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Adaptive Control of Robot Manipulator Motion

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Abstract

This paper presents algorithms for continuous-time direct adaptive control of robot manipulators. Lyapunov theory is used for controller design and stability investigation. Algorithms for rapid continuous-time adaptive control are presented.

Key words
Adaptive control, Lyapunov stability, L^2–stability, Robot control, Rapid parameter estimation.

Classification system and/or index terms (if any)

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Introduction

Robotic systems intended for autonomous operation need an ability of adaptation to new and rapidly changing operating conditions. In such a situation various automatic control methods become important. A standard regulator, however, might be insufficient for successful solutions of control problems. It is therefore of interest to consider adaptive control for complex multi-input multi-output regulators.

The term 'adaptive control' is here used in the terminology of control theory [4]. An adaptive regulator is itself able to tuning and self-modification in continuous operation to increase flexibility and operation autonomy.

The vast literature on adaptive control of linear systems is only partly applicable to the control problems of robotics [4], [11]. In general, multi-input multi-output adaptive control is the desirable tool to solve problems of coupled motion [7]. The nonlinear robot dynamics with rapidly changing operating conditions also make the adaptive control problems difficult.

There is however an advantage compared to the setting of adaptive control of linear systems. The structural information is considerable and there are usually only few unknown parameters. The adaptive control problems of robotics are thus meaningful to consider and a special literature has appeared in this field [9], [14], [1], [10].

Vukabratović *et al* [15] developed a linear estimation model suitable for identification of the payload of a partially known robot system. Craig *et al* [3] applied ideas of model reference adaptive control and developed a regulator and stability proofs. Slotine and Li [13] approached the problem in a similar way but with weaker assumptions. They presented a regulator that is linear in the parameters and without any requirement of acceleration measurement.

Problem statement

The following objections could be raised against the solution of Craig *el* [3]. First, it requires measurement of the angular acceleration of the manipulator joints. Second, the algorithm involves matrix inversion of the moment of inertia matrix $\widehat{M}(q,t)$ containing estimated parameters. This is computationally difficult and time consuming. Third, the parameter estimation problem is not solved in a quite satisfactory way because the algorithm needs an additional, fairly complicated reset action of parameter estimates to avoid control problems. Finally, the reference signal is restricted to be a signal generated by a strictly positive real linear system transfer function.

Slotine and Li [13] recognized the problem with acceleration measurement and the matrix inversion and tried to solve this problem in a similar setting. Their technical innovation makes use of the skew-symmetric system matrix properties and thereby eliminates the problems of measurement and computation. Stability properties are however not quite satisfactory with respect to position errors. Elimination of steady-state errors is not guaranteed in their fundamental algorithm. The authors attempt to modify the algorithm [13] (sec. 2.2.2) to obtain stability but then make formal errors. They formulate a "Lyapunov function candidate" containing a linear combination of velocity and position error state vectors ($s = \dot{\tilde{q}} + \Lambda\tilde{q}$). There is however a 'forgotten' subspace of the state. The suggested Lyapunov function candidate is not a function of the complete state vector and is therefore not formally correct. A formal requirement is that the Lyapunov function is a function of all state vector components and not only a subset thereof. Moreover, the authors incorrectly claim [13; p. 51] global asymptotic stability although no parameter convergence can be guaranteed.

EXAMPLE 1

Define with the notation of [13] the transformed state vector

$$s = \dot{\tilde{q}} + \Lambda\tilde{q}, \qquad \Lambda = \Lambda^T > 0 \tag{1}$$

with the associated Lyapunov function candidate

$$V_s = s^T H(q) s \tag{2}$$

Introduce also the functions

$$s_\perp = -\Lambda\dot{\tilde{q}} + \tilde{q}; \qquad V_\perp = s_\perp^T H(q) s_\perp \tag{3}$$

Slotine and Li [13] show correctly that $V_s(t)$ and $s(t)$ converge to zero as the time t increases. However, the state vector $s_\perp$ orthogonal to s is not represented in the function V_s. Simulations shows that $V_\perp(t)$ develops irregularly with time also when s is very small, see Fig 1. In some simulations of the example $V_\perp$ remains constant and rather large. Sometimes it slowly tends towards zero for non-zero initial conditions.

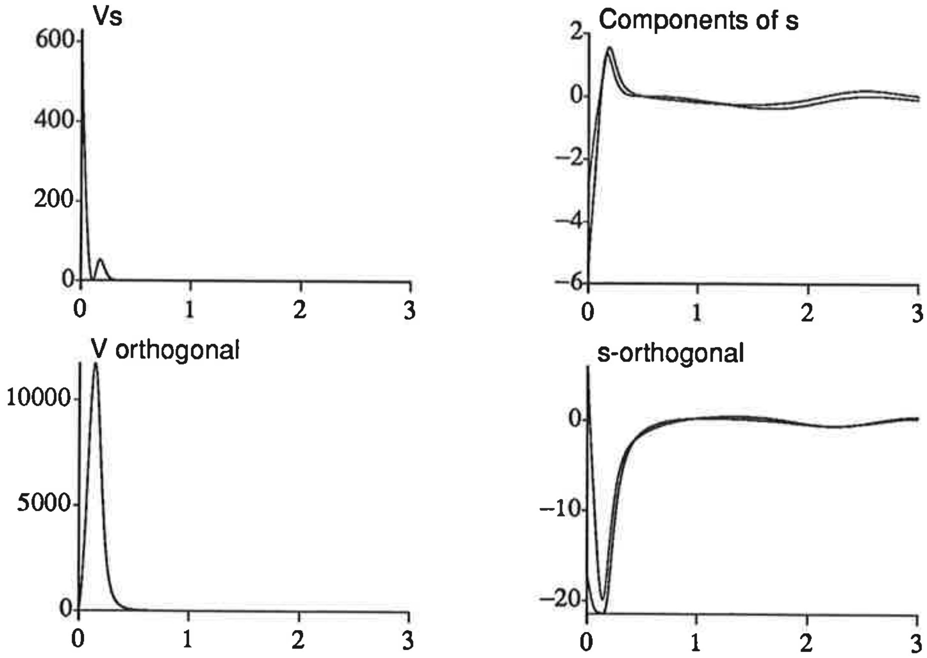

Figure 1. Simulation of the example from Slotine and Li ([13]; app. 1) with $\Lambda = 5I_{2\times 2}$ and non-zero initial conditions. The upper graph shows the "Lyapunov function" V_s. The lower left graph shows $V_\perp$. State vector components depicted to the right. All graphs vs. time [s].

