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NELME has been tested and compared with a quasi-Newton algorithm without derivatives and with the Powell-Brent algorithm.

The result of the tests shows that it is hard to rank the algorithms.

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Abstract

This paper presents a Fortran IV subroutine called NELME that is a straightforward implementation of the simplex method for function minimization proposed by Nelder and Mead. NELME is written and documented according to the rules of the Scandinavian Control Library.

NELME has been tested and compared with a quasi-Newton algorithm without derivatives and with the Powell-Brent algorithm.

The result of the tests shows that it is hard to rank the algorithms.

1. INTRODUCTION

This paper presents a Fortran IV subroutine called NELME that is a straightforward implementation of the Nelder and Mead search. NELME has been tested and compared with a quasi-Newton algorithm without derivatives and with the Powell-Brent algorithm.

Chapter 2 describes NELME and the computer codes can be found in Appendix A. Chapter 3 treats the problem of evaluating NELME and test results are presented. Appendix B contains additional test results.

2. NELME

NELME is a Fortran IV subroutine that is a straightforward implementation of a simplex method for function minimization proposed by Nelder and Mead (1964). In this implementation the reflection coefficient is chosen to 1, the contraction coefficient to 0.5 and the expansion coefficient to 2.

NELME is written and documented according to the rules of the Scandinavian Control Library and consequently the subroutine within itself contains all adequate information how it is used. A listing of the subroutine NELME can be found in Appendix A.

3. EVALUATION

In Himmelblau (1972) chapter 5 and in Brent (1973) section 7.7 the problem of evaluate algorithms for unconstrained nonlinear programming is discussed and a number of algorithms are evaluated. The inventors have of course evaluated their method and Himmelblau (1972) discusses explicitly the method of Nelder and Mead.

Five different functions were used to test NELME. The functions, whose minimum value is zero, were:

(1) Rosenbrock's parabolic valley (Rosenbrock (1960))

$$y = 100(x_2-x_1^2)^2 + (1-x_1)^2,$$

starting point (-1.2,1),
minimum point (1,1)

(2) Wood's function (Colville (1968))

$$y = 100(x_2-x_1^2)^2 + (1-x_1)^2 + 90(x_4-x_3^2)^2 + (1-x_3)^2$$
$$+ 10.1((x_2-1)^2 + (x_4-1)^2) + 19.8(x_2-1)(x_4-1),$$

starting point (-3,-1,-3,-1),
minimum point (1,1,1,1)

(3) Fletcher and Powell's helical valley (Fletcher and Powell (1963))

$$y = 100(x_3-10\theta(x_1,x_2))^2 + (\sqrt{(x_1^2+x_2^2)} - 1)^2 + x_3^2$$

$$\text{where } 2\pi\theta(x_1,x_2) = \begin{cases} \arctan(x_2/x_1), & x_1 > 0 \\ \pi + \arctan(x_2/x_1), & x_1 < 0 \end{cases}$$

starting point (-1,0,0)
minimum point (1,0,0)

(4) Powell's quartic function (Powell (1962))

$$y = (x_1 + 10x_2)^2 + 5(x_3-x_4)^2 + (x_2-2x_3)^4 + 10(x_1-x_4)^4,$$

starting point (3,-1,0,1)
minimum point (0,0,0,0)

(5) A quadratic function with truncated linear terms

$$y = x_1^2 + e(|x_1|) + 5\,x_2^2 + e(x_2),$$

where $e(z) = \text{sign}(z)\cdot n$, n is the largest integer $\leq |z|$,
starting points (± 2, ± 2),
minimum point (0,0)

The properties of functions 1-4 are discussed in Brent (1973).

The progress of NELME on function 1-5 has been studied for different sizes of the initial simplex. The number of iterations, the number of function evaluations and the value of the test quantity (TESTQ) were noted when the function value had been reduced to 10^{-j} for j = 1, 3, 5, 7. TESTQ (to be compared in the algorithm with the desired accuracy in the minimum value) is given by

$$\text{TESTQ} = \sqrt{\frac{1}{n+1}\sum_{i=1}^{n+1} (f(x_i)-f(x_c))^2}$$

where n is the dimension of the optimization problem, the x_i:s are the vertices and x_c the centroid of the polyhedron and $f(\cdot)$ the function to be minimized.

The results can be found in Appendix B. The maximum number of times the loss function could be evaluated was set to 2000. If a number in the column "number of evaluations required" is 2000 or greater, it means that the polyhedron has degenerated (to a point) before reaching the minimum point.

In order to get a feeling for the relative efficiency of NELME, the problems 1-5 were solved with the help of NUFLET and POWBRE. NUFLET is based on a quasi-Newton method without derivatives. The method is described in Fletcher (1971). POWBRE is an implementation of a version of Powell's algorithm, modified by Brent (1973). Powell's algorithm is a conjugate direction method without derivatives. The computer codes (in Fortran) can be found in Källström (1978). NUFLET and POWBRE have some additional parameters that have to be set. The values of these parameters have been chosen according to Källström (1978),
i.e.

in NUFLET	DFN = -0.2	(estimate of the likely reduction to be obtained in f(x) is $0.2\|f(x_o)\|$. DFN is only used on the first iteration so an order of magnitude estimate suffices)
	XM=[1,...,1]	(no scaling wanted)
	HH=10^{-3}	(the step length used when calculating the gradient is 10^{-3})
	EPS=10^{-5}	(the accuracy required in x_i is 10^{-5})
	MODE=1	(the initial estimate of the Hessian matrix is set to the unit matrix)

and

in POWBRE	DIST=1	(estimated distance from initial approximation to minimum)
	SCALE=1	(no scaling wanted)
	TOL=10^{-6}	(wanted relative accuracy in x)
	MODE=1	(the algorithm is started with the coordinate axes as search directions)
	ILLCO=.TRUE.	(the problem is supposed to be illconditioned)
	NSTOP=1	(number of iterations without progress before termination)

The results for NELME with VDIST=1 (initial size of the simplex), NUFLET and POWBRE on functions 1-4 are given in Table 1-4.

Table 1

Number of iterations (n_i) and number of function evaluations (n_f) to reduce Rosenbrock's function to < 10^{-j}.

j	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
1	59	185	23	99	37	102
2	75	231	29	140	47	127
3	82	253	31	152	52	140
4	83	256	32	158	52	140
5	86	268	33	164	52	140
6	88	276	34	170	57	152
7	89	279	35	176	57	152

Table 2

Number of iterations (n_i) and number of function evaluations (n_f) to reduce Wood's function to < 10^{-j}.

j	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
1	30	105	68	664	253	681
2	35	125	72	704	265	709
3	35	125	73	714	270	719
4	81	269	74	724	287	758
5	94	310	75	734	287	758
6	103	344	76	744	294	777
7	108	358	77	754	299	787

Table 3

Number of iterations (n_i) and number of function evaluations (n_f) to reduce Fletcher and Powell's helical valley to < 10^{-j}.

j	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
1	154	480	18	102	43	110
2	172	533	24	158	57	143
3	185	576	28	191	63	158
4	189	590	29	199	63	158
5	193	606	29	199	67	166
6	196	616	30	207	67	166
7	200	632	31	215	73	180

Table 4

Number of iterations (n_i) and number of function evaluations (n_f) to reduce Powell´s quartic function to < 10^{-j}.

j	NELME		NUFLET		POWBRE	
	n_i	n_f	n_i	n_f	n_i	n_f
1	31	99	9	61	38	91
2	56	177	10	67	43	101
3	68	206	12	79	48	111
4	70	218	22	161	55	130
5	76	238	23	171	65	152
6	78	246	25	191	72	171
7	85	276	27	211	77	182

The results speak for themselves and only a few comments will be given here. As seen from the results in Appendix B the size of the initial simplex has a significant effect on the speed of the convergence. Wood's function is an illustrative example. The result shown in Table 2 (the size of the initial simplex is 1) is very good, but NELME fails for some sizes of the initial simplex. POWBRE too had difficulties and stopped at (-0.988, 0.986, -0.949, 0.913) and declared that this point is a minimum. But when NSTOP was increased to 5, POWBRE found the minimum point and it is these results that are given in Table 2. Brent (1973) reports that with DIST=10 POWBRE after 191 linear searches and 452 function evaluations had reduced the function value to $6 \cdot 10^{-14}$.

This dependence, which is not *a priori* known, makes it difficult to rank NELME, NUFLET and POWBRE. Function 5 is discontinuous when x_1 or x_2 is an integer and one may suspect that NUFLET and POWBRE might fail. As seen from the results in Table 5 this is the case.

If no special information is available that can confirm the result of a "minimization algorithm" to be a close approximation to the solution, how can it then be decided whether the solution of the problem is found or not? This is a hard question. This problem is discussed in Murray (1972). He gives the following advice:

1) Check the rate of convergence
2) Restart the routine
3) Try other starting points, input parameters, rescale the variables
4) Try a different method

Table 5

Number of iterations (n_i) and number of function evaluations (n_f) to reduce function 5 to 10^{-7} for different starting points.

starting point	NELME n_i	NELME n_f	NUFLET n_i	NUFLET n_f	POWBRE n_i	POWBRE n_f
(2, 2)	28	94	8	44	12	34
(2,-2)	24	86	failed 1)		failed 2)	
(-2, 2)	31	104	8	69	12	34
(-2, -2)	26	90	12	87	failed 3)	

1) stopped at (1.9996, -2.0098)
2) stopped at $(-1.53\cdot 10^{-3}, -2)$
3) stopped at $(1.01\cdot 10^{-3}, -2)$

4. REFERENCES

Brent, R.P. (1978): Algorithms for minimization without derivatives, Prentice-Hall, Inc. Englewood Cliffs, N.Y. (7.7).

