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Åström, Karl Johan; Neumann, Linda; Gutman, Per-Olof

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# A Comparison of Robust and Adaptive Control

K.J. Åström  
L. Neumann  
P.O. Gutman

Department of Automatic Control  
Lund Institute of Technology  
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## A COMPARISON OF ROBUST AND ADAPTIVE CONTROL

K.J. Aström

Department of Automatic Control, Lund Institute of Technology,  
Box 118, 221 00 Lund, Sweden

L. Neumann and P.O. Gutman

EL-OP, Israel Electro-Optics Industries, POB 1165, Rehovot 76110, Israel

**Abstract.** Design of control systems for plants with considerable parameter uncertainty can be approached by robust control and adaptive control. Robust design methods gives fixed compensators with satisfactory performance over a specified range of plant parameter variations. Adaptive design methods extract knowledge of the plant parameters on line and redesigns the control law. These two approaches are compared in the paper.

### 1. INTRODUCTION

This paper considers control of systems with unknown or time varying dynamics. It is attempted to understand the possibilities and limitations of controlling such system with linear constant parameter controllers, and to compare with adaptive control. One goal is to indicate when one type of control is advantageous over the other.

The problem of plant uncertainty inspired Black's invention of the feedback amplifier. It has in different forms been in the mainstream of feedback control research ever since. The problem is now frequently popularized as robust control. The problem of parameter variations cannot be dealt with in isolation. A proper approach must also include other aspects of control systems design like command following, load disturbances and measurement noise. In this paper the main emphasis is, however, on the parameter variations.

Some fundamental issues that characterize the control of uncertain systems are discussed in Section 2. A few illustrative examples are given. Different ways to formulate design problems which can cope with parameter variations are mentioned in Section 3. The main emphasis is on the Horowitz design method. Conditions for this method to work for a realistic class of plants are stated in

essential that the perturbations are such that  $M(s)$  is stable.

There is a convenient frequency domain criterion which guarantees the stability of a closed loop system under unstructured uncertainty. This can be stated as the inequality:

$$\left| \frac{1 + L_0(j\omega)}{L_0(j\omega)} \right| > m(\omega) \quad (2.3)$$

where  $L_0(s)$  is the loop transfer function. The inverse of (2.3) gives the Horowitz bound which will be introduced later in Section 3. Unstructured uncertainties and consequences for design are discussed in Kimura (1984), Åström (1985), Doyle and Stein (1981), and Morari and Doyle (1986).

The graph topology is suitable for determining a neighbourhood of a linear system for investigating stability robustness, see Vidyasagar (1985).

In this paper we will use combinations of structured and unstructured uncertainty. It will be assumed that the low frequency characteristics of the plant are described by structured uncertainty but we will allow unstructured uncertainty at high frequency.

### Simultaneous stabilization

General conditions for existence of robust controls are not known. It is therefore of interest to know necessary conditions. Simultaneous stabilization is such a condition. It is defined as to find a fixed gain compensator that stabilizes all plants in a given set (Vidyasagar, 1985). For sets that include unstable plants robust control implies simultaneous stabilization. Criteria for robust stabilization are given in Kimura (1984), Lehtomäki et al. (1984), Vidyasagar (1985), Ghosh (1985), Bialas (1985), Saberi (1985). Simple conditions involve the notion of high frequency gain which is defined as follows.

Definition 1: The high frequency gain  $k$  of a plant  $P(s)$  is defined as:

$$k \triangleq \lim_{s \rightarrow \infty} s^d P(s) \quad (2.5)$$

where  $d$  denotes the excess of poles over zeros. □

family as shown in Yaniv (1986c).

The sign of the high frequency gain must be known when a linear time invariant compensator is designed. This is however no longer necessary if nonlinear compensators are used. See Nussbaum (1983) and Mårtensson (1986).

### 3. THE HOROWITZ DESIGN METHOD

There are several design methods which can give robust control laws. One technique has been proposed by Doyle and Stein (1981). It is based on the LQG methodology. By adjusting the weighting matrices in the LQG problem, a "loop transfer recovery" is achieved. Doyle and Stein (1981) also presents a method to analyze robustness when the plant uncertainty is unstructured, the so called  $\sigma$ -method which is based on minimum and maximum singular values. Morari and Doyle (1986) give an overview of several methods. Another approach is to describe the system by a set of models (Fan et al. 1985). The design methods used in this case are frequently based on multiobjective optimization. In this paper we will use the Horowitz design method, Horowitz and Sidi (1972). This was originally developed for structured perturbation. An extension to constructed perturbations will be given.

#### The Method

The Horowitz design method is a direct descendent of Bode's work on feedback amplifiers Bode (1945), where the key problem was to design feedback systems that could cope with the changing characteristics of vacuum tube amplifiers, see Horowitz (1963). The key ideas are the following: Assuming that the output and the reference are both available for measurement, a two degree-of-freedom structure, consisting of a feedback compensator, and a prefilter is proposed. The feedback loop is used to stabilize the plant (if necessary), to reduce the sensitivity to plant variations, and to reject disturbances. The prefilter is used to shape the nominal transmission from reference to output, thereby giving a required set point response. See Figure 3.1.

