NSF-STU Workshop on Adaptive Control

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1985

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):

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NSF-STU Workshop on Adaptive Control

9 - 11 July 1984

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY
APRIL 1985
NSF-STU Workshop on Adaptive Control

Department of Automatic Control
Lund Institute of Technology
Lund Sweden

9 - 11 July 1984

Edited by Tore Hågglund
**Title and subtitle**

NSF-STU Workshop on Adaptive Control

**Abstract**

The National Science Foundation (NSF) and the Swedish Board for Technical Development (STU) have signed an agreement for cooperation. Both agencies are currently supporting research in adaptive control. Within this framework, a workshop was held at the Department of Automatic Control at the Lund Institute of Technology, Lund, Sweden, on July 9-11 1984. This report contains abstracts, copies of the viewgraphs and a summary of the discussions.

**Document name**
Final report

**Date of issue**
April 1985

**Document number**
CODEN: LUTFD2/(TFRT-3175)/1-175/(1985)

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**Sponsoring organization**
The National Science Foundation (NSF)
The Swedish Board for Technical Development (STU)

**Key words**

**Classification system and/or index terms (if any)**

**Supplementary bibliographical information**

**ISSN and key title**

**Language** | Number of pages | Recipient's notes
---|---|---
English | 175 | |

**Security classification**

Distribution: The report may be ordered from the Department of Automatic Control or borrowed through the University Library 2, Box 1010, S-221 03 Lund, Sweden, Telex: 33248 lubbis lund.
Introduction

The National Science Foundation (NSF) and the Swedish Board for Technical Development (STU) have signed an agreement for cooperation. Both agencies are currently supporting research in adaptive control. The purpose of the seminar was to bring the research in the supported projects together to provide a perspective of the field, its accomplishments and deficiencies and to find directions for future research.

The laboratory for Information and Decision Systems at the Massachusetts Institute of Technology, directed by Professor Sanjoy Mitter, and the Department of Automatic Control at Lund Institute of Technology, Lund, Sweden, directed by Professor Karl Johan Åström, have long maintained close informal contact on research problems of common interest. There is interest on both sides in formalizing this arrangement so that exchanges of faculty, staff (and possibly students) can take place. The workshop on Adaptive Control was organized within this framework.

The workshop program was discussed with Dr. Abraham Haddad, former Program Director of the Systems Theory and Operations Research Program of the National Science Foundation and Mr Ove Berkefelt Program Director for at the Swedish Board of Technical Development. They were in agreement with the aims and objectives of the workshop.

The workshop was held at the Department of Automatic Control at the Lund Institute of Technology, Lund, Sweden, on July 9 - 11, 1984, this week followed the IFAC Congress in Budapest.

The workshop was informal. There were 14 US and 20 Swedish participants. The formal presentations covered industrial needs and experiences, applications, stability theory, system identification, stochastic adaptive control, unmodeled dynamics and new directions as well as many informal discussions. The workshop was viewed very favourable from the participants who found it a stimulating intellectual experience.

A brief assessment of the field can be summarized as follows. There has been research in adaptive control for at least 30 years. The field is, however, still not in very good shape. There is a proliferation of ideas and techniques, and a lack of coherence. Recently there has, however, been some limited theoretical results. Adaptive techniques are also starting to be used in microprocessor based controllers. It appears as a good research field because theoretical results are badly needed to get insight, to structure the problem, and to unify the field. There is also a considerable industrial interest to use adaptive techniques in many different fields. Several new products have recently been announced. There are several strong research groups in the field, both in the United States and in Europe. The theoretical aspects have been emphasized in U.S. research. In Europe the theoretical research has however also been blended with practice. There are new application areas emerging, e.g. in robotics.

This report contains abstracts, copies of the viewgraphs and a summary of the discussions.
Dedication

This report is dedicated to the memory of Dr. Howard Elliott a prominent researcher in adaptive control. He contributed significantly to the success of the workshop which was the last formal meeting in which he participated.
Program

MONDAY - APPLICATIONS

9.00 Introduction and Welcome
Berkefelt
Cooperation between NSF and STU

9 - 12 Industrial Products

Egardt
The ASEA-Novatune system

Bengtsson
Experiences with the ASEA-Novatune

Åström
Automatic tuning of simple regulators

Bååth
The NAF - Autotuner

13 - 15 Applications

Stein
History and issues in adaptive flight control

Olsson, Rundqwist
Self-tuning control of dissolved oxygen concentration in activated sludge systems

Elliott
Adaptive pole placement for robots and servomechanisms

15 - 17 Discussion
Where do we stand with respect to applications? What algorithms are being used? What things work? What are the difficulties? What tricks are used?

Sternby
Some desirable features of industrial adaptive controllers
TUESDAY - THEORY

8 - 10  Stability Theory
Wittenmark  Self-tuning regulator with increased prediction horizon
Morse  A universal control capable of stabilizing any single-input, single-output, minimum phase linear system of relative degree \( \leq 2 \)
Byrnes  Adaptive stabilization of linear multivariable systems
Johansson  Lyapunov functions, cost functions and adaptive control

10 - 12  System Identification
Söderström  Instrumental variable methods for systems operating in closed loop with application to adaptive control
Solo  Adaptive spectral factorization
Ljung  Frequency domain properties of identified transfer functions

13 - 15  Stochastic Adaptive Control
Varaya  Multi-armed bandits
Kumar  On self-tuning to the optimal controller
Millnert  A comparison of some control strategies for systems with fast parameter variations
Hägglund  Recursive estimation of slowly time-varying parameters

15 - 17  Discussion
Where does the theory stand? What results are needed? Do the results cover the problems brought up by practice? What problems can we hope to solve?
WEDNESDAY - ROBUSTNESS AND NEW DIRECTIONS

8 - 11  **Unmodeled Dynamics and Robustness**

Kokotovic  Robustness of (MRAS) adaptive control
Sastry  Parameter convergence in model reference adaptive control and its impact on robustness
Trulsson  On adaptive control with prescribed robustness properties
Rohrs  On living with the positive real condition
Bertsekas  Distributed asynchronous algorithms for deterministic and stochastic optimization

11 - 12  **Demonstration of Control Laboratory**

13 - 15  **New Directions**

Åström  Expert Control
Arzén  Experiments with Expert Control

15 - 17  **Discussion**

Future directions.
**List of Participants**

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**Technical secretaries**

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Cooperation between NSF and STU

Ove Berkefelt
STU
Stockholm

Nearly three years ago NSF and STU reached a general agreement on cooperation in research. This agreement covers all scientific areas where STU and NSF are funding research projects.

For a small country like Sweden with limited resources in personnel and money it is of course difficult to find areas where our research is of adequate level and size compared to US. I am therefore very pleased to note that we have recently managed to arrange workshops in two areas within the field of electronics, computers and systems sciences. About two months ago we had a joint NSF-STU workshop on computer based vision and this week we will have this workshop on adaptive control.

It is interesting to compare these two areas. Computer based vision is a new science. By a generous budget we have managed to create a good scientific level in a short time, approximately 5 years.

Adaptive control on the other side has been built up gradually during a long time and I would say more thanks to excellent and devoted researchers than to a generous budget. In any case we have in adaptive control enough research results, enough researchers and enough applications to attract the interest from NSF and US researchers and this is in a time when, as I understand it, NSF is putting heavier conditions on research cooperation with other countries than earlier.

The purpose of this workshop is apart from the exchange of research results between the two countries to find out whether adaptive control is an area where we can find possibilities for future cooperation. A joint US-Swedish project on adaptive control would have a great chance to get funding at least from STU. Unfortunately my counterpart from NSF is not here so that we can hear NSF's opinion.

This workshop happens to be very suitably located in time. We are at present at STU planning a new national program in information technology. We are trying to establish which areas in systems and computer sciences where we can compare with other countries and where scientific results are likely to lead to industrial applications and progress. I hope this workshop will help in this respect.

Let me end by saying that I hope that our American guests will have a pleasant time at Lund for some days and that this workshop will lead to deeper contacts between Swedish and US researchers and to a future more or less formalised cooperation.
Egardt gave an overview of the system hardware, containing process interfaces etc.

The application program is written in a block oriented language. This is the industrial control engineers look at processes and control. Besides the selftuning regulator the language contains arithmetics, logics and other functions like PID.

The signal types in the language are integer and real, and there are modules like selectors, arithmetic operations, logic, delays, interface modules, filters, regulators etc.

All modules are available in the system library. The program is entered via a simple hand terminal or a standard terminal in a laboratory or at the installation site.

The programmer selects sampling intervals and priorities for the different control tasks. Except standard clock interrupt it's possible to use software interrupts or pulse counters to determine the sampling instants.

The system contains three different adaptive regulators built around the same algorithm. The difference between the regulators is the degree of flexibility offered to the user. The regulators are

| STAR1 | Basic, least complex regulator |
| STAR2 | Medium complex regulator |
| STAR3 | Complex regulator |

STAR3 is the most frequently used regulator. STAR2 and STAR3 both contain feedforward, but in STAR3 the number of parameters in the control law is selected by the user.

The signals connected to the STAR3 module are the manual control signal, the feedback signal (process output), the reference value and the feedforward signal. The control value can be given both absolute and incremental limits.

Integer signals determine the mode of the regulator, i.e. adaptation can be switched off, saved parameters can be restored etc. Unconnected inputs are given default values.

The regulator uses a minimum variance algorithm with least squares identification. Besides, one closed loop pole can be defined, the prediction horizon and the sampling interval can be chosen, and the integral action can be switched on and off. A penalty can be introduced on the control output.
Reference:

DEFINING A MODULE

Module identity is only defined in a block if it can be the last module used in block.

DEFINING A BLOCK

NEW
ID
TIME
PRIORITY

New block is created.
Already defined block.
Base execution time.

CUT
DELETE
TIME

Block is going to be deleted.
Time clock executed block.

DELETE
TIME

Block (connected to any module)
EXAMPLE

BLOCK 1 PRIORITY 1 PERIOD 4
BLOCK 2 PRIORITY 2 PERIOD 2
BLOCK 3 PRIORITY 3 PERIOD 1

1 2 3 4 5 6 7 x Time base

EXECUTION TIME
Experiences with the ASEA-Novatune

Gunnar Bengtsson

ASEA AB
Västerås, Sweden

A number of feasibility studies for adaptive controls have been made at Asea during the past 10 years. The adaptive system Novatune was announced in 1982. A lot of experience has been gained during the past years use of it. About 70 systems are currently in operation. The systems cover a wide range of applications in paper mills, steel mills, boilers, waste water treatment, building automation, and looms. The installations of Novatunes is currently increasing rapidly. To do this it has been necessary to train a number of people in the commissioning of the system. A series of courses has been designed to give the proper background for customers and ASEA engineers.

There are several reasons why the Novatune system has got a good acceptance. In many process industries ordinary PID regulators are frequently badly tuned. As a result of this they are often switched to manual mode. There are a number of critical control loops where there are tangible economical benefits by reducing variations in quality variables. Experience indicates that reductions in standard deviations by a factor of two compared with a welltuned PID is quite common. The reason for this is that there frequently are time delays which the Novatune handles better than PID. The improvements in comparisons with poorly tuned PID are of course more favorable. Improvements in variances with an order of magnitude have been found in several cases when feedforward can be applied. Effective use of feedforward requires however good models which have to be updated. It is thus a good case for adaptive control.

Two applications are described in some detail, a cold rolling mill and a chemical reactor.

The rolling mill is a typical batch process there are roughly speaking three phases, startup, full speed operation and breaking. The Novatune was applied to the gauge control loop. The screw position was controlled using feedback from a gauge sensor after the rolls and by feedforward from a gauge sensor in front of the rolls. The main disturbances are variations in gauge and hardness. The time constants and the time delay varies with a factor of 25 over the operation range. The variations in the time delay are handled by having a speed sensor and by introducing length as the independent variable instead of time. The actual sampling period in the regulator will thus vary with speed from 40 ms at full speed to several seconds at slow speeds. The adaptive regulator performed significantly better than a conventional PID regulator with feedforward.

The first Novatune application was made in connection with temperature control in a chemical reactor. The temperature fluctuations were reduced by an order of magnitude mainly due to feedforward operation. The application is critical with
respect to safety and production. The possibility of storing parameters which will give a safe performance of the closed loop system and reinitializing the adaptation using these parameters was incorporated in the system. This application is described in more detail in [1].

The key problem areas that have been found have to do with nonlinearities like friction, dead zone and hysteresis. It may be a good idea to have more flexible ways of modeling these in the regulator. The unstable zeros which appear when the sampling rate is increased is another problem of practical importance.

Reference:

Adaptation to changing dynamics

Adaptive feedback

Dead-Time

ASEA NOVATUNE are now running in several plants:
- Pulper Mill
- Pulper Drier
- Winder
- Chemical Reactor
- Skin Pass Mill
- Cold Rolling Mill
- Rotary Kiln Drier
### Reference List - 1984-03-10

**STEEL/METALLURGIC**
- Krupp Bochum (Germany) - Skin Pass Mill
- CPWAS (Sweden) - Strip Tension
- Sandvik (Sweden) - AAC
- SSAB (Sweden) - Mold Level Control
- Falk (Italy) - Induction Furnace
- Arradini [ ] -

**PULP/PAPER**
- Norra (Sweden) - Pulp Drying
- Abat [ ] - Consistency Control
- EKA (France) - Retention Control
- Nyte (Sweden) - Wall Trimmer
- Espanaveden (Sweden) -
- St. Regis (USA) -
- Boise Cascade (Canada) -
- Bowater (Canada) -
- Albany (Australia) -
- Mondi (South Africa) -
- Kalista (Sweden) -
- Polson (Scotland) -

**FOOD**
- SEA (Sweden) - Boost Pulp Dryer
- African Products (South Africa) - Maize Drying

**CHEMICAL**
- Borol (Sweden) - Chemical Reactor
- Teijinshu (Japan) - Rubber
- Cementa (Sweden) - Cement Kiln
- Borol (Sweden) - Total Instrumentation

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**BOILER**
- Monekilde (Denmark) - Boiler Control
- Manasse (Sweden) - Total Instrumentation

**WATER**
- Kuggala (Sweden) - Waste Water Control

**MACHINERY**
- Rudolphs Hospital (Sweden) - Total Instrumentation

**CHEMICAL**
- Asea NOVATUNE (Global) - Noise Control

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**Reduced Installation Costs**

**Conventional controllers:**

- Increased production capacity
- Decreased production costs
- Improved quality
- Reduced installation costs
- Reduced maintenance costs

**ASEA NOVATUNE controllers:**

- Comissioning

- Disturbed production
- Undisturbed production
Chemical Reactor

Reactor Control System Structure

Reactor Control

Combining parallel valves
Reactor Control
Performance

Skin Pass Mill

Elongation Control
Performance
Automatic tuning of simple regulators

K.J. Åström and T. Hägglund
Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

Abstract.

Procedures for automatic tuning of regulators of the PID type are described. The methods are based on a simple identification method which gives critical points on the Nyquist curve of the open loop transfer function. The key idea is a scheme which provides automatic excitation of the process which is nearly optimal for estimating the desired process characteristics. The methods proposed are primarily intended to tune simple regulators of the PI(D) type. In such applications they will of course inherit the limitations of the PI(D) algorithms. They will not work well for problems where more complicated regulators are required. The proposed algorithms may be used in several different ways. They may be incorporated in single loop controllers to provide an option for automatic tuning. They may also be used to provide a solution to the long-standing problem of safe initialization of more complicated adaptive or self-tuning schemes. In contrast to other methods based on self-tuning control, they do not require apriori information about time scales.

References

AUTOMATIC TUNING OF SIMPLE REGULATORS
K.J. Åström
LUND - SWEDEN

1. INTRODUCTION
2. THE BASIC IDEA
3. CONDITIONS FOR OSCILLATION
4. REGULATOR DESIGN
5. PRACTICAL ISSUES
6. EXPERIMENTS
7. CONCLUSIONS

SELF-TUNING CONTROL

INTRODUCTION

�� BACKGROUND
ADAPTIVE CONTROL RESEARCH
ROBUSTNESS
UNMODELED DYNAMICS
PRIOR INFORMATION
REACTIONS FROM INDUSTRY
�� ADAPTATION VS TUNING
�� THE PID STRUCTURE
�� STR & MRAS APPROACHES
LS+MU
LS+PP
ELS+LQG
�� A NEW APPROACH

THE BASIC IDEA

�� DESCRIBE THE PROCESS IN TERMS OF CRITICAL GAIN $k_c$ AND CRITICAL PERIOD $t_c$.
�� FIND METHODS FOR DETERMINING $k_c$ AND $t_c$.
�� APPLY DESIGN METHOD BASED ON $k_c$ AND $t_c$
EX: ZIEGLER NICHOLS
$k = \frac{k_c}{2}, T_i = \frac{k_c}{2}, T_d = \frac{k_c}{8}$
DETERMINATION OF $k_c$ & $t_c$

**RELAY CONTROL**

\[ y = \frac{u}{G(s)} \]

\[ k_c \approx \frac{a_d}{\pi a_e} \]

\[ \omega_n = \frac{a_c}{a_e} \]

**PROPERTIES**

- **AUTOMATIC GENERATION OF NEAR OPTIMAL INPUT**
- **GOOD POSSIBILITIES FOR CONTROLLING THE OUTPUT AMPLITUDE DURING THE EXPERIMENT**
- **SAFE PROCEDURE FOR STABLE SYSTEMS**

**THE DESCRIBING FCN APPROX.**

\[ H(e^{j\theta}) = \sum_{m=-\infty}^{\infty} \frac{1}{n + j\omega_m} \left(1 - e^{-j\omega_m}\right) G(j\omega_m) \]

\[ \omega_m = \frac{2\pi m}{n} \quad 5\pi = \frac{1}{n} \quad \text{gives} \]

\[ H(-1) = \sum_{m=-\infty}^{\infty} \frac{2}{n + j(\pi + 2\pi m)} G\left(i\frac{\pi + 2\pi m}{n}\right) \]

\[ = \sum_{m=-\infty}^{\infty} \frac{2}{n + j(\pi + 2\pi m)} \text{Im}\left\{ G\left(i\frac{\pi + 2\pi (m+1)}{n}\right)\right\} \]

\[ \approx \frac{4}{\pi} \text{Im}\left\{ G\left(i\frac{\pi}{n}\right)\right\} \]

**THEORY:** $H(-1) = 0$

**APPROXIMATION**

\[ k_c \approx \frac{a_d}{\pi a_e} \]
EXAMPLE 1

\[ G(s) = \frac{1}{s} e^{-sT} \]

\[ H(z) = \frac{2T + (h-1)}{z - 1} \]

\[ H(-1) = 0 \Rightarrow h = 2T \]

Period \( T_p = 2h = 4T \)

\[ \text{arg} \ G(i\omega_c) = -\frac{\pi}{2} - \omega_c T = -\frac{\pi}{2} \]

\( \Rightarrow \omega_c = \frac{\pi}{2T} \quad T_p = \frac{2\pi}{\omega_c} = 9T \)

EXAMPLE 2

\[ G(s) = \frac{1}{s(s+1)(s+a)} \]

\[ \text{arg} \ G(i\omega_c) = 0 \Rightarrow \omega_c = \sqrt{a} \]

\[ H(-1) = -\frac{h}{2a} + \frac{1}{a} \left[ \frac{1-e^{-1}}{1+e^{-h}} - \frac{1}{a} \frac{1-e^{-ah}}{1+e^{-ah}} \right] \]

\[ H(-1) = 0 \Rightarrow h \approx \frac{\sqrt{a}}{\sqrt{a}} \quad \text{approx} \]

\[ T_p = \frac{3\pi}{\sqrt{a}} = 6.92 \quad \frac{T_0}{a} \]

CONDITIONS FOR PERIODIC SOLUTIONS

EXAMPLE:

\[ G(s) = \frac{b}{s+a} \quad a, b > 0 \]

\[ H(s) = \frac{b(1-e^{-sT})}{z - e^{-sT}} \]

\[ \text{H}(s) = -\frac{b(1-e^{-sT})}{1+e^{-sT}} = -\frac{e}{s} \]

\[ T = 2\pi = \frac{2\pi}{a} \ln \frac{bd-e}{bd+e} \approx \frac{4\pi}{abc} \]

\[ Z\{u\} = \frac{d}{z+1} \quad Z\{v\} = -\frac{e}{z+1} \]

\[ H(\frac{T}{2},-1) = -\frac{e}{d} \]
EXAMPLE

\[
\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -\omega & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \omega \end{bmatrix} u
\]

\[y = \begin{bmatrix} 1 & 0 \end{bmatrix} x\]

\[H(t, z) = \frac{2 - 2\omega t z + 1}{z^2 - 2\omega t z + 1}\]

\[H(t, -1) = -\frac{\omega t}{1 + \omega t} = -\frac{\varepsilon}{\delta}\]

\[T = 2\pi = \frac{\omega}{\varepsilon} \text{ arccos } \frac{\varepsilon}{\delta - \varepsilon}\]

\[\varepsilon = 0 \Rightarrow T = \frac{2\pi}{\omega} \cdot \frac{1}{2} = \frac{\pi}{\omega}\]

HOW TO DETERMINE \( t_c \) & \( a \)

ZERO CROSSINGS & PEAK DETECTION

ZERO CROSSINGS & CORRELATION

LEAST SQUARES

\[\Sigma \left[ y(t) - b y(t-h) + y(t-2h) \right]^2 \text{ Min}_b\]

\[b = 2 \omega \left( \frac{h}{t_c} \right) \cdot 2\pi\]

\[\Sigma \left[ y(t) - a_1 \sin \omega t - a_2 \cos \omega t \right]^2\]

\[\omega = \frac{2\pi}{t_c}\]

EXAMPLE:

\[G(s) = \frac{1}{s^2}\]

\[H(t, s) = \frac{s^2}{s^2 + \frac{2+1}{2}}\]

\[H(t, -1) = 0\]

\( \Rightarrow \) NO PERIODIC SOLUTION WITH HYSTERESIS

\( \Rightarrow \) PERIODIC SOLUTIONS WITH ARBITRARY PERIOD WITH IDEAL RELAY

ZIEGLER-NICHOLS RULE

CRITICAL GAIN \( K_c \)

- \( \frac{1}{4} \) PERIOD \( T_c \)

\( K \) \( T_i \) \( T_d \)

P 0.5\( K_c \)

PI 0.4\( K_c \) 0.7\( T_c \)

PID 0.6\( K_c \) 0.5\( T_c \) 0.12\( T_c \)

MODIFIED RULES:

BETTER DAMPING?
CONTROL DESIGN

**AMPLITUDE MARGIN DESIGN**

\[ G_r(s) = \frac{k}{s} \left[ 1 + \frac{1}{s T_i} + s T_d \right] \]

\[ = k \left[ 1 + \frac{1}{s T_s} \right] \left( 1 + s^2 T_i T_d \right) \]

\[ k = \frac{K_m}{A_m}, \quad T_d = \frac{1}{\omega_i T_i}, \quad T_i \text{ arbitrary} \]

**PHASE MARGIN DESIGN**

\[ \frac{1}{\omega_i G(i\omega)} \]

\[ G(i\omega) \]

\[ A \]

\[ B \]

PICK \( k, T_i \) AND \( T_d \) TO MOVE \( A \) TO \( B \)

REGULATOR STRUCTURE

```
+-----------------+
| PID             |
| Process         |
| u              |

\[ y \]
```

PRACTICAL ISSUES

**MEASUREMENT NOISE**

**ADJUSTMENT OF AMPLITUDE OF OUTPUT**

**SATURATION OF ACTUATORS**

**WHAT PRIOR INFORMATION IS NEEDED?**

**HYSTERESIS**

EXPERIMENTS

**GOALS**

WHEN & HOW WILL IT WORK?

