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Energy-Based Interference Analysis of Heterogeneous Packet Radio Networks

André Stranne, Ove Edfors, Bengt-Arne Molin

Abstract—While the use of radio technology for wireless data communications has increased rapidly, the wide variety of radio interfaces being used has made interference investigations hard to perform. With that in mind, we present a novel approach for analyzing packet radio communications, applicable to interfering heterogeneous networks, which leads to tractable analytical expressions. The core of the approach is an analytical framework modeling each network with individual properties for the packet types and the channel sets used, while taking path loss between all network nodes into account. Furthermore, we present a derivation of closed-form expressions for the throughput of the networks, thus allowing for the investigation of important mechanisms limiting network and system performance—the expressions enable fast and flexible analysis to be performed without extensive computer simulations or measurement campaigns.

To illustrate the use of the framework and the strength of the closed form expressions we analyze a heterogeneous example system consisting of one IEEE 802.11b network and multiple Bluetooth networks that use multiple packet types. In the analysis we also take the adjacent channel interference into account when calculating network throughput as functions of number of interferers in the system. Furthermore, we present a derivation of closed-form expressions for the throughput of the networks, thus allowing for the investigation of important mechanisms limiting network and system performance—the expressions enable fast and flexible analysis to be performed without extensive computer simulations or measurement campaigns.

It is anticipated that the use of radio communications in both licensed and unlicensed spectrum will continue to grow in the coming years. Therefore, it is important to develop flexible methods of analyzing interfering radio networks. In this paper we present an analytical framework for calculating the performance of interfering packet radio networks (PRNs), as illustrated in Fig. 1. The framework can be used to analyze systems consisting of networks with different radio interface properties, which makes it suitable for applications to coexistence problems in unlicensed frequency bands.

Previous work addressing interference problems in unlicensed bands has mainly been based on simulations, although some analytical results have been reported. Analytical results for the coexistence of multiple Bluetooth networks have been presented by, e.g., El-Hoiydi in [1]. These results are based on packet collisions in time and frequency and are limited to single packet lengths. The extension to multiple packet lengths is treated by Florén et al. in [2], [3] and by Lin et al. in [4], [5]. Howitt performs a more detailed analysis of Bluetooth networks using a single packet length in [6], considering, e.g., radio link properties and adjacent-channel interference (ACI), which is also done by Souissi et al. in [7] and by Zürbes in [8] by means of simulations. Pasolini also considers single packet lengths and radio link properties in [9] but does not take ACI into account. In [10], Cordeiro et al. present analytical results based on the assumption that the number of packet collisions are Poisson distributed, and in [11], [12] detailed analyses of the coexistence between Bluetooth and IEEE 802.11 networks are presented.

In this paper we present a general framework for interference analyses, which can be applied to, e.g., IEEE 802.11b and Bluetooth, as shown in [13] and [14]. The detailed derivation of the closed form expressions for the throughput of the interfering networks used in those papers is presented here along with an analysis of a system consisting of a single IEEE 802.11b network and multiple Bluetooth networks using multiple packet types with different lengths. It should however be noted that the framework is not limited to analysis of IEEE 802.11b and
Bluetooth networks but can be used to analyze systems consisting of a wide class of interfering PRNs with different packet type properties, spectral shapes of the carriers, etc.

The outline of the paper is as follows: In Section II the system model is described. The description has been divided into three parts where the first part presents the model of the networks and the transmission of packets. This is basically an extended version of the network model used in the collision-based analysis in [3]. The second part describes the reception of packets and the assumptions about when packets are successfully received, and the third part presents the throughput expression, which is the performance measure under consideration. In Section III closed form expressions for the throughput of the interfering networks are derived, and in Section IV the framework is applied to a heterogeneous example system consisting of one IEEE 802.11b network and multiple Bluetooth networks, to illustrate the use of the framework and the strength of the closed form expressions. Finally, in Section V, we give some concluding remarks and discuss future work.

II. SYSTEM MODEL

A. Network Model

A system is defined to consist of $N$ interfering networks, each of which is defined to consist of an arbitrary number of communicating units. The units within each network are assumed to be synchronized to transmit exactly one packet at a time. By making this assumption we will obtain performance results for networks with full traffic load in the worst case interference environment. It should be noted that the network model is easily extended to allow for less than full traffic load, e.g., by introducing "empty" packet types during which no transmissions take place.

The assumption that there is exactly one packet transmitted at a time within a network also implies that there is no interference within a network—only between networks. This simplifies the method of analysis at the expense of not being able to directly analyze networks with internal interference. Analysis of networks with internal interference from simultaneous packet transmissions, such as, e.g., IEEE 802.11b networks, must therefore be given some special considerations, as demonstrated in Section IV.

Furthermore, units in different networks transmit independently, which means that there is no coordination between networks. In this way, the networks are unaware of each other and sometimes interfere by transmitting packets simultaneously in overlapping frequency bands. This assumption will make it possible to analyze heterogeneous systems of interfering networks and to scale up the number of networks in a simple way.

The packets are modeled as power bursts carrying information between transmitter and receiver units within networks, as shown in Fig. 2. Each burst has a limited duration in time, $T$, and following the burst the transmitter is idle for a period of time, $d$. The information content forwarded up to higher layers of the communication stack is transmitted in the packet payload of length $l$ with a data rate of $D$ bits per time unit. The header part of the packet, representing the overhead content such as MAC header information, preambles, cyclic redundancy check (CRC) information, etc., is $h$ time units long. In addition, $L$ denotes the total length of the packet, including the idle time.

Each network with index $j$ in the system transmits packets taken from a set of $M_j$ available packet types, where each packet type $i$ is described by the parameters, $h_{ji}$, $l_{ji}$, $d_{ji}$, and $D_{ji}$, introduced above. Furthermore, the packet types in the set are selected randomly for transmission, each with a probability $r_{ji}$.

