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A COMPARISON BETWEEN ROBUST AND ADAPTIVE CONTROL OF UNCERTAIN SYSTEMS

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Abstract. Design of control systems for plants with considerable parameter uncertainty can be approached by robust control and adaptive control. Robust design methods gives fixed compensators with satisfactory performance over a specified range of plant parameter variations. Adaptive design methods extract knowledge of the plant parameters on line and redesigns the control law. These two approaches are compared in the paper.

1. INTRODUCTION

This paper considers control of systems with unknown or timevarying dynamics. It is attempted to understand the possibilities and limitations of controlling such system with linear constant parameter controllers, and to compare with adaptive control. One goal is to indicate when one type of control is advantageous over the other. The problem of plant uncertainty inspired Black's invention of the feedback amplifier. It has in different forms been in the mainstream of feedback control research ever since. The problem is now frequently popularized as robust control. The problem of parameter variations cannot be dealt with in isolation. A proper approach must, however, also include other aspects of control systems design like command following, load disturbances and measurement noise. In this paper the main emphasis is, however, on the parameter variations.

Some fundamental issues that characterize the control of uncertain systems are discussed in Section 2. A few illustrative examples are given. Different ways to formulate design problems which can cope with parameter variations are presented in Section 3. The methods of Horowitz, Doyle and Stein, and the multimodel approach are included. Conditions for the methods to work are stated. In Section 4 the adaptive approach is briefly described, as well as conditions for it to work. Section 5 contains three examples, solved both with the Horowitz method and with an adaptive method. The examples are: a simple servo, a system with time delay, and a robotic example. The discussion in Section 6 covers a comparison of the various conditions for robust and adaptive control, and a few conclusions are drawn from the examples. Section 7 finally various suggests ways to combine adaptive and robust control.

2. FUNDAMENTAL ISSUES

Some fundamental limitations that uncertainties in process dynamics imposes on the performance of a control system will be discussed in this section. A closely related problem is how much information is required about a control system in order to make a control design. Since the robust design methods are based on the Nyquist curve at the plant it is essential to know if knowledge about the Nyquist curve alone suffices for design. Since the transfer function is analytic apart from singularities it is in principle sufficient to know the transfer function in an arbitrarily small region. The full transfer function can then in principle be recovered by analytic continuation. This argument requires that the transfer function is known exactly. The situation is very different when the transfer function is only known approximatively as is shown by the following example.

Example 2.1

Consider a system with the loop transfer function

$$G(s) = \frac{1}{s(s+1)}$$

The corresponding closed loop system is clearly stable. Now consider the system with the loop transfer function

$$G_\varepsilon(s) = \frac{s-a-\varepsilon}{s-a} G(s)$$

The transfer function G and G_ε can be made arbitrarily close by choosing ε sufficiently small. The closed loop system which corresponds to G_ε is, however, unstable. □

The example shows that in order to guarantee stability it is not enough to know the simplified Nyquist curve. It is also necessary to know if there are some unstable right half plane poles which are close to cancellation. This is also clear from the Nyquist stability criterion which states that the closed loop system is stable if the number of encirclements of a standard contour equals the number of singularities of the open loop transfer function in the right half plane. It is thus essential to know the unstable process poles explicitly.

In the robust design method introduced in the following section it is crucial to know the sign of the high frequency gain. The following result illustrates why this is necessary.

Example 2.2

An integrator whose sign is not known cannot be controlled using high gain robust control. □

Uncertainties in the time delay poses another essential constraint. This is illustrated by the following example.

Example 2.3 - Time delays

Consider a linear plant where the major uncertainty is due to variations in the time delay. Assume that the time delay varies between T_{min} and T_{max}. Furthermore assume that it is required to keep the variations in the phase margin less than 20°. It then follows that the cross-over frequency ω_c must satisfy

$$\omega_c \le \frac{0.35}{T_{max} - T_{min}}$$

The uncertainty in the time delay thus induces an upper bound to the achievable cross-over frequency.

