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ANALYSIS AND SIMULATION OF A MACRO
ECONOMIC MODEL

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ABSTRACT.

A monetarist model for the U.S. economy (the St Louis model) is considered. A state space representation is given together with elementary analysis. The model response to various inputs has been simulated giving results consistent with economic theory. No drastic sensitivity to parameter changes was found.

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APPENDIX I: A Monetarist Model for Economic
Stabilization

APPENDIX II: Program Listing

1. INTRODUCTION.

In 1973 a course on "Automatic Control and Economic Policy" was given at the divisions of Automatic Control and Economics, University of Lund. As a separate project in connection with the course an economic model was analysed and simulated. This report summarizes the results of the project. As a suitable model the so called St Louis model was chosen, Andersen and Carlson (1970 a), given in Appendix I. A short review of the model is given in Section 2.

State space representations of the model are developed in Section 3, where also some of the properties of the model are investigated. Section 4 deals with simulations of the model.

Programs are listed in Appendix II.

2. THE ST LOUIS MODEL OF THE U.S. ECONOMY.

The St Louis model of the U.S. economy was developed for the analysis of economic stabilization and focuses on the influence of monetary expansion on total spending. The model exists in several variants. In the sequel the following sets of equations are used:

$$\Delta Y_t = y_0 + \sum_{i=0}^4 m_i \Delta M_{t-i} + \sum_{i=0}^4 e_i \Delta E_{t-i} \quad (1)$$

$$\Delta P_t = p_0 + \sum_{i=0}^8 d_i D_{t-i} + \sum_{i=0}^8 v_i \Delta M_{t-i} \quad (2)$$

$$\Delta X_t = \Delta Y_t - \Delta P_t \quad (3)$$

$$D_t = \Delta Y_t - (X_t^F - X_{t-1}) \quad (4)$$

$$G_t = \frac{X_t^F - X_t}{X_t^F} \quad (5)$$

$$U_t = u_0 + 0.04G_t + 0.28G_{t-1} \quad (6)$$

$$R_t^L = r_0^L - 0.06\dot{M}_t + 1.42Z_t + \sum_{i=0}^{16} x_i^L \dot{X}_{t-i} + \sum_{i=0}^{16} p_i^L \left(\frac{\dot{P}_{t-i}}{U_{t-i}/4} \right) \quad (7)$$

$$R_t^S = r_0^S - 0.17\dot{M}_t + 0.92Z_t + \sum_{i=0}^{10} x_i^S \dot{X}_{t-i} + \sum_{i=0}^{10} p_i^S \left(\frac{\dot{P}_{t-i}}{U_{t-i}/4} \right) \quad (8)$$

where the endogenous variables are

ΔY_t = change in total spending (nominal GNP)

ΔP_t = change in price level (GNP price deflator)

D_t = demand pressure

ΔX_t = change in output (real GNP)

R_t^S = short-term market interest rate

R_t^L = long-term market interest rate

U_t = unemployment rate

G_t = GNP gap

and the exogenous variables are

ΔM_t = change in money stock

ΔE_t = change in high-employment Federal expenditures

X_t^F = potential (full employment) output.

The sampling interval is a quarter of a year. The dotted variables are defined analogously to

$$\dot{p}_t = \left(\frac{p_t}{p_{t-1}} \right)^4 - 1$$

and described as annual rate of change in the specific variable in quarter t . $y_0, m_1, e_1, p_0, d_1, v_1, r_0^L, x_1^L, u_0, p_1^L, r_0^S, x_1^S$ and p_1^S are parameters estimated from quarterly data during the period 1953(5) - 1969. The least squares method was used. Z_t is a dummy variable, 0 or 1. The numerical values of the parameters are given in Andersen and Carlson (1970 a, b).

The equations are discussed in detail in Andersen and Carlson (1970 a). Equation (1) is the total spending equation. In order to get a system which is nonlinear only in some of its outputs the price equation (2) was taken from Andersen and Carlson (1970 b). (3) - (5) are pure identities for the total spending, the demand pressure and the GNP gap respectively. Equation (6) defines the unemployment rate and equations (7) and (8) are the interest rate equations.

In Fig. 2.1 a block chart of the equations (1) - (4) is given.

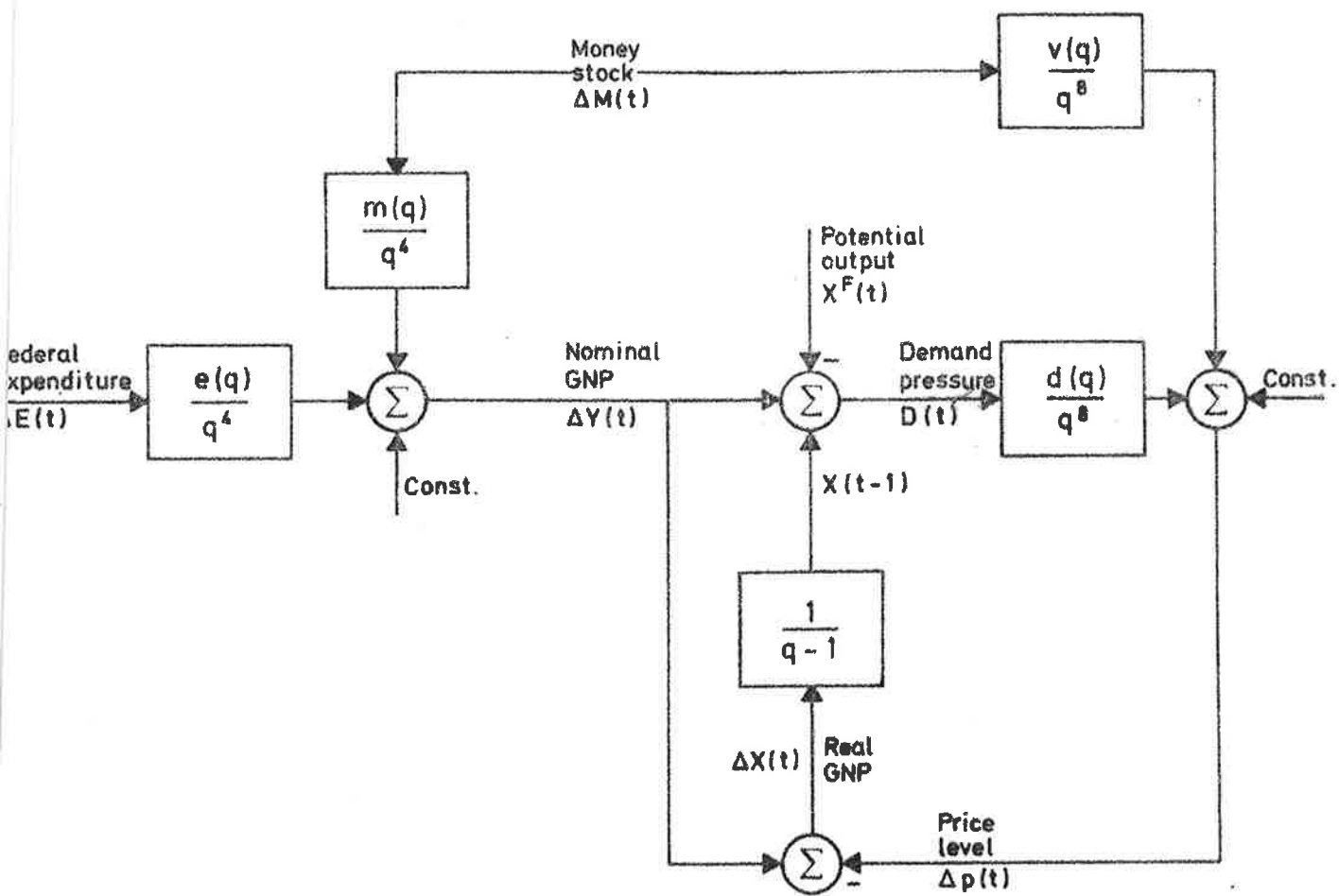


Fig. 2.1 - Block chart of the model equations (1) - (4).

$$e(q) = \sum_{i=0}^4 e_i q^{4-i}$$

$$m(q) = \sum_{i=0}^4 m_i q^{4-i}$$

$$v(q) = \sum_{i=0}^8 v_i q^{8-i}$$

$$d(q) = \sum_{i=0}^8 d_i q^{8-i}$$

q shift operator

3. STATE SPACE REPRESENTATIONS OF THE ST LOUIS MODEL.

There are many possible ways to transform a system to state space form for the same system. Considering only equations (1) - (4) the model consists of linear difference equations only. The remaining equations (5) - (8) can be considered as definitions of additional output variables, which depend in a nonlinear fashion on the other variables. In the following mainly the linear part will be considered, i.e. equations (1) - (4).

We will give two different state space models. Firstly, a straightforward transformation from the resulting vector difference equation to a state space model is performed. The result is a model of order sixteen. Secondly we will use the special block form shown in the block chart in Fig. 2.1. to derive a state space model of order thirteen.

Generally, a state space model must be completely controllable and observable to be of minimal order. In this case only the controllability condition is relevant, since the observability condition will depend upon the number of delayed variables we intend to describe in the actual model.

3.1. State Space Model I.

First the difference equations (1) - (4) are transformed to a standard reduced form vector difference equation. Due to the identities (3) and (4) one of the three variables $\Delta X_t (X_t)$, ΔP_t or ΔY_t can be eliminated and considered as an output variable, and furthermore D_t can be eliminated from Eq. (2).

From (3)

$$\Delta Y_t = X_t - X_{t-1} + \Delta P_t$$

and from (4)

$$D_t = \Delta P_t + X_t - X_t^F \quad (10)$$

Using (9) and (10) the equations (1) and (2) will be

$$X_t = X_{t-1} - \Delta P_t + Y_0 + \sum_{i=0}^4 m_i \Delta M_{t-i} + \sum_{i=0}^4 e_i \Delta E_{t-i} \quad (11)$$

$$\Delta P_t = p_0 + \sum_{i=0}^8 d_i (\Delta P_{t-i} + X_{t-i} - X_{t-i}^F) + \sum_{i=0}^8 v_i \Delta M_{t-i} \quad (12)$$

if the variable ΔY_t is eliminated.

The standard vector difference equation form is

$$\sum_{i=0}^P A_i y_{t-i} = \sum_{i=0}^P B_i u_{t-i} + K \quad (13)$$

where y denotes the output vector, u the input vector and K a constant vector. A_i and B_i are matrices of suitable dimensions. If now

$$y = \begin{pmatrix} X \\ \Delta P \end{pmatrix} \quad \text{and} \quad u = \begin{pmatrix} \Delta M \\ \Delta E \\ X^F \end{pmatrix}$$

(11) and (12) directly give

$$A_0 = \begin{pmatrix} 1 & 1 \\ -d_0 & 1-d_0 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -1 & 0 \\ -d_1 & -d_1 \end{pmatrix}$$

$$A_i = \begin{pmatrix} 0 & 0 \\ -d_i & -d_i \end{pmatrix} \quad i = 2, \dots, 8$$

$$B_i = \begin{pmatrix} m_i & e_i & 0 \\ v_i & 0 & -d_i \end{pmatrix} \quad i = 0, \dots, 4$$

$$B_i = \begin{pmatrix} 0 & 0 & 0 \\ v_i & 0 & -d_i \end{pmatrix} \quad i = 5, \dots, 8$$

$$K = \begin{pmatrix} Y_0 \\ P_0 \end{pmatrix}$$

In order to transform to reduced form, i.e. $A_0 = I$, all coefficients must be multiplied by

$$A_0^{-1} = \begin{pmatrix} 1-d_0 & -1 \\ d_0 & 1 \end{pmatrix}$$

Including ΔY_t and D_t as output variables would not influence on the transformation to state space form since all variables in (9) and (10) also are in Eqs. (11) and (12).