The suggested Lyapunov function candidate of [13; 2.2.2.] is not formally correct. Simulations verify the existence of dynamics not modelled in the stability investigation. The arguments presented in [13] for a claim on global asymptotic stability are thus not valid. □

It is the purpose of this paper to provide Lyapunov functions for analysis and design of stable solutions to the problem of direct adaptive control of robotic manipulators when velocity and position measurements are available.

Manipulator dynamics

We model the manipulator dynamics as a set of n rigid bodies connected [2]. Consider the equations:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \qquad (4)$$

We have used the following notations where time arguments have been omitted.

q	Joint angular positions	$(\dim q = n \times 1)$
$\dot{q}$	Joint angular velocities	$(\dim \dot{q} = n \times 1)$
$\ddot{q}$	Joint angular acceleration	$(\dim \ddot{q} = n \times 1)$

τ	Joint torques		$(\dim \tau = n \times 1)$
$M(q)$	Moment of inertia	$M(q) = M^T(q) > 0$	$(\dim M = n \times n)$
$C(q, \dot{q})$	Coriolis, centripetal and frictional forces		$(\dim C = n \times n)$
$G(q)$	Gravitational forces		$(\dim G = n \times n)$

(5)

It is assumed that the positions q and velocities $\dot{q}$ but not the accelerations $\ddot{q}$ are available for measurement. It is further assumed that the control input is equal to the torque vector τ. It is assumed that the matrices M, C, G have a known structure and contain constant but unknown parameters. The matrices M, C, G are linear in the unknown parameters.

Control objective

The desired reference trajectory for the manipulator to follow is assumed available as bounded functions of time in terms of joint accelerations $\ddot{q}_r$, angular velocities $\dot{q}_r$, and angular positions q_r. In the case where accelerations and velocities are not known they may be conveniently generated with a reference signal r as input to a reference model of the type

$$\ddot{q}_r + K_D \dot{q}_r + K_P q_r = K r \qquad (\dim q_r = \dim r = n \times 1) \tag{6}$$

The dynamic system (6) with the $n \times n$-matrices K_D, K_P, K chosen to obtain desired properties of stiffness, damping, and gain should be stable. This reference model need not be strictly positive real as is often required in the adaptive control literature [4]. A stable, nonlinear reference model is also feasible. Define the errors of accelerations, velocities, and positions as

$$\begin{pmatrix} \ddot{\tilde{q}} \\ \dot{\tilde{q}} \\ \tilde{q} \end{pmatrix} = \begin{pmatrix} \ddot{q} - \ddot{q}_r \\ \dot{q} - \dot{q}_r \\ q - q_r \end{pmatrix} \tag{7}$$

The control objective is to follow a given, bounded reference trajectory $\dot{q}_r, q_r$ without position errors $\tilde{q}$, or velocity errors $\dot{\tilde{q}}$.

Lyapunov design

The most straightforward approach to solve the control problem is via estimation of the unknown coefficients of M, C, G of (4) by making a separation of the unknown matrix coefficients from the signals. This is possible because (4) is linear in the unknown parameters. Reordering of (4) gives an equation where θ denotes the vector of p unknown parameters and φ, φ_0 denote functions of signal variables.

$$\tau = \varphi(\ddot{q}, \dot{q}, q)\theta + \varphi_0(\ddot{q}, \dot{q}, q); \qquad \dim\theta = p \times 1, \quad \dim\varphi = n \times p, \quad \dim\varphi_0 = n \times 1. \quad (8)$$

where φ, φ_0 are computable provided that $\ddot{q}, \dot{q}, q$ are available for measurement. The acceleration $\ddot{q}$ is however often not available. In the absence of acceleration measurement it is desirable to estimate the unknown parameters θ and solve the control problem without the impossible computation of φ, φ_0.

We use Lyapunov theory to do this because a successful solution will determine not only the solutions of controller and estimator design but also the associated stability properties [5], [9], [12], [6].

A state space description

Let $\widehat{\theta}$ denote the estimate of the constant but unknown parameters θ and let $\widetilde{\theta}$ denote associated the parameter errors. This gives

$$\widetilde{\theta} = \widehat{\theta} - \theta, \qquad \dot{\widetilde{\theta}} = \dot{\widehat{\theta}} \tag{9}$$

The full error state space representation is found as

$$\widetilde{x}(t) = \begin{pmatrix} \dot{\widetilde{q}}^T(t) & \widetilde{q}^T(t) & \widetilde{\theta}^T(t) \end{pmatrix}^T; \qquad \dim\widetilde{x} = (2n + p) \times 1 \tag{10}$$