The Horowitz design method has been successfully used in a considerable number of practical applications, among others Ashworth and Towill (1982), Horowitz et al. (1983), and Gutman et al. (1986).

### Design Procedure

Step 1: Determine the closed loop specifications as upper and lower limits on the gain of the closed loop transfer function. Phase tolerances can also be included. The differences between the limits (in decilog or dB) are called TOLERANCES. Also determine the desired disturbance rejection, see Equation (3.3c). This may be frequency dependent.

Step 2: Determine the plant uncertainty by specifying the range of variation of the plant transfer function for different frequencies. For each frequency the plant uncertainty gives rise to a TEMPLATE.

Notice that no feedback is necessary at those frequencies where the tolerances exceed the plant uncertainty. If this is the case for all frequencies, an open loop control could be considered.

Step 3: Given the tolerances and the templates, calculate constraints on the open loop transfer function  $L(j\omega)$ . In the complex plane the constraint for each frequency will take the form of a border between an allowed and a forbidden region for the nominal compensated open loop system. The borders are called HOROWITZ BOUNDS.

Step 4: Design a feedback compensator so that the compensated open loop transmission satisfies the tolerances. This is conveniently done in a Nichols chart. A series compensator composed of minimum phase stable transfer functions is added until the loop transfer function for each frequency is on the correct side of the bound, or until the task is considered impossible.

Step 5: Design a prefilter so that all transfer functions lie within the closed loop specifications. This design is conveniently done in a Bode chart.

It is straightforward to calculate the Horowitz bounds. Several computer programs are available for the design, e.g. the interactive program HORPAC,



$$\left\{ \begin{array}{ll} 0 < A(\omega) < |T(j\omega)| < B(\omega), & \text{for } \omega < \omega_c \\ |T(\omega)| \leq \frac{C}{\omega^r}, & \text{for } \omega > \omega_c \end{array} \right. \quad \begin{array}{l} (3.3a) \\ (3.3b) \end{array}$$

$$\left| \frac{1}{1 + G(j\omega)P(j\omega)} \right| < x, \quad x > 1 \quad (\text{disturbance rejection}) \quad (3.3c)$$

where  $\omega_c$  defines the end of the "low frequency region". Assume that  $\omega_c \leq \omega_H$ , and let  $[\omega_c, \omega_H]$  be the "cross-over frequency region".

For a given  $\omega$  find the set of complex numbers  $E(\omega) \triangleq \{\alpha(\omega)\}$ :

$$\left\{ \alpha(\omega) : \Delta \left| \frac{\alpha(\omega)P(j\omega)}{1 + \alpha(\omega)P(j\omega)} \right| \leq B(\omega) - A(\omega) \text{ for } \omega < \omega_c \right. \quad (3.4)$$

$$\left. \text{and } \left| \frac{1}{1 + \alpha(\omega)P(j\omega)} \right| \leq x \text{ for all } P(s) \in P(s, p) \right\}$$

where  $\Delta$  denotes the variation over the set of plants. The difference  $B(\omega) - A(\omega)$  is the "tolerance" for  $\omega$ . We have the following result:

**Theorem 1:** Necessary and sufficient conditions to achieve the closed loop specifications (3.3) with stable, minimum phase, strictly proper compensators  $G(s)$  and  $F(s)$  are that:

- (a) if  $\omega_c < \omega_H$ , there exists an  $\omega \in [\omega_c, \omega_H]$  such that the set  $E(\omega)$  is simply connected, or, if  $\omega_c = \omega_H$ ,  $E(\omega_c^-) \cap E(\omega_H^+)$  is non-empty, (where  $\omega^-$  ( $\omega^+$ ) denotes an infinitesimal neighbourhood to the left (right) of  $\omega$ ).
- (b) that there exists a continuous function  $\beta(\omega)$  that satisfies the Bode gain-phase relations, such that for each  $P(s) \in P(s, p)$ ,  $\beta(\omega) \cdot P(j\omega)$  satisfies the general Nyquist stability criterion and such that  $\beta(\omega) \in E(\omega)$  for each  $\omega$ . □

**Remark 1:**

The assumption that  $\omega_c \leq \omega_H$  is necessary since if  $\omega_H < \omega_c$ , it is impossible to find a  $G(s)$  that satisfies (3.3a) for  $[\omega_H, \omega_c]$ . Knowledge of the sign of the high frequency gain is necessary for stabilizability. See section 2. □

