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An Adaptive Observer for Dynamical Ship Position Control Using Vectorial Observer Backstepping

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Abstract— **In this paper, we propose an adaptive observer for dynamically positioned ships, that can be used together with the controller shown by Fossen and Grøvlen [3], to design an observer-based adaptive control scheme. The resulting closed-loop system is globally asymptotically stable with respect to the ship positions and velocities, and globally stable with respect to the unknown parameters.**

I. INTRODUCTION

Fossen and Grøvlen proposed an observer-based backstepping method that allows the decomposition of nonlinear output feedback control into an observer and a state feedback control [3]. However, the observer design does not cover unstable ship dynamics, and an extension for these cases has been proposed [2], under a detectability condition. The adaptive observer proposed is a modified version of the reduced-order observer proposed by Erlic and Lu [1] for manipulator control, and does not require any condition for its application, except a bound for the unknown parameters. In this case, however, a full-order observer is required, in order to have a good filtering of x and y, which are measured by DGPS, with a noise in the range of 1-3 [m]. The yaw angle ψ is assumed to be measured by using a gyro compass, which is quite accurate (the noise being less than 0.1 [deg]). Furthermore, the proposed adaptive observer permits implementation of an adaptive version of the control law proposed in [3]. In this paper, we will show derivation of an observer for output feedback control of the ship positioning dynamics considered by Fossen and Grøvlen [3], [2]. An adaptive version of the observer will be considered.

II. PROBLEM FORMULATION AND ASSUMPTIONS

We use system models and problem formulations from [3], [2].

Ship Model and Properties: The earth-fixed positions (x, y) and yaw angle ψ of the vessel is expressed in vector form as $\eta = [x, y, \psi]^T$, and the body-fixed velocities are represented by the vector $\nu = [u, v, r]^T$. The elements in η and ν describe the surge, sway, and yaw modes, respectively. Using the problem formulation from [3], we have the following system model

$$\begin{aligned} \dot{\eta} &= J(\eta)\nu & (1)\\ M\dot{\nu} + D\nu + K\eta &= \tau, & (2) \end{aligned}$$

with the Jacobian matrix

$$\begin{aligned} J(\eta) &= \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} & (3)\\ &= e^{\boldsymbol{J}\psi}, \quad \boldsymbol{J} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & (4) \end{aligned}$$

and the inertia matrix

$$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix} > 0 \qquad (5)$$

and positive definite matrices

$$\begin{aligned} D &= \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}, & (6)\\ K &= \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix} & (7) \end{aligned}$$

$J(\eta)$ is the yaw rotation matrix, M is the inertia matrix, K represents the mooring forces and τ is the control vector of forces from the thruster system [3]. We suppose that some parameters of the matrices M, D, K are unknown but constant, and that positions η only are available to measurement. For purposes of adaptation, it is suitable to reformulate Eq. (2) as

$$M\dot{\nu} + D\nu + K\eta = \tau = \varphi_0(\dot{\nu}, \nu, \eta) + \varphi(\dot{\nu}, \nu, \eta)\theta \qquad (8)$$

where $\theta \in \mathbb{R}^p$ is the unknown parameter vector, supposing bounds for M, D and K, to be known, that is

$$\begin{aligned} 0 &< M_{min} < \|M\| < M_{max} & (9)\\ 0 &< D_{min} < \|D\| < D_{max} & (10)\\ 0 &< \|K\| < K_{max} & (11) \end{aligned}$$

Note that $J^{-1}(\eta) = J^T(\eta)$, and $\|J(\eta)\| = 1$.

Remark 1: As in [2], [3], Eq. (2) can be rewritten as

$$\dot{\nu} = A_1\eta + A_2\nu + B\tau \tag{12}$$

where $A_1 = -M^{-1}K, \quad A_2 = -M^{-1}D, \quad B = M^{-1}$, but, from a viewpoint of parameter identification, Eq. (2) is a better description of the system. If, for instance, only the inertia matrix M is unknown, using (2) we shall have to estimate only the matrix M, but using (12) we shall have to estimate A_1, A_2 and B, because all these matrices contain M.