USER REACTION

**MEANS**

LSI 11/03, Apple II,

Intel 8086

Analogue computer simulation

Laboratory processes

Industry

Flow

Temperature

Level

Composition

**RESULTS**
PI - AUTO-TUNER

\[ G(s) = \frac{1}{(1+0.25s)^4} \]

![Simulation of a PI auto-tuner applied to a process with the transfer function \(G(s) = \frac{1}{(1+0.25s)^4}\).](image)

VARIATIONS IN PROCESS TIME CONSTANTS

\[ G(s) = \frac{1}{(1+5T)^4} \]

- **T = 5**
- **T = 1**
- **T = 0.2**

![Simulation of a PI auto-tuner applied to a process with the transfer function \(G(s) = \frac{1}{(1+5T)^4}\).](image)

VARIATIONS IN PROCESS GAIN

\[ G(s) = \frac{k}{(1+0.25s)^4} \]

- **k = 0.1**
- **k = 1**
- **k = 10**

![Simulation of a PI auto-tuner applied to a process with the transfer function \(G(s) = \frac{k}{(1+0.25s)^4}\).](image)

EFFECTS OF NOISE

![Simulation of the system with variable noise level. The standard deviation of the measurement noise is 0.1 and 0.2 in A and B respectively.](image)
CONCLUSIONS

A SIMPLE IDEA

SEEM TO WORK WELL

ROBUST

IMPLICATIONS FOR MORE GENERAL ADAPTIVE SYSTEMS

GETS YOU IN THE BALL PARK

RESEARCH ISSUES

AUTOMATIC TUNING OF SIMPLE REGULATORS

K. J. Åström
LUND - SWEDEN

1. INTRODUCTION
2. THE BASIC IDEA
3. AMPLITUDE-MARGIN DESIGN
4. PHASE-MARGIN DESIGN
5. EXPERIMENTS
6. CONCLUSIONS
The NAF - Autotuner

Lars Bååth
NAF Controls AB
Stockholm

Among other products for instrumentation and control NAF Control AB manufactures the Control and Information system NAF-Unic. As a special version of the software of the systems NAF-Unic S SDM-10 and NAF-Unic S SDM-20 the NAF-Autotuner can be obtained.

The NAF-Unic S SDM-20 system consists of one central unit, one or two color screens and function keyboards, one alphanumeric keyboard for program development, one printer and one tape recorder. The system has the capacity of handling 30 analog and 30 digital inputs, 16 digital and 16 analog outputs and up to 45 PID-controllers. With 45 PID-loops the system loop time is 250 ms. The expanded version can handle 240 inputs and 128 output signals.

The system can be programmed with function modules such as PID-controllers, a deadtime controller, limiters, alarm modules, adders, multipliers, logical modules and so on. In total there are about 30 different modules. The software of the system can contain 220 function modules. The system also contains a PLC-system which is integrated with the analog control system. It is possible to program the system on line.

A major goal in the design of this system has been high system security. Several measures have been taken to achieve this, for example

- All PC-boards are duplicated
- Checksum calculation of configuration in RAM and code in EPROM
- A triple serial bus for internal communication is used
- Self diagnostic routines for the inputs and outputs

The NAF-Unic S SDM-20 and SDM-10 systems also have incorporated an autotuner to help the operator in tuning the PID-controllers. A major design effort has been to simplify operation of the autotuner. All 45 PID regulator loops can use the Autotuner. The Autotuner has been developed by NAF-Controls in collaboration with Karl Johan Åström and Tore Hägglund at the Department of Automatic Control at Lund Institute of Technology. The principle of the Autotuner is to replace the normal PID-controller by a relay controller. A system with a relay controller starts to oscillate. If the period and amplitude of this oscillation are measured, the critical gain \( k_c \) and the critical period time \( T_c \) can be computed, because the describing function of the relay is known. When \( k_c \) and \( T_c \) are known a PID-controller can easily be designed.

The use of NAF-autotuner is very simple. Tuning can be started on operator command or by a digital signal. When no previous tuning is done, the operator must bring the process up to desired reference value in MAN-mode and start tuning.
If there is a controller that already has been tuned, just start tuning. Tuning can be interrupted by setting the controller in MAN- or AUTO-mode. When tuning is ready, the controller is automatically set in AUTO-mode with new PID-parameters.
SDM-20. System Features and Capacity

- handles both analogue and digital signals
  - 60 input signals — analogue or digital
  - 32 output signals — analogue or digital
- for up to 45 control loops
- supervision of up to 8 external single loop controllers
- optional signal can be connected to a pen recorder for long-term trends
- central loop and 250 ms

- software with 210 function modules
- advanced PLC system for interlocking and sequence control
- expandable to 240 input and 120 output signals
- very high system security
- PLC - system integrated with analogue control system by means of 256 digital signals used for communication
- on-line programming

NAF Control and Information System

- NAF-Unic S SDM-20
- NAF-Unic S SDM-30
- DEC PDP
- NAF-Uniview

NAF-Unic S SDM-20 with options

- up to eight single loop controllers can be connected to the system as well
- optional signal can be connected to a pen-recorder via the alphanumeric keyboard

Central Unit
Video Display Unit SIM-10
Printer SRM-10-2
Tape Recorder SRM-10-1
Operator Keyboard SHM-10-1
Programming Keyboard SHM-10-2
**NAF-AUTOTUNER in Unit S**

- Major design effort: Simplify operation of AUTOTUNER.
- Unit S incorporates 45 P/D controlled with AUTOTUNER.

**OPERATION**

- Tuning can be started on operator command or by a digital signal.
- Start of tuning is only permitted if control error is sufficiently small.

**First Time Tuning**

1. Slowly bring up the process up to desired reference value; in MAN-mode, and start tuning.

**Previous Tuning**

2. Start Tuning

- Tuning ready → Controller automatically set in AUTO with new P/D-parameters.
- Tuning can be interrupted by setting controller in MAN- or AUTO-mode.

**TUNE → MAN**

- Control signal returns to its value prior to start of tuning.

**TUNE → AUTO**

- Controller returns to AUTO-mode with old P/D-parameters.

---

**Fig. 2** Group display with controller selected for tuning

---

**Table:**

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Min.</th>
<th>Max. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P-Value</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>I-Value</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>D-Value</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>Controller</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>Mode</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>Setpoint</td>
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<td>100</td>
</tr>
<tr>
<td>7</td>
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<td>100</td>
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<tr>
<td>8</td>
<td>Feedback</td>
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<td>100</td>
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<tr>
<td>9</td>
<td>Loop Gain</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>Tuning Mode</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>
History and issues in adaptive flight control

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This paper attempts to answer three questions: Why are there no adaptive flight control systems in modern aircrafts? Why has adaptive theory been irrelevant to all attempted designs? What is happening to change this around?

Inspection of aircraft physics reveals that the linearized aircraft dynamics depend typically on three parameters, dynamic pressure, Mach number and angle of attack. If these parameters are known it is straightforward to design flight control systems which accomplishes desired goals. Two competing alternative designs are discussed: the adaptive schemes and systems based on air data scheduling. The history is described from the point of view of competition between these schemes. An important aspect is that the control systems are flight critical in new aircraft. This means that rigid safety measures are required. The space shuttle is another example. It requires four identical control channels plus a fifth backup channel. These channels are not adaptive. The adaptive systems described include the ones used in the X15, F-111 and the F-8.

The tradeoff between performance and stability are reviewed in order to discuss quantitatively the influence of plant uncertainty on feedback design. The plant uncertainties are separated into structured uncertainty which corresponds to the rigid body dynamics and unstructured uncertainties which correspond to flexure modes. Adaptive control can only deal with the structured part. Some assumptions on the unstructured dynamics must be made beforehand. It is interesting that the frequency ranges are surprisingly similar for a wide range of aircrafts. The airframe typically has resonances at 40 rad/s, the nonstationary aerodynamics around 60 rad/s actuator dynamics is around 12 rad/s, presampling filters and delays around 100 rad/s.

It is concluded that adaptive control can help only with the structured model errors but that it must work in the presence of unstructured model uncertainties. Some knowledge about the unstructured model errors must be available. This knowledge must be included in the design of the adaptive system.

The main conclusion is that the adaptive control has lost the competition against gainscheduling for regular flight control. Some potential problems where adaptive control may apply are when airdata is impractical (small missiles and reentry vehicles), where air data will not work (flexure and flutter control).

It is suggested that some work in the adaptive field is devoted to understand the engineering fixes that are made to make the systems work and particularly that unstructured uncertainties are considered.
References


CONVENTIONAL AIR-DATA-SCHEDULED CONTROL

COMPETING FLIGHT CONTROL ALTERNATIVES

\[ x = A(p, M) + B(q, M) + d \]

- Some fore/aft stabilizers change with angle-of-attack: \( \frac{d}{dM} = \frac{\dot{\theta}}{\dot{\phi}} \)
- Dynamic pressure: \( M = \frac{1}{2} \rho V^2 \)

- Forces/transients move with pitch: \( \dot{M} = \frac{1}{2} \rho V^2 \)

Sensors and Actuators in Aircraft and Control Laws

ADAPTIVE ALTERNATIVE

SONE UNITS OF MEASUREMENT TO CHANGE

ADAPTIVE FLIGHT CONTROL

HISTORY AND ISSUES
**The Basic Feedback Problem**

![Diagram of feedback system](image)

**Resulting Fundamental "Fact of Life"**

Good Feedback Performance

\( G \circ K > I \) over a given frequency range

Is possible if and only if

Model uncertainties are sufficiently small

\( \Phi[L] < 1 \) in the same frequency range

We have two options

- Know our plant before hand, or
- Learn it as we go

**Uncertainties in FDLTI Models**

![Diagram of uncertainty analysis](image)

**Two Types of Errors**

Structured - Errors which can be eliminated by adjusting parameters to best match the plant

Unstructured - Errors which remain

\[ e = G(u) - G(e^*) u \]

\[ = (G + L) u - G(u) \]

\[ = L Gu \Rightarrow \Phi[L] = \max \frac{|e|}{|Gu|} \]

Some mechanisms: neglected dynamics, actuator/sensor resonances, disturbances, time delays, non-linear behavior, etc.
MORE "FACTS OF LIFE"

- Adaptive control can help with structured errors only
- Adaptive control must work in the presence of unstructured errors
- We must know (or assume) the $\mathcal{F}(L)$ curve before hand
  - freq range over which $G_v$ can be large
  - bounds on $G_v$ outside that range

ADAPTIVE CONTROL TASKS

**Task 1**
Learn $G_v$ (explicitly or implicitly)

**Task 2**
Implement control law with $G_v$
Consistent with $\mathcal{F}(L)$ curve and with expected $C_v$ errors

**Task 3**
Protect against unstructured errors
1. Don't let them confuse the learning process
2. Don't make $G_v$ large where $\mathcal{F}(L) > 1$

F-8C UNSTRUCTURED UNCERTAINTIES

- Actuator satur. $\sim 12 R/I$
- Average structural $\sim 50 N/S$ resonances
- Unsteady aerodynamics $\sim 10 N/S$
- Sampling delays / pre-filters $\sim 10$

ADAPTIVE CONTROL SOLUTIONS

VIA STOCHASTIC OPTIMIZATION

**Optimization Problem**

Given:

$$\dot{x} = A(x) + B(u) + \eta$$
$$y = C(x) + \eta$$

Find:

$$u = P \hat{x} + F y + \delta$$

To minimize:

$$J = \mathbb{E} \left\{ \frac{1}{T} \int_0^T L(x, u) dt \right\}$$

Features:

- **Theoretically Optimal**
- Generally unsolvable
- Tasks 1 & 2 are optimally blinded (dual error)
- Task 3 is not appreciated

Solutions are (probably) easily confused by unstructured uncertainties
ADAPTIVE CONTROL SOLUTIONS
VIA STABILITY THEORY

MODEL REFERENCE STRUCTURE

FEATURES:
- THEORETICALLY GLOBALY STABLE
- TASK 1: ACHIEVED IMPLICITLY BY ADJUSTING $N_1, N_2, D$
  TO CANCEL PLANT
  \[ \frac{N_2 D}{D_p (N_1 + D) + N_1 N_2} = \frac{N_m}{D_m} \]
- TASK 2: ACHIEVED BY APPROPRIATE SELECTION OF $M(s)$
- TASK 3: NOT APPLIED

ADAPTIVE CONTROL SOLUTIONS
VIA CERTAINTY EQUIVALENCE

FEATURES:
- NO THEORETICAL PROPERTIES
- TASK 1: ACHIEVED BY EXPERT IDENTIFICATION
  OR 'BEST FIT' C. VECTOR
- TASK 2: ACHIEVED BY PRE-DESIGNED COMPENSATOR
  $K(s, p)$ WITH $p = \hat{e}$
- TASK 3: ACHIEVED BY
  'BEST FIT' FUNCTION AND TEST SIGNAL $u^*_T$
  CONFINED TO DEFINED RANGE

APPR. RULE
 \[ L \text{DESIGNED SUCH THAT } \varepsilon \to 0 \]

ASSUMPTIONS REQUIRED
FOR STABILITY/CONVERGENCE PROGRRA

PLANT: \[ G'(s) = \frac{N_p(s)}{D_p(s)} \to \frac{1}{\varepsilon} \text{ AS } s \to \infty \]

ASSUMPTIONS:
1. $N_p(s)$ IS STABLE
2. $D_p(s)$ HAS KNOWN MAX DEGREE
3. $\varepsilon$ HAS KNOWN SIGN
4. RELATIVE DEGREE $m$ IS KNOWN

IMPLICATIONS:
- GOOD UNSTRUCTURED UNCERTAINTIES
- HIGH GAIN FEEDBACK WITHOUT
  ADAPTATION WILL ACHIEVE MODEL
  FOLLOWING

F-BC PROGRAM CONCLUSIONS

- ADAPTIVE FLIGHT CONTROLS STILL CAN'T
  BEAT AIRDATA
  - MOST 'MODERN' CONCEPTS DID NOT
    WARRANT FLIGHT TESTING
  - THE MC. CONCEPT BARELY MATCHES
    AIRDATA PERFORMANCE

- ADAPTIVE FLIGHT CONTROLS NEED TEST
  SIGNALS (SUFFICIENTLY RICH INPUTS)
  - PILOTS HATE THESE

- ADAPTIVE FLIGHT CONTROL DESIGNERS SHOULD
  TURN TO PROBLEMS BEYOND THE
  CAPABILITY OF AIRDATA
**Potential Adaptive Problems**

**Where Airdata is Impractical**
- Small Missiles
- Reentry

**Where Airdata Won't Work**
- Direct Force Modes
- Flexure/Flutter Control

**Pie-in-the-Sky**
- Surface Reconfiguration
- Large Space Structures

**Status**
- Adapt systems exist
- Bounded Aero/inertial data
- Wind out

**Flexure/Flutter Control**

- Airplanes have speed limits
- Active control can extend limits by augmenting aeroelastic damping
  - Dynamics between 10-1000 R/Sec
  - Highly detailed, poorly known
  - No viable control solutions today

**Flexure/Flutter Models**

- Sense each resonance individually
- Close simple rate loops around each mode

**Ideally Flexure/Flutter Control is Easy**

- Nyquist diagram
simple adaptive flutter control

ideal sensor synthesis

real mode sensors are synthesized from various real sensors

adaptation in flutter control?

flutter detection is a more unstable?
if so, which one?
get & plot component with good flutter signature available
which mode design"

syntesises, etc.

vapor, etc.

which mode design with

gain & plot components available

which flutter mode is ideal?
RAW ACCELEROMETERS

Figure 33. Transient Responses for Run 7 of Table 13
(MLE, Case 1, 1.3 Vp) (continued)

Figure 34. Transient Responses for Run 7 of Table 13
(MLE, Case 1, 1.3 Vp) (continued)

PROGRESS IN ADAPTIVE THEORY

• OLD ENGINEERING RIES GET SERIOUS
  THEORETICAL ATTENTION
  - DEADZONES Pederson, Orlicki
  - RETARDATION Kreiss, Pecor, Johnson, Keffer
  - SAMPLE RATES Astrom et al., Parks

• NEW APPLICATION OF OLD THEORETICAL TOOLS
  - AVERAGING ANALYSES Astrom, K Avenue

• REFORMULATIONS
  - INCLUSION OF UNSTUCTURE UNTERTAINTIES Kautt, Johnson

CONCLUSIONS

BAD NEWS

WE LOST THE COMPETITION TO
SENSORS BUILDERS FOR ALL CONVENTIONAL
FLIGHT CONTROL PROBLEMS

GOOD NEWS

THERE ARE A FEW PROBLEMS WHERE
OUR TECHNOLOGY OFFERS HOPE

BAD NEWS

WE CAN'T SOLVE THESE PROBLEMS (YET)

GOOD NEWS

THE THEORY IS COMING ALONG TO HELP!
Self tuning control of the dissolved oxygen concentration in activated sludge systems

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Lund Institute of Technology
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The activated sludge process is recognized as the most common and major unit process for the reduction of organic waste. An overview of the control problems is found in [1].

A fully structured model of an activated sludge system is very complex and includes the following phenomena,

* the degradation of degradable pollutants, containing both organic carbon, phosphorus and nitrogen;
* cell growth and basal metabolism;
* the oxygen requirements of the system;
* the flow regime in the aeration basin;
* the representation of the settler and clarifier performance;
* the effect of secondary parameters, such as temperature, pH and toxic or inhibitory substances.

The goal for the reactor operation is of course to degrade the degradable pollutants. However, the operation must be such that organisms with the preferred floc formation are produced, thus giving desired clarification and thickening properties. Otherwise the operation of the system will fail, even if the degradation is efficient.

The DO concentration is an essential variable of the activated sludge process. It has a significant influence on both the plant operation economy and on the biological activity, and consequently on the quality of the effluent water.

A detailed derivation of the equations can be found in [1]. In a dispersed plug flow reactor the resulting DO dynamics can be described by

\[
\frac{\partial c}{\partial t} = \frac{E}{\partial z^2} \frac{\partial c}{\partial z} - n \frac{\partial c}{\partial z} + k_L a (c^* - c) - R \tag{1}
\]
where

\[ z = \text{length along the reactor} \]
\[ c(z,t) = \text{dissolved oxygen concentration} \]
\[ c^* = \text{dissolved oxygen saturation concentration} \]
\[ k_a = \text{oxygen mass transfer rate} = f(u), \ u = \text{air flow rate} \]
\[ E = \text{dispersion coefficient} \]
\[ v = \text{stream velocity} \]
\[ R = \text{oxygen uptake rate} \ (\text{respiration rate}) \]

The phenomena influencing the DO concentration have widely different timescales.

The control of DO as a physical variable does not require any in-depth knowledge of the microbial dynamics. The problem of finding the right DO set-point has been discussed e.g. in [3].

There are some important reasons for self-tuning control. The oxygen transfer rate is approximately proportional to the control signal, the air flow rate. Therefore a self-tuner can compensate for different "time constants" at different operating levels. Moreover, the oxygen transfer rate is time varying on a day-to-day time scale. The respiration R varies on an hourly time scale and is the main reason for control. However, R is interesting to know for other reasons. This can be part of the estimation scheme of a self-tuner. If the fact is used, that \( k_a \) and R vary in different time scales they can be estimated simultaneously.

Since last year full scale experiments of DO control have been performed at the Käppala wastewater treatment plant at Lidingö, outside Stockholm. The plant serves the northern suburbs of Stockholm and has a flow rate of about 2-4 m³/sec. A Novatune controller has been installed to take care of both the DO control loop and the air production system.

The figure shows some important features of the control. A constant setpoint of the DO concentration is given to the STR, sampled every 10 minutes. The DO sensor is fed into the controller and the control signal is cascaded with a local analog controller to adjust a throttle valve for the air flow rate. A limiting switch tells the STR if the valve saturates.

The pressure control is currently based on constant pressure set-point. It is kept via guide vanes on 3 of the 6 compressors. This does not give full control authority, and discontinuities of the control signals cannot be avoided. A pressure optimized will be added as soon as a sensor of the throttle valve angle can be measured. Then the pressure will be minimized so as to keep the valve as open as possible.