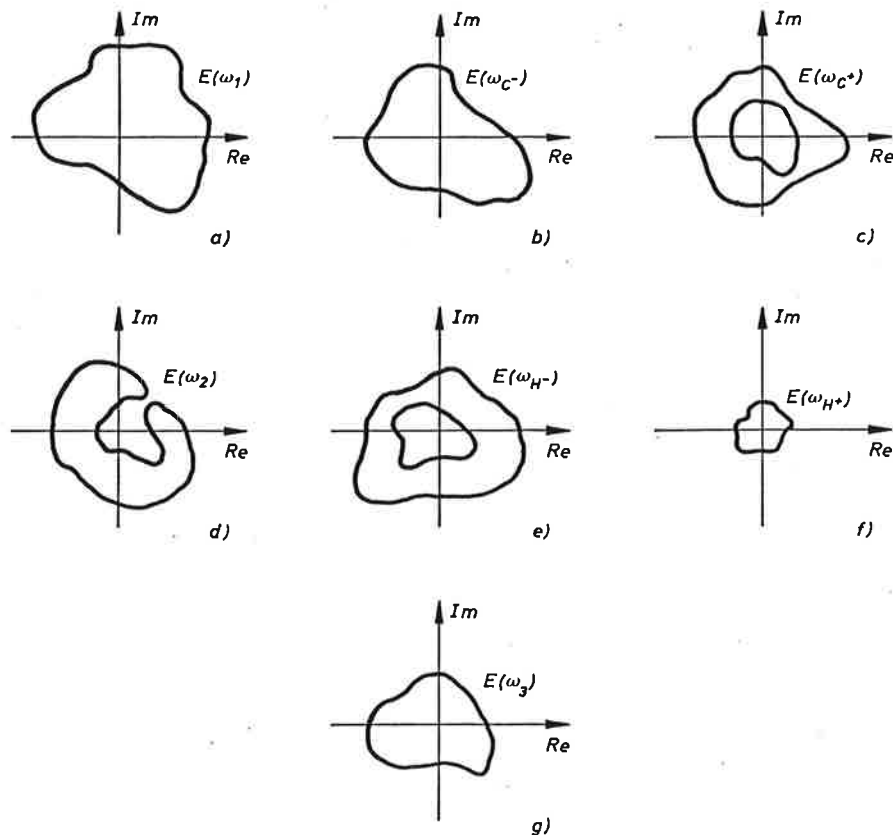


Figure 3.2.  $E(\omega)$  for some values of  $\omega$ .  $\omega_1 \in$  low frequency region.  $\omega_2 \in$  cross over frequency region.  $\omega_3 \in$  high frequency region. Note that  $E(\omega^+)$  is disjoint and that  $E(\omega^-) \subseteq E(\omega^+)$ . Note that  $E(\omega_H^-)$  is disjoint and that  $E(\omega_H^+) \subseteq E(\omega_H^-)$ . Note that  $E(\omega_3) = E(\omega_H^+)$ .

different frequencies.

From the above arguments it is clear that it is possible to select continuous functions  $\beta(\omega)$ , with  $\beta(\omega) \in E(\omega)$  for each  $\omega$ , separately in the low frequency region, and high frequency region. If  $\omega_c < \omega_H$ , the

where  $P_{\text{nom}}(\omega)$  is a nominal plant model.  $\square$

**Remark 6:**

Conditions (b) and (c) impose conditions on the interplay between the plant uncertainty (3.1) and the closed loop specifications (3.3). Recall that according to Bode's relations e.g. Ashworth, (1982), gain decrease per frequency unit is coupled to the phase (cf. e.g.  $1/s$  and  $1/s^2$ ). Hence, the required open loop gain decrease per frequency unit over the cross-over frequency region must not be such that the open loop phase violates the stability criterion (b). For example if the required gain decrease is of the order  $1/s^3$ , then the open loop phase has to be  $-270^\circ$ , and the Nyquist criterion will be violated for an open loop stable  $P(s)$ .

The passage from "high" open loop gains near  $\omega_c$  to "low" open loop gains near  $\omega_H$  must take place over a suitable frequency span. This imposes the following limit on the phase variations of the plant templates  $P_L(j\omega, p)$  in the cross over frequency region:

$$\Delta \arg P_L(j\omega, p) \Big|_{\omega_c \leq \omega_1 < \omega_2 \leq \omega_H} < (360 - \phi)^\circ \quad (3.6)$$

where  $\phi/2$  denotes the phase margin given by (3.3c)). An example of a "fat" template that must be placed either at a high or at a low open loop gain is given in Figure 3.3.

The theorem gives an insight into the trade-off between plant uncertainty and closed loop specifications. By relaxing the tolerances in (3.3) and (3.4), the sets  $E(\omega)$  may grow, and conditions (b) and (c) are easier to satisfy. Conversely, it might be found that the plant uncertainty has to be decreased, and that a robust control alone is not sufficient. One possibility is then to use on-line identification to improve plant knowledge, and to implement an adaptive controller. Note in particular that the theorem indicates the frequency range where the plant uncertainty should be reduced.  $\square$

**Remark 7:**

If the plant is non-minimum phase arbitrary specifications (3.3) cannot be satisfied, see Horowitz and Sidi (1978, Horowitz and Liao (1984). In practical

Horowitz (1980) that the optimal  $G(s)$  lies on the boundary of  $E(\omega)$  for all  $\omega < \omega_c$ . Gera and Horowitz (1980) also gives an algorithm how to compute the optimal  $G(s)$ .  $\square$

When applying the Horowitz design method in practice, only a finite number of plant cases is considered, Gutman and Neumann (1985). These must be chosen judiciously. It is not sufficient to consider only cases where the parameters assume their extreme values. This is realized by studying the template and Nichols chart of Figure 3.3.

