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Adaptive friction compensation in DC motor drives

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ADAPTIVE FRICTION COMPENSATION IN DC MOTOR DRIVES

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Abstract. The problem of friction compensation in direct current motor drives is considered. The static and viscosity friction are assumed to have asymmetric characteristics. The friction torque effects are compensated by a feedback which results in an "idealized" linear model. The control design can, then, be based on the linear model. Through adaptive compensation the performance of the closed-loop system is improved over the non-adaptive case, where parameter uncertainties may be high. The final control law resulting from this scheme is a combination of: a fixed linear controller based on the linearized model and an adaptive part which compensates for nonlinear friction effects and model parameters uncertainty. The proposed ideas have been implemented and tested on a laboratory prototype with good results. The control law is implemented on an IBM-PC. The paper describes the ideas, the algorithm and the experimental results.

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1. Introduction

Adaptive control has predominantly dealt with generic models where all parameters are unknown. Such an approach has the advantage that it is general but also the disadvantage that many parameters have to be estimated. Much of the work on adaptive control has also been confined to linear systems. There are in practice many adaptive problems where the system can be described as partially known in the sense that part of the system dynamics is known and another unknown. In this paper we consider a problem of this type, namely, a servo with nonlinear friction. Friction, which is always present to some degree causes, difficulties and gives rise to poor properties in precision servos in robotics and other applications.

Control of a speed motor servo with friction is considered in this paper. It is assumed that static and viscosity frictions can be described as nonlinear functions of the angular velocity. The input-output of the asymmetric nonlinearity is one-to-one correspondence. This static friction characteristics allows to split the motor model into two distinct modes of operations, over each a linear representation is given. This model isolate the friction torque effects in order to cancel them using feedback compensation.

Adaptive friction compensation has been considered before. It was treated with model reference techniques in (Gilbart and Winston, 1974), and more recently by (Walrath, 1984). This paper differs in the friction model and in the adaptive control law used. To see previous work on adaptive control in similar nonlinear systems, refer to (Kung and Womack, 1984).

The adaptive scheme introduced in this paper attempts to use the maximum a priori information available from the system: the structure of the nonlinear block and some knowledge of the model parameters. It seems natural to use adaptive schemes (explicit identification) which utilizes this a priori information. The estimation algorithm estimates only that part of the system which is related to the nonlinear nature and to the model-parameter uncertainty. The estimates are used to compensate for the friction-torque effects and a linear control design is thus based on a linearized model. The final control structure can be viewed as a combination of a fixed linear controller and a feedback adaptive compensation.

The paper is organized as follows: Section 2 discusses some of the friction models proposed in the literature and establishes the opposition-torque model as a piecewise constant function of the angular velocity. Opposition torque includes static and viscosity torques. Section 3 discusses the friction compensation policy proposed. Section 4 describes the control design for known parameters; the friction torques are cancelled by a feedback compensation and a pole-placement linear design is based on a the linearized model. Friction characteristics are asymmetrical and may change as a function of the motor geometry and environmental variations. Section 5 proposes an adaptive version of the fixed friction compensation and studies one possible design path. The proposed ideas have been implemented on a laboratory prototype in combination with a PC/IBM computer; the experiments are shown in Section 6. Finally the conclusions are given in Section 7.

2. Mathematical models

A DC motor with a permanent magnet was used in our experiments. These types of motors are commonly used in robots and precision servos. The motor is provided with an electronic amplifier with current feedback. This arrangement permit also to emulate the motor as a current drive one by eliminating the time constant due to the induced voltage. If all inertias are reflected to the motor axis the motor can be described by the following model:

$$J \frac{d\omega}{dt} = K I(t) - T_f(t) + T_l(t) \quad (1)$$

Where J is the total moment of inertia reflected to the motor axis, K is the current constant, I is the motor current, T_f the friction torque and T_l the load disturbance torque. For the purpose of the investigation of the friction compensation, phenomenas like compliance and torque ripple are not included in the model (1).