III. Observer Design and Stability Analysis

We propose the following adaptive observer for the system (1) and (2):

$$\begin{aligned} \dot{\widehat{\eta}} &= J(\eta)\widehat{\nu} + K_1(\eta - \widehat{\eta}) && (13)\\ \dot{\widehat{\nu}} &= \widehat{M}^{-1}(\tau - \widehat{D}\widehat{\nu} - \widehat{K}\widehat{\eta}) + K_2(\nu - \widehat{\nu}) && (14)\\ \dot{\widehat{\theta}} &= -\Gamma\varphi^T(\xi, \widehat{\nu}, \widehat{\eta})[\nu - \widehat{\nu}] && (15) \end{aligned}$$

where $\widehat{\eta}, \widehat{\nu}, \widehat{\theta}$ are the position, velocity, and parameter estimates, respectively, $\Gamma = \Gamma^T > 0$ a gain matrix

$$\xi(\widehat{\eta}, \widehat{\nu}, \widehat{\theta}, \tau) = \widehat{M}^{-1}(\tau - \widehat{D}\widehat{\nu} - \widehat{K}\widehat{\eta}),\ K_1 > 0,\ K_2 > 0$$

K_1, K_2 being constant gain matrices. Subtracting (13) from (1), and (14) from (2), we have the observation error dynamics

$$\begin{aligned} \dot{\widetilde{\eta}} &= J(\eta)\widetilde{\nu} - K_1\widetilde{\eta} && (16)\\ M\dot{\widetilde{\nu}} &= -\widetilde{M}(\dot{\widehat{\nu}} - K_2\widetilde{\nu}) - \widetilde{D}\widehat{\nu} - \widetilde{K}\widehat{\eta} \\ &- D\widetilde{\nu}K\widetilde{\eta} - MK_2\widetilde{\nu} \end{aligned}$$

where $\widetilde{\eta} = \eta - \widehat{\eta}$ and $\widetilde{\nu} = \nu - \widehat{\nu}$ are the position and velocity estimation errors, respectively, and $\widetilde{M} = M - \widehat{M}, \quad \widetilde{D} = D - \widehat{D}, \quad \widetilde{K} = K - \widehat{K}$. Let us define the parameter estimation error $\widetilde{\theta} = \theta - \widehat{\theta}$ and consider the following Lyapunov function candidate

$$V(\widetilde{\eta}, \widetilde{\nu}, \widetilde{\theta}) = \frac{1}{2}(\widetilde{\eta}^T\widetilde{\eta} + \widetilde{\nu}^T M\widetilde{\nu} + \widetilde{\theta}^T\Gamma^{-1}\widetilde{\theta}) > 0 \tag{17}$$

its time derivative along the solutions of Eq. (16) being

$$\begin{aligned} \dot{V} &= \widetilde{\eta}^T\dot{\widetilde{\eta}} + \widetilde{\nu}^T M\dot{\widetilde{\nu}} + \widetilde{\theta}^T\Gamma^{-1}\dot{\widetilde{\theta}} \\ &= -\widetilde{\eta}^T K_1\widetilde{\eta} - \widetilde{\nu}^T(D + MK_2)\widetilde{\nu} + \widetilde{\eta}^T(J(\eta) - K)\widetilde{\nu} \\ &- \widetilde{\nu}^T(\widetilde{M}\xi + \widetilde{D}\widehat{\nu} + \widetilde{K}\widehat{\eta}) + \widetilde{\theta}^T\Gamma^{-1}\dot{\widetilde{\theta}}. && (18) \end{aligned}$$

Using the property (8) and noting that $\dot{\widetilde{\theta}} = -\dot{\widehat{\theta}}$ for constant parameters, (18) becomes

$$\begin{aligned} \dot{V} &= -\widetilde{\eta}^T K_1\widetilde{\eta} - \widetilde{\nu}^T(MK_2 + D)\widetilde{\nu} + \widetilde{\eta}^T(J(\eta) - K)\widetilde{\nu} \\ &- \widetilde{\theta}^T(\varphi^T(\xi, \widehat{\nu}, \widehat{\eta})\widetilde{\nu} + \Gamma^{-1}\dot{\widehat{\theta}}), && (19) \end{aligned}$$

and furthermore, using Eq. (15) and assumptions (9), (10) and (11), we have

$$\begin{aligned} \dot{V} &\le -\underline{\sigma}_1\|\widetilde{\eta}\|^2 - (\underline{\sigma}_2 + D_{min})\|\widetilde{\nu}\|^2 \\ &+ (1 + K_{max})\|\widetilde{\eta}\|\|\widetilde{\nu}\|, && (20) \end{aligned}$$