□

3. APPROACHES TO ROBUST DESIGN

Different approaches to design of robust control systems will be summarized in this chapter. Although we are emphasizing the effect of parameter variations it should be kept in mind that the control should also be capable of dealing with command following, load disturbances and measurement noise.

The Horowitz Design Method

This method is a direct descendent of Bodes work on feedback amplifiers (Bode, 1945), where the key problem was to design a feedback system that could cope with the changing characteristics of vacuum tube amplifiers, see (Horowitz, 1963).

The key ideas of the Horowitz design are the following: Assuming that the output and the reference are both available for measurement, a two degree-of-freedom structure, consisting of a feedback compensator, and a prefilter is proposed. The purpose of the feedback loop is to stabilize the plant (if necessary), to reduce the sensitivity to plant variations, and to reject disturbances. The prefilter is designed to shape the nominal transmission from reference to output.

One consequence of feedback is that measurement noise is led back to the input channel, possibly leading to saturation. Therefore it is essential to keep the bandwidth and the gain of the feedback loop as small as possible while satisfying the specifications on disturbance rejection and sensitivity reduction.

A key idea in the Horowitz design method is design to specifications. Horowitz calls his method "quantitative" in contrast to most other methods (Horowitz, 1975a). Originally the method was developed for minimum phase SISO plants (Horowitz, 1972). It has been extended in various degrees to non-minimum phase plants, (Horowitz, 1984), plants with saturations, cascaded plants, time varying plants, non-linear plants, multi-variable plants, etc. This method is still very much subject of further research and extensions to new types of systems, see e.g. Yaniv (1986a).

The major drawback of the method is that except for certain classes of systems it is impossible to know a priori if the desired closed loop specifications are attainable. In such cases the design becomes a trial-and-error procedure, and the designer never knows if a more skilful designer would have been able to satisfy the specifications. The advantage is, however, that a systematic design procedure is available which allows the designer to gain insight into the trade-offs between different specifications, such as closed loop sensitivity and bandwidth, complexity of regulator, measurement noise amplification, etc.

Design Procedure

The basic design procedure may be summarized in five steps:

Step 1: Determine the closed loop specifications as upper and lower limits on the gain of the closed loop transfer function. The differences between the limits (in decilog or dB) are called TOLERANCES. Various suggestions have been made how to translate time domain specifications into specifications on the closed loop gain (Horowitz, 1972). There exists no systematic procedure to do so. Notice in particular that if the uncertain plant is such that the specifications are satisfied for all plant cases, then an open loop control could be considered.

Step 2: Determine the plant uncertainty. For each frequency the plant uncertainty gives rise to a TEMPLATE, i.e. a set in the complex plane. The plant may be given in parametric form. The uncertainty is then specified as bounds for each parameter. Alternatively, a number of experimental transfer function may be acquired. The templates are then read directly for each frequency of interest.

Step 3: Given the tolerances and the templates, calculate constraints on the open loop including the feedback compensator. In the complex plane the constraint for each frequency will take the form of a border between an allowed and forbidden region for the nominal compensated open loop system. We call the borders HOROWITZ BOUNDS.

Step 4: Design a feedback compensator so that the compensated open loop transmission satisfies the bounds. This is conveniently done in a Nichols chart. Minimum phase and stable links are included, deleted and changed until the loop transfer function for each frequency is on the correct side of the bound, or until the task is considered impossible.

Step 5: Design a prefilter so that the total nominal or average transfer function from reference to output is the desired one. This design is conveniently done in a Bode chart.

Notice that phase tolerances could be included in Step 1. Disturbance rejection specifications are normally entered for high frequencies in Step 3 in order to produce high frequency bounds; such specifications can also be introduced for lower frequencies.

It has been shown in (Horowitz 1978, 1984) that the design procedure will always give a solution if the plant is minimum-phase, if the sign of the high frequency gain is known and if the parameter uncertainties are such that for every gain, $0 \le \arg G(i\omega) \le 2\pi - \varphi(\omega)$, where φ is positive, for frequencies around the crossover frequency ω_c.