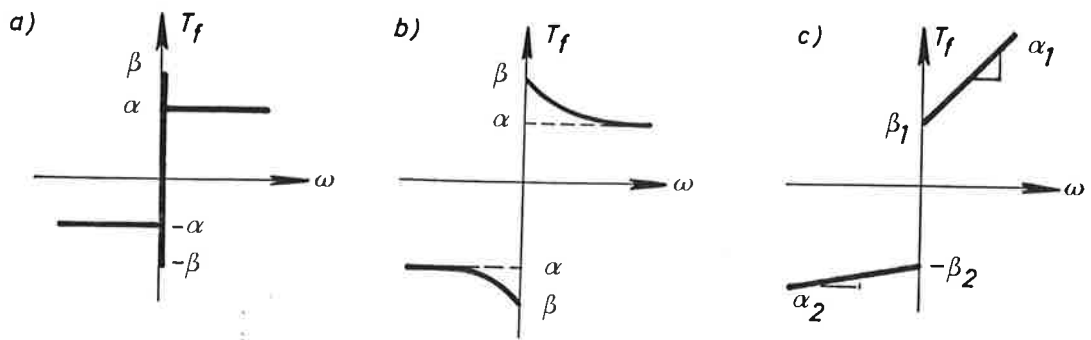


Figure 1. Different friction models.

FRICTION MODELS

Friction models have been extensively discussed in literature, see Dahl (1977), Walrath (1984) and Gilbert and Wilston (1974). In spite of this there is a considerable disagreement on the proper model structure. It is well established that the friction torque is a function of the angular velocity, there is however a disagreement in the character of the function. In the classical coulomb/stiction friction model there is a constant friction torque opposing the motion when $\omega \neq 0$. For zero velocity the stiction will oppose all motions as long as the torques are smaller in magnitude than the Stiction torque. This model is represented in Fig. 1A. The coulomb-friction/Stiction model has been well established in connection with slow speeds in numerically controlled machines.

The model shown in Fig.1B was proposed by Gilbert and Wilston (1974). A very different model was proposed by Walrath (1984). He based his model on experimental studies of a stabilized platform with ball bearings on the gimbals. Walrath proposes the model

$$\dot{T}_f(t) + aT_f(t) = T_c \text{sgn}(\omega)$$

where the parameter a depends on ω . Notice that this model introduces additional dynamics but that it does not include any static friction characteristics. The friction model used in our studies is shown in Fig.1C This model includes Coulomb friction and viscous friction. The friction curve is, however, not symmetric. The possible imperfections in the motor geometry and unbalance on

the motor shaft motivate the asymmetrical characterization of the friction.

The model can be described as

$$T_f(\omega) = \begin{cases} \alpha_1\omega + \beta_1 & \omega > 0 \\ \alpha_2\omega - \beta_2 & \omega < 0 \end{cases} \quad (2)$$

Neglecting the load disturbances torque and the resonances modes of the motor couplings, the motor operation can thus be described by (1) where the friction torque is given by (2).

3. Friction compensation

The nonlinear friction limits the performance of the closed-loop system. The influences of the nonlinearities can to some extent be reduced by high gain linear feedback. This is suggested in Wu and Paul (1980). This approach has however some severe limitations because the nonlinearities will dominate any linear compensation for small errors. The effects of the friction can also be alleviate by mounting force sensors, which measure the friction levels, and using them in a linearizing feedback loop around the torque motor, as suggested in Handlykken and Turner (1980). The selection of the adequate technics to compensate for the friction torques depends on the choice of the friction model. For the dynamic model proposed by Walrath (1984), it is possible to predict the friction behavior and compensate it using feedforward. An alternative approach is to reduce the effects of the friction terms by a nonlinear compensation. It is easy to see how this can be done. Neglecting the load disturbance torques T_l the equation (1) and (2) can be written as

$$J \frac{d\omega}{dt} = KI(t) - T_f(\omega) \quad (3)$$

Introduce

$$I(t) = u(t) + \frac{\hat{T}_f(\omega)}{\hat{K}} \quad (4)$$

Where $u(t)$ is a new control variable, \hat{T}_f is an estimate of the function T_f and \hat{K} is an estimate of the current constant K . Then

$$J \frac{d\omega}{dt} = Ku(t) + \left\{ \frac{K}{\hat{K}} \hat{T}_f(\omega) - T_f(\omega) \right\} \quad (5)$$

If the estimates are good the terms within the curly brackets vanish and the system obtained with the nonlinear feedback looks like a frictionless motor. It is therefore natural to call the feedback (4) a friction compensation. The success of the compensation clearly depends on how accurate estimates of k and T_f can be provided. The parameter K is the torque constant of the motor Its average value can be found from catalogues. It can also be measured. A complication is that K is not a constant. For many motors K will also depend on the relative oscillation of the rotor and stator at high frequencies (ripple torque). The friction torque T_f is a function of the angular velocity. To obtain \hat{T}_f it is thus important to know the shape of the function and to have a good estimate of the angular velocity ω . In our investigation we have used functions of the form (2). This simple model makes it possible to deal with variations and asymmetries of the friction torque which is not possible with the other models.