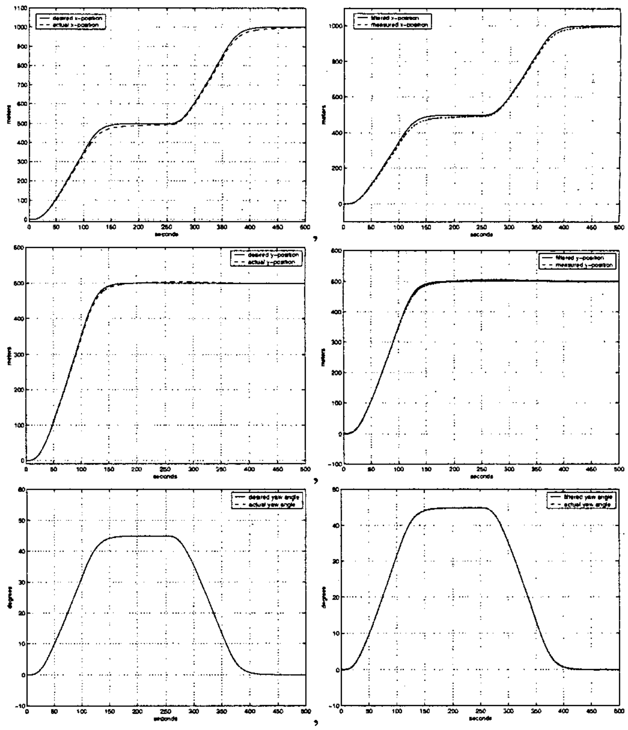

Fig. 1. DP of a supply vessel: tracking of a time-varying reference trajectory (left); DP of a supply vessel: measured and filtered positions (right)

where $\underline{\sigma}_1 = \lambda_{min}(K_1)$ and $\underline{\sigma}_2 = \lambda_{min}(K_2^T M^T + MK_2)/2$. Rewriting Eq. (20) as

$$\dot{V} \le -[\|\widetilde{\eta}\|, \|\widetilde{\nu}\|]Q(\underline{\sigma}_1, \underline{\sigma}_2)[\|\widetilde{\eta}\|, \|\widetilde{\nu}\|]^T, \tag{21}$$

it can be verified readily that Q is positive definite if

$$\underline{\sigma}_1 > \frac{(1 + K_{max})^2}{4(\underline{\sigma}_2 + D_{min})}, \quad \underline{\sigma}_2 > 0 \tag{22}$$

and in this case we have global asymptotic stability with respect to the ship positions and velocities, and global stability with respect to the unknown parameters.

Remark 2: The observer (13), (14) and (15) is not directly implementable because of the presence of the unknown signal ν into the equations (14) and (15). However, a discrete-time approximation of the above observer can be implemented as shown in Appendix A.

Observer Backstepping

Referring to [3], we define a smooth reference trajectory $\eta_d = [x_d, y_d, \psi_d]^T$ satisfying

$$\ddot{\eta}_d, \dot{\eta}_d, \eta_d \in \mathcal{L}_\infty. \tag{23}$$

Since the measurement of η is affected by sensor noise and the observer guarantees that $\widehat{\eta} \to \eta$, the tracking error $\eta - \eta_d$

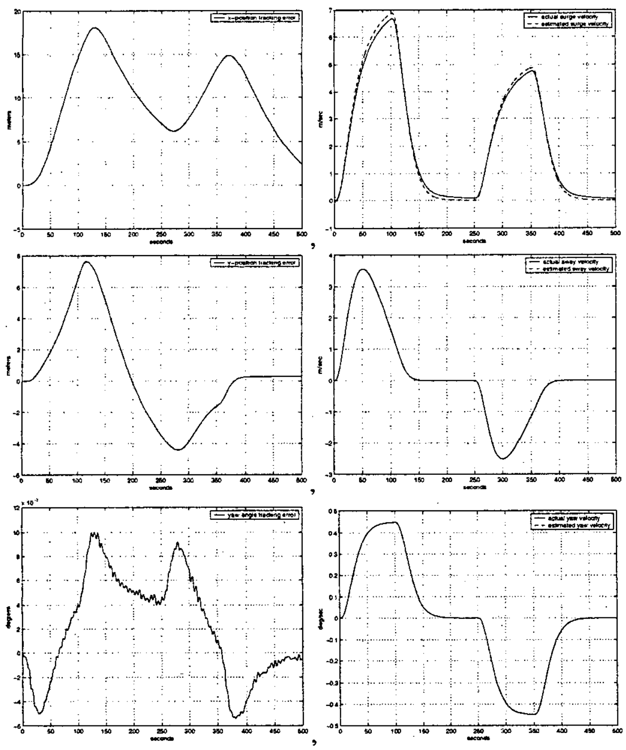

Fig. 2. DP of a supply vessel: tracking errors (left); actual and estimated velocities (right).

is replaced by $\widehat{\eta} - \eta_d$, and is used for vectorial observer backstepping. Defining $z_1 = \widehat{\eta} - \eta_d$ we have

$$\dot{z}_1 = J(\eta)\widehat{\nu} + K_1\widetilde{\eta} - \dot{\eta}_d. \tag{24}$$