Colville, A.R. (1968): A comparative study of nonlinear programming codes, IBM New-York Scientific Center, Report 320-2949.

Elmqvist, Tyssö, Wieslander (1976): Programming and documentation rules for subroutine libraries - designed for the Scandinavian Control Library, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden. CODEN:LUTFD2/(TFRT-3139)/1-041/(1976).

Fletcher, R., and Powell, M.Y.D. (1963): A rapidly convergent descent method for minimization, The Computer Journal, Vol. 6, p. 163,

Fletcher. R. (1971): Fortran subroutines for minimization by quasi-Newton methods, Report AERE-R7125, Harwell.

Himmelblau, D.M. (1972): Applied nonlinear programming, Mc Gray-Hill Book Company, New York (p. 148-157, 190-217).

Källström, C.G. (1978): LISPID-user´s manual, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden. CODEN: LUTFD2/(TFRT-7147)/1-147/(1978)

Murray, W. (1972): Numerical methods for unconstrained optimization, Academic Press, London (p. 119-122).

Nelder, Y.A. and Mead, R. (1964): A simplex method for function minimization, The Computer Journal, Vol. 7, p. 308.

Powell, M.Y.D. (1962): An iterative method for finding stationary value of a function of several variables, The Computer Journal, Vol. 5. p. 147.

Rosenbrock, H. (1960): An automatic method for finding the greatest or least value of a function, The Computer Journal, Vol.3, p.175.

