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Simple Drum-Boiler Models

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Abstract
This paper describes a simple nonlinear models for a drum-boiler. The models are derived from first principles. They can be characterized by a few physical parameters that are easily obtained from construction data. The models also require steam tables for a limited operating range, which can be approximated by polynomials. The models have been validated against experimental data. A complete simulation program is provided.

1. Introduction

There are many models of drum-boilers in the literature. See the reference list. The models described in this paper are derived from first principles. They are characterized by a few parameters only which can be obtained from first principles. The models are validated by comparison with extensive plant data.

A key feature of a drum-boiler is that there is a very efficient energy and mass transfer between all parts that are in contact with the steam. The mechanism responsible for the heat transfer is boiling and condensation. A consequence of this is that it is a very good approximation to assume that all water steam and metal is in thermal equilibrium. This means that the total energy can be represented by a global energy balance. The validity of this approximation has been shown by many modeling exercises.

The paper is organized as follows. A first order model is presented in Section 2. This model is obtained from a global energy balance for the total plant. The model has one state variable which is chosen as the drum pressure. This model has the same structure as the model presented in Åström and Eklund (1972). The parameters are, however, obtained from first principles. To model the drum water level it is necessary to account for the shrink and swell phenomena. This is done in Section 3. A third order model is obtained. This model has drum pressure, water volume and steam quality in the risers as state variables. The model exhibits a complex behaviour in spite of being of low order. Simulation of step responses are presented in Section 4.

2. A First Order Model

Because of the efficient heat and mass transfer due to boiling and condensation all parts of the system which are in contact with the steam will be in thermal equilibria. It is therefore natural to describe the plant with global mass and energy balances as was done in Åström and Eklund (1972). The global energy balance can be written as

$$\frac{d}{dt}[\varrho_s h_s V_{st} + \varrho_w h_w V_{wt} + m c_p T] = P + q_{fw} h_{fw} - q_s h_s \qquad (1)$$

where ϱ denotes specific density, h enthalphy, V volume and q mass flow. The indices s, w and fw refers to steam, water and feedwater respectively. The total mass of the metal tubes is m, the specific heat is c_p and the average metal temperature is T. The input power from the fuel is denoted by P. The total steam volume is given by

$$V_{st} = V_{drum} - V_w + a_m V_r \qquad (2)$$

where V_{drum} is the drum volume, V_w the volume of water in the drum, V_r the riser volume and a_m the average steam-water volume ratio. The total water volume is

$$V_{wt} = V_w + V_{dc} + (1 - a_m) V_r \qquad (3)$$

The right hand side of equation (1) represents the energy flow to the system from fuel and feedwater and the energy flow from the system via the steam. Since all parts are in thermal equilibria the state of the system can be represented by one variable which we choose as the steam pressure. Using steam tables the variables ϱ_s, ϱ_w, h_s and h_w can then be expressed as functions of steam pressure. Similarly T can be expressed as a function of pressure by assuming that T is equal to the saturation temperature of steam which corresponds to p.

This model represents the dynamics due to input power well. When the feedwater flow or the steam flow is changed it is, however, necessary to also take into account that the water and steam masses are also changing. This can be accounted for with a global massbalance.

$$\frac{d}{dt}[\varrho_s V_{st} + \varrho_w V_{wt}] = q_{fw} - q_s \qquad (4)$$

The dynamics which describe how the drum pressure is influenced by input power, feedwater flow and steam flow is well captured by equations (1) and (4).

The derivative of the total water volume (dV_{wt}/dt) can be eliminated between equations (1) and (4). Multiplication of (4) by h_w and subtracting from (1) gives

$$\begin{aligned} h_c \frac{d}{dt}(\varrho_s V_{st}) + \left[\varrho_s V_{st}\frac{dh_s}{dt} + \varrho_w V_{wt}\frac{dh_w}{dt} + mc_p\frac{dT}{dt}\right] \\ = P - q_{fw}(h_w - h_{fw}) - q_s h_c \end{aligned} \qquad (5)$$

The condensation enthalpy $h_c = h_s - h_w$ has also been introduced. If the boiler is provided with a good level control system the total water volume (V_{wt}) and the total steam volume (V_{st}) do not change much. Equation (5) can then be simplified to

$$e_{11}\frac{dp}{dt} = P - q_{fw}(h_w - h_{fw}) - q_s h_c \qquad (6)$$

where

$$e_{11} = h_c V_{st}\frac{d\varrho_s}{dp} + \varrho_s V_{st}\frac{dh_s}{dp} + \varrho_w V_{wt}\frac{dh_w}{dp} + mc_p\frac{dT_s}{dp}$$

Apart from steam table data it is thus sufficient to know total steam and water volumes and total metal mass. The model (6) is identical to the model in Åström and Eklund (1972). Notice however that in this case the parameters are obtained from construction data. Also notice that the term

$$q_c = -\frac{1}{h_c}\left[\varrho_s V_{st}\frac{dh_s}{dt} + \varrho_w V_{wt}\frac{dh_w}{dt} + mc_p\frac{dT_s}{dt}\right]$$

can be interpreted as the total condensation flow. It is observed that the terms dh_s/dp and dh_w/dp are key quantities in predicting the energy and mass transfer between steam and water. These terms also appeared in the drum-boiler model of Morton and Price (1977).

3. Shrink and Swell

For some control tasks e.g. drum level control it is necessary to model the dynamics of the drum level. This is more difficult because of the shrink and swell effect. To describe this it is necessary to account for the distribution of steam and water and the transfer of mass and energy between steam and water.

The steam-water distribution varies along the risers. Partial differential equations are needed to describe this properly. To keep a finite dimensional model we will assume that the shape of the distribution is known. The assumed shape is based on solving the partial differential equations in the steady state. This gives a linear distribution of the steam-water mass ratio along the risers. We will therefore assume that the ratio varies

$$x(\xi) = x_r\xi \quad 0 \le \xi \le 1 \tag{7}$$

where ξ is a normalized length coordinate along the risers and x_r is the steam-water mass ratio at the riser outlet. The transfer of mass and energy between steam and water by condensation and evaporation is a key element in the modelling. When modelling steam and water separately the transfer must be accounted for explicitly. This can be avoided by writing joint balance equations for water and steam. The global mass balance for the riser section is

$$\frac{d}{dt}(\varrho_s a_m V_r) + \frac{d}{dt}(\varrho_w(1-a_m)V_r) = q_{dc} - q_r \tag{8}$$

where q_r is the total mass flow out of the risers. The global energy balance for the riser section is

$$\begin{aligned}
&\frac{d}{dt}(\varrho_s h_s a_m V_r) + \frac{d}{dt}(\varrho_w h_w(1-a_m)V_r) = \\
&P + q_{dc}h_w - x_r q_r h_s - (1-x_r)q_r h_w = \\
&P + q_{dc}h_w - x_r q_r h_c - q_r h_w
\end{aligned} \tag{9}$$

The flow out of the risers (q_r) can be eliminated by multiplying equation (8) by $-(h_w + x_r h_c)$ and adding to equation (9). Hence

$$\begin{aligned}
&\frac{d}{dt}(\varrho_s h_s a_m V_r) - (h_w + x_r h_c)\frac{d}{dt}(\varrho_s a_m V_r) \\
&+ \frac{d}{dt}(\varrho_w h_w(1-a_m)V_r) - (h_w + x_r h_c) \\
&\frac{d}{dt}(\varrho_w(1-a_m)V_r) = P - x_r h_c q_{dc}
\end{aligned}$$