The main idea of backstepping is to choose one of the state variables as virtual control. It turns out that

$$\xi_1 = J(\eta)\widehat{\nu} = z_2 + \alpha_1 \tag{25}$$

is an appropriate choice for the virtual control, ξ_1 being defined as the sum of the next error variable z_2, and α_1 interpreted as a stabilizing function. Hence

$$\dot{z}_1 = z_2 + \alpha_1 + K_1\widetilde{\eta} - \dot{\eta}_d. \tag{26}$$

We choose the following stabilizing function

$$\alpha_1 = -C_1 z_1 - D_1 z_1 + \dot{\eta}_d \tag{27}$$

where C_1 is a constant strictly positive feedback design matrix, usually diagonal, and D_1 is a positive diagonal damping matrix defined as

$$D_1 = \begin{bmatrix} d_1 k_1^T k_1 & 0 & 0 \\ 0 & d_2 k_2^T k_2 & 0 \\ 0 & 0 & d_3 k_3^T k_3 \end{bmatrix} \tag{28}$$

where $d_i > 0\,(i = 1\ldots3)$, and $k_i\,(i = 1\ldots3)$ are the column vectors of $K_1^T = [k_1, k_2, k_3]$. The damping term $-D_1 z_1$ has been added because $K_1\widetilde{\eta}$ in (24) can be treated as a disturbance term to be compensated for. Then we can write

$$\dot{z}_1 = -(C_1 + D_1)z_1 + z_2 + K_1\widetilde{\eta}. \tag{29}$$

To specify the desired dynamics of z_2, we have from Eq. (25)

$$\begin{aligned} \dot{z}_2 &= \dot{\xi}_1 - \dot{\alpha}_1 = J(\eta)\dot{\widehat{\nu}} + \dot{J}(\eta)\widehat{\nu} + (C_1 + D_1)\dot{z}_1 - \ddot{\eta}_d \\ &= -(C_1 + D_1)^2 z_1 + (C_1 + D_1)(z_2 + K_1\widetilde{\eta}) \\ &- \ddot{\eta}_d + \dot{J}(\eta)\widehat{\nu} \\ &+ J(\eta)(-\widehat{M}^{-1}\widehat{K}\widehat{\eta} - \widehat{M}^{-1}\widehat{D}\widehat{\nu} + \widehat{M}^{-1}\tau + K_2\widetilde{\nu}) \end{aligned} \tag{30}$$

Defining

$$\rho = \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix}, \quad S(\rho) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{31}$$

and $\widetilde{\rho} = \rho - \widehat{\rho}$, we can write

$$\dot{J}(\eta) = J(\eta)S(\rho) = J(\eta)S(\widetilde{\rho}) + J(\eta)S(\widehat{\rho}) \tag{32}$$

and

$$\begin{aligned} \dot{J}(\eta)\widehat{\nu} &= J(\eta)S(\widetilde{\rho})\widehat{\nu} + J(\eta)S(\widehat{\rho})\widehat{\nu} \\ &= J(\eta)T(\widehat{\nu})\widetilde{\nu} + J(\eta)S(\widehat{\rho})\widehat{\nu} \end{aligned} \tag{33}$$

where

$$T(\widehat{\nu}) = \begin{bmatrix} 0 & 0 & -\widehat{v} \\ 0 & 0 & \widehat{u} \\ 0 & 0 & 0 \end{bmatrix}. \tag{34}$$

Substituting (33) into (30), yields

$$\begin{aligned} \dot{z}_2 &= -(C_1 + D_1)^2 z_1 + (C_1 + D_1)(z_2 + K_1\widetilde{\eta}) \\ &- \ddot{\eta}_d + J(\eta)T(\widehat{\nu})\widetilde{\nu} + J(\eta)S(\widehat{\rho})\widehat{\nu} \\ &+ J(\eta)(-\widehat{M}^{-1}\widehat{K}\widehat{\eta} - \widehat{M}^{-1}\widehat{D}\widehat{\nu} + \widehat{M}^{-1}\tau + K_2\widetilde{\nu}). \end{aligned} \tag{35}$$

Now we choose the control law as follows

$$\begin{aligned} \tau &= -\widehat{M}J^T(\eta)\Big[-(C_1 + D_1)^2 z_1 + (C_1 + D_1)z_2 \\ &- \ddot{\eta}_d + C_2 z_2 + D_2 z_2 + z_1\Big] \\ &- \widehat{M}S(\widehat{\rho})\widehat{\nu} + \widehat{K}\widehat{\eta} + \widehat{D}\widehat{\nu}, \end{aligned} \tag{36}$$

where C_2 is a constant strictly positive feedback design matrix, usually diagonal. Substituting (36) into (35), we have

$$\dot{z}_2 = -C_2 z_2 - D_2 z_2 - z_1 + \Omega_1\widetilde{\eta} + \Omega_2\widetilde{\nu} \tag{37}$$