The results hitherto are encouraging but the evaluations of the biological properties due to control have just started. An expansion of the control to the whole activated sludge system will be made during the Fall of 1984.
References


SELF TUNING CONTROL OF THE DISSOLVED OXYGEN CONCENTRATION IN ACTIVATED SLUDGE SYSTEMS

GUSTAF OLSSON
LARS RUNDEVIST

THE ACTIVATED SLUDGE PROCESS

INFLUENT WATER CONTAINS
BIODEGRADABLE POLLUTANTS
NON-BIODEGRADABLE POLLUTANTS
CHEMICALS
TOXIC MATERIAL
INERT MATERIAL

MICROORGANISMS REACT WITH POLLUTANTS AND OXYGEN TO FORM MORE CELL MASS
CARBON DIOXIDE
WATER

OXYGEN IS SUPPLIED FROM DIFFUSERS

Introduction
Dissolved Oxygen Dynamics
Why Self Tuning Control?
The Koppala Plant
Evaluation of Results

CONTENT
INCENTIVES FOR CONTROL

- RISING OPERATING COSTS (600% BETWEEN 1971 AND 1984)
  - ENERGY (AIR SUPPLY, PUMPING)
  - CHEMICALS
  - PERSONNEL

- STRICTER EFFLUENT CONTROL

- TIME VARYING INFLUENT
  - HYDRAULICS
  - CONCENTRATION
  - COMPOSITION

POSITIVE DRIVING FORCES

- BETTER PROCESS KNOWLEDGE
- IMPROVED INSTRUMENTS
- COMPUTER COSTS AND COMPUTER PERFORMANCE
- BETTER CONTROL METHODS

THE DISSOLVED OXYGEN CONCENTRATION

- ECONOMY - MINIMIZE THE CONCENTRATION
- SET POINT - FORMATION OF DIFFERENT ORGANISMS
  - LIMITATION OF GROWTH
- MIXING - DO CONCENTRATION AND MIXING ARE COUPLED
  - FLOC FORMATION
- MULTI REACTOR SYSTEM - ANAEROBIC AND AEROBIC ZONES IN SERIES
- PROFILES OF DO - CAN GENERALLY NOT CONTROL THE PROFILE
  ALONG THE REACTOR
Dissolved Oxygen Dynamics

In complete mix reactor - mass balance

\[ \frac{dc}{dt} = \frac{q_{in}}{V} c_{in} - \frac{q_{out}}{V} c + \frac{\alpha \cdot u (c^* - c)}{c} - R \]

- Oxygen transfer
- \( \alpha \) varies slowly (days - weeks)
- Respiration
- \( R \) depends on biological growth and maintenance
  - Varies quickly (minutes - hours)

Why Self-Tuning Control of DO?

- Bilinear system - time constant varies with operating level
- \( \alpha \) varies slowly
- Want to know the respiration for other purposes:
  - Estimate from the mass balance!
- Feed forward signals can be introduced from influent
  (COD, TOC)
THE KAPPALA PLANT

- Serves the northern part of Stockholm
- About 500 000 persons
- Influent flow rate 2-4 m³/sec
- Six parallel basins
- Electric bill about 1 Mkr/year
INTRODUCTION
Dissolved Oxygen Dynamics
Why Self Tuning Control?
The KPPALA Plant
Evaluation of Results
EVALUATION OF METHANE

- Control Authority
  Discontinuities when blower is turned off
  Difficulty in obtaining certain ranges
- Throttle Valve
  Position must be measured
  Valve must be fitted with a suitable device
- Water Quality
  Being evaluated by Institute of Surface Chemistry
- Detection of Toxic Inputs
- Expansion to all six reactions
Adaptive pole placement for robots and servomechanisms

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Robots or mechanical manipulators are multi-degree of freedom systems whose dynamics are often nonlinear. Furthermore, because such systems pick up and move loads of various shapes and weights, the dynamics can also be viewed as time varying. The difficulties of working with nonlinear dynamics and time variation lead researchers to consider adaptive control as an alternative. Many recent papers have applied adaptive control schemes originally developed for unknown linear dynamics to such systems[1]-[4]. In this presentation we will begin by showing that this is a potentially dangerous design methodology. That is, adaptive controllers of this type will not perform as expected if the nonlinearities are dominant.

With this as motivation, we then develop a new approach to adaptive control of manipulators where the controller adapts to load changes but not nonlinearities. Rather the nonlinearities are included in design of the adaptive control law. The approach first uses nonlinear feedback to cancel the effect of the nonlinearities. Linear feedback is then used to place closed loop poles. The controller is intended for digital computer implementation and is discrete in nature. Most importantly, the design is based upon discrete time model for the nonlinear dynamics which is derived using Euler approximations for derivatives.

As a step toward understanding the stability and performance properties of such an adaptive scheme, one must first determine the stability and performance properties of the corresponding fixed computer control algorithm designed using the same Euler approximation model. To this end, we restrict our attention to the case of a single link or single degree of freedom manipulator. Assuming torque to be supplied by a D.C. motor, this boils down to design of a D.C. servo system. However, because the link represents an asymmetric load rotating in a gravitational field the resulting model is still nonlinear. In the case where the link rotates in a horizontal plane the model becomes linear. By considering this simple case of computer control of a linear servo, it is shown that fixed computer control algorithms for pole placement, designed using Euler approximation models are stable for a very wide range of model parameters, sample rates and closed pole locations.

Furthermore, it is shown that considerable performance improvements can be obtained when designing zero cancelling controllers such as are used in model matching. In particular the problems which arise because sample data models have zeros on or near the unit circle can be avoided. Since many adaptive control schemes are based upon fixed zero cancelling control strategies these results are also of potential importance in the design of adaptive sample data systems.

During the course of the presentation, simulations will be presented
demonstrating the potential performance of the new nonlinear adaptive control algorithm on a three degree of freedom manipulator. In addition experimental results will be presented for a hardware implementation on a one degree of freedom servo system in both the linear and nonlinear cases.

References


ADAPTIVE POLE PLACEMENT FOR ROBOTS AND SERVO MECHANISMS

by

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OUTLINE OF PRESENTATION

I. INTRODUCTION
II. LINEAR VS. NONLINEAR ADAPTIVE CONTROL CASE STUDY - 1 DOF
III. GENERAL ALGORITHM FOR CONTROL OF MANIPULATORS 2 DOF Case
IV. 3 DOF Case and Engineering simplifications
V. Issues in Stability Analysis

VI. Experimental Results

Problems in Manipulator Control
* Changing Dynamics
  load changes
  environmental changes
* Nonlinearities
  coordinate transformations
  dynamics

Why Adaptive Control?

Control
Unknown or Changing Process
Adaptation Algorithm
Adjustable Controller

* Changing Dynamics
General Form of Nonlinear Dynamics

\[ y' + Ky + G(y)y' = u \]

- \( y \): vector of joint angles
- \( u \): vector of motor torques
- \( G(y) \): Jacobian matrix
- \( K \): damping matrix

Control Problem

Find \( u(t) \) so that \( y(t) \) tracks a reference trajectory \( \hat{y}(t) \).

II. Comparison of Two Approaches

Single-Link Case

\[ T = \frac{mL^2}{3} \]

\[ J = ml^2 \]

\[ G = mg \]

\[ J\ddot{y} + K\dot{y} + G\cos y = u \]

\[ u_k = y_{kr} \]

\[ u(t) = u_k + K \sin(y-y_k) \]

\[ \Theta_k \]

\[ \phi_k \]

\[ \phi_k = [y_{kr}, y_k, y_{kr}, \cos y_k] \]

\[ \Theta_k = [\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, G] \]

\[ \dot{\hat{y}} = \frac{y_{kr} - y_k}{T} \]

\[ \ddot{\hat{y}} = \frac{(y_{kr} - 2y_k + y_{kr})}{T^2} \]

\[ \frac{1}{T} (y_{kr} - 2y_k + y_{kr}) + \frac{K}{T} (y_k - y_{kr}) + G\cos y_k = u \]

\[ \phi_k = [y_{kr}, y_k, y_{kr}, \cos y_k] \]

\[ \Theta_k = [\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, G] \]

General Approach

Let the system have multiple components

\[ u(x) = \tilde{u}(x) + e(x) \]

\[ \tilde{u}(x) \]: adaptive control

\[ e(x) \]: control error

Advantages

- [Advantages listed here]
Deadbeat (Minimum Variance) Control

Assume \( y_k^* \) to be tracked
know one step ahead
\( (f_k = y_{k+1}^*) \)

Choose
\[
U_k = \phi_k^T \theta_k \quad k > 0
\]
\[
\phi_k^T = [y_k^*, y_k, y_{k-1}, \cos y_k]
\]

Guarantees \( y_k = y_k^* \) \( k > 1 \)

For linearised model
\[
U_k = \phi_k^T \hat{\theta}_k \quad \hat{\phi}_k^T = [y_k^*, y_k, y_{k-1}, u_m]
\]

Adaptive Implementation
\[
U_k = \phi_k^T \hat{\theta}_k \quad \hat{\theta}_k = \text{Est}(\theta)
\]
\[
U_k = \phi_k^T \hat{\theta}_k \quad \hat{\theta}_k = \text{Est}(\theta)
\]

Least Squares Estimation
\[
e_k = U_k - \phi_k^T \beta_k \quad (s = u_k - \phi_k^T \gamma)
\]
\[
\beta_k = \beta_{k-1} + \frac{P_{k-2} \phi_k \beta_{k-1}}{\lambda + \phi_k^T P_{k-2} \phi_k}
\]
\[
P_{k+1} = \left[ P_{k-2} + \frac{P_{k-2} \phi_k \phi_k^T P_{k-2}}{\lambda + \phi_k^T P_{k-2} \phi_k} \right] \frac{1}{\lambda}
\]
\[
P_{k} = \iota I \quad \iota > 0
\]
\( 0 < \lambda \leq 1 \) (exponential weighting)

linear case \( \phi_k \theta_k \rightarrow \phi_k \hat{\theta}_k \)

---

Figure 1. Response of non-linear, a), b), and linear c), d) when
a), b), c), d).
Typical Linear Model

\[ y = \frac{1}{\epsilon^2} \frac{d^4 y}{dx^4} \]

In simple case

\[ y' + A_y x + A_y y = B u + E h \]

Model Matching Control

\[ \dot{x} = A x + B u \]

Equations for Two-Link Planar Manipulator

\[ \begin{bmatrix}
    \dot{\theta}_1 \\
    \dot{\theta}_2
\end{bmatrix} =
\begin{bmatrix}
    \frac{1}{m_2} c_{\alpha_2} \sin(\beta_{\alpha_2}) - \frac{1}{m_1} c_{\alpha_1} \sin(\beta_{\alpha_1}) \\
    \frac{1}{m_2} c_{\alpha_2} \sin(\beta_{\alpha_2}) - \frac{1}{m_1} c_{\alpha_1} \sin(\beta_{\alpha_1})
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
\]

\[ J(u) = m_1 \dot{\theta}_1 + m_2 \dot{\theta}_2 + D \dot{\theta}_2 + D \dot{\theta}_1 + D \dot{\theta}_2 + D \dot{\theta}_1 \]

\[ \alpha = \frac{1}{2} \left[ \begin{array}{c}
    0 \\
    0 \\
    0 \\
    0
\end{array} \right] \]

\[ \alpha = \frac{1}{2} \left[ \begin{array}{c}
    0 \\
    0 \\
    0 \\
    0
\end{array} \right] \]
Adaptive Implementation

1) Estimate $\Theta_i$ using the errors

$$e_{ix} = \phi_{ix}^T \Theta_i - u_{ix}$$

and two parallel sequential least squares estimators (one for $\Theta_i$ one for $\Theta_{ix}$)

Two estimation problems of size 5 (as 4x5 for mean)

4=3

2) Use $\Theta_i$ to generate

$$J(k) = \text{Est}(J(u_{ix}))$$

$$\Theta_{ix} = \text{Est}(\Theta_i) \text{ for } i=3,4,5$$

3) Use

$$u_{ix} = D_{ix}v_{ix} + D_{ix}S_{ix} + D_{ix}e_{ix} + \frac{1}{2}J(k)[v_{ix} - 2v_{ix} - E_{ix} - \Theta_{ix} + \Theta_i]$$
I. THREE LINK CASE:

\[ \begin{align*}
\dot{\theta}_1 & = \text{mass link } i \\
L_i & = \text{length link } i \\
\alpha_k & = \text{dist. to c.g. link } k \\
I_k & = \text{mom. Inertia link } k \\
\end{align*} \]

**Equation 1**

\[ \begin{align*}
D_1 \ddot{\theta}_1 + D_2 \ddot{\theta}_2 + D_3 \ddot{\theta}_3 & + D_4 \left[ c_{e_2} (\dot{\theta}_2 - \dot{\theta}_1) - 5 \theta_2 (\dot{\theta}_2 - \dot{\theta}_1) \right] \\
& + D_5 \left[ c_{e_3} (\dot{\theta}_3 - \dot{\theta}_1) - 5 \theta_3 (\dot{\theta}_3 - \dot{\theta}_1) \right] \\
& + D_6 \left[ c_{e_4} (\dot{\theta}_4 - \dot{\theta}_1) - 5 \theta_4 (\dot{\theta}_4 - \dot{\theta}_1) \right] \\
& + D_7 \left[ \theta_1 + D_{10} S (\theta_{10} + \theta_1) \right] = T_1 \\
\end{align*} \]

**Equation 2**

\[ \begin{align*}
D_1 \ddot{\theta}_1 + D_2 \ddot{\theta}_2 & + D_3 \left[ c_{e_2} (\dot{\theta}_2) - 5 \theta_2 \right] \\
& + D_4 \left[ c_{e_3} (\dot{\theta}_3) - 5 \theta_3 \right] \\
& + D_5 \left[ c_{e_4} (\dot{\theta}_4) - 5 \theta_4 \right] \\
& + D_6 \left( \theta_1 + \theta_2 + \theta_3 - \theta_4 - \theta_1 \right) + D_{10} S (\theta_{10} + \theta_1) = T_2 \\
\end{align*} \]

**Equation 3**

\[ \begin{align*}
D_1 \ddot{\theta}_1 + D_2 \ddot{\theta}_2 & + D_3 \left[ c_{e_2} (\dot{\theta}_2) - 5 \theta_2 \right] \\
& + D_4 \left[ c_{e_3} (\dot{\theta}_3) - 5 \theta_3 \right] \\
& + D_5 \left[ c_{e_4} (\dot{\theta}_4) - 5 \theta_4 \right] \\
& + D_6 \left( \theta_1 + \theta_2 + \theta_3 - \theta_4 - \theta_1 \right) + D_{10} S (\theta_{10} + \theta_1) = T_3 \\
\end{align*} \]

**Net Result**

Estimate:

\[ \begin{align*}
\Theta &= [D_1, D_2, D_3] \\
\Theta' &= [D_4, D_5, D_6] \\
\Theta'' &= [D_7, D_8, D_9] \\
\end{align*} \]

\[ \begin{align*}
X_i \Theta' &= Y_i \\
\text{Measurable Signals} \\
\end{align*} \]

**Control Strategy**

As before
II. DIGITAL CONTROLLER DESIGN

**Using Euler Approx for SERVO**

1. Euler Approx
2. Recall usual approach
3. Model matching control based on (2)
4. Pole placement using error feedback
5. Pole placement using error feedback

- \( u_n = \frac{r_n}{r_n - (\alpha + \beta)u_n} \)
- \( u_n = \frac{r_n}{r_n - (\alpha + \beta)u_n} \)
- \( u_n = \frac{r_n - (\alpha + \beta)u_n}{r_n - (\alpha + \beta)u_n} \)
- \( u_n = \frac{r_n}{r_n - (\alpha + \beta)u_n} \)
- \( u_n = \frac{r_n}{r_n - (\alpha + \beta)u_n} \)

**Questions:**
- Will new design be stable?
- Look at closed loop pole locations when Euler Based Controller applied to Servo. Note!
- How does new design perform compared to Step Inv. Model?
V. STABILITY ANALYSIS

Can we justify methodology with any Theory?

Outline of proof:

Use analysis methodology of Lemma 2.3, and Combine, must show

\[ \text{Input} + K x \text{Output} \]

\[ e = \text{Input} - \text{Error} \]

Can Show:

\[ (\text{Input} + \text{Output}) = \text{Output} \]

\[ \text{Input} = \text{Output} \]

Problem: I # of DOF \geq 2 then

\[ \phi_n \sim y_n^2 \]
Some desirable features of industrial adaptive controllers

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Abstract

After a short introduction, three different physical processes will be described, for which adaptive control have been tried or considered. They are: a level control for a tank in a pulp production plant, an autopilot for ships, and a pressure and flow control system for a medical treatment system (artificial kidney). It will be discussed why adaptive control could be useful in these cases, and how it really works in two of them with certain algorithms.

Each process and its operators represent different demands on the adaptive control system such as robustness, variations in model structure, interpretability of identified parameters. These demands will be discussed for each process separately. This results in a small list of some desirable features of industrial adaptive controllers.
Some desirable features of industrial adaptive controllers:

- Introduction
- Level control
- Autopilot
- Dialysis machine
- Summary

Algorithm:
- Minimum Variance
- Least Squares Est. (b_0 fixed)
- Feed forward (from chip feed)
- T_s = 1 min.
- k = 3, 5, 6
- λ = 0.96 - 0.995

Level control

- Simple level control with unknown actuator dynamics
- Time delay 2-3 min.
- On PID control, T_s = 5 sec, no feed forward
- Digital controller gives setpoint to analog controller

Experience:
- PID structure (NB=0) robust, works well
- Difficult to adjust k if NB=0
- Desirable to be able to compare parameters with PID control
- Excess of parameters causes covariance blow-up (feed forward)
Algorithm:
* Optimal LQ control
* Spectral factorization
* ELS estimation
* Forced integrator
* $tr P \leq \text{constant}$
* $NA = 1$ (+ known integrator)
* time delay $\leq NB$ (= 3)
* $T_s = 5$ sec. (3 sec.)
* $\lambda = 0.99 - 0.9999$

Experiences:
* Integral action important (quick)
  * Natural criterion good as motivation
  * Essential to be able to handle different levels of noise
  * Quarterly sea difficult - should separate models for dynamics & disturbance
  * Would like to fix physical parameters

Dialysis machine
* Control of: - Temperature
  - Conductivity
  - Flow
  - Pressure
* Simple time constant: 0.5 - 5 sec.
* Dynamics vary with:
  - Aging of pumps (Gain)
  - Filter type (time constant)
  - Flow
  - Blood composition
Demands on adaptive control

- **SAFETY!**
  - No risk for sudden or slow loss of control accuracy.
  - Robustness
- Should identify filter once/treatment compensate for pump aging = long term drift
  - Separate known models and unknown parts
  - Some physical parameters fixed

- **PID-structure** good for servicability.

**Some desirable features**

- **Safety**

- Avoid manual tuning of critical parameters (k).
- Robustness for model errors (e.g. rudder machine)
- PID-structure (\(N_\theta = 0\)) robust (?) and well-known
- Automatic detection of parameter excess (Covariance blow-up)
- Possibility to fix physical parameters
- Separation dynamics-disturbance
- Capability to handle varying noise levels

- Most cope with non-minimum phase

- Include analog prefilter in self-tuner
- Feedforward essential
Self-tuning regulator with increased prediction horizon

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The basic self-tuning regulator based on least squares estimation and minimum variance control is not suited for systems with zeros outside the unit circle. In the presentation it is shown that the basic algorithm often can be used also for this type of systems by increasing the prediction horizon of the regulator.

Examples are given which show how the algorithm can be used for non-minimum phase systems. Possible convergence points are discussed and some new results concerning local convergence of the algorithm are presented.

The new insight into the properties of the algorithm explains why it can be applied to a wide range of practical problems provided that some design parameters are correctly chosen.

Reference

SELF-TUNING REGULATOR WITH INCREASED TIME HORIZON

INTRODUCTION

THE BASIC ALGORITHM

INCREASED PREDICTION HORIZON

ANALYSIS

SIMULATIONS

CONCLUSIONS

THE BASIC ALGORITHM

Process

\[ A^2(q^{-1})y(k) = B^2(q^{-1})u(k-d_0) + C^2(q^{-1})e(k) \]

Algorithm

LS: \[ y(k+d) = R^2(q^{-1})u(k) + S^2(q^{-1})y(k) \]

MV: \[ u(k) = - \frac{S^2}{R^2} y(k) \]

PROPERTIES

If convergence then

1. \( R_y(z) = 0 \) \( z = d_0, \ldots, d_0 \deg S^2 \)
   \( R_u(z) = 0 \) \( z = d_0, \ldots, d_0 \deg R^2 \)

2. Sufficiently complex regulator
   \( \Rightarrow \) Minimum variance control

Convergence of

\( a \) Minimum phase plant
\( b \) \( \frac{1}{C(\omega)} - \frac{1}{2} \) strictly positive real

Local stability if

\( C(\zeta) > 0 \)

for all \( \zeta \); such that \( B(\zeta) = 0 \)

INCREASED PREDICTION HORIZON

Use \( \Delta > d_0 \) in the estimation

What will happen?

\( \Rightarrow \) Good thing to do in practice

\( \Rightarrow \) Property 1 still valid but with \( d_0 \) replaced by \( \Delta \), Wittenmark (1973)

\( \Rightarrow \) Can handle some non-minimum phase processes
ANALYSIS

A(q) y(k) = B(q) u(k) + C(q) e(k)

\[ \text{deg } A = \text{deg } B \]

R(q) u(k) = -S(q) y(k)

Closed loop system

\[ y(k) = \frac{C R}{AR + BS} e(k) = \frac{F}{q^a} e(k) \]

MA process

Pole placement interpretation

Minimum variance (B^* = B, A_w = q^{d_w}, A_C)

\[ R = BF \]

\[ AF + S = q^{d_r} C \]

\[ y(k) = \frac{BF}{q^{d_r} C} e(k) \]

deg R = n-1

deg S = n-1

INCREASED TIME HORIZON

Assume d = n + \alpha + 1

\[ AR + BS = q^{\alpha} C \]

\[ y(k) = \frac{FR}{q^{\alpha+1} C} e(k) \]

deg R = n-1

deg S = n-1

Comparison

* Same degree of regulator

* No cancellation of process zeros, compare Åström (1970)

* Closed loop system is MA of order \( \alpha = n-1 \)

RESULTS

1. If convergence then there is a possible convergence point that corresponds to MA process of order n-1

2. Local convergence can be analysed

SIMULATIONS

Example 1

\[ y(k) - y(k-1) = u(lk) + 1.1 u(k-1) + e(k) - 0.5 e(k-1) \]

Output | Input
--- | ---
![Output 1](image1.png) | ![Input 1](image2.png)

\( d = 1 \)

\( d = 2 \)

Optimal
**Example 2**

\[ \dot{y} = uu - t \]

\[ y(k+1) + y(k) = (u-k)u(k) + I y(k-1) + e(k-1) + e(k) \]

Non minimum phase if \( t > h \)

Simulation of ODE for the LS case

**Example 3**

\[ A(1) = 2(1-0)(1-0.5) \]

\[ B(1) = 3(1-2)(1-0.5) \]

\[ C(1) = 3(1-0.5) \]

\[ \dot{u}(k) = -3u + 3.5 \]

\[ 1 + r + r^2 + r^2 = y(k) \]

Accumulated loss

- **d=2**
- **d=3**

Optimal 1.84 1.31 1.51

C - polynomial
A universal control capable of stabilizing any single-input, single-output, minimum phase linear system of relative degree \( \leq 2 \)

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Within the past few years there have been developed several smooth dynamical controllers, not requiring "sufficiently rich" probing signals, which are capable of stabilizing any process which can be modelled by a linear system with transfer function of the form

\[
T(s) = g \frac{\alpha(s)}{\beta(s)}
\]

where \( g \) is a nonzero constant, and \( \alpha(s) \) and \( \beta(s) \) are monic, coprime polynomials, provided it can be assumed that

1. \( \alpha(s) \) is a strictly stable polynomial (i.e., \( \Sigma \) is minimum phase)
2. a bound \( n \geq \text{degree } \beta(s) \) is known
3. the relative degree \( n' = \text{degree } \beta(s) - \text{degree } \alpha(s) \) is known exactly
4. the sign of \( g \) is known

Since the preceding assumptions are very restrictive, there is ample motivation to see if controller structure can in some way be modified so that at least some of these assumptions can be avoided.