APPENDIX A

Computer Codes

NELME 16

NEWX 24

PNELME 25

```
C       NAME: NELME                                      NUMBER:
C       -----                                            -------
C
C       SUBTITLE: THE FLEXIBLE POLYHEDRON SEARCH OF NELDER AND MEAD
C       ---------
C
C       LANGUAGE: FORTRAN IV
C       ---------
C
C       KEYWORDS: NONLINEAR PROGRAMMING, NONLINEAR OPTIMIZATION,
C       --------- PARAMETRIC OPTIMIZATION, NELDER-MEAD SEARCH
C
C       IMPLEMENTOR: SVEN ERIK MATTSSON                  DATE: 1978-06-19
C       ------------                                     -----
C
C       INSTITUTE: DEPARTMENT OF AUTOMATIC CONTROL
C       ---------- LUND INSTITUTE OF TECHNOLOGY, SWEDEN
C
C       ACCEPTED:                                        VERSION:
C       ---------                                        --------
C
C=======================================================================
C
C       PURPOSE
C       =======
C
C       TO FIND THE MINIMUM OF A FUNCTION F(X) OF SEVERAL VARIABLES
C       WITHOUT CALCULATING DERIVATIVES.
C
C       USAGE
C       =====
C
C       PROGRAM TYPE: SUBROUTINE
C       -------------
C
C       ARGUMENTS:
C       ----------
C
C       NELME(FUNC,X,FMIN,N,MODE,VDIST,MAXFN,EPS,IERR,IPRINT,PRINT,WX,WF)
C
C       FUNC    - NAME OF SUBROUTINE WHICH COMPUTES THE LOSS FUNCTION
C                 TO BE MINIMIZED, (I)
C       X       - REAL VECTOR, SIZE(N), (I/O)
C                 SEE NOTE 1 AND 2!
C       FMIN    - REAL CONTAINING, IF SUCCESS, THE MINIMUM VALUE, (O)
C                 SEE NOTE 2!
C       N       - ACTUAL DIMENSION OF THE OPTIMIZATION PROBLEM, (I)
C                 N>1.
C       MODE    - INDICATOR TO CONTROL THE CALCULATION OF THE STARTING
C                 POLYHEDRON, (I)
C                 HAS TO BE 0 OR 1. SEE NOTE 1!
C       VDIST   - DISTANCE BETWEEN TWO VERTICES IN THE STARTING
C                 POLYHEDRON, (I)
C                 SEE NOTE 1!
C       MAXFN   - MAXIMUM NUMBER OF TIMES FUNC CAN BE CALLED, (I)
C                 MAXFN>N+1
C       EPS     - STOPPING CRITERION, (I)
C                 SEE NOTE 2 AND 3!
C       IERR    - ERROR INDICATOR, (O)
C                 -1: THE SEARCH IS TO BE CONTINUED, USED ONLY IN PRINT.
C                  0: SUCCESS
C                  1: INCORRECT CALL OF NELME
C                     N<2
C                     INVALID MODE-VALUE
```

```
C             MAXFN(N+1
C           2: FUNC HAS BEEN CALLED WITH AN ILLEGAL X AND THIS
C              X CAN BE FOUND IN NELME'S WX(.,N+2). SEE NOTE 2!
C           3: FUNC CALLED MAXFN TIMES. SEE NOTE 2!
C  IPRINT - PRINT INDICATOR. (I)
C           PRINTING OCCURS ON ENTRY, EVERY ABS(IPRINT) ITERATIONS
C           AND ON EXIT.
C           ALL INTERMEDIATE PRINTING CAN BE SUPPRESSED BY
C           SETTING IPRINT)MAXFN.
C           ALL PRINTING CAN BE SUPPRESSED BY SETTING IPRINT=0.
C  PRINT  - NAME OF SUBROUTINE WHICH PERFORMS THE PRINTING. (I)
C           IF IPRINT=0, PRINT IS NOT CALLED.
C  WX     - WORK AREA, SIZE(N,N+4)
C           SEE NOTE 1!
C  WF     - WORK AREA, SIZE(N+4)
C
C  NOTE:
C  -----
C  1) THE INITIAL VALUES CAN BE GIVEN IN TWO DIFFERENT WAYS:
C     MODE=0: NELME CALCULATES AN INITIAL POLYHEDRON WITH X AS THE
C             CENTROID AND WITH THE DISTANCE BETWEEN TWO VERTICES
C             EQUAL TO VDIST AND STORES THESE VERTICES IN THE
C             FIRST N+1 COLUMNS OF WX. THIS IS THE NORMAL MODE.
C     MODE=1: THE INITIAL POLYHEDRON IS GIVEN IN THE N+1 FIRST
C             COLUMS OF WX. THIS MODE CAN BE USED IF THE ROUTINE IS
C             TO BE RESTARTED. VDIST IS NOT USED IN THIS MODE.
C  2) AFTER THE EXIT FROM NELME AND IF IERR IS NOT EQUAL TO 1, THE
C     RESULTS OF THE SEARCH ARE IN X AND FMIN.
C  3) THE STOPPING CRITERION USED FOR HALTING THE ROUTINE IS
C     SQRT(SUM((F(X(I))-F(XC))**2)/(N+1)) < EPS,
C     WHERE THE X(I):S ARE THE VERTICES AND XC THE CENTROID OF THE
C     POLYHEDRON AND F(.) THE FUNCTION TO BE MINIMIZED.
C     THE SUCCESS OF THE CRITERION DEPENDS ON THE POLYHEDRON NOT
C     BECOMING TOO SMALL IN RELATION TO THE CURVATURE OF THE
C     SURFACE UNTIL THE FINAL MINIMUM IS REACHED.
C
C
C  USER SUPPLIED SUBROUTINES:
C  --------------------------
C
C  SUBROUTINE FUNC(X,F,FX,FXX,N,ICONT,IERR1,IFXX)
C
C  DEFINES THE FUNCTION F(X) TO BE MINIMIZED IN NELME.
C  X      - ARGUMENT, SIZE(N),(I)
C  F      - RETURNED FUNCTION VALUE, (O)
C  FX     - GRADIENT VECTOR, SIZE(N),(O)
C           NOT USED IN NELME
C  FXX    - HESSIAN MATRIX, SIZE(N,N), DIMENSIONED(IFX,.), (O)
C           NOT USED IN NELME
C  N      - DIMENSION OF X, (I)
C  ICONT  - CONTROL VARIABLE, (I)
C             0: COMPUTE F
C             1: COMPUTE F, FX
C             2: COMPUTE F, FX, FXX
C           ICON IS SET TO ZERO AT CALLING FROM NELME
C  IERR1  - ERROR INDICATOR OF FUNC, (O)
C             0: SUCCESS
C             1: DIFFICULTIES
C            -1: ILLEGAL CALL OF FUNC. FX OR FXX WANTED (ICONT= 1,2)
C                BUT THE THE COMPUTATION OF FX OR FXX IS NOT
C                IMPLEMENTED IN FUNC
C  IFXX   - DIMENSION PARAMETER OF IFXX, (I)
C
C  THE ACTUAL SUBROUTINE NAME MUST BE DECLARED EXTERNAL IN THE
```

```
C     PROGRAM THAT CALLS NELME. IF ICONT IS 0 (1) THEN FX AND FXX
C     (FXX) MUST NOT BE USED IN FUNC IN ORDER TO MAKE IT POSSIBLE
C     TO CALL FUNC WITH DUMMIES.
C
C
C     SUBROUTINE PRINT(WX,WF,N,NITE,NFN,TESTQ,IPRINT,IERR)
C
C     AT THE BEGINNING OF THE FIRST ITERATION, ON EVERY SUBSEQUENT
C     ABS(IPRINT) ITERATIONS AND ON EXIT OF NELME PRINT IS CALLED BY
C     NELME IN ORDER TO PRINT A MESSAGE. IF IPRINT=0, PRINT IS NOT
C     CALLED AND PRINT MAY BE A DUMMY.
C
C     WX     - DEFINES THE POLYHEDRON, SIZE(N,N+4),DIMENSIONED(N,.),
C              (I)
C              THE FIRST N+1 COLUMNS OF WX CONTAIN THE VERTICES OF
C              THE POLYHEDRON.
C     WF     - VECTOR CONTAINING THE VALUES OF THE OBJECTIVE FUNCTION
C              AT THE VERTICES, SIZE(N+4), (I)
C     N      - ACTUAL DIMENSION OF THE OPTIMIZATION PROBLEM, (I)
C     NITE   - ITERATION NUMBER, (I)
C     NFN    - NUMBER OF CALLS OF FUNC, (I)
C     TESTQ  - OBTAINED STOPPING CRITERION, (I)
C     IPRINT - PRINT INDICATOR, (I)
C              SEE THE ARGUMENT LIST OF NELMF.
C     IERR   - ERROR INDICATOR OF NELME, (I)
C              SEE THE ARGUMENT LIST OF NELME.
C
C     THE ACTUAL SUBROUTINE NAME MUST BE DECLARED EXTERNAL IN THE
C     PROGRAM THAT CALLS NELME. IF IPRINT IS SET TO ZERO AND A DUMMY
C     IS GOING TO BE USED AS PRINT AND IF THIS DUMMY IS NOT DECLARED
C     EXTERNAL, IT NEED NOT BE INCLUDED AT LOADING.
C     THE SUBROUTINE PNELME IN THE LIBRARY CAN BE USED AS THE ACTUAL
C     SUBROUTINE PRINT.
C
C
C     METHOD
C     ======
C
C     THIS ROUTINE IS A STRAIGHTFORWARD IMPLEMENTATION OF A FLEXIBLE
C     POLYHEDRON SEARCH DESCRIBED IN NELDER,J.A. AND MEAD,R. (1964).
C     THE REFLECTION COEFFICIENT IS CHOSEN TO 1, THE CONTRACTION
C     COEFFICIENT TO 1/2 AND THE EXPANSION COEFFICIENT TO 2.
C     THE METHOD OF CALCULATING A STARTING POLYHEDRON CAN BE FOUND IN
C     HIMMELBLAU,D.M. (1972).
C
C
C     REFERENCES:
C     -----------
C     1. NELDER,J.A. AND MEAD,R. (1964) 'A SIMPLEX METHOD FOR FUNCTION
C        MINIMIZATION', THE COMPUTER JOURNAL, VOL. 7, P. 308
C     2. HIMMELBLAU,D.M. (1972) 'APPLIED NONLINEAR PROGRAMMING',
C        MC GRAW HILL BOOK COMPANY, NEW YORK, P. 148-157
C
C
C     CHARACTERISTICS
C     ===============
C
C     SIZE:
C     -----
C     PDP 15: 1716
C     PDP 11: 1570
C
C     LIBRARY SUBROUTINES REQUIRED: RMACON, MMOVE
C     -------------------------------------------
```

```
C
C      INTERNAL SUBROUTINES REQUIRED: NEWX
C      ------------------------------------
C
C-----------------------------------------------------------------------
C
C
       SUBROUTINE NELME(FUNC,X,FMIN,N,MODE,VDIST,MAXFN,EPS,IERR,
      *                 IPRINT,PRINT,WX,WF)
C
       DIMENSION X(1),WX(N,1),WF(1)
C
C       ALFA : REFLECTION COEFFICIENT
C      -BETA : CONTRACTION COEFFICIENT
C      -GAMMA: EXPANSION COEFFICIENT
C
       DATA ALFA,BETA,GAMMA/1.,-0.5,-2./
       DATA ICONT /0/
C
C      VALIDITY TEST AND CALCULATION OF INITIAL VALUES
C
       NPLUS1=N+1
       NPLUS2=N+2
       NPLUS3=N+3
       NPLUS4=N+4
       FLN=FLOAT(N)
       FLNPL1=FLOAT(NPLUS1)
       A=100.0*RMACON(3)
       IF(EPS.LT.A) EPS=A
       IERR=1
       IF(N.LT.2 .OR. MAXFN.LE.NPLUS1) RETURN
C
C      IS THE INITIAL POLYHEDRON GIVEN OR HAS IT TO BE CALCULATED?
C
       IF(MODE.EQ.1) GO TO 50
       IF(MODE.NE.0) RETURN
C
C      CALCULATE THE INITIAL POLYHEDRON. LET X BE THE CENTROID AND
C      VDIST THE DISTANCE BETWEEN TWO VERTICES AND STORE THE VERTICES
C      IN THE FIRST N+1 COLUMNS OF WX
C
       VDIST1=VDIST
       IF(ABS(VDIST1).LE.A) VDIST1=A
       A=SQRT(2.0)
       B=SQRT(FLNPL1)
       D3=-VDIST1/(A*B)
       C=VDIST1/(A*FLN)
       B=B-1.
       D2=C*B+D3
       D1=C*(B+FLN)+D3
       DO 40 I=1,N
       A=X(I)
       B=A+D2
       WX(I,NPLUS1)=A+D3
       DO 30 J=1,N
       WX(I,J)=B
    30 CONTINUE
       WX(I,I)=A+D1
    40 CONTINUE
C
C      NOW THE INITIAL POLYHEDRON EXISTS. CALCULATE THE VALUE OF THE
C      OBJECTIVE FUNCTION AT THE VERTICES OF THE INITIAL POLYHEDRON.
C
    50 CONTINUE
```

```
      IERR=-1
      DO 60 IX=1,NPLUS1
      CALL FUNC(WX(1,IX),WF(IX),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
      IF(IERR1.NE.0) GO TO 400
   60 CONTINUE
C
C     NFN-NUMBER OF CALLS OF FUNC
C     NITE-ITERATION NUMBER
C     NPRINT-COUNTER TO CONTROL PRINT
C     IABSPR-ABS(IPRINT)
C
      NFN=NPLUS1
      NITE=0
      IABSPR=IABS(IPRINT)
C
C     PRINT START MESSAGE.
C
      TESTQ=0.
      DO 65 J=NPLUS2,NPLUS4
      WF(J)=0.
      DO 65 I=1,N
      WX(I,J)=0.
   65 CONTINUE
      IF(IPRINT.NE.0) CALL PRINT(WX,WF,N,NITE,NFN,TESTQ,IPRINT,IERR)
      NPRINT=IABSPR
C
C     END OF INITIALIZATION
C
C-----------------------------------------------------------------------
C
C     START OF AN ITERATION LOOP
C
   70 CONTINUE
      NITE=NITE+1
C
C     FIND THE VERTICES WITH THE HIGHEST AND LOWEST VALUE OF
C     THE OBJECTIVE FUNCTION AND LET WX(.,NPLUS2) BE THE CENTROID OF