The packet sequences transmitted within each network are assumed to start at random instants in time. Consequently, no starting point in time is assumed to be more probable than any other.

The locations in the spectral domain of the packets transmitted by network $j$ are assumed to be randomly selected from a set of $q_j$ carrier frequencies. All carrier frequencies used by network $j$ are assumed to be selected with equal probabilities $1/q_j$. This allows for packet-based frequency hopping networks to be included in the analysis. Note that by setting $q_j = 1$, no frequency hopping is used by network $j$.

When a packet is transmitted, the energy from the transmission is distributed in a certain way in time and frequency relative to the carrier frequency. In this paper we assume that the power transmitted during a packet burst of length $T$ is constant over time, while the spectral shape of each network’s transmissions can be arbitrarily chosen. E.g., the transmit power spectrum can be modeled as a piecewise constant function in frequency, which leads to packet transmissions modeled as shown in Fig. 3.

On the receiver side, a specified filter is assumed to be used to filter out interfering transmissions. The shape of this filter is referred to as the receiver channel selectivity and can be arbitrarily modeled for each network in the system. The channel selectivity is simply the power attenuation as a function of frequency.

To handle the leakage of power between different networks and channels in a simple way, we focus on the packet receptions in one of the networks, referred to as the reference network, and consider all the other networks as interferers. When the performance of the reference network has been calculated, a new network is selected as reference network and the calculations are repeated for that network.

The leakage of power between adjacent channels used by the reference network and an interfering network is given by the
combination of transmit power spectrum of the interfering signal, and the channel selectivity of the receiving reference network unit. Let the received power spectral density of the interfering signal be denoted by \( P_r(f) \), and the channel selectivity of the receiving unit \( G_r(f) \). Then, the received interfering power is given by

\[
I = \int_{-\infty}^{\infty} P_r(f) G_r(f) df. \tag{1}
\]

Since each network in the system is allowed to use frequency hopping over a set of channels, the leakage of power between all pairs of overlapping channels must be calculated using (1) and accounted for in the system model. This is done by introducing one coupling matrix for each of the interfering networks, where the received powers are stored as elements for all pairs of channels used. Consider a reference network interfered by a network with index \( j \). The power received by a reference network unit on channel \( f_{\text{ref}} \), when an interfering unit in network \( j \) is transmitting on channel \( f_{\text{int}} \), is simply stored as an element at row \( f_{\text{int}} \) and column \( f_{\text{ref}} \) in the coupling matrix. This element is from now on denoted by \( I_{j}(f_{\text{int}}, f_{\text{ref}}) \).

### B. Packet Reception Model

When two or more of the networks in the system transmit packets simultaneously on overlapping frequency channels, as illustrated in Fig. 4, there will be interference and packets may be lost. This is the only way in which the networks are assumed to interact, and the interaction will now be described in detail.

The performance of the communications within each network depends on what is assumed to cause packets to be lost. In this paper we make the assumption that the outcome of packet receptions is determined by the per-packet average signal-to-noise and interference ratio (SNIR), \( \gamma \), at the receivers. More specifically, we assume that if the useful signal for a packet is not strong enough compared to the noise and interference, the packet will be lost. To find the probability of a packet received with \( \gamma \) above a specified threshold \( \gamma_{\text{min}} \), i.e., the probability of successful packet reception, the probability density function (PDF) of \( \gamma \) must be determined.

We consider static channels between all network units and no fading, which means that the received useful power \( C \) at a receiver is a fixed deterministic quantity given by the transmitted useful power, the distance attenuation, and some additional losses at the receiver. Note that the power levels, distance attenuations and the losses may be arbitrarily chosen for all individual pairs of networks and packet types. The average SNIR of a packet can be written as

\[
\gamma = \frac{N_{\text{noise}} + E_{I,\text{tot}}}{(L - d)}, \tag{2}
\]

where \( N_{\text{noise}} \) is a deterministic noise power parameter and \( E_{I,\text{tot}} \) is the total interfering energy. Considering the reception of a reference packet, the stochastic variable \( E_{I,\text{tot}} \) represents the total amount of interfering energy received from all other packet transmissions that occur simultaneously in the system. Since the transmitted power is assumed to be constant (over time) during a packet transmission, the received interfering energy is a weighted sum of the length of the overlaps of power bursts and the corresponding interfering powers at the receiver of the reference packet. The received interfering power depends on which channels are used by the networks and the leakage of power between the channels through the coupling matrix \( I \).

We will now assume that whenever a packet is received with a total amount of interfering energy \( E_{I,\text{tot}} \) above a given threshold, \( E_{I,\text{max}} \), the packet reception fails and the data in the payload is lost. Thus, the probability of successful packet reception given \( E_{I,\text{tot}} = \epsilon \) can be expressed as

\[
Pr\{\text{success}|E_{I,\text{tot}} = \epsilon\} = u(E_{I,\text{max}} - \epsilon), \tag{3}
\]

where \( u(\cdot) \) is the Heaviside step function.

In this paper we have chosen to use the step function for the conditional probability in (3) because of its simple form. It should however be noted that if desired, it is possible to use more detailed models that better reflect the decreasing probability of successful reception given increasing amounts of interfering energy, \( E_{I,\text{tot}} \).

### C. Throughput

A network in the system will have its communication performance affected by the interference from other networks in the vicinity. As a measure of communication performance we will use the network throughput. The definition adopted here for the throughput is the amount of data received per unit time in the payload, times the fraction of channel time used for successful payload transmissions [3]. This performance measure can be associated with the achievable data rate of the communication within a network.