This can be simplified to

$$\begin{aligned}
&h_c(1-x_r)\frac{d}{dt}(\varrho_s a_m V_r) + \varrho_w(1-a_m)V_r\frac{dh_w}{dt} \\
&- x_r h_c\frac{d}{dt}(\varrho_w(1-a_m)V_r) + \varrho_s a_m V_r\frac{dh_s}{dt} = P - x_r h_c q_{dc}
\end{aligned} \tag{10}$$

Drum Level

To calculate the drum level it is necessary to know the average steam-water volume ratio in the risers (a_m). We have

$$x = \frac{\varrho_s a}{\varrho_s a + \varrho_w(1-x)}$$

Solving this equation for a we get

$$a = a(x) = \frac{\varrho_w x}{\varrho_s + (\varrho_w - \varrho_s)x}$$

Assume that the steam-water mass ratio is linear along the riser as expressed by equation (7). The average steam-water volume ratio in the risers is

$$\begin{aligned}
a_m &= \int_0^1 a(x_r\xi)d\xi = \frac{1}{x_r}\int_0^1 a(x_r\xi)d(x_r\xi) \\
&= \frac{1}{x_r}\int_0^{x_r} a(x)dx \\
&= \frac{\varrho_w}{\varrho_w - \varrho_s}\left[1 - \frac{\varrho_s}{(\varrho_w - \varrho_s)x_r}\ell n\left(1 + \frac{\varrho_w - \varrho_s}{\varrho_s}x_r\right)\right]
\end{aligned} \tag{11}$$

We can now obtain the following equation for the drum level

$$\ell = \frac{V_w + a_m V_r}{A} \tag{12}$$

where A is the wet surface of the drum. This equation tells that the drum level is composed of two terms, the total amount of water in the drum, and the displacement due to changes of the steam-water ratio in the risers. The model has the same basic form as the water level model in Bell and Åström (1979). This model was, however, developed heuristically and not from first principles.

Downcomer Flow

The flow through the downcomers (q_{dc}) can be obtained from a momentum balance. In natural circulation boilers the flow is driven by the difference between the densities of water and steam. A momentum balance gives

$$a_m V_r(\varrho_w - \varrho_s) = \frac{1}{2}k\,q_{dc}^2 \tag{13}$$

where k is a friction coefficient. The riser flow q_r can be calculated from equation (8). We get

$$q_r = q_{dc} - \frac{d}{dt}(\varrho_s a_m V_r) - \frac{d}{dt}(\varrho_w(1-a_m)V_r) \tag{14}$$

4. Simulations

The equations derived in Section 3 will now be summarized. The state equations are given by (1), (4) and (8). The state variables are chosen as drum pressure p, water volume in drum V_w and average steam quality at riser outlet x_r. Equation (1), (4) and (8) can then be written as

$$\begin{cases}
e_{11}\frac{dp}{dt} + e_{12}\frac{dV_w}{dt} + e_{13}\frac{dx_r}{dt} = P + q_{fw}h_{fw} - q_s h_s \\
e_{21}\frac{dp}{dt} + e_{22}\frac{dV_w}{dt} + e_{23}\frac{dx_r}{dt} = q_{fw} - q_s \\
e_{31}\frac{dp}{dt} + e_{33}\frac{dx_r}{dt} = P - q_{dc}x_r h_c
\end{cases} \tag{15}$$

where

$$\begin{aligned}
e_{11} &= \left(\frac{d\varrho_s}{dp}h_s + \varrho_s\frac{dh_s}{dp}\right)V_{st} + \left(\frac{d\varrho_w}{dp}h_w + \varrho_w\frac{dh_w}{dp}\right)V_{wt} + \\
&\quad mc_p\frac{dT_s}{dp} \\
e_{12} &= \varrho_w h_w - \varrho_s h_s \\
e_{13} &= (\varrho_s h_s - \theta_w h_w)V_r\frac{da_m}{dx_r} \\
e_{21} &= \frac{d\varrho_s}{dp}V_{st} + \frac{d\varrho_w}{dp}V_{wt} \\
e_{22} &= \varrho_w - \varrho_s \\
e_{23} &= (\varrho_s - \varrho_w)V_r\frac{da_m}{dx_r} \\
e_{31} &= \left[(1-x_r)h_c\frac{d\varrho_s}{dp} + \varrho_s\frac{dh_s}{dp}\right]a_m V_r \\
&\quad + \left[\varrho_w\frac{dh_w}{dp} - x_r h_c\frac{d\varrho_w}{dp}\right](1-a_m)V_r \\
e_{33} &= [(1-x_r)\varrho_s + x_r\varrho_w]h_c V_r\frac{da_m}{dx_r}
\end{aligned} \tag{16}$$

To execute the simulation equation (15) has to be solved for the derivatives of the state variables. The right hand side of (15) contain input variables P, q_{fw} and q_r, and functions of the state variables. Notice that downcomer flow q_{dc} is given by equation (13). A detailed description of the simulation is given in the code in the Appendix.

Parameters

The model is characterized by the variables

V_{drum}	drum volume
V_r	riser volume
V_{dc}	downcomer volume
m	total metal mass
c_p	specific heat of metal
k	friction coefficient

and the functions $\varrho_s(p)$, $\varrho_w(p)$, $h_s(p)$, $h_w(p)$, $T_s(p)$, $h_{fw}(p)$ which are obtained from steam tables. Quadratic approximations to the steam tables are given in the program listing in the Appendix.

Equilibrium Conditions

Equilibrium conditions are obtained from (15). Hence

$$q_{fw} = q_s \tag{17}$$

$$P = q_s(h_s - h_{fw}) \tag{18}$$

$$P = q_{dc}x_r h_c \tag{19}$$

The equilibrium value of the drum pressure can be determined from equation (18) since h_s and h_{fw} depend on the pressure.

Dynamic Response

Responses to steps in fuel flow and steam flow are given in Figures 1 and Figure 2. The simulations illustrate the dynamic features that are captured by the model. Figure 1 shows the response to a step change in fuel flow. The pressure responds like a pure integrator. The total amount of water in the drum increases because steam is generated in the risers. The total amount of steam in the risers increases because of the increased steam generation. The steam quality in volume ratios increases initially but it will later decrease because of the compression effect.

The drum level increases rapidly at first but the rate of increase decreases. The downcomer flow matches the steam fraction volume ratio. There is an instantaneous increase of the riser flow at the beginning of the step. The riser flow will then decrease at the same rate as the downcomer flow. Figure 2 shows the response to a step change in steam flow. The global effect is that the pressure and the volume will respond like integrators. There will, however, be a swell effect because of the initial evaporation of steam.

5. Conclusions

This paper has presented simple models for a drum boiler system. The models capture the major dynamical behaviour. They are derived from first principles and require only a few physical parameters that are easily obtained from construction data and steam tables. The behaviour has been shown by simulating step responses to fuel and steam flow changes. Reasonable results are obtained even for the difficult problems of predicting circulation flow and drum water level shrink and swell. The model can easily be augmented by equations for turbine and electrical output given in Åström and Eklund (1972, 1975) or Bell and Åström (1979) to produce a simple model for a complete boiler-turbine alternator system. A strong feature is that the model capture the essence of the steam generation in a heated pipe. It has also been used successfully to model steam generation in a nuclear plant. It can also be adapted to model once-through boilers.