#### 4. ADAPTIVE CONTROL

When comparing robust and adaptive control it is essential that similar control problems are discussed. For this reason we will discuss an adaptive regulator for a deterministic control problem. The regulator is based on the principle of certainty equivalence, i.e. the control design is based on the assumption that the identified process parameters are the correct ones. It is assumed that the process to be controlled can be modeled by

$$A(q)y(t) = B(q)u(t) \quad (4.1)$$

where  $u$  is the control variable  $y$  the controlled output and  $A$  and  $B$  are polynomials in the forward shift operator. Assume that it is desired to find a control law such that the relation between the command signal and the output is given by

$$A_m(q)y(t) = B_m(q)u(t) \quad (4.2)$$

Furthermore let the observer polynomial be  $A_o$ . When the parameters are known and certain technical conditions are satisfied the design procedure can be expressed as follows.

##### Design Procedure

Factor the polynomial  $B$  as  $B^+B^-$ , where  $B^+$  is monic stable and well damped. Determine polynomials  $R$  and  $S$  which satisfy the equation

$$AR_1 + B^-S = A_o A_m \quad (4.3)$$

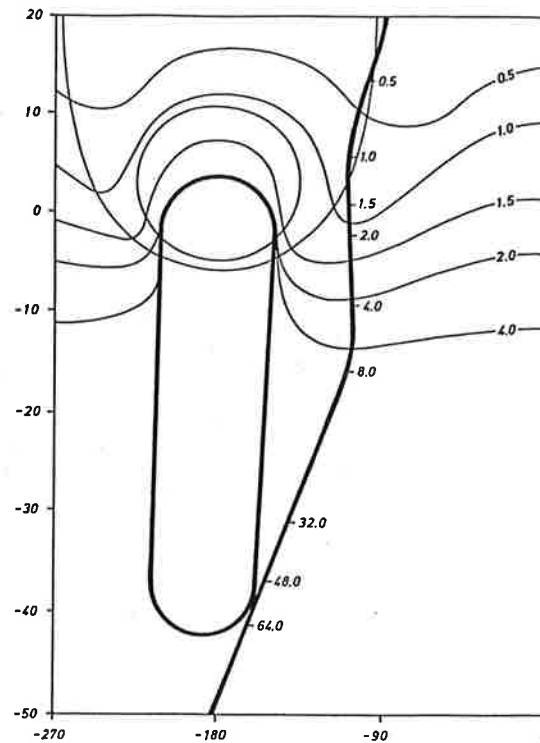


Figure 5.1. Nichols chart (dB vs. degrees) displaying bounds and the compensated nominal open loop for Example 1.  $P_{\text{nom}}(s) = 1/(1+2s)^2$ .

Consider a process characterized by

$$P(s) = k/(1+Ts)^2, k \in [1, 4], T \in [0.5, 2]$$

Let the aim of the control system is to make the output follow step commands well and give zero steady state error for constant load disturbances for all parameters in the given range.

#### Robust design:

The closed loop gain tolerances are given in Table 5.1. The disturbance rejection ( $= x$  in equ. 3.3c) is required to be at least 6 dB.

Table 5.1: Closed loop gain specifications for Example 1.

$\omega$ [rad/s]	0.1	0.2	0.5	1.0	1.5	2	4	$\geq 5$
upper [dB]	0.1	0.2	0.8	1.5	2.0	0	-12	-18
lower [dB]	-0.1	-0.2	-0.5	-1.0	-3.0	-8	-25	free

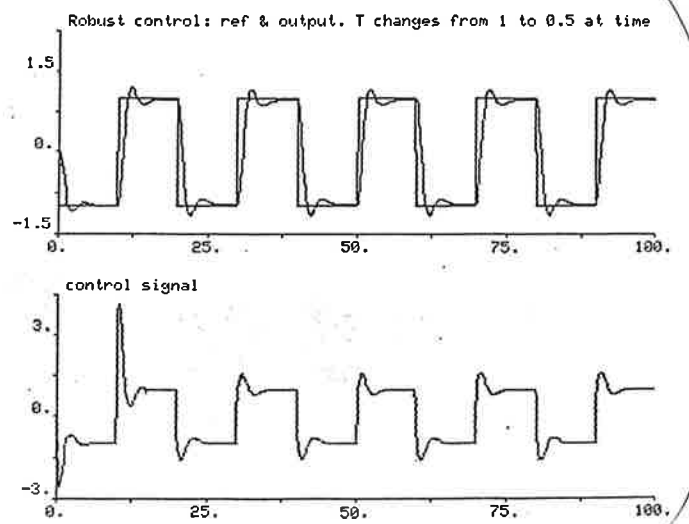


Figure 5.3. Simulation of the robust control system. The gain is changed from 1 to 4 at time  $t=15s$ . The time constant is 1.