where

$$\Omega_1 = (C_1 + D_1)K_1 \tag{38}$$

$$\Omega_2 = J(\eta)(T(\widehat{\nu}) + K_2) \tag{39}$$

The damping matrix D_2 is defined in terms of Ω_1 and Ω_2 as

$$\begin{aligned} D_2 = \ & \mathrm{diag}[d_4(\omega_1^T\omega_1 + \omega_4^T\omega_4), \quad d_5(\omega_2^T\omega_2 + \omega_5^T\omega_5), \\ & d_6(\omega_3^T\omega_3 + \omega_6^T\omega_6)] \end{aligned} \tag{40}$$

where $\Omega_1^T = [\omega_1, \omega_2, \omega_3]$, $\Omega_2^T = [\omega_4, \omega_5, \omega_6]$ and $d_i > 0\,(i = 4\ldots6)$.

IV. Closed-Loop Stability Analysis

We can write the error dynamics as

$$\begin{aligned}\dot{z} &= -(C_z + D_z + E)z + W_1\widetilde{\eta} + W_2\widetilde{\nu} &(41)\\ \dot{\widetilde{\eta}} &= J(\eta)\widetilde{\nu} - K_1\widetilde{\eta} &(42)\\ M\dot{\widetilde{\nu}} &= -\widetilde{M}(\dot{\widehat{\nu}} - K_2\widetilde{\nu}) - \widetilde{D}\widehat{\nu} - \widetilde{K}\widehat{\eta}\\ &- D\widetilde{\nu} - K\widetilde{\eta} - MK_2\widetilde{\nu} &(43)\end{aligned}$$

where

$$\begin{aligned} z &= \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad C_z = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} &(44)\\ D_z &= \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} &(45)\\ W_1 &= \begin{bmatrix} K_1 \\ \Omega_1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0 \\ \Omega_2 \end{bmatrix}. &(46)\end{aligned}$$

Consider the following Lyapunov function candidate

$$V(z,\widetilde{\eta},\widetilde{\nu},\widetilde{\theta}) = \frac{1}{2}(z^T z + \widetilde{\eta}^T\widetilde{\eta} + \widetilde{\nu}^T M\widetilde{\nu} + \widetilde{\theta}^T\Gamma^{-1}\widetilde{\theta}) \quad (47)$$

its time derivative along the solutions of (41), (42) and (43) is

$$\begin{aligned}\dot{V} &= -z^T C_z z - z^T D_z z + z^T W_1\widetilde{\eta} + z^T W_2\widetilde{\nu}\\ &- \widetilde{\eta}^T K_1\widetilde{\eta} - \widetilde{\nu}^T(MK_2 + D)\widetilde{\nu} + \widetilde{\eta}^T(J(\eta) - K)\widetilde{\nu}\\ &- \widetilde{\theta}^T(\varphi^T(\xi,\widehat{\nu},\widehat{\eta})\widetilde{\nu} + \Gamma^{-1}\dot{\widehat{\theta}}) &(48)\end{aligned}$$

where we have used the fact that $z^T E z = 0$. Now, using (15), and adding the zero terms

$$\begin{aligned}\frac{1}{4}(\widetilde{\eta}^T G_1\widetilde{\eta} - \widetilde{\eta}^T G_1\widetilde{\eta}) &= 0 &(49)\\ \frac{1}{4}(\widetilde{\nu}^T G_2\widetilde{\nu} - \widetilde{\nu}^T G_2\widetilde{\nu}) &= 0 &(50)\end{aligned}$$

(48) becomes

$$\begin{aligned}\dot{V} &= -z^T C_z z - z^T D_z z + z^T W_1\widetilde{\eta} + z^T W_2\widetilde{\nu}\\ &- \frac{1}{4}(\widetilde{\eta}^T G_1\widetilde{\eta} + \widetilde{\nu}^T G_2\widetilde{\nu}) - \widetilde{\eta}^T(K_1 - \frac{1}{4}G_1)\widetilde{\eta}\\ &- \widetilde{\nu}^T(MK_2 + D - \frac{1}{4}G_2)\widetilde{\nu}\\ &+ \widetilde{\eta}^T(J(\eta) - K)\widetilde{\nu}. &(51)\end{aligned}$$

Defining the matrices G_1 and G_2 as

$$G_1 = g_1 I, \quad G_2 = g_2 I \quad (52)$$

where

$$g_1 = \sum_{i=1}^{6}\frac{1}{d_i}, \quad g_2 = \sum_{i=1}^{3}\frac{1}{d_{i+3}} \quad (53)$$