There are several interactive computer programs for the Horowitz design method. One of them is HORPAC (Gutman, 1985).

Other Design Methods

There are other design methods which can give robust control laws. One technique has been proposed by Doyle and Stein. This method is based on the LQG methodology. Another approach is to describe the system by a set of models (M_k) and it is attempted to find a constant gain regulator which will give satisfactory performance for all models in the class. The design methods used in this case are frequently based on multiobjective optimization.

4. ADAPTIVE CONTROL

When comparing robust and adaptive control it is essential that similar control problems are discussed. For this reason we will discuss an adaptive regulator for a deterministic control problem. It is assumed that the process to be controlled can be modeled by

$$A(q)y(t) = B(q)u(t) \tag{4.1}$$

where u is the control variable y the controlled output and A and B are polynomials in the forward shift operator. Assume that it is desired to find a control law such that the relation between the command signal and the output is given by

$$A_m(q)y(t) = B_m(q)u(t) \tag{4.2}$$

Furthermore let the observer polynomial be A_o. When the parameters are known and certain technical conditions are satisfied the design procedure can be expressed as follows.

Design Procedure

Factor the polynomial B as B^+B^-, where B^+ is monic stable and well damped. Determine polynomials R and S which satisfy the equation

$$AR_1 + B^-S = A_oA_m \tag{4.3}$$

The control law is then

$$Ru = Tc - Sy \tag{4.4}$$

where $R=R_1B^+$ and $T=A_oB_m/B^-$. □

The equation (4.3) has many solutions. Under the compatibility conditions a solution which is realizable can always be found. In the case when there are known disturbances characterized by the polynomial A_d a solution where A_d divides R can be found. This corresponds to the internal model principle.

An Adaptive control law can now be obtained as follows.

ALGORITHM
Repeat the following steps at each sampling period:

Step 1. Update the estimates of the parameters of the model (4.1) by some recursive estimation method.

Step 2. Apply the design procedure to obtain the polynomials R, S and T of the control law (4.4) and the control signal u. □

In the examples presented in Section 5 we will use recursive least squares as an estimation method. The signals u and Y will be high pass filtered before they are entered into the estimator. It will be assumed that the polynomial A_d which characterizes the disturbances is $A_d = q-1$. This ensures that the regulator will have integral action. There are of course many other choices.

5. EXAMPLES

In this Section we will give examples of robust and adaptive designs.

Example 1: A simple servo. The servo model is taken from (Aström, 1979):

$$P(s)=k/(1+Ts)^2, k \in [1,4], \quad T \in [0.5,2] \tag{5.1}$$

The system is thus of second order with variations in gain and time constants. The aim of the control system is to make the servo follow step commands well in face of the above specified parameter variations, and constant load disturbances.

Robust design: The closed loop gain tolerances are shown in Fig. 1. Step 3 in Horowitz design procedure gives the bounds shown in Fig. 2. A suitable compensator can now be designed by trial and error. Fig. 2. shows one compensator which satisfies the bounds. The corresponding controller has the transfer

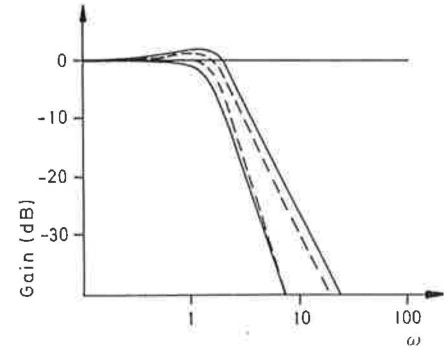

Fig. 1 Closed loop gain tolerance specifications, and envelope of final transfer functions from reference to output for Example 1.