The error dynamics of the manipulator may be obtained from (4), (6), and (7) as a state space description where the derivative of $\widetilde{x}$ is

$$\dot{\widetilde{x}}(t) = \begin{pmatrix} \ddot{\widetilde{q}}(t) \\ \dot{\widetilde{q}}(t) \\ \dot{\widetilde{\theta}}(t) \end{pmatrix} = \begin{pmatrix} -M^{-1}(q)C(q,\dot{q}) & 0_{n\times n} & 0_{n\times p} \\ I_{n\times n} & 0_{n\times n} & 0_{n\times p} \\ 0_{p\times n} & 0_{p\times n} & 0_{p\times p} \end{pmatrix} \widetilde{x}(t) +$$

$$+ \begin{pmatrix} -\ddot{q}_r - M^{-1}(q)(G(q) + C(q,\dot{q})\dot{q}_r) \\ 0_{n\times n} \\ 0_{p\times n} \end{pmatrix} + \begin{pmatrix} M^{-1}(q)\tau \\ 0 \\ \dot{\widehat{\theta}}(t) \end{pmatrix} \tag{11}$$

or with shorter notation

$$\dot{\tilde{x}}(t) = A(q,\dot{q})\tilde{x}(t) + B_0(\ddot{q}_r,\dot{q}_r,\dot{q},q) + B_c(q;\tau,\dot{\hat{\theta}}) \tag{12}$$

where B_c contains the free control variables τ and $\dot{\hat{\theta}}$ available for assignment of the control law.

A Lyapunov function candidate

A Lyapunov function candidate must represent all relevant states of the investigated system. It must also increase with increasing magnitude of the state vector in all directions in the state space [5]. A quadratic function of $\tilde{x}$ fulfils these criteria. Introduce Lyapunov function candidates with the following composition scheme:

1: Choose the constant positive definite $n \times n$ matrices $P_{qq} = P_{qq}^T > 0$, $\Omega = \Omega^T > 0$, and the $p \times p$-matrix $P_{\theta\theta} = P_{\theta\theta}^T > 0$.

2: Compose the $(2n+p) \times (2n+p)$ weighting matrix

$$P_D(q) = \begin{bmatrix} M(q) & 0 & 0 \\ 0 & P_{qq} & 0 \\ 0 & 0 & P_{\theta\theta} \end{bmatrix} \tag{13}$$

3: Compose the full-rank $(2n+p) \times (2n+p)$ matrix U for state-space transformations

$$U = \begin{bmatrix} I_{n\times n} & P_{12} & 0_{n\times p} \\ 0_{n\times n} & I_{n\times n} & 0_{n\times p} \\ 0_{p\times n} & 0_{p\times n} & I_{p\times p} \end{bmatrix}; \qquad P_{12} = P_{qq}^{-1}\Omega \tag{14}$$

with $\dim U = \dim P_D = (2n+p) \times (2n+p)$

4: Choose the Lyapunov function candidate

$$V(\tilde{x}(t)) = \frac{1}{2}\tilde{x}^T(t)P_0(q)\tilde{x}(t); \qquad P_0(q) = U^T P_D(q) U \tag{15}$$

□

Lyapunov design

The time derivative of the Lyapunov function candidate is then

$$\dot{V}(\tilde{x}(t)) = \frac{1}{2}\tilde{x}^T(t)\dot{P}_0(q)\tilde{x}(t) + \tilde{x}^T(t)P_0(q)\dot{\tilde{x}}(t) \tag{16}$$

It is necessary to make $\dot{\tilde{x}}$ such that the function $\dot{V} \leq 0$ in order to obtain a stable system. The following lemma solves this problem.

LEMMA 1

Let $P_{qq}, P_{\theta\theta}, \Omega, D$ be positive definite matrices and define $P_{12} = P_{qq}^{-1}\Omega$. Let θ denote the vector of unknown parameters of (4). Let ψ, ψ_0 be defined from the relation

$$\psi(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r)\theta + \psi_0(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r) =$$

$$= -\frac{1}{2}\dot{M}(\dot{\tilde{q}} + P_{12}\tilde{q}) + M(q)(\ddot{q}_r - P_{12}\dot{\tilde{q}}) + C(q,\dot{q})\dot{q} + G(q); \qquad \dim\psi = n \times p \tag{17}$$

For any choice of $P_{qq} = P_{qq}^T > 0$, $P_{\theta\theta} = P_{\theta\theta}^T > 0$, $\Omega = \Omega^T > 0$, $D = D^T > 0$ there is an adaptive control law

$$\dot{\widehat{\theta}}(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r) = -P_{\theta\theta}^{-1}\psi^T(\dot{\tilde{q}} + P_{12}\tilde{q}) \tag{18}$$

$$\tau(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r; \widehat{\theta}) = \psi\widehat{\theta} + \psi_0 - (D + P_{qq}\Omega^{-1}P_{qq})(\dot{\tilde{q}} + P_{12}\tilde{q}) + P_{qq}\tilde{q} \tag{19}$$

such that the function (15) is a Lyapunov function with the time derivative

$$\dot{V} = -\begin{pmatrix} \dot{\tilde{q}}^T & \tilde{q}^T \end{pmatrix} Q \begin{pmatrix} \dot{\tilde{q}} \\ \tilde{q} \end{pmatrix}; \qquad Q = \begin{pmatrix} D + P_{qq}\Omega^{-1}P_{qq} & DP_{qq}^{-1}\Omega \\ \Omega P_{qq} D & \Omega P_{qq}^{-1} D P_{qq}^{-1}\Omega \end{pmatrix} > 0 \tag{20}$$

□

Proof: See appendix 1.

Remark:

The interpretation of P_{qq} is that of stiffness while D, Ω represent the damping terms. The matrices P_{qq}, D, Ω may be chosen independently for e.g. purposes of performance tuning.