Consider a reference network with a set of \( M \) packet types. Denoting the probability of successful reception of a reference packet of type \( m \) by \( Pr\{\text{success}; m\} \), the average number of successfully transferred bits of payload data per packet is

\[
\sum_{m=1}^{M} r_m D_m l_m \Pr\{\text{success}; m\}, \tag{4}
\]

where we have used the notation introduced in Section II-A. Since the average length of the packets used by the reference network, including the idle time interval, is

\[
\sum_{n=1}^{M} r_n L_n, \tag{5}
\]
the network throughput, \( R \), can be defined as the average number of successfully transferred payload bits per packet (4), divided by the average packet length (5),

\[
R = \frac{\sum_{m=1}^{M} r_m D_m m \Pr\{\text{success}; m\}}{\sum_{n=1}^{N} r_n L_n}.
\] (6)

When the performance of a group of networks or the system as a whole is of interest, the system throughput can be defined as the sum of the throughput quantities for the individual networks in the group. Note that if the received useful signal is strong and there is no interference from other networks in the system, \( \Pr\{\text{success}; m\} = 1 \) for all \( M \) packet types, and \( R \) will attain its maximum value for this set of parameters.

Now that the performance measure has been defined, the interference analysis consists of finding an expression for the quantity \( \Pr\{\text{success}; m\} \) for the probability of successful reception of a packet type \( m \) in (6), since all other quantities are known. This will be done by using the stochastic variable \( E_{I,\text{tot}} \) for the total interfering energy from interfering packets received during the reception of the type \( m \) packet (from now on referred to as the reference packet).

### III. Interference Analysis

The initial expression for the probability of successful reception of a reference packet, \( \Pr\{\text{success}\} \), is formed by conditioning on the outcomes of \( E_{I,\text{tot}} \) and using the law of total probability. Thus,

\[
\Pr\{\text{success}\} = \int_{e=0}^{\infty} f_{E_{I,\text{tot}}}(e) \Pr\{\text{success}|E_{I,\text{tot}} = e\} de.
\] (7)

where \( f_{E_{I,\text{tot}}}(e) \) is the PDF of \( E_{I,\text{tot}} \). The conditional probability for successful reception in (7) is given by the energy threshold (3).

Since it was assumed in the system model that all the \( N-1 \) interfering networks transmit packets independently, the PDF of the total interfering energy,

\[
E_{I,\text{tot}} = E_{I,1} + E_{I,2} + \cdots + E_{I,N-1},
\] (8)

can be obtained by convolution of the PDFs for the interfering energy from each of the interferers, as

\[
f_{E_{I,\text{tot}}}(e) = f_{E_{I,1}}(e) * f_{E_{I,2}}(e) * \cdots * f_{E_{I,N-1}}(e).
\] (9)

In (8), \( E_{I,j} \) is the stochastic variable for the interfering energy from network \( j \), and in (9), \( \ast \) denotes convolution of the PDFs of \( E_{I,j} \), namely \( f_{E_{I,j}}(e) \). If the system of interfering networks is heterogeneous, each network \( j \) will produce its own type of interference characterized by \( f_{E_{I,j}}(e) \) in (9). This means that once a general expression for the received interfering energy from a single interferer has been derived, it can be used in (9) to calculate the PDF of the total received interference from all interferers in the system.

In the following derivations, we focus on a single interfering network \( j \). Since the same interferer is considered throughout the derivations, the expressions can be simplified by suppressing the index \( j \) on \( E_{I,j} \). Furthermore, we will derive an expression for the cumulative distribution function (CDF) of \( E_I \) instead of the PDF. The CDF can be used to calculate the corresponding PDF in practical cases.

Before we proceed with the detailed analysis, the core of the derivation of the CDF of \( E_I \) can be described in the following way: Events that determine the CDF, such as the type of interfering packets that are transmitted, starting points in time of the transmissions and the selected frequency channels, are introduced by marginalization and by using the definition of conditional probability, basically in the same way as was done with the outcomes of \( E_{I,\text{tot}} \) in (7). Based on these events we move forward in time, following the possible transmissions from the single interferer and building a recursive expression for the CDF of \( E_I \). The recursion terminates when a chain of events has produced a sequence of interfering packets that has passed the ending of the reference packet, since the interfering packet transmissions after the ending of the reference packet transmission will not influence the received interference.

Consider a reference network transmitting a reference packet with an active interval of length \( T = h + l \) and a single interfering network. To find the CDF of \( E_I \), denoted by \( \Pr\{E_I \leq e\} \), we first introduce the condition on which channel \( f_{\text{ref}} \) the reference packet is transmitted. Thus,

\[
\Pr\{E_I \leq e\} = \frac{1}{q_{\text{ref}}} \sum_{f_{\text{ref}}=1}^{q_{\text{ref}}} \Pr\{E_I \leq e|f_{\text{ref}} = f_{\text{ref}}\},
\] (10)

where \( q_{\text{ref}} \) is the number of channels used by the reference network. To simplify the coming expressions, we will suppress the condition on which channel was selected by the reference network in the following.

To continue, introduce the condition on during which packet type, transmitted by the interfering network, the reference packet transmission starts. Then,

\[
\Pr\{E_I \leq e|f_{\text{ref}} = f_{\text{ref}}\} = \sum_{k=1}^{M} \Pr\{E_I \leq e|K = k\} \Pr\{K = k\},
\] (11)

where \( \Pr\{K = k\} \) denotes the probability of a start of a reference packet transmission during the transmission of an interfering packet type \( k \). It is given by

\[
\Pr\{K = k\} = \frac{r_k L_k}{\sum_{i=1}^{M} r_i L_i},
\] (12)

which essentially is the fraction of time occupied by type \( k \) packets.