Prompted by this, we have just discovered that the 7-dimensional control system consisting of sensitivity function \( T = [\theta_u \theta_y \theta]' \), where

\[
\begin{align*}
\theta_u &= -\lambda \theta_u + u \\
\theta_y &= -\lambda \theta_y + y.
\end{align*}
\]

filtered sensitivity function \( \phi = [\phi_u \phi_y \theta]' \), where

\[
\begin{align*}
\phi_u &= -\lambda \phi_u + \theta_u \\
\phi_y &= -\lambda \phi_y + \theta_y.
\end{align*}
\]

parameter adjustment law

\[
k = \phi y.
\]

Gain \( N(x) = x \cos(x) \), where \( x = k'k \), and feedback law
\[ u = N(x)k'\theta + \left(\frac{\partial N(x)}{\partial x}\right) (k'\phi)^2 y + N(x)\phi'\psi y \\
= x \cos(x) (k'\theta + \phi'\psi y) + (\cos(x) - x \sin(x)) (k'\phi)^2 y \\
= N(x)k'\theta + \phi' \frac{d}{dt} (N(x)k) \]

stabilizes any relative degree 2, minimum phase system of any dimension. What's surprising is that this control can also stabilize any relative degree 1 minimum phase system of any dimension. In other words, to achieve stability with the above control, it is only necessary to know that the minimum phase system to be controlled has relative degree not exceeding 2; i.e., it is not necessary to know relative degree exactly only an upper bound is required. It is natural to speculate that this should be true in general. In other words, with apriori knowledge of an upper bound \( \bar{n} \), it should be possible to construct a smooth control system not incorporating a probing signal, which can stabilize any minimum phase system of any dimension, provided the system's relative degree does not exceed \( \bar{n} \).
GENERAL ISSUES

Stability is of central importance - why?

What should be the role of a probing signal?

Should we insist on "smooth algorithms"?

OBJECTIVES:

- Tracking: \( t \to 0 \) as \( t \to 0 \)
- Internal stabilization - all states bounded to 0

Control Class

1. Smooth finite dimensional dynamical system
2. Arbitrary bounded reference input - "efficiency rate" probing not compensated for disturbance!

Plant Model Assumptions - linear system with transfer function \( g_p(\xi_p(1)/\xi_p(2)) \)

1. \( \xi_p(1) \) stable - minimum phase
2. Bonded \( \xi_p(2) \) degree \( p_1(a) \) in phase
3. Rel. degree \( n_p = \deg \xi_p(2) - \deg \xi_p(1) \) known
4. Sign \( g_p \) known

Base Problem: Given \( \dot{y} = ay + g(x, y) \), with \( \alpha \) and \( g \) unknown and \( g \neq 0 \), does there exist an integer \( m \geq 0 \) and smooth functions \( f: \mathbb{R}^m \to \mathbb{R}^m \)
\( h: \mathbb{R}^m \to \mathbb{R} \) such that the closed loop system:
\[
\dot{y} = ay + g(x, y) \\
\dot{x} = f(x, y)
\]
is stable in the sense of for any causal state \((x, y)\), there exist a solution \((x(0), y(0))\) bounded on \((0, \infty)\) and \( y(0) \to \) as \( t \to \infty \)?

If sign \( g \) known, the classical adaptive control with \( n = 1 \), \( f = g \), \( h = -(\beta g(x, y))x \) stabilizes:
\[
\dot{y} = (a-\beta g(x, y))y \\
\dot{x} = x^2
\]
The control \( u = y^2 \sin(y) \) [inspired by Woukand] also stabilizes:
\[
\dot{y} = (a + g \sin(g))y
\]
but \( y \) doesn't necessarily \( y \to 0 \) due to multiple equilibria.

Negative Results

If \( \alpha \neq 0 \) and \( f(x, y) \) and \( h(x, y) \)
are constrained to be quadratic polynomials in \( x \) and \( y \), then stabilization of
\[
\begin{align*}
\dot{y} &= ay + g(x, y) \\
\dot{x} &= f(x, y)
\end{align*}
\]
Is impossible.

If \( \alpha \neq 0 \) and \( f(x, y) \) and \( h(x, y) \)
are constrained to be rational functions in \( x \) and \( y \), then stabilization of \( y \) is impossible.

Open Problem: Generalize the above by proving that Neubeck's assertion is true for rational controllers of any dimension \( m \geq 0 \).
Willem's Phase Rule: It is possible to stabilize
\( y = ay + gu \) with a smooth 1-dimensional
controller.

**Simul- Control**
\[
U = N(x)y, \quad N(x) = x^2 \cos(x)
\]
\[
\dot{x} = y^2
\]
\[
y = (a + gx^2 \cos(x))y
\]
\[
\dot{y}^2 = 2(a + gx^2 \cos(x))y^2
\]

**Analysis (Nichols)**
\[
\dot{y}^2 = 2(a + gx^2 \cos(x))y^2
\]
\[
\dot{y}(t) - p(0) = (a + m^2 \cos(x)(\dot{x})
\]
\[
\dot{y}(t) = \pi(x(t)) + y(0)
\]
\[
\pi(\mu) = 2x_0 + 2(x_0 - \mu) + 2.4\mu(x_0 - \mu)
\]
\[
\pi(\mu) = 2x_0 + 2(x_0 - \mu) + 2.4\mu(x_0 - \mu)
\]
\[
\pi(\mu) = 2x_0 + 2(x_0 - \mu) + 2.4\mu(x_0 - \mu)
\]
\[
= \text{dominates}
\]
\[
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= \text{dominates}
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\[
= \text{dominates}
\]

**General Parameterizations**
- Observer Based (MACO, etc.)
- Dynamic Comparator Based

et (Q,1) be any (n-1)-dimensional \((\mathbb{R}^n)\)-dimensional) controllable pair with a stable, definite, semi-definite function \(\Theta = [\Theta_x, \Theta_y, y] \) where
\[
\Theta_x = \Phi_x + bu \quad \Theta_y = \Phi_y + by
\]
\[
\gamma = \Phi_x + \Phi_y
\]

**Lemma:** For each \(n\)-dimensional, minimum phase, system of relative degree \(n \leq n_R\) and each \(\Theta_x, \Theta_y\), stable polynomial \(\Phi_x + \Phi_y\) there exist a moment contour \(\gamma\), a symmetric, stable transfer matrix \(T\), a constant proper, stable transfer function \(L\) and a constant vector \(\Phi_x + \Phi_y\) such that
\[
y = \frac{1}{\gamma} \left( g_x(u, y) + L \right)
\]
\[
\Theta = T(\Phi_x + \Phi_y) + E + \Phi_x
\]

Where \(E\) is a linear combination of decaying exponent.
Adaptive stabilization of linear multivariable systems

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Framework:
We have a system known only imprecisely, and we want to find a "universal stabilizing controller" which makes the output \( y(t) \) of the system \( \to 0 \) as \( t \to \infty \), while the parameters in the controller stay bounded.

I. The SISO case
Theorem. Consider the SISO system \( \dot{x} = Ax + Bu; \ y = cx \). Assume it is minimum phase and \( \Re(b) > 0 \). Define the controller \( k = y^*, \ u = -ky \). Then, for all \( (x_0, k_0) \) it is true that \( k_t \to k_\infty < \infty \) and \( x_t \to 0 \) as \( t \to \infty \).

II. The MIMO case
Theorem. Consider the \( m \times m \) system \( \dot{x} = Ax + Bu; \ y = Cx \) and suppose that \( \det G(s) = 0 \Rightarrow \Re(s) < 0 \) and that \( \text{spec}(CB) \subset C^+ \). Then, the controller \( k = \|y\|^2 \), \( u = -ky \) satisfies \( x_t \to 0 \) and \( k_t \to k_\infty \) as \( t \to \infty \).

The proof consists of two ideas: First multivariable root-locus methods are employed to show that the eigenvalues will go into the left half plane, then "frozen analysis" as below shows that this is sufficient to deduce stability. An example of multivariable root locus was analyzed.

III. Frozen Eigenvalue Analysis
Theorem. Consider the system \( \dot{x} = Ax - k(t)Bx \), where (i) for \( k \gg 0 \) \( A - kB \) is stable, and, (ii) \( k(t) \to +\infty \) as \( t \to \infty \). Then each solution \( x(t) \) tends to \( 0 \) exponentially.

IV. Modifications and Extensions
The standard assumptions of necessary a priori knowledge for adaptive stabilization of an unknown SISO system are:

1) minimum phase
2) \( n \leq N \), i.e. we have an upper bound on the order of the system
3) \( n^* \), the relative degree of the system, is known
4) \( \text{sign}(cA^{n^*-1}b) \) is known

We have shown that 2) is not needed, actually under weak condition the results can be generalized to a Hilbert space context. By a variant of Nussbaum's result, it was demonstrated that 4) is not needed either.
V. Necessary Conditions for Adaptive Stabilization

**Metapriniciple.** "Whatever can be done adaptively can be done if we know (A,b,c)."

**Framework:** Let the adaptive regulator be a $n_z$-dimensional linear system with internal state $z$, and dependent on a parameter $k_t$ which is updated according to $k = f(k,y)$. By convergence we mean that $(x_t,z_t,k_t) \rightarrow (0,0,k_\infty)$ as $t \rightarrow \infty$.

**Theorem.** If we have a convergent parameter adaptive stabilization scheme, then $n_z \geq n - 1$.

**Theorem.** Suppose $g(s) = c(sI - A)^{-1}b$ is minimum phase and of relative degree $\hat{n} \leq n$. Then, if $p(s)$ is minimum phase of degree $n^*$, the regulator

$$k(s) = \frac{k\beta^n p(s)}{(s + \beta)^n}, \quad k = y^2, \quad \beta = e^k$$

will stabilize $g(s)$. 
Adaptive Stabilization of Linear Multivariable Systems

Hypotheses: I. \( q(s) = c(sI - A)^{-1}b = 0 \)
\[ \Rightarrow \text{Re}(s) < 0 \]
minimum phase
II. \( q'(s) = cb > 0 \)

Theorem: Suppose
\[ x = Ax + bu \]
\[ y = cx \]
is minimum phase with \( cb > 0 \). Define the controller
\[ u_c = y^2 , \quad u = -hy \]
Then (1) - (2) satisfies \( y(x_0, x_0) \):
(i) \( x_1 \to 0 \) \( t \to \infty \)
(ii) \( x_2 \to 0 \) \( t \to \infty \)

Proof: \( cb > 0 \) \( \Rightarrow \) \( b \neq 0, \text{ker c} \)

\[ \ker c + \text{span} f = \mathbb{R}^n \]
\[ \langle x_1, x_2 \rangle = x \in \mathbb{R}^n \]

\[ x_1 = A_{11} x_1 + A_{12} y \]

\[ x_2 = A_{21} x_1 + A_{22} y - \beta z y \]

\[ \beta z y \]

1.3. \( \text{sign} (\beta) = \text{sign} (cb) > 0 \), \( \text{spec}(A_{11}) = \mathbb{C}(g(s)) \subset \mathbb{C} \)

\[ \beta = cb. \]

Proof 1: \( q(s) = 0 \) \( \Rightarrow \) \( q(\beta s) = 0 \)

where \( x(0) = 0 \)

Therefore, at \( s = s_0 \) we have
\[ s_0 \neq A_{11} \bar{\bar{x}}_1(s_0) \]
and \( s_0 \in \text{spec}(A_{11}) \).

Proof 2: \( A_{11} = A + b \bar{u} : \mathbb{V}^* / \mathbb{K}^* \to \mathbb{V}^* \)
where \( \bar{u} = kw_c \)
\[ \mathbb{P}^* = \{ 0 \} \]
Claim: It suffices to prove $|k_4| \leq M$, because

1. $k_4$ bounded $\Rightarrow \lim k_4 = k_0$

2. $A \Rightarrow y_4 \in L^2(0, \infty)$

3. $B \Rightarrow (x_4)_t \in L^2(0, \infty)$

4. $A, B, C \Rightarrow \int y \in L^2(0, \infty) \Rightarrow y_4 \to 0$

5. $A, B \Rightarrow x_4 \in L^2(0, \infty) \Rightarrow (x_4)_t \to 0$

Lemma $k_4$ is bounded

Proof. Set $V(x_4, y_4, k_4) = \frac{1}{2} y^2$

$\dot{V} = y \dot{x}_4 + \frac{\lambda_2}{\lambda_3} y^2 - \beta k_4 y$

$\frac{\lambda_1}{\lambda_3} = \frac{\int y A_{31} x_{4_t} dt + A_{33} k_4}{\int y A_{31} x_{4_t} dt + A_{33} k_4}$

Claim $\int y A_{31} x_{4_t} dt \leq c_1 + c_2 \int y^2 dt$

$\int y^2 \leq \alpha \int y^2 + c_1$

Claim $\int \int y^2 \leq \beta k_4 (T) + c_1$

$|k_4| \leq M$

II. Multivariable Linear Systems

$x = Ax + Bu$, $y = Cx$

$G(s) = G(sI - A)^{-1}B$

Theorem (B-W) Suppose $\det G(s) = 0$

$\Rightarrow \text{Re}(s) < 0 \text{ and } \text{spec}(C^c) \subset C^+$

Then, the controller

$\begin{align*}
\dot{k}_4 &= \|y\|^2, & u &= -k_4 \\
&\text{satisfies} & x_4 &\to 0, & k_4 &\to k_0
\end{align*}$

Postscript

1. Multivariable Root-Loci: "Frozen Analysis" \Rightarrow Stability

2. $\dot{\lambda}(s) = s^2$

Multivariable Root-Loci d'apres MacFarlane Frozen Eigenvalue Analysis d'apres B& Marder
Thus we ask 5 questions
1) Where do the root-loci start?
2) Where do the root-loci end?
3) How do they get there?

Example: \( G(s) = \frac{1}{(s^2)(s+1)(s+2)} \)

\[ K_h = \begin{bmatrix} \frac{h}{s} & 0 \\ 0 & h \end{bmatrix}, \quad u = -kyp \]

1) \( s_1(s) \in \text{Poles} \) \( G(s) = \{-1, -2\} \)
2) \( \lim_{s \to \infty} s_1(s) = \infty \), \( \text{Zeros} G = \{0\} \)
3) How do they get there?

\[ \begin{array}{c|c}
 & -2 & -1 \\
\hline
x & x & \\
\end{array} \]

N.B. 1. \( 0 \leq k \leq 1.25 \) stable
2. \( 1.25 \leq k \leq 2.5 \) unstable
3. \( k > 2.5 \) stable

In particular, \( k \to s_1(k) \) is not 1-1!!

\[ f(s, k) = 1 + kg(s) = 0 \Rightarrow k = -\frac{1}{g(s)} \]

Indeed \( \det (I + k G(s)) = 0 \)

Set \( q = -\frac{1}{k} \)

\( \det (qI - G(s)) = 0 \)

In our example

\[ f(s, k) = 1.25 - 2s^2 + 3s^3 + 2s^4 \]

\[ g(s) = (2s - 3) \pm \sqrt{1 - 24s^2} \]

The algebraic functions have a branch point at \( s = \frac{1}{2}\sqrt{24} \)

Similarly, we compute

\[ s_\pm(q) = \frac{(2 - 3q) \pm \sqrt{9q^2 - 9q^2}}{2q} \]

\[ q_\pm(-2) = \infty, q_\pm(-1) = \infty \]

\[ g_\pm(-2) = \frac{-2 + \sqrt{149}}{2} \]

Consider \( f(s, q) = 0 \). This defines the algebraic curve / Riemann surface \( X \)

\[ X < S^2 \times S^2, \quad \text{proj}_1 (s, q) = s \]

Graph of \( g_+ \)

Graph of \( g_- \)
Newton's Theorem: \( f(s, q) = 0 \) has \( n \) branches.

At any branch point, each \( s; q \) can be developed into a Newton-Puiseux expansion: 
\[
\begin{align*}
\frac{d}{ds} s; q &= s_0 + \sum_{k=1}^{n} \frac{d^k f}{ds^k} (s_0, q) s^{k-1} + \cdots \\
\text{The sum of the orders} &= n.
\end{align*}
\]

**Application:**
1. Plot Loci Plots (Bretschneider-Muirhead)
2. General Position Loci:
   - Lemma: Given \( G(s) \), \( s \in \mathbb{C} \). There exists \( u = ky \) such that \( G_k(s) \) has distinct poles.
3. Adaptive Control: \( G(s) \) is a linear system, suppose \( \det G(s) = 0 \Rightarrow s \in \mathbb{C}^+ \), \( \text{spec} G(s) \in \mathbb{C} \)
   - Then \( u = ky \), \( k = \|y\|^2 \)
   - Is self-tuning. Thus:
     - \( k_2 = k_{20} \)
     - \( x_2 \to 0 \)

**Frozen Eigenvalue Analysis**

**Theorem:** Consider the system
\[
\dot{x} = Ax - k(t) Bx
\]
where
1. For \( k \to 0 \), \( A - k B \) is stable.
2. \( k(t) \to 0 \)

Each solution to \((A)\) tends to zero exponentially.

**Proof:** Let \( x \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m} \).

Consider \((x_i, y) = (k_i + \text{spec } B, y) \in \mathbb{R}^m\) and (i)

\[
\begin{align*}
\dot{x} &= A x + A_2 y + \mathbf{B} \theta(\mathbf{y}) \\
\dot{y} &:= A_3 x + A_{42} y + \mathbf{B} \theta(\mathbf{y}) \quad \text{for } \theta(\mathbf{y}) \\
\text{spec } A &\subset \text{spec } (A_3, B_1) \\
\text{II. } s \in \text{spec } (A_3) \implies \det \left(C_i A_i + C_i B_i A_i \right) x - \mathbf{B} \min(\beta \mathbf{y}) &\leq 0
\end{align*}
\]

Choose coordinates \( z = \mathbf{T} y \) so that \( \beta \) is diagonal.

Then,
\[
\frac{d}{dt} \left( \frac{1}{2} \|z\|^2 \right) \leq x^T (A C_1 C_1^T + C_1^T C_1 A^T) x - \beta \|y\|^2 \\
\leq -c \|y\|^2 \\
\|z\|^2 \leq c \|y\|^2 \\
\|z\|^2 \leq c \|y\|^2 \\
\text{for } \mathbf{z} \to 0.
\]

**Frozen eigenvalues:** Suppose \( \lambda \) is a

- \( \lambda \in \mathbb{C}^+ \)
- \( \lambda \in \mathbb{C}^- \)

Choose coordinates \( z = \mathbf{T} y \).
**Theorem** If we have a convergent parameter adaptive stabilization scheme, then $n_2 \geq n - 1$.

**Proof**

\[
\begin{align*}
(x, p, k) & \rightarrow w(x) \\
(\dot{w}, \ddot{w}) & = f(x, p, k)
\end{align*}
\]

**Lemma 1** $u = \dim w(x, p, k) = 0$.

*Proof* Follows from Reduction Theorem of Shoshitaishvili / Polis-Takens:

\[
\begin{align*}
\dot{x} &= f(x) \\
\frac{\partial f}{\partial p} &= 0 \\
\frac{\partial f}{\partial k} &= 0
\end{align*}
\]

**Lemma 2** $\text{spec}(A + bT c, b + c) = 0$.

**Lemma 3** Consider $g(s) = \frac{1}{s^n - s^{n-1} ... - 1}$

Suppose $(F_0, G_0, H_0, J_0)$ stabilizes (internally) $g(s)$ and has degree $n_2$. Then, $n_2 \geq n - 1$.

**Theorem (B^2)** Suppose $g(s)$ is minimum phase and has relative degree $\overline{n}$.

If $p(s)$ is minimum phase of degree $n^*$, then

\[
\begin{align*}
\frac{1}{s^n - s^{n-1} ... - 1} & = \frac{\beta}{(s + \beta)^n} \\
\end{align*}
\]

**Sketch** Assume $\alpha, n^* > 0$.

\[
\begin{align*}
C(s) &= \frac{\beta}{(s + \beta)^n} \\
\end{align*}
\]

$p(s)$ Hurwitz of degree $n^* - 1$

\[
\begin{align*}
g(s) = \frac{n(s)}{d(s)}
\end{align*}
\]

**N.B.** In closed loop, this is the same as

\[
\begin{align*}
C'(s) &= \frac{\beta}{(s + \beta)^n} \\
\end{align*}
\]

compensating

\[
\begin{align*}
g'(s) &= \frac{\beta p(s) n(s)}{d(s)}
\end{align*}
\]

Frozen eigenvalue analysis: stable for $\beta > 0$ - Algebraic function theory but $g'(s)$ has relative degree $n^* - 1$!
Modifications and Extensions

\[ q(s) = e^{(T - A)^{-1} b} = \frac{p(s)}{q(s)} \]

1. minimum phase
2. \( n \leq N \) upper bound on order of system
3. \( n^* = \text{deg } q - \text{deg } p \) is known
4. \( \text{sign}(c A^{n^* - 1} b) \) is known

2) is not needed, infact \( n = \infty \),

\[ \begin{align*}
    x &= A x + b u, \\
    y &= c x
\end{align*} \]

\( A : \mathbb{R}(A) \to \mathbb{H} \) densely defined

+ pure point spectrum

Lemma 1. A + b² pure point spectrum

- Weyl von Neumann Theorem

2. spec A + b² = \( A(p(c(t))) \)

\( A + b² \) stable: Hille-Beurling Thm

Necessary Conditions in Adaptive Stabilization

"Whatever can be done adaptively can be done if we know \((A, b, c)\)"?

\[ \text{Adaptive stabilization with smooth nonlinear controllers of dim } \leq N \text{ implies (classical) stabilization with linear compensators of dim } \leq N. \]

Proposition \( \exists \) is a collection of minimum phase systems which can be stabilized by some linear system of order \( \leq N \), then for all \( \lambda \) ?

\[ n^* (\lambda) \leq N + 1. \]

Corollary For minimum phase systems, an upper bound on \( n^* \) is necessary for adaptive stabilization.