THEOREM 1

The robot manipulator (4) with constant but unknown parameters θ together with the adaptive control law

$$\dot{\widehat{\theta}}(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r) = -P_{\theta\theta}^{-1}\psi^T(\dot{\tilde{q}} + P_{12}\tilde{q})$$

$$\tau(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r; \widehat{\theta}) = \psi\widehat{\theta} + \psi_0 - (D + P_{qq}\Omega^{-1}P_{qq})\dot{\tilde{q}} - DP_{qq}^{-1}\Omega\tilde{q} \tag{21}$$

is stabilized to its reference trajectories q_r with decreasing position and velocity errors $\tilde{q}$, $\dot{\tilde{q}}$ for any choice of the matrices $P_{qq} = P_{qq}^T > 0$, $P_{\theta\theta} = P_{\theta\theta}^T > 0$, $\Omega = \Omega > 0$, $D = D^T > 0$

The system is uniformly globally stable in the sense of Lyapunov. The manipulator link velocity and position errors $\dot{\tilde{q}}$, $\tilde{q}$ are L^2-stable and L^∞−stable if the reference trajectories $\dot{q}_r$, q_r are bounded. □

Proof:

The uniform stability in the sense of Lyapunov follows from the existence of a negative semidefinite Lyapunov function derivative as shown in lemma 1. The control law (21) can be calculated from (14), (18), and (19).

Finite initial conditions and q_r, $\dot{q}_r \in L^\infty$ mean that $V(\tilde{x}(0))$ is bounded. A finite value of the Lyapunov function V necessarily means a finite magnitude of the tracking errors $\tilde{q}$, $\dot{\tilde{q}}$. The L^∞-stability follows from the fact that the Lyapunov function can only decrease with time.

When initial conditions are bounded and $q_r, \dot{q}_r \in L^\infty$ it follows that V is bounded and

$$\int_0^\infty \begin{pmatrix} \dot{\tilde{q}}^T & \tilde{q}^T \end{pmatrix} Q \begin{pmatrix} \dot{\tilde{q}} \\ \tilde{q} \end{pmatrix} dt \leq \int_0^\infty -\dot{V}(\tilde{x}(t))dt \leq V(\tilde{x}(0)) < \infty \tag{22}$$

The L^2-stability follows because $\dot{V}$ is negative definite for non-zero tracking errors $\tilde{q}$, $\dot{\tilde{q}}$. □

Proposition 1

The derivative of V is negative definite for non-zero tracking errors $\tilde{q}$, $\dot{\tilde{q}}$. The system is therefore uniformly asymptotically stable with respect to the manipulator positions and velocities for constant parameters θ.

Proof: This follows from (20) and [5] (chapter 55, def. 55.2).

Remark The Lyapunov function derivative (20) is negative semidefinite with respect to $\tilde{x}$. No parameter convergence can be guaranteed.

A simulated example

We consider the two-link example from [2] (sec. 6.7) with masses m_1, m_2 [kg], lengths l_1, l_2 [m], angles q_1, q_2 [rad], and torques τ_1, τ_2 [Nm]. The end-effector load m_2 is assumed to vary rather drasticly. The equations are:

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q); \qquad \theta = m_2 \tag{23}$$

$$M(q) = \begin{pmatrix} m_2 l_2^2 + 2m_2 l_1 l_2 c_2 + (m_1 + m_2) l_1^2 & m_2 l_2^2 + m_2 l_1 l_2 c_2 \\ m_2 l_2^2 + m_2 l_1 l_2 c_2 & m_2 l_2^2 \end{pmatrix} \tag{24}$$

$$-\frac{1}{2}\dot{M}(q,\dot{q}) = \begin{pmatrix} m_2 l_1 l_2 s_2 \dot{q}_2 & \frac{1}{2} m_2 l_1 l_2 s_2 \dot{q}_2 \\ \frac{1}{2} m_2 l_1 l_2 s_2 \dot{q}_2 & 0 \end{pmatrix} \tag{25}$$

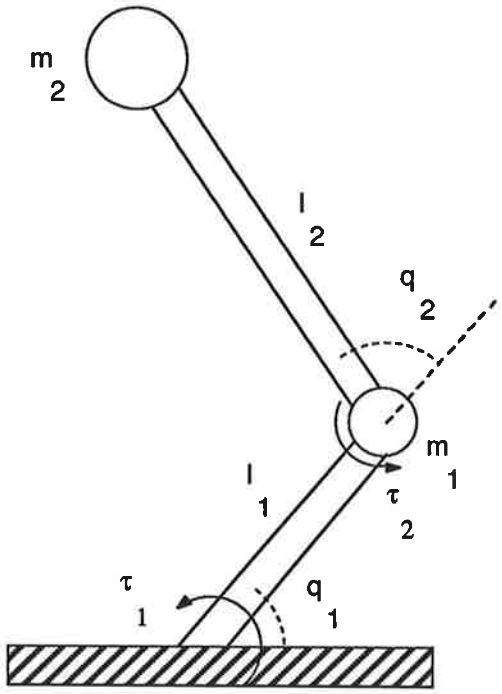

Figure 2. A two-link manipulator with masses m_1 and m_2.

$$C(q,\dot q) = \begin{pmatrix} -2m_2 l_1 l_2 s_2 \dot q_2 & -m_2 l_1 l_2 s_2 \dot q_2 \\ m_2 l_1 l_2 s_2 \dot q_1 & 0 \end{pmatrix} \tag{26}$$

$$G(q) = \begin{pmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{pmatrix} \tag{27}$$

with the short notation $c_2 = \cos(q_2)$, $c_{12} = \cos(q_1 + q_2)$ etc.