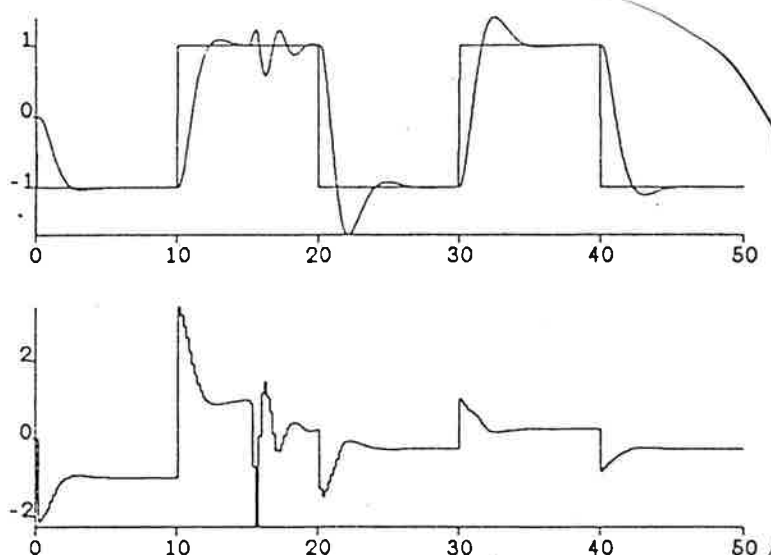


Figure 5.4. Simulation of the adaptive control system. The gain is changed from 1 to 4 at time  $t=15s$ . The time constant is 1.

Figure 5.4.

**Comparison:** A comparison of the Figure 5.3 with Figure 5.4 and other simulations, Åström et al. (1987b) gives some of the characteristics of the different approaches. The robust control responds much faster to parameter variations than the adaptive system. In Figure 5.4 it takes three transients after a parameter change before the adaptive system has adjusted to the changed parameters. The robust system responds instantaneously. The response of the robust systems is not perfect, but within specifications. The

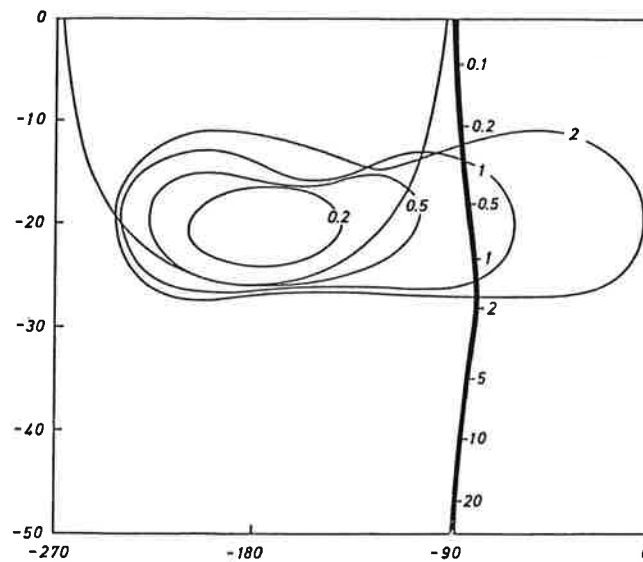


Figure 5.5 Nichols chart (dB vs. degrees) displaying bounds and the compensated nominal open loop for the integrator with delay.  $P_{\text{nom}}(s) = 1$ .

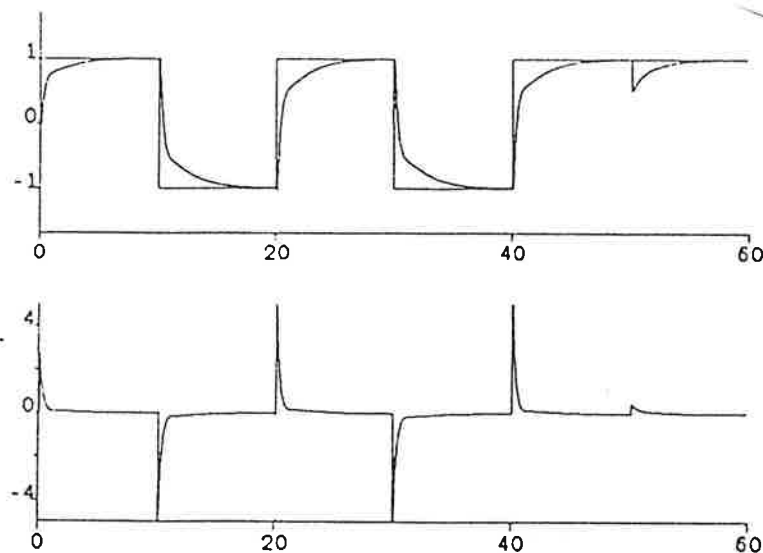


Figure 5.6. Step response and step disturbance rejection for control with the robust regulator when  $T_d = 0.1$ .

$$F(s) = \frac{(1+s)}{\left(1 + \frac{s}{3}\right)}$$

Simulations of the time responses for the extreme values of the time delay are shown in Figure 5.6 and Figure 5.7. A more complex controller that satisfied the bounds was attempted, but the time response were inferior, see Aström, (1987b).