Ex. \( \dot{x} = a x + b u \), \( \dot{z} = z^2 \), \( y = -\text{sign}(1) \cdot z \)

Instead, set \( u = \pm(h) y \), \( R \). Mundelmann

\[ \begin{align*}
    z(h) & = -1 \\
    x & = 10 \\
    y & = 0
\end{align*} \]

so that Cesaro mean \( C(z(h)) \) satisfies

\[ \lim_{h \to 1} \frac{\int_0^T C(z(h))}{h} = \infty \]

Ex. \( \dot{x} = x - b x \), \( \dot{z} = x^2 \)

\[ \text{W}^3 - \text{stable manifold} \]

\[ \text{dim}(W^3) = \text{dim} \]

\( (x, y, b) \to (0, k) \)

\[ \dot{x} = A x + b u, \ y = c x \]

\[ \dot{z} = f(k, y) \]

\[ u = H(k) z + J(k) y \]

\[ (x_t, z_t, b_t) \to (0, 0, k) \] — Convergence
\[
\begin{align*}
   x_1 &= A_{11} x_1 + A_{12} y_2 \\
   \dot{y}_2 &= A_{21} x_1 + A_{22} y_2 - ke^{-k \eta} e^{-k \eta} \\
   \dot{z} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} (z - x) + \begin{bmatrix} 0 \\ \cdots \end{bmatrix} \\
   \dot{x} &= y_2^2
\end{align*}
\]

**Note.**
- \( x_1 \) is stable.
- \( \dot{z} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} (z - x) + \begin{bmatrix} 0 \\ \cdots \end{bmatrix} \)

are stable; 

minimum phase

\( e^b \to \infty \) or bounded solution.

\[
\frac{d}{dt} \|y\|^2 = 2 x_1 y_2 + 2 x_2 y_2^2 - ke^{-k \eta} e^{-k \eta} y_2
\]

\[
\Rightarrow \|y\| \to \infty \quad \text{or bounded.}
\]

\[
\Rightarrow x_1, y_2 \to 0 \quad k \to k_0
\]
Lyapunov functions, cost functions, and adaptive control

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Abstract

Lyapunov functions are derived for a class of discrete time adaptive systems. The derivations have been performed under the following assumptions: Reference value = 0, no non-minimum phase zeros of the control object, well damped desired closed loop poles. The parameter estimation is made via a gradient type of algorithm which makes use of the output error.

It is shown that the considered adaptive system have the following properties.

- Global stability in the sense of Lyapunov
- Exponential convergence to zero of the state vector and the output error
- Monotone convergence (not necessarily to zero) of the parameter errors

References

LYAPUNOV FUNCTIONS
COST FUNCTIONS
AND
ADAPTIVE CONTROL

ROLF JOHANSSON
LTH

STABILITY CRITERIA
What conditions for stability prevail in adaptive control theory?
- Strictly positive real transfer functions
- Persistency of excitation
- \( \| \phi \| \) limited

Criticism
- Implicit conditions
- No link between stability and convergence

Remedy?
- Try to stick to "old" concepts like
  - Lyapunov functions
  - Cost criteria
  - Gain margin

Problem
Stability of discrete-time
direct adaptive control

Previous work
Lyapunov functions, SPR - Parks
BIBO - Goodwin-Ramadge-Caines
Egardt

My approach
Deterministic case
State-space
Lyapunov function

New results
Stability: Lyapunov-stability
Convergence: Exponential (Global)

CONTENTS
- Introduction
- Direct adaptive control
- What should a state-space model contain?
- Parameter convergence
- Error and state dynamics
- Lyapunov function
- Stability
- Convergence
- Example
- Conclusions
DIRECT ADAPTIVE CONTROL

OBJECT, PLANT
\[ y(t) = \frac{A^*(q)}{R^*(q)} u(t) \]

CONTROL LAW
\[ R^*(q) u(t) = - S^*_y(q) y(t) \]

CLOSED-LOOP POLES ASSIGNMENT
\[ R^*_a A^*_m + S^*_y B^*_m = T_{11} A^*_m B^*_m \]

PARAMETRIZATION
\[ y = \frac{b_0 q^k B^*_m}{T_{11} A^*_m B^*_m} [u(t) + \Theta^T \varphi(t)] \]

CONTROL LAW
\[ u(t) = - \Theta^T(q) \varphi(t) \]

PARAMETER ESTIMATION
\[ \hat{\Theta}(t) = \hat{\Theta}(t-k) + \gamma(t-k) \varphi(t-k) e^T(t) \]
\[ e^T(t) = T_{11}^*(q^*) A^*_m(q^*) \quad e = y \]
\[ \gamma(t) = \frac{1}{\beta^0 \| \varphi(t) \|^2} \]
\[ 0 < \frac{b_0}{\beta^0} < 2 \]

IDENTIFICATION DYNAMICS
\[ \begin{cases} \dot{\hat{\Theta}}(t) = \hat{\Theta}(t-k) + \gamma(t-k) \varphi(t-k) e^T(t) \\ e^T(t) = b_0 q^k \left[ - \Theta^T(t) \varphi(t) \right] \end{cases} \]
\[ \| \hat{\Theta}(t) \|^2 - \| \hat{\Theta}(t-k) \|^2 = \]
\[ = \gamma \frac{e^2(t)}{\| \varphi(t-k) \|^2} = \]
\[ = \gamma \| \hat{\Theta}(t-k) \|^2 \cos^2 \alpha(t-k) \]
\[ \alpha = \text{Angle between } \varphi \text{ and } \hat{\Theta} \]

\[ \hat{\Theta}(t-k), \ldots, \hat{\Theta}(t) \]

STATE-SPACE?

Dynamics
Control object \[ x, y, u \]
Regulator \[ u, y \]
Estimator \[ \Theta, \Theta \]
\[ \varphi, \varphi(t-k) \]

\[ \Theta, \hat{\Theta} \rightarrow x, y, u, \varphi, \varphi(t-k) \]

Parameters
Inputs
Outputs
Regression vectors

State at time t includes \[ \tilde{\Theta}(t+k-1), \ldots, \tilde{\Theta}(t) \]

Introduce \[ z(t) = [\tilde{\Theta}(t+k-1), \ldots, \tilde{\Theta}(t)]^T \]

\[ V_{\Theta}(z(t)) = \| z(t) \|^2 = z^T(t) z(t) \]

\[ V_{\Theta}(z(t+1)) - V_{\Theta}(z(t)) = \]
\[ = -\gamma \| \tilde{\Theta}(t) \|^2 \cos^2 \alpha(t) \]
\[ \alpha = \text{Angle between } \varphi, \tilde{\Theta} \]

Standard proof of parameter convergence
ERROR DYNAMICS

More difficult?

Control object \( u, y, x \)
Regulator \( u, y \)
Regression vectors \( \varphi \)

IDEA #1

Relate all of them to \( \xi(t) \)?

\[
\begin{align*}
    u(t) & = A^*(q^*) \xi(t) \\
    y(t) & = B^*(q^*) \xi(t) \\
    x(t) & = [x_1(t), \ldots, x_n(t)]^T \\
    \text{where} \quad x_i(t) & = \xi(t-i) \\
    \varphi(t) & = [u(t-1), \ldots, u(t-k+1), y(t), \ldots]^T \\
    & = M \times (t)
\end{align*}
\]

How does \( \xi(t) \) develop?

\[
\xi(t) = \ldots = \frac{1}{T_1^X A_m^x B_m^x} [-\varphi^T(t) \varphi(t)]
\]

State-space formulation

\[
x(t+1) = Fx(t) + G(-\varphi^T(t) \varphi(t))
\]

Remember that

\[
\varphi = M \times
\]

State-space representation

\[
x_i(t) \triangleq \xi(t-i)
\]

\[
[n \times 1]^{[m \times n]}
\]

\[
[y_{(t)}] = \begin{bmatrix}
-a_1 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
\]

\[
y_{(t)} = [0 \ldots 0 \ b_0 \ b_1 \ldots] \times (t)
\]

Controllable canonical form

Direct adaptive control reparametrization of \( \xi(t) \)?

\[
\xi = \frac{R^X A^x + S^X B^x}{T_1^X A_m^x B_m^x} \xi = \frac{1}{T_1^X A_m^x B_m^x} [u + \Theta^T \varphi]
\]

State-space formulation

\[
[n \times 1]^{[m \times n]}
\]

\[
T_1^X A_m^x B_m^x = 1 + p \varphi_1^T \ldots P_{m-1} \varphi^T
\]

\[
x(t+1) = Fx(t) + G(-\varphi^T(t) \varphi(t))
\]
A STATE VECTOR

Parameter errors
\[ z(t) = [\theta^T(t+1), \ldots, \theta^T(t)]^T \]

Error dynamics, regulator etc
\[ x(t) = [\xi(t-1), \ldots, \xi(t-n)]^T \]

\[ X(t) = [z^T(t), x^T(t)]^T \]

\[ X(t+1) = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} X(t) + \begin{bmatrix} L(X(t)) \\ 0 \end{bmatrix} \]

\[ L[X(t)] = \gamma(\varphi(t)) \beta \varphi(t) \]

\[ \nu[X(t)] = -\tilde{\theta}^T(t) \varphi(t) \]

\[ \nu_X(x(t)) = \ln[1 + \|Qx(t)\|^2] \]

\[ \nu_X(x(t+1)) - \nu_X(x(t)) \leq \]

\[ \leq \nu_X \|\tilde{\theta}(t)\|^2 \cos^2 \alpha(t) \]

\[ \alpha - \text{angle between } \tilde{\theta} \text{ and } \varphi \]

Matching with
\[ \nu_\theta(z(t)) = \sum_{i=0}^{k-1} \|\tilde{\theta}(t+i)\|^2 \]
gives
\[ \nu[X(t)] = \sum_{i=0}^{k-1} \|\tilde{\theta}(t+i)\|^2 +\]

\[ + K \ln[1 + \|Qx(t)\|^2] \]

Lyapunov function

We have a linear system with
\[ \nu[X(t)] = -\tilde{\theta}^T(t) \varphi(t) \]
as a formal input \( \varphi \)
\( \nu \) is bilinear in state vector components?

IDEA # 2

A logarithmic function would dissolve the bilinear product
\[ \nu_X(x(t)) = \ln[1 + x^T(t)Q^TQx(t)] \]

\( Q \) - weighting matrix

\[ \nu[X(t)] = \sum_{i=0}^{k-1} \|\tilde{\theta}(t+i)\|^2 + \ln[1 + \|Qx(t)\|^2] \]

\[ \nu[X(t+1)] - \nu[X(t)] \leq \]

\[ \leq \nu_X \|\tilde{\theta}(t)\|^2 \cos^2 \alpha(t) \]

Conclusions - Stability

• Stability in the sense of Lyapunov

• Hard bound on output error from \( \|Qx(t)\|^2 < e \)

• No assumptions on persistent excitation
CONVERGENCE

- $\hat{\Theta}(t), \hat{\Theta}(t-1), \ldots, \hat{\Theta}(t-k+1)$
  do not necessarily converge to $\Theta$
- $\|x\| \to 0$

Assume

$$\hat{\Theta}(t) = \hat{\Theta}(t-k) + \frac{1}{\beta_0 \| \phi(t-k) \|} \phi(t-k) e(t)$$

for all $\|\phi\| \neq 0$. Then

**Theorem 2**

$$\sup \| Q x(t) \|^2 \leq C(t_0) e^{-\delta(t-t_0)}$$

$V[\hat{X}(t_0)] < C_0$

with

$$C(t_0) = \exp \left[ C_0 / k \right]$$

$\delta \in (0, 1)$

RESTRICTIONS

- Regulators only
- Only well damped closed-loop poles
  $$T_1 A_0 B_0 = 1 + p q^{-1} + \ldots + p_n q^{-n}$$
  $$\| Q^{-1} p \| < \frac{1}{2} (1 - j)$$
- $\hat{\Theta}(t) = \hat{\Theta}(t-k) + \frac{1}{\beta_0 \| \phi(t-k) \|} \phi(t-k) e(t)$
  may become large
- Prior knowledge of
  - $k$ - time delay
  - $\beta_0$ - estimation of gain
  - $0 < \frac{\beta_0}{\beta_0} < 2$

Example

Develop an adaptive dead-beat control strategy

$$R^* A^* B^* = 1$$

$$\begin{align*}
A^*(q^{-1}) &= 1 + \alpha q^{-1} + \beta q^{-2} \\
B^*(q^{-1}) &= b_o q^{-3} \\
R^*(q^{-1}) &= 1 + r_1 q^{-1} + r_2 q^{-2} \\
S^*(q^{-1}) &= s_o + s_1 q^{-1} \\
\end{align*}$$

$$y = b_o q^{-3} [u + e(t) p]$$

$$\Theta = [r_1, r_2, s_o, s_1]^{T}$$

$$[\phi]_m = [u(t-1), u(t-2), y(t), y(t-1)]$$
\[ u(t) = A^T(q^t) \dot{q}(t) = \left[ I + \alpha q^t + \beta q^{2t} \right] \dot{q}(t) \]
\[ y(t) = B^T(q^t) \dot{q}(t) = b_0 q^{2t} \dot{q}(t) \]

\[
\begin{bmatrix}
  u(t-1) \\
  u(t-2) \\
  y(t) \\
  y(t-1)
\end{bmatrix} = 
\begin{bmatrix}
  1 & \alpha & \beta & 0 \\
  0 & 1 & \alpha & \beta \\
  0 & 0 & b_0 & 0 \\
  0 & 0 & 0 & b_0
\end{bmatrix} 
\begin{bmatrix}
  x(t-1) \\
  x(t-2) \\
  y(t-1) \\
  y(t-2)
\end{bmatrix}
\]

\[ \lambda_{\max}(M^TM) \leq 2(1 + \alpha^2 + \beta^2 + b_0^2) \]

**Identification**

\[ \hat{\theta}(t) = \hat{\theta}(t-1) + \frac{1}{\beta \| p(t-1) \|^2} p(t-1) e_t(t) \]
\[ e_t(t) = y(t) - b_0 q^{2t}(u + \delta^T \dot{q}) \]
\[ e_t(t) = b_0 q^{2t}(-\delta^T \dot{q}) \]

**State Vector**

\[ \begin{bmatrix} \hat{\theta}(t+2) \\ \hat{\theta}(t+1) \\ \hat{\theta}(t) \end{bmatrix}^T = z(t) \]

\[ V_\theta(x(t)) = x^T(t) z(t) = \frac{2}{\alpha} \| \hat{\theta}(t+1) \|^2 \]

\[ x(t) = [\xi(t-1) \ldots \xi(t-d)]^T \]
\[ v_x(x(t)) = \ln \left[ 1 + \| Qx(t) \|^2 \right] = \ln \left[ 1 + \sum_{i=2}^d \xi(t-i) p_i(t-i) \right] \]
\[ \mathbf{x}(t) = [\hat{\theta}(t+2) \ldots \hat{\theta}(t) x(t)] \]
\[ V(x(t)) = \sum_{i=0}^2 \| \hat{\theta}(t+i) \|^2 + \ln \left[ 1 + \| Qx(t) \|^2 \right] \]
CONCLUSIONS

- Global stability in the sense of Lyapunov
- Exponential convergence of error (globally)

What remains?

- Include reference value
- LS-identification
- Include noise and disturbances
Instrumental variable methods for systems operating in closed loop with application to adaptive control

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Instrumental variable methods are normally designed for systems operating in open loop. The reason is that the instruments are typically computed by filtering the input signal. Such an approach does not work if the system operates under feedback control, since then the instruments and disturbances becomes correlated. However, if there is a measurable setpoint (which anyhow is a modest identifiability condition) then this signal can be used for constructions of instruments. Some ways to achieve this are described.

Recent theory of instrumental variable methods show that the estimates are asymptotically Gaussian distributed. The instruments and data prefilters influence the covariance matrix of the estimates. They can in particular be chosen so that an optimal accuracy is obtained. This result is extended to the case of closed loop operation. A key part is then to use the "noisefree" part of the input and the output, i.e. the part that depends of the setpoint but not on the disturbances. The optimal instruments are easily computed once these noisefree signals (or consistent estimates thereof) and the noise correlation properties are known.

An optimal IV method can also be designed on a minimax basis. Minimization is then with respect to the instruments and maximization with respect to the disturbance properties. Also this IV variant is closely related to the noisefree input and output data.

It is further discussed how the above results for a time invariant feedback can be used for design of adaptive control system. Some few numerical simulated examples are also presented.

References

INSTRUMENTAL VARIABLE METHODS FOR CLOSED LOOP SYSTEMS

MOTIVATION

IV METHOD
- DEFINITION
- CLOSED LOOP SYSTEMS
- CONSISTENCY
- EXAMPLES

OPTIMAL IV METHOD
- OPTIMAL COV MATRIX
- MAXIMUM OPTIMALITY
- EXAMPLES

EXTENSION TO MULTIVARIABLE SYSTEMS

APPLICATION TO ADAPTIVE CONTROL
- APPROACHES
- EXAMPLES

CONCLUSIONS

MOTIVATIONS

INSTRUMENTAL VARIABLE METHODS GIVE
CONSISTENCY IN PRESENCE OF
"ARBITRARY" DISTURBANCES

POTENTIALLY USEFUL FOR INDIRECT ADAPTIVE CONTROL
RECURSIVE IMPLEMENTATION EASY
QUICK CONVERGENCE
FEW PARAMETERS

BASIC NOTATIONS

SYSTEM ($A$)

\[ A(q^{-1})y(t) = B(q^{-1})u(t) + v(t) \]

\[ v(t) = H(q^{-1})e(t) + E(e(t)) + \lambda^T e(t) \]

\[ y(t) = g^T (t) \theta + w(t) \]

MODEL ($M$)

\[ A(q^{-1})y(t) = B(q^{-1})u(t) \]

\[ y(t) = \phi^T (t) \theta \]

EXPERIMENTAL CONDITION ($X'$)

\[ R(q^{-1})w(t) = -S(q^{-1})y(t) + T(q^{-1})w(t) \]

$w(t)$ SETPOINT

$A_0, B_0, A, B, R, S, T$ POLYNOMIALS IN $q^{-1}$

$H$ RATIONAL FUNCTION IN $q^{-1}$

$\phi(t) = [-y(t-1) \ldots y(t-n\lambda) u(t-1) \ldots u(t-n\beta)]^T$
**IV Method**

\[ \hat{\theta}_q = \left[ \sum_{t=1}^{N} z(t) F(q^{-1}) w(t) \right]^{-1} \sum_{t=1}^{N} z(t) F(q^{-1}) y(t) \]

- \( z(t) \) VECTOR OF INSTRUMENTS
- \( F(q^{-1}) \) (SCALAR) PREFILTER

**Examples**

1) \( z(t) = [w(t-1) \ldots w(t-na-nb)]^T \)
   \( F(q^{-1}) = 1 \)

2) \( z(t) = [K_1(q^{-1}) w(t) \ldots K_{na-nb}(q^{-1}) w(t)]^T \)

**Consistency Analysis**

\[ \hat{\theta}_q = \left( \sum_{t=1}^{N} z(t) F(q^{-1}) w(t) \right) \left( \sum_{t=1}^{N} z(t) F(q^{-1}) w(t) \right)^{-1} \sum_{t=1}^{N} z(t) F(q^{-1}) w(t) y(t) \]

**Conditions**

1) \( R \neq \sum_{t=1}^{N} z(t) F(q^{-1}) w(t) \)
   NONSINGULAR

2) \( \theta = \sum_{t=1}^{N} z(t) F(q^{-1}) w(t) \)
   ASSUME

3) \( z(t), w(t) \) INDEPENDENT
   FOR ALL \( t \) AND \( s \)

4) \( w(t) \) PERSISTENTLY EXCITING

**Accuracy**

\[ \sqrt{N} \left( \hat{\theta}_q - \theta \right) \overset{\text{DIST}}{\rightarrow} \mathcal{N}(0, P_{IV}) \]

\[ P_{IV} = A R^{-1} S R^{-1} \]

\[ S = \mathbb{E}[F(q^{-1})H(q^{-1}) z(t)][F(q^{-1})H(q^{-1}) z(t)]^T \]
**OPTIMAL COVARIANCE MATRIX**

\[ P_{IV} = \text{OPT} \begin{bmatrix} E(t)^{T} \lambda(t) \Sigma(t) \lambda(t)^{T} \end{bmatrix} \]

\[ \hat{u}(t) = E\{p(t) \mid \hat{w}(t-1), \hat{w}(t-2) \} \]

\[ \tilde{u}(t) \text{ DISTURBANCE FREE PART OF } u(t) \]

\[ \hat{u}(t) = E\{p(t) \mid \hat{w}(t-1), \hat{w}(t-2) \} \]

\[ y(t) = \begin{bmatrix} \hat{u}(t) \end{bmatrix} A \begin{bmatrix} \hat{u}(t) \end{bmatrix} + \begin{bmatrix} \hat{e}(t) \end{bmatrix} \]

**EXAMPLE OF OPTIMALITY**

- **SYSTEM**
  \[ y(t) = x(t-1) + u(t-1) + e(t) + \text{noise} \]

- **REGULATOR**
  \[ w(t) = \begin{bmatrix} \hat{w}(t) \end{bmatrix} \]

- **CASE 1**
  \[ z(t) = \begin{bmatrix} \hat{w}(t) \end{bmatrix} \]

- **CASE 2**
  \[ \text{OPTIMAL IV} \]

**OPTIMAL IV METHODS**

- **CASE II MINIMAL COVARIANCE MATRIX**
  \[ z(t) = H^{-1}(q^{-1}) \hat{w}(t) \]

- **CASE III MINMAX IV METHOD**
  \[ z(t) = \hat{w}(t) \quad F(q^{-1}) = I \]

**EXTENSIONS**

- **MULTIVARIABLE SYSTEMS**

- **nz \neq n0** (OVERDETERMINED IV EQUATIONS)

- **CASE 1**
  \[ z(t) = \begin{bmatrix} \hat{w}(t-1) \end{bmatrix} \]

- **CASE 2**
  \[ \text{OPTIMAL IV} \]

**EQUATION FOR**

\[ nz \neq n0 \quad z(t) = \lambda^{-1} H^{-1}(q^{-1}) \tilde{e}(t)^{T} \]

\[ F(q^{-1}) = H^{-1}(q^{-1}) \]
APPLICATION TO ADAPTIVE CONTROL

ASSUMPTIONS

1. POLE ASSIGNMENT DESIGN
   CLOSED LOOP CHAR POL $p_0(q^{-1})$

2. COVARIANCE OPTIMAL IV
   WITH $H(q^{-1})$ DESIGN VARIABLE

APPLICATION TO ADAPTIVE CONTROL

Algorithm

1. ESTIMATE $A$ AND $B$ BY RECURSIVE IV
2. SOLVE $A P_k + B S_k = P_0$
   
   $$R_k(q^{-1}) = P_0(q^{-1})$$
   FOR $k = 1$

3. COMPUTE $u(t)$ FROM
   
   $$R_k(q^{-1}) u(t) = -S_k(q^{-1}) y(t-T) w(t)$$

4. COMPUTE $\hat{v}(t), \hat{\xi}(t)$ FROM
   
   $$\hat{v}(t) = \frac{A \hat{x}(q^{-1})}{P_0(q^{-1})} w(t)$$
   $$\hat{\xi}(t) = \frac{A \hat{x}(q^{-1})}{P_0(q^{-1})} w(t)$$

5. COMPUTE $z(t)$ AS
   
   $$z(t) = \left(1-q^{-1}\right) \left(-\hat{y}(t-1)...-\hat{y}(t-na) \hat{u}(t-1)...\hat{u}(t-nb)\right)^T$$

NOTE $H(q^{-1}) = \frac{1}{1-q^{-1}}$

[ORIGIN ACTION FOR RESET WINDUP]
Adaptive spectral factorization

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Despite the great amount of work in the last ten years on the behaviour of various recursive parameter estimation and self-tuning control schemes the situation is disappointing, though a number of global convergence results are available they usually entail restrictions that amount to knowledge of the true system.