The expression for the probability of received interfering energy below \( e \), given start of the reference packet during a type \( k \) packet, \( \Pr\{E_I \leq e|K = k\} \), can be found by conditioning on the exact starting point of the reference packet within the type \( k \) packet. Then,

\[
\Pr\{E_I \leq e|K = k\} = \int_{z=0}^{L_k} f_{Z|K}(z|K = k) \cdot \Pr\{E_I \leq e|K = k, Z = z\} dz,
\] (13)

\(^1\) These expressions are used since the focus of the paper is on the ensemble average properties of the system. For more information, see Appendix A in [15].
where $Z$ is defined to be the time from the start of the type $k$ packet to the start of the reference packet, as shown in Fig. 5a, and $f_{Z|K}(z|K=k)$ is the conditional PDF of $Z$. To continue, we use a uniform distribution for the starting points of the reference packet within the interfering type $k$ packet,

$$f_{Z|K}(z|K=k) = \frac{1}{L_k},$$

(14)

which is a consequence of our assumption that all starting points are equally probable.

Since the expression for $\Pr\{E_I \leq e|K = k, Z = z\}$ in (13) depends on whether the start of the reference packet occurs in the active interval or the idle time interval of the type $k$ packet, it is convenient to separate the integral in (13) into the corresponding regions. Then, using (14), we have

$$\Pr\{E_I \leq e|K = k\} = \frac{1}{L_k} \left( \int_{z=0}^{L_k-d_k} \Pr\{E_I \leq e|K = k, Z = z, A\}dz + \int_{z=L_k-d_k}^{L_k} \Pr\{E_I \leq e|K = k, Z = z, G\}dz \right),$$

(15)

where the events $A$ and $G$ mean that the reference packet starts in active and idle time intervals, respectively.

The next step is to make a variable transformation to $Y = Z + T - L_k$, where $Y$ (for $Y \geq 0$) represents the remaining time interval after the end of the interfering packet to the end of the reference packet, as illustrated in Fig. 5a. The reason for introducing $Y$ is that it is easier to interpret than $Z$ when deriving the remaining relations. As shown below, the total received energy can be separated in two contributions—one contribution from the initial type $k$ packet, and one with the contribution of interfering energy from the remaining interval after the type $k$ packet where the reference packet is still vulnerable to interference. Note that the realizations of $Z$ can only be positive, whereas the realizations of $Y$ can be both positive and negative.

Performing the transformation, (15) can be written as

$$\Pr\{E_I \leq e|K = k\} = \frac{1}{L_k} \left( \int_{y=T-d_k}^{T} \Pr\{E_I \leq e|K = k, Y = y, A\}dy + \int_{y=T-d_k}^{T} \Pr\{E_I \leq e|K = k, Y = y, G\}dy \right).$$

(16)

To proceed, the received interfering energy $E_I$ over the whole reference packet can be separated in two parts as

$$E_I = E_{I,k} + E_{I,y},$$

(17)

where $E_{I,k}$ is the part of the interfering energy received from the initial type $k$ packet transmission, and $E_{I,y}$ is the part of the energy received in the interval $y$ following the initial packet. Then, the first integrand in (16) can be written as

$$\Pr\{E_I \leq e|K = k, Y = y, A\} = \Pr\{E_{I,k} + E_{I,y} \leq e|K = k, Y = y, A\} = \Pr\{E_{I,y} \leq e - E_{I,k}|K = k, Y = y, A\}.$$ (18)

By conditioning on which channel the interfering type $k$ packet was transmitted on, we have

$$\Pr\{E_{I,y} \leq e - E_{I,k}|K = k, Y = y, A\} = \frac{1}{q_j} \sum_{f_k=1}^{q_j} \Pr\{E_{I,y} \leq e - E_{I,k}|K = k, Y = y, A, F_k = f_k\},$$

(19)

where $F_k$ is the stochastic variable for the channel selected by the interfering network $j$.

Now that we have conditioned on which channels were selected for the interfering packets, the following arguments are used in the derivation of the expression in the sum of (19): If the transmission of the interfering packet ends after the ending of the reference packet, there is a fixed amount of interference received. On the other hand, if the transmission of the interfering packet ends before the ending of the reference packet, the following interfering packets may contribute to the received interference. Then, the next interfering packet must be considered, and the ending of that packet relative to the reference packet must be investigated in the same way as described above. This leads to a recursive formulation of the probability functions as shown below.

Using the arguments given above, we start by investigating (19) to eventually end up with a recursive expression. If $y \leq 0$, all the interference received comes from the type $k$ packet transmission, and thus, $E_{I,y} = 0$, since the transmission of the reference packet will be finished before the transmission of the interfering packet. The amount of interfering energy received is a product of the received interfering power, which is given by the coupling matrix $I_{j}$, and the length of the overlap. Two situations can occur depending on $y$, as shown in figures 5b and 5c. If $|y| \leq d_k$, the received energy is $E_{I,k} = I_{j}(f_k, f_{ref})(T - d_k - y)$, and if $|y| > d_k$ the energy is instead $E_{I,k} = I_{j}(f_k, f_{ref})T$. Fig. 5. Different cases of overlaps between a reference packet and interfering packets.
On the other hand, if $y > 0$, as illustrated in Fig. 5d, $E_{I,k} = I_j(f_k, f_{\text{ref}})(T - d_k - y)$. In that case, however, the packet transmissions following the type $k$ packet can produce more interference in the interval $y$, which means that $E_{I,y}$ is not always zero. Thus, making use of the Heaviside step function to write the probability function in a compact form,

$$Pr \{ E_{I,y} \leq e - E_{I,k} | K = k, Y = y, A, F_k = f_k \} = u(-y) u(d_k + y) u(e - I_j(f_k, f_{\text{ref}})(T - d_k - y))$$

$$+ u(-y) u(-d_k + y) u(e - I_j(f_k, f_{\text{ref}})T)$$

$$+ u(y) \cdot Pr \{ E_{I,y} \leq e - I_j(f_k, f_{\text{ref}})(T - d_k - y) | Y = y \},$$

(20)

where $Pr \{ E_{I,y} \leq e - I_j(f_k, f_{\text{ref}})(T - d_k - y) | Y = y \}$ is the probability of an energy $E_{I,y}$ below $e - I_j(f_k, f_{\text{ref}})(T - d_k - y)$ received in the interval $y$. Note that the interference received in $y$ is independent of the realizations of $K, A$ and $F_k$.