Let

$$\begin{aligned} A_1' &= A_0^{-1} \cdot A_1 & i &= 1, \dots, 8 \\ B_1' &= A_0^{-1} \cdot B_1 & i &= 1, \dots, 8 \end{aligned} \quad (14)$$

Two possible ways to get state space equivalents of the vector difference equation (13) (assuming $K = 0$) are:

$$\begin{cases} x(t+1) = \begin{bmatrix} -A_1 & I & 0 & \dots & 0 \\ -A_2 & 0 & I & & 0 \\ \vdots & & & \ddots & \\ -A_p & 0 & 0 & \dots & 0 \end{bmatrix} x(t) + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ \vdots \\ \bar{B}_p \end{bmatrix} u(t) \\ y(t) = [I, 0, \dots, 0]x(t) + B_0 u(t) \end{cases}$$

where $\bar{B}_i = B_i' - A_i' B_0'$ and

$$\begin{cases} x(t+1) = \begin{bmatrix} 0 & \dots & 0 & x & | & & & x \\ 1 & & 0 & x & | & & & x \\ \vdots & \ddots & \vdots & \vdots & | & & 0 & \vdots \\ 0 & \dots & 1 & x & | & & & x \\ \hline & & & x & | & 0 & \dots & 0 & x \\ & & & x & | & 1 & & \cdot & x \\ & 0 & & \vdots & | & \vdots & \ddots & \vdots & \vdots \\ & & & x & | & 0 & & 1 & x \end{bmatrix} x(t) + \begin{bmatrix} x & x & x \\ x & x & x \\ \vdots & \vdots & \vdots \\ x & x & x \\ x & x & x \\ x & x & x \\ \vdots & \vdots & \vdots \\ x & x & x \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 0 & 0 & \dots & 1 & | & & & 0 \\ \hline & & 0 & & | & 0 & 0 & \dots & 1 \end{bmatrix} x(t) + B_0 u(t) \end{cases}$$

where the x 's denote coefficients which are directly given by A_i' and \bar{B}_i . The second form is given by Wall (1972).

$$B = \begin{bmatrix} -0.172 & -0.006 & 0.010 \\ -0.301 & -0.009 & 0.016 \\ -0.389 & -0.011 & 0.020 \\ -0.434 & -0.011 & 0.020 \\ -0.381 & -0.565 & 0.017 \\ 0.493 & -0.444 & 0.011 \\ 1.340 & 0.013 & 0.003 \\ 2.862 & 1.053 & -0.034 \\ \hline 0.172 & 0.006 & -0.010 \\ 0.301 & 0.009 & -0.016 \\ 0.389 & 0.011 & -0.020 \\ 0.434 & 0.011 & -0.020 \\ 0.437 & 0.022 & -0.017 \\ 0.368 & 0.016 & -0.011 \\ 0.282 & 0.002 & -0.003 \\ 0.127 & -0.153 & 0.010 \end{bmatrix}$$

$$C = \left[\begin{array}{ccc|ccc} 0 & \dots & 1 & & & 0 \\ \hline & & & 1 & & \\ 0 & & & & 0 & \dots & 1 \end{array} \right]$$

$$D = \begin{bmatrix} 1.313 & 0.570 & -0.024 \\ 0.034 & -0.013 & 0.024 \end{bmatrix}$$

A lower order model will be given in the next section.

3.2. State Space Model II.

Using the special block form of the system shown in Fig. 2.1, it is possible to derive a model of lower order. We will then use the following facts:

- o Eq. (1) can be described by 4 states,
- o Eq. (2) can be described by 8 states if $D(t)$ is considered as input signal,
- o one additional state is needed to describe $D(t)$. The Eqs. (3) and (4) will be used for this purpose.

Consider first Eq. (1) and introduce

$$x_k(t) = \sum_{i=k}^4 \left\{ m_i \Delta M_{t+k-i-1} + e_i \Delta E_{t+k-i-1} \right\} \quad (15)$$

for $k = 1, 2, 3, 4$. By straightforward manipulations it can be shown that

$$\begin{cases} x_k(t+1) = x_{k+1}(t) + m_k \Delta M_t + e_k \Delta E_t ; k = 1, 2, 3 \\ x_4(t+1) = m_4 \Delta M_t + e_4 \Delta E_t \end{cases} \quad (16)$$

The same procedure is performed on Eq. (2), i.e. introduce

$$x_{5+k}(t) = \sum_{i=k}^8 \left\{ d_i D_{t+k-i-1} + v_i \Delta M_{t+k-i-1} \right\} \quad (17)$$

In the same way as above we have

$$\begin{cases} x_{5+k}(t+1) = x_{6+k}(t) + v_k \Delta M_t + d_k D_t ; k = 1, 2, \dots, 7 \\ x_{13}(t+1) = d_8 D_t + v_8 \Delta M_t \end{cases} \quad (18)$$

We have now introduced 12 states in all. From the definitions (15) and (17) of the state variables and Eqs. (1) - (2), we have

$$\Delta y_t = y_0 + x_1(t) + m_0 \Delta M_t + e_0 \Delta E_t \quad (19)$$

$$\Delta p_t = p_0 + x_6(t) + v_0 \Delta M_t + d_0 D_t$$

Introduce

$$x_5(t) = - X_{t-1} - y_0 \quad (20)$$

Using (3), (4) and (19) we have

$$D_t = x_1(t) - x_5(t) - X_t^F + m_0 \Delta M_t + e_0 \Delta E_t \quad (21)$$

$$\begin{aligned} \Delta p_t = & d_0 x_1(t) + x_6(t) - d_0 x_5(t) + (v_0 + d_0 m_0) \Delta M_t + \\ & + d_0 e_0 \Delta E_t - d_0 X_t^F + p_0 \end{aligned} \quad (22)$$

A substitution of (21) into (18) gives the final form for these equations:

$$\begin{aligned} x_{5+k}(t+1) = & x_{6+k}(t) + d_k x_1(t) - d_k x_5(t) + (v_k + m_0 d_k) \Delta M_t + \\ & + d_k e_0 \Delta E_t - d_k X_t^F; \quad k = 1, 2, \dots, 8 \end{aligned} \quad (23)$$

$$\begin{aligned} x_{13}(t+1) = & d_8 x_1(t) - d_8 x_5(t) - d_8 X_t^F + (d_8 m_0 + v_8) \Delta M_t + \\ & + d_8 e_0 \Delta E_t \end{aligned}$$

Finally we have for $x_5(t)$, using (19), (22) and (3)

$$\begin{aligned}
x_5(t+1) &= -x_t - y_0 = x_5(t) - \Delta x_t = x_5 + \Delta p_t - \Delta y_t = \\
&= (d_0 - 1)x_1(t) + (1 - d_0)x_5(t) + x_6(t) + \\
&\quad + (v_0 + d_0 m_0 - m_0)\Delta M_t + (d_0 - 1)e_0 \Delta E_t - d_0 x_t^F - \\
&\quad - (y_0 - p_0)
\end{aligned} \tag{24}$$

The state equations are summarized below. The state variables are:

$$x_k(t) = \sum_{i=k}^4 \left\{ m_i \Delta M_{t+k-i-1} + e_i \Delta E_{t+k-i-1} \right\} \quad k = 1, 2, 3$$

$$x_5(t) = -x_{t-1} - y_0$$

$$x_{5+k}(t) = \sum_{i=k}^8 \left\{ d_i D_{t+k-i-1} + v_i \Delta M_{t+k-i-1} \right\} \quad k = 1, 2, \dots, 8$$

and the state space model:

$$x_k(t+1) = x_{k+1}(t) + m_k \Delta M_t + e_k \Delta E_t \quad k = 1, 2, 3$$

$$x_4(t+1) = m_4 \Delta M_t + e_4 \Delta E_t$$

$$\begin{aligned}
x_{5+k}(t+1) &= x_{6+k}(t) + d_k x_1(t) - d_k x_5(t) + (v_k + m_0 d_k) \Delta M_t + \\
&\quad + d_k e_0 \Delta E_t - d_k x_t^F
\end{aligned}$$

$$\begin{aligned}
x_{13}(t+1) &= d_8 x_1(t) - d_8 x_5(t) - d_8 x_t^F + (d_8 m_0 + v_8) \Delta M_t + \\
&\quad + d_8 e_0 \Delta E_t
\end{aligned}$$

$$\begin{aligned}
 x_5(t+1) = & (d_0-1)x_1(t) + (1-d_0)x_5(t) + x_6(t) + \\
 & + (v_0 + (d_0-1)m_0)\Delta M_t + (d_0-1)e_0\Delta E_t - \\
 & - d_0x_t^F - (y_0 - p_0)
 \end{aligned}$$

The structure of the model is more easily seen from the matrix form shown in Fig. 3.1.

3.3. Simple Analysis.

The positions of the eigenvalues of A will essentially determine the transient behaviour of the model. The characteristic polynomial is easily obtained from state space model II, since the system matrix A has a convenient block triangular form. The upper diagonal block is on Jordan form and the lower one on canonical form. The characteristic polynomial becomes

$$\begin{aligned}
 p(s) = \det(sI-A_{11})\det(sI-A_{22}) = \\
 = s^4(s^9 + (1-d_0)s^8 + d_1s^7 + \dots + d_8)
 \end{aligned}$$

The stability properties of the open loop system is thus determined by the coefficients d_i in Eq. (2), i.e. how demand pressure affects prices. For the numerical values on d_1 given in Andersen and Carlson (1970 b), the eigenvalues are shown in Fig. 3.2 for the discrete time system and the corresponding continuous time system.

$$\begin{aligned}
 \mathbf{x}(t+1) = & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ v_0 + m_0(d_0 - 1) \\ v_1 + m_0 d_1 \\ v_2 + m_0 d_2 \\ v_3 + m_0 d_3 \\ v_4 + m_0 d_4 \\ v_5 + m_0 d_5 \\ v_6 + m_0 d_6 \\ v_7 + m_0 d_7 \\ v_8 + m_0 d_8 \end{pmatrix} \Delta M_t + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_0(d_0 - 1) \\ d_1 e_0 \\ d_2 e_0 \\ d_3 e_0 \\ d_4 e_0 \\ d_5 e_0 \\ d_6 e_0 \\ d_7 e_0 \\ d_8 e_0 \end{pmatrix} \Delta E_t + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -d_0 \\ -d_1 \\ -d_2 \\ -d_3 \\ -d_4 \\ -d_5 \\ -d_6 \\ -d_7 \\ -d_8 \end{pmatrix} X_t^F + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P_0 - Y_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
 \end{aligned}$$

Fig. 3.1 - State space model II. State equations.

$$\begin{pmatrix} \Delta Y_t \\ \Delta P_t \\ \Delta X_t \\ D_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d_0 & 0 & 0 & 0 & -d_0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1-d_0 & 0 & 0 & 0 & d_0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} m_0 & e_0 & 0 & Y_0 \\ v_0 + d_0 m_0 & d_0 e_0 & -d_0 & P_0 \\ m_0(1-d_0) - v_0 & e_0(1-d_0) & d_0 & Y_0 - P_0 \\ m_0 & e_0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \Delta M_t \\ \Delta E_t \\ X_t^F \\ C \end{pmatrix}$$

State space model II. Some important output variables.

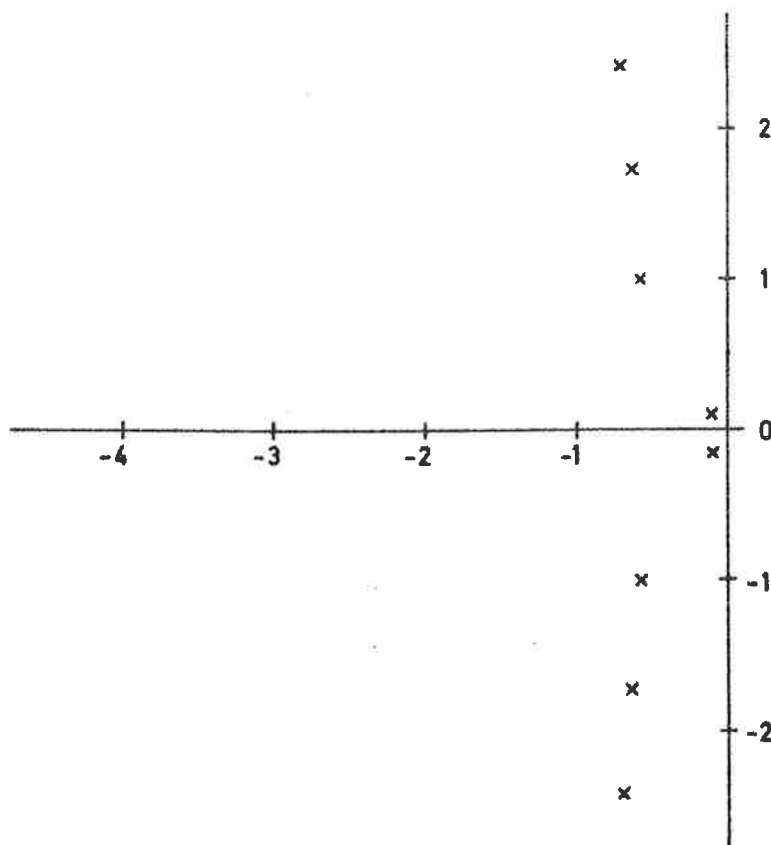
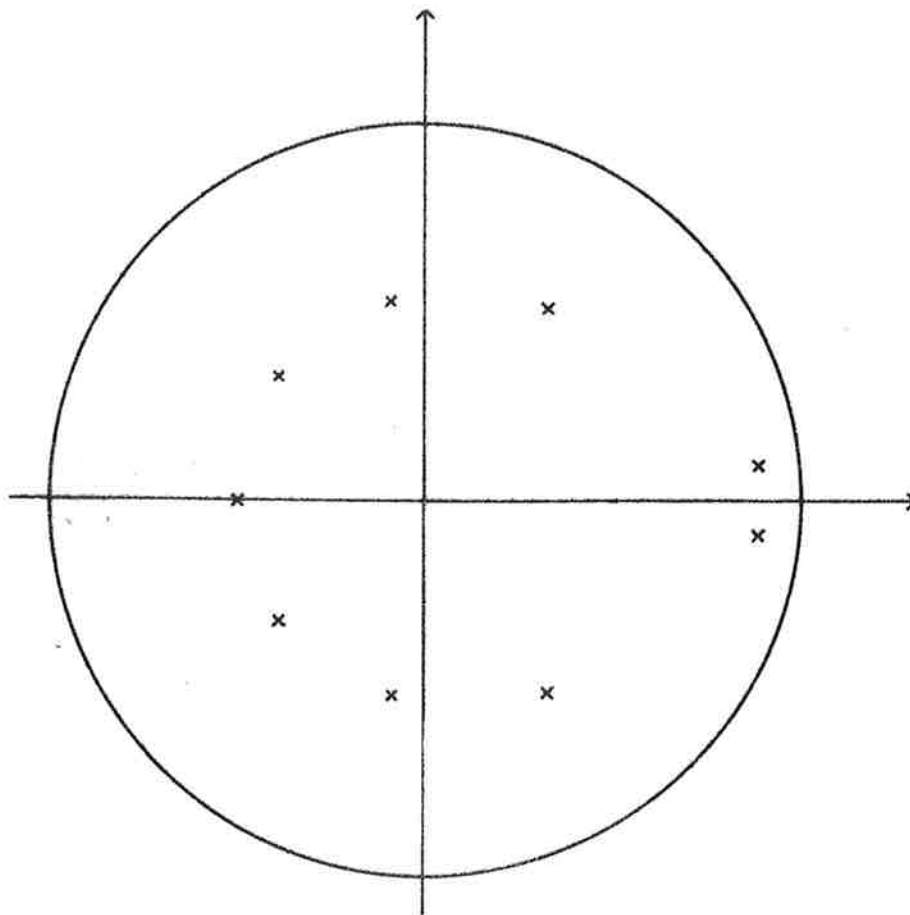