The velocity estimate has been generated by a tachometer or by a Kalman-filter. In our first experiments it was simply attempted to introduce a friction compensation based on (2) where the parameters were adjusted manually. The experiments performed was simply to adjust the parameters α_1 , β_1 , α_2 and β_2 so that the motor behaved like a frictionless motor. These experiments clearly indicated the necessity of having different parameters for different directions of rotation. The experiments also showed that it was possible to archive friction compensation using a friction model like (2) except for very slow tracking rates. It was also found that the coefficients in the friction (2) varied with the temperature and time. They may also varied with changes of the operation conditions. This motivates making the friction compensation adaptive.

4. Control design

Although the main thrust of this paper is to discuss friction compensation it is necessary to also add a conventional feedback loop to evaluate the final results. A natural approach is to design the feedback loop under the assumptions of perfect friction compensation. The system is then described by

$$J \frac{d\omega}{dt} = Ku(t) \quad (6)$$

and it is easily verified that the control law

$$u(t) = K_r [-\omega(t) + \frac{1}{T_i} \int_0^t (\omega_r(\tau) - \omega(\tau)) d\tau] \quad (7)$$

with

$$K_r = \frac{2\xi\omega_0 J}{K} \quad \text{and} \quad T_i = \frac{K}{J\omega_0^2} \quad (8)$$

gives a closed loop system with the transfer function

$$G(s) = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2} \quad (9)$$

An equivalent discrete time control law at the form

$$u(t) = u(t-h) + s_0 [\omega_r(t) - \omega(t-h)] + s_1 [\omega_r(t-h) - \omega(t-h)] \quad (10)$$

was actually used in the experiments. The parameters of the regulator were determined by pole-placement desing Åström and Wittenmark, (1984) to obtain the discrete time equivalent of the closed loop transfer function (9).

5. Adaptive friction compensation

In a typical servo application the moment of inertia J and the current constant K may be regarded as Known. With the reservation discussed in Section 3. To obtain the friction compensation it is necessary to obtain estimates of the friction torque functions T_f . With the representation (2) this reduces to estimation of the parameters α_1 , α_2 , β_1 and β_2 .

Alternative Parameter Estimation Methods (adaptive predictors)

Since the parameters α_1 , α_2 , β_1 and β_2 naturally appears in the continuous time formulation it is natural to estimate the parameters in this form. Standard linear parameters estimation methods may be obtained to the equation

$$J \frac{d\tilde{\omega}}{dt} = K\tilde{I}(t) - \alpha_1 \tilde{\omega} - \beta_1 \quad (11)$$

Where $\tilde{\omega}$ denotes a filtered version of ω , i.e.

$$\frac{d\tilde{\omega}}{dt} + c\tilde{\omega} = c\omega$$

with appropriate initial conditions. Since the filter is chosen to be stable, its initial conditions will vanish as time grows, then equation (11) becomes an equivalent representation of the motor model (1-2) and holds for all times. The sampled filtered signals and standard recursive estimation methods are used to generate the estimates. The parameter c in the filter can be optimized based on knowledge of the noise and the Known parameters. More details of this technique can be found in Eykhoff (1974), Young (1965, 1969), Pernebo (1978), Johansson (1983), (1985) and Canudas (1985).

Another alternative is to derive a zero-order hold model for the motor representation (1-2). This gives

$$\omega(t) + a_{1i}\omega(t-h) = b_{oi}I(t-h) + b_{1i} \quad (12)$$

Where

$$a_{1i} = e^{-\alpha_i h/J}$$

$$b_{0i} = \frac{K}{\alpha_i} (1 - e^{-\alpha_i h/J})$$

$$b_{1i} = -\frac{\beta_i}{\alpha_i} (1 - e^{-\alpha_i h/J}) \quad i = 1, 2$$

With this approach it became necessary to estimate six parameters instead of four.

Another possibility is to sample with a such short rate that the derivative can be approximated by forward differences, i.e.

$$\omega(t+h) = \omega(t) + \frac{h}{J} [KI(t) - \alpha_i \omega(t) - \beta_i] \quad (13)$$

This approximation retains the number of parameters of the physical model. However its utilization may be constraint by computation time limitations. Other alternatives are Tustin approximation, etc.

Estimation Algorithms

All estimation algorithms can be characterized by the error model

$$\epsilon(t_i) = f(t_i) - \varphi^T(t_i)\theta \quad (14)$$

where the function f and the regression vector φ are functions of the data and θ is the vector of the unknow parameters. A recursive least-squares algorithm is then given by the normal equations. Fig.2 shows the closed loop scheme with the adaptive friction compensation.