$$\psi = \begin{pmatrix} l_1^2 v_1 + l_2^2(v_1 + v_2) + l_1 l_2 c_2 (2v_1 + v_2) + l_1 l_2 s_2 \dot q_2 (u_1 + \frac{1}{2}u_2 - 2\dot q_1 - \dot q_2) + g(l_2 c_{12} + l_1 c_1) \\ (l_2^2 + l_1 l_2 c_2) v_1 + l_2^2 v_2 + l_1 l_2 s_2 (\dot q_1^2 + \frac{1}{2}\dot q_2 u_1) + + l_2 g c_{12} \end{pmatrix} \tag{28}$$

$$\psi_0 = \begin{pmatrix} m_1 l_1^2 v_1 + m_1 l_1 g c_1 \\ 0 \end{pmatrix} \tag{29}$$

$$u = \dot q + P_{12}\tilde q \qquad v = \ddot q_r - P_{12}\dot{\tilde q}; \tag{30}$$

Choose $P_{qq} = 5I_{2\times 2}$, $\Omega = 10I_{2\times 2}$, $D = 2.5I_{2\times 2}$, and $P_{\theta\theta} = 0.25$. The resulting control law is then

$$\dot{\hat\theta} = -4\psi^T \begin{pmatrix} \dot q_1 - \dot q_{1r} \\ \dot q_2 - \dot q_{2r} \end{pmatrix} - 8\psi^T \begin{pmatrix} q_1 - q_{1r} \\ q_2 - q_{2r} \end{pmatrix}$$

$$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \hat\theta + \begin{pmatrix} \psi_{01} \\ \psi_{02} \end{pmatrix} - 5 \begin{pmatrix} \dot q_1 - \dot q_{1r} \\ \dot q_2 - \dot q_{2r} \end{pmatrix} - 5 \begin{pmatrix} q_1 - q_{1r} \\ q_2 - q_{2r} \end{pmatrix} \tag{31}$$

$m_1 = 1$ [kg], $m_2 =?$ [kg], $l_1 = 1$ [m], $l_2 = 1$ [m]

The simulations were made with a sudden change from $m_2 = 1$ [kg] to $m_2 = 10$ [kg] at time $t = 1$ during the transient of recovery from the initial conditions, see Fig 3. Notice

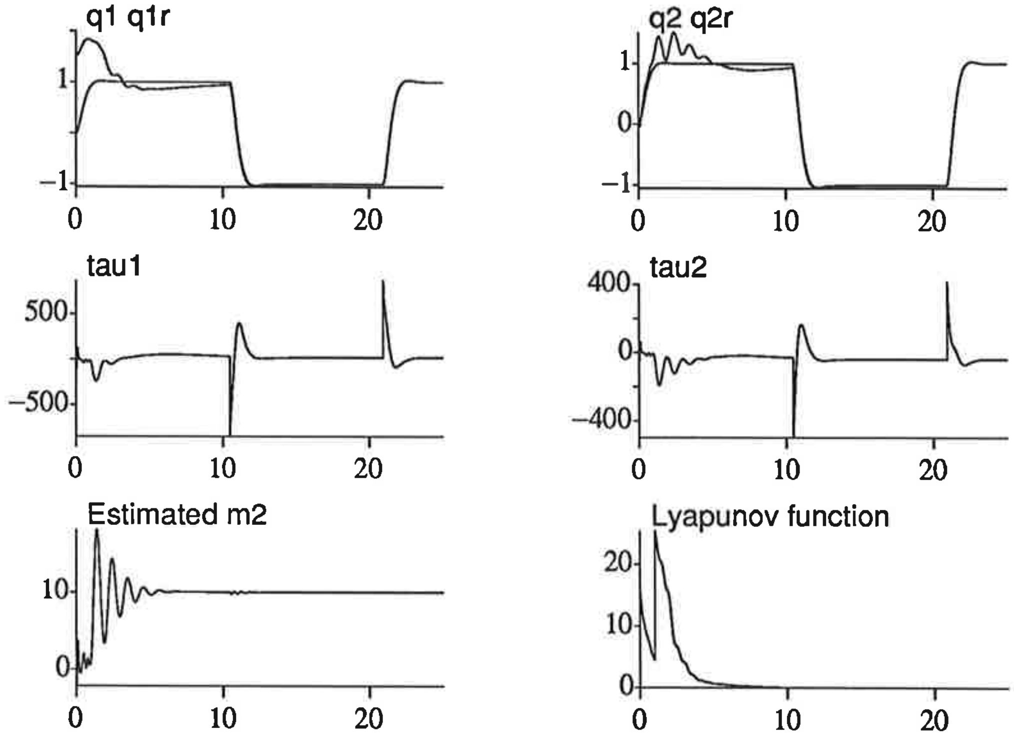

Figure 3. Simulation of the robot (23) with the adaptive control law (31). Upper graphs show q_1, q_{1r} and q_2, q_{2r}, respectively. Middle graphs show τ_1 and τ_2. Lower left graph shows the estimate $\widehat{\theta}$ of m_2. The lower right graph shows the Lyapunov function (15) that decreases everywhere except at time $t = 1$ [s] due to the sudden change of the payload m_2. All graphs vs. time [s].

that the Lyapunov function decreases everywhere except at time $t = 1$ due to the sudden change of the payload m_2.

When the accelerations are measurable

It was shown that the Lyapunov function decreases as long as there is an error in the velocity or position states of the manipulator compared to the reference trajectories. This is not necessarily valid for the parameter errors. Craig *et al* [3] showed a similar result although with stronger assumptions than here. Assume therefore that the acceleration is available for measurement as considered in [3]. We now show how to significantly improve the parameter stability and convergence properties.

