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Abstract

The simple relay auto-tuners are based on the idea that the ultimate gain and the ultimate period can be determined from a simple relay feedback experiment. This tuning principle has proven very successful in practical industrial applications. This paper proposes a new tuning method which is also based on a relay feedback experiment. A new method to process the data from the experiment gives a sampled data model. Many different analog and digital design methods can be applied to the model including dead-time compensation, pole placement etc. The method also offers interesting possibilities to determine the process time delay and to validate the model. The method is also ideally suited for pre-tuning of more complicated adaptive algorithms. The paper describes the basic idea and gives some examples.

Key words
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A New Auto-tuning Design

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Abstract. The simple relay auto-tuners are based on the idea that the ultimate gain and the ultimate period can be determined from a simple relay feedback experiment. This tuning principle has proven very successful in practical industrial applications. This paper proposes a new tuning method which is also based on a relay feedback experiment. A new method to process the data from the experiment gives a sampled data model. Many different analog and digital design methods can be applied to the model including dead-time compensation, pole placement etc. The method also offers interesting possibilities to determine the process time delay and to validate the model. The method is also ideally suited for pre-tuning of more complicated adaptive algorithms. The paper describes the basic idea and gives some examples.

Keywords: Auto-tuning; dead-time compensation; relay oscillations.

1. Introduction

In spite of many advances in control theory simple regulators of the PID type are still widely used. See Deshpande and Ash (1981) and McMillan (1983). Significant efforts have lately been devoted to provide automatic tuning of such regulators, see e.g., Bristol (1977) and Åström and Hägglund (1988). A new method for automatic tuning of simple regulators was introduced in Åström and Hägglund (1984a). The idea was to determine the critical period and the critical gain from a simple relay feedback experiment and to use Ziegler-Nichols type of control design methods to find the parameters of PID regulators. Several industrial products based on this idea are now available on the market, see Åström and Hägglund (1988). The scheme does, however, inherit the fundamental limitations of the Ziegler Nichols designs. The underlying difficulty being that the design is based on knowledge of only one point of the Nyquist curve of the open loop system. The consequences of this are analyzed in Åström and Hägglund (1984b) and Hägglund and Åström (1985) where some remedies are also suggested.

A different approach to the problem is taken in this paper. The idea is to still use the relay feedback experiment to obtain the process knowledge required. Instead of only using the amplitude and the period of the oscillation we will however now also use the shape of the oscillation under feedback control. A new technique for estimating a process model based on the wave-form of the oscillation is proposed. This is described in Section 2. Many different methods can be used for the control design. A simple pole placement method is given in Section 3. This allows test of the idea. It can be expected to outperform conventional PID designs because the response speed is easily adjusted and it can also handle systems with time-delays. There are several interesting practical aspects of the scheme. One is that it is possible to adjust the input filtering with respect to the closed loop bandwidth. The other is that the model can be validated based on properties of the wave-form. An alarm can then be generated if the model is not proper with respect to the specifications. This is discussed in Section 4.

2. Parameter Estimation

To design digital control laws it is necessary to know the sampling period and a discrete time process model. To determine a process model experimentally a sampling period is first chosen, perturbation signals are then introduced and the process model is then obtained using some parameter estimation method. In self-tuning control the perturbations are generated by the conventional feedback and the parameters are estimated recursively. The sampling period is a crucial parameter both in conventional parameter estimation and in adaptive control. Prior knowledge about the time-scale of the process and the closed loop system are required to determine the sampling period. This fact has for a long time been a stumbling block for automatic modeling and adaptive control.

An interesting tuning method which bypasses the difficulties discussed above was proposed in Åström and Hägglund (1984a). The idea is that most plants will exhibit a periodic oscillation under relay feedback. The amplitude and the period of the oscillation can then be used to tune a regulator based on conventional Ziegler-Nichols type designs.

The tuning methods discussed in Åström and Hägglund (1984a) are only based on knowledge of the amplitude and the period of the oscillation. We will here show that conventional sampled data models can be determined using the wave-form of the oscillation.