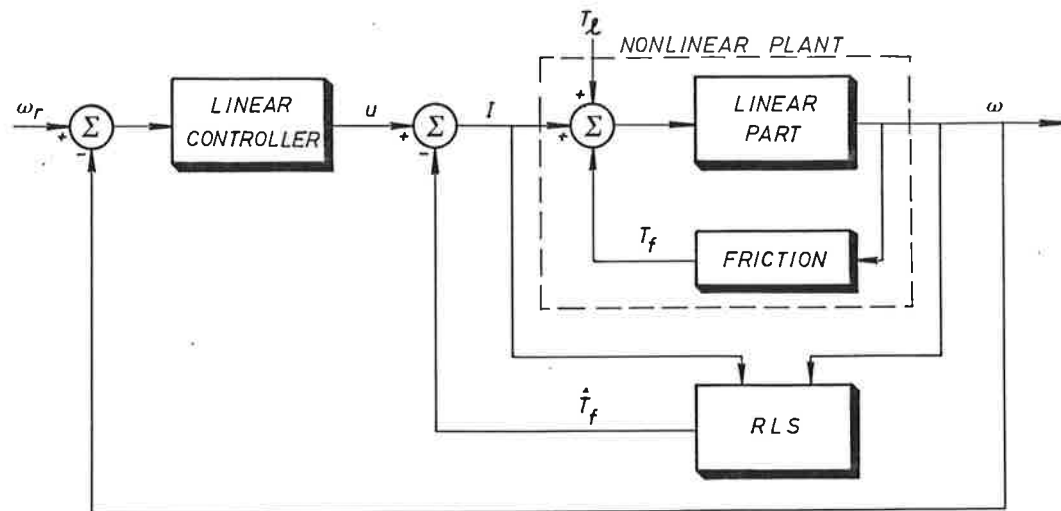


Figure 2. Block diagram of a motor controller with adaptive friction compensation.

A possible design path.

The previous sections covered different frictions models and alternative methods to construct adaptive predictors. The necessity to add conventional feedback loop in order to evaluate the effectiveness of the friction compensation was also mentioned. To illustrate one possible path to implement the previous ideas, let choose a discrete time predictor and a pole-placement control design policy. We can then proceed as follows. The discrete time models, (12) or (13), can be reformulated as the following general model:

$$\omega(t+h) = \begin{cases} (A_1 + \tilde{A}_1) \omega(t) + (B_1 + \tilde{B}_1) I(t) + \delta_1 & \text{if } \omega(t) > 0 \\ (A_2 + \tilde{A}_2) \omega(t) + (B_2 + \tilde{B}_2) I(t) + \delta_2 & \text{if } \omega(t) < 0 \end{cases} \quad (15)$$

Where A and B are polynomials in the delay operator of the appropriate order. The polynomials A_1, B_1, A_2, B_2 are the known-model part of the plant. $\tilde{A}_1, \tilde{B}_1, \tilde{A}_2, \tilde{B}_2$ are the unknown-model part of the plant which is provided by the model uncertainty and by the nonlinear feedback of the process. The operator q^{-1} indicate a delay operation of one period h .

Notice that, for the model (12) the friction characteristics (α_i, β_i) cohabit with the motor constant (k, J) , it is thus not possible to have an independent cancellation of the friction effects as it was mentioned in Section 3.

We can always let $A_1=A_2=A$ and $B_1=B_2=B$. The difference in each case will be absorbed by the corresponding polynomial uncertainty \tilde{A}_i and \tilde{B}_i . Then equation (15) can be reduced to a more compact form:

$$\omega(t+h) = A\omega(t) - BI(t) + g(t) \quad (16)$$

where:

$$g(t) = g_1(t)m(t) + g_2(t)(1-m(t))$$

$$g_1(t) = \tilde{A}_1 y(t) + \tilde{B}_1 I(t) + \delta_1$$

$$g_2(t) = \tilde{A}_2 y(t) + \tilde{B}_2 I(t) + \delta_2 \quad (17)$$

and

$$m(t) = \begin{cases} 1 & \text{if } \omega(t) > 0 \\ 0 & \text{if } \omega(t) < 0 \end{cases} \quad (18)$$