Fig. 3.2 - Eigenvalues of the discrete and corresponding continuous St Louis model.

4. SIMULATIONS.

In this section the computer programs will be described and the results from some simulations will be discussed.

4.1. Programs.

The equations describing the St Louis model were simulated on a PDP 15 computer. The following programs were written:

USEC	Main program for the simulation
INDAT	Guides user at the input of data
OUTDAT	Prints initial data and results from the simulations on line printer
COEF	Gives numeric values to the coefficients of the equations
SIMUS	Simulates the model one step of time

Listings of the programs are given in Appendix II. Results from a simulation were stored in a disk file. This file could be plotted on the display after the simulation.

Change of money stock (ΔM_t) and change in high-employment Federal expenditures (ΔE_t) were the input signals. In the computer program it was possible to make a step change or a ramp change in these variables.

4.2. Initial Values.

The following variables were initialized by user:

- o Change in money stock ($\Delta M_t, \Delta M_{t-1}, \dots, \Delta M_{t-8}$).
- o Change in high-employment Federal expenditures ($\Delta E_t, \Delta E_{t-1}, \dots, \Delta E_{t-4}$).
- o Demand pressure ($D_t, D_{t-1}, \dots, D_{t-8}$).
These variables were usually initialized with 0 billions of dollars.
- o GNP gap (G_{t-1}). The initial value was usually 2%.
- o Annual rate of change in money stock (\dot{M}_t). The initial value was usually 4%.
- o Annual rate of change in output (GNP in 1958 prices), ($\dot{X}_t, \dot{X}_{t-1}, \dots, \dot{X}_{t-16}$). These variables were initially given the same value, usually 4%.
- o Annual rate of change in prices ($\dot{P}_t, \dot{P}_{t-1}, \dots, \dot{P}_{t-16}$).
These variables were initially given the same value, usually 4%.
- o Unemployment ($U_t, U_{t-1}, \dots, U_{t-16}$). These variables were initially given the same value, usually 4%.
- o Potential (full employment) output (X_t^F). The initial value was usually 914 billions of dollars.
- o Output (real GNP), (X). The initial value was usually 900 billions of dollars.
- o Money stock (M). The initial value was usually 900 billions of dollars.
- o Price level (P). The initial value was usually 900 billions of dollars.
- o Annual rates of change in X^F , ΔM and ΔE ($ADXF$, ADM and ADE resp.). These variables were usually set to 4%.

- o Changes in input signals
 - Step in ΔM , ΔE (DMPRO and DEPRO resp.)
 - Ramp in ΔM , ΔE (ADMI and ADEI resp.)

4.3. Step Response When ΔM Is Altered.

The model was run for 80 sample points (sampling interval was equal to a quarter of a year) with ΔM increased 4% each year from 2 billion dollars at the start. After the 80 sample points ΔM was increased with 50%, and then the slow 4% rise was resumed. The reason for having the step in the middle of the run was to have the transients due to bad choices of initial values damped out before the step was made. In Figs. 4.1 - 4.6 the effects of this step in ΔM are shown.

Figure 4.1 shows the direct effects on ΔY of the change in ΔM which is to be expected from the total spending equation. The response in ΔP is smoother due to the dynamics involved in the price equation. From Fig. 4.2 it follows that there is a marked decrease in unemployment. This decrease begins immediately after the step in ΔM , but the full effect is not reached until after about five years. Figure 4.4 shows that the increase in ΔM results in a narrowing between the curves for X^F and X . This is also seen in the curve for G in Figure 4.5.

4.4. Step Response When ΔE Is Altered.

The same type of experiment was made with a step in ΔE , see Figures 4.7 - 4.12. After 80 sample points ΔE was increased with 300%. This large (and perhaps unrealistic) change was made because the smaller changes hardly had any effect at all. In Fig. 4.7 the qualitative difference between a change in ΔM and in ΔE is seen. While the change in ΔM has a lasting effect the influence of the change in ΔE has almost vanished after a few years. This type of behaviour is seen throughout the equations. Unemployment, Fig. 4.8, is influenced only for about 5 years, then it goes back almost to the same level as before, in contrast to lasting effect obtained from a change in ΔM .

4.5. Sensitivity Test.

It is quite difficult to get good estimates of the parameters in economic models. This is due to the fact that usually relatively few data are measured. Therefore it is important to study how sensitive the simulation results are with respect to parameter changes.

The price equations have the most important influence on the dynamics of the system. Especially the d -parameters are important since these are the only ones that enter the characteristic equation (cf Section 3.3).

It is desirable that the parameters should be changed in accordance with the covariances of the estimates. These are, however, not available and therefore the parameters are varied in a straightforward manner.

In Figures 4.13 - 4.14 the effect of various variations

in the price equation parameters is shown.

It is concluded that the dynamic behaviour is not very sensitive to parameter variations.

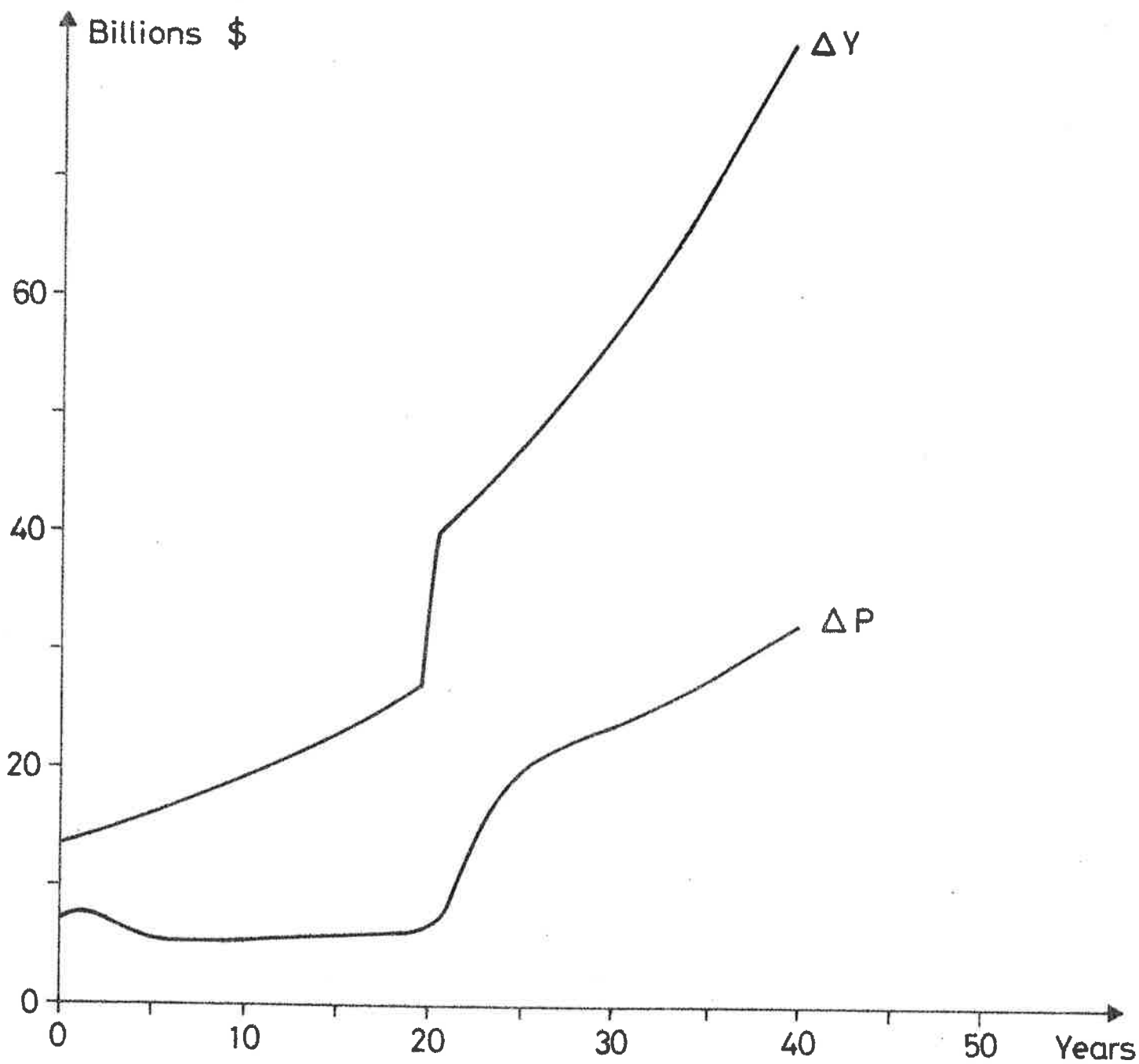


Fig. 4.1

ΔY (change in GNP in current prices, billions of dollars) and ΔP (the part of ΔY which is due to price changes, billions of dollars) as functions of time when ΔM is increased 50% after 80 samples (= 20 years).