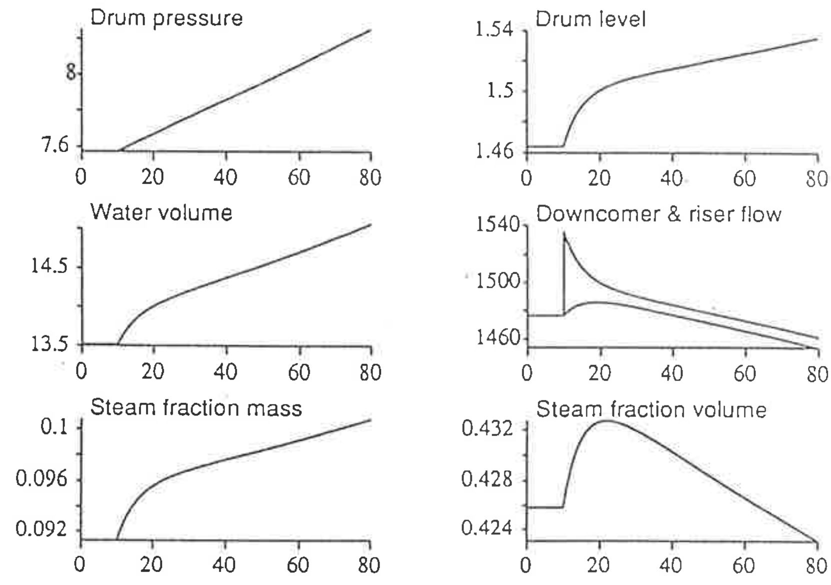

Figure 1. Responses to a step in fuel flow.

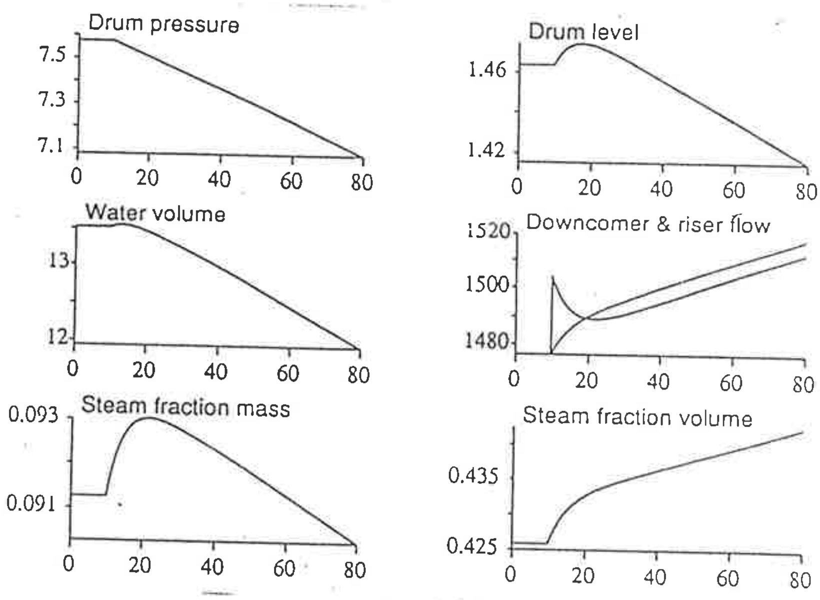

Figure 2. Response to a step in steam flow.

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Appendix

```
CONTINUOUS SYSTEM DRUM
"Nonlinear third order model for drum-downcomer-
riser
"Author K J Astrom 870805

INPUT pow qfw tfw qs
OUTPUT dl am
STATE p Vw xr
DER dp dVw dxr
```

```
"pow    Power from fuel                      [MW]
"qfw    Feedwater flow                       [kg/s]
"tfw    Feedwater temperature                [deg C]
"qs     Steam flow                           [kg/s]

"dl     Drum level                           [m]
"am     Steam quality volume ratio
"qc     Condensate flow (total)              [kg/s]
"qcr    Condensate flow (risers)             [kg/s]

"p      Drum pressure                        [MPa]
"Vw     Drum water volume                    [m*m*m]
"xr     Steam quality at riser outlet

"Properties of steam and water in saturated state

hs = a01+(a11+a21*(p-10))*(p-10)
dhsdp = a11+2*a21*(p-10)
a01: 2.728E6
a11: -1.792E4
a21: -924.0

rs = a02+(a12+a22*(p-10))*(p-10)
drsdp = a12+2*a22*(p-10)
a02: 55.43
a12: 7.136
a22: 0.224

hw = a03+(a13+a23*(p-10))*(p-10)
dhwdp = a13+2*a23*(p-10)
a03: 1.408E6
a13: 4.565E4
a23: -1010.0

rw = a04+(a14+a24*(p-10))*(p-10)
drwdp = a14+2*a24*(p-10)
a04: 691.35
a14: -1.867
a24: 0.081

ts = a05+(a15+a25*(p-10))*(p-10)
dtsdp = a15+2*a25*(p-10)
a05: 311.0
a15: 7.822
a25: -0.32

"Properties of water in subcritical state

"hd = hw+(a06+a16*(p-10))*(td-ts)
"dhddp = dhwdp+a16*(td-ts)-(a06+a16*(p-10))*dtsdp
"cp = a06+a16*(p-10)
a06: 5900
a16: 250

"rd = rw+(a07+a17*(p-10))*(td-ts)
"drddp = drwdp+a17*(td-ts)-(a07+a17*(p-10))*dtsdp
"drddt = a07+a17*(p-10)
"a07: 2.4
"a17: 0.2

hfw = hw+(a06+a16*(p-10))*(tfw-ts)
hc = hs-hw
hr = xr*hs+(1-xr)*hw

"Drum level
lw=Vw/adrum
lr=am*Vr/adrum
dl = lr+lw

"Average steam quality volume ratio
s2 = rs/(xr*(rw-rs))
s3 = 1+xr*(rw/rs-1)
am = rw/(rw-rs)*(1-s2*ln(s3))
damdx = rw*s2*(ln(s3)/(xr*(rw-rs))-1/s3/rs)

"Circulation flow
s1 = 2*(rw-rs)*Vr*am/k
qdc = sqrt(s1)
qr=qdc-(am*drsdp+(1-am)*drwdp)*Vr*dp+(rw-rs)
*Vr*damdx*dxr

"Total condensation flow
qc = (rs*Vst*dhsdp+rw*Vwt*dhwdp)*dp/hc

"Condensation flow in risers
qcr = (rs*am*Vr*dhsdp+rw*(1-am)*Vr*dhsdp)*dp/hc

"Equations for derivatives of state variables
Vst = Vdrum - Vw + am*Vr
Vwt = Vw + Vdc + (1-am)*Vr
e11 = Vst*(hs*drsdp rs*dhsdp)+Vwt*
(hw*drwdp+rw*dhwdp)
e12 = hw*rw-hs*rs
e13 = (hs*rs-hw*rw)*Vr*damdx
b1 = pow*1e6+qfw*hfw-qs*hs
e21 = Vst*drsdp+Vwt*drwdp
e22 = rw-rs
e23 = (rs-rw)*Vr*damdx
b2 = qfw-qs
e31 = ((1-xr)*hc*drsdp+rs*dhsdp)*am*Vr+
(rw*dhwdp-xr*hc*drwdp)*(1-am)*Vr
e32 = 0
e33 = ((1-xr)*rs+xr*rw)*hc*Vr*damdx
b3 =  pow*1e6-qdc*xr*hc

"Solve linear equation for derivatives of state
vector p1 = e21/e11
e221 = e22-e12*p1
e231 = e23-e13*p1
b21 = b2-b1*p1

p2 = e31/e11
e321 = -e12*p2
e331 = e33-e13*p2
b31 = b3-b1*p2

p3 = e321/e221
e332 = e331-e231*p3
b32 = b31-b21*p3

dxr = b32/e332
dVw = (b21-e231*dxr)/e221
dp = (b1-e12*dVw-e13*dxr)/e11

"Parameters
adrum: 20
vdrum: 40
vr: 37
vdc: 19
k: 0.01

"Initials
p: 7.576
Vw:13.521
xr:0.091263

END
```