As shown in Appendix B, the quadratic form

$$\begin{aligned} Q &= -z^T D_z z + z^T W_1\widetilde{\eta} + z^T W_2\widetilde{\nu}\\ &- \frac{1}{4}(\widetilde{\eta}^T G_1\widetilde{\eta} + \widetilde{\nu}^T G_2\widetilde{\nu}) \le 0 &(54)\end{aligned}$$

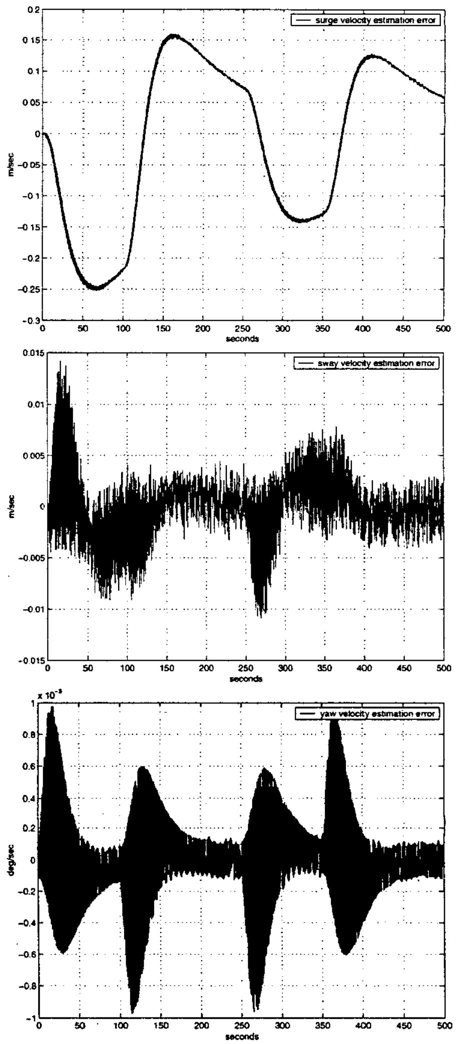

Fig. 3. DP of a supply vessel: velocity estimation errors

Hence, Eqs. (51)-(54) and we can write

$$\begin{aligned}\dot{V} &\le -z^T C_z z - \widetilde{\eta}^T(K_1 - \frac{1}{4}G_1)\widetilde{\eta} &(55)\\ &- \widetilde{\nu}^T(MK_2 + D - \frac{1}{4}G_2)\widetilde{\nu} + \widetilde{\eta}^T(J(\eta) - K)\widetilde{\nu}\end{aligned}$$

and using assumptions (9), (10), (11) we have

$$\begin{aligned}\dot{V} &\le -z^T C_z z - (\underline{\sigma}_1 - \frac{1}{4}g_1)\|\widetilde{\eta}\|^2 &(56)\\ &- (\underline{\sigma}_2 + D_{min} - \frac{1}{4}g_2)\|\widetilde{\nu}\|^2 + (1 + K_{max})\|\widetilde{\eta}\|\|\widetilde{\nu}\|\\ &= -z^T C_z z - [\|\widetilde{\eta}\|, \|\widetilde{\nu}\|]\bar{Q}(\underline{\sigma}_1, \underline{\sigma}_2)[\|\widetilde{\eta}\|, \|\widetilde{\nu}\|]^T.\end{aligned}$$

It can be verified that $\bar{Q}$ is positive definite if

$$\underline{\sigma}_1 > \frac{1}{4}g_1 + \frac{(1+K_{max})^2}{4(\underline{\sigma}_2 + D_{min} - \frac{1}{4}g_2)} \quad (57)$$

$$\underline{\sigma}_2 > max\Big[0, \quad (\frac{1}{4}g_2 - D_{min})\Big] \quad (58)$$

and in this case we have global asymptotic stability with respect to the ship positions and velocities, and global stability with respect to the unknown parameters.

Remark 3: Using (65), (66) and (67) for the implementation of the adaptive observer, the implementation of controller (36) involves simply the calculation of $\tau(t)$ at time instant $t = i\Delta$.

V. Simulation Results

To show the performance of the proposed adaptive observer-controller, we consider the case of dynamic positioning of an off-shore supply vessel [4, Fig.8], described by

$$M = \begin{bmatrix} 5.3122 \cdot 10^6 & 0 & 0 \\ 0 & 8.2831 \cdot 10^6 & 0 \\ 0 & 0 & 3.7454 \cdot 10^9 \end{bmatrix}$$
$$D = \begin{bmatrix} 5.024 \cdot 10^4 & 0 & 0 \\ 0 & 2.7229 \cdot 10^5 & -4.399 \cdot 10^6 \\ 0 & -4.399 \cdot 10^6 & 4.1894 \cdot 10^8 \end{bmatrix}$$
$$K = 0 \quad (59)$$

We suppose that the inertial parameter m_{11} is unknown, that is $\theta = m_{11}$. The observer-controller parameters are chosen according to

$$K_1 = 10^{-3}I, \quad K_2 = 10^{-3}I, \quad \Gamma = 10^3,$$
$$C_1 = 0.1I, \quad C_2 = 0.1I, \quad d_i = 0.1\,(i = 1, \ldots, 6).$$

Reference trajectories are generated by using a third-order filter with poles in -0.1, that is

$$F(s) = \frac{0.1^3}{(s+0.1)^3} \quad (60)$$

Furthermore, the sampling time Δ is 0.1 [s], and white noise is added to the measurements in order to illustrate the filtering properties of the observer. Results in Figs. 1–4 show a good performance of the proposed adaptive observer-based controller.