For the model (13) the above defined polynomials are; $A=1$, $B=hK/J$, and $\tilde{A}_i=h\alpha_i/J$, $\tilde{B}_i=0$, $\delta_i=h\beta_i/J$ for $i=1, 2$. Then the terms $g(t)$ contains the friction effects to be cancelled. By the arguments discussed in Section 3, the nonlinear model (16) can be linearized if the following control law is applied;

$$I(t) = u(t) + \bar{g}(t) \quad (19)$$

where $\bar{g}(t)$ is equal to $g(t)/B$. Replacing the above control law into the process model (16) gives,

$$\omega(t+h) = A\omega(t) + Bu(t) \quad \text{or} \quad \bar{A}\omega(t) = q^{-1}Bu(t) \quad ; \quad \bar{A} = 1 + q^{-1}A \quad (20)$$

The most general linear controller is giving by the following equation:

$$Ru(t) = T\omega_r(t) - S\omega(t) \quad ; \quad \omega_r(t) = A_m(1)\bar{\omega}_r(t) \quad (21)$$

where $\bar{\omega}_r(t)$ is the reference signal, $\omega(t)$ the process output and $u(t)$ the input applied to the linearized system (20). A_m is the polynomial which describe the desired closed-loop characteristics. And the R, S, T polynomials will be found by solving the Diophantine equation:

$$R\bar{A} + q^{-1}BS = TBA_m \quad ; \quad T = A_n A_o \quad ; \quad R = DR' \quad (22)$$

where A_o is the observer polynomial, A_n a notch filter and D the internal model (These polynomials can be included or not, the simplest case : $A_o = A_n = D = 1$). Integral action can be included to improve robustness in the closed-loop system. This is achieved by putting $D = 1 - q^{-1}$.

The adaptive nonlinear compensation algorithm based on the same previous linear design philosophy can be described by the next sets of equations:

Adaptive predictor:

$$\hat{\omega}(t+h|t) = A\omega(t) + Bu(t) - \hat{g}(t) \quad (23)$$

where:

$$\hat{g}(t) = \phi(t)^T \hat{\theta}(t) = [\phi_1(t)^T h(t), \phi_2(t)(1-m(t))] \cdot \begin{bmatrix} \hat{\theta}_1(t) \\ \hat{\theta}_2(t) \end{bmatrix}$$

$$\begin{aligned} \phi_1(t)^T &= [\omega(t), \dots, \omega(t-hn_{a1}^-), u(t), \dots, u(t-hn_{b1}^-), 1] \\ \phi_2(t)^T &= [\omega(t), \dots, \omega(t-hn_{a2}^-), u(t), \dots, u(t-hn_{b2}^-), 1] \end{aligned} \quad (24)$$

$$\begin{aligned} \hat{\theta}_1(t)^T &= [\tilde{a}_1^1(t), \dots, \tilde{a}_{na1}^1(t-hn_{a1}^-), \tilde{b}_0^1(t), \dots, \tilde{b}_{nb1}^1(t-hn_{b1}^-), \delta_1] \\ \hat{\theta}_2(t)^T &= [\tilde{a}_1^2(t), \dots, \tilde{a}_{na2}^2(t-hn_{a2}^-), \tilde{b}_0^2(t), \dots, \tilde{b}_{nb2}^2(t-hn_{b2}^-), \delta_2] \end{aligned} \quad (25)$$

Prediction error:

$$e(t) = \omega(t) - \hat{\omega}(t|t-h) = g(t-h) - \hat{g}(t-h) \quad (26)$$

Parameter estimation algorithm: Use a RLS algorithm.

Adaptive control law:

$$Ru(t) = T\omega_r(t) - S\omega(t) + R\hat{g}(t) \quad ; \quad \hat{g}(t) = \hat{g}(t)/B \quad (27)$$

Typical assumptions of the pole-placement design are needed: Coprimeness between the polynomials \bar{A} , B and the stability of $(B + \bar{B}_i)^{-1}$ for $i=1,2$. The closed-loop properties of the above set of equations are analyzed in, Canudas (1985). The previous algorithm has been implemented in our experimental set. The result are described in the following section.

6. Experiments

The ideas have been tested experimentally on a simple servo. The experimental set up is shown in Figure 3. It consists of a servo composed of a DC motor with gear and load. The motor speed is measured using a tachometer. There is friction in the motor bearings and in the gear train. The friction can also be increased by a simple mechanical arrangement. The first experiments were performed using dedicated analog hardware which was built up using operational amplifiers. In this experiment it was attempted to reduce the friction by a fixed nonlinear compensator as was discussed in Section 3. A nonlinear friction compensation of the form (4) was introduced and the parameters were adjusted manually. It was found that friction compensation is indeed possible but that the parameters of the friction compensation depend on the operating condition. The adjustment of the parameters of the friction compensation is also quite critical. Figure 4 shows that degradation of the closed-loop responses may occur if the friction's parameters are not chosen properly.