Simple Drum-Boiler Models

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Simple Drum-Boiler Models

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6. Conclusions

INTRODUCTION

Motivation

Simple physics based models
for system studies

Experimental verification

Industrial collaboration with
Sydkraft AB Malmö Sweden

Progress

Slow painstaking
Eklund 1968
Åström Eklund 1972, 1975
Bell and Åström 1979
Bell and Åström 1987

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Global Energy Balance

$$\frac{d}{dt}\left[\rho_s h_s V_{st} + \rho_w h_w V_{wt} + m c_p T\right] = P + q_{fw} h_{fw} - q_s h_s \tag{1}$$

Total Steam Volume

$$V_{st} = V_{drum} - V_w + a_m V_r \tag{2}$$

Total Water Volume

$$V_{wt} = V_w + V_{dc} + (1 - a_m) V_r \tag{3}$$

Global Mass Balance

$$\frac{d}{dt}\left[\rho_s V_{st} + \rho_w V_{wt}\right] = q_{fw} - q_s \tag{4}$$

Eliminate dV_{wt}/dt between (1) and (4)

$$h_c \frac{d}{dt}\left(\rho_s V_{st}\right) + \left[\rho_s V_{st} \frac{dh_s}{dt} + \rho_w V_{wt} \frac{dh_w}{dt} + m c_p \frac{dT}{dt}\right] = P - q_{fw}\left(h_w - h_{fw}\right) - q_s h_c \tag{5}$$

$$h_c \frac{d}{dt}(\rho_s V_{st}) + \left[\rho_s V_{st} \frac{dh_s}{dt} + \rho_w V_{wt} \frac{dh_w}{dt} + mc_p \frac{dT}{dt}\right] = P - q_{fw}(h_w - h_{fw}) - q_s h_c \tag{5}$$

Rewritten as

$$e_{11} \frac{dp}{dt} = P - q_{fw}(h_w - h_{fw}) - q_s h_c \tag{6}$$

$$e_{11} = h_c V_{st} \frac{d\rho_s}{dp} + \rho_s V_{st} \frac{dh_s}{dp} + \rho_w V_{wt} \frac{dh_w}{dp} + mc_p \frac{dT_s}{dp}$$

Total condensation flow

$$q_c = \frac{1}{h_c}\left[\rho_s V_{st} \frac{dh_s}{dt} + \rho_w V_{wt} \frac{dh_w}{dt} + mc_p \frac{dT_s}{dt}\right]$$

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THE VOID MODEL

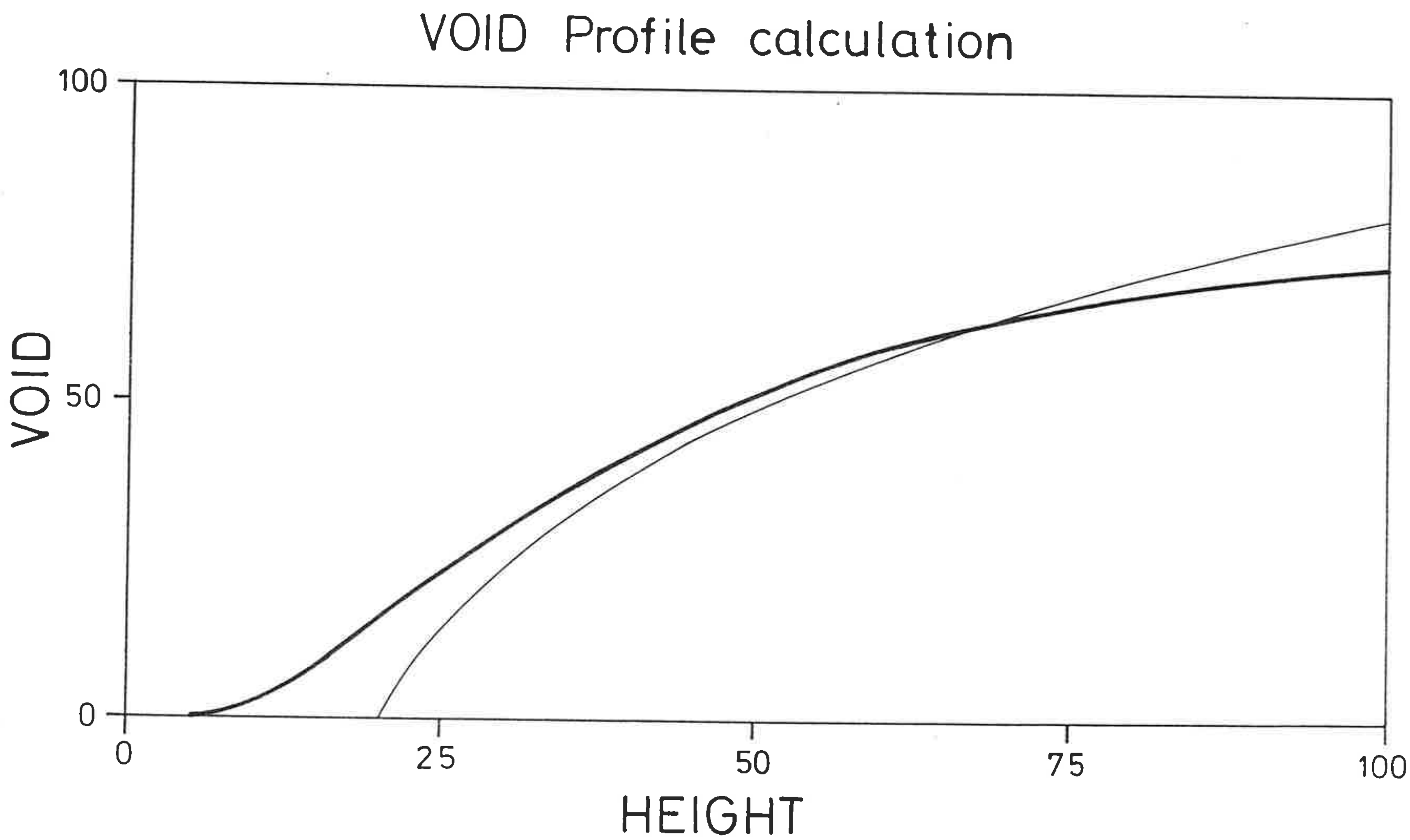
A distributed parameter system

Assuming a void distribution gives a lumped parameter model

The PDEs gives a steady state solution with a linear steam water mass ratio

Use static relation also for dynamics

Model explored for nuclear reactor models where elaborate simulation models are available


VOID Profile calculation
100
50
0
VOID
0
25
50
75
100
HEIGHT

Mass Balance for Riser Section

$$\frac{d}{dt}(\rho_s a_m V_r) + \frac{d}{dt}(\rho_w (1 - a_m) V_r) = q_{dc} - q_r \tag{8}$$

Energy Balance

$$\begin{aligned}&\frac{d}{dt}(\rho_s h_s a_m V_r) + \frac{d}{dt}(\rho_w h_w (1 - a_m) V_r)\\ &\quad = P + q_{dc} h_w - x_r q_r h_s - (1 - x_r) q_r h_w\\ &\quad = P + q_{dc} h_w - x_r q_r h_c - q_r h_w\end{aligned} \tag{9}$$

Eliminate q_r between (8) and (9)

$$\begin{aligned}&\frac{d}{dt}(\rho_s h_s a_m V_r) - (h_w + x_r h_c)\frac{d}{dt}(\rho_s a_m V_r)\\ &\quad + \frac{d}{dt}(\rho_w h_w (1 - a_m) V_r) - (h_w + x_r h_c)\\ &\quad \frac{d}{dt}(\rho_w (1 - a_m) V_r) = P - x_r h_c q_{dc}\end{aligned}$$