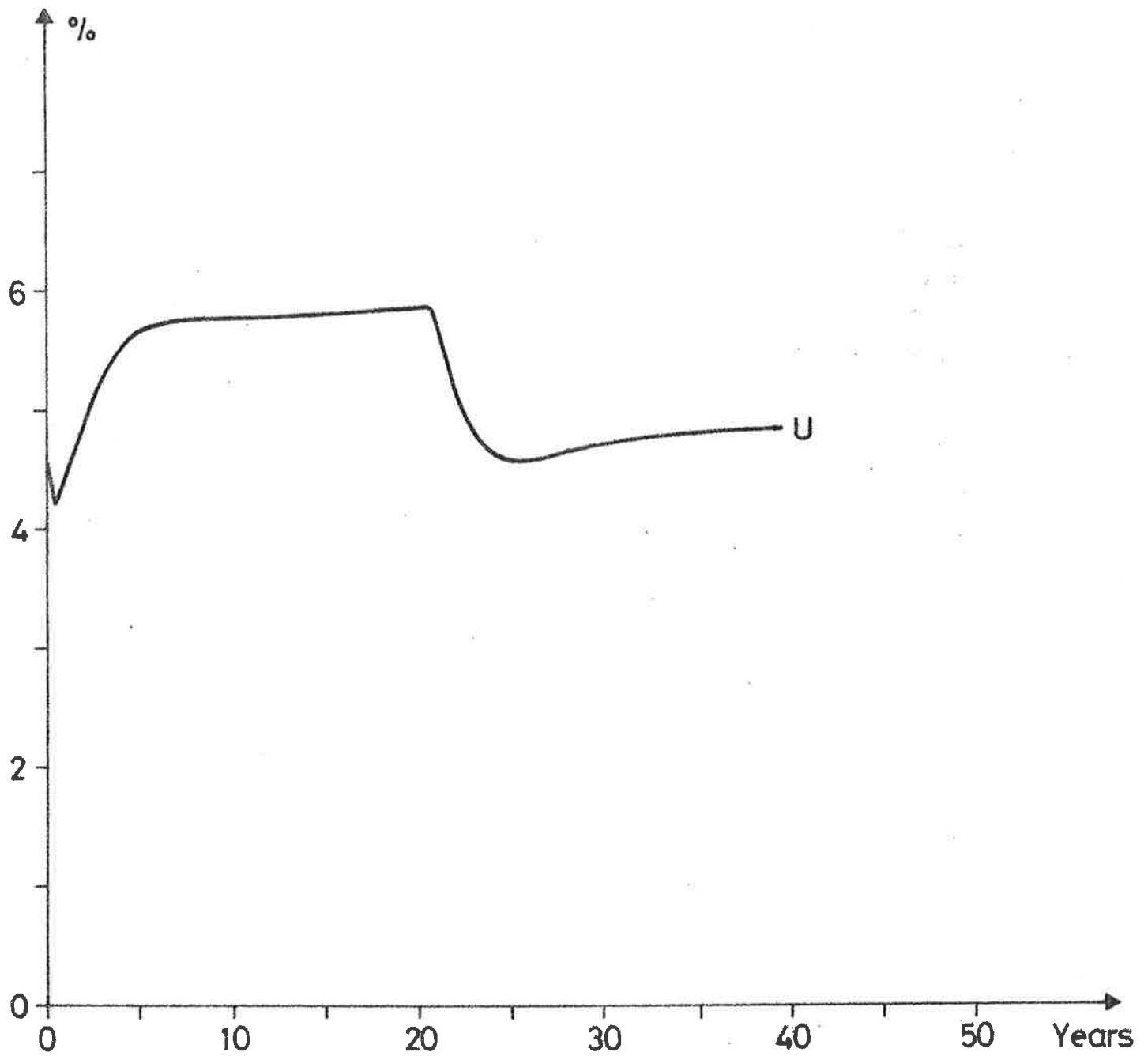


Fig. 4.2

Unemployment, U (as a per cent of labor force), as function of time when ΔM is increased 50% after 80 samples.

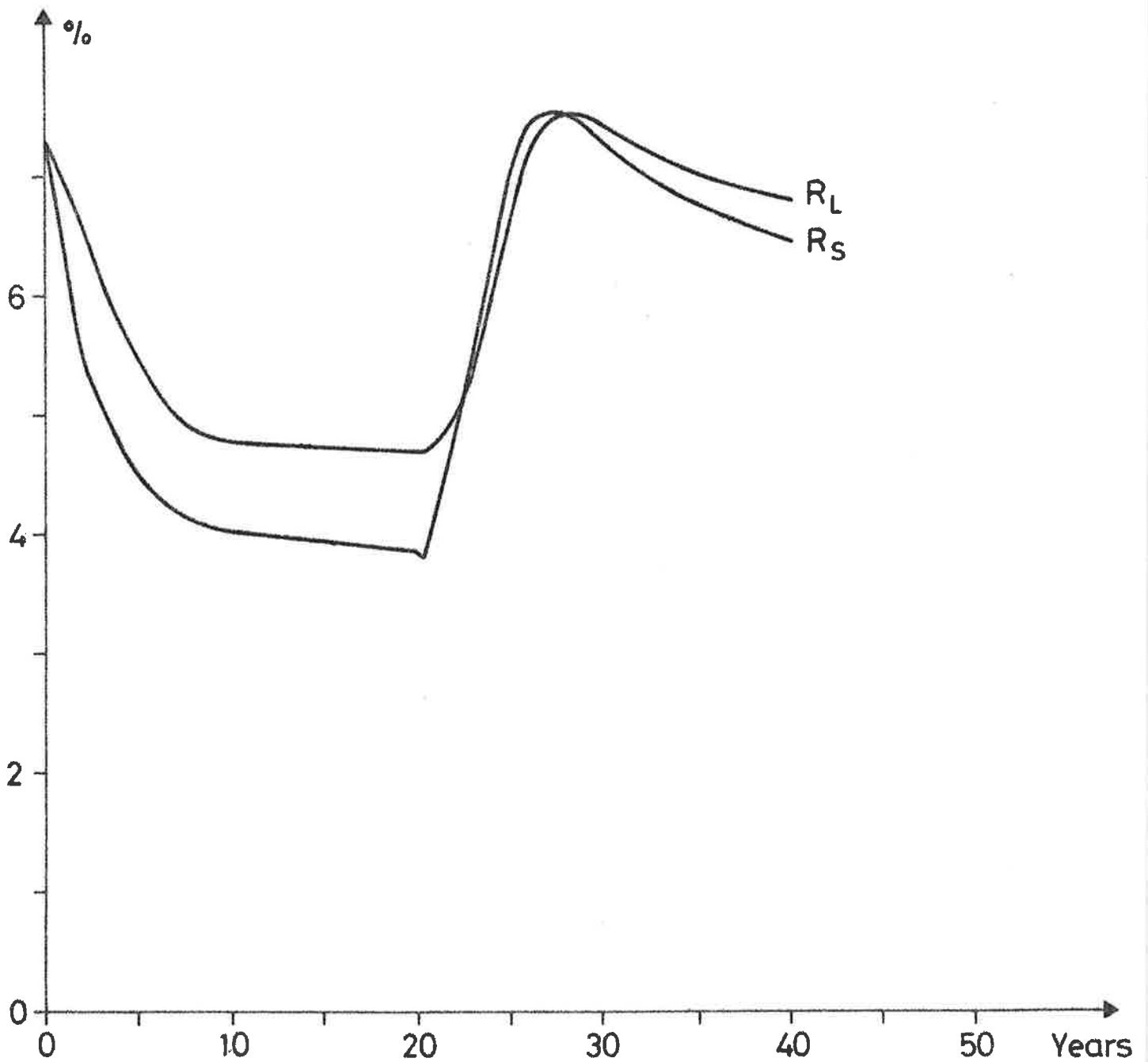


Fig. 4.3

Long term interest rate, R_L , and short term interest rate, R_S , as functions of time when ΔM is increased 50% after 80 samples.

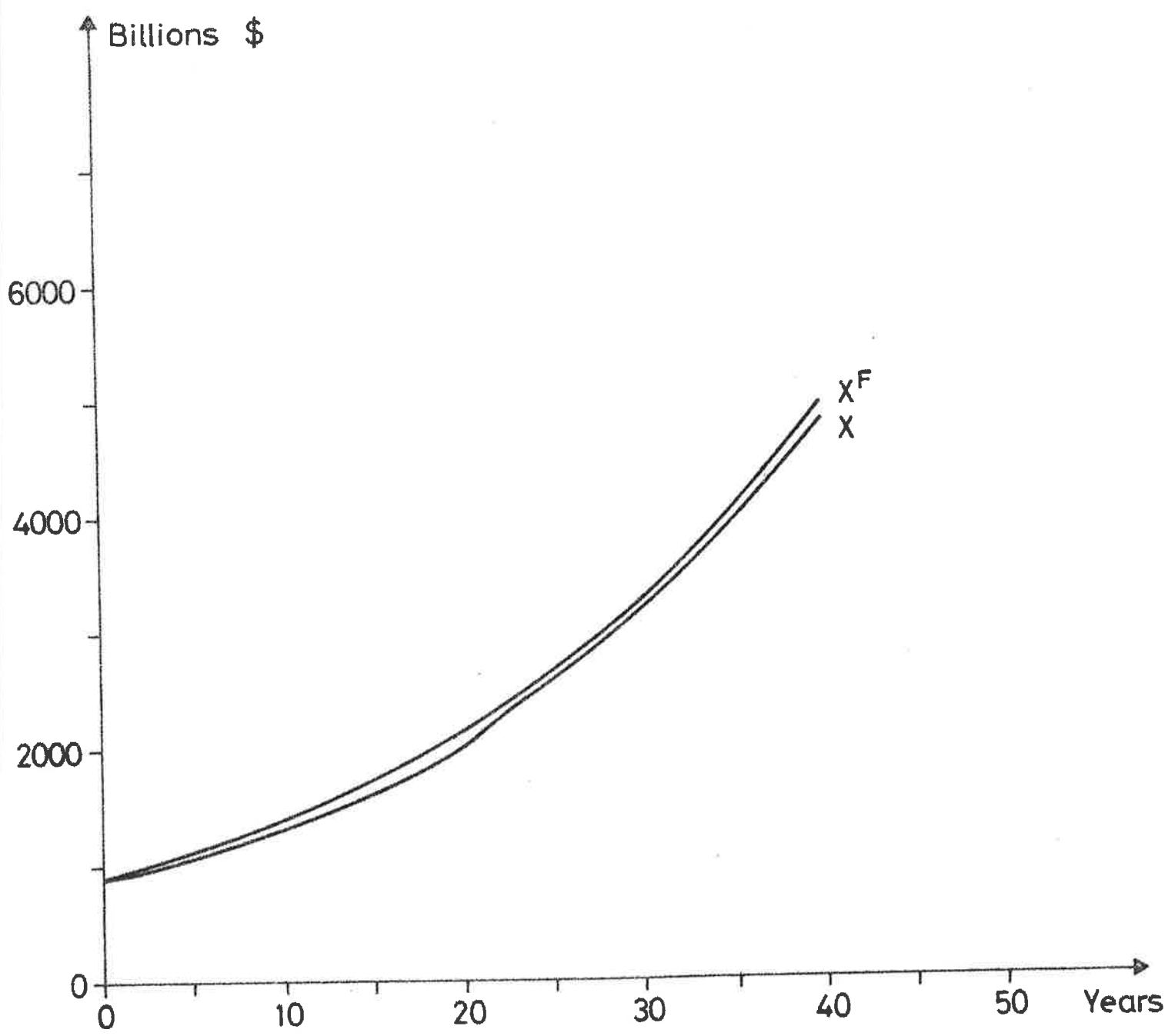


Fig. 4.4
 X^F (potential, full employment GNP, billions of dollars) and X (real GNP, billions of dollars) as functions of time when ΔM is increased 50% after 80 samples.

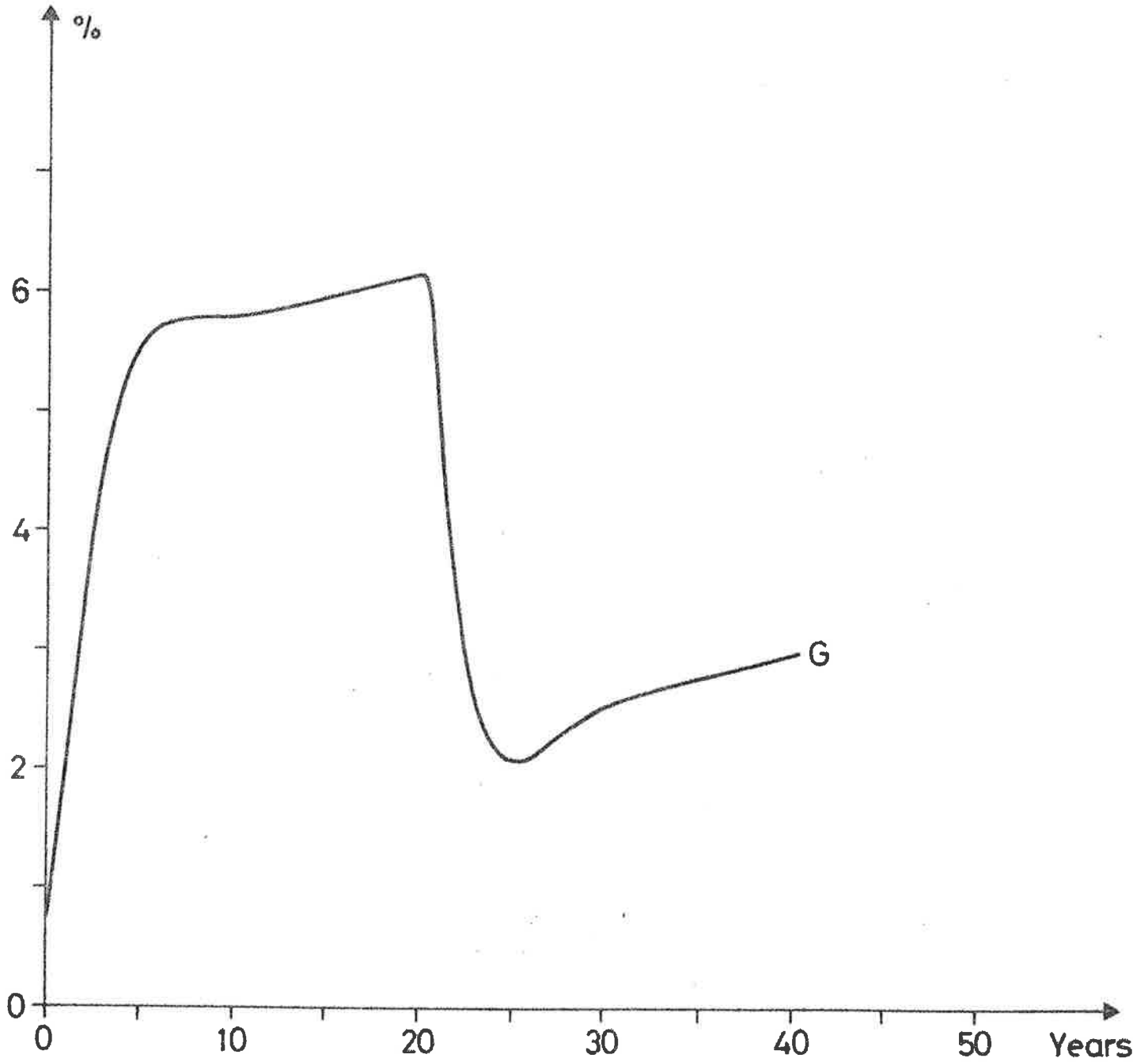


Fig. 4.5

$G (= 100(x^F - x)/x^F)$ as a function of time when ΔM is increased 50% after 80 samples.