Thus, the positive real condition cannot be checked unless the true system parameters are known. Further, monitoring schemes cannot be properly designed unless the true system parameters are known. Also, even for the globally convergent algorithms, various internal filters are not guaranteed to be stable.

In this work some building blocks for algorithm design are suggested. The Levinson, Burg or Lattice algorithm guarantees stability for autoregressive (AR) models. Wilson’s (1969) Newton Raphson scheme for spectral factorization guarantees stability of the spectral factor iterate at each iteration. Finally, general regression enjoys a bounded posterior error power property. The use of these building blocks together with the idea of split recursions is illustrated by developing a number of algorithms for ARMA and ARMAX recursive estimation (no iteration is involved).

One of those schemes (called RF2) is shown to be globally convergent. This scheme is then used to develop a convergent self-tuning Kalman filter. The algorithm is free of the criticism mentioned above.

Finally, the three tools above are combined to produce a self-tuning LQG controller. It enjoys some stability properties but no convergence proof is available.

Reference

ADAPTIVE SPECTRAL FACTORIZATION

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* REVIEW
1. MODELS
2. RECURSIVE PARAMETER ESTIMATION
3. STABILITY OF ALGORITHMS
4. SPECTRAL FACTORIZATION

* PRESENT
1. FOUR RECURSIVE SCHEMES (SPLIT RECURSIONS)
   1. ONE CONVERGES GLOBALY
   2. A CONVERGENT SELF-TUNING RALFAN FILTER
   2. USE IN SELF-TUNING CONTROL (LQG)
   1. A GENERAL MONITORING SCHEME

* MODELS
1. ARMA
   \[ A(z) \hat{y}_k = B(z)e_k \]
   \[ A(z) = 1 - a_1 z^{-1} + a_2 z^{-2} \]
   \[ B(z) = b_0 + b_1 z^{-1} + z^{-2} \]

2. ARMAX or CARMA
   \[ A(z) \hat{y}_k = B(z)e_k \]
   \[ A(z) = 1 - a_1 z^{-1} + a_2 z^{-2} \]
   \[ B(z) = b_0 + b_1 z^{-1} + z^{-2} \]

3. ARX or CAR (NOT ARMA)
   \[ a(z) \hat{y}_k = b(z)e_k \]
   \[ a(z) = 1 - a_1 z^{-1} + a_2 z^{-2} \]
   \[ b(z) = b_0 + b_1 z^{-1} + z^{-2} \]

4. TFAARMA
   \[ y_k = b(z) \hat{y}_k + e_k \]
   \[ A(z) = 1 - a_1 z^{-1} + a_2 z^{-2} \]
   \[ b(z) = b_0 + b_1 z^{-1} + z^{-2} \]

5. FIR or REGRESSION
   \[ y_k = b(z) \hat{y}_k + e_k \]

* RECURSIVE PARAMETER ESTIMATION

* WAYS TO GENERATE GAIN \( \theta \) GRADIENT
   (1) PCRA
   (2) PLR
   (3) EKF
   (4) IV

* AIDS FOR AN ALGORITHM
  STABILITY e.g., ERACE P没收
  CONVERGENCE e.g., CAKE FILTER 
  EFFICIENCY e.g., FALFAC AND MINV
STABILITY PROPERTIES OF SINC SIGNAL PROCESSING ALGORITHMS

(1) LEVINSON

OR BING OR LATTICE

ENHANCE OR POLYNOMIAL IS STABLE

(2) WELSON'S ALGORITHM FOR SPECTRAL FACTORIZATION

SOLVE \[ |x(t)|^2 \sigma^2 = x_k L \]

AT EACH ITERATION \( \beta_k(\ell) \) IS STABLE

WRITE ? OR BAYER

(3) REREGRESSION ERROR POWER IS DATA POWER

DATA \( x_k \) REGRESSION \( x_k \)

\[ p_k = p_{k-1} + \lambda_k \Delta x_k \Delta x_k \]

\[ \Delta^{-1} p_{k-1} + \Delta x_k \Delta x_k = \Delta^{-1} \Delta x_k \]

\[ \lim \Delta^{-1} \Delta x_k = \lim \Delta^{-1} \Delta x_k \]

SPECTRAL FACTORIZATION

\[ \| x_k \|^2 = \| x_k \|^2 = 1 + \sum \Delta x_k \Delta x_k \]

EQUIVALENTLY

\[ \| x_k \|^2 = 1 + \sum \Delta x_k \Delta x_k \]

FREQUENCY DOMAIN - BAYER

STATE SPACE VERSION IS POSITIVE REAL EQUATION

\[ F = F_0 e^{a \tau} + g v \]

\[ x = A x + b \]

\[ v = 1 - l H b \]

ALGORITHMIC SOLUTIONS

(3) LINEARLY CONVERGENT (BAYER, )

(4) AUTOMATICALLY CONVERGENT (WILSON, BAYER)

OFFLINE SF - LINEAR SCHEME

FREQUENCY DOMAIN - BAYER

STATE SPACE VERSION IS POSITIVE REAL EQUATION

\[ F = F_0 e^{a \tau} + g v \]

\[ x = A x + b \]

\[ v = 1 - l H b \]

\[ q_l(q) = V_l((1 + l / 8 I - F)^{-1} A v_l) \]

\[ F_{\text{opt}} = F_0 e^{a \tau} + g v \]

\[ x = A x + b \]

\[ v = 1 - l H b \]

\[ q_l(q) = V_l((1 + l / 8 I - F)^{-1} A v_l) \]
OFFLINE SF - QUADRIC SCHEMES

* FREQUENCY DOMAIN (WILSON, 1969)

SOLVE
\[
\begin{align*}
\delta_f & = \frac{\delta_f}{\delta_f(0)} - \frac{\delta_f}{\delta_f(1)} \\
\delta_f(0) & = \frac{\delta_f}{\delta_f(1)} \\
\delta_f(1) & = \frac{\delta_f}{\delta_f(0)} \\
\end{align*}
\]

SO
\[
T(\xi) \delta_f = \delta(1)
\]

STABILITY
\[
\mu(\xi) = \frac{\mu(\xi)}{\mu(1)}
\]

* TIME DOMAIN (STATE SPACE) HOLLER, 1973

\[
\begin{align*}
\delta_f &= \frac{F}{k} F + B \psi A \psi^T \\
\psi &= 0 \\
\theta &= m - F \frac{k}{h} h \\
\psi &= 1 - h \frac{k}{h} h \\
\delta_f(0) &= \psi (1 + h^2) \psi^{-1} \\
\psi &= \psi (1 + h^2) \psi^{-1}
\end{align*}
\]

DERIVATION + PROPERTIES OF WILSON'S ALGORITHM

* GIVEN \( A \) CHOOSE \( B = B + dB \), \( \Rightarrow \)

\[
\begin{align*}
|B| &= |B + dB| \\
|B| &= |B - dB| \quad \text{ON \( |B| = 1 \)}
\end{align*}
\]

NOTE SET
\[
\delta_0 = \pm |B| \delta
\]

\[
\begin{align*}
& \text{RHF}_1 \\
& \text{RHF}_2
\end{align*}
\]

- 1

- 11
RECALL \( A_L = 0 \cdot 0 \cdot l + 0 \cdot l + \rho \)

**TROUBLE WITH ON-LINE \( S \) F**

**RECALL** \( A_L = 1 \cdot 0 \cdot l - 0,1, 1 \cdot 0 \cdot l + \rho \)

1) **GET** \( \hat{b}_x \) AND \( \hat{b}_y = \frac{1}{2} b_x + \frac{1}{2} b_y \)

2) **STABILITY CHECK OF** \( \hat{b}_y \)

3) **IF IT FAILS** **FORM**

\[ \hat{b} = |\hat{b}_x| \]

**AND RE-DO ONE STEP OF** \( S \) **F**

**THEN** \( \hat{b}_y = \frac{1}{2} \hat{b}_x + \frac{1}{2} \hat{b}_y \)

**IS GUARANTEED STABLE**

**DOES NOT AFFECT CONVERGENCE**

**PROPERTY** | **AR** | **MA** | **TRANSVERSAL** | **CONVERGENCE**
--- | --- | --- | --- | ---
RHF | | | | |
RHF | | | | |
RF | | | | |
RF | | | | |

**SELF-TUNING KALMAN FILTER**

**JUST SLOT RECURSIVE ESTIMATES**

\[ \hat{x}_{k|k} = \hat{F}_k \hat{x}_{k-1|k} + \hat{K}_k e_k N_k \]

\[ e_k = y_k - H_k \hat{x}_{k+1|k} \]

**WE WILL IN** \( S \) **F**

\[ \hat{x}_{k+1|k} = \hat{F}_k \hat{x}_{k-1|k} \]

\[ V_k = V_{k-1|k} \]

\[ \hat{F}_k = \begin{bmatrix} \hat{A}_k & 0 \\ \hat{B}_k \end{bmatrix} \]

**RETURN TO** \( S \) **F**

**CONVERGENCE** \( x_{k+1|k} - \hat{x}_{k+1|k} \to 0 \)

**TROUBLEOUS BUT STRAIGHTFORWARD**

**SINCE** \( \hat{x}_{k} \to g - a = d - a \)
ARMAX!

\[(1 - a(z^{-1})) y_k = b(z^{-1}) u_k + (1 + d(z^{-1})) e_k\]

USE (TV) H. TO GET \( \theta, \delta \)

- POOR TRANSIENT BEHAVIOR?
- TOO LARGE \( \hat{\theta} \)

- SO ADD R - STEP \( \rightarrow \) IVHR

- REGRESS \( y_k \) ON \( y_k = (\hat{\theta}_{k-1}, y_{k-1})' \)
- \( s_k = \hat{\theta}_k(e^{z}) s_{k-1} + \hat{\theta}_k(e^{z}) u_k \)
- THEN \( \hat{\theta}_k = y_k - \hat{\theta}_k(e^{z}) u_k \)
  \( \frac{1}{1 + \hat{\theta}_k(e^{z})} \)

- HAS BOUNDED POWER
- FORM \( \hat{\theta}_k = (1 + \hat{\theta}_k(e^{z}) \hat{\theta}_k \)

AND USE RF

\( \Rightarrow \) R, HFG

\( b_{1-1} = \epsilon^{2 \epsilon} \)

\( d_{k+1} \alpha_k + \alpha_{k-1} / \alpha_k = \beta_{k+1} \alpha_k^2 + 1 \) (1)

\( |d_{k+1}| = |d_{k-1} + d_{k-1} | \geq 70 \) (2)

\[ \Rightarrow \Re (d_{k+1} / \alpha_k) = 1 + \frac{\alpha_{k+1} \alpha_k}{\beta_{k+1} \alpha_k^2} \geq 1 \]

\( \leq 1 + \frac{1}{1 + \epsilon^{2 \epsilon}} \) (3)

IF \( d_k \) HAS NO ZEROS INSIDE \( |z| = 1 \)

ROUH \( \rightarrow \) \( d_{k+1} \) DOES NOT \( \rightarrow \) \( d_{k+1} / d_k \) ANALYTIC IN \( |z| = 1 \)

\( \Rightarrow \Re (d_{k+1} / \alpha_k) = 1 \) HARMONIC

\( \Rightarrow \) CONSTANT \( |z| = 1 \) OR MAX \( \hat{\theta}_k \) FOR \( \alpha_{k+1} \)

ALSO (4) \( \Rightarrow \) \( \alpha_k \) HAS CONSTANT SIGN (HE)

\( \Rightarrow (1 + \alpha_k) / \alpha_k \leq |1 + \epsilon^{2 \epsilon} | \)

\( \alpha_k = \epsilon^{2 \epsilon} (1 - \epsilon^{2 \epsilon}) \rightarrow 0 \)

\( \Rightarrow \alpha_k / \alpha_k = \alpha_k / \alpha_k : \alpha_k \leq \epsilon^{2 \epsilon} (1 - \epsilon^{2 \epsilon}) \rightarrow 0 \)

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\[ \Pi^2 = \Pi^4 (1 + 2 \sigma s / \beta_0) = \Pi^4 (\Pi^2 + \sigma^2) \]

\[ = \Pi^4 \frac{\beta_{\alpha}}{\alpha_{\alpha}} (1 + 2 \sigma s / \beta_0) \leq \pi^4 \frac{\beta_{\alpha}}{\alpha_{\alpha}} (\beta_{\alpha} / \beta_0) \]

\( \beta_{\alpha}^{\pi_\pi^{(1 - \epsilon)}} = (\beta_{\alpha}^{\pi_\pi^{(1 - \epsilon)}}) \leq (\beta_{\alpha}^{\pi_\pi^{(1 - \epsilon)}}) \)

\( \Pi^4 \frac{\beta_{\alpha}}{\alpha_{\alpha}} (1 + 2 \sigma s / \beta_0) \leq \beta_{\alpha}^{\pi_\pi^{(1 - \epsilon)}} \)

\( \Rightarrow \sigma s < \epsilon \Pi^4 \frac{\beta_{\alpha}}{\alpha_{\alpha}} (1 + 2 \sigma s / \beta_0) \)

\( \Rightarrow \sigma s < \epsilon \Pi^4 \frac{\beta_{\alpha}}{\alpha_{\alpha}} (1 + 2 \sigma s / \beta_0) \)

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ADAPTIVE SPECTRAL FACTORIZATION

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Despite the great amount of work in the last ten years on the behavior of various recursive parameter estimation and self-tuning control schemes the situation is disappointing. Though a number of global convergence results are available they usually entail restrictions that amount to knowledge of the true system.

Thus, the positive real condition cannot be checked unless the true system parameters are known. Further, monitoring schemes cannot be properly designed unless the true system parameters are known. Also, even for the globally convergent algorithms, various internal filters are not guaranteed to be stable.

In this work some building blocks for algorithm design are suggested. The Levinson, Burg or Lattice algorithm guarantees stability for autoregressive (AR) models. Wilson's (1969) Newton Raphson scheme for spectral factorization guarantees stability of the spectral factor iterate at each iteration. Finally, general regression enjoys a bounded posterior error power property. The use of these building blocks together with the idea of split recursions is illustrated by developing a number of algorithms for ARMA and ARMAX recursive estimation (no iteration is involved).

One of those schemes (called RF$_2$) is shown to be globally convergent. This scheme is then used to develop a convergent self-tuning Kalman filter. The algorithm is free of the criticisms mentioned above.

Finally, the three tools above are combined to produce a self-tuning LQG controller. It enjoys some stability properties but no convergence proof is available.

Reference

Consider the problem of identifying the transfer function of the system

\[ y(t) = G_0(q)u(t) + v(t) \]

where \( y \) is output, \( u \) is input and \( v \) is a stationary disturbance with spectrum \( \Phi_v(\omega) \).

The input spectrum is supposed to be \( \Phi_u(\omega) \). From data up to time \( N \) an estimate \( \hat{G}_N(\omega) \) is formed. One method for this is a k-step ahead prediction error method in a given model set

\[ y(t) = G(q, \theta)u(t) + H(q, \theta)e(t), \]

\[ \hat{G}_N(e^{i\omega}) = G(e^{i\omega}, \hat{\theta}_N) \]

\[ \hat{\theta}_N = \arg \min_\theta \frac{1}{N} \sum_{t=1}^{N} \epsilon_F^2(t, \theta) \]

\[ \epsilon_F(t, \theta) = L(q) \epsilon(t, \theta) \]

\[ \epsilon(t, \theta) = H^k(q, \theta) [y(t) - G(q, \theta) u(t+kT)] \]

\[ H(q, \theta) = \bar{H}_k(q, \theta) + q^{-k} \bar{H}_k(q, \theta) \quad H^k = H_k H^{-1} \]

Several design variables are involved in this method, like \( k \) (the prediction horizon), \( T \) (the sampling horizon), \( H \) (the noise model) and \( \Phi_u \).

Then

\[ \hat{G}_N(e^{i\omega}) \to G^*(e^{i\omega}) \quad \text{w.p.1 as } N \to \infty \]

where \( G^*(e^{i\omega}) \) is "essentially" determined as the closest function to \( G_0(e^{i\omega}) \) in the model set, as measured in the weighted \( L_2 \)-norm (over the frequencies \(-\pi/T \leq \omega \leq \pi/T\)) with weighting function.
\[ Q(\omega) = \Phi_u(\omega) \cdot I_L(e^{i\omega}) l^2 \cdot I_W(e^{i\omega}) l^2 \]

This can be used for a more or less formal discussion of optimal choices of design variables.

References

L. Ljung: Estimation of transfer functions. Report LiTH
L. Ljung: Asymptotic variance expressions for identified transfer function estimates. Report LiTH
B. Wahlberg & L. Ljung: Design variables for bias distribution of identified transfer functions. Report LiTH
FREQUENCY DOMAIN PROPERTIES OF IDENTIFIED TRANSFER FUNCTION ESTIMATES

LENART HUNG 
BO WALTHER

PROBLEM:
True system:
y(t) = G(w) u(t) + v(t)

Observed data:
Zn = u(n), y(n), ..., u(N), y(N)
Model: \( \hat{G}_N(w) \)

WHAT CAN WE SAY ABOUT \( \hat{G}_N(e^{j\phi}) - G_c(e^{j\phi}) \)
AS A FUNCTION OF \( \phi \)?

1. BIAS 
2. RANDOM ERRORS

Let
\[ G_c(e^{j\phi}) = E \hat{G}_N(e^{j\phi}) \quad (E \text{ w.r.t. } \{V_N, v(N)\}) \]
\[ G_c(e^{j\phi}) - G_c(e^{j\phi}) \quad \text{BIAS} \]
\[ \hat{G}_N(e^{j\phi}) - G_c(e^{j\phi}) \quad \text{RANDOM error} \]

\[ \hat{G}_N - G_c = \hat{G}_N - G_c(e^{j\phi}) + G_c(e^{j\phi}) - G_c(e^{j\phi}) \]

"Traditional analysis neglects "

Typically, \( \text{bias} \left[ \hat{G}_N(e^{j\phi}) - G_c(e^{j\phi}) \right] \) \( \in \mathbb{R} \), \( P(w) \)

Expressions for \( P \), ...

Here, we shall concentrate on bias

2. PREDICTION ERROR METHODS

Choose:

1. \( T \) Sampling interval
2. \( \Phi_N(\omega) \) input spectrum
3. Model set \( G = \{ G(k, 0) \} \)
4. \( \Phi_N(\omega) \) noise model set \( N = \{ H(k, 0) \} \)
5. Prediction horizon \( k \)
6. Filter \( L(\omega) \)

Then
\[ \hat{E}_N(\omega) = \text{bias} \left[ \hat{G}_N(e^{j\phi}) \right] \]

1. Bias error & random error
2. k-step ahead prediction error methods
3. Conventional analysis design issues
4. A frequency domain expression for the limiting estimate
5. Choice of \( u \)
6. Conclusions
3. Design issues

$\hat{G}_n(\varepsilon)$ depends on all the listed 6 choices.

What are 'optimal' choices of
- Noise model
- Prediction horizon
- Prefilter
- Input spectrum
- Sampling interval

Traditional analysis: "optimal" = min var.