It is then possible to combine the above adaptive control with prediction error estimation based on (8). Compute the prediction error

$$\varepsilon(\ddot{q}, \dot{q}, q; \widehat{\theta}) = \widehat{\tau}(\ddot{q}, \dot{q}, q) - \tau(\ddot{q}, \dot{q}, q) = \varphi(\ddot{q}, \dot{q}, q)\widetilde{\theta} \tag{32}$$

and add a term to the parameter estimation law (18) so that

$$\dot{\widehat{\theta}}(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r) = -P_{\theta\theta}^{-1}\psi^T(\dot{\widetilde{q}} + P_{12}\widetilde{q}) - P_{\theta\theta}^{-1}\varphi^T(\ddot{q}, \dot{q}, q)\varepsilon(\ddot{q}, \dot{q}, q; \widehat{\theta}) \tag{33}$$

Then the derivative of the Lyapunov function is modified to

$$\dot{V} = -\begin{pmatrix} \dot{\widetilde{q}}^T & \widetilde{q}^T \end{pmatrix} Q \begin{pmatrix} \dot{\widetilde{q}} \\ \widetilde{q} \end{pmatrix} - \varepsilon^T \varepsilon(\ddot{q}, \dot{q}, q; \widehat{\theta}); \qquad Q > 0 \tag{34}$$

Much better stability properties for the parameter estimation are thus accomplished. The Lyapunov function derivative $\dot{V}$ is now negative semidefinite with respect to non-zero parameter errors $\widetilde{\theta}$. There is now a global asymptotic stability if the $p \times p$-matrix $\varphi^T\varphi$ is positive definite.

Proposition 2

The robot manipulator (4) with the adaptive control law (21), (33) is asymptotically stable if

$$\varphi^T\varphi > 0 \tag{35}$$

Rank conditions on $\varphi^T\varphi$ set a constraint on the maximal number of identified parameters ($\dim \varphi^T\varphi = p \times p$) with possible global asymptotic stability to $p \leq n$. □

EXAMPLE 2

A reordering of (23) as shown in (8) gives

$$\varphi = \begin{pmatrix} (l_2^2 + 2l_1l_2c_2 + l_1^2)\ddot{q}_1 + (l_2^2 + l_1l_2c_2)\ddot{q}_2 - (2l_1l_2s_2\dot{q}_1\dot{q}_2 + l_1l_2s_2\dot{q}_2^2) + (l_2gc_{12} + l_1gc_1) \\ (l_2^2 + l_1l_2c_2)\ddot{q}_1 + l_2^2\ddot{q}_2 + l_1l_2s_2\dot{q}_1^2 + l_2gc_{12} \end{pmatrix} \tag{36}$$

and

$$\varphi_0 = \begin{pmatrix} m_1l_1^2\ddot{q}_1 + m_1l_1gc_1 \\ 0 \end{pmatrix} \tag{37}$$

The vector φ is zero only when no torques are applied. The matrix $\varphi^T\varphi$ is therefore never singular except when only gravitation torques affect the robot arm.

$$\tau = \varphi_0 = m_1l_1^2 \begin{pmatrix} \ddot{q}_1 + (g/l_1)c_1 \\ 0 \end{pmatrix} \tag{38}$$

The identification (33) with support of prediction error estimation is thus globally asymptotically stable whenever there are torques applied. A simulated example in Fig. 4 with $P_{qq} = 2I_{2\times2}$, $\Omega = 2I_{2\times2}$, $P_{\theta\theta} = 1$ shows improvement of performance compared to Fig 3.

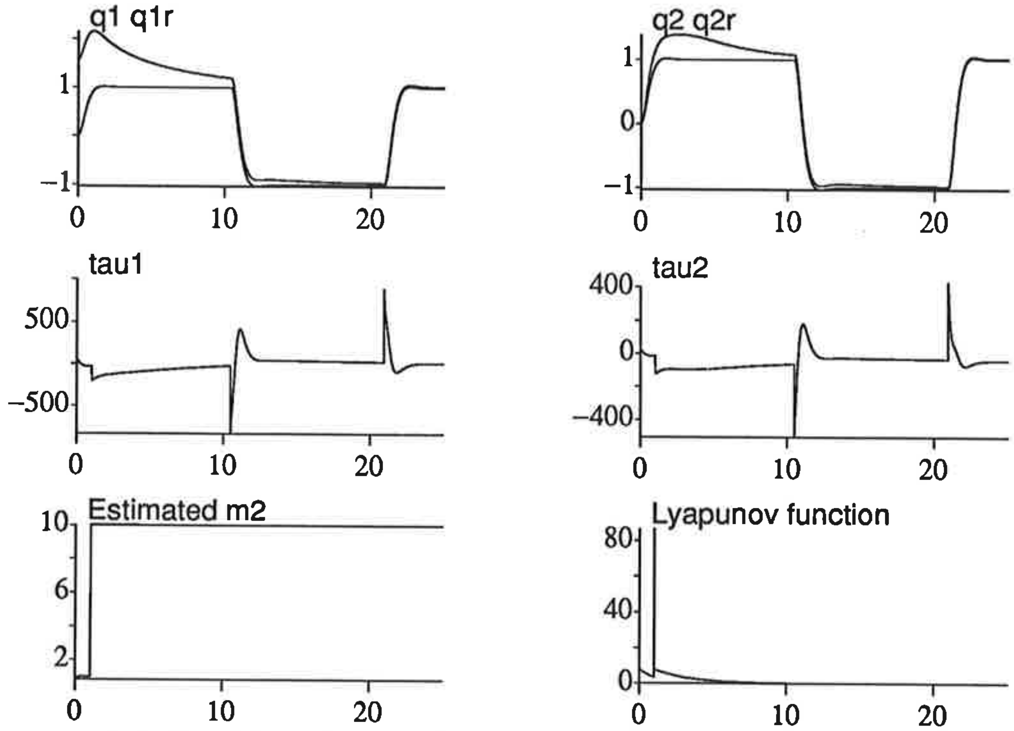

Figure 4. Simulation of the robot (23) with the adaptive control law (33). Upper graphs show q_1, q_{1r} and q_2, q_{2r}, respectively. Middle graphs show τ_1 and τ_2. Lower left graph shows the estimate $\widehat{\theta}$ of m_2. The lower right graph shows the Lyapunov function (15) The lower right graph shows the Lyapunov function (15) that decreases everywhere except at time $t = 1$ [s] due to the sudden change of the payload m_2. All graphs vs. time [s].