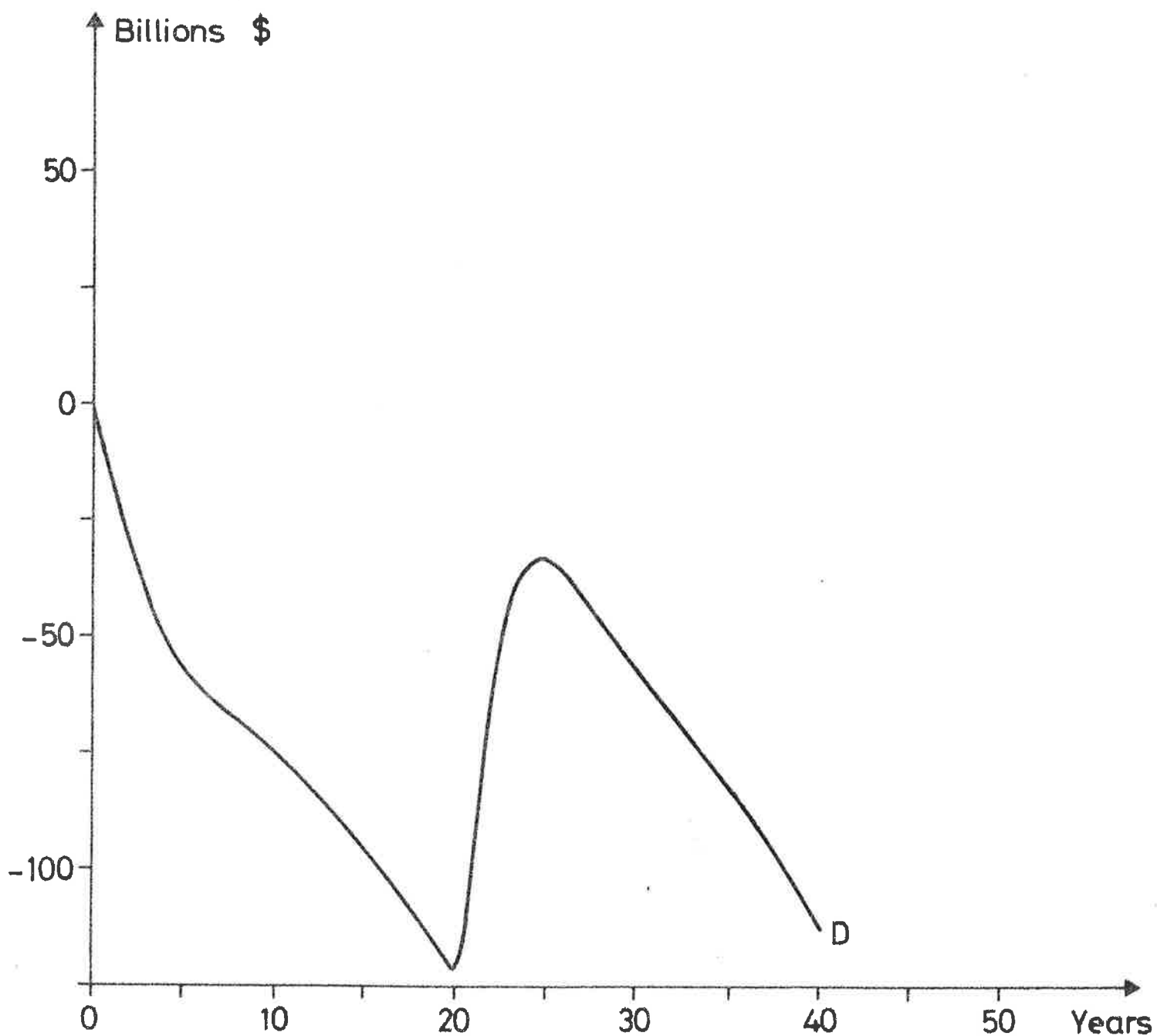


Fig. 4.6

Demand pressure, D , as a function of time when ΔM is increased 50% after 80 samples.

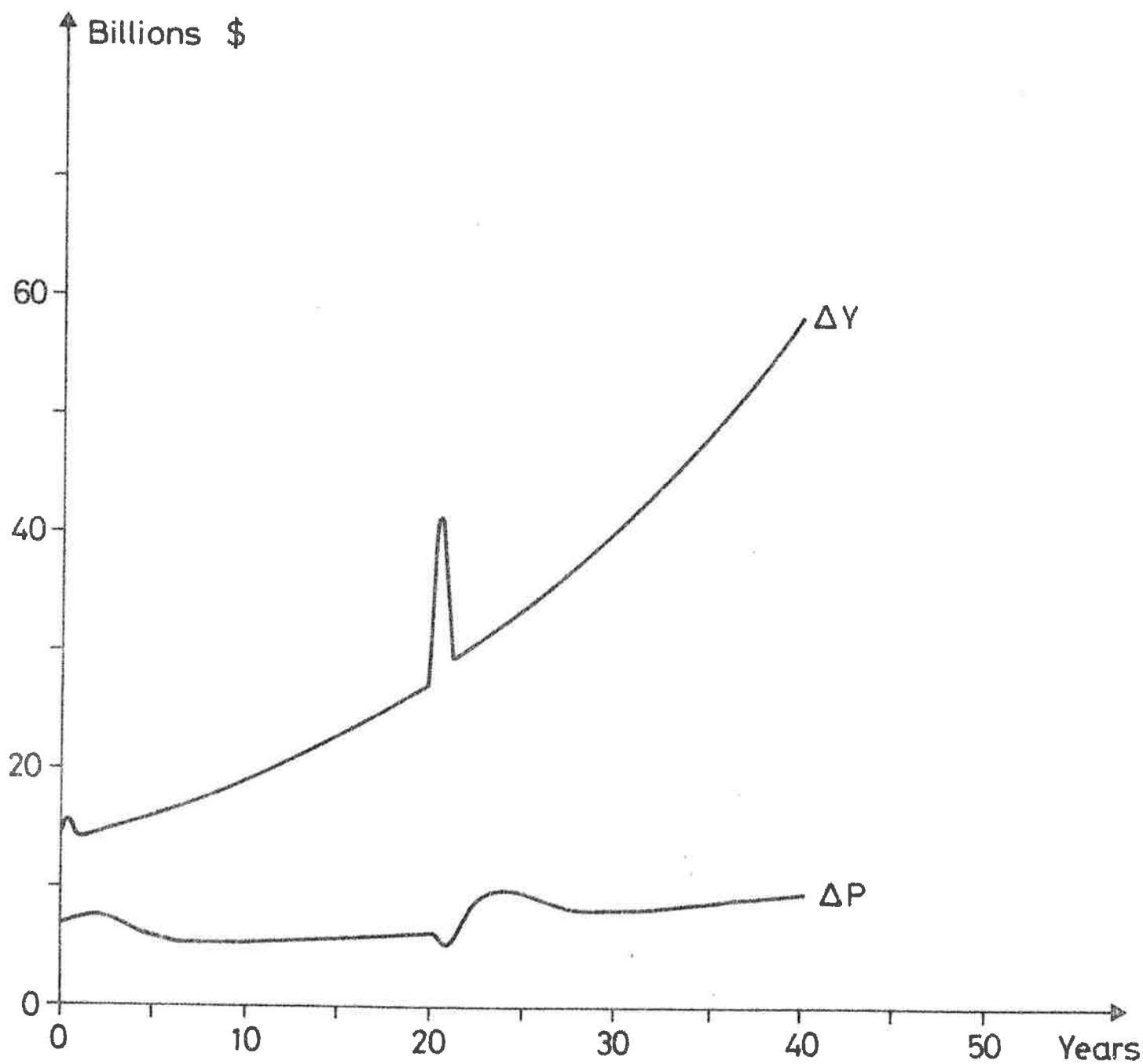


Fig. 4.7

ΔY (change in GNP in current prices, billions of dollars) and ΔP (the part of ΔY which is due to price changes, billions of dollars) as functions of time when ΔE is increased 300% after 80 samples (= 20 years).

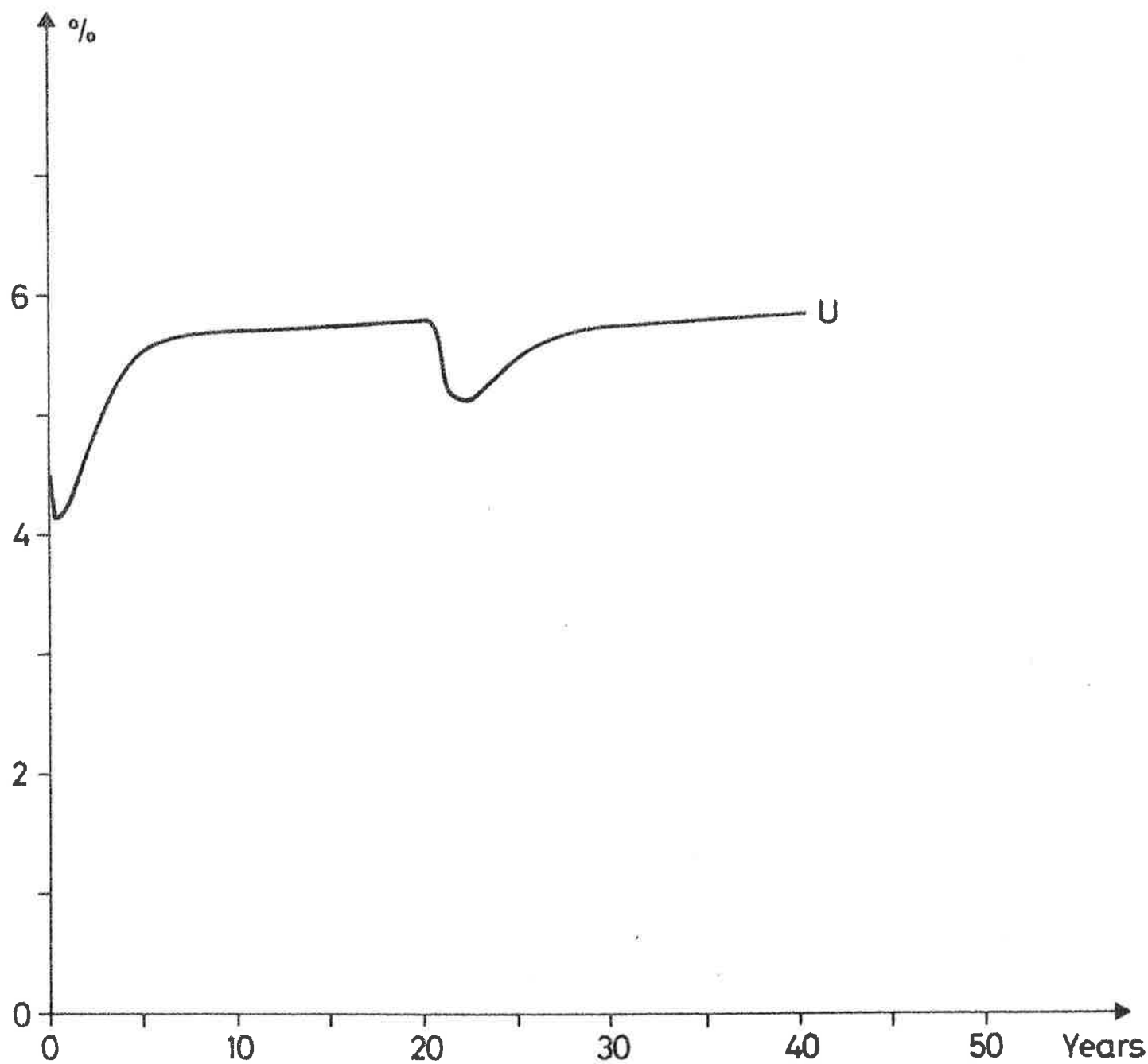


Fig. 4.8

Unemployment, U (As a per cent of labor force), as function of time when ΔE is increased 300% after 80 samples.