VI. Discussion

The approach presented here aimed towards an extension of Fossen and Grøvlen [3] covering the case of unstable ship dynamics, parameter uncertainties and smooth time-varying parameters. As the stability analysis of the the system under influence of disturbances is not complete, this objective is not quite fulfilled. It remains to show effects of measurement noise with changes in the stability analysis of the closed-loop system and the observer.

Another issue is the effect of discretization and discrete approximation with effects on stability and performance. Limits to stability of the discrete approximation remain to be analyzed.

VII. Conclusion

In this paper an adaptive observer has been proposed and combined with an adaptive controller for dynamically positioned ship control. Global asymptotic stability of both the observer and the control law and global stability of the parameter update law have been proven by applying Lyapunov stability theory. In order to have a good filtering of noisy position measurements, a full-order observer has been used. Although only an approximate implementation of the proposed adaptive observer-controller is possible, this solution overcomes the difficulties in designing adaptive observers for nonlinear systems in which the unknown parameters and the unmeasured states are coupled. Therefore, the approximated implementation of this control scheme approaches the real one as the sampling interval approaches zero. The proposed adaptive observer does not require any conditions for its application, except a bound for the unknown parameters. In particular it is an extension of the scheme proposed in [3], as it covers unstable ship dynamics, parameter uncertainties and smooth time-varying parameters. Furthermore simulation results show good filtering and tracking properties also in presence of highly noise contaminated measurements.

Appendix A—Discrete-Time Approximation of the Adaptive Observer

Integrating (13), (14) and (15), we have

$$\begin{aligned}
\widehat{\eta}(t) &= \widehat{\eta}(t_0) + \int_{t_0}^{t} [J(\eta)\widehat{\nu} + K_1(\eta - \widehat{\eta})]\mathrm{d}t \\
\widehat{\nu}(t) &= \widehat{\nu}(t_0) + \int_{t_0}^{t} [\xi(\widehat{\eta}, \widehat{\nu}, \widehat{\theta}, \tau) - K_2\widehat{\nu}]\mathrm{d}t \\
&+ \int_{\eta(t_0)}^{\eta(t)} K_2 J^T(\eta)\mathrm{d}\eta \qquad (61)\\
\widehat{\theta}(t) &= \widehat{\theta}(t_0) + \Gamma\int_{t_0}^{t} \varphi^T(\xi, \widehat{\nu}, \widehat{\eta})\widehat{\nu}\mathrm{d}t \\
&- \Gamma\int_{\eta(t_0)}^{\eta(t)} \varphi^T(\xi, \widehat{\nu}, \widehat{\eta})J^T(\eta)\mathrm{d}\eta,
\end{aligned}$$

and replacing t_0 with $t - \Delta$, $\Delta > 0$, we can write

$$\widehat{\eta}(t) = \widehat{\eta}(t-\Delta) + \int_{t-\Delta}^{t} [J(\eta)\widehat{\nu} + K_1(\eta - \widehat{\eta})]\mathrm{d}t \quad (62)$$

$$\begin{aligned}
\widehat{\nu}(t) &= \widehat{\nu}(t-\Delta) + \int_{t-\Delta}^{t} [\xi(\widehat{\eta}, \widehat{\nu}, \widehat{\theta}, \tau) - K_2\widehat{\nu}]\mathrm{d}t \\
&+ \int_{\eta(t-\Delta)}^{\eta(t)} K_2 J^T(\eta)\mathrm{d}\eta \qquad (63)
\end{aligned}$$

$$\begin{aligned}
\widehat{\theta}(t) &= \widehat{\theta}(t-\Delta) + \Gamma\int_{t-\Delta}^{t} \varphi^T(\xi, \widehat{\nu}, \widehat{\eta})\widehat{\nu}\mathrm{d}t \\
&- \Gamma\int_{\eta(t-\Delta)}^{\eta(t)} \varphi^T(\xi, \widehat{\nu}, \widehat{\eta})J^T(\eta)\mathrm{d}\eta. \qquad (64)
\end{aligned}$$