Discussion

There are two problems of adaptive control that often need separate treatments although the literature sometimes fails to distinguish between the two cases. Adaptive control of an object with constant but unknown parameters is different from the case when the system parameters have rapid variations. To remain stable for arbitrary parameter variations it is in general necessary to demand global asymptotic stability, *i.e.*, also parameter estimation stability. A factor $\dot{\theta}$ may otherwise result in a positive term of $\dot{V}$ and hence a system with limit cycles or instability. 'Slow' parameter variations may however often be sufficiently modelled as constant, unknown parameters. The results of Craig *et al* [3], Slotine and Li [13] , and theorem 1 of this paper all belong to this latter category. 'True' adaptive control with requirements both on stable regulation and parameter tracking for arbitrary variation in the system parameters is only met by the adaptive control law (19), (33).

The algorithms of this paper provide PD-type controllers with adaptation. The P- and D-actions can be chosen to provide damping and stiffness to assure good properties during adaptation transients as well as for non-adaptive operation. First, bandwidth or response time is determined by choice of P_{qq} while the damping is chosen with Ω and D. Regular tuning trade-offs between performance and noise sensitivity are applicable.

There is an important advantage compared to standard direct adaptive control. The choice of a closed-loop specification does not interfere with the parameter estimation. Re-adaptation is therefore not necessary for each new choice of closed-loop properties. The matrices $P_{qq}, P_{\theta\theta}, D, \Omega$ are significant for the choice of adaptation bandwidth but do not interfere with the closed-loop bandwidth which is determined by $\ddot{q}_r, \dot{q}_r, q_r$. Very rapid adaptation is thus possible.

Some non-standard features of adaptive control appear in this context: Structural knowledge is fully utilized so that only few parameters need identification. Partial knowledge of the nonlinear control object can thus be helpful to track rapidly varying parameters. Notice also that the estimation is based upon predictions of the control input, [7], and not of the output as is common in adaptive control of linear systems.

Conclusions

We have presented an adaptive control algorithm for robotic manipulators. Lyapunov functions and L^2−bounds are presented. Uniform global asymptotic stability with respect to the manipulator positions and velocities is guaranteed for constant, unknown parameters. The algorithm is suitable for rapid adaptation to rapidly changing system parameters. The identification may be supported by prediction error estimation when acceleration measurement is available. The adaptation properties improve considerably and we can show uniform parameter stability.

The manipulator operation bandwidth and the adaptation bandwidth may be chosen independently via the reference signal generator and the weighting matrices $P_{qq}, P_{\theta\theta}, D, \Omega$ of the algorithm. A new manipulator operation bandwidth may thus be chosen without necessary re-adaptation or other harmful interference. The matrices P_{qq} and D are performance tuning parameters that are free to choose with a guarantee of closed-loop stability. Rapid adaptation with damping as a stability safeguard is thus provided.

The contributions of this paper compared to earlier work are the following: Our assumptions are the same as in Slotine and Li [13] but our algorithm provides better stability properties with respect to both positions and velocities. Our algorithm contains the algorithm of [13] as a degenerate case ($\Omega \rightarrow 0$, $P_{qq} \rightarrow 0$). In contrast to Craig *et al* [3] we avoid requirements of acceleration measurements and positive real reference models.

Finally, with acceleration measurement available we have shown system stability including parameter stability. Stable, 'true' adaptive control of robots with system parameter variations is thus provided. A capability of accurate, rapid adaptation is demonstrated. A 'criterion' for uniform asymptotic system stability is given.

Appendix 1

$$\widetilde{x}^T(t) = \begin{pmatrix} \dot{\widetilde{q}}^T(t) & \widetilde{q}^T(t) & \widetilde{\theta}^T(t) \end{pmatrix}^T \tag{10}$$

Introduce the Lyapunov function candidate

$$V(\widetilde{x}(t)) = \frac{1}{2}\widetilde{x}^T(t)P_0(q)\widetilde{x}(t) \tag{15}$$

The time derivative of the Lyapunov function candidate is

$$\dot{V}(\widetilde{x}(t)) = \frac{1}{2}\widetilde{x}^T(t)\dot{P}_0(q)\widetilde{x}(t) + \widetilde{x}^T(t)P_0(q)\dot{\widetilde{x}}(t) \tag{A1.1}$$

The first term is

$$\frac{1}{2}\widetilde{x}^T(t)\dot{P}_0(q)\widetilde{x}(t) = (U\widetilde{x}(t))^T\dot{P}_D(q)(U\widetilde{x}(t))^T \tag{A1.2}$$

or explicitly

$$\frac{1}{2}\widetilde{x}^T(t)\dot{P}_0(q)\widetilde{x}(t) = (U\widetilde{x}(t))^T \begin{pmatrix} \frac{1}{2}\dot{M}(q) & 0_{n\times n} & 0_{n\times p} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times p} \\ 0_{p\times n} & 0_{p\times n} & 0_{p\times p} \end{pmatrix} (U\widetilde{x}(t)) \tag{A1.3}$$

and contains the time derivative of the inertia matrix $M(q)$. Further calculations give

$$\dot{V} = (U\widetilde{x})^T\Big((P_DUAU^{-1} + \dot{P}_D)(U\widetilde{x}) + P_DUB_0 + P_DUB_c\Big) \tag{A1.4}$$