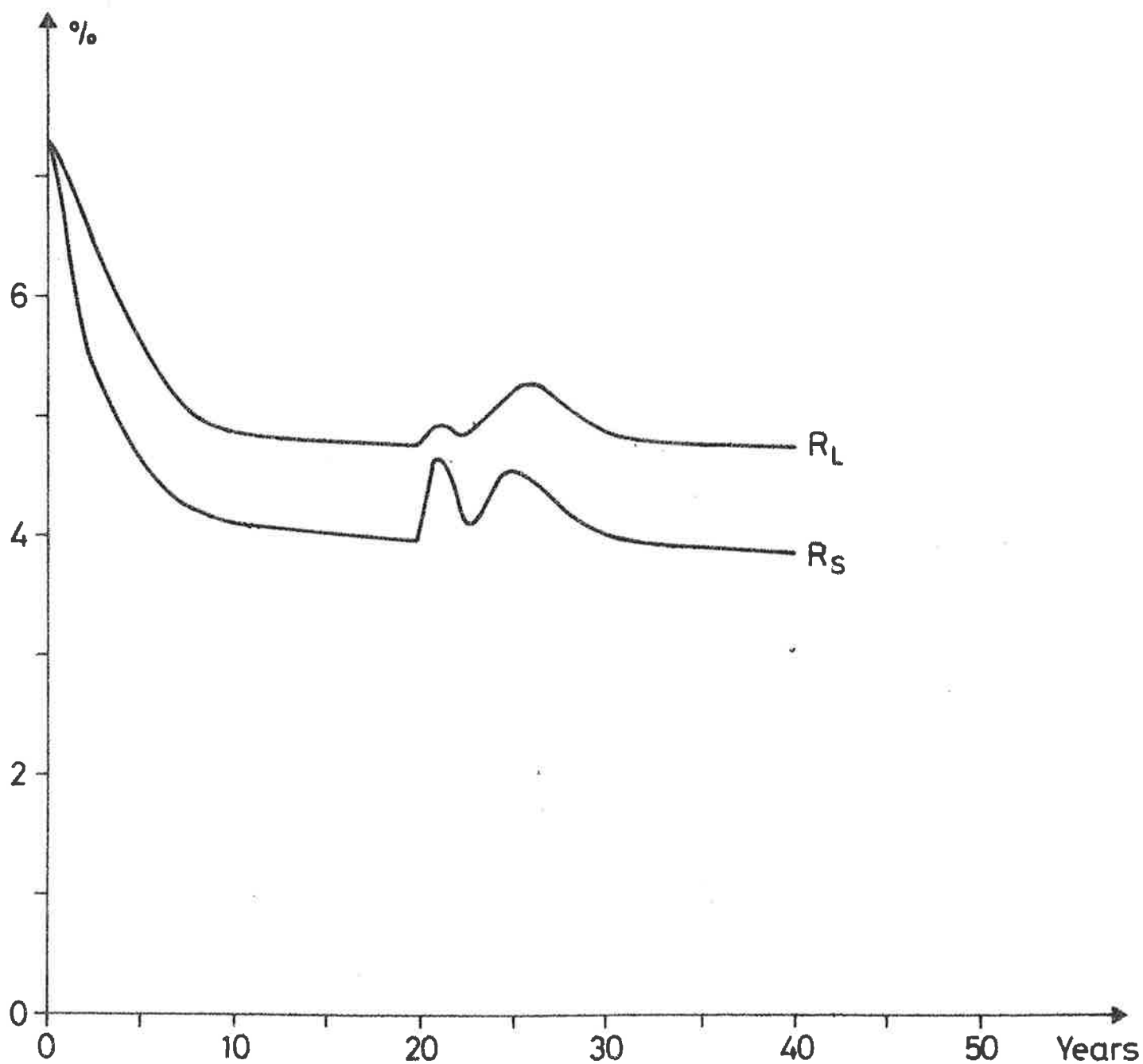


Fig. 4.9

Long term interest rate, R_L , and short term interest rate, R_S , as functions of time when ΔE is increased 300% after 80 samples.

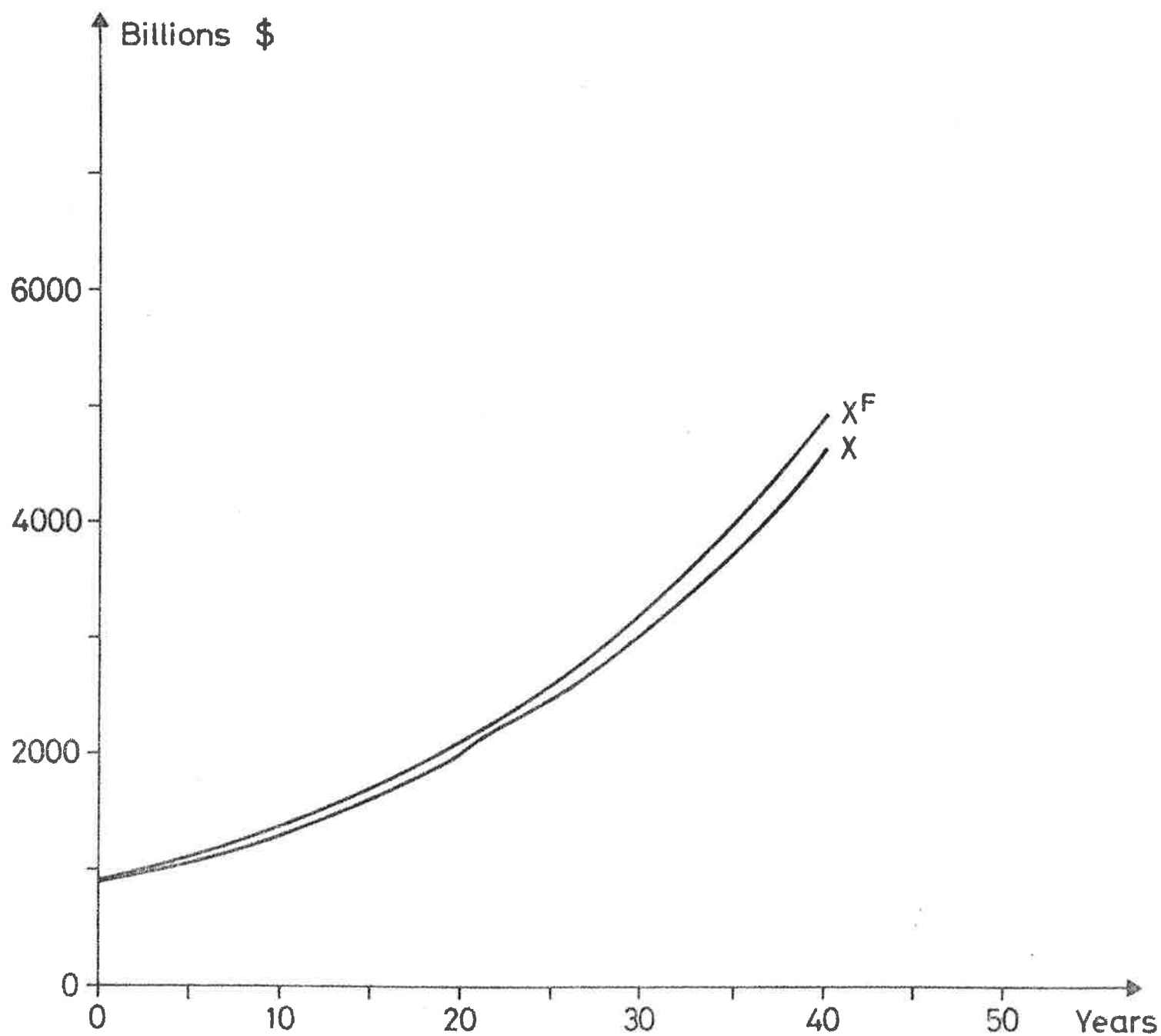


Fig. 4.10

X^F (potential, full employment GNP, billions of dollars) and X (real GNP, billions of dollars) as functions of time when ΔE is increased 300% after 80 samples.

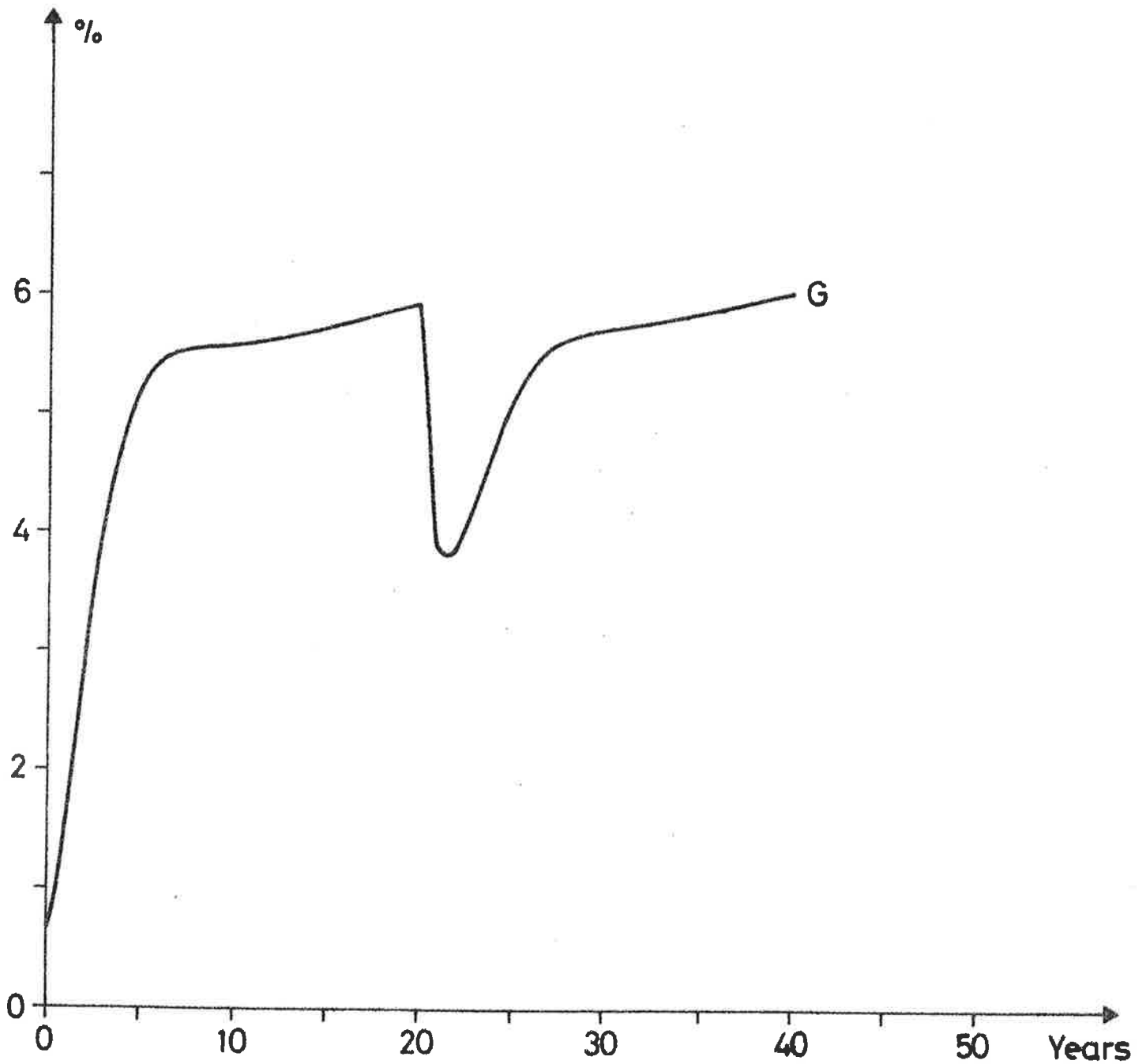


Fig. 4.11

$G (= 100(x^F - x)/x^F)$ as a function of time when ΔE is increased 300% after 80 samples.