The starting point is that a relay feedback experiment in stationarity gives periodic input output signals as shown in Figure 1. The period of the oscillation is approximately the ultimate period under proportional feedback. This period can be used as a basis for selecting the sampling period. Assume that we choose $2n$ samplings per period. Since the signals are periodic, it is sufficient to consider a half-period. The input signal is then given by

$$u_0 = u_1 = \ldots = u_n = 1 \qquad (1)$$

Without loss of generality it has been assumed that the relay gives an output with unit amplitude. The output
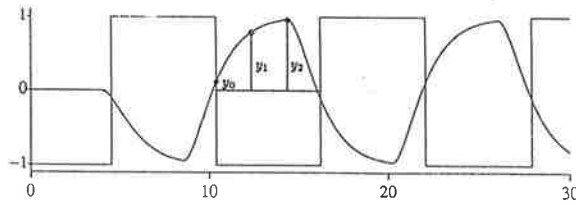
Figure 1. Input and output signals obtained from an experiment with relay feedback.

signal is given by

$$y_0, y_1, \ldots y_n \qquad (2)$$

A standard input-output model for a linear system can be written as

$$A(q)y(k) = B(q)u(k) \qquad (3)$$

Where $A(q)$ and $B(q)$ are polynomials in the forward shift operator with

$$d = \deg A - \deg B$$

The estimation problem is thus to determine the model (3) from the data (1) and (2). A simple scheme for doing this will now be developed. First observe that in steady state the Z-transforms of the signals (1) and (2) can be written as

$$U(z) = -\frac{z^n + z^{n-1} + \cdots z}{z^n + 1} = -\frac{E(z)}{z^n + 1}$$

$$Y(z) = \frac{y_d z^n + y_{d+1} z^{n+1} + \cdots + y_{d+n-1} z}{z^d(z^n + 1)} = \frac{D(z)}{z^d(z^n + 1)}$$

The particular representation of the Z-transform of the periodic function y is chosen to be causally compatible with the model (3). Notice also that

$$y_{n+k} = -y_k$$

It follows from equation (3) that

$$Y(z) = \frac{B(z)}{A(z)} U(z) + \frac{Q(z)}{A(z)}$$

where the polynomial $Q(z)$ corresponds to initial conditions which give the steady state periodic output. Introducing the expressions for $Y(z)$ and $U(z)$ above we get

$$\frac{D(z)}{z^d(z^n + 1)} = -\frac{B(z)E(z)}{(z^n + 1)A(z)} + \frac{Q(z)}{A(z)}$$

or

$$A(z)D(z) + z^d B(z)E(z) = z^d(z^n + 1)Q(z) \qquad (4)$$

This is an equation for determining the polynomials A, B, and Q. Equating coefficients of equal powers of z we get $n + \deg A + 1$ linear equations to determine the coefficients in the model. The number of unknown parameters in the polynomials A, B, and Q is $3\deg(A) - 2d + 2$. To determine these parameters, it is therefore necessary to have $n \geq 2(N - d) + 1$. In the case $n > 2(N - d) + 1$, the number of equations is larger than the number of unknown parameters. It is therefore suggested to use some kind of minimization technique to determine the parameters in this case. It is thus straightforward to determine the coefficients of the process model (3) from the wave-form $y_0, y_1, \ldots, y_n$ of the periodic solution. The procedure is illustrated by some examples:

EXAMPLE 1
Consider the process model

$$y(t + 1) = ay(t) + b_1 u(t) + b_2 u(t - 1) \qquad (5)$$

which corresponds to sampling of a first order system with a time-delay less than one sampling period. In this case we have

$$A(z) = z(z - a)$$
$$B(z) = b_1 z + b_2$$

The model has 3 parameters. To determine these it is thus necessary to have $n \geq 3$. For $n = 3$ the equation (4) becomes

$$z(z - a)(y_1 z^3 + y_2 z^2 - y_0 z) + z(b_1 z + b_2)(z^3 + z^2 + z)$$
$$= z(z^3 + 1)(q_0 z + q_1)$$