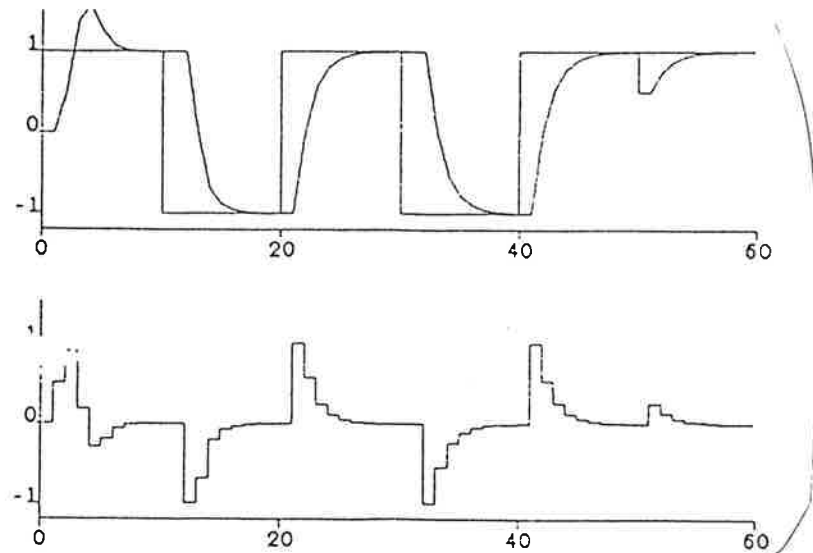


Figure 5.9. Step response and step disturbance rejection for control with the adaptive regulator when  $T_d = 1$ .

#### Comparison:

Figure 5.6 and Figure 5.7 show that although the frequency domain specifications for the robust design are almost satisfied there is quite a variation in the step responses when the time delay is changing. The responses of the adaptive system are much better. Apart from the time delay there is practically no difference between the responses shown in Figure 5.8 and Figure 5.9. Also notice the differences in the control signals. The high gain nature of the robust regulator is clearly noticeable.

#### Example 3: An industrial robot arm.

A simple model of a robot arm was used in this case. The transfer function from the control input (motor current) to measurement output (motor angular velocity) is

$$P(s) = \frac{\frac{km}{J_m} s^2 + d \cdot \frac{km}{J_m} J_a \cdot s + k \cdot \frac{km}{J_m} J_a}{s^3 + d \cdot \frac{J_m + J_a}{J_m \cdot J_a} s^2 + k \cdot \frac{J_m + J_a}{J_m \cdot J_a} s}$$

with  $J_a \in [0.0002, 0.002]$ ,  $J_m = 0.002$ ,  $d = 0.0001$ ,  $k = 100$ , and  $km = 0.5$ . The moment of inertia  $J_a$  of the robot arm varies with the arm angle. Bode plots of the plant gain for the extreme values of the arm inertia  $J_a$  are given in Figure 5.10. The purpose of the control system is to control the angular velocity step responses at various arm angles.



The open loop design is shown in Figure 5.11. A feedback compensator which satisfies the specifications is is:

$$G(s) = \frac{125 \left(1 + \frac{s}{50}\right) \left(1 + \frac{s}{300}\right)}{s \left(1 + \frac{s}{800}\right) \left(1 + \frac{s}{5000}\right)}$$

This compensator is essentially a PI regulator with a lead filter. The design of the prefilter presented some difficulties. It was possible to get the closed loop transfer function envelope within the specifications by choosing a unit gain prefilter with a pole at -60, a zero at -200, and a resonance at 200 rad/s with a relative damping of 0.25. However, individual transfer functions inside the envelope had well attenuated resonance peaks which may cause oscillations or "wobbling" Horowitz and Sidi (1972). Indeed, simulations revealed that the arm angular velocity wobbled slightly, while the measured motor angular velocity behaved smoothly. Moreover, the settling time was 0.3 seconds for the largest arm inertia, i.e. much slower than the designed bandwidth would seem to imply. The prefilter was therefore redesigned with the help of simulations so that acceptable, and non-oscillating step responses with a settling time of 0.3s were achieved for all arm angle inertias. The final prefilter is:

$$F(s) = \frac{\left(1 + \frac{s}{1000}\right)}{\left(1 + \frac{s}{26}\right) \left(1 + \frac{s}{200}\right) \left(1 + \frac{s}{200}\right)}$$

Simulated responses are shown in Figure 5.12 and Figure 5.13.