C     ALL THE VERTICES EXCLUDING THAT WITH THE HIGHEST VALUE.
C
      IHIGH=NPLUS1
      ILOW=NPLUS1
      DO 80 I=1,N
      IF(WF(I).GT.WF(IHIGH)) IHIGH=I
      IF(WF(I).LT.WF(ILOW)) ILOW=I
   80 CONTINUE
      DO 100 I=1,N
      A=-WX(I,IHIGH)
      DO 90 J=1,NPLUS1
      A=A+WX(I,J)
   90 CONTINUE
      WX(I,NPLUS2)=A/FLN
  100 CONTINUE
C
C     CALCULATE THE FUNCTION VALUE AT WX(.,NPLUS2).
C
      NFN=NFN+1
      IX=NPLUS2
      CALL FUNC(WX(1,NPLUS2),WF(NPLUS2),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
      IF(IERR1.NE.0) GO TO 500
C
C     CALCULATE THE REFLECTION POINT
C     WX(.,NPLUS3)=WX(.,NPLUS2)+ ALFA*(WX(.,NPLUS2)-WX(.,IHIGH))
C     AND VALUE (WF(NPLUS3)) AT THAT POINT.
C
```

```
      CALL NEWX(WX(1,NPLUS2),WX(1,IHIGH),ALFA,N,WX(1,NPLUS3))
      NFN=NFN+1
      IX=NPLUS3
      CALL FUNC(WX(1,NPLUS3),WF(NPLUS3),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
      IF(IERR1.NE.0) GO TO 500
C
C     TEST FUNCTION VALUE AT REFLECTION POINT
C
      IF(WF(NPLUS3).GE.WF(ILOW)) GO TO 120
C
C     WF(NPLUS3) < WF(ILOW).
C     TRY EXPANSION! CALCULATE EXPANSION POINT
C     WX(.,NPLUS4)= WX(.,NPLUS2)+GAMMA*(WX(.,NPLUS2)-WX(.,NPLUS3))
C     AND VALUE (WF(NPLUS4)) AT THAT POINT.
C
      CALL NEWX(WX(1,NPLUS2),WX(1,NPLUS3),GAMMA,N,WX(1,NPLUS4))
      NFN=NFN+1
      IX=NPLUS4
      CALL FUNC(WX(1,NPLUS4),WF(NPLUS4),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
      IF(IERR1.NE.0) GO TO 500
C
C     TEST FUNCTION VALUE AT EXPANSION POINT.
C
      IF(WF(NPLUS4).GE.WF(ILOW)) GO TO 110
C
C     ACCEPT WX(.,NPLUS4) AS A NEW VERTEX OF THE POLYHEDRON.
C     REPLACE WX(.,IHIGH) BY WX(.,NPLUS4).
C
      CALL MMOVE(WX(1,NPLUS4),WX(1,IHIGH),N,1,0,N,N)
      WF(IHIGH)=WF(NPLUS4)
      GO TO 170
C
C     ACCEPT WX(.,NPLUS3) AS A NEW VERTEX OF THE POLYHEDRON.
C     REPLACE WX(.,IHIGH) BY WX(.,NPLUS3).
C
  110 CONTINUE
      CALL MMOVE(WX(1,NPLUS3),WX(1,IHIGH),N,1,0,N,N)
      WF(IHIGH)=WF(NPLUS3)
      GO TO 170
C
C     AT THIS POINT OF THE PROGRAM IT IS KNOWN THAT WF(NPLUS3) >
C     WF(ILOW), BUT IS WF(NPLUS3) > WF(I) FOR ALL I=1,...,N+1 AND
C     I NOT EQUAL TO IHIGH ?
C
  120 CONTINUE
      DO 130 I=1,NPLUS1
      IF(WF(NPLUS3).GT.WF(I)) GO TO 130
      IF(I.EQ.IHIGH) GO TO 130
      GO TO 110
  130 CONTINUE
C
C     YES, WF(NPLUS3) > WF(I) FOR ALL I=1,...,N+1 AND I=/IHIGH, BUT
C     IS WF(NPLUS3) > WF(IHIGH) ?
C
      IF(WF(NPLUS3).GE.WF(IHIGH)) GO TO 150
C
C     NO, WF(NPLUS3) < WF(IHIGH). ACCEPT WX(.,NPLUS3) AS A NEW VERTEX
C     OF THE POLYHEDRON.
C     REPLACE WX(.,IHIGH) BY WX(.,NPLUS3).
C
      CALL MMOVE(WX(1,NPLUS3),WX(1,IHIGH),N,1,0,N,N)
      WF(IHIGH)=WF(NPLUS3)
      IHIGH=NPLUS1
      DO 140 I=1,N
```

```
      IF(WF(I).GT.WF(IHIGH)) IHIGH=I
  140 CONTINUE
C
C     TRY CONTRACTION! CALCULATE CONTRACTION POINT
C     WX(.,NPLUS4)=WX(.,NPLUS2)+BETA*(WX(.,NPLUS2)+WX(.,IHIGH))
C     AND VALUE AT (WF(NPLUS4) THAT POINT.
C
      CALL NEWX(WX(1,NPLUS2),WX(1,IHIGH),BETA,N,WX(1,NPLUS4))
      NFN=NFN+1
      IX=NPLUS4
      CALL FUNC(WX(1,NPLUS4),WF(NPLUS4),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
      IF(IERR1.NE.0) GO TO 500
C
C     TEST FUNCTION VALUE AT CONTRACTION POINT.
C
      IF(WF(NPLUS4).GT.WF(IHIGH)) GO TO 150
C
C     WF(NPLUS4) < WF(IHIGH). ACCEPT WX(.,NPLUS4) AS A NEW VERTEX OF
C     THE POLYHEDRON.
C     REPLACE WX(.,IHIGH) BY WX(.,NPLUS3).
C
      CALL MMOVE(WX(1,NPLUS4),WX(1,IHIGH),N,1,0,N,N)
      WF(IHIGH)=WF(NPLUS4)
      GO TO 170
C
C     REDUCTION: REPLACE ALL WX(.,I) BY WX(.,ILOW)-0.5*(WX(.,ILOW)-
C                WX(.,I))
C
  150 CONTINUE
      NFN=NFN+N
      DO 160 IX=1,NPLUS1
      IF(IX.EQ.ILOW) GO TO 160
      CALL NEWX(WX(1,ILOW),WX(1,IX),-0.5,N,WX(1,IX))
      CALL FUNC(WX(1,IX),WF(IX),DUM1,DUM2,N,ICONT,IERR1,IDUM2)
      IF(IERR1.NE.0) GO TO 500
  160 CONTINUE
C
C     TEST OF CONVERGENCE
C
C     CALCULATE THE TEST QUANTITY
C     TESTQ=SQRT(SUM((WF(I)-WF(NPLUS2))**2/(N-1)))
C
  170 CONTINUE
      A=0
      DO 180 I=1,NPLUS1
      B=WF(I)-WF(NPLUS2)
      A=A+B*B
  180 CONTINUE
      TESTQ=SQRT(A/FLNPL1)
      IF(TESTQ.LT.EPS) GO TO 200
C
C     THE CONVERGENCE CRITERION IS NOT FULFILLED.
C     TEST NUMBER OF FUNCTION EVALUATIONS
C
      IF(NFN.GE.MAXFN) GO TO 190
C
C     PRINT A MESSAGE AND DO A NEW ITERATION.
C
      NPRINT=NPRINT-1
      IF(NPRINT.GT.0 .OR. IPRINT.EQ.0) GO TO 70
      CALL PRINT(WX,WF,N,NITE,NFN,TESTQ,IPRINT,IERR)
      NPRINT=IABSPR
      GO TO 70
C
```

```
C      END OF AN ITERATION LOOP
C------------------------------------------------------------------------
C
C      FUNC HAS BEEN CALLED MORE THAN MAXFN TIMES.
C
  190 CONTINUE
      IERR=3
      GO TO 205
C
C      THE CONVERGENCE CRITERION IS FULFILLED, BUT THIS DOES NOT
C      CERTAINLY MEAN THAT A LOCAL MINIMUM IS FOUND.
C
  200 CONTINUE
      IERR=0
C
C      OUTPUT THE RESULT, PRINT A MESSAGE AND RETURN.
C
  205 CONTINUE
      DO 210 I=1,NPLUS1
      IF(WF(I).LT.WF(ILOW)) ILOW=I
  210 CONTINUE
      CALL MMOVE(WX(1,ILOW),X,N,1,0,N,N)
      FMIN=WF(ILOW)
      IF(IPRINT.NE.0) CALL PRINT(WX,WF,N,NITE,NFN,TEST0,IPRINT,IERR)
      RETURN
C
C      FUNC HAS BEEN CALLED WITH AN ILLEGAL X (WX(.,IX)) IN THE
C      INITIALIZATION PART.
C
  400 CONTINUE
      TEST0=0.
      DO 410 J=IX,NPLUS1
      WF(J)=0.
  410 CONTINUE
      DO 420 J=NPLUS2,NPLUS4
      WF(J)=0.
      DO 420 I=1,N
      WX(I,J)=0.
  420 CONTINUE
C
C      FUNC HAS BEEN CALLED WITH AN ILLEGAL X (WX(.,IX)).
C      MOVE THIS X TO WX(.,NPLUS2).
C
  500 CONTINUE
      IERR=2
      IF(IX.NE.NPLUS2) CALL MMOVE(WX(1,IX),WX(1,NPLUS2),N,1,0,N,N)
      GO TO 205
      END
```

```
      SUBROUTINE NEWX(X1,X2,ALFA,N,X3)
C
C     CALCULATES X3=X1+ALFA*(X1-X2)
C
      DIMENSION X1(1),X2(1),X3(1)
      DO 10 I=1,N
      X3(I)=X1(I)+ALFA*(X1(I)-X2(I))
   10 CONTINUE
      RETURN
      END
```

```
C       NAME: PNELME                                           NUMBER:
C       ----                                                   ------
C
C       SUBTITLE: I/O ROUTINE OF THE SUBROUTINE NELME
C       --------
C
C       LANGUAGE: FORTRAN IV
C       --------
C
C       KEYWORDS:
C       --------
C
C       IMPLEMENTOR: SVEN ERIK MATTSSON                        DATE: 1978-06-19
C       -----------                                            ----
C
C       INSTITUTE: DEPARTMENT OF AUTOMATIC CONTROL
C       ---------  LUND INSTITUTE OF TECHNOLOGY, SWEDEN
C
C       ACCEPTED:                                              VERSION:
C       --------                                               -------
C
C=============================================================================
C
C       PURPOSE
C       =======
C
C       TO BE USED AS THE ACTUAL SUBROUTINE PRINT IN THE SUBROUTINE NELME.
C       SEE THE DOCUMENTATION OF THE SUBROUTINE NELME.
C
C       USAGE
C       =====
C
C       PROGRAMTYPE: SUBROUTINE
C       -----------
C
C       ARGUMENTS: SEE THE DOCUMENTATION OF THE SUBROUTINE NELME.
C       ---------
C
C       NOTE:
C       ----
C
C       IN THE DOCUMENTATION OF NELME IT IS DESCRIBED HOW THE CALLING OF
C       PRINT IN NELME IS CONTROLLED.
C       THE PRINTING STARTS ON A NEW PAGE WITH THE TEXT
C       'PRINT FROM NELME'.
C       THE PRINTING IS IN THE FORM
C
C       ITERATION NO.=I6 NO. OF CALLS OF FUNC=I6 STOPPING CRITERION=G6.3
C       F(X)      X
C       'F(X1)' 'X1(1)' 'X1(2)' ...' X1(7)'
C               'X1(8)' 'X1(9)' ...  X1(14)'
C                .................. ........
C                ............... X1(N+1)'
C       'F(X2)' 'X2(1)' ......................
C                .............................
C        .....................................
C       'F(XNPLUS1)'..........................
C                         ....................
C
C       THE FUNCTION VALUES ('FX(XI)' ) HAVE THE FORMAT G14.7 AND THE
C       COORDINATES OF THE VERTICES (XI(J)) HAVE THE FORMATS G16.7.
C       IF IPRINT<0, THE PRINTING OF ALL VERTICES AND THEIR FUNCTION
C       VALUES IS SUPPRESSED AND ONLY THE VERTEX WITH THE LOWEST VALUE
C       AND ITS FUNCTION VALUE IS PRINTED.
```

```
C      ON EXIT FROM NELME A SHORT MESSAGE IS PRINTED THAT TELLS WHY
C      NELME EXITS.
C
C-----------------------------------------------------------------------
C
C
       SUBROUTINE PNELME(WX,WF,N,NITE,NFN,TESTQ,IPRINT,IERR)
C
       DIMENSION WX(N,1),WF(1)
       DATA LOUT/6/
C
       NPLUS1=N+1
C
C      ON ENTRY OF NELME, START ON A NEW PAGE.
C
       IF(NITE.EQ.0) WRITE(LOUT,500)
   500 FORMAT(1H1,16HPRINT FROM NELME/1X,16(1H*)//)
C
C      PRINT NITE, NFN AND TESTQ.
C
       WRITE(LOUT,510) NITE,NFN,TESTQ
   510 FORMAT(/1X,14HITERATION NO.=,I6,2X,21HNO. OF CALLS OF FUNC=,I6,
      *        2X,19HSTOPPING CRITERION=,G10.3)
       WRITE(LOUT,520)
   520 FORMAT(1X,4HF(X),11X,1HX)
C
C      IS THE PRINTING OF ALL VERTICES SUPPRESSED?
C
       IF(IPRINT.LT.0) GO TO 10
C
C      NO, PRINT ALL VERTICES AND THEIR VALUES.
C
       ISTART=1
       IEND=NPLUS1
       GO TO 30
C
C      YES, PRINT ONLY THE VERTEX WITH THE LOWEST VALUE.
C      FIND THIS VERTEX.
C
    10 CONTINUE
       ILOW=NPLUS1
       DO 20 I=1,N
       IF(WF(I).LT.WF(ILOW)) ILOW=I
    20 CONTINUE
       ISTART=ILOW
       IEND=ILOW
    30 CONTINUE
       DO 40 I=ISTART,IEND
    40 WRITE(LOUT,530) WF(I),(WX(J,I),J=1,N)
   530 FORMAT(1X,G14.7,7G16.7/(15X,7G16.7))
C
C      IF IERR=1, THE SEARCH OF BETTER VALUES IS GOING TO BE CONTINUED.
C
       IF(IERR.EQ.-1) RETURN
C
C      IF IERR=0, THE STOPPING CRITERION IS SATISFIED.
C
       IF(IERR.NE.0) GO TO 50
       WRITE(LOUT,540)
   540 FORMAT(//1X,28HSTOPPING CRITERION SATISFIED/1H1)
       RETURN
C
C      IF IERR=2, FUNC HAS BEEN CALLED WITH AN ILLEGAL X THAT CAN BE
C      FOUND IN WX(.,NPLUS2).
```

```
C
   50 CONTINUE
      IF(IERR.NE.2) GO TO 60
      NPLUS2=N+2
      WRITE(LOUT,550) (WX(I,NPLUS2),I=1,N)
  550 FORMAT(//1X,40HFUNC HAS BEEN CALLED WITH THE ILLEGAL X=
     *        /(1X,8G16.7))
      WRITE(LOUT,560)
  560 FORMAT(1H1)
      RETURN
C
C     IERR=3.
C
   60 WRITE(LOUT,570)
  570 FORMAT(//1X,23HFUNC CALLED MAXFN TIMES/1H1)
      RETURN
      END
```