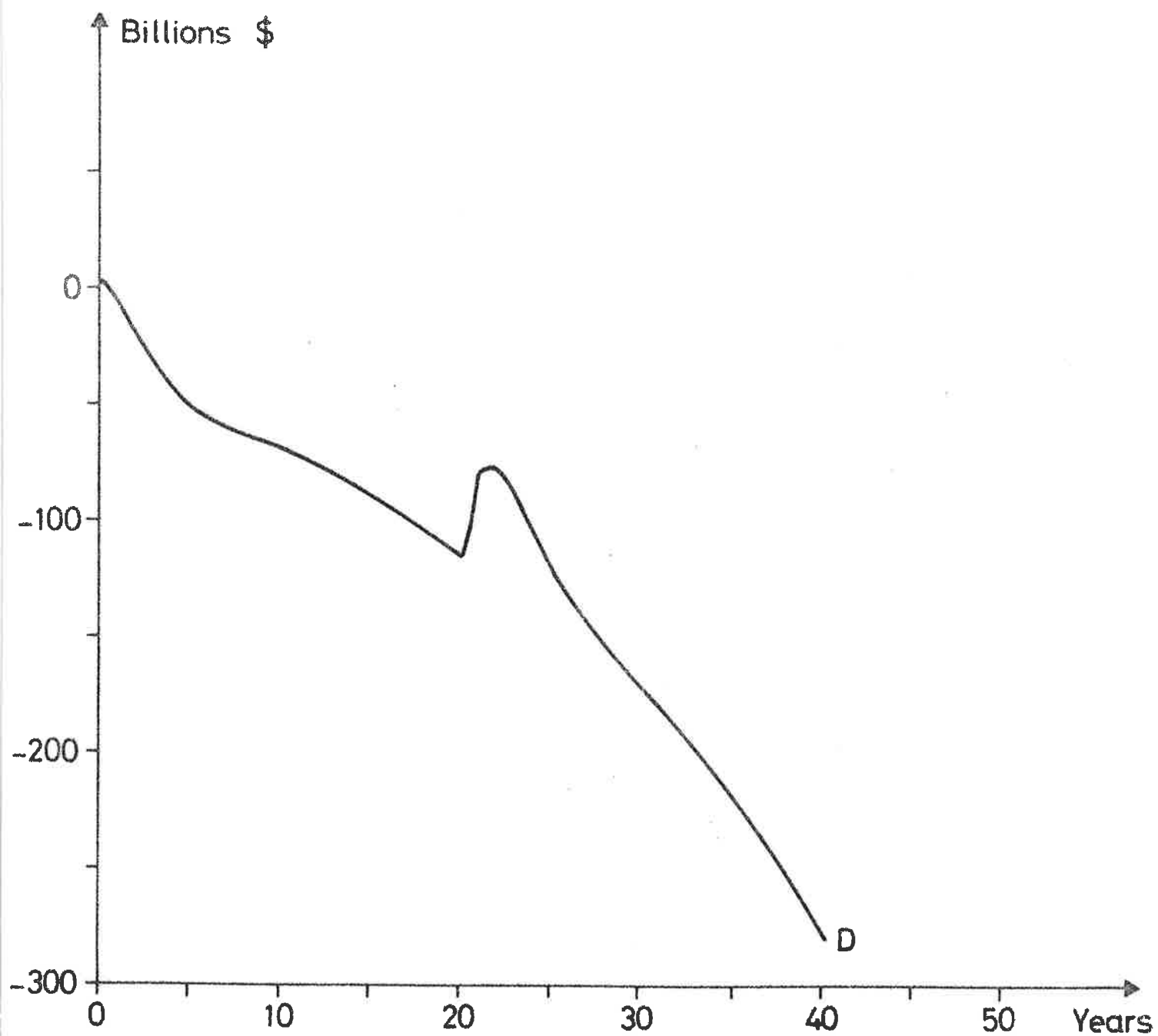


Fig. 4.12

Demand pressure, D, as a function of time when ΔE is increased 300% after 80 samples.

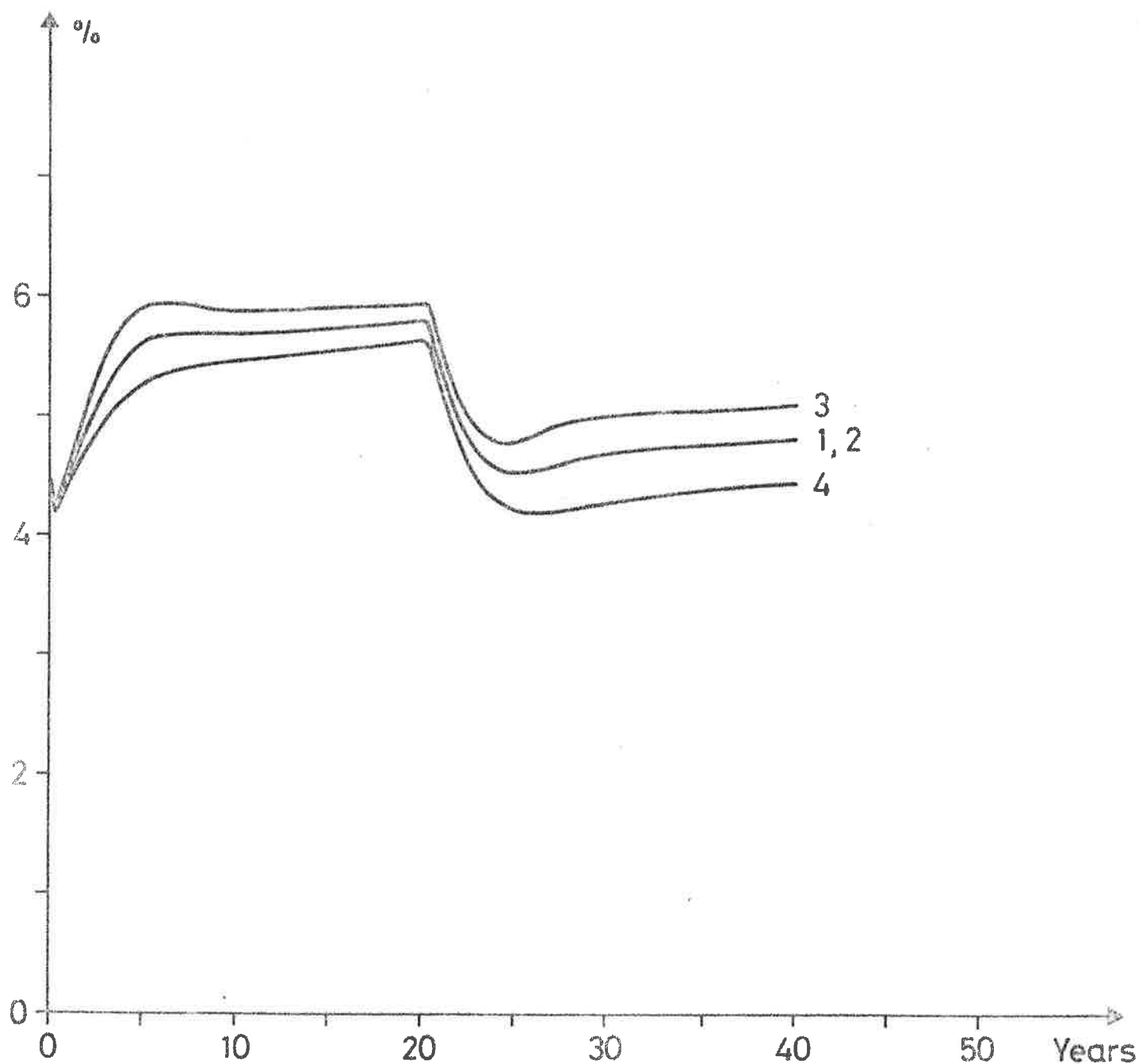


Fig. 4.13

Unemployment for various parameter values.

Curve 1: Nominal parameter values as given in Table XI, Appendix I. This curve is identical to Fig. 4.2.

Curve 2: Parameters d_2 and v_0 are increased by 50% from nominal values.

Curve 3: Parameters d_1 and v_1 are increased by 10%.

Curve 4: Parameters d_1 and v_1 are decreased by 10%.

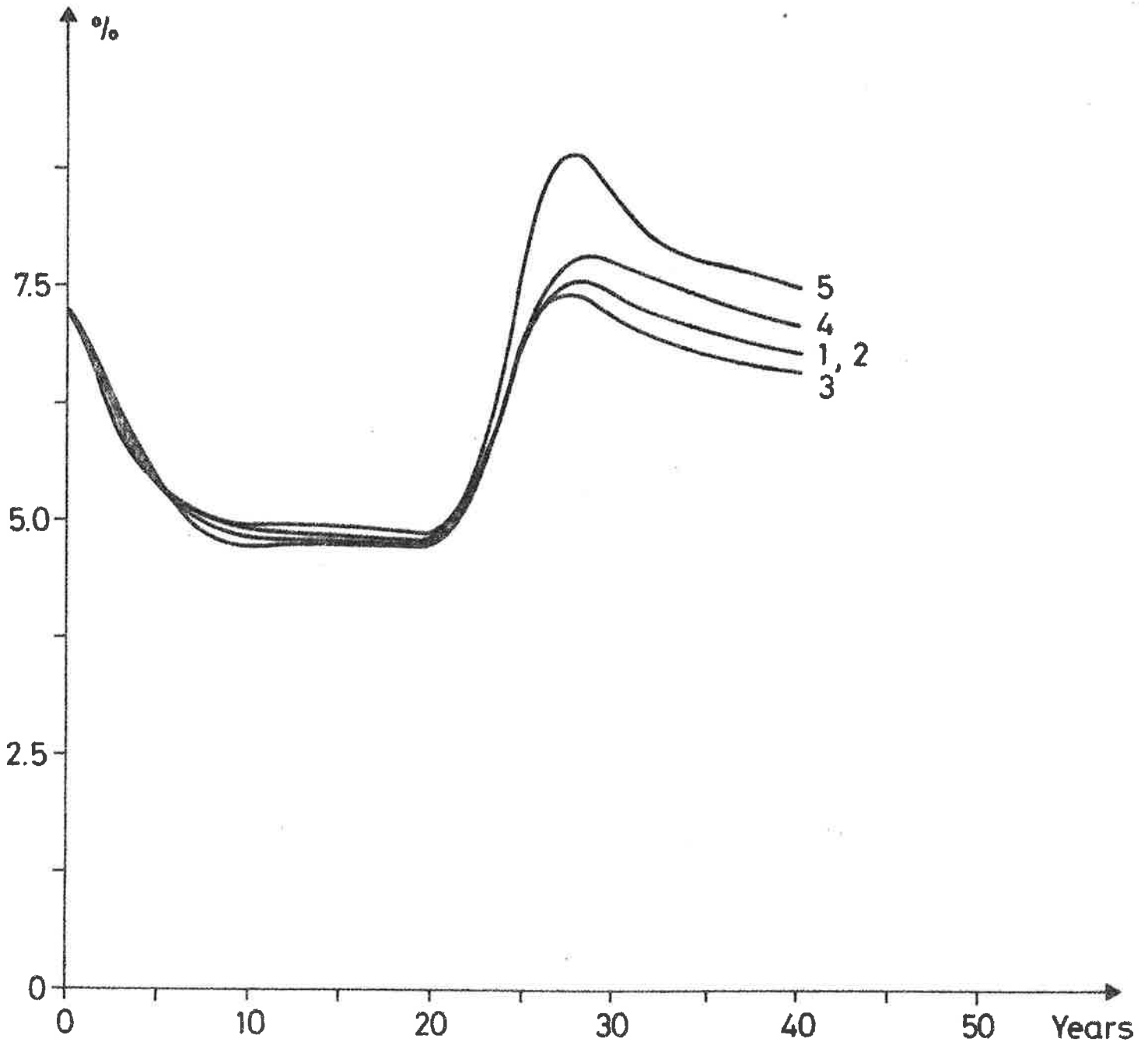


Fig. 4.14

Long term market interest rate for various parameter values.

Curve 1: Nominal parameter values as given in Table XI, Appendix I. This curve is also given in Fig. 4.3.

Curves 2-4: as in Fig. 4.13.

Curve 5: Parameters d_1 are increased by 25% from nominal values and parameters v_1 are decreased by 25%.

5. CONCLUSIONS.

Monetary policy has become the most important means for economic stabilization in economies with well developed financial markets. Previously fiscal actions dominated as stabilization instrument. In the model that has been discussed here GNP is depending on monetary stock and to some extent, on government spending. GNP, in its turn, influences e.g. output (real GNP) and employment in the short run, while only price level is affected in the long run.

The computer simulations gave a reasonable behaviour of the model in spite of some arbitrariness at the choice of initial values and magnitudes of step changes. Stability analysis showed that the open loop system was stable.

It is attractive to use macro-economic models in practice as basis for financial decisions. It seems most suitable to use a state space representation. Two variants of state space representations for the St Louis model have been presented here. It is then possible to study different control strategies for economic stabilization using standard theory of automatic control. Still some problems remain to be studied, for instance how monetary stock is influenced by different actions in monetary policy, e.g. changes in the rate of discount, operations in open market or changes in cash and liquidity limits for banks.

6. REFERENCES.

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- Andersen, L. C., and Carlson, K.M. (1970 b):
Supplementary Materials accompanying "A Monetarist Model for Economic Stabilization", Federal Reserve Bank of St Louis Review, April, 1970.
- Wall, K.D. (1972):
A Useful State Space Realisation for the Reduced Form, PREM Working Paper (KW 18), Nov., 1972.