Assuming that Δ is sufficiently small, (62), (63) and (64) suggest a discrete implementation of the proposed observer as follows

$$\begin{aligned}
\widehat{\eta}(i) &= \widehat{\eta}(i-1)+ \\
&+ \Delta(J(i-1)\widehat{\nu}(i-1)+K_1\widetilde{\eta}(i-1)) \quad (65)\\
\widehat{\nu}(i) &= (I-\Delta K_2)\widehat{\nu}(i-1)+\Delta\xi(i-1) \\
&+ K_2 J^T(i-1)(\eta(i)-\eta(i-1)) \quad (66)\\
\widehat{\theta}(i) &= \widehat{\theta}(i-1)+\Gamma\varphi^T(i-1)[\Delta\widehat{\nu}(i-1) \\
&- J^T(i-1)(\eta(i)-\eta(i-1))] \quad (67)
\end{aligned}$$

Remark 4: Obviously (65), (66) and (67) are only an approximation of the proposed observer (13), (14) and (15). However, they are implementable and stand for a good representation of the observer if the sampling interval Δ is sufficiently small.

APPENDIX B—PROOF OF INEQUALITY (54)

Consider the quadratic form of Eq. (54) expanded as

$$\begin{aligned}
Q = &- z^T D_z z + z^T W_1\widetilde{\eta} + z^T W_2\widetilde{\nu} \\
&- \frac{1}{4}(\widetilde{\eta}^T G_1\widetilde{\eta} + \widetilde{\nu}^T G_2\widetilde{\nu}) \\
&= -z_1^T D_1 z_1 - z_2^T D_2 z_2 \quad (68)\\
&+ z_1^T K_1\widetilde{\eta} + z_2^T\Omega_1\widetilde{\eta} + z_2^T\Omega_2\widetilde{\nu} - \frac{g_1}{4}\widetilde{\eta}^T\widetilde{\eta} - \frac{g_2}{4}\widetilde{\nu}^T\widetilde{\nu}
\end{aligned}$$

Using definitions (28), (38), (39) and (40) together with $z_1 = [\bar{z}_1, \bar{z}_2, \bar{z}_3]^T$ and $z_2 = [\bar{z}_4, \bar{z}_5, \bar{z}_6]^T$, Eq. (69) can be rewritten as the following negative definite quadratic form

$$\begin{aligned}
Q &= -z^T D_z z + z^T W_1\widetilde{\eta} + z^T W_2\widetilde{\nu} \\
&- \frac{1}{4}(\widetilde{\eta}^T G_1\widetilde{\eta} + \widetilde{\nu}^T G_2\widetilde{\nu}) \\
&= -\sum_{i=1}^{3}\Big[d_i(\bar{z}_i k_i - \frac{1}{2d_i}\widetilde{\eta})^T(\bar{z}_i k_i - \frac{1}{2d_i}\widetilde{\eta}) \quad (69)\\
&+ d_{i+3}(\bar{z}_{i+3}\omega_i - \frac{1}{2d_{i+3}}\widetilde{\eta})^T(\bar{z}_{i+3}\omega_i - \frac{1}{2d_{i+3}}\widetilde{\eta}) \\
&+ d_{i+3}(\bar{z}_{i+3}\omega_{i+3} - \frac{1}{2d_{i+3}}\widetilde{\nu})^T(\bar{z}_{i+3}\omega_{i+3} - \frac{1}{2d_{i+3}}\widetilde{\nu})\Big] \\
&\leq 0 \quad (70)
\end{aligned}$$

because all the quadratic terms in (69) are less than or equal to zero.

VIII. REFERENCES

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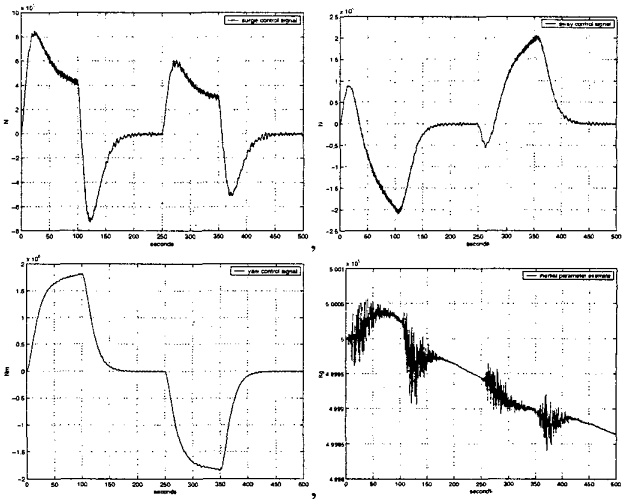

Fig. 4. DP of a supply vessel: control signals and inertial parameter estimate

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