APPENDIX B

Results from the minimization of five functions using NELME.

Function 1	(Rosenbrock´s function)	29
2	(Wood´s function)	37
3	(Fletcher and Powell´s helical valley)	45
4	(Powell´s quartic function)	53
5	(A quadratic function with truncated linear terms)	61

If a number in the column "number of evaluations required" is 2000 or greater, it means that NELME failed. The polyhedron has degenerated before reaching the minimum point.

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2
UNTIL F(X) < 1.E-1 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	64	204	0.141E-01
0.2	51	159	0.213E-01
0.3	70	214	0.294E-01
0.4	60	191	0.406E-01
0.5	66	209	0.288E-01
0.6	59	191	0.184E-01
0.7	72	220	0.150E-01
0.8	47	151	0.443E-01
0.9	70	220	0.285E-02
1.0	59	185	0.195E-01
1.1	63	200	0.547E-02
1.2	63	198	0.536E-02
1.3	51	164	0.712E-01
1.4	64	202	0.651E-01
1.5	48	149	0.207E-01
1.6	58	187	0.181E-01
1.7	63	198	0.498E-01
1.8	63	192	0.155E-01
1.9	55	175	0.359E-02
2.0	72	229	0.925E-02
2.1	59	183	0.338E-01
2.2	53	171	0.342E-01
2.3	69	214	0.133E-01
2.4	58	187	0.249E-01
2.5	64	202	0.464E-01
2.6	86	271	0.161E-01
2.7	89	272	0.242E-01
2.8	80	252	0.668E-01
2.9	81	255	0.538E-01
3.0	92	284	0.457E-01
3.1	8	32	0.110
3.2	10	38	0.743E-01
3.3	6	26	0.877
3.4	1	8	145.
3.5	6	28	0.311
3.6	5	24	2.38
3.7	102	321	0.115E-01
3.8	99	309	0.990E-02
3.9	30	98	0.171E-01
4.0	43	143	0.345E-01

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2
UNTIL F(X) < 1.E-1 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	113	354	0.371E-01
4.2	95	297	0.495E-01
4.3	111	353	0.411E-02
4.4	96	306	0.295E-01
4.5	84	271	0.198E-01
4.6	88	280	0.274E-01
4.7	93	296	0.261E-01
4.8	112	344	0.133E-01
4.9	92	288	0.162E-01
5.0	92	286	0.133E-01
5.1	87	277	0.122E-01
5.2	94	295	0.205E-01
5.3	105	321	0.241E-01
5.4	87	275	0.733E-01
5.5	105	330	0.861E-02
5.6	101	316	0.413E-01
5.7	111	356	0.354E-02
5.8	95	296	0.304E-01
5.9	96	309	0.720E-02
6.0	90	285	0.113E-01
6.1	105	326	0.506E-01
6.2	87	276	0.620E-01
6.3	67	213	0.215E-01
6.4	57	184	0.335E-01
6.5	73	236	0.302E-01
6.6	113	354	0.409E-01
6.7	95	296	0.129E-01
6.8	108	343	0.258E-01
6.9	102	325	0.744E-01
7.0	103	327	0.737E-01
7.1	92	283	0.412E-01
7.2	96	305	0.264E-01
7.3	109	341	0.313E-01
7.4	122	387	0.666E-02
7.5	95	306	0.259E-01
7.6	102	316	0.391E-01
7.7	116	361	0.232E-01
7.8	111	346	0.116E-01
7.9	110	343	0.153E-01
8.0	100	307	0.174E-01

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2
UNTIL F(X) < 1.E-3 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	83	263	0.105E-02
0.2	70	216	0.201E-03
0.3	85	261	0.859E-03
0.4	77	246	0.191E-02
0.5	81	255	0.786E-03
0.6	72	233	0.179E-02
0.7	85	259	0.219E-02
0.8	67	212	0.149E-02
0.9	89	273	0.519E-03
1.0	82	253	0.366E-03
1.1	82	261	0.734E-03
1.2	80	251	0.192E-02
1.3	77	242	0.506E-03
1.4	80	252	0.117E-02
1.5	61	188	0.357E-02
1.6	75	240	0.933E-03
1.7	79	247	0.310E-03
1.8	74	226	0.112E-01
1.9	73	231	0.131E-02
2.0	84	266	0.766E-03
2.1	89	274	0.672E-03
2.2	68	218	0.845E-03
2.3	84	257	0.199E-02
2.4	68	217	0.633E-02
2.5	73	231	0.334E-02
2.6	102	321	0.656E-02
2.7	101	311	0.198E-02
2.8	94	298	0.455E-03
2.9	90	284	0.354E-02
3.0	117	363	0.448E-03
3.1	29	98	0.114E-02
3.2	50	164	0.117E-02
3.3	38	123	0.993E-03
3.4	32	111	0.881E-03
3.5	33	111	0.787E-04
3.6	28	96	0.606E-03
3.7	115	360	0.320E-02
3.8	121	375	0.332E-03
3.9	51	162	0.149E-03
4.0	65	210	0.191E-02

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2
UNTIL F(X) < 1.E-3 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	121	381	0.487E-02
4.2	111	350	0.260E-03
4.3	141	444	0.679E-03
4.4	110	349	0.258E-02
4.5	102	326	0.839E-03
4.6	101	320	0.202E-02
4.7	109	343	0.846E-03
4.8	121	371	0.350E-02
4.9	101	317	0.131E-02
5.0	109	340	0.592E-03
5.1	100	317	0.120E-02
5.2	113	351	0.132E-02
5.3	118	364	0.134E-02
5.4	103	326	0.387E-03
5.5	120	379	0.201E-02
5.6	116	364	0.441E-03
5.7	134	422	0.341E-03
5.8	111	346	0.893E-03
5.9	113	362	0.515E-03
6.0	105	331	0.161E-02
6.1	129	400	0.601E-03
6.2	101	321	0.152E-02
6.3	78	248	0.140E-02
6.4	76	243	0.160E-02
6.5	101	322	0.675E-03
6.6	133	415	0.102E-02
6.7	112	346	0.696E-03
6.8	129	409	0.164E-02
6.9	111	356	0.976E-03
7.0	119	381	0.156E-02
7.1	112	349	0.173E-02
7.2	109	346	0.996E-03
7.3	137	430	0.869E-04
7.4	148	467	0.107E-02
7.5	111	355	0.175E-02
7.6	119	373	0.560E-03
7.7	130	407	0.433E-03
7.8	129	402	0.965E-03
7.9	126	390	0.153E-02
8.0	123	380	0.151E-02