### Adaptive Design

In this particular problem the essential uncertainty is in one parameter only, the moment of inertia. It is then natural to try to make a special adaptive design where only this parameter is estimated.

The adaptive regulator is designed based on a simplified model. Neglecting the elasticity in the robot arm the system can be described by

$$J \frac{d\omega}{dt} = k_m I \quad (5.1)$$

where  $J = J_a + J_m$  is the total moment of inertia and  $k_m$  the current gain of the

$$\begin{cases} k_r = \frac{2\xi\omega J}{k_m} \\ T_i = \frac{2\xi}{\omega} \end{cases} \quad (5.2)$$

gives a closed loop system with the characteristic equation

$$s^2 + 2\xi\omega s + \omega^2 = 0$$

The regulator parameters are thus related to the model by simple equations. Notice that the integration time  $T_i$  does not depend on the moment of inertia and that the regulator gain should be proportional to the moment of inertia.

A root-locus calculation shows that the design based on the simplified model will work well if

$$\omega < \omega_c = \zeta \left( \frac{kJ_m}{J_a^2} \right)^{1/2}$$

The most critical case occurs for  $J_a=0.002$ . It implies that  $\omega$  must be less than 20 rad/s.

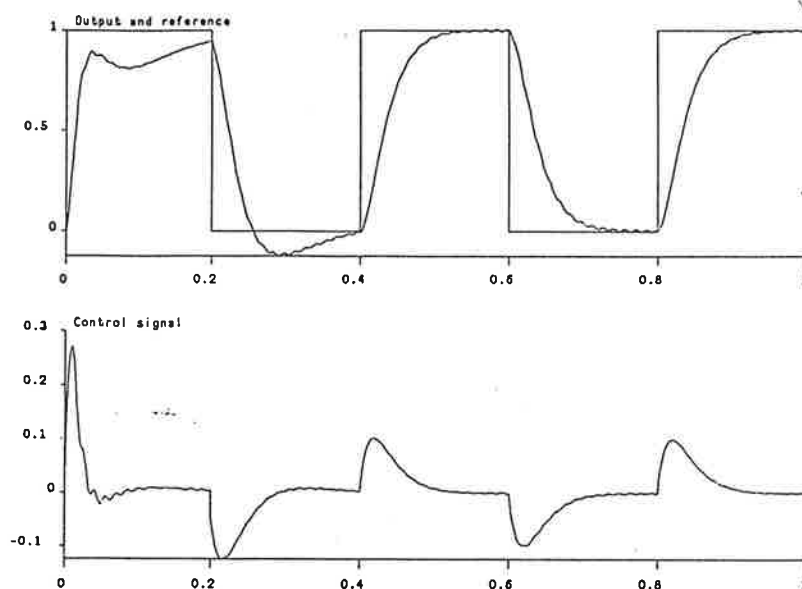
The fact that the design is based on a simplified model limits the closed loop bandwidth. A fast response to command signals can still be obtained by use of feedforward compensation. For this purpose let the desired response to angular velocity commands be given by

$$G_m = \frac{\omega_m^2}{s^2 + 2\xi\omega_m s + \omega_m^2} \quad (5.3)$$

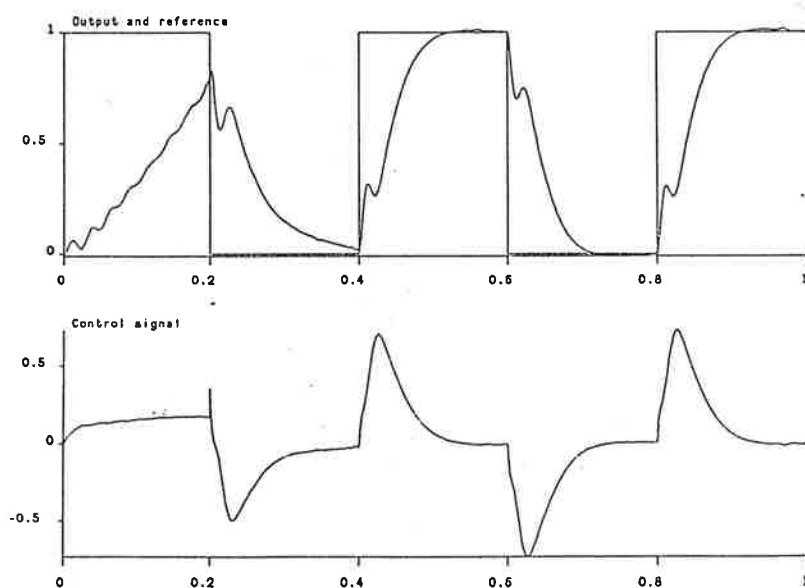
With the process model (5.1) the feedforward transfer function is given by

$$G_{FF} = \frac{G_m}{G_o} = \frac{J\omega_m}{k_m} \cdot \frac{\omega_m s}{s^2 + 2\xi\omega_m s + \omega_m^2}$$

This feedforward compensator can be implemented as



**Figure 5.14.** Simulation of the tailored adaptive systems response, with the arm inertia  $J_a=0.0002$ . The regulator parameters are initially tuned for  $J_a=0.002$ .