It follows immediately that $q_1 = 0$ and the equation then reduces to

$$(z - a)(y_1 z^2 + y_2 z - y_0) + (b_1 z + b_2)(z^2 + z + 1) = q_0(z^3 + 1)$$

Equating coefficients of equal powers of z we get 4 equations to determine the unknowns a, b_1, b_2, and q_0. These equations are

$$y_1 + b_1 = q_0$$
$$-ay_1 + y_2 + b_1 + b_2 = 0$$
$$-ay_2 - y_0 + b_1 + b_2 = 0$$
$$ay_0 + b_2 = q_0$$

The equations have a solution if $y_1 \neq y_2$. It is given by

$$a = \frac{y_0 + y_2}{y_1 - y_2}$$

$$b_1 = \frac{y_0^2 - y_1^2 + y_2^2 + y_0 y_1 + y_0 y_2 + y_1 y_2}{2(y_1 - y_2)}$$

$$b_2 = \frac{-y_0^2 + y_1^2 + y_2^2 + y_0 y_1 - y_0 y_2 - y_1 y_2}{2(y_1 - y_2)}$$

$$q_0 = \frac{y_0^2 + y_1^2 + y_2^2 + y_0 y_1 + y_0 y_2 - y_1 y_2}{2(y_1 - y_2)}$$

$\square$

In Example 1 it was assumed that the time delay was less than one sampling period. We will now consider a problem where the time delay is between one and two sampling periods.

EXAMPLE 2
Consider a first order system where the time delay is between one and two sampling intervals. The sampled model of such a system is

$$y(t + 1) = ay(t) + b_1 u(t - 1) + b_2 u(t - 2) \qquad (6)$$

In this case we have

$$A(z) = z^2(z - a)$$
$$B(z) = b_1 z + b_2$$

The equation (4) becomes

$$z^2(z - a)(y_2 z^3 + y_0 z^2 - y_1 z) + z^2(b_1 z + b_2)(z^3 + z^2 + z)$$
$$= z(z^3 + 1)(q_0 z + q_1)$$

It follows immediately that $q_1 = 0$ and the equation then reduces to

$$(z - a)(y_2 z^2 + y_0 z - y_1) + (b_1 z + b_2)(z^2 + z + 1) = q_0(z^3 + 1)$$

Equating coefficients of equal powers of z we get 4 equations to determine the unknowns a, b_1, b_2, and q_0. These equations are

$$y_2 + b_1 = q_0$$
$$-ay_2 + y_0 + b_1 + b_2 = 0$$
$$-ay_0 - y_1 + b_1 + b_2 = 0$$
$$ay_+ b_2 = q_0$$

The equations have a solution if $y_0 \neq -y_2$. It is given by

$$a = \frac{y_1 - y_0}{y_0 + y_2}$$

$$b_1 = \frac{y_0^2 + y_1^2 - y_2^2 - y_0 y_1 - y_0 y_2 + y_1 y_2}{2(y_0 + y_2)}$$

$$b_2 = \frac{y_0^2 - y_1^2 + y_2^2 + y_0 y_1 + y_0 y_2 + y_1 y_2}{2(y_0 + y_2)}$$

$$q_0 = \frac{y_0^2 + y_1^2 + y_2^2 - y_0 y_1 + y_0 y_2 + y_1 y_2}{2(y_0 + y_2)}$$

$\square$

In Example 2 it was assumed that the time delay was less than one sampling period. We will now consider a problem where the time delay is between one and two sampling periods.