where

$$P_DUAU^{-1} + \dot{P}_D = \begin{pmatrix} \frac{1}{2}\dot{M} - C + M(q)P_{12} & (C - M(q)P_{12})P_{12} & 0_{n\times p} \\ P_{qq} & -P_{qq}P_{12} & 0_{n\times p} \\ 0_{p\times n} & 0_{p\times n} & 0_{p\times p} \end{pmatrix} \tag{A1.5}$$

$$P_DUB_c = \begin{pmatrix} \tau \\ 0 \\ P_{\theta\theta}\dot{\widehat{\theta}} \end{pmatrix} \tag{A1.6}$$

$$P_DUB_0 = \begin{pmatrix} -M(q)\ddot{q}_r - G(q) - C(q,\dot{q})\dot{q}_r \\ 0_{n\times n} \\ 0_{p\times n} \end{pmatrix} \tag{A1.7}$$

Recall ψ, ψ_0 of (17) and define the functions

$$\tau_\theta(t) = \psi\widehat{\theta} + \psi_0 = -\frac{1}{2}\widehat{\dot{M}}(\dot{\widetilde{q}} + P_{12}\widetilde{q}) + \widehat{M}(q)(\ddot{q}_r - P_{12}\dot{\widetilde{q}}) + \widehat{C}(q,\dot{q})\dot{q} + \widehat{G}(q) \tag{A1.8}$$

$$\tau_c(t) = -(D + P_{qq}\Omega^{-1}P_{qq})(\dot{\widetilde{q}} + P_{12}\widetilde{q}) + P_{qq}\widetilde{q} \tag{A1.9}$$

Choose the control input as

$$\tau = \tau_\theta + \tau_c \tag{A1.10}$$

The torque is composed of two components τ_θ, τ_c . The first component is computed via the estimated unknown parameters while τ_c may be computed without these parameters. Consider the quantity

$$\tau_\theta - (\psi\theta + \psi_0) = \tau_\theta + \frac{1}{2}\dot{M}(\dot{\tilde{q}} + P_{12}\tilde{q}) - M(q)(\ddot{q}_r - P_{12}\dot{\tilde{q}}) - C(q,\dot{q})\dot{q} - G(q) =$$

$$= -\frac{1}{2}\widetilde{\dot{M}}(\dot{\tilde{q}} + P_{12}\tilde{q}) + \widetilde{M}(q)(\ddot{q}_r - P_{12}\dot{\tilde{q}}) + \widetilde{C}(q,\dot{q})\dot{q} + \widetilde{G}(q) = \psi(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r)\widetilde{\theta} \quad (A1.11)$$

Collecting terms ....

$$\dot{V} = (U\tilde{x})^T\Big((P_D U A U^{-1} + \dot{P}_D)(U\tilde{x}) + P_D U B_0 + P_D U B_c\Big) =$$

$$= -(U\tilde{x})^T \begin{pmatrix} D + P_{qq}\Omega^{-1}P_{qq} & -P_{qq} & 0_{n\times p} \\ -P_{qq} & \Omega & 0_{n\times p} \\ 0_{p\times n} & 0_{p\times n} & 0_{p\times p} \end{pmatrix} (U\tilde{x}) + (U\tilde{x})^T \begin{pmatrix} \psi(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r)\widetilde{\theta} \\ 0 \\ P_{\theta\theta}\dot{\widehat{\theta}} \end{pmatrix}$$

$$(A1.12)$$

The last term is eliminated if the adaptation law is chosen as

$$\dot{\widehat{\theta}}(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r) = -P_{\theta\theta}^{-1}\psi^T(\dot{\tilde{q}} + P_{12}\tilde{q}) \quad (A1.13)$$

so that for constant parameters θ it holds that

$$P_{\theta\theta}\dot{\widetilde{\theta}}(\ddot{q}_r, \dot{q}, \dot{q}_r, q, q_r) + \psi^T(\dot{\tilde{q}} + P_{12}\tilde{q}) = 0 \quad (A1.14)$$

and with $\tilde{x}$ of (7)

$$\dot{V} = -\tilde{x}^T \begin{pmatrix} D + P_{qq}\Omega^{-1}P_{qq} & DP_{qq}^{-1}\Omega & 0_{n\times p} \\ \Omega P_{qq}^{-1}D & \Omega P_{qq}^{-1}DP_{qq}^{-1}\Omega & 0_{n\times p} \\ 0_{p\times n} & 0_{p\times n} & 0_{p\times p} \end{pmatrix} \tilde{x} = \tilde{x}^T \begin{pmatrix} Q & 0_{2n\times p} \\ 0_{p\times 2n} & 0_{p\times p} \end{pmatrix} \tilde{x}$$

$$(A1.15)$$

It remains to show that the matrix Q of (17) is positive definite. Make the Cholesky factorization $\Omega = R^T R$, $D = L_1^T L_1$ with R, L_1 invertible. With

$$L = \begin{pmatrix} L_1 & L_1 P_{qq}^{-1}\Omega \\ -R^{-T}P_{qq} & 0_{n\times n} \end{pmatrix} \quad (A1.16)$$

one easily verifies that $Q = L^T L$. Since L is invertible the conclusion $Q > 0$ follows. The second equality follows from the definition (14) and $P_{12} = P_{qq}^{-1}\Omega$ so that

$$Q = Q^T = \begin{pmatrix} D + P_{qq}\Omega^{-1}P_{qq} & DP_{qq}^{-1}\Omega \\ \Omega P_{qq}^{-1}D & \Omega P_{qq}^{-1}DP_{qq}^{-1}\Omega \end{pmatrix} \quad (A1.17)$$

The robot (1) with the adaptation law (18) therefore results in a negative semidefinite Lyapunov function.

□

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