4. The Limiting Estimate

$\hat{G}_n \to \theta^* = \arg \min\ E \left[ \varepsilon^2 (\hat{G}_n, G) \right] \quad \text{w.p.}$

$\theta^* = \arg \min_{\theta} E \left[ \sum \left[ (G - G(\theta))^2 \cdot \varepsilon^2 \cdot \left| W_\varepsilon (\theta) \right|^2 \right] d\omega \right]$

Fixed noise model: $H(\omega) = H^*(W_\varepsilon (\theta) + W_0^*)$

Indep. parameterized $G(\theta) = G(\theta) : H(\omega) \equiv H(\gamma)$

$\theta^* = \arg \min_{\theta} E \left[ \sum \left[ (G - G(\theta))^2 \cdot \varepsilon^2 \cdot \left| W_\varepsilon (\theta) \right|^2 \right] d\omega \right]$

$\theta^* = \arg \min_{\theta} E \left[ \sum \left[ (G - G(\theta))^2 \cdot \varepsilon^2 \cdot \left| W_\varepsilon (\theta) \right|^2 \right] d\omega \right]$

General case:

$\theta^* = \arg \min_{\theta} E \left[ \sum \left[ (G - G(\theta))^2 \cdot \varepsilon^2 \cdot \left| W_\varepsilon (\theta) \right|^2 \right] d\omega \right]$

Formal design problem:

design variables:

$D = \{ \hat{G}_n, \theta^* \}$

$\theta^* = \theta^*(D)$

design criterion

$J(D) = \int \left[ G(\theta^*(D)) - G \right]^2 \left| W_\varepsilon (\theta^*(D)) \right|^2 d\omega$

$\min \ J(D)$

Recall:

$\theta^*(D) = \arg \min_{\theta} \int \left[ G(\theta) - G \right]^2 \left| W_\varepsilon (\theta) \right|^2 \left| W_\varepsilon (\theta) \right|^2 d\omega$

SOLUTION: $\theta^* = \hat{G}_n \left[ \left| W_\varepsilon (\theta) \right|^2 \left| W_\varepsilon (\theta) \right|^2 \right]_{\text{ac}}$
System:
\[ m(x) = 2.14 y(t-1) + 7.85 y(t-2) - 0.49 y(t-3) + 0.24 y(t-4) + \
0.01 u(t-3) + 0.0034 u(t-1) + 0.0012 u(t-1) - \
0.000576 u(t-6) \]

Model:
\[ y(t) = y(t-1) + a_x y(t-2) \]

Input: PRBS - white noise

Output error: \( R^2 = 1 \)

Least squares

\[ \text{LP filtered} \quad [0, 0.1] \]

\[ \text{BP filtered} \quad [0.1, 0.5] \]
CONCLUSION

The bias distribution is determined by a frequency domain weighting function that is influenced by the noise model, the prediction horizon, the prefilter, the input spectrum and the sampling interval.
Multi-armed bandits

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Contents
1) Bandit process
2) Multiarmed
3) Index
4) Adaption

1. A bandit process is \( \{(x(1), F(1)); (x(2), F(2))\ldots\} \), where
\( x(s) \) = reward on the \( s \)th continuation
\( F(s) = \sigma \)-field of information after \( s-1 \) plays.

Ex Coin toss
\( x(s) \in \{0, 1\} \). \( F(0) = \) prior information about success
\( F(s) = F(0) \cup \{x(0)\ldots x(s-1)\} \)

2. A multiarmed bandit is defined according to
\[ \{x^i(s), F^i(s)\} \quad i = 1, \ldots, n \]
\[ F^i(\omega) \sqsupseteq F^j(\omega) \quad i \neq j \]
\[ t - 1 = t^1 + \ldots + t^n \; ; \; F^i(t^1+1) \cup \ldots \cup F^i(t^n+1) = F(t) \]

The control \( u(t) \in \{1\ldots n\} \)
\( u(t) \) is the decision which bandit that shall continue.

\[ u(t) = i \rightarrow \begin{cases} R(t) = x^i(t^i+1) \\ F(t+1) = F(t) \cup F^i(t^i+1) \end{cases} \]

The control \( u(t) \) has to maximize the total reward

\[ V_{\beta}(\Pi) = \max \sum_{t=1}^{\infty} \beta^t R(t) \quad 0 < \beta < 1 \]
$\beta$ close to zero --- we want to maximize the reward in the first play(s).

$\beta$ close to unity --- all rewards are equally valuable

There is a conflict between getting immediate rewards and learning more about the other bandits.

3. Index of a bandit process \(\{x(s), F(s)\}\)

\[
\nu_\beta(s) = \max_{\tau \geq 1} \frac{E \left[ \sum_{t=s}^{\tau-1} \beta^t x(t) \mid F(s) \right]}{E \left[ \sum_{t=s}^{\tau-1} \beta^t \mid F(s) \right]} = \frac{\text{Acc. reward}}{\text{"Acc. time"}}
\]

\(\tau\) ranges over the stopping times of \(F(s)\). The best policy is to continue the bandit with the largest index (multiarmed case).

Notice that the calculation of \(\nu_\beta\) for a bandit involves only the bandit itself.

Two coins \(\theta_1\) and \(\theta_2\)

Fact about index \(\nu_\beta(s) = \nu_\beta(p)\) (distribution of success probability).

1) Expected reward just now

\[\nu_0(p) = \bar{\theta} = \exp \theta\]

2) \(\nu_1(p) = \max \theta \quad \text{(the most favourable possible event)}\)

\[
\theta_{\min} = \theta_{\max}
\]

\[
\nu_\beta(p_{\theta^*}) - \text{second coin}
\]

\[
\nu_\beta(p_{\theta^1}) - \text{first coin}
\]

\[
\theta_{\min}, \theta_{\max}
\]

\[
\text{choose first coin}
\]

\[
\text{choose second coin}
\]
Optimal solution for $\beta = 1, r \to \infty$
(infinite horizon problem) [Kelly Am. stat. 1981]

LFR = least failure rule: Try the one that currently looks best. As soon as it fails (no reward), choose another one.

4.

The procedure above constitutes a kind of adaptation.

Unsolved problems: Transient behavior
Length of the first two phases.

References

On self-tuning to the optimal controller

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Summary

The question of whether some adaptive controllers automatically tune themselves to the optimal control law is examined.

First we consider an adaptive control law consisting of a Stochastic Approximation (or Stochastic Gradient) parameter estimator followed by a minimum variance control law. Under some conditions it is shown that, see [1.2].

i) the parameter estimates converge to a random multiple of the true parameter,

ii) and so the adaptive control law converges to the true minimum variance control law,

iii) even though the standard persistency of excitation condition is not satisfied.

Next we consider the same parameter estimator followed by a linear quadratic control law. Ljung's O.D.E.'s are examined to show that though the parameter estimates may converge, they will not generally converge to an optimal control law.

In fact, the result for the case of the minimum variance control law rests on a fortuitous mathematical coincidence, and it is unlikely that self-tuning to the optimal will occur for general cost criteria, see [3].

References


W. Lin, P. R. Kumar and T. I. Seidman: Will the Self-tuning Approach Work for General Cost Criteria?
**SELF - TUNING**

**Question:** Does Self-Tuning Take Place?

Will the adaptive controller converge to the optimal controller?

**Result:** No: Optimal Controller

**General Cost Criterion**

Speculative: Self-Tuning Result is not possible in general by straightforward schemes.

---

**STEM**

\[
\begin{align*}
\dot{x}_k &= \left( \begin{array}{c} 2x_{k-1} + u_{k-1} + c_{x}W_{k-1} \end{array} \right) + v_{k-1} \\
\text{MINIMUM VARIANCE CONTROL LAW} \\
\dot{x}_k &= \frac{1}{2} \left[ (1 + \epsilon) \dot{x}_k + (1 - \epsilon) \dot{x}_k \right] + v_{k-1} \\
\text{MORE CONVENIENTLY} \\
\dot{x}_k &= \left( \begin{array}{c} \dot{x}_k, \ldots, \dot{x}_k \end{array} \right)^T \\
\dot{u}_k &= \left( \begin{array}{c} \dot{u}_k, \ldots, \dot{u}_k \end{array} \right)^T \\
\text{ADAPTIVE SCHEME} \\
\text{ESTIMATE} : & \hat{\theta}_k \\
\text{CONTROL} : & \phi_k^T \hat{\theta}_k = 0 \\
\text{Note:} & \text{One do not need absolute} \\
\text{Eqn ex.} & \theta_k \to \theta^* \\
\end{align*}
\]

---

**STOCHASTIC APPROXIMATION**

**LOCAL STOCHASTIC GRADIENT ALGORITHM**

\[
\begin{align*}
\hat{\theta}_{k+1} &= \hat{\theta}_k + \frac{1}{2} \lambda \left( \gamma_{k+1} - \phi_k^T \hat{\theta}_k \right) \\
\text{MORE CONVENIENTLY} \\
\hat{\theta}_{k+1} &= \hat{\theta}_k + \frac{1}{2} \lambda \left( \gamma_{k+1} - \phi_k^T \hat{\theta}_k \right) \\
\text{U_n chosen such that} & \phi_k^T \hat{\theta}_k = 0 \\
\text{WHAT DOES SUCH AN ALGORITHM DO?} \\
\dot{\theta}_k &= \dot{\theta}_k + \frac{1}{2} \lambda \left( \gamma_{k+1} - \phi_k^T \hat{\theta}_k \right) \\
\text{Hence} & \hat{\theta}_{k+1} \perp \phi_k \\
\end{align*}
\]

---

**T: OPTIMAL SELF-TUNING**
Conclusions

- \( \hat{\theta}_n \to k \theta^* \) where \( k \) is a random scalar.

- \( \hat{\theta}_n \to \theta^* \): Does not converge to true parameter.

- Regulator converges to optimal

**SELF-TUNING**

\[
\sum_{i=1}^{n} \left| \| \theta_n - \theta^* \| \right| \geq \sum_{i=1}^{n} \left| \| \theta_n - \theta^* \| \right|
\]

Convergence impossible at \( \theta^* \).

**Open Questions**

- Proof of self-tuning
  - i) LS scheme
  - ii) Delay
  - iii) Tracking

- Rate of convergence
  - Control, Information Theory

- Estimation theory
  - Min. P. E., Max. Entropy, etc.

- More sophisticated control laws.

**Status**

- Much progress since 1973
- Much remains to be done.
Why did the variance case work? 

\[
\frac{da}{dt} = k(a, b)
\]

Hope: If there are other undesirable equilibrium points, are they repellers? 

\[
\frac{da}{dt} = k(a, b)
\]

Integral curves: 

\[
\frac{da}{dt} = k(a, b)
\]

No: \(a + b k(a, b) = 0 + b k(0, 0)\) only if \(a(0) = 0\) 

\[
\frac{da}{dt} = k(a, b)
\]

Closed loop identification problem: 

Equilibrium points of ODE 

\[
\{[\cdot + b k(a, b)] : a + b k(a, b)\} \
\]

\[
\{ [\cdot + b k(a, b)] : a + b k(a, b) \}
\]

Does question: \(a, b k(a, b)\) converge to such \(a, b\)?
ENTATIVE CONCLUSIONS

Fundamental Closed Loop IDENTIFICATION

PROBLEM

\[ \theta_0 = K(\theta) \]

\[ \theta = T(\theta, K(\theta)) = \theta^* \]

\[ \hat{\theta} = \frac{1}{N} \sum_{k=1}^{N} \theta(k) \]

3. Minimize \[ \theta(1) \log J(\theta) + \sum_{k=1}^{N} \{y(k) - \hat{g}(k)\}^2 \]

Proof of Self-Tuning to Minimum Variance Controller

Self-Tuning probably not possible for any other cost criteria.

Method to obtain self-tuning sorts of results in general
A comparison of some control strategies
for systems with fast parameter changes

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This seminar is concerned with adaptive control of systems with abrupt changes in the parameters. First an algorithm for recursive identification of the parameters in a special model-class, suitable for modeling sudden time variations, is presented. Then it is shown how this identification procedure can be used for adaptive control. To illustrate the algorithm and also to discuss some points on adaptive control a comparison study is then referenced.

The basic idea behind the algorithm is to use several parameter sets to model the system. The different parameter vectors correspond to different typical modes of the system. By combining estimation and detection techniques it is possible to estimate the different parameter vectors describing the system.

The purpose of the comparison study was to compare some strategies for adaptive control. Among the "competitors" were for instance a self-tuning regulator (LS with forgetting factor combined with a pole-placement procedure), a time-invariant robust regulator, the regulator mentioned above and an algorithm based on an identification procedure called AFMM (adaptive forgetting through multiple models). A conclusion from the tests was that it can be useful to design regulators which saves information for later use.

References

A COMPARISON OF SOME CONTROL STRATEGIES FOR SYSTEMS WITH FAST PARAMETER CHANGES
Mika Målnert
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A contest between some strategies for adaptive control

THE CONTEST

the plant

A parameter estimation algorithm
A \textit{SM} sum (LC + pole-placement)

A constant regulator minimizing a variance-type criterion

A regul-\textit{ar} minimizing the criterion
\[ E((\gamma - r)^2 + \lambda a^2) \]

The MR algorithm

The \textit{SFAM} algorithm (forgetting through multiple models)

\( \Theta(t+1) = \Theta(t) + W(t) \)
\[ y(t) = p(t) \Theta(t) + e(t) \]

\[ W(t) = \begin{cases} 
V(t) & \text{w.p. } q \\
0 & \text{w.p. } 1-q 
\end{cases} \]

\( \text{COV} \, Y(t) = R_1 \)

\( p(h(t) | \gamma) = \text{Gaussian Sum} \text{ based on } \bar{e}_i(t), \quad i=1,...,m \)

\( \bar{\theta}_i = \text{least squares} \)

At each time

\( \bar{\theta}_i(t) \text{ with smallest prob.} \)

\( \bar{\theta}_i(t) \text{ with largest prob.} \)

\( \bar{\theta}_i(t) \text{ "Test Pilot"} \)

\( \text{Alert } R_1 = R_1 \)
Recursive estimation of slowly time-varying parameters

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Abstract

The problem to extend traditional algorithms for estimation of constant parameters, such as the least squares algorithm, to capture even the case of time-variable parameters has become important because of their use in adaptive control. Several ad hoc methods have been proposed to handle slowly time-varying parameters. Previous methods, such as the use of a forgetting factor, are here discussed from an information handling point of view. A new method is presented, which is based on the idea to retain a constant amount of information in the estimator. The method is shown to avoid well-known problems associated with other, more heuristic schemes. Analysis as well as simulation experiments are presented.

References

Recursive Estimation Of

Slowly Time-Varying Parameters

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Time-Varying Parameters

Process Model:
\[ y(t) = \Theta(t-1)^T \phi(t) + \epsilon_n(t) \]

If the parameters to be estimated or the noise level vary, they vary slowly and/or seldom compared with the time constants of the system.

Two Cases:
1. Large Parameter Changes
2. Slow Parameter Changes

LS Estimation

Minimize \[ \sum_{i=1}^{n} \frac{1}{\sigma_n(i)} \left( y(i) - \Theta(i)^T \phi(i) \right)^2 \]

Problem: How to choose \( \omega \)

\[ y(i) = \Theta(i-1)^T \phi(i) + \epsilon_n(i) = \]
\[ = \Theta(i-1)^T \phi(i) + \left[ \Theta(i-1) - \Theta(i-1)^T \right] \phi(i) + \epsilon_n(i) = \]
\[ = \Theta(i-1)^T \phi(i) + \epsilon_m(i, i) + \epsilon_n(i) \quad \text{i.e.} \]
model error noise

\[ \Sigma(i, i)^2 = \Sigma_m(i, i)^2 + \Sigma_n(i)^2 \]

Try to choose \( \omega = \Sigma_n \)

Example: Constant Parameters and Constant Noise Level

\[ \epsilon_m(i, i)^2 = \left[ \Theta(i-1) - \Theta(i-1)^T \right] \phi(i) = 0 \]
\[ \Rightarrow \Sigma_m(i, i)^2 = 0 \]
\[ \Rightarrow \Sigma(i, i)^2 = \Sigma_n(i)^2 = \sigma^2 \]

All measurements have the same weight.

Example: Exponentially Increasing Model Error Variance and Constant Noise Level

\[ \Sigma_m(i, i)^2 = \left( \frac{\lambda}{\lambda-1} \right) \psi \quad \Sigma_n(i)^2 = \sigma^2 \]
\[ \Rightarrow \Sigma(i, i)^2 = \left( \frac{\lambda}{\lambda-1} \right) \psi \]

Constant forgetting factor \( \lambda \)
**Slow Parameter Changes**

Discount past data in such a way that a constant desired amount of information is retained, if the parameters are constant.

**Information:** $P^t$

**Goal:** $P \rightarrow \alpha \cdot I$

\[
P^t = P_n + \text{alg} \cdot q^T - \alpha \cdot q^T P_n \Rightarrow \alpha > 0
\]

\[
P = \frac{P_n + \text{alg} \cdot q^T P_n}{(\alpha \cdot q^T + q^T P_n q)

**Compare:**

\[
P^t = P_n + \text{alg} \cdot q^T - (1 - \alpha) P_n
\]

**Convergence**

If $\alpha$ not too large (cf $\lambda < 0$)

Then

\[
W(t) = \sum_{i=1}^{n} [\lambda_i^2(t) - \lambda_i^2(t)]^2 \lambda_i(t) = \text{eig}(P_n)
\]

is decreasing.

$W(t) \rightarrow 0 \Rightarrow P(t) \rightarrow \alpha \cdot I$

**Excitation Condition** $\Rightarrow W(t) \rightarrow 0$

**Choice of $\alpha(t)$**

**Stationarity:** $\alpha_s = \frac{1}{\sqrt{t}}$

$\alpha_s \geq 0$

**Theorem:**

If

\[
0 \leq \alpha_s \leq \frac{1}{\text{alg} \cdot q^T P_n q}
\]

Then

* $P$ stays positive definite
* $\alpha_s \cdot P \rightarrow 0$ is decreasing

In each iteration, try to obtain

\[
\frac{P^t}{x^T x} = \alpha \quad \alpha = \rho_n \rho_n
\]

But fulfil the limitation \(*)

**Example - Industrial Robot**

\[
\frac{d\omega}{dt} = T_e + T_d + T_b
\]

$\omega(t+1) = \omega(t) + \frac{1}{J} [T_e + T_d + T_b]$

$T_d = k_1 \cdot \text{sign}(\omega)$

$T_e = \frac{1}{J} \cdot \text{alg} \cdot (\omega_{\text{ref}} - \omega) = \frac{\text{alg}}{J} \cdot \text{sign}(\omega)$

\[
T_b = \frac{1}{J} \cdot \text{alg} \cdot (\omega_{\text{ref}} - \omega) = \frac{\text{alg}}{J} \cdot \text{sign}(\omega)
\]
Figure A.1 - The estimated parameters and the estimated noise variance.

Figure B.1 - The output- and input-signals of the system, and the residuals with.

Figure C.1 - The test signal \( u(t) \), the additive noise \( n(t) \) and the decoupling measure \( \text{AD} \).

Figure D.1 - The elements of the \( F \)-matrix.
Robustness of (MRAS) adaptive control

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The topic of this lecture was robustness of (MRAS) adaptive control subject to high frequency unmodeled dynamics of the process model. This area of stability problems can be classified in three categories: HCG-high controller gain, HAG-high adaptive gain and HF-high frequency input. Here only the adaptation loop is considered i.e. HAG- and HF-instabilities. In this way the resulting system to analyze remains linear, time-varying.

The main idea is to approximate the high frequency dynamics with a right half plane zero. (Cf. Padé approximation of a timedelay.) Consider the example below. The approximation is valid for \( \mu s < < 1 \).

\[
G(s) = \frac{1}{s+1} \frac{1}{s+1} (1 - \frac{\mu s}{1+\mu s}) \approx \frac{1}{s+1} (1-\mu s) = \frac{1+\mu}{s+1} - \mu \tag{1}
\]

For stability analysis the differentiating effect of the RHP zero is stressed while for synthesis the throughput effect gives a clue how to improve robustness.

The MRAS scheme was analysed on the process above (1) using a first order reference model with unknown gain.

![Figure 1](image)

**Fig. 1.** The MRAS system and the transformed error system (2).

After the approximation (1) of the process and change of coordinates to to an error system, the following equations are obtained:
\[
\frac{dx}{dt} = \begin{bmatrix}
-1 + \mu yr^2 & r - \mu r^2 \\
-yr & 0
\end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \mu yr^2 \\ -yr \end{bmatrix} e
\]  \hspace{1cm} (2)

where

\( x \) = difference in output between the actual and tuned system.
\( y \) = difference in controller gain in reference to the tuned system.

HAG instability is shown in the following way. Assume \( r = R \text{ const. and } e = 0 \). The characteristic equation gives the stability condition on the adaptive gain \( \nu R^2 < \mu \).

To reveal HF-input instability, choose \( r = R \sin(\omega t) \) and assume \( \mu R^2 < 1 \), \( e = 0 \). Simulation shows a slow drift in \( x \) and a limit cycle in \( x \). Approximation of the equations (2) for slow adaptation, i.e. \( \nu R^2/\sqrt{1+\omega^2} \) is sufficiently small, gives the stability condition \( \omega^2 < \mu \). One interpretation derived from the approximated equations is that instability is reached when the noise-to-signal ratio is larger than unity. Another interpretation is that the positive real condition for the plant is violated at high frequencies due to the RHP zero.

For the combined problem of HAG and HF stability, a detailed analysis shows that the joint stability condition is somewhat conservative.

In the approximation (1) it is seen that the unmodeled high frequency dynamics can be seen as a negative throughput. A simple device to increase the stability properties of the MRAS scheme could therefore be to introduce a positive bypass \( \beta \). Analysis by means of a Lyapunov function shows that stability is achieved for \( \beta > \mu \). The positive real condition for the plant with bypass is also satisfied for these \( \beta \).

The analysis shows that a bypass increases stability properties against unmodeled high frequency dynamics. However, a tracking error is introduced, which is difficult to predict when \( \beta \neq \mu \). For instance, when \( \beta \) increases toward \( \mu \), the tracking error can decrease or increase depending on the frequency of the reference signal.

Reference

Can the simplest adaptive loop be unstable? Yes, in two ways:

1. Fast (HAG)
2. Slow (HF)

A method to stabilize the loop (discussions with Åström, Bitmead, Rohrs, et al.)