The results in Figure 4 were obtained by simulation. Similar phenomena could be found also when experimenting with the servo.

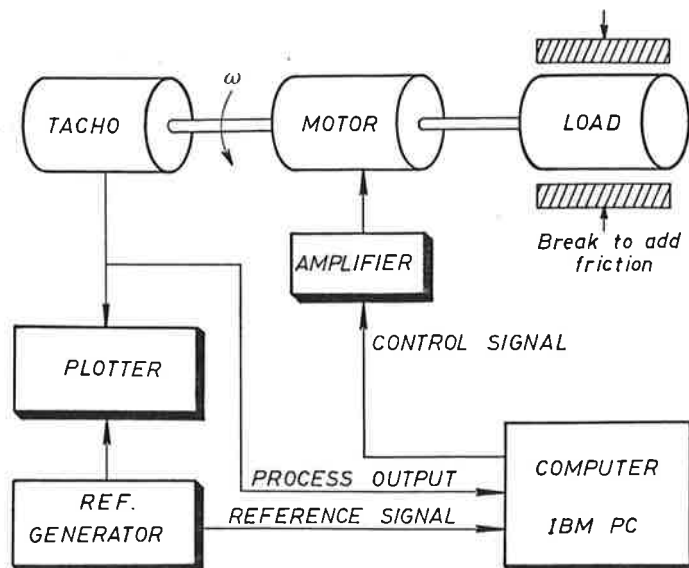


Figure 3. The experimental set up.

The experiments with adaptive friction compensation were performed under computer control. An IBM PC-XT with the 8087 floating point chip and Data Translation AD and DA converters were used. The major part of the software was written in Microsoft Pascal. The MetaWINDOW package was used for the graphics. Concurrency was obtained by using a simple scheduler written in assembler. This allowed the control program to run in the foreground and graphics and man-machine communication in the background. The major software functions are listed in the Appendix A. The minimal sampling rate is 55 ms. For more details of the implementation aspects, see Braun (1985).

Tracking experiments were carried out with a constant gain regulator without friction compensation and a controller with adaptive friction compensation. Some typical results are shown in Figure 5. The upper traces in the figure shows the tracking performance with a linear regulator. Notice the deterioration in performance as the friction is increased. The lower traces show the corresponding curves for a regulator with adaptive friction compensation. The improvements are quite noticeable.