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2
UNTIL F(X) < 1.E-5 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	90	286	0.168E-04
0.2	82	254	0.691E-05
0.3	93	285	0.591E-05
0.4	85	271	0.176E-04
0.5	85	270	0.270E-04
0.6	76	248	0.684E-04
0.7	90	276	0.308E-04
0.8	72	230	0.103E-04
0.9	99	304	0.579E-04
1.0	86	268	0.726E-05
1.1	87	279	0.135E-04
1.2	84	266	0.114E-04
1.3	83	259	0.109E-04
1.4	86	273	0.762E-05
1.5	69	214	0.159E-04
1.6	80	257	0.192E-04
1.7	86	271	0.121E-04
1.8	80	248	0.555E-05
1.9	78	249	0.191E-04
2.0	90	285	0.184E-04
2.1	102	316	0.683E-04
2.2	77	249	0.127E-04
2.3	90	277	0.499E-04
2.4	75	240	0.278E-04
2.5	80	255	0.408E-04
2.6	109	345	0.212E-04
2.7	105	326	0.530E-04
2.8	100	318	0.236E-04
2.9	95	303	0.638E-05
3.0	123	382	0.164E-04
3.1	35	118	0.472E-05
3.2	55	180	0.138E-03
3.3	43	139	0.823E-04
3.4	41	139	0.193E-04
3.5	42	140	0.164E-04
3.6	36	122	0.913E-05
3.7	123	386	0.198E-04
3.8	123	392	0.624E-03
3.9	63	201	0.672E-05
4.0	[illegible]	234	0.104E-04

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2
UNTIL F(X) < 1.E-5 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	131	415	0.222E-05
4.2	117	370	0.121E-04
4.3	145	459	0.404E-04
4.4	119	379	0.232E-05
4.5	107	344	0.159E-04
4.6	106	337	0.375E-04
4.7	115	364	0.123E-04
4.8	128	395	0.561E-04
4.9	107	337	0.306E-04
5.0	113	355	0.249E-04
5.1	104	331	0.205E-03
5.2	119	372	0.840E-05
5.3	123	380	0.549E-05
5.4	116	367	0.523E-04
5.5	123	389	0.170E-03
5.6	121	381	0.912E-05
5.7	144	455	0.734E-05
5.8	117	367	0.877E-05
5.9	116	369	0.378E-03
6.0	110	347	0.443E-04
6.1	137	427	0.196E-04
6.2	107	341	0.142E-04
6.3	85	270	0.243E-04
6.4	76	243	0.160E-02
6.5	106	336	0.149E-03
6.6	140	439	0.483E-05
6.7	119	368	0.922E-05
6.8	137	436	0.580E-05
6.9	117	377	0.806E-05
7.0	121	389	0.807E-04
7.1	122	382	0.396E-05
7.2	117	373	0.954E-05
7.3	145	454	0.652E-04
7.4	153	484	0.299E-05
7.5	118	375	0.119E-03
7.6	126	395	0.333E-04
7.7	138	433	0.599E-05
7.8	132	414	0.154E-04
7.9	133	411	0.810E-05
8.0	131	405	0.950E-05

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2
UNTIL F(X) < 1.E-7 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	97	308	0.273E-06
0.2	89	276	0.731E-07
0.3	96	296	0.138E-06
0.4	94	303	0.113E-06
0.5	88	281	0.289E-05
0.6	81	267	0.574E-07
0.7	95	293	0.611E-06
0.8	76	244	0.808E-07
0.9	105	324	0.347E-06
1.0	89	279	0.259E-06
1.1	90	291	0.289E-06
1.2	88	280	0.889E-06
1.3	87	274	0.211E-06
1.4	89	285	0.472E-07
1.5	74	233	0.463E-07
1.6	83	269	0.447E-06
1.7	89	281	0.130E-05
1.8	88	274	0.740E-07
1.9	83	265	0.924E-07
2.0	94	300	0.262E-06
2.1	109	341	0.133E-06
2.2	82	266	0.239E-06
2.3	96	299	0.131E-06
2.4	86	276	0.432E-07
2.5	86	276	0.108E-06
2.6	113	360	0.767E-07
2.7	109	342	0.341E-06
2.8	105	336	0.147E-06
2.9	98	315	0.145E-06
3.0	128	401	0.979E-07
3.1	38	129	0.223E-06
3.2	60	200	0.554E-07
3.3	46	151	0.244E-05
3.4	45	153	0.114E-06
3.5	47	158	0.199E-06
3.6	40	137	0.708E-07
3.7	126	398	0.354E-06
3.8	129	404	0.297E-06
3.9	65	209	0.420E-06
4.0	77	252	0.434E-06

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2
UNTIL F(X) < 1.E-7 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1.2, 1.0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	137	435	0.206E-06
4.2	123	392	0.790E-07
4.3	151	480	0.205E-07
4.4	124	396	0.214E-06
4.5	121	390	0.233E-07
4.6	112	358	0.843E-07
4.7	118	375	0.727E-06
4.8	133	413	0.128E-06
4.9	113	358	0.997E-07
5.0	118	373	0.451E-07
5.1	110	353	0.624E-07
5.2	129	404	0.838E-07
5.3	125	388	0.127E-05
5.4	122	387	0.798E-07
5.5	141	449	0.466E-07
5.6	126	400	0.302E-07
5.7	151	478	0.162E-06
5.8	121	382	0.843E-07
5.9	123	395	0.264E-07
6.0	116	367	0.209E-06
6.1	142	445	0.272E-06
6.2	114	364	0.113E-06
6.3	91	290	0.113E-06
6.4	83	271	0.813E-07
6.5	114	364	0.100E-06
6.6	143	451	0.210E-06
6.7	124	385	0.773E-06
6.8	142	453	0.978E-06
6.9	118	381	0.223E-05
7.0	126	408	0.290E-07
7.1	125	394	0.582E-07
7.2	122	390	0.670E-07
7.3	149	468	0.831E-06
7.4	158	502	0.139E-06
7.5	122	390	0.975E-06
7.6	131	415	0.481E-07
7.7	143	451	0.100E-06
7.8	137	432	0.230E-06
7.9	138	429	0.200E-06
8.0	136	423	0.186E-06

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2+
90*(X(4)-X(3)**2)**2+(1-X(3))**2+10.1*((X(2)-1)**2+(X(4)-1)**2)+
19.8*(X(2)-1)*(X(4)-1)
UNTIL F(X) < 1.E-1 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON,
STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	82	240	0.488E-02
0.2	183	541	0.951E-02
0.3	48	146	0.355E-01
0.4	50	161	0.592E-01
0.5	46	150	0.926E-02
0.6	86	276	0.153E-01
0.7	34	119	0.176
0.8	49	152	0.142
0.9	32	107	0.209
1.0	30	105	0.397E-01
1.1	40	135	0.404E-01
1.2	33	108	0.677E-01
1.3	33	125	0.259E-01
1.4	670	2001	0.105E-05
1.5	863	2000	0.148E-05
1.6	951	2000	0.477E-06
1.7	565	1656	0.197E-01
1.8	452	1324	0.190E-03
1.9	316	944	0.877E-02
2.0	250	748	0.244E-01
2.1	264	764	0.160E-01
2.2	954	2000	0.369E-06
2.3	950	2000	0.369E-06
2.4	956	2001	0.674E-06
2.5	950	2000	0.640E-06
2.6	947	2000	0.302E-06
2.7	286	821	0.100E-01
2.8	101	305	0.190E-01
2.9	70	226	0.207E-01
3.0	59	192	0.368E-01
3.1	72	233	0.729E-02
3.2	346	1018	0.161E-01
3.3	250	733	0.145E-01
3.4	123	398	0.252E-01
3.5	136	415	0.414E-01
3.6	157	477	0.109E-01
3.7	105	337	0.844E-02
3.8	101	329	0.148E-01
3.9	57	184	0.860E-02
4.0	37	131	0.930E-01

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2+
90*(X(4)-X(3))**2+(1-X(3))**2+10.1((X(2)-1)**2)+(X(4)-1)**2)
19.8*(X(2)-1)*(X(4)-1)
UNTIL F(X) < 1.E-1 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	196	604	0.224E-01
4.2	267	804	0.953E-02
4.3	24	94	0.650E-01
4.4	656	2001	0.506E-04
4.5	349	1021	0.248E-01
4.6	325	965	0.541E-02
4.7	384	1149	0.584E-02
4.8	292	877	0.862E-02
4.9	291	858	0.582E-01
5.0	268	799	0.212E-01
5.1	363	1081	0.264E-02
5.2	369	1106	0.127E-01
5.3	245	726	0.492E-01
5.4	325	961	0.161E-01
5.5	252	755	0.624E-02
5.6	895	2001	0.475E-05
5.7	320	968	0.467E-02
5.8	253	772	0.552E-01
5.9	130	409	0.118E-01
6.0	133	412	0.295E-01
6.1	138	438	0.648E-02
6.2	221	670	0.503E-01
6.3	244	716	0.336E-01
6.4	71	226	0.154E-01
6.5	83	267	0.225E-01
6.6	222	652	0.352E-01
6.7	283	854	0.230E-01
6.8	271	794	0.725E-01
6.9	67	222	0.134E-01
7.0	33	123	0.185E-01
7.1	24	91	0.332
7.2	17	69	1.31
7.3	932	2001	0.302E-06
7.4	64	219	0.175E-02
7.5	58	199	0.189E-01
7.6	67	221	0.122E-01
7.7	66	216	0.430E-01
7.8	55	193	0.217E-01
7.9	75	241	0.391E-01
8.0	152	469	0.186E-01