APPENDIX I

A Monetarist Model for
Economic Stabilization

A Monetarist Model for Economic Stabilization*

by LEONALL C. ANDERSEN and KEITH M. CARLSON

THE MONETARIST VIEW that changes in the money stock are a primary determinant of changes in total spending, and should thereby be given major emphasis in economic stabilization programs, has been of growing interest in recent years. From the mid-1930's to the mid-1960's, monetary policy received little emphasis in economic stabilization policy. Presumed failure of monetary policy during the early years of the Great Depression, along with the development and general acceptance of Keynesian economics, resulted in a main emphasis on fiscal actions—Federal Government spending and taxing programs—in economic stabilization plans. Monetary policy, insofar as it received any attention, was generally expressed in terms of market rates of interest.

Growing recognition of the importance of money and other monetary aggregates in the determination of spending, output, and prices has been fostered by the apparent failure of stabilization policy to curb the inflation of the last half of the 1960's. Sharply rising market interest rates were interpreted to indicate significant monetary restraint, while the Revenue and Expenditure Control Act of 1968 was considered a major move toward fiscal restraint.

Despite these policy developments, total spending continued to rise rapidly until late 1969, and the rate of inflation accelerated. Those holding to the monetarist view were not surprised by this lack of success

in curbing excessive growth in total spending, largely because the money stock grew at a historically rapid rate during the four years ending in late 1968. Economic developments from 1965 through 1969 were in general agreement with the expectations of the monetarist view.

This article develops a model designed to analyze economic stabilization issues within a framework which focuses on the influence of monetary expansion on total spending. Most of the major econometric models have not assigned an important role to the money stock or to any other monetary aggregate.¹ Furthermore, most econometric models contain a large number of behavioral hypotheses to be empirically estimated and integrated with each other, because they are designed to aid in understanding the determination of many economic magnitudes. By comparison, the model presented in this article is quite small. It is designed to provide information on the most likely course of movement of certain strategic economic variables in response to monetary and fiscal actions.

*Offering helpful suggestions throughout the study were Denis Karnosky of this bank, William P. Yohe of Duke University and Visiting Scholar at this bank, 1969-70, and David Fand of Wayne State University. Susan Smith provided programming assistance and Christopher Babb and H. Albert Margolis advised on statistical problems. The authors thank the following for their comments on earlier drafts, without implying their endorsement of either the methods of analysis or the conclusions: F. Gerard Adams, Philip Cagan, E. Gerald Corrigan, Richard Davis, Ray Fair, Edgar Fiedler, Milton Friedman and members of the Money and Banking Workshop at the University of Chicago, Edward Gramlich, Harry G. Johnson, John Kalchbrenner, Edward Kane, Michael Keran, Allan Meltzer, Franco Modigliani, George Morrison, David Ott, Joel Popkin, Thomas Saving, Roger Spencer, Henry Wallich, Clark Warburton, Manfred Wilms, and Arnold Zellner.

¹Frank de Leeuw and Edward M. Gramlich, "The Federal Reserve-MIT Econometric Model," *Federal Reserve Bulletin* (January 1968), pp. 11-40, and "The Channels of Monetary Policy: A Further Report on the Federal Reserve MIT Econometric Model," *Federal Reserve Bulletin* (June 1969), pp. 472-91; James S. Duesenberry, Cary Fromm, Lawrence R. Klein, and Edwin Kuh (ed.), *The Brookings Quarterly Econometric Model of the United States* (Chicago: Rand McNally, 1965), and *The Brookings Model: Some Further Results* (Chicago: Rand McNally, 1969); Michael K. Evans and Lawrence R. Klein, *The Wharton Econometric Forecasting Model*, 2nd Enlarged Edition (Philadelphia: University of Pennsylvania, 1968); Maurice Liebenberg, Albert A. Hirsch, and Joel Popkin, "A Quarterly Econometric Model of the United States: A Progress Report," *Survey of Current Business* (May 1966), pp. 423-56; Daniel M. Suits, "The Economic Outlook for 1969," in *The Economic Outlook for 1969*, Papers presented to the Sixteenth Annual Conference on the Economic Outlook at The University of Michigan (Ann Arbor: University of Michigan, 1969), pp. 1-26. For a discussion of the role of money in these models, see David I. Fand, "The Monetary Theory of Nine Recent Quarterly Econometric Models of the United States," forthcoming in the *Journal of Money, Credit, and Banking*.

The model presented here is the authors' own version of how monetary and fiscal actions influence the economy. Other economists (including those of a monetarist persuasion) may prefer to develop certain aspects of the model in a different way. Two such modifications are presented in Appendix C. The model is considered open to revision, but is presented at this time with a view to stimulating others to join in quantifying relationships that are generally associated with the monetarist view.

This article is divided into five major sections. A general monetarist view of the response of spending, output, and prices to monetary and fiscal actions is summarized first. Next, the specific features of the model are discussed within a formal framework of analysis. Statistical estimates of the model's parameters are presented in the third section. The fourth section tests the performance of the model with several dynamic simulation experiments. Finally, by simulating the response of the economy to alternative rates of monetary expansion, an illustration is provided of how the model can be used for current stabilization analysis.

General Monetarist View

The general monetarist view is that the rate of monetary expansion is the main determinant of total spending, commonly measured by gross national product (GNP).² Changes in total spending, in turn, influence movements in output, employment, and the general price level. A basic premise of this analysis is that the economy is basically stable and not necessarily subject to recurring periods of severe recession and inflation. Major business cycle movements that have occurred in the past are attributed primarily to large swings in the rate of growth in the money stock.

This view regarding aggregate economic relationships differs from prevailing views which consider aggressive policy actions necessary to promote stability. Monetarists generally hold that fiscal actions, in the absence of accommodative monetary actions, exert little net influence on total spending and therefore have little influence on output and the price level. Government spending unaccompanied by accommodative monetary expansion, that is, financed by taxes or borrowing from the public, results in a crowding-out of private expenditures with little, if any,

net increase in total spending. A change in the money stock, on the other hand, exerts a strong independent influence on total spending. Monetarists conclude that actions of monetary authorities which result in changes in the money stock should be the main tool of economic stabilization. Since the economy is considered to be basically stable, and since most major business cycle movements in the past have resulted from inappropriate movements in the money stock, control of the rate of monetary expansion is the means by which economic instability can be minimized.

The theoretical heritage of the monetarist position is the quantity theory of money.³ This theory dates back to the classical economists (particularly David Ricardo) in the early 1800's. The quantity theory in its simplest form is characterized as a relationship between the stock of money and the price level. Classical economists concentrated on the long run aspects of the quantity theory in which changes in the money stock result in changes only in nominal magnitudes, like the price level, but have no influence on real magnitudes like output and employment.

The quantity theory of money in its modern form recognizes the important influence that changes in the money stock can have on real magnitudes in the short run, while influencing only the price level in the long run. The modern quantity theory postulates that in the short run a change in the rate of growth in money is followed with a moderate lag by changes in total spending and output, while changes in the price level follow with a somewhat longer lag.⁴ These changes in total spending, output, and prices are in the same direction as the change in the rate of monetary expansion.

The modern quantity theory still accepts the long-run postulates of its older version. A change in the rate of monetary expansion influences only nominal magnitudes in the long run, namely, total spending (GNP) and the price level. Real magnitudes, notably

²General references on the monetarist view are Karl Brunner, "The Role of Money and Monetary Policy," this *Review* (July 1968), pp. 9-24; David L. Fand, "Some Issues in Monetary Economics," this *Review* (January 1970), pp. 10-27, and "A Monetarist Model of the Monetary Process," forthcoming in the *Journal of Finance*.

³The classic work on the quantity theory is Irving Fisher, *The Purchasing Power of Money* (New York: Macmillan, 1911). For an extensive review of the quantity theory literature, see Arthur W. Marget, *The Theory of Prices: A Re-examination of the Central Problems of Monetary Theory* (New York: Prentice-Hall, 1938), volume II, pp. 3-133.

⁴Many of the ideas prevalent in current monetarist doctrine can be found in the writings of Clark Warburton in the 1940's and early 1950's. Many of his important articles have been reprinted in his *Depression, Inflation, and Monetary Policy, Selected Papers, 1945-1953* (Baltimore: The Johns Hopkins Press, 1966). See also Milton Friedman, (ed.), *Studies in the Quantity Theory of Money* (Chicago: University of Chicago Press, 1958), and Lloyd W. Mints, *Monetary Policy in a Competitive Society* (New York: McGraw-Hill, 1951).

output and employment, are unaffected.⁵ Following the short-run responses to a change in the rate of monetary growth, total spending and the price level grow at rates determined by the rate of increase in money, while output moves toward and resumes a long-run growth path. Such growth in output is little influenced by the rate of monetary expansion. Instead, it is determined by growth in the economy's productive potential, which depends on growth of natural resources, capital stock, labor force, and productivity.

General Form of the Model

A summary of the model is presented in algebraic form in Exhibit I, along with a listing of variables classified as to whether they are endogenous or exogenous to the model (for a graphical illustration of the model, see Appendix B). This general form of the model summarizes its essential features, ignoring problems of dimensionality and lag length.

Equations of the Model

Equation (1) is the total spending equation. The change in total spending (ΔY) is specified as a function of current and past changes in the money stock (ΔM) and current and past changes in high-employment Federal expenditures (ΔE). This general specification represents the reduced form for that class of structures which has ΔM and ΔE as exogenous variables. In this form the total spending equation remains uncommitted as to structure; it is potentially consistent with both Keynesian and quantity theory models.⁶ (The magnitude and significance of the estimated parameters determine whether the data conform more closely to a Keynesian or a quantity theory).

Equation (2) specifies the change in the price level (ΔP) as a function of current and past demand pressures (D) and anticipated price changes (ΔP^A). Demand pressure is defined in equation (3) as the change in total spending minus the potential increase in output ($X^P - X$). The price equation is an alternative to the standard short-run Phillips curve relation generalized to include changes in total spending and anticipated prices.⁷ (See Appendix A for further development of this relationship.)

Equation (4) defines a change in total spending in terms of its components, the part associated with changes in the price level (ΔP) and the part associated with changes in output (ΔX).⁸ With ΔY determined by equation (1), and ΔP by equation (2), ΔX can be derived from equation (4).

Equation (5) specifies the market rate of interest (R) as a function of current changes in the money stock (ΔM), current and past changes in output

⁷See Roger W. Spencer, "The Relation Between Prices and Employment: Two Views," this *Review* (March 1969), pp. 15-21.

⁸The change in the price level, ΔP , and the change in output, ΔX , are defined in dollar units so that their sum is equal to the change in total spending, ΔY .

⁵See Milton Friedman, "The Role of Monetary Policy," *American Economic Review* (March 1968), pp. 1-17.

⁶For further discussion of the structural versus the reduced form of a model, see Michael Keran, "Monetary and Fiscal Influences on Economic Activity - The Historical Evidence," this *Review* (November 1969), pp. 5-24; Edward M. Gramlich, "The Usefulness of Monetary and Fiscal Policy as Discretionary Stabilization Tools," forthcoming in the *Journal of Money, Credit and Banking*; and Richard C. Davis, "How Much Does Money Matter? A Look at Some Recent Evidence," Federal Reserve Bank of New York, *Monthly Review* (June 1969), pp. 119-31.

Exhibit I

MODEL IN ALGEBRAIC FORM

- (1) Total Spending Equation
 $\Delta Y_t = f_1(\Delta M_t \dots \Delta M_{t-n}, \Delta E_t \dots \Delta E_{t-n})$
- (2) Price Equation
 $\Delta P_t = f_2(D_t \dots D_{t-n}, \Delta P_t^A)$
- (3) Demand Pressure Identity
 $D_t = \Delta Y_t - (X_t^P - X_{t-1})$
- (4) Total Spending Identity
 $\Delta Y_t = \Delta P_t + \Delta X_t$
- (5) Interest Rate Equation
 $R_t = f_3(\Delta M_t, \Delta X_t \dots \Delta X_{t-n}, \Delta P_t, \Delta P_t^A)$
- (6) Anticipated Price Equation
 $\Delta P_t^A = f_4(\Delta P_{t-1} \dots \Delta P_{t-n})$
- (7) Unemployment Rate Equation
 $U_t = f_5(G_t, G_{t-1})$
- (8) GNP Gap Identity
 $G_t = \frac{X_t^P - X_t}{X_t^P}$

Endogenous Variables	Exogenous Variables*
ΔY_t = change in total spending (nominal GNP)	ΔM_t = change in money stock
ΔP_t = change in price level (GNP price deflator)	ΔE_t = change in high-employment Federal expenditures
D_t = demand pressure	X_t^P = potential (full-employment) output
ΔX_t = change in output (real GNP)	
R_t = market interest rate	
ΔP_t^A = anticipated change in price level	
U_t