Adaptive system

Tuned system

Tuned error $e^* = y^* - y_m$

Right-half-plane zero instead of HF parasitics

At low frequencies, let $\eta = -\mu$

FACTS FROM STABILITY THEORY

FACT 1

$$\lim_{T \to \infty} \int_T^0 \text{trace } A(t) \, dt = +\infty$$

is sufficient for instability of $\dot{x} = A(t) \, x$

FACT 2

If $\dot{x} = A(t) \, x$ is exponentially stable, then

$$\dot{\hat{x}} = (A(t) + B(t)) \, x$$

is exponentially stable when

$$\int_0^t \| B(t) \| \, dt < c, \quad t + c, \quad \forall t > c$$

for some constants $c_1$ and $c_2$.!
FAST ADAPTATION ($\mu^2$ SMALL)

\[ e = y - y^* \]
\[ \psi = k - k^* \]

Linear, let $e^* = 0$

\[
\begin{bmatrix}
\dot{e} \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
-1 + \mu \gamma r^2 & r \\
-\gamma r & 0
\end{bmatrix} \begin{bmatrix}
e \\
\psi
\end{bmatrix}
\]

NECESSARY FOR STABILITY
\[ \mu \gamma \lambda < 1 \quad \lambda = \lim_{t \to \infty} \frac{1}{T} \int_{t-T}^{t} r(t) dt \quad (\text{average}) \]

SUFFICIENT FOR STABILITY ($\mu^2$ SMALL)
\[ \mu \gamma r^2 t < 1 \quad t \]

\[ V_1 = \frac{1}{2} e^2 + \frac{1}{2\gamma} \psi^2, \quad \dot{V}_1 = (-1 + \mu \gamma r^2) e^2 \]

\[ \mu^2 = - \]

\[ r = \sin \omega t \]

\[ \mu \gamma < 1 \]

SUFFICIENT ($\mu^2$ SMALL)

\[ \frac{\mu^2}{\gamma} < 1 \quad \text{necessary} \]

\[ r = \sin t \quad \mu = 0.1 \quad \gamma = 1 \]

\[ r = \sin 0.5t \quad \mu = 0.1 \quad \gamma = 24 \]
SLOW ADAPTATION (γ and μ γr² SMALL)

\[
\begin{align*}
\dot{\psi} &= -\gamma + \gamma (1/2) r \\
\dot{r} &= \gamma (-r \gamma + \mu r^2)
\end{align*}
\]

WHEN \( \gamma = 0 \)
\( \kappa(t) = c(t) \psi(t) \), \( \psi(t) = \text{const.} \)

DEFINE \( S(t) \) SUCH THAT
\( \kappa(t) = c(t) \psi(t) + \gamma S(t) \)

\[
\mu < \frac{\lambda_2}{\lambda_1}
\]

NECESSARY AND SUFFICIENT CONDITION FOR SLOW ADAPTATION STABILITY

\[
\mu < \frac{\lambda_2}{\lambda_1} = \frac{\text{average of } r c}{\text{average of } r^2}
\]

REQUIRES LOW FREQUENCY REFERENCE INPUTS
\[
r(t) = \sum_{i=1}^{N} R_i \sin(w_i t + \theta_i)
\]
\[
\mu < \frac{\lambda_2}{\lambda_1} = \frac{\sum_{i=1}^{N} \frac{1}{1 + \omega_i^2} R_i^2}{\sum_{i=1}^{N} R_i^2}
\]

SUFFICIENT \( \frac{1}{\mu} > \omega_i^2 \), \( \omega_i < \frac{1}{\sqrt{\mu}} \) \( \forall i \in [1, N] \)

\[
r = \sin(4t) \quad \mu = 0.1 \quad \gamma = 1.0
\]
CONCLUDING REMARKS

* SIMPLEST ADAPTIVE SYSTEM EXHIBITS TWO TYPES OF INSTABILITY CAUSED BY ADAPTATION.

* THESE TWO MECHANISMS PLUS LINEAR HIGH CONTROLLER GAIN CAN EXPLAIN ALL PHENOMENA OBSERVED IN MORE COMPLEX ADAPTIVE SYSTEMS.

* BOUNDS DERIVED HERE WILL GENERALIZE

\[
\begin{align*}
\gamma \tau^2 &< 1 \\
\omega^2 \tau^2 &< 1 \\
\text{ave}(r_c - \omega_r^2) &> 0 \\
\text{ave}(\omega_c^2 - \omega_r^2) &> 0
\end{align*}
\]
Parameter convergence issues in MRAC

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This contribution addresses parameter convergence in continuous time model reference adaptive control. The scheme of Narendra et al. is considered. The reported result can be summarized as follows:

- The technique of generalised harmonic analysis is employed to translate the persistent excitation condition on the signal vector into an equivalent condition on the exogenous reference signal only for parameter convergence.

- Partial convergence results for the case where the reference signal is not sufficiently rich are derived.

- Connections with robustness are illustrated by a first order example with output plant disturbance and plant parameter variation.
PARAMETER CONVERGENCE
ISSUES
IN MODEL REFERENCE ADAPTIVE
CONTROL

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with M. Bodson

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If \( \theta^T = [\theta_0, \theta_1, \theta_2, \ldots]^T \) such that

\[ \phi \in \mathbb{R}^n \] PARAMETER ERROR

then (Anderson, Moore-Kand Henrik...) \( \| \phi \| \rightarrow 0 \) exponentially \( \Rightarrow \exists \bar{\lambda}, \bar{\gamma} > 0 \)

\[ \{ \sum_{i=0}^{\infty} \| \phi \|^2 \} \quad \text{VS. EXCITATION} \]

If \( \theta^T = [\theta_0, \theta_1, \theta_2, \theta_3]^T \) then there exist

\( 
\) PARAMETER CONVERGENCE

CONDITIONS ON INDEPENDENT REF SIGNAL

(1) PARTIAL CONVERGENCE RESULTS

(2) CONNECTIONS WITH ROBUSTNESS

CONVAJUS: GENERALIZED HARMONIC ANALYSIS.

A LA WIENER

\[ \begin{align*}
R_w(\omega) & = \frac{\sigma^2}{2\pi^2} \left[ \frac{\sin^2 \frac{\omega \delta}{2}}{\omega^2 \delta^2} \right]
\end{align*} \]
\[
\begin{align*}
\text{(Supers.)} & \geq 2m, \text{ bottom.} \\
\text{Sup}_{\text{y}} \tilde{R}^2 (\omega) + 3(\lambda_{\text{y}} (\omega)) = C \\
\frac{\partial \tilde{R}^2 (\omega)}{\partial \lambda_{\text{y}} (\omega)} &= \frac{\lambda_{\text{y}} (\omega) - \lambda_{\text{y}} (\omega)}{\partial \lambda_{\text{y}} (\omega)} = C
\end{align*}
\]
On adaptive control with prescribed robustness properties

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We will describe a frequency domain method for regulator design. The resulting regulator has a stability margin in terms of the Nyquist curve which is independent of the system. This is a useful property in an adaptive control application, where the controller at every step is designed for a recursively identified model of the system, for the following reason. Since the Nyquist curve is known it is known at which frequencies it is important to have a good model of the system in order to have a good control result. Then the input-output data can be filtered through filters which emphasizes these frequencies and this will lead to a model of the system which is best there. (See B. Wahlberg and L. Ljung (1984)).

The main idea of the method is to obtain a given Nyquist curve by a cancellation of the system poles and zeroes. Therefore the basic version can only be applied to stable minimum phase systems. By a slight modification it is however possible to handle also pure integrators and discrete time real unstable zeroes.

Reference

On an adaptive control algorithm with prescribed robustness properties.

A frequency domain method.

- There will always be unmodeled dynamics.
- Most important to have a good model of the system around the cross-over frequency.
- Filtering can be used to distribute the bias.

A replacement: The Nyquist curve depends almost as much on the system as on the desired closed loop poles. That may be a problem in an adaptive control application.

An other design method.

Choose some "ideal" open loop reference system $S/R_o$.

Use the control:

$$u(t) = -\frac{S_o}{R_o} \cdot \frac{A}{B} (y(t) - r(t))$$

Observations

- Same robustness against multiplicative errors in $\frac{S_o}{R_o}$ for all systems (given by $\frac{S_o}{R_o}$).
- In order to have (4) realizable $\frac{S_o}{R_o}$ and $\frac{A}{B}$ must be of the same type.
  (Same pole-zeroes or same number of delays.)
- Only single minimum phase systems can be handled. With some modifications, also integrators and real discrete time zeros can be treated.)
Suppose that
\[ \frac{B}{A} = \frac{g^{-d} B_D B_S}{A} \]
where
\[ B_I = k \cdot (1 - t^{-b}) \quad b > 0 \]
Then choose \( S_0 \) with \( d+1 \) pure delays:
\[ \frac{S_0}{R_0} = \frac{2^{-d-1}}{B_S} \]
as reference loop gain.
and use the control
\[ u(t) = - \frac{S_0 A}{R_0 B_S} (y(t) - r(t)) \]
This will lead to the loopgain
\[ G_d G = \frac{S_0 A}{R_0 B_S} \cdot \frac{B_D B_S g^{-d}}{A} = \frac{S_0 g^{-d-1}}{R_0} \cdot \frac{B_D}{B_S} \text{nice} \]

Reference open loop system:
\[ G(s) = \frac{0.125 s^{-2} (1 + s)}{(1 - s)} \]

- 30° phase margin
- integral action
- gain decreasing to zero
- poles at \(-0.27\) \(0.127\) \(0.127\) \(0.127\)

An example:
System:
\[ y(t) = \frac{g^{-d} (1 + 0.25 t^{-2})}{(7 - 0.85 t^{-2})^2} (7 - 0.25 t^{-2}) \]
Almost second order

Reference system
pole placement in the origin

pole placement at \(s^*\) with integral action
Conclusions

- Frequency domain (robustness) properties which are independent of the system model.
- No problems with common factors in \( A \) and \( B \).
- Simple.
- The disturbance rejection properties may be bad.
- The choice of reference system may need some initial testing.

- Pole placement in the origin
- Pole placement at "p"
- Reference system
- Pole placement at "p" with integral action.

\[ p = -0.17 \text{ ; } 0.15 \pm 0.15 \]
On living with the positive real condition

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In this lecture an example is given on a simple model reference adaptive system for which instability is achieved for certain command signals. A modification of the adaptation is suggested to overcome the problem.

In the example the reference model is of 1st order while the process is of 3rd order. The adjustable parameters are the feedforward gain \( k_r \) and the feedback gain \( k_y \). If \( k_y = k_v \) is known and the reference signal \( r(t) \) is a step \( r_0 \) then the closed loop system is linear and time invariant. It is then easy to see that for \( r_0 \) sufficiently large the closed loop system will become unstable unless the process transfer function is positive real. An easy way to eliminate this problem is to use a normed adaptation gain \( g_r = g_{r_0} / r \) instead of \( g_r \). If the reference signal is a sinusoidal \( r(t) = \sin \omega_0 t \) then simulations show that for frequencies larger than a certain frequency \( \omega_0 \) the system will be driven into instability.

The proposed method is based on frequency domain arguments. The idea is to turn off the adaptation for sufficiently high frequencies in the reference input. For these frequencies the adaptation is done with respect to a benign model with positive real transfer function \( G_{\text{good}}(s) \).

![Diagram](image)

**Fig.** The "conditioned plant".

The following assumptions are made:

- The sign of the process gain is known.
- The process is minimum phase.
- The relative degree of the process is known.
- The controller uses \( 2n^* \) parameters, where \( n^* \) is an upper bound on the process order.
Simulations of the MRAS when applied to the "conditioned plant" show that the parameter drifts due to high frequency command signals are eliminated. There are however no theoretical results on the robustness of the modified controller.
1. The positive real condition appears to be necessary as well as sufficient.

Living with Positive Madness

2. There are measures which can be taken which may be taken to allow us to live with the positive real condition.

The adaptive controller's setup

\[ P^* = \frac{b}{s^3 + a_2 s^2 + a_1 s + a_0} \]

Adaptive controller's error system

\[ \frac{b}{s^3 + a_2 s^2 + a_1 s + (a_0 - k_y^*)} \]

Stability determined by characteristic equation

\[ s(s^3 + a_2 s^2 + a_1 s + (a_0 - k_y^*)) + \beta g_r r = 0 \]

Case 1

\[ k_y^* \text{ known; } r(t) = r, \text{ a constant} \]

\[ \begin{bmatrix} e \\ \dot{e} \\ \ddot{e} \\ \dddot{e} \\ \tilde{k}_r \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -(a_0 - k_y^*) & -a_1 & -a_2 & 0 \\ -g_r r & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \\ \ddot{e} \\ \dddot{e} \\ \tilde{k}_r \end{bmatrix} \]

3 \( r \) which will cause instability iff 
\[ \arg P^*(j\omega) > 90 \text{ degs for some } \omega \]

One should use 
\[ g_r = \frac{g_r^*}{r^2(t)} \]
**ERRCR System can be Viewed as Follows:**

If true plant were first order, then there would be feedback parameter \( k_T \) so that \( P^* \) would be positive real or passive.

Then the passivity theorem would say that the loop is stable.

For the case where the plant is properly modeled, the stability result holds despite the fact that the feedback portion of the loop may have large gain.

Example: Assume \( r(t) \) and \( e(t) \) are sinusoidal of the same frequency.

Note: that this instability is independent of the reference model used to form the error.

The instability cannot be explained by any inability of the adaptive system to match the model with a well behaved nominal system.

For example, Case 2 could have arisen from the following system.

We know from Parks that creating a positive real operator is sufficient for stability. These results indicate that the positive real condition is necessary.
Assumptions needed in stability proofs of adaptive algorithms:

1. The sign of $g_p$ is known
2. The zeroes of $B(s)$ are in the left half plane
3. The relative degree of the plant is known exactly
4. The controller uses $2n^*$ parameters where $n^*$ is an upper bound on the plant order.

*THESE ASSUMPTIONS CAN NOT BE MET IN PRACTICAL SITUATIONS*

**Instability** can result in the presence of unmodeled dynamics due to an infinite gain operator in the feedback loop.
CONCLUSION

Using proper conditioning of the type described here, adaptive control can be made robust for any frequency reference input.

CHALLENGES

1. The inputs used here were exactly sufficiently exciting. What are the problems with under excitation and over excitation?

2. The problems with disturbances are not eliminated by conditioning.
Distributed asynchronous algorithms for deterministic and stochastic optimization

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There is presently a great deal of interest in distributed implementations of various iterative algorithms whereby the computational load is shared by several processors while coordination is maintained by information exchange via communication links. In most of the work done in this area the starting point is some iterative algorithm which is guaranteed to converge to the correct solution under the usual circumstances of centralized computation in a single processor. The computational load of the typical iteration is then divided in some way between the available processors, and it is assumed that the processors exchange all necessary information regarding the outcomes of the current iteration can begin.

The mode of operation described above may be termed synchronous in the sense that each processor must complete its assigned portion of an iteration and communicate the results to every other processor before a new iteration can begin. This assumption certainly enhances the orderly operation of the algorithm and greatly simplifies the convergence analysis. On the other hand synchronous distributed algorithms also have some obvious implementation disadvantages such as the need for an algorithm initiation and iteration synchronization protocol. Furthermore the speed of computation is limited to that of the slowest processor. It is thus interesting to consider algorithms that can tolerate a more flexible ordering of computation and communication between processors. Such algorithms have so far found applications in computer communication networks, e.g., ARPANET and other networks designed like it where processor failures are common and it is quite complicated to maintain synchronization between the nodes of the entire network as they execute real-time network functions such as the routing algorithm.

Processor network environments for which weakly coordinated distributed computation seems particularly advantageous typically possess one or more of the following characteristics all of which involve occurrence of some type of unpredictable event.

1/ Computation nodes and communication links are subject to frequent and/or unexpected failures. (For example packet radio networks).

2/ Computation nodes have different and/or time varying speeds of execution. (For example each processor is assigned to a perhaps time varying number of tasks involving computation loads which are not fixed a priori).

3/ Computations at various nodes is event driven. (For example in data collection or sensor networks where the timing, and ordering of measurements may not be predictable.).
It is possible to consider various degrees of coordination in different types of distributed algorithms. An interesting question is to determine the minimum degree of coordination needed in a given algorithm in order to obtain the correct solution. To this end we consider an extreme model of asynchronous distributed algorithms where by computation and communication are performed at each processor completely independently of the progress in the other processors. It is perhaps surprising that even under these chaotic circumstances it is still possible to solve correctly important classes of problems. An account of progress made in this direction is given in a survey jointly written with J. Tsitsiklis and M. Athans (1983). An analysis is given in (Bertsekas, 1982) for broad classes of dynamic programming problems and in (Bertsekas, 1983) for more general fixed point problems involving contractions and monotonicity assumptions. Further related work is (Tsitsiklis, Bertsekas, and Athans, 1983), and (Tsitsiklis, 1983).

References

Expert control

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There has been substantial progress in theory and practice of automatic control through application of mathematical analysis and numerics. Nonnumerical data processing has, however, so far only had marginal influence on control systems. The purpose of this paper is to identify possible uses of expert system techniques in implementation of control systems. It is first observed that actual implementation of control laws often involves a substantial amount of heuristic logic. This is true for simple regulators as well as for more sophisticated multivariable control loops. The paper shows that the heuristic logic may be replaced by an expert system. This leads to simplifications in implementation as well as new capabilities in the control system. Selected basic elements of an expert system are presented. Stochastic dynamic programming offers a framework in which the heuristics can be embedded. This points to requirements for a new artificial intelligence approach for heuristic planning under uncertainty. The ideas are illustrated by examples: a smart PID regulator, a self-tuner with safety jackets and a pole-placement adaptive regulator which can by itself determine suitable pole locations. Once the expert system approach is taken it is possible to obtain control systems with new functions. This is illustrated by the smart PID regulator which incorporates learning.

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SUCCESS STORIES
Experts and data available
Problem Scope
Combinatorics
Incremental progress

PRIOR KNOWLEDGE
delay
sampling period
regulator complexity
forgetting factor
initial estimates
bounds on control

OPERATOR CLASSES
Main Monitor
Backup Control
Minimum variance control
Estimation
Tuning
Learning

Main monitor:
stability-supervisor
control-quality-supervisor

Back-up control:
pid-control
auto-tune

Fixed gain MV control:
minimum-variance-control
minimum-variance-supervisor
ringing-detector
degreesupervisor

Estimation:
parameter-estimation
excitation-supervisor
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jump-detector

Self-tuning:
self-tuning-regulation

Learning:
get-regulator-parameters
put-regulator-parameters
store-regulator-parameters
test-scheduling-hypothesis
smooth-table-entries

\[ A y_t = B u_{t-d} + C e_t \]
\[ R u = -S y \]
\[ z^{d-1}C = A R + B S \]
\[ y_t = f_0 e_t + \ldots + f_{d-1} e_{t-d+1} \]
\[ F = R / B \]
Main monitoring table

Backup control Table.

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<tr>
<th>#</th>
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<th>u</th>
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<th>y</th>
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Minimum variance control table

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ALGORITHM ORCHESTRATION

Control Algorithms
Diagnosis Algorithms
Logic and sequencing
Tables for learning

IMPLEMENTATION

Vax 11/780
ES in Lisp
Algorithms in Pascal
Concurrency

EXPERIMENTS

- SMART PID
- INTELLIGENT STR
- AUTOMATIC $\omega_B$ CHOICE

JUDGEMENT

Ideas probably much more important for large complex systems.
But let us do simple things first.

CONCLUSIONS

- New control laws
- Many algorithms
- Control and diagnosis
- Learning

WHY USE ES?

Simple coding
Separate algorithms and logic
Experiments with expert control

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Abstract:
Expert control refers to a control system where an expert system is used to orchestrate a collection of control and identification algorithms. This is done in real time. Adaptive control algorithms need large safety jackets of logic to work in practice. Existing adaptive algorithms perform well locally but require a priori information about time delay, system order etc. to do so. Simple algorithms exist that can provide some of this information. An expert system is well suited for implementation of logic. Heuristics and rules of thumb are also easily implemented.

A testbench for experiments is presented. The controller is divided in two parts. One algorithm library written in Pascal and one expert system written in OPS4 and Lisp. These parts are implemented as communicating concurrent processes. The communication is done with mailboxes and messages. A typical message is to start or stop an algorithm. OPS4 is a rule based, forward chaining expert system framework.

An experiment is presented where the level of a water tank is controlled by a PID controller with Ziegler-Nichols auto tuning and gain scheduling.

Reference:
EXPERIMENTS WITH EXPERT CONTROL

- Introduction
- Implementation
- Experiments
- Demonstration

Programming Languages

Symbolic Data Processing, AI-tradition => Lisp, Prolog

Numerical algorithms => Pascal, Fortran, ADA

Concurrency

Expert System in real time.

MOTIVATION

- "Safety Jacket"
- Many algorithms
- Heuristics & rules of thumb
- Make full use of a priori information
- Division: Logic => Algorithm
- Workbench

IMPLEMENTATION

- VAX 11/780, VMS
- Pascal, Franz Lisp

[Diagram]

Expert part Lisp
Algorithm part Pascal

DA
PD

Man-Machine I/O
LISP
Algorithm part

- Library of control, identification and supervision algorithms with a few well specified operations upon.

- Control Loop

```java
while true do
  begin
    if mail-in-box then readmail;
    for all algorithms do
      if algorithm_is_active then
        execute_algorithm;
        wait;
    end;
  end;
```

EXPERIMENTS

1) "Smart" PID that can decide whether
   - The process can be controlled by PI
   - PID
   - The process needs a more complex controller

2) STR that determines the prediction horizon automatically

—

Expert System Part

- Existing Expert System Framework

OPS 4

Consists of

- Working Memory
- Production - IF- Control Structure

- Forward Chaining
- Rule format:
  - Condition
  - Action
- Recognize-act cycle

DEMONSTRATION

PID with Ziegler-Nichols autotuning and gain scheduling

Algorithms

- PID
- Relay
- Relay guard
- Noise estimator

≈ 50-60 rules.
Notes from the discussion

A short summary of the discussion on Wednesday afternoon is given here. The conclusions given below should not be taken as declarations everyone agreed on, but rather as a couple of interesting statements brought up during the discussion. Some statements were accepted by most participants, other maybe by very few.

I. Process control versus aircraft control

Process control and aircraft control are two totally different issues. In aircraft control, lots of time, money and work are spent to obtain a very high performance. In process control, the control work must often be both fast and cheap, and high performance is often not so important. Stability is often enough. Furthermore, these systems have different types of dynamics. These are some differences which has influenced the success of adaptive control in the process industry, and the lack of success in the aircraft control.

II. Parametrization

Current parametrizations used in adaptive control are not good - we are probably using them only because we know how to solve the identification problem for these parametrizations.

Approximations are best done on an input-output basis, in the frequency domain. Unstructured inaccuracy would e.g. be given as nominal phase and amplitude curves ± ranges. State-space parametrization is not good for approximation. Therefore, the state-space realization part should be left to the end of the design procedure.

State-space is excellent for rigid body dynamics.

You cannot have an adaptive theory unless you have a feedback theory that deals with plant uncertainty. Feedback reduces tolerance bound.

III. Is identification essential to adaptive control?

Identification is essential to adaptive systems. You have to obtain knowledge of the system from measurements.

From a theoretical point of view, identifiability may not be essential, but for robustness it may. Otherwise we do not catch the whole dynamics. As long as you get the I/O-map, this is enough. The number of parameters may lead to numerical problems. Even if the I/O map is not changing, the internal parameters may change. This is a numerical problem.

IV. How to take care of apriori knowledge

Prefiltering may be one way to include apriori knowledge to decrease the number of parameters.

We would like to remove as many critical parameters, such as nonminimum phase and time delay, as possible. The relay method can solve the time delay problem, at least for systems with monotone step responses.
A lot of tricks are currently added to take care of non-typical situations. It should be useful to get insight from theory why we need those tricks. Some problems are related to robustness.