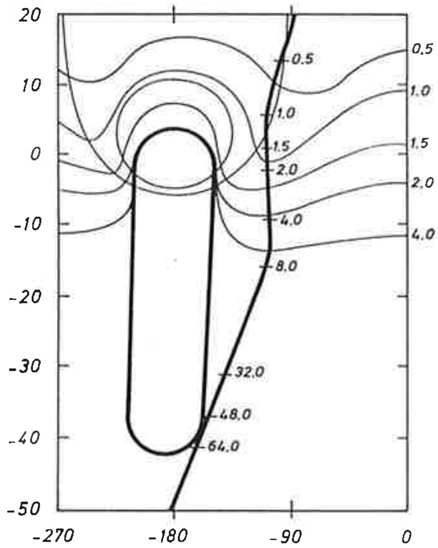

Fig. 2 Nichols chart (dB vs. degrees) displaying bounds and the compensated nominal open loop for Example 1.

function

$$G(s) = \frac{4\cdot 10^7(s+0.25)(s+1.5)}{s(s+30)(s+500s+250000)}$$

An integrator is included to satisfy the bounds more easily and to ensure rejection of low frequency disturbances. A suitable prefilter is

$$F(s) = \frac{2.89}{s^2+1.87s+2.89}$$

Fig. 1 shows that this prefilter gives a closed loop system which satisfies the tolerance specifications. The time domain properties of the compensated system are illustrated by simulation. Fig. 3 shows the response when the gain k is changed from 1 to 4 at time t=15s and Fig. 4 shows the response when the time constant T is changed from 1 to 0.5 at time t=15s.

Adaptive control: An explicit 2nd order pole placement Self Tuning Regulator is suggested in (Aström, 1979). The procedure described in Section 4 is used. A second order model is estimated. The desired response is specified to be of second order with the bandwidth 1.5 and the relative damping 0.707. The sampling interval

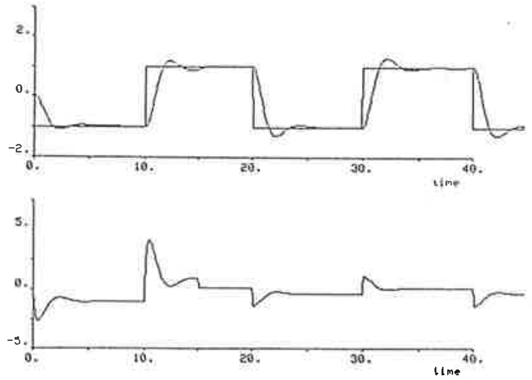

Fig. 3 Simulation of the robust control system. The gain is changed from 1 to 4 at time t=15s. The time constant is 1.

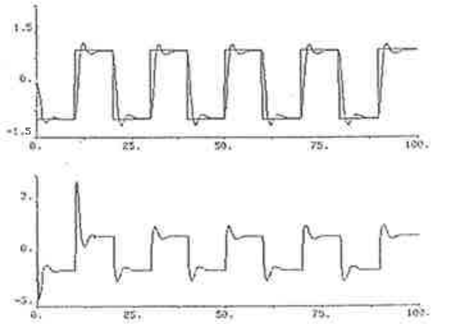

Fig. 4 Simulation of the robust control system. The time constant T changes from 1 to 0.5 at time t=15s. The gain is k=1.

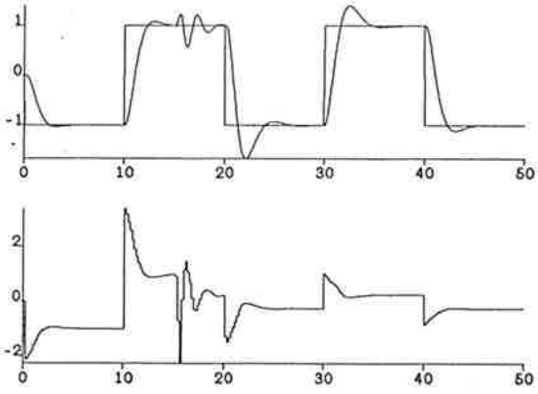

Fig. 5 Simulation of the adaptive control system. The gain is changed from 1 to 4 at time t=15s. The time constant is 1.

is 0.3s. The performance of the adaptive system when the gain and time constants are changed are shown in Fig. 5 and Fig. 6. These figures are the same experiments as were shown in Fig. 3 and Fig. 4 for the robust regulator.