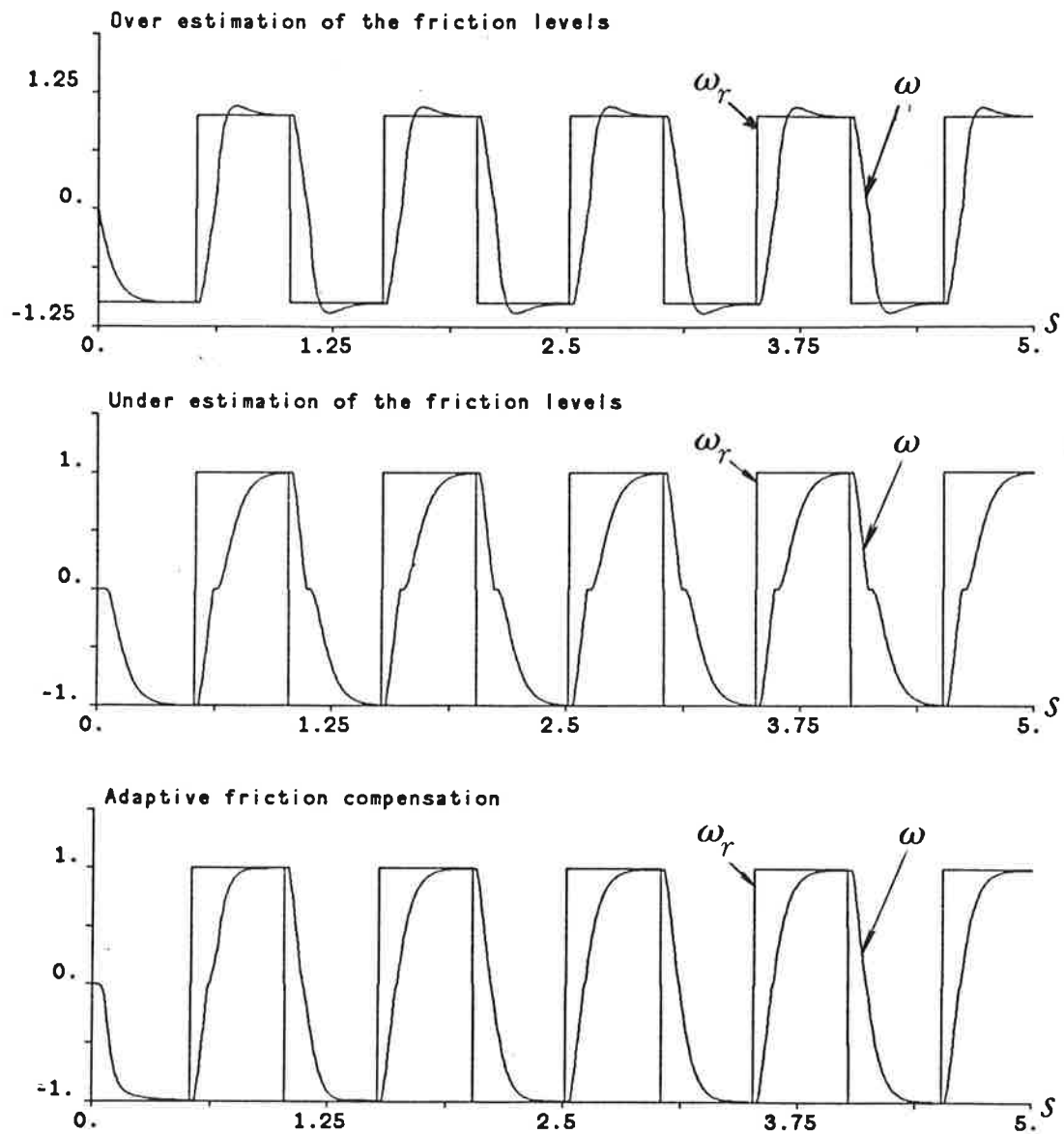


Figure 4. Response of a system with a nonlinear friction with improperly chosen parameters and the same system with adaptive friction compensation. Simulation results (process output ω , reference signal ω_r).