The second integrand in (16) is handled in the same way as the first, which means that

$$Pr \{ E_I \leq e | K = k, Y = y, G \} = Pr \{ E_{I,y} \leq e - E_{I,k} | K = k, Y = y, G \},$$

(21)

but in this case $E_{I,k} = 0$ since the reference packet started transmission in the idle time interval of the type $k$ packet. For $y > 0$ the successive packet transmissions may contribute to the received interference, and thus,

$$Pr \{ E_{I,y} \leq e - E_{I,k} | K = k, Y = y, G \} = u(-y) u(e) + u(y) Pr \{ E_{I,y} \leq e | Y = y \}.$$ (22)

Consequently, the integrands in (16) can be expressed as probabilities for an interfering energy $E_{I,y}$ below $e'$, $Pr \{ E_{I,y} \leq e' | Y = y \}$, received in an interval $y$, given a start of an interfering packet transmission at $y = 0$.

To continue, an expression for the probability function, $Pr \{ E_{I,y} \leq e' | Y = y \}$, appearing in (20) and (22), must be found. To find an expression for this probability function, we condition on the successive packet transmissions by the interferer. Thus, introducing the next transmitted packet type $\Lambda$ by marginalization, we have

$$Pr \{ E_{I,y} \leq e' | Y = y \} = \sum_{\lambda=1}^{M} Pr \{ E_{I,y} \leq e' | Y = y, \Lambda = \lambda \} Pr(\Lambda = \lambda | Y = y).$$

(23)

We find that since the successive transmissions are independent, as specified in the system model, the probability for selecting packet type $\Lambda$ for the next transmission is given by

$$Pr(\Lambda = \lambda | Y = y) = r_\lambda.$$ (24)

Furthermore, the probability for an interfering energy $E_{I,y}$ below $e'$, received in the interval $y$, given that the next transmitted packet is of type $\lambda$, $Pr \{ E_{I,y} \leq e' | Y = y, \Lambda = \lambda \}$, can be found by once more conditioning on which frequency channel was selected for the type $\lambda$ packet transmission. Thus,

$$Pr \{ E_{I,y} \leq e' | Y = y, \Lambda = \lambda \} = \frac{1}{q_j} \sum_{f_k=1}^{q_j} Pr \{ E_{I,y} \leq e' | Y = y, \Lambda = \lambda, F_\lambda = f_k \}.$$ (25)

The CDF, $Pr \{ E_{I,y} \leq e' | Y = y, \Lambda = \lambda, F_\lambda = f_k \}$, is the probability for an interfering energy $E_{I,y}$ below $e'$, received in a given interval $y$ with the next packet type $\lambda$ transmitted on channel $f_\lambda$. As can be seen in figures 5e and 5f, this probability can be calculated directly if the packet type $\lambda$ is larger than $y$.

The probability of interfering energy will then depend on the length of the idle time interval of the type $\lambda$ packet. If $y \leq L_\lambda - d_\lambda$, the only possible amount of interfering energy received in $y$ is $I_j(f_\lambda, f_{\text{ref}})y$, as shown in Fig. 5e. If $y > L_\lambda - d_\lambda$, the only possible amount of interfering energy is instead $I_j(f_\lambda, f_{\text{ref}})(L_\lambda - d_\lambda)$, as indicated in Fig. 5f.

If the packet type $\lambda$ is shorter than $y$, the successive transmissions determine the amount of interference. Then, the probability for an interfering energy $E_{I,y}$ below $e'$, received in $y$, can be expressed as the probability for an interfering energy below $e' - I_j(f_\lambda, f_{\text{ref}})(L_\lambda - d_\lambda)$, received in the interval $y - L_\lambda$. Thus,

$$Pr \{ E_{I,y} \leq e' | Y = y, \Lambda = \lambda, F_\lambda = f_k \} = u(e' - I_j(f_\lambda, f_{\text{ref}})y)(L_\lambda - d_\lambda - y) + u(e' - I_j(f_\lambda, f_{\text{ref}})(L_\lambda - d_\lambda)) u(y - (L_\lambda - d_\lambda))$$

$$+ u(L_\lambda - y) \cdot Pr \{ E_{I,y} \leq e' - I_j(f_\lambda, f_{\text{ref}})(L_\lambda - d_\lambda) | Y = y - L_\lambda \}$$

$$\cdot u(y - L_\lambda),$$ (26)

where the probability function $Pr \{ E_{I,y} \leq e' - I_j(f_\lambda, f_{\text{ref}})(L_\lambda - d_\lambda) | Y = y - L_\lambda \}$ is given by (23) with new arguments, resulting in a recursive expression.

A significant part of the complexity of the expression presented above is a result of allowing for multiple packet types to be used by the networks. However, as demonstrated in [3], accounting for the networks’ use of multiple packet types is important in the investigation of the interference between packet radio networks, and specifically heterogeneous systems.