EXAMPLE 3
Consider a first order system where the time delay is between two and three sampling intervals. The sampled model of such a system is

$$y(t+1) = ay(t) + b_1 u(t-2) + b_2 u(t-3) \qquad (7)$$

in this case we have

$$A(z) = z^3(z-a)$$
$$B(z) = b_1 z + b_2$$

The model has 3 parameters. To determine these it is thus necessary to have $n \geq 3$. The equation (4) becomes

$$z^3(z-a)(y_0 z^3 - y_1 z^2 - y_2 z) + z^3(b_1 z + b_2)(z^3 + z^2 + z)$$
$$= z^3(z^3 + 1)(q_0 z + q_1)$$

It follows immediately that $q_1 = 0$ and the equation then reduces to

$$(z-a)(-y_0 z^2 - y_1 z - y_2) + (b_1 z + b_2)(z^2 + z + 1) = q_0(z^3 + 1)$$

Equating coefficients of equal powers of z we get 4 equations to determine the unknowns a, b_1, b_2, and q_0. These equations are

$$-y_0 + b_1 = q_0$$
$$ay_0 - y_1 + b_1 + b_2 = 0$$
$$ay_1 - y_2 + b_1 + b_2 = 0$$
$$ay_2 + b_2 = q_0$$

The equations have a solution if $y_0 \neq y_1$. It is given by

$$a = \frac{y_1 + y_2}{y_0 - y_1}$$

$$b_1 = \frac{y_0^2 - y_1^2 - y_2^2 - y_0 y_1 + y_0 y_2 + y_1 y_2}{2(y_0 - y_1)}$$

$$b_2 = \frac{-y_0^2 - y_1^2 + y_2^2 + y_0 y_1 + y_0 y_2 - y_1 y_2}{2(y_0 - y_1)}$$

$$q_0 = \frac{-y_0^2 - y_1^2 - y_2^2 + y_0 y_1 + y_0 y_2 + y_1 y_2}{2(y_0 - y_1)}$$

$\square$

A dead-beat observer is used in all the examples. A more robust design is obtained by choosing observer poles that are removed from the origin, e.g., at $z = 0.2$, see Lennartson (1987).

3. Control Design

When a process model of the form (3) is available there are many design methods that can be used to obtain a control law. A pole placement design where natural frequency ω and relative damping ζ of the dominant poles are specified is, e.g., one alternative. The design parameters can be chosen automatically. Parameter ζ can be fixed and frequency ω can be chosen as $\omega = 2\pi/T_c$ for systems with low order dynamics. For systems with a large pole excess this value of ω is, however, too large. It is better to reduce ω by a factor of two. In those cases where ω is reduced we may also consider to make a new experiment at a lower frequency. Otherwise the input signal is not ideal for the parameter estimation, see Åström and Wittenmark (1984). This design will now be determined for the systems in Examples 1, 2, and 3.

EXAMPLE 4
Consider the system (5) obtained in Example 1, i.e.,

$$A(z) = z(z-a)$$
$$B(z) = b_1 z + b_2$$

Assume that the desired closed loop characteristic polynomial is given by

$$z^2 P(z) = z^2(z^2 + p_1 z + p_2)$$

where

$$p_1 = -2e^{-\zeta \omega h} \cos\left(\zeta \omega h \sqrt{1 - \zeta^2}\right)$$
$$p_2 = e^{-2\zeta \omega h}$$

This corresponds to a second order response with relative damping ζ and frequency ω. To obtain a pole placement design with integral action we have to solve the Diophantine equation

$$A(z)(z-1)R_1(z) + B(z)S(z) = z^2 P(z)$$

It is straightforward to find the following minimal degree solution

$$R(z) = (z-1)(z + r_1)$$
$$S(z) = s_0 z^2 + s_1 z$$

where

$$s_0 = \frac{1}{1-a}\left(\frac{P(1)}{B(1)} - a\frac{P(a)}{B(a)}\right)$$

$$s_1 = \frac{a}{a-1}\left(\frac{P(1)}{B(1)} - \frac{P(a)}{B(a)}\right)$$

$$r_1 = -\frac{b_2}{a}s_1$$

The control law is then given by

$$u(t) = t_0 y_{\text{ref}} - s_0 y(t) - s_1 y(t-h)$$
$$+ (1-r_1)u(t-h) + r_1 u(t-2h)$$

where

$$t_0 = s_0 + s_1$$

$\square$

EXAMPLE 5

Consider the system (6) obtained in Example 2, i.e.,

$$A(z) = z^2(z - a)$$
$$B(z) = b_1 z + b_2$$

Assume that the desired closed loop characteristic polynomial is given by

$$z^4 P(z) = z^4(z^2 + p_1 z + p_2)$$

with p_1 and p_2 as before. To obtain a pole placement design with integral action we have to solve the Diophantine equation

$$A(z)(z - 1)R_1(z) + B(z)S(z) = z^4 P(z)$$

It is straightforward to find the following minimal degree solution

$$R_1(z) = z^2 r_1 z + r_2$$
$$S(z) = s_0 z^3 + s_1 z^2$$

where

$$s_0 = \frac{1}{1-a}\left(\frac{P(1)}{B(1)} - a^2 \frac{P(a)}{B(a)}\right)$$

$$s_1 = \frac{a}{a-1}\left(\frac{P(1)}{B(1)} - a\frac{P(a)}{B(a)}\right)$$

$$r_2 = -\frac{b_2}{a}s_1$$

$$r_1 = \frac{1+a}{a}r_2 - \frac{b_1}{a}s_1 - \frac{b_2}{a}s_0$$

The control law is then given by

$$u(t) = t_0 y_{ref} - s_0 y(t) - s_1 y(t-h) + (1 - r_1)u(t-h)$$
$$+ (r_1 - r_2)u(t-2h) + r_2 u(t-3h)$$

where

$$t_0 = s_0 + s_1$$

(8)

□

EXAMPLE 6

Consider the system (7) obtained in Example 3, i.e.,

$$A(z) = z^3(z - a)$$
$$B(z) = b_1 z + b_2$$

Assume that the desired closed loop characteristic polynomial is given by

$$z^6 P(z) = z^6(z^2 + p_1 z + p_2)$$

with p_1 and p_2 as before. To obtain a pole placement design with integral action we have to solve the Diophantine equation

$$A(z)(z - 1)R_1(z) + B(z)S(z) = z^6 P(z)$$

The degree solution is given by

$$R_1(z) = z^3 + r_1 z^2 + r_2 z + r_3$$
$$S(z) = s_0 z^4 + s_1 z^3$$

where

$$s_0 = \frac{1}{1-a}\left(\frac{P(1)}{B(1)} - a^3 \frac{P(a)}{B(a)}\right)$$

$$s_1 = \frac{a}{a-1}\left(\frac{P(1)}{B(1)} - a^2 \frac{P(a)}{B(a)}\right)$$

$$r_3 = -\frac{b_2}{a}s_1$$

$$r_2 = \frac{1+a}{a}r_3 - \frac{b_1}{a}s_1 - \frac{b_2}{a}s_0$$

$$r_1 = \frac{(1+a)}{a}r_2 - \frac{1}{a}r_3 - \frac{b_1}{a}s_0$$

The control law is then given by

$$u(t) = t_0 y_{ref} - s_0 y(t) - s_1 y(t-h)$$
$$+ (1 - r_1)u(t-h) + (r_1 - r_2)u(t-2h)$$
$$+ (r_2 - r_3)u(t-3h) + r_3 u(t-4h)$$

where

$$t_0 = s_0 + s_1$$

□

It is now straightforward to obtain a tuning procedure as follows:

ALGORITHM 1

Step 1. Run a relay feedback experiment.

Step 2. Determine the model parameters from the waveform as expressed by (5), (6), and (7).

Step 3. Compute the regulator parameters from the equations given in this section. □

Remark. Since the time delay is not known, it will be estimated from the position of the maximum point of the wave-form. If the system has the transfer function

$$G(s) = k\frac{e^{sL}}{1 + sT}$$

(9)

then the maximum of the output appears L time units after a switch in the input. With high order dynamics the maximum will typically occur earlier. The maximum of the output can be used to estimate the time delay in the following way. Let t_{max} be the distance from the extremum of the output to the previous switch of the input. The value of d should then be chosen so that

$$d \geq \frac{t_{max}}{h}$$

where h is the sampling period.

There are several relations which can be used for model validation. It follows from the calculations in Examples 1, 2, and 3 that

$$a^1 a^2 a^3 = -1$$

Where a^1, a^2 and a^3 are the parameters a obtained in the different calculations. The expression implies that one of the a^i:s always is positive. This means that only two models are reasonable. This can also help in determination of the time delay.

The parameters b_j^i satisfy the following relations:

$$b_2^2 = \frac{y_1 - y_2}{y_0 + y_2}b_1^1$$

$$b_1^3 = \frac{y_1 - y_2}{y_0 + y_1}b_2^1$$

$$b_1^3 = \frac{y_0 + y_2}{y_0 - y_1}b_2^2$$

Also notice that a proper model must have positive b_i parameters.

Lack of Identifiability

The output of the system is close to sinusoidal for certain processes. This may happen when the transfer function falls off rapidly with increasing frequency. In such a case it is not possible to determine more than two parameters in a model. When three amplitude values are used, as in Examples 1, 2, and 3, it is easy to verify that the amplitudes are related through

$$y_0 - y_1 + y_2 = 0$$

Table 1. Parameters of models having different delay.

d	a	b_1	b_2	k	T	L
1	−6.11	−3.59	−2.16			
2	0.636	0.128	0.586	2.0	4.3	3.6
3	0.257	0.554	0.202	1.0	1.4	4.7

Based on this relation it is easy to develop a criterion that tells if the results are reliable. It may, e.g., be required that the quantity

$$\alpha = \frac{|y_0 - y_1 + y_2|}{\max_i |y_i|} \tag{10}$$

is sufficiently large. If the test quantity is too small the additional information can be derived from the process gain that can be determined from a static experiment.

4. An Example

To illustrate the design procedure we will show the results when it is applied to a process with the transfer function

$$G(s) = \frac{e^{-4s}}{(s+1)^2}$$

The results of a relay experiment with the process is shown in Figure 1. The relay amplitude was 1 and the hysteresis level 0.1. With six samples per period we get

$$y_0 = -0.106$$
$$y_1 = 0.782$$
$$y_2 = 0.956$$

The reason why y_0 is slightly less than −0.1 is because of the sampling. The criterion (10) becomes $\alpha = 0.07$, which means that there is a reasonable content of higher harmonics in the output. Calculation of the parameters of a first order model gives the results shown in Table 1. The model for $d = 1$ can immediately be excluded since the parameter a is negative. To choose among the models we also compute the parameters in the model (9), that corresponds to the discrete time system (5), (6), or (7). These parameters are given by

$$k = \frac{b_1 + b_2}{1 - a}$$
$$T = -\frac{h}{\ln a}$$
$$L = hd + T \ln \frac{ab_1 + b_2}{b_1 + b_2}$$

The numerical values are given in Table 1. The maximum of the waveform in the relay experiment occurs 4 s after the input change. Since the sampling period is 1.94 s, we get $t_{max}/h = 2$. The model with $d = 2$ is therefore chosen. If the undamped closed loop frequency is chosen to correspond to the period of the relay oscillation and if the relative damping is specified to $\zeta = 0.707$ the control law is given by (8). This regulator can be interpreted as a PI controller with dead-time compensation. The parameters are

$$s_0 = 0.925 \qquad r_2 = 0.665$$
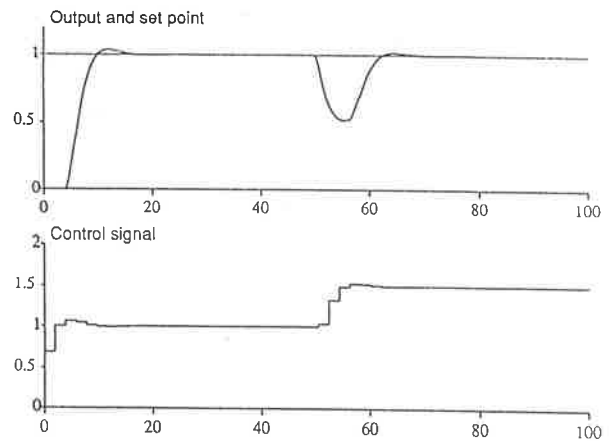$$s_1 = -0.232 \qquad r_3 = 0.182$$
$$r_1 = 0.553$$

Figure 2. Response of the closed loop system obtained when the design procedure is applied to a system with the transfer function $G(s) = e^{-4s}/(s+1)^2$.

Disregarding the dead-time compensation the regulator has an equivalent integration time $T_i = 2.6$. This can be compared with the values $k = 0.23$ and $T_i = 1.9$, which are obtained by a dominant pole design of a PID regulator, see Åström and Hägglund (1988). Figure 2 shows how the closed loop system responds to commands and load disturbances. The controller gives good performance both with respect to command following and load disturbance rejection.

5. Practical Aspects

There are many practical aspects of the regulator that are similar to the conventional auto-tuner. It is useful to introduce hysteresis in the relay to avoid chattering. The hysteresis amplitude can be determined by measuring the noise level in steady state. The relay amplitude can be adjusted to avoid too large deviations during the relay experiment. The criterion (10) can also be used to influence the hysteresis. If (10) is too low the hysteresis should be increased.

Proper signal conditioning is very important for good control. For digital control laws it is particularly important to have good anti-aliasing filters. See e.g., Åström and Wittenmark (1984). Since this filter will depend on the sampling period it is normally difficult to make a good a priori choice. With the design procedure proposed in this paper the sampling period is determined automatically. It is then possible to match also the antialiasing filter to the particular conditions.

If the sampling period is h then the Nyquist frequency is π/h rad/s. Assuming that we are using a second order Butterworth filter, to obtain a gain reduction of 16 at the Nyquist frequency the filter bandwidth should then be chosen as $\pi/(4\omega)$.

The following procedure is used to obtain a filter whose bandwidth is easily adjusted. Select a high sampling rate and an appropriate analog prefilter. Then implement a digital filter which does the additional filtering required and sample the output of this filter with period h. In this way it is easy to obtain the appropriate filtering. It is probably also a good idea to repeat the relay experiment with the digital prefilter in the loop.

Based on some preliminary experiments it is expected that this form of adaptive prefiltering will substantially improve the quality of the control.

6. Conclusions

This paper proposes a simple method for tuning digital control laws directly. It is believed that these methods are superior to the techniques based on continuous time PID algorithms for the following reasons: There are fewer approximations involved, because a discrete time model is fitted directly. The information about the full waveform is used, not only amplitude and frequency. It is also easy to include an adjustment of the response rate in the system simply by letting the operator choose ζ and ω. The algorithm can also cope with systems having time delays. The algorithm also allows adaptive prefiltering.

Extensive simulation has shown that the method works very well for low order systems with time delay. This can of course be expected. For system with a large pole excess the direct approach does not work so well. There are several reasons for this. The output signal is almost sinusoidal which means that only two parameters can be determined. It is however possible to arrange the relay experiment so that the steady state gain can also be determined. The model (9) can then be determined.

There are some key design issues to be explored further. A major issue is the model complexity required. It is our guess that the simple model used in the examples will go a long way. It is a good idea to introduce some observer dynamics. This may be chosen as a function of the noise level. A design based on predictive control may also be considered.

Apart from uses as a PID regulator the algorithm is also ideally suited for initialization of adaptive regulators. In this case the initialization is executed under tight feedback conditions. The algorithm gives initial parameter estimates as well as estimates of sampling periods as well as an estimate of the achievable bandwidth.

7. References

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