of order 1 it follows that the parameters k_r and k_y are not identifiable. The equilibrium set for the parameters is then given by the plane

$$2k_r - 2k_y - 1 = 0 \tag{5.3}$$

An arbitrary small disturbance can then make the parameters drift in this subspace. The system becomes unstable as soon as k_y comes outside the interval (0,17.03). A typical example is shown e.g. in Fig. 4-19A and in Section 5.2.5 in Rohrs (1983).

Rohrs also found difficulties due to a combination of a constant reference signal and a constant disturbance. Using the method of averaging the parameters are then approximatively described by

$$\frac{dk_r}{dt} = - kr_0 \left[\frac{2k_r r_0 + d_0}{1+2k_y} - r_0 \right]$$

$$\frac{dk_y}{dt} = k \left[\frac{2k_r r_0 + d_0}{1+2k_y} \right] \left[\frac{2k_r r_0 + d_0}{1+2k_y} - r_0 \right]$$

where r_0 is the constant reference value and d_0 is a constant disturbance of the process output. These equations hold only if $0 < k_y < 16.09$.

The equilibrium set is then

$$2k_y - 2k_r - 1 + d_0/r_0 = 0 \tag{5.4}$$

Compared to (5.3) the measurement error then gives a shift in the equilibrium set. Notice that the shift is proportional to d_0/r_0.

In the special case $r_0 = 0$ the equilibrium set is k_r arbitrary and $k_y = \infty$. The parameter k_y is in fact given by the differential equation

$$\frac{dk_y}{dt} = \frac{kd_0^2}{(1+2k_y)^2}$$

which has the solution

$$4k_y^3 + 3k_y^2 + 3k_y = 4kd_0^2 t$$

The gain k_y will thus increase monotonically towards $+\infty$ with increasing t. When k_y becomes 17.03 the closed loop becomes unstable.

Once the basic difficulty is understood it is easy to find a remedy. Simply make sure that persistent excitation is obtained by signals in the right frequency range either by adding perturbation signals or by updating parameters only when there is proper excitation. See Section 6. The equilibrium set for the parameters is then a point and small disturbances will only generate small parameter deviations.

6. DESIGN OF ROBUST ADAPTIVE SYSTEMS

Having developed some insight into the mechanisms that cause instability of an adaptive system design of a robust adaptive system will now be discussed briefly. More details on this problem are found in Åström (1980a,b). The point of view that many adaptive system can be regarded as a combination of a control design for systems having known parameters and a recursive parameter estimation problem is taken. The robustness of the deterministic design problem which is the basis of the design calculations is first discussed. The robustness of recursive estimation and of the combined problem are then treated.

Robust Control Design

Robustness properties are conveniently discussed in terms of the loop gain. See the Bode plot of a typical loop gain in Fig. 1. The loop gain is unity at the cross-over frequency ω_c. A common engineering practice which is now well supported by theory Horowitz (1963), Doyle and Stein (1981) boils down to the following: Make the loop gain high below

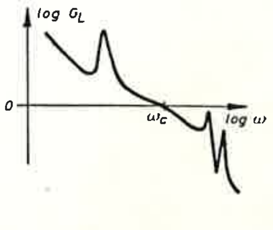

Fig. 1. - Bode diagram of the loop gain.

the cross-over frequency and make sure that the loop gain falls off rapidly above the cross-over frequency. A high loop gain for low frequencies is obtained by introducing integral action or some resonant system which gives a high gain for special frequency bounds as is indicated in Fig. 1. The rapid roll-off for high frequencies is necessary to ensure that unmodeled high frequency dynamics will not cause difficulties. Computer controlled systems should always be provided with antialiasing filters to eliminate signal transmission above the Nyquist frequency. The high frequency roll-off for a digital regulator is thus significantly influenced by the sampling period.

A quantitative statement of the above discussion for a design like the MRAS, which is based on pole placement, can be obtained as follows. Consider a regulator designed for the process model

$$y(t) = G(p)u(t) = \frac{B(p)}{A(p)} u(t) \tag{6.1}$$

where A(p) and B(p) are polynomials. Let T/S be the ratio of the feedforward and the feedback transfer functions obtained from a design procedure which gives a closed loop system with the transfer function G_m.

THEOREM 1

Consider a system with the transfer function G_o. The closed loop system obtained using a design procedure based on the model G is then stable if

$$\left|G_o - G\right| < \left|\frac{G}{G_m}\right| \cdot \left|\frac{T}{S}\right| \tag{6.2}$$

on the imaginary axis and at infinity.

□

The theorem is proven for discrete time systems in Åström (1980c). It also follows from the result of Doyle and Stein (1981). Other theorems of similar nature are given in Mannerfelt (1981) and Åström and Wittenmark (1984).

The left-hand side of the inequality (6.2) is the error in the model transfer function. The right-hand side contains quantities which can be computed when the design calculations have been performed. Notice that G is the open loop pulse transfer function of the plant model and that G_m is the pulse transfer function from the command signal to the output.

The detailed character of the inequality (6.2) is highly problem dependent. Some general characteristics can, however, be found by inspection. It follows that it is necessary to have a model which gives an accurate description of the process for frequencies around the cross-over frequency.