IV. APPLICATION EXAMPLE

To illustrate how the presented analytical framework can be used, we analyze a heterogeneous system consisting of one IEEE 802.11b network and multiple Bluetooth networks, as illustrated in Fig. 1. We start in Section IV-A by identifying the network parameters required for the analysis. Then, in Section IV-B, we calculate the IEEE 802.11b and Bluetooth network throughput as functions of number of interfering Bluetooth networks in the system.

A. Bluetooth and IEEE 802.11b Network Models

Starting with Bluetooth, six of the packet types defined in the standard [16] are used. We will assume that the Bluetooth networks use either the DH1, DH3 and DH5 packet types, or the
DM1, DM3 and DM5 packet types, which are 1, 3 and 5 time slots long respectively, where a time slot is 625 $\mu$s. The DMx packet types carry payloads which are protected by a rate 2/3 forward error correction (FEC) code, and the DHx packet types carry payloads which are unprotected. In both cases we assume that the packet types are equally probable of being selected for transmissions within the Bluetooth networks, and that the following parameters apply

| Packet selection prob. $r$ | $[1/3 \ 1/3 \ 1/3]$ | 
| Header length $h$ | $[150 \ 160 \ 160]$ $\mu$s |
| Payload length $l$ | $[200 \ 1450 \ 2700]$ $\mu$s |
| Idle time ("guard interval") $d$ | $[275 \ 265 \ 265]$ $\mu$s, |

and since FEC coding with rate 2/3 is used only in the DMx case,

Payload bit rate $D_{DHx} = [1 \ 1 \ 1]$ bits/$\mu$s |

Payload bit rate $D_{DMx} = [2/3 \ 2/3 \ 2/3]$ bits/$\mu$s.

The channel set used by the Bluetooth networks consists of $q_{BT} = 79$ channels with all channels equally probable of being selected. The Bluetooth networks use packet-based frequency hopping over the channels in the channel set, which means that there is a pseudo-random change of channel after each packet transmission.

The link budget for the Bluetooth networks is assumed to be given by

- Radiated power $EIRP = 0$ dBm
- Path loss (ref. units) $PL_{ref} = 40$ dB
- Path loss (int. units) $PL_{inter} = [40 \ 50 \ 54]$ dB
- Receiver loss $L_r = 2$ dB
- Min. received SNIR $\gamma_{\min, DHx} = 20$ dB
- Min. received SNIR $\gamma_{\min, DMx} = 18$ dB,

where the path loss figures 40, 50 and 54 dB correspond to typical path loss for distances of 1, 3 and 5 meters in a line-of-sight (LOS) situation at 2.4 GHz [17]. We have assumed a transmit power of 2 dBm (Bluetooth power class 2) and transmitter losses of 2 dB to obtain an effective isotropically radiated power (EIRP) of 0 dBm. The (15,10) shortened Hamming code used with the DMx packet types is assumed to provide a coding gain of 2 dB at the expense of lower payload data rate. The propagation loss between the units of the reference network (i.e., the communicating units in a test network) has been set to 40 dB, corresponding to a typical distance of approximately 1 m. In the calculations below we will consider three cases where the distance between the units of the reference network and the units of the interfering networks (i.e., the distance to the interferers) is either 1, 3 or 5 meters.

To find the parameter $E_{t,\max}$ we assume that the Bluetooth receivers in the system are characterized by the parameters

- Noise figure $F_{sys} = 20$ dB
- Noise bandwidth $B = 60$ dB/Hz
- Ref. noise pow. dens. $N_0 = -174$ dBm/Hz,

which results in a noise power of $N_{noise} = F_{sys} + B + N_0 = -94$ dBm. It should be noted that current Bluetooth receiver implementations generally have lower noise figures than 20 dB, which means that we, from this point of view, somewhat underestimate the performance of the receivers.

For successful packet reception, the received SNIR must be above the specified threshold $\gamma_{\min}$, and using (2) the corresponding thresholds for the amounts of tolerable interfering energy for the Bluetooth packet types become

$E_{t,\max, DHx} = [0.22 \ 1.0 \ 1.8] \ \mu$J

$E_{t,\max, DMx} = [0.35 \ 1.6 \ 2.9] \ \mu$J.

The IEEE 802.11b networks [18] use a single wideband carrier in the ISM-band at 2.4 GHz and no frequency hopping. Consequently, we define the channel set used by each IEEE 802.11b network to consist of only one channel. In this example we assume that only the 11 Mbit/s mode of transmission is used for transmitting packets, and that the RTS-CTS handshaking mechanism is not used. We also assume that three types of packets are used with 40, 500 and 1500 bytes of payload data, and that they are equally likely to be selected for transmission.

The carrier sense multiple access mechanism with collision avoidance (CSMA/CA) used in IEEE 802.11b must be given some special consideration, since it affects the idle time, $d$, between packet transmissions. With short preambles the total overhead adds up to $121 \ \mu$s for all three packet types, and the lengths of the payloads become 30 $\mu$s, 364 $\mu$s and 1091 $\mu$s. However, the idle time between consecutive packets is random in CSMA/CA. In fact, it also depends on the probability for successful packet reception since a random back-off procedure is initiated whenever a packet is lost. Thus, for IEEE 802.11b we obtain the parameters

- Packet selection prob. $r = [1/3 \ 1/3 \ 1/3]$
- Header length $h = [121 \ 121 \ 121]$ $\mu$s
- Payload length $l = [30 \ 364 \ 1091]$ $\mu$s
- Idle time $d =$ Determined by $Pr\{success\}$
- Payload bit rate $D = [11 \ 11 \ 11]$ bits/$\mu$s.

The determination of $d$ can be made by the use of a Markov model, as will be shown in Section IV-B. When a packet has been successfully received it is acknowledged by an ACK-frame with length $b_{ACK} + l_{ACK} = 106$ $\mu$s. However, to further simplify the analysis, we assume that no ACK-frame is transmitted.