Comparison: A comparison of the Fig. 3 with Fig. 5 and Fig. 4 with Fig. 6 gives some of the characteristics of the different approaches. It is clear that the robust control responds much faster and cleaner to parameter variations than the adaptive system. In Fig. 5 and Fig. 6 it takes three transients after a parameter change before the adaptive systems have adjusted to the changed parameters. The robust system responds

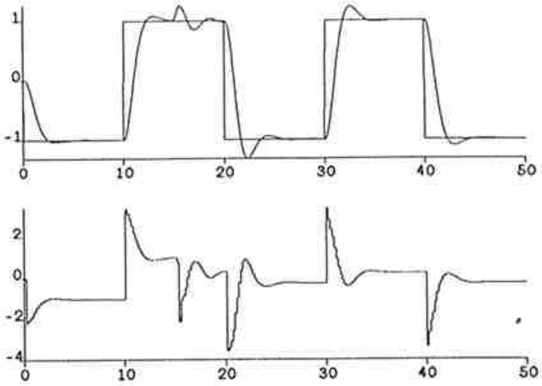

Fig. 6 Simulation of the adaptive control system. The time constant T changes from 1 to 0.5 at time t=15s.The gain is k=1.

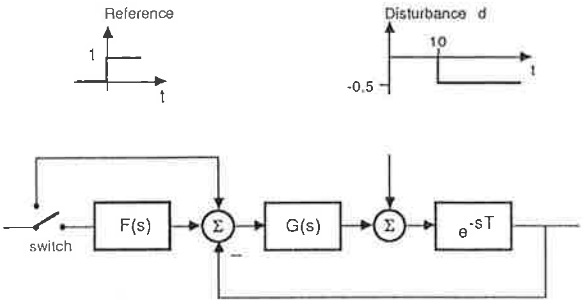

Fig. 7. Control system configuration for the integrator with time delay.

instantaneously. The responses of the robust systems are not perfect although they are within the specifications. See e.g. Fig. 4 where there is a distorsion in the step response. The responses of the adaptive systems are better when they have adapted to the changed conditions.

Example 2: An integrator with variable time delay. The plant is a simple non-minimum phase system. It is also of interest because it is a typical part of many industrial processes.

$$P(s) = \frac{1}{s}\exp(-sT), \quad T \in [0,1] \qquad (5.2)$$

The system is assumed to be subject to step disturbance inputs at the plant input. The aim of the control system is to follow step commands as fast as possible, and to eliminate the effect of the load disturbances as fast as possible.

Robust design: The system configuration is shown in Fig. 7. The unknown time delay causes a phase uncertainty that will violate the maximum phase condition of Section 3 for high frequencies. The attainable bandwidth is therefore limited. The maximally attainable crossover frequency is about 3 rad/s. It seems impossible, however, to find a rational, proper, stable and minimum-phase compensator that achieves this, since the phase advance demands and the gain limitation demands seem incompatible (cf. Bode's relations, Bode (1945)). Instead a simple lead-lag network is suggested that gives a crossover frequency of 0.7 rad/s (see Fig. 8):

$$G(s) = \frac{0.6\left(1 + \frac{s}{1.3}\right)}{s\left(1 + \frac{s}{2}\right)}$$

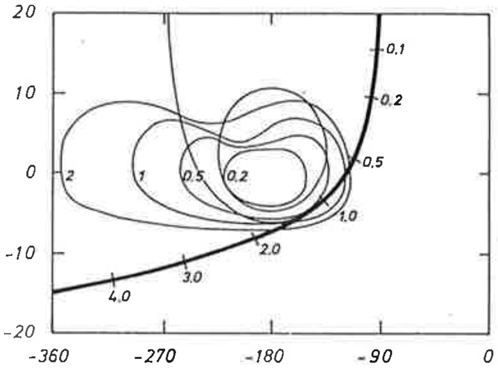

Fig. 8 Nichols chart (dB vs. degrees) displaying bounds and the compensated nominal open loop for the integrator with delay.