**Figure 5.15.** Simulation of the tailored adaptive systems response, with the arm inertia  $J_a=0.002$ . The regulator parameters are initially tuned for  $J_a=0.0002$ .

The parameter  $h$  was chosen to 0.1s in the simulations. The figures show that the system adapts to a good response after two transients. Notice the different magnitudes of the control signal for the cases of low and high inertia.

parameter variation in order to ensure stability Kreisselmeier (1986). The adaptive systems will also give better responses to set point changes and load variations when the parameter estimates have converged, if the underlying design model is sufficiently correct. This is particularly noticeable for non-minimum phase systems. Systems obtained by the Horowitz design will in general have higher loop gains which makes them more sensitive to noise. The Horowitz design as presented here is meant to be implemented in by analog hardware. If a digital implementation is desired, the method is slightly modified. Also notice that a few examples are not sufficient for a comparison between two design methods. More work is therefore necessary.

Robust and adaptive control are two complementary ways to deal with process uncertainty. Robust control gives a fixed gain regulator which is designed to be insensitive to specific parameter variations. Adaptive control deals with uncertainty by reducing the uncertainty by recursive parameter estimation. The adaptive control laws are normally derived using the certainty equivalence principle. This means that a model is determined and a regulator is designed as if the model was exact. The design is based on low order simple models. The fact that this procedure may lead to difficulties when the estimated model is inaccurate is now well understood. A key difference between robust and adaptive and robust control is that robust control leads to a high order fixed gain regulator while adaptive control leads to a time varying compensator of lower order.

It seems attractive to combine robust and adaptive control. This can be done in many different ways. One possibility is to use a robust control design as the underlying design method in an adaptive system. This would intuitively be better than to use the certainty equivalence principle. The parameter estimation will also reduce plant uncertainty and thereby make the robust design easier.

It is difficult to use robust control in this way because the Horowitz design method in its present form is difficult to implement as an analytic procedure which can be used on line. The method thus requires development of analytic versions. The approach suggested by Doyle and Stein (1981) may be an alternative. See also Gawthrop (1985). There are however other ways to combine robust and adaptive control.

- Aström, K.J. (1979): Simple self-tuners I. CODEN: LUTFD2/(TFRT-7184) (TFRT-7184)/1-63). Dept. of Automatic Control, Lund Inst. of Technology, Sweden.
- Aström K.J. (1985): Adaptive Control - a Way to Deal with Uncertainty: in Ackerman, J Uncertainty and control↓ Lecture Notes in Control and Information Sciences, 70, Berlin: Springer Verlag.
- Aström K.J. (1987a): Adaptive feedback control. Proc. IEEE, vol. 75, 185-217.
- Aström K.J., Neumann L., Gutman P.O: (1987b): Robust and adaptive control - a comparison for uncertain systems. Report. Dept. Aut. Contr., Lund Inst. Tech., Lund, Sweden.
- Bialas, S. and J. Garloff (1985): Stability of polynomials under coefficient perturbation. IEEE Aut. Contr., 30, no. 3, 310-312.
- Bode, H.W. (1945): Network Analysis and Feedback Amplifier Design, Van Nostrand, New York.
- Brockett R.W. (1976): Some geometric questions in the theory of linear systems. IEEE Trans. Aut. Contr., vol. 21, no. 4, 449-455.
- D'Azzo J.J., Houpis C.H. (1981): Linear Control Systems Analysis and Design, Conventional and Modern. 2nd ed. Tokyo: Mc Graw-Hill.
- Doyle, J.C. and G. Stein (1981): Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis. IEEE Trans. Auto. Control, vol. AC-26, pp. 4-16.
- Fan M.K.H., Walrath C.D., Tits A.L., Nye W.T., Rimer M., Grant R., and Levine W.S.: (1985): Two case studies in optimization-based computer-aided design of control systems. Proc. 2nd IEEE Control System Society Symp. on Computer Aided Control System Design. Santa Barbara, Ca., March 13 - 15.
- Gawthrop P.J. (1985): Comparative Robustness of Non-adaptive and Adaptive Control. IEE Control 85, 1.

Yaniv O., and P.O. Gutman, L. Neumann (1986a): An algorithm for adaptation of a robust controller to reduced plant uncertainty. 2nd IFAC Workshop on Adaptive Systems in Control and Signal Processing. Lund, Sweden, July 1-3.

Yaniv, O. and I.M. Horowitz (1986b): A Quantitative Design Method for MIMO Linear Feedback Systems Having Uncertain Plants. Int. J. Contr., 43, 401-421.

Yaniv, O (1986c): A Quantitative Design Method for MIMO uncertain plants to achieve prescribed diagonal dominant closed loop minimum phase tolerances. Submitted to Int. J. Contr.