To continue, the IEEE 802.11b link budget is assumed to be given by

- Radiated power $EIRP = 20$ dBm
- Path loss (ref. units) $PL_{ref} = 60$ dB
- Path loss (int. units) $PL_{inter} = [40 \ 50 \ 54]$ dB
- Receiver loss $L_r = 2$ dB
- Min. received SNIR $\gamma_{\min} = 10$ dB.

Here we have assumed an EIRP of 100 mW. The propagation loss between the units of the IEEE 802.11b network has been set to 60 dB corresponding to a distance of approximately 10 m using the path loss model introduced above.

Furthermore, the IEEE 802.11b receivers are assumed to be characterized by the parameters

$S_0 + b = 40$ dB and $a = 2.0$. 

\footnote{We have used the path loss model in eq. (12) in [17]. For simplicity we have chosen the parameters $S_0 + b = 40$ dB and $a = 2.0$.}
I_j = P_r \cdot \left[ 0 \ 0 \ 0 \ 0 \ 0.5 \ 1 \ 1 \ 0.5 \ 0 \ 0 \ldots \right] \text{mW}

Fig. 6. Model of overlapping IEEE 802.11b and Bluetooth channels, and the corresponding coupling matrix \( I_j \).

Noise figure \( F_{\text{sys}} = 7 \text{ dB} \)
Noise bandwidth \( B = 74 \text{ dBHz} \)
Ref. noise pow. dens. \( N_0 = -174 \text{ dBm/Hz} \),

which results in a noise power of \( N_{\text{noise}} = -93 \text{ dBm} \). From (2), the thresholds for the amounts of tolerable interfering energy for the three packet lengths used by the IEEE 802.11b networks are \( E_{I,\text{max}} = \begin{bmatrix} 0.95 & 3.1 & 7.6 \end{bmatrix} \text{ pJ} \).

Note that the IEEE 802.11b network is assumed only to use clear channel assessment (CCA) mode 2 in this example, which means that the network nodes consider the channel busy and hold transmissions only when other IEEE 802.11b carriers are detected. For simplicity, we have also assumed that there is no internal interference within the IEEE 802.11b network, which basically means that we consider an IEEE 802.11b network with only two units and traffic in only one direction.

Lastly, we need to model the leakage of power between overlapping frequency channels. To illustrate how this is done, consider the simple model of a 22 MHz wide IEEE 802.11b channel and a set of 1 MHz wide Bluetooth channels shown in Fig. 6. Let the IEEE 802.11b network be the interferer and let the Bluetooth network be the reference network. The integration in (1) is evaluated for each of the overlapping channels, resulting in the coupling matrix \( I_j \) of size 1-by-79 shown in Fig. 6, where \( P_r \) is the total received interfering power per MHz from the IEEE 802.11b network transmissions. In this application example we use the more detailed models of the transmit power spectra and the receiver channel selectivities of the Bluetooth and IEEE 802.11b transceivers shown in Fig. 7, which roughly correspond to what is given in the respective specifications [16], [18]. The calculations of the coupling matrices \( I_j \) are however performed as described above.

B. Interference Analysis

Using the network parameters specified in the previous section we will now analyze a heterogeneous system consisting of one IEEE 802.11b network and multiple Bluetooth networks. First, we investigate how the throughput of the single IEEE 802.11b network depends on the total number of interfering Bluetooth networks and the distance to the networks, and then we calculate the throughput of a Bluetooth network interfered by the IEEE 802.11b network and other Bluetooth networks in the system. For convenience we use only one distance attenuation parameter, \( L_{\text{PL,interf}} \), controlling the density of the interfering networks.\(^3\) In the general case, the individual distance attenuation parameters between all interfering networks and within each network can be set arbitrarily.

First, consider a single IEEE 802.11b reference network in the vicinity of interfering Bluetooth networks, as illustrated in Fig. 8. Due to the random back-off mechanism used in CSMA/CA, the idle time between IEEE 802.11b packets is affected by the transmissions from the interfering Bluetooth networks. Whenever an IEEE 802.11b packet transmission fail due to interfering Bluetooth transmissions, random back-off is initiated and the idle time is increased in the IEEE 802.11b network. Consequently, given a certain number of Bluetooth networks at a certain distance from the IEEE 802.11b network, the idle time in the IEEE 802.11b network must first be calculated.

To calculate the IEEE 802.11b idle time parameter \( d \), we use a Markov model with six states corresponding to the different sizes [18] of the contention window (CW) in number of slots, \( \text{CWSize} = \begin{bmatrix} 31 & 63 & 127 & 255 & 511 & 1023 \end{bmatrix} \), where a slot is 20 \( \mu \)s. The state diagram is illustrated in Fig. 9. The transition probabilities in the Markov model are given by the packet type selection probabilities, \( r \), and the successful packet reception probabilities, \( Pr\{\text{success}\} \), for the three IEEE 802.11b packet types.

\(^3\)We disregard the geometrical problem of organizing the units in the system according to the specified distances.
To calculate $\Pr\{\text{success}\}$ for each of the three IEEE 802.11b packet types we use (7). E.g., if there is only a single interfering Bluetooth network present, we obtain the successful packet reception probabilities $[0.74 \ 0.71 \ 0.65]$ for a distance of 1 m to the Bluetooth network units, $[0.78 \ 0.83 \ 0.89]$ for 3 m, and $[1.0 \ 1.0 \ 1.0]$ for 5 m. Note that for 1 m, shorter packets have higher probabilities of successful receptions, whereas for 3 m, the situation is reversed. For a distance of 5 m to the Bluetooth network, all three packet types are received correctly.