Simplify to

$$\begin{aligned}&h_c(1 - x_r)\frac{d}{dt}(\rho_s a_m V_r) + \rho_w (1 - a_m) V_r \frac{dh_w}{dt}\\ &\quad - x h_c \frac{d}{dt}(\rho_w (1 - a_m) V_r) + \rho_s a_m V_r \frac{dh_s}{dt}\\ &\quad = P - x_r h_c q_{dc}\end{aligned} \tag{10}$$

Drum Level

Average steam-water volume ratio

$$x = \frac{\rho_s a}{\rho_s a + \rho_w (1-x)}$$

Solving with respect to a

$$a = a(x) = \frac{\rho_w x}{\rho_s + (\rho_w - \rho_s)\, x}$$

Assume

$$x(\xi) = x_r \xi \qquad 0 \leq \xi \leq 1 \tag{11}$$

Hence

$$\begin{aligned} a_m &= \int_0^1 a(x_r\xi) d\xi = \frac{1}{x_r}\int_0^1 a(x_r\xi) d(x_r\xi) \\ &= \frac{1}{x_r}\int_0^{x_r} a(x) dx \\ &= \frac{\rho_w}{\rho_w - \rho_s}\left[1 - \frac{\rho_s}{(\rho_w - \rho_s)x_r}\,\ell n\left(1 + \frac{\rho_w - \rho_s}{\rho_s} x_r\right)\right] \end{aligned} \tag{11}$$

$$\ell = \frac{V_w + a_m V_r}{A} \tag{12}$$

Downcomer Flow

Momentum balance

$$a_m V_r(\rho_w - \rho_s) = \frac{1}{2} k \, q_{dc}^2 \tag{13}$$

Riser flow from (8)

$$q_r = q_{dc} - \frac{d}{dt}(\rho_s a_m V_r) - \frac{d}{dt}(\rho_w (1 - a_m) V_r) \tag{14}$$

Summary

$$\frac{d}{dt}\left[\rho_s h_s V_{st} + \rho_w h_w V_{wt} + mc_p T\right] = P + q_{fw} h_{fw} - q_s h_s \tag{1}$$

$$\frac{d}{dt}\left[\rho_s V_{st} + \rho_w V_{wt}\right] = q_{fw} - q_s \tag{4}$$

$$\begin{aligned} &h_c(1 - x_r)\frac{d}{dt}(\rho_s a_m V_r) + \rho_w (1 - a_m) V_r \frac{dh_w}{dt} \\ &- x h_c \frac{d}{dt}(\rho_w(1 - a_m)V_r) + \rho_s a_m V_r \frac{dh_s}{dt} \\ &= P - x_r h_c q_{dc} \end{aligned} \tag{10}$$

Choose p, V_w, and x_r as state variables.

Simulation Model

$$\begin{cases} e_{11}\dfrac{dp}{dt} + e_{12}\dfrac{dV_w}{dt} + e_{13}\dfrac{dx_r}{dt} = P + q_{fw}h_{fw} - q_s h_s \\ e_{21}\dfrac{dp}{dt} + e_{22}\dfrac{dV_w}{dt} + e_{23}\dfrac{dx_r}{dt} = q_{fw} - q_s \\ e_{31}\dfrac{dp}{dt} + e_{33}\dfrac{dx_r}{dt} = P - q_{dc}x_r h_c \end{cases} \tag{15}$$

$$
\begin{aligned}
e_{11} &= \left(\frac{d\rho_s}{dp}h_s + \rho_s\frac{dh_s}{dp}\right)V_{st} \\
&\quad + \left(\frac{d\rho_w}{dp}h_w + \rho_w\frac{dh_w}{dp}\right)V_{wt} + mc_p\frac{dT_s}{dp} \\
e_{12} &= \rho_w h_w - \rho_s h_s \\
e_{13} &= (\rho_s h_s - \rho_w h_w)\,V_r \\
e_{21} &= \frac{d\rho_s}{dp}V_{st} + \frac{d\rho_w}{dp}V_{wt} \\
e_{22} &= \rho_w - \rho_s \\
e_{23} &= (\rho_s - \rho_w)\,V_r\frac{da_m}{dx_r} \\
e_{31} &= \left[(1-x_r)h_c\frac{h\rho_s}{dp} + \rho_s\frac{dh_s}{dp}\right]a_m V_r \\
&\quad + \left[\rho_w\frac{dh_w}{dp} - x_r h_c\frac{d\rho_w}{dp}\right](1-a_m)V_r \\
e_{33} &= \left[(1-x_r)\rho_s + x_r\rho_w\right]h_c V_r\frac{da_m}{dx_r}
\end{aligned}
\tag{16}
$$

Parameters

V_{drum}	drum volume
V_r	riser volume
V_{dc}	downcomer volume
m	total metal mass
c_p	specific heat of metal
k	friction coefficient

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Step in Fuel Flow

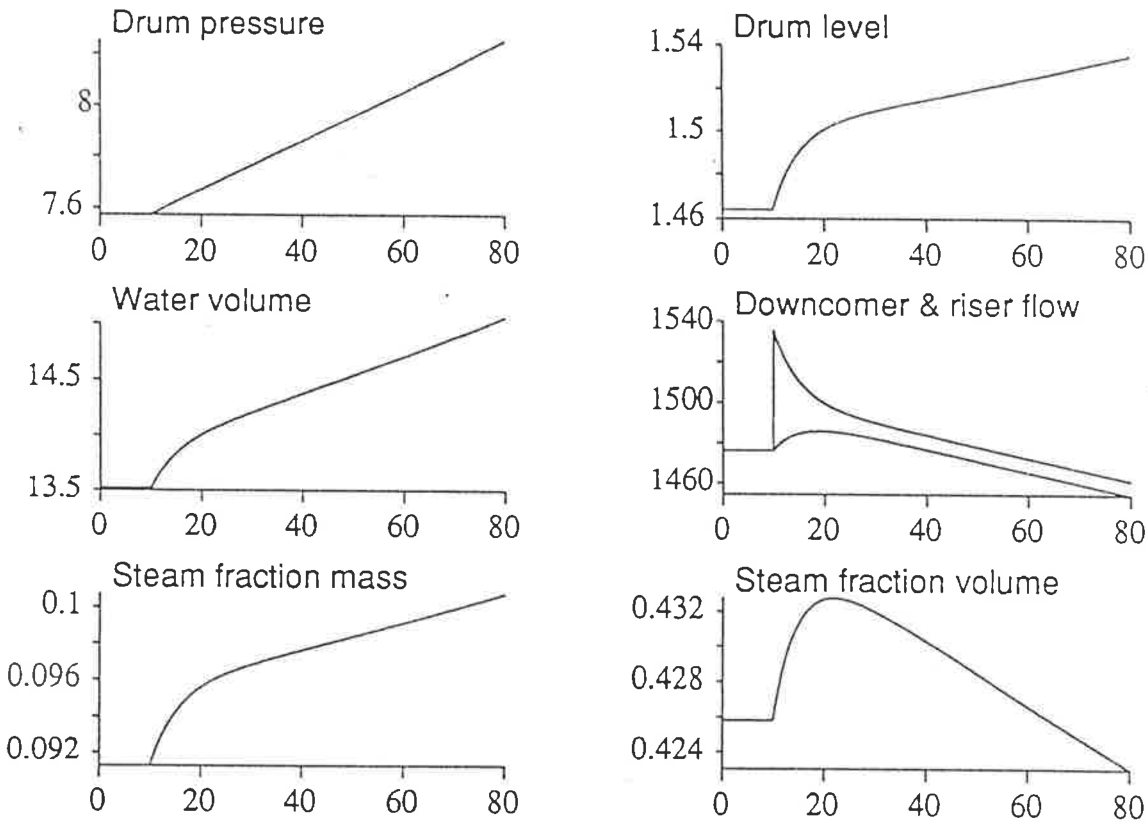

Figure 1. Responses to a step in fuel flow.