It may be possible to get a slightly higher crossover frequency without too great an effort. To increase the bandwidth of the transfer function from reference to output, the following prefilter was tried:

$$F(s) = \frac{(1+s)}{\left(1 + \frac{s}{3}\right)}$$

In the simulations, step command following and step disturbance rejection was tested for various values of T, with and without the prefilter. The results are shown in Fig. 9 and Fig. 10.

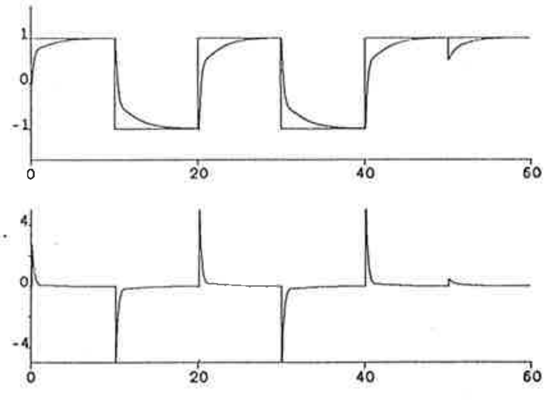

Fig. 9. Step response and step disturbance rejection for control with the robust regulator when T_d =0.1.

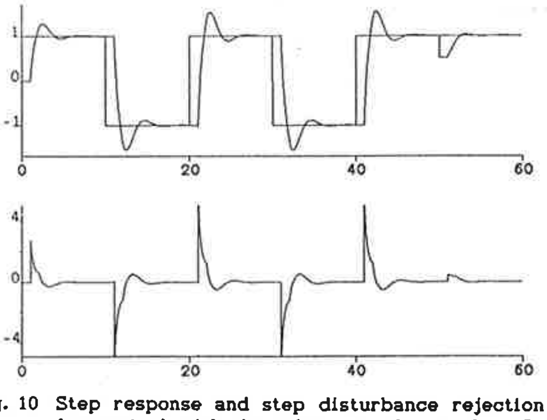

Fig. 10 Step response and step disturbance rejection for control with the robust regulator when T_d =1.

Adaptive control: The model

$$y(t+h) = y(t) + b_1 u(t) + b_2 u(t-h)$$

was used in the adaptive regulator. The sampling period was chosen as 1s which is adequate to cover the variations in the time delay with the model. A shorter sampling period will require a more complicated model. The polynomials A_m and A_o were chosen as

$$A_m(z) = z - a \text{ and } A_o(z) = z$$

The procedure described in Section 4 was then applied. The results obtained are show in Fig. 11 and Fig. 12.

Comparison: Since the system in this example is non-minimum phase Horowitz design procedure is not guaranteed to give a solution. Fig. 9 and Fig. 10 also shows that although the frequency domain specifications are satisfied there is quite a variation in the step responses when the time delay is changing. The responses of the adaptive system are much better. Apart from the time delay there is practically no difference between the responses shown in Fig. 11 and Fig. 12. Also notice the differences in the control signals. The high gain nature of the robust regulator is clearly noticeable.

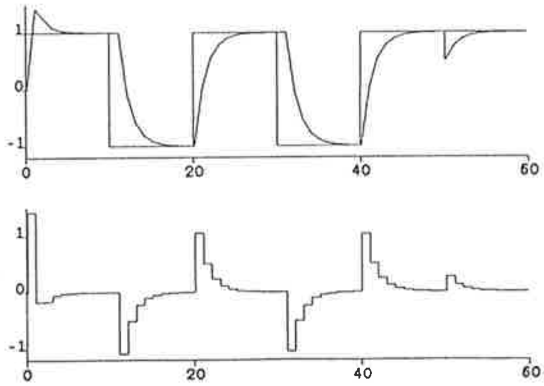

Fig. 11. Step response and step disturbance rejection for control with the adaptive regulator when T_d =0.1.