Robust estimation

When a parameter estimator is used in an adaptive scheme it is important to make sure that good estimates are obtained. The necessity for taking precautions so that poor models are not generated by bad data is amply illustrated by Rohrs' examples.

To guarantee a stable closed loop system in the case of known parameters it follows from Theorem 1 that precision of the model transfer function is needed for frequencies in the neighbourhood of the cross-over frequency. To ensure this it is therefore necessary that the input signal has a sufficient energy content in that frequency band. This can be monitored using the system shown in Fig.2. The conditions for persistent excitation, Åström and Bohlin (1965), can be monitored instead of the signal energies as shown in Fig.2. Filtering of the signals before they are introduced into the estimator also helps.

If the power of the useful signal component is less than the noise power there are two options: Excitation signals may be introduced or the parameter estimation may be switched off. See Ioannou and Kokotovic (1982). Guided by the results of Egardt (1979a) and Peterson and Narendra (1982) it is reasonable to estimate only when the absolute level of the useful input energy is above a certain level. These safe-guards can be regarded as an implementation of the common sense rule: Do not estimate unless the data is good.

There are other safe-guards of a similar nature to make sure that the data used for estimation is always good by excitation or that the parameter estimation is only made when the data is reasonable. The difficulties due to high frequency reference signals and measurement noise noticed by Rohrs and others (1981,1983) will not arise if the precautions discussed above are taken.

Robust adaptive control

To obtain a robust adaptive control algorithm it is necessary to use both robust control and robust estimation. It is also necessary to make sure that the equivalent gain in the adaptive loop is sufficiently small.

In the adaptive problem there are also some new trade-offs to be made. Consider for example the robustness properties obtained by having a high open loop gain at low frequencies. This may be obtained by having integral action in the control loop. It can also be obtained via adaptation. An adaptive regulator with enough parameters will automatically introduce a high gain at those frequencies when there are low frequency disturbances.

I have often found it beneficial to use a design method which gives a high gain at low frequencies and use adaptation only to find the characteristics around the cross-over frequency. This has the additional advantage that fewer parameters are needed. It speeds up the estimation, and the degrees of the polynomials are kept low

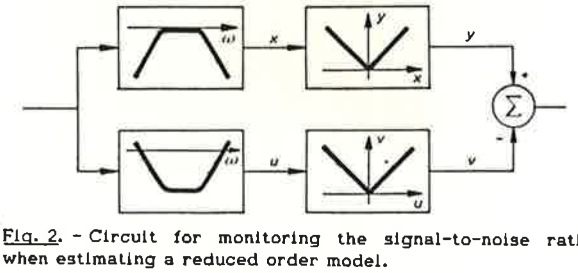

Fig. 2. - Circuit for monitoring the signal-to-noise ratio when estimating a reduced order model.

which improves the inherent numerical problems with polynomial representation. One possibility is to estimate a model of the type

$$A(q)F(q)y(t) = B(q)F(q)u(t-1) + C(q)e(t) \quad (6.3)$$

where F(q) is an operator having high pass characteristics.

7. CONCLUSIONS

The "counterexamples" to adaptive control published by Rohrs and others (1981,1982,1983) have been scrutinized. It has been found that the instabilities observed are caused either by high equivalent adaptation gain, high frequency command signals or measurement noise. Explanations of the different mechanisms are given and remedies are suggested. The explanations are different from the arguments given by Rohrs and others (1981, 1983) which are based on the notion of high gain. Instead the arguments are based on a characterization of the equilibrium sets for the parameters. The instability mechanisms due to high frequency command signals and measurement noise are both due to the fact that the equilibrium sets contain parameter values which render the closed loop unstable. The reason for this is that the input signal does not excite the process properly. The problems can be avoided by filtering and by monitoring the excitation and either introducing perturbation signals or to switch off the estimation when the excitation is not proper. The instability due to too high equivalent adaptation gain is avoided by modifying the parameter updating algorithms.

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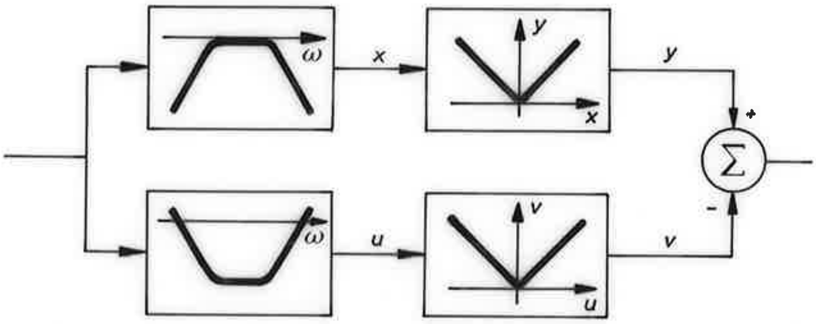

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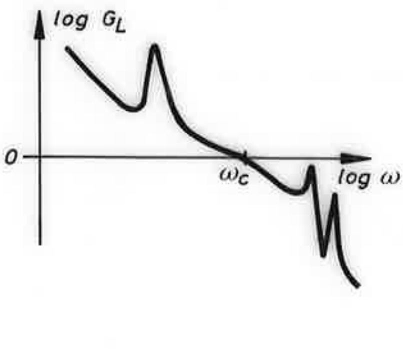

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