Step in Steam Flow

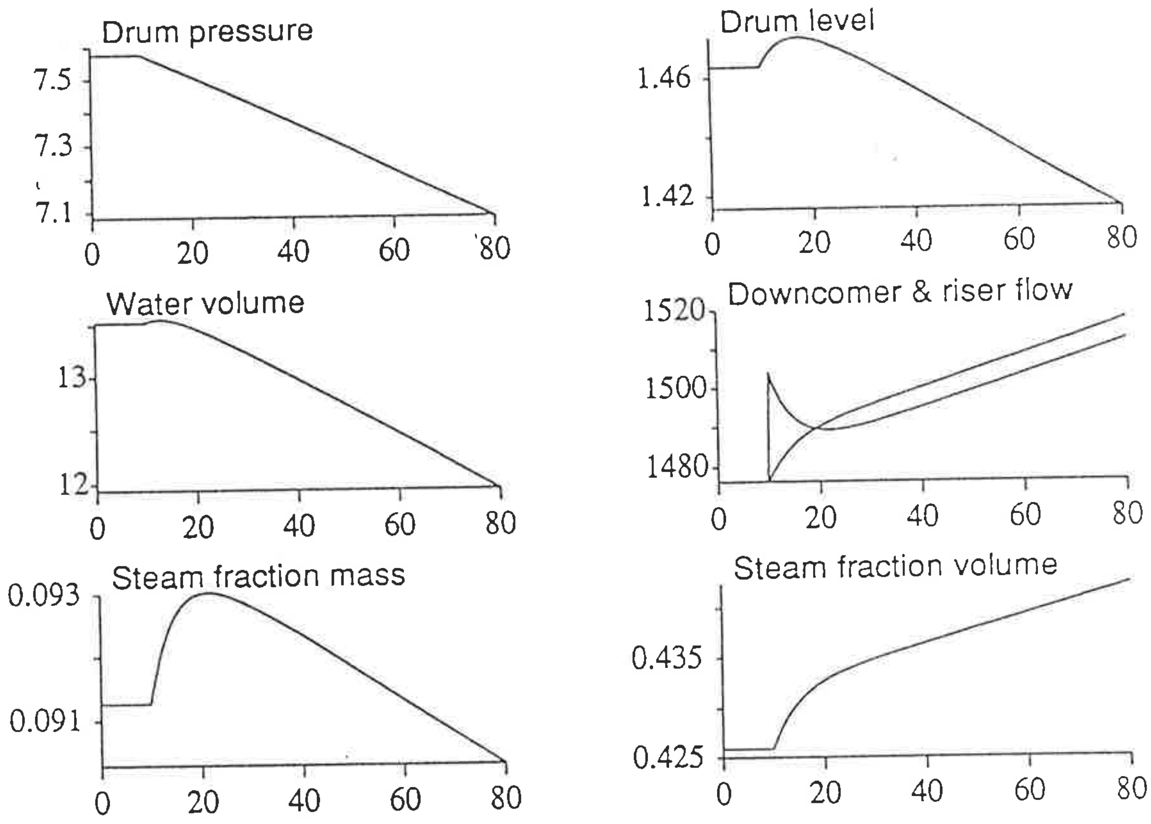

Figure 2. Response to a step

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EXPERIMENTS

P16-G16 at Öresundsverket

Steinmuller boiler Stal-Laval turbine. Active power 160MW.

Controllers disconnected. PRBS like perturbations introduced in fuel flow, feedwater flow and steam valve at high and low load

Fuel Flow Changes at Low Load

88.02.05 - 17:15:22 nr: 1
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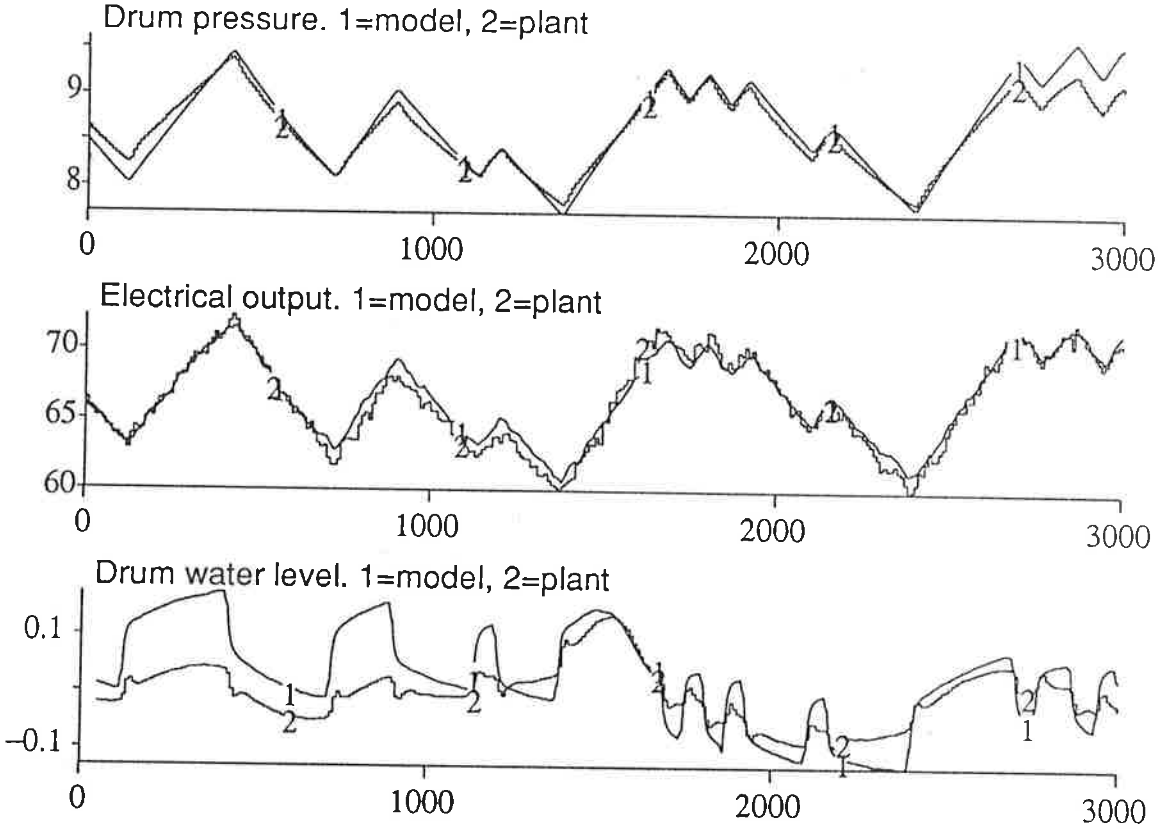