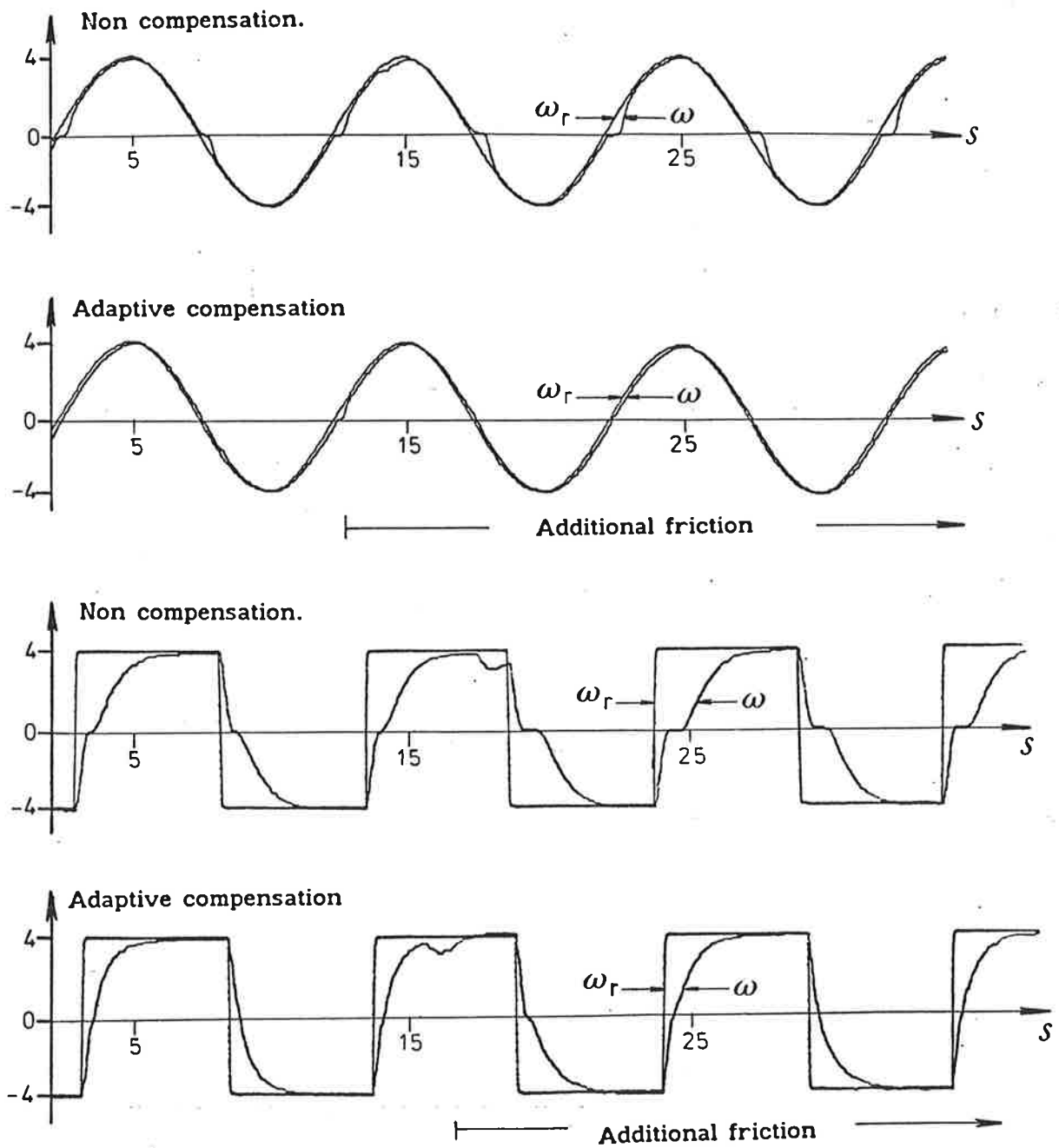


Figure 5. Results of tracking using linear regulator and regulator with adaptive friction compensation for two different types of reference signals. Experimental results (process output ω , reference signal ω_r).

7. Conclusions

Although high quality servos of the type that is found in robots and systems for tracking and pointing are largely described by linear models their performance is often limited by nonlinear phenomena like friction and backlash. In this paper we have discussed the possibilities of improving the performance of a servo by nonlinear compensation of friction. Models for friction have been reviewed. Several different models have been proposed. A particular form was chosen based on experiments on a servo. It was found to be essential to have a model which is asymmetric in the angular velocity. Different ways to compensate for friction have been discussed. Different ways to estimate the coefficients of the friction model have also been investigated. The adaptive techniques have been found superior because the friction depends on the operating conditions. Adaptive friction compensation is a special case of adaptive control of partially known systems. Such problems can conveniently be approached from the continuous time point of view because this is the natural way to formulate what is known about the system. Adaptive friction compensation has been applied to an experimental system. Its benefits have been demonstrated clearly in experiments on a servo where the control law was implemented on an IBM PC. With regards to future work it seems appropriate to investigate the structure of the friction models in more detail since this seems to be an issue where there is considerable disagreement in the literature. The availability of a friction model with appropriate structure is also crucial for the performance of the adaptive friction compensation.

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Appendix A. Description of the program AFRICO.

The program AFRICO (adaptive friction compensation) performs real-time tasks based on a simple real-time scheduler (foreground-background), see Mattsson (1978) and Braun (1985). The program is divided into four parts:

1. Initialization of the program. The I/O board and the graphics system are initialized. Default values are read from a specified file. Different default values can thus easily be used in different experiments.

2. Modification of the parameters. Parameters can be changed interactively using a menu.

3. Experiment. The regulator runs in the foreground until the function key F1 is pressed. Control is then passed to the background program.

4. Evaluation of the experiment. The data stored during the experiment can be investigated under menu control. Signals and estimates can be plotted. Parameters can be displayed and initialized.

The possible flows through the program are shown in Fig. A.1. The man-machine communication and graphics are in the background. The control algorithm runs in the foreground, which has the following sequence:

1. Input of the reference signal $\omega_r(t)$ and the process output $\omega(t)$.
2. Computation of the control law $u(t)$ using the previous estimates.
3. Output of the control signal $u(t)$.
4. Update the recursive least-squares estimates.
5. Store variables, which may be plotted later.
6. Update of the variables.

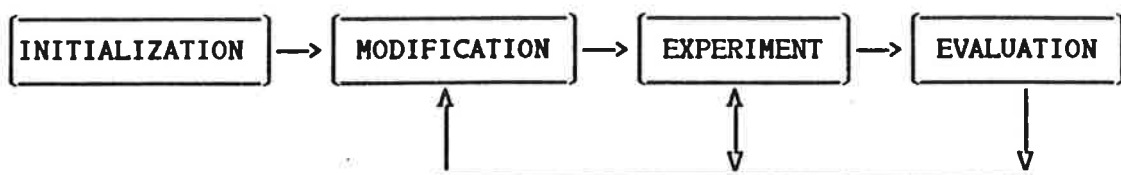


Fig. A.1: Possible flows through the program AFRICO