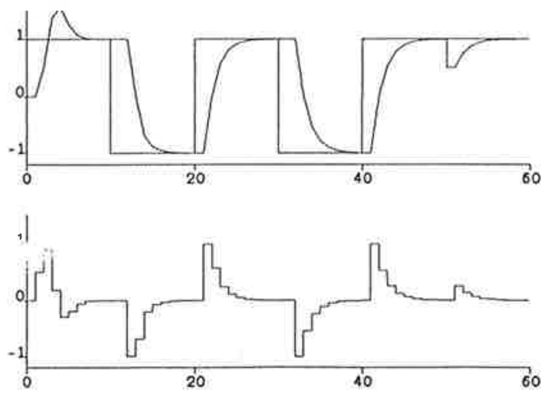

Fig. 12 Step response and step disturbance rejection for control with the adaptive regulator when T_d =1.

Discussion

Based on the examples shown in this paper and other that we have solved a few observations can be made about robust and adaptive control. Robust design will in general give systems that respond faster to variations in process parameters. They will, however, only respond well to variations inside the design specifications. The adaptive systems respond more slowly to parameter changes. The adaptive systems give better responses to set point changes and load variations when the parameter estimates have converged. This is particularly noticeable for non-minimum phase systems. The robust systems will in general have higher loop gains which makes them more sensitive to noise. Also notice that a few examples are not sufficient for a comparison between two design methods. More work is therefore necessary.

6. ROBUST AND ADAPTIVE CONTROL

Robust and adaptive control are two complementary ways to deal with process uncertainty. Robust control gives a fixed gain regulator which is designed to be insensitive to specific parameter variations. Adaptive control deals with uncertainty by reducing it through parameter estimation. The adaptive control laws are normally derived using the certainty equivalence principle. This means that a model is determined and a regulator is designed as if the model was exact. The fact that this procedure may lead to difficulties when the estimated model is inaccurated is now well understood. From this point of view it seems attractive to combine robust and adaptive control. This can be done in many different ways.

One possibility is to use a robust control design as the underlying design method in an adaptive system. This would intuitively be better than to use a certainty equivalence principle. The parameter estimation will also reduce plant uncertainty and thereby make the robust design easier. To use robust control in this way is difficult because the Horowitz design method in it present form is difficult to implement as an analytic procedure which can be used on line. The approach thus requires development of analytic versions. The approach suggested by (Doyle and Stein, 1981) may be an alternative. See also (Gawthrop 1985). There are however other ways to combine robust and adaptive control.

Another possibility is to apply parameter adaptation of a basic robust design. See (Yaniv, 1986b). The idea is to modify the parameters but not the structure of the original robust design when tighter uncertainty sets are identified on-line.

A robust gain scheduling procedure can be developed as follows. Divide the total plant parameter uncertainty set into subsects. Design off-line a robust controller for each subset. Use an on-line estimator to determine in what set the system currently is, and apply the appropriate controller. This method has certainly been used a long time in flight control. Problems of switching between controllers must be solved. The global stability problem must also be considered.

Robust feedback can also be combined with adaptive prefiltering. It is clear from the examples in Section 5 that it is easier to design a suitable robust feedback compensator than a prefilter. Adaptive control has also been very successful in tuning feedforward gains where there are no problems with closed loop stability. By having a fixed parameter robust compensator inside the loop, stability is easily analyzed. The closed loop system exhibits less variations than the original plant. The adaptive prefilter thus has an easier task to find the parameters for a feedforward compensator which gives the desired response to command signals. An adaptive prefilter also simplifies the feedback design.

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