Turbine Valve Changes at Low Load

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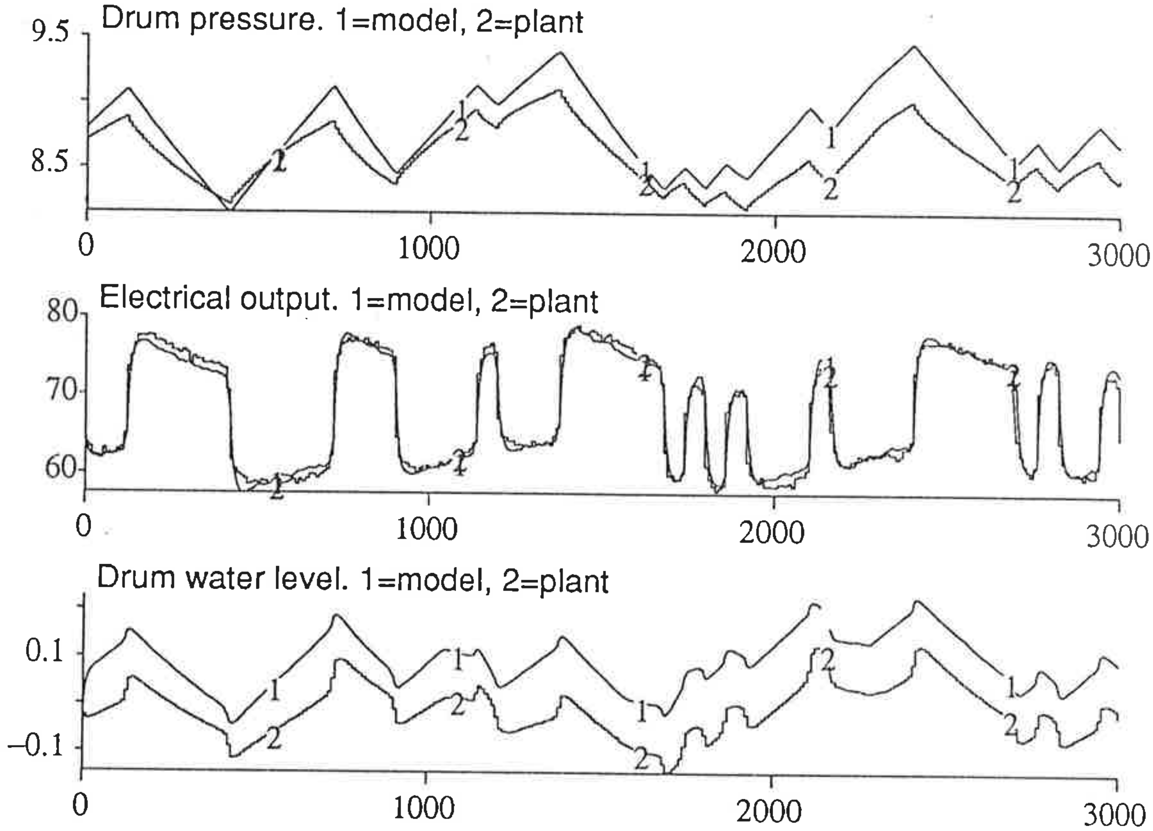

Feedwater Flow Changes at Low Load

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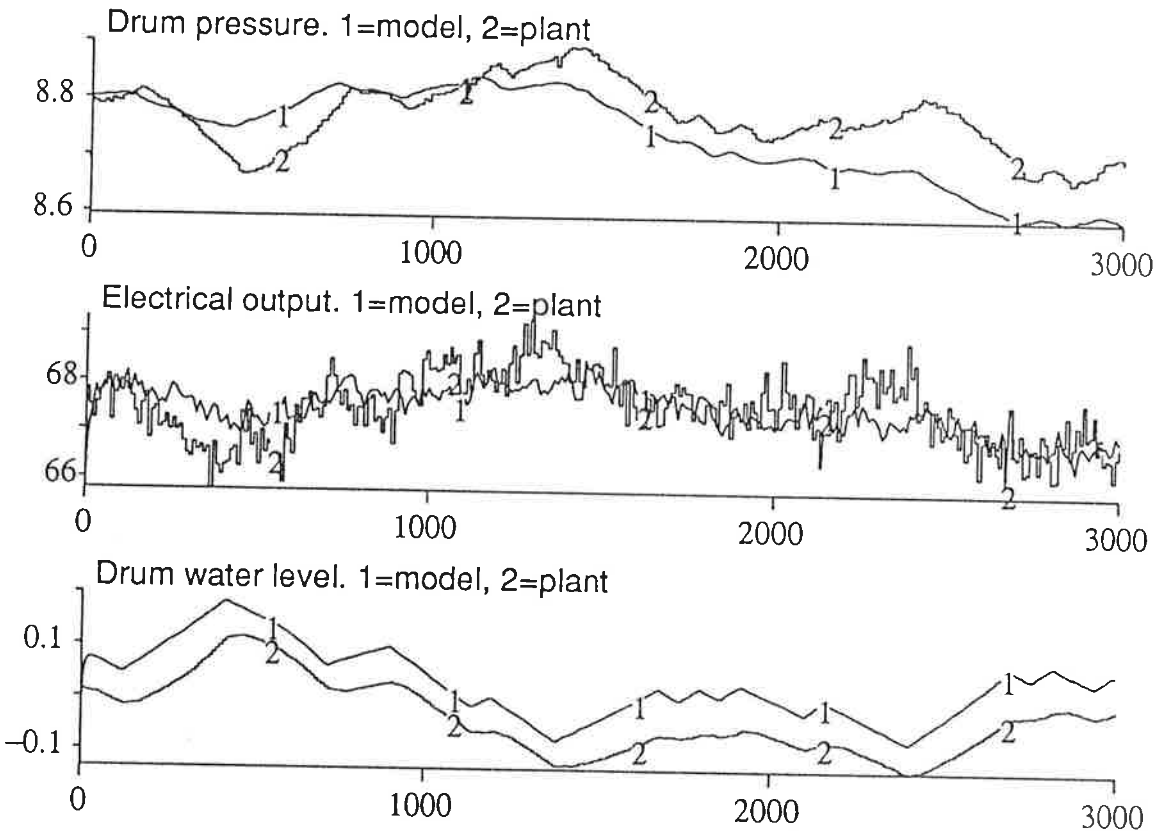

Fuel Flow Changes at High Load

88.02.05 - 19:16:05 nr: 1
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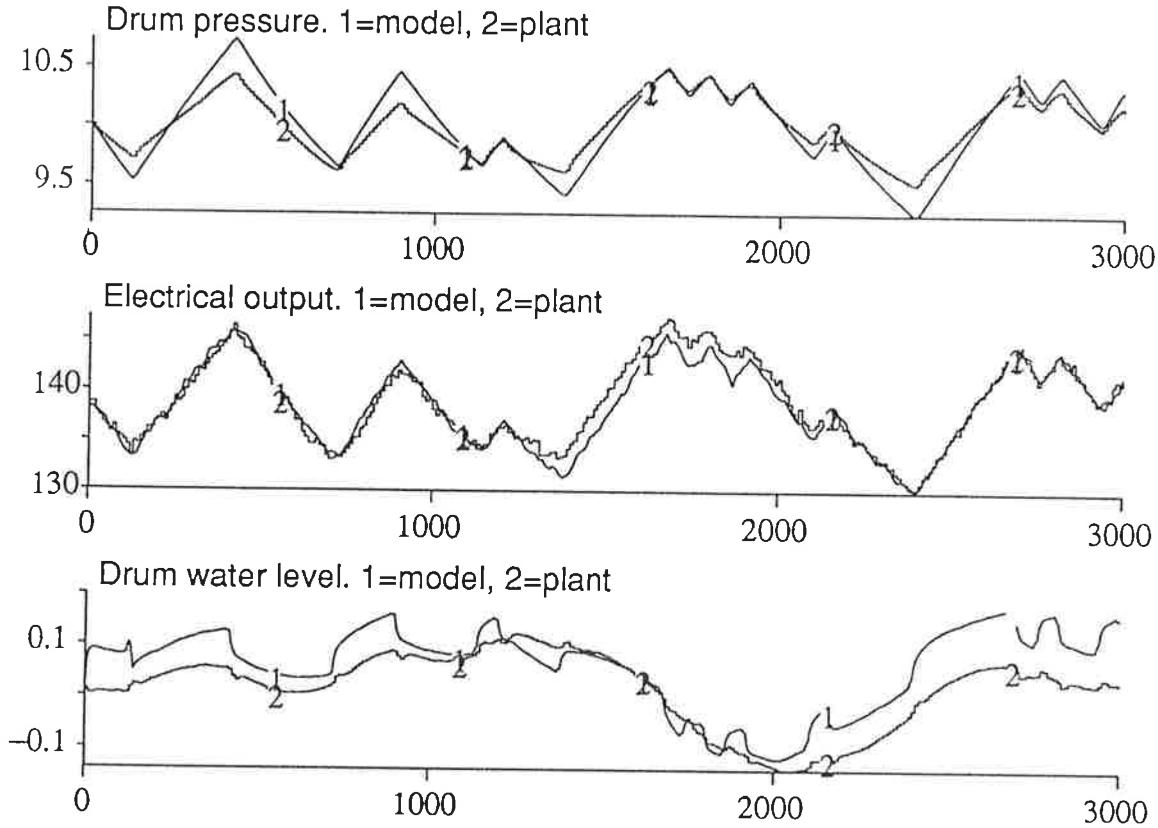