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2+
90*(X(4)-X(3))**2+(1-X(3))**2+10.1((X(2)-1)**2)+(X(4)-1)**2)
19.8*(X(2)-1)*(X(4)-1)
UNTIL F(X) < 1.E-3 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	211	613	0.321E-04
0.2	217	644	0.171E-02
0.3	101	306	0.673E-03
0.4	115	358	0.150E-04
0.5	129	385	0.998E-02
0.6	127	394	0.427E-03
0.7	129	395	0.600E-04
0.8	134	397	0.283E-03
0.9	101	317	0.292E-03
1.0	35	125	0.616E-02
1.1	126	393	0.412E-03
1.2	92	293	0.613E-03
1.3	128	397	0.159E-02
1.4	670	2001	0.105E-05
1.5	863	2000	0.148E-05
1.6	951	2000	0.477E-06
1.7	658	1918	0.282E-03
1.8	588	1713	0.228E-03
1.9	378	1125	0.583E-03
2.0	324	967	0.479E-03
2.1	302	884	0.304E-03
2.2	954	2000	0.369E-06
2.3	950	2000	0.369E-06
2.4	956	2001	0.674E-06
2.5	950	2000	0.640E-06
2.6	947	2000	0.302E-06
2.7	335	964	0.881E-03
2.8	146	445	0.106E-02
2.9	134	421	0.924E-04
3.0	81	254	0.851E-03
3.1	101	311	0.156E-02
3.2	389	1146	0.794E-03
3.3	320	941	0.135E-02
3.4	263	804	0.185E-03
3.5	149	458	0.897E-03
3.6	204	608	0.103E-02
3.7	161	499	0.238E-03
3.8	127	415	0.816E-03
3.9	80	256	0.138E-03
4.0	75	250	0.147E-02

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2+
90*(X(4)-X(3))**2+(1-X(3))**2+10.1((X(2)-1)**2)+(X(4)-1)**2)
19.8*(X(2)-1)*(X(4)-1)
UNTIL F(X) < 1.E-3 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	217	668	0.243E-02
4.2	309	925	0.623E-03
4.3	108	349	0.563E-03
4.4	656	2001	0.506E-04
4.5	390	1139	0.452E-03
4.6	412	1214	0.563E-03
4.7	422	1255	0.934E-03
4.8	355	1058	0.156E-03
4.9	303	897	0.241E-02
5.0	339	1016	0.463E-03
5.1	458	1351	0.184E-03
5.2	391	1173	0.455E-03
5.3	284	852	0.242E-04
5.4	350	1039	0.408E-03
5.5	354	1054	0.945E-03
5.6	895	2001	0.475E-05
5.7	373	1125	0.384E-03
5.8	413	1242	0.560E-03
5.9	204	635	0.770E-04
6.0	160	495	0.623E-03
6.1	179	557	0.331E-02
6.2	282	840	0.229E-02
6.3	320	949	0.425E-03
6.4	124	389	0.129E-02
6.5	141	436	0.210E-02
6.6	332	973	0.304E-03
6.7	347	1037	0.657E-02
6.8	308	900	0.247E-02
6.9	104	334	0.647E-03
7.0	102	328	0.667E-03
7.1	86	277	0.346E-03
7.2	77	260	0.566E-03
7.3	932	2001	0.302E-06
7.4	105	332	0.110E-02
7.5	128	409	0.736E-04
7.6	116	362	0.213E-03
7.7	88	284	0.101E-02
7.8	78	263	0.733E-03
7.9	98	316	0.793E-03
8.0	186	574	0.467E-03

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2+
90*(X(4)-X(3))**2+(1-X(3))**2+10.1((X(2)-1)**2)+(X(4)-1)**2)
19.8*(X(2)-1)*(X(4)-1)
UNTIL F(X) < 1.E-5 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	259	748	0.790E-05
0.2	232	695	0.436E-05
0.3	127	380	0.569E-05
0.4	146	441	0.499E-05
0.5	156	483	0.825E-06
0.6	140	437	0.108E-04
0.7	154	471	0.309E-05
0.8	154	457	0.624E-05
0.9	132	412	0.141E-04
1.0	94	310	0.789E-05
1.1	139	436	0.453E-05
1.2	104	332	0.667E-05
1.3	141	438	0.180E-04
1.4	670	2001	0.105E-05
1.5	863	2000	0.148E-05
1.6	951	2000	0.477E-06
1.7	677	1985	0.804E-06
1.8	614	1787	0.145E-04
1.9	386	1151	0.274E-04
2.0	338	1009	0.592E-05
2.1	315	923	0.200E-04
2.2	954	2000	0.369E-06
2.3	950	2000	0.369E-06
2.4	956	2001	0.674E-06
2.5	950	2000	0.640E-06
2.6	947	2000	0.302E-06
2.7	376	1086	0.533E-05
2.8	163	499	0.208E-04
2.9	154	477	0.827E-05
3.0	115	358	0.724E-05
3.1	123	383	0.438E-05
3.2	444	1313	0.275E-05
3.3	328	975	0.639E-05
3.4	279	855	0.563E-05
3.5	179	553	0.474E-05
3.6	220	654	0.585E-05
3.7	173	536	0.683E-05
3.8	139	456	0.219E-04
3.9	112	354	0.201E-05
4.0	90	304	0.299E-05

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2+
90*(X(4)-X(3))**2+(1-X(3))**2+10.1((X(2)-1)**2)+(X(4)-1)**2)
19.8*(X(2)-1)*(X(4)-1)
UNTIL F(X) < 1.E-5 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	502	2000	0.817E-08
4.2	320	959	0.130E-04
4.3	125	400	0.684E-05
4.4	656	2001	0.506E-04
4.5	404	1184	0.108E-04
4.6	424	1254	0.615E-05
4.7	434	1296	0.197E-04
4.8	368	1100	0.231E-04
4.9	498	2001	0.274E-08
5.0	360	1080	0.582E-05
5.1	470	1383	0.522E-04
5.2	413	1244	0.363E-05
5.3	409	1190	0.202E-04
5.4	378	1122	0.600E-05
5.5	369	1108	0.501E-05
5.6	895	2001	0.475E-05
5.7	379	1147	0.532E-04
5.8	445	1346	0.147E-05
5.9	224	690	0.964E-05
6.0	198	609	0.499E-05
6.1	233	713	0.637E-05
6.2	310	930	0.170E-04
6.3	336	999	0.930E-05
6.4	153	478	0.448E-05
6.5	149	466	0.889E-05
6.6	370	1093	0.146E-05
6.7	360	1086	0.682E-05
6.8	317	933	0.437E-04
6.9	119	388	0.782E-05
7.0	134	430	0.322E-05
7.1	104	336	0.262E-05
7.2	96	318	0.487E-05
7.3	932	2001	0.302E-06
7.4	118	376	0.860E-05
7.5	145	454	0.242E-04
7.6	129	403	0.130E-04
7.7	95	312	0.751E-05
7.8	117	376	0.668E-05
7.9	118	375	0.139E-04
8.0	199	615	0.947E-05

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2+
90*(X(4)-X(3))**2+(1-X(3))**2+10.1((X(2)-1)**2)+(X(4)-1)**2)
19.8*(X(2)-1)*(X(4)-1)
UNTIL F(X) < 1.E-7 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	272	789	0.796E-07
0.2	242	729	0.127E-06
0.3	142	428	0.495E-07
0.4	149	456	0.314E-06
0.5	175	541	0.345E-07
0.6	148	467	0.135E-06
0.7	170	521	0.142E-06
0.8	169	504	0.111E-06
0.9	155	485	0.128E-06
1.0	108	358	0.288E-06
1.1	151	475	0.374E-07
1.2	120	385	0.512E-07
1.3	151	475	0.175E-06
1.4	670	2001	0.105E-05
1.5	863	2000	0.148E-05
1.6	951	2000	0.477E-06
1.7	682	2001	0.882E-06
1.8	626	1829	0.396E-07
1.9	399	1199	0.295E-07
2.0	352	1059	0.540E-07
2.1	324	953	0.306E-06
2.2	954	2000	0.369E-06
2.3	950	2000	0.369E-06
2.4	956	2001	0.674E-06
2.5	950	2000	0.640E-06
2.6	947	2000	0.302E-06
2.7	391	1133	0.685E-07
2.8	168	517	0.132E-05
2.9	170	535	0.198E-07
3.0	124	390	0.562E-07
3.1	146	453	0.784E-07
3.2	456	1352	0.105E-06
3.3	338	1011	0.841E-07
3.4	297	915	0.383E-07
3.5	192	594	0.559E-07
3.6	228	679	0.431E-06
3.7	181	567	0.126E-06
3.8	151	499	0.124E-06
3.9	146	460	0.315E-07
4.0	110	362	0.313E-06

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(2)-X(1)**2)**2+(1-X(1))**2+
90*(X(4)-X(3))**2+(1-X(3))**2+10.1((X(2)-1)**2)+(X(4)-1)**2)
19.8*(X(2)-1)*(X(4)-1)
UNTIL F(X) < 1.E-7 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-3,-1,-3,-1).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	502	2000	0.817E-08
4.2	334	1014	0.469E-07
4.3	136	438	0.106E-06
4.4	656	2001	0.506E-04
4.5	418	1232	0.386E-07
4.6	445	1322	0.743E-07
4.7	444	1332	0.170E-06
4.8	376	1132	0.133E-06
4.9	498	2001	0.274E-08
5.0	373	1126	0.388E-07
5.1	485	1434	0.116E-06
5.2	429	1295	0.126E-06
5.3	420	1226	0.240E-06
5.4	391	1165	0.134E-06
5.5	377	1139	0.717E-07
5.6	895	2001	0.475E-05
5.7	388	1182	0.112E-06
5.8	477	1438	0.874E-07
5.9	237	735	0.957E-07
6.0	224	694	0.590E-07
6.1	247	758	0.116E-06
6.2	337	1014	0.348E-07
6.3	342	1021	0.680E-06
6.4	172	539	0.180E-06
6.5	163	516	0.732E-07
6.6	386	1143	0.785E-07
6.7	374	1140	0.526E-07
6.8	324	966	0.253E-06
6.9	132	429	0.102E-06
7.0	143	459	0.327E-06
7.1	118	382	0.894E-07
7.2	110	369	0.670E-07
7.3	932	2001	0.302E-06
7.4	128	414	0.124E-06
7.5	166	522	0.675E-07
7.6	142	450	0.813E-07
7.7	104	348	0.820E-07
7.8	122	396	0.323E-06
7.9	133	423	0.792E-07
8.0	207	644	0.134E-06