The transition probabilities in the state diagram in Fig. 9 are then calculated, and once the transition probabilities have been obtained, the stationary state probabilities of the Markov model are calculated. These calculations are performed for all three distances and all the number of interferers under consideration. E.g., in the case of a single interfering Bluetooth network at 1 m distance to the IEEE 802.11b network units, the stationary state probabilities $[0.70 \ 0.21 \ 0.06 \ 0.02 \ 0.01 \ 0.0]$ are obtained. As the distance to the interferers is then increased, the states corresponding to smaller CWs become more probable.

Finally, using the stationary state probabilities, the probabilities for different values of the parameter $d$ are calculated. The idle time between two IEEE 802.11b packets consists of a SIFS interval of $10 \mu s$, an ACK-frame time of $106 \mu s$, a DIFS interval of $50 \mu s$, and a uniformly distributed random number of slots, $A$, from 0 to CWSize. Thus, with known stationary state probabilities, the probabilities for all possible values of $d = \text{SIFS} + \text{ACK} + \text{DIFS} + A \cdot 20 \mu s$ can be calculated and used together with the other system parameters in (6) to calculate IEEE 802.11b network throughput in the presence of Bluetooth interferers. However, including all the individual values of $d$ in the analysis is not feasible, since the computational complexity grows with the number of packet types used. Therefore, to reduce the number of parameters $d$, we evaluate the analytical expression using only the mean value of $d$ for each CW size$^4$. This results in idle time parameters $d = [476 \ 796 \ 1436 \ 2716 \ 5276 \ 10396] \mu s$ in the IEEE 802.11b network.

With known idle time distributions in the IEEE 802.11b network, the throughput can be calculated. The IEEE 802.11b network throughput results from the calculations are shown in Fig. 10 as a function of number of interfering Bluetooth networks for the three different distances to the interferers. Note that the throughput of the IEEE 802.11b network is independent of the use of DHx or DMx packets in the Bluetooth networks since both types of packets are assumed to be of the same lengths and transmitted with the same powers. It can be seen in the figure that when the Bluetooth networks are close to the IEEE 802.11b reference network, throughput decreases very rapidly with the number of interferers. With the interferers at a distance of 1 m from the reference network, the IEEE 802.11b throughput is reduced to about 50% for a single interfering Bluetooth network. For distances of 3 and 5 meters, the corresponding numbers of interferers are 2 and 9, respectively. Here, the throughput of the IEEE 802.11b network is affected by the Bluetooth interference both in terms of packet losses due to collisions, but also indirectly in terms of larger idle times when packet losses occur.

To analyze the performance of a Bluetooth reference network in the vicinity of the IEEE 802.11b network and the other interfering Bluetooth networks in the system, we use the same method as described above, calculating first the influence of the Bluetooth interference on the idle time distribution in the IEEE 802.11b network, and then the Bluetooth reference network throughput as it is interfered by the IEEE 802.11b network and the other Bluetooth networks in the system. In Fig. 11 the throughput of a Bluetooth reference network using either the DHx packets or the DMx packets has been plotted as functions of number of interfering networks for distances between reference and interfering units of 1, 3 and 5 m. Note that for a single interfering network in the figure, there is only the interfering IEEE 802.11b network present, and then, for numbers of interferers from 2 and above, there are also additional Bluetooth interferers. It can be seen in the figure that the three curves for the DHx and DMx packet type cases, indicate significant throughput reductions due to the interference from the IEEE 802.11b network, whereas the interference from the Bluetooth interferers has a lower impact on the performance reduction. For short distances to the interfering networks, however, the impact of the interference from the Bluetooth networks becomes more severe, mainly due to the adjacent channel interference.

In this example, we have considered only a single IEEE 802.11b network. It is however possible to analyze a system with multiple IEEE 802.11b networks [14] if their carriers are sufficiently separated in frequency, or if they are sufficiently far

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$^4$Calculations performed on a similar system, where the IEEE 802.11b network only use the 1500 bytes packet type instead of the three types defined in this example, show that increasing the number of idle time parameters $d$ per CW size from 1 through 5 yields almost identical throughput results.
have shown above, the presented analytical framework can be independent. From each other, so that their transmissions can be assumed to be independent.

The system model used does not take into account receiver nonlinearities limiting the performance of, especially, Bluetooth receivers under strong interference. Taking these into account would result in lower calculated performance. Even so, as we have shown above, the presented analytical framework can be useful since it enables fast and straightforward investigations of heterogeneous systems of interfering networks.

V. CONCLUSIONS

In this paper we have presented a framework with closed form expressions for the throughput of interfering PRNs. The framework allows for analysis of heterogeneous networks transmitting multiple packet types over different channels sets, taking also radio link properties into account. An example system consisting of one IEEE 802.11b network and multiple Bluetooth networks was analyzed to illustrate the use of the closed form expressions and the strength of the framework. For the example system, CDFs of received interfering energy from the interferers were calculated and used to obtain IEEE 802.11b and Bluetooth network throughput as functions of numbers of Bluetooth interferers.

Using the analytical framework important mechanisms limiting the performance of heterogeneous interfering radio networks can be investigated. This can be useful in the analysis of existing radio networks, but also in the design of new radio interfaces. The closed form expressions provide a powerful tool for general understanding of the interference mechanisms and fast and flexible evaluations of network and system throughput without the need to perform extensive computer simulations.

The strength of the framework lies in the choice of system model, which has been designed to capture interference effects in systems of networks that use different types of packets with different lengths and amounts of useful and overhead data, that are transmitted with different transmit power levels using different widths and shapes of the transmit power spectrum, and that are received with different receiver sensitivities. Some extensions of the model, e.g., to handle less than full traffic load, can easily be incorporated in the analysis, while other extensions, e.g., to include other propagation models and fading, are more complex and leave room for future work.

REFERENCES