Turbine Valve Changes at High Load

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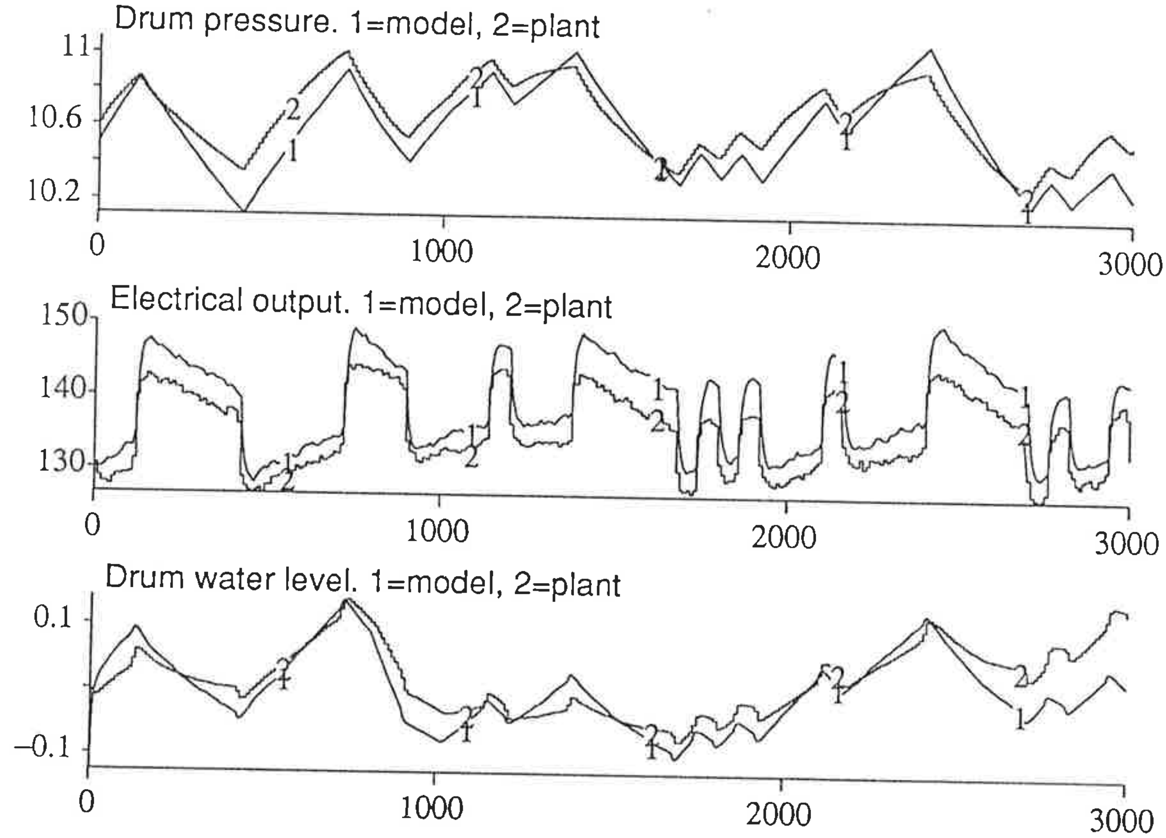

Feedwater Flow Changes at High Load

88.02.05 - 19:45:37 nr: 1
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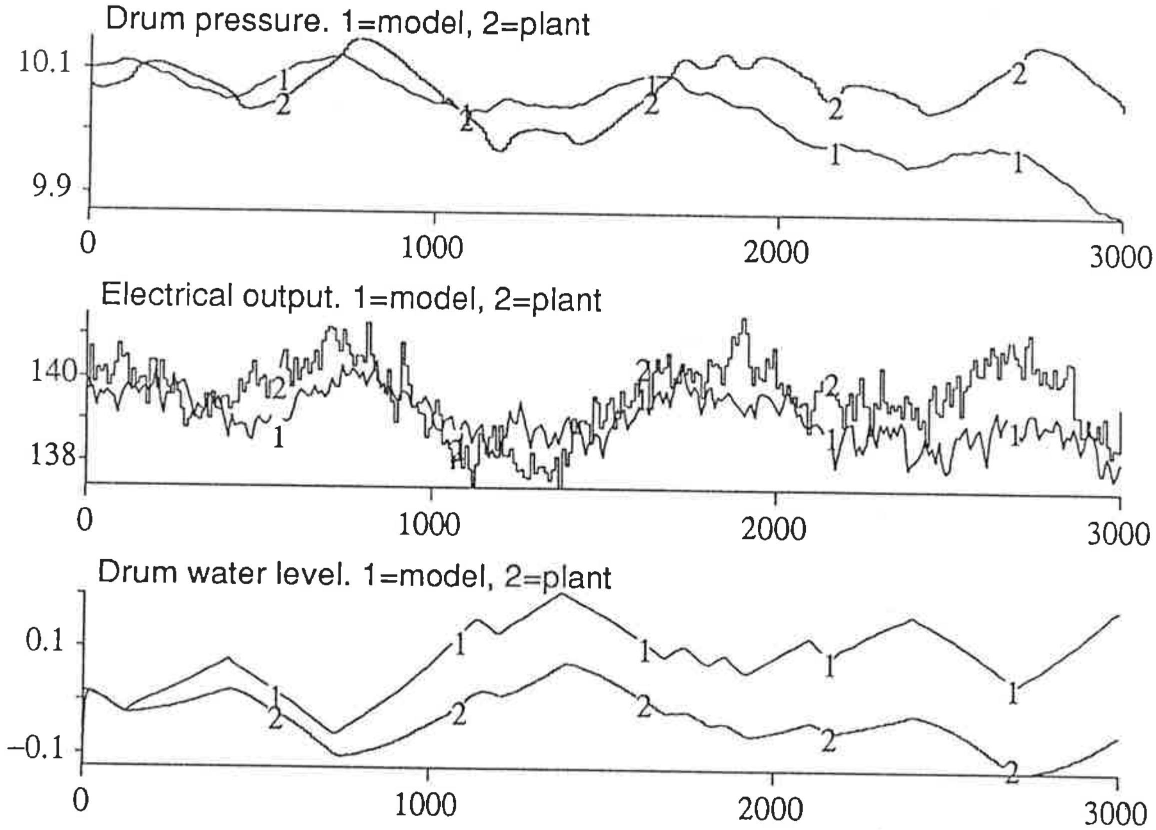

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CONCLUSIONS

Promising results

Some details remain

Further experiments

Simplifications

Control design