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(3)-10*ATAN2(X(2),X(1))/(2*PI))**2+
(SQRT(X(1)**2+X(2)**2)-1)**2+X(3)**2
UNTIL F(X) < 1.E-1 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1,0,0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTO OF NELME
0.1	203	630	0.819E-01
0.2	198	609	0.278E-01
0.3	142	428	0.144E-01
0.4	153	471	0.553E-01
0.5	141	438	0.107E-01
0.6	114	364	0.983E-01
0.7	866	2001	0.119E-06
0.8	797	2001	0.000
0.9	146	436	0.681E-01
1.0	154	480	0.603E-02
1.1	184	566	0.143
1.2	148	441	0.147E-01
1.3	90	286	0.406E-02
1.4	81	258	0.471E-01
1.5	88	277	0.102
1.6	205	640	0.520E-01
1.7	117	361	0.348E-01
1.8	23	80	0.353E-01
1.9	23	90	0.443E-01
2.0	12	48	0.465
2.1	12	48	0.279
2.2	10	38	2.45
2.3	10	38	2.50
2.4	24	83	0.707E-01
2.5	98	314	0.307E-01
2.6	31	105	0.693E-02
2.7	8	30	4.04
2.8	8	30	4.26
2.9	18	69	0.619E-02
3.0	21	77	0.127E-01
3.1	26	92	0.122E-01
3.2	42	136	0.204E-01
3.3	31	112	0.574E-02
3.4	47	158	0.913E-03
3.5	29	103	0.396E-02
3.6	31	112	0.250E-01
3.7	31	107	0.407E-01
3.8	40	136	0.557E-02
3.9	48	160	0.254E-01
4.0	54	169	0.135E-01

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(3)-10*ATAN2(X(2),X(1))/(2*PI))**2+
(SQRT(X(1)**2+X(2)**2)-1)**2+X(3)**2
UNTIL F(X) < 1.E-1 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1,0,0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	42	137	0.267E-01
4.2	50	159	0.351E-01
4.3	26	89	0.101
4.4	33	107	0.364E-01
4.5	49	163	0.130E-01
4.6	30	103	0.373E-01
4.7	52	167	0.908E-01
4.8	48	158	0.497E-01
4.9	38	122	0.189E-01
5.0	82	260	0.184E-01
5.1	38	129	0.694E-02
5.2	70	231	0.198E-01
5.3	32	116	0.442E-01
5.4	48	161	0.205E-01
5.5	42	138	0.142E-01
5.6	23	86	0.167
5.7	31	109	0.571E-01
5.8	38	133	0.616E-01
5.9	109	333	0.116E-01
6.0	36	118	0.556E-01
6.1	35	121	0.589E-01
6.2	64	202	0.342E-01
6.3	37	134	0.208E-01
6.4	44	150	0.203E-01
6.5	53	176	0.169E-01
6.6	51	171	0.149E-01
6.7	62	199	0.324E-01
6.8	84	262	0.665E-01
6.9	111	355	0.656E-01
7.0	68	224	0.449E-01
7.1	96	300	0.885E-02
7.2	65	212	0.102
7.3	56	192	0.733E-02
7.4	85	267	0.486E-01
7.5	40	141	0.229E-01
7.6	56	179	0.367E-01
7.7	50	168	0.126E-01
7.8	99	313	0.231E-01
7.9	59	187	0.351E-01
8.0	81	253	0.329E-01

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(3)-10*ATAN2(X(2),X(1))/(2*PI))**2+
(SQRT(X(1)**2+X(2)**2)-1)**2+X(3)**2
UNTIL F(X) < 1.E-3 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1,0,0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	228	708	0.455E-03
0.2	235	719	0.686E-03
0.3	186	561	0.496E-03
0.4	194	591	0.968E-03
0.5	194	593	0.128E-03
0.6	191	601	0.138E-03
0.7	866	2001	0.119E-06
0.8	797	2001	0.000
0.9	194	579	0.754E-03
1.0	185	576	0.555E-03
1.1	248	758	0.648E-03
1.2	174	525	0.302E-03
1.3	148	467	0.288E-03
1.4	90	287	0.184E-02
1.5	124	387	0.126E-02
1.6	251	784	0.673E-03
1.7	142	442	0.188E-02
1.8	30	103	0.365E-02
1.9	42	139	0.117E-02
2.0	56	185	0.646E-03
2.1	77	251	0.603E-03
2.2	75	243	0.199E-03
2.3	84	270	0.118E-03
2.4	65	212	0.171E-03
2.5	134	425	0.797E-03
2.6	59	195	0.428E-03
2.7	49	165	0.183E-02
2.8	64	204	0.940E-03
2.9	51	164	0.192E-02
3.0	51	162	0.110E-02
3.1	57	179	0.117E-02
3.2	79	246	0.569E-03
3.3	74	236	0.451E-02
3.4	80	261	0.346E-03
3.5	88	285	0.264E-03
3.6	68	222	0.281E-02
3.7	88	274	0.164E-02
3.8	89	282	0.370E-03
3.9	86	267	0.444E-03
4.0	95	296	0.197E-03

PRINT FROM NELME
****

MINIMIZATION OF F(X)=100*(X(3)-10*ATAN2(X(2),X(1))/(2*PI))**2+
(SQRT(X(1)**2+X(2)**2)-1)**2+X(3)**2
UNTIL F(X) < 1.E-3 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1,0,0).

SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
4.1	54	176	0.432E-02
4.2	92	287	0.199E-02
4.3	68	216	0.252E-02
4.4	101	313	0.534E-03
4.5	90	288	0.694E-03
4.6	56	189	0.516E-03
4.7	90	282	0.197E-02
4.8	84	273	0.409E-04
4.9	73	233	0.442E-03
5.0	126	391	0.108E-02
5.1	69	219	0.749E-03
5.2	127	403	0.624E-03
5.3	64	217	0.586E-03
5.4	126	397	0.396E-03
5.5	118	380	0.114E-03
5.6	79	255	0.429E-03
5.7	88	272	0.292E-03
5.8	118	369	0.115E-02
5.9	121	371	0.645E-03
6.0	74	228	0.150E-02
6.1	58	197	0.812E-03
6.2	108	344	0.655E-03
6.3	80	266	0.191E-02
6.4	111	353	0.633E-03
6.5	110	346	0.638E-03
6.6	113	358	0.370E-03
6.7	111	351	0.401E-03
6.8	114	355	0.784E-04
6.9	124	401	0.340E-03
7.0	130	404	0.352E-03
7.1	156	478	0.729E-03
7.2	113	363	0.299E-03
7.3	87	284	0.100E-02
7.4	136	421	0.377E-03
7.5	109	353	0.187E-02
7.6	84	272	0.499E-03
7.7	102	329	0.312E-03
7.8	110	349	0.330E-02
7.9	135	417	0.325E-03
8.0	115	352	0.757E-03

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PRINT FROM NELME
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MINIMIZATION OF F(X)=100*(X(3)-10*ATAN2(X(2),X(1))/(2*PI))**2+
(SQRT(X(1)**2+X(2)**2)-1)**2+X(3)**2
UNTIL F(X) < 1.E-5 FOR DIFFERENT SIZES OF THE INITIAL POLYHEDRON.
STARTING POINT IS (-1,0,0).
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SIZE OF THE INITIAL POLYHEDRON	NUMBER OF ITERATIONS REQUIRED	NUMBER OF EVALUATIONS REQUIRED	TESTQ OF NELME
0.1	235	732	0.623E-04
0.2	254	774	0.213E-04
0.3	201	604	0.140E-04
0.4	206	632	0.908E-05
0.5	203	622	0.148E-04
0.6	209	654	0.176E-04
0.7	866	2001	0.119E-06
0.8	797	2001	0.000
0.9	201	604	0.209E-04
1.0	193	606	0.672E-05
1.1	269	822	0.181E-04
1.2	186	560	0.620E-05
1.3	166	522	0.132E-04
1.4	161	498	0.469E-05
1.5	140	433	0.173E-04
1.6	260	815	0.880E-05
1.7	150	473	0.648E-05
1.8	60	196	0.138E-04
1.9	75	240	0.154E-04
2.0	79	261	0.719E-05
2.1	99	319	0.202E-04
2.2	87	280	0.315E-04
2.3	93	299	0.114E-04
2.4	80	255	0.366E-05
2.5	147	467	0.636E-05
2.6	67	221	0.733E-04
2.7	57	196	0.811E-05
2.8	71	229	0.146E-04
2.9	60	195	0.533E-05
3.0	60	191	0.109E-04
3.1	90	280	0.225E-04
3.2	96	303	0.400E-05
3.3	116	367	0.276E-05
3.4	112	356	0.402E-05
3.5	107	346	0.778E-05
3.6	76	251	0.620E-05
3.7	100	315	0.143E-04
3.8	103	327	0.128E-04
3.9	100	313	0.468E-05
4.0	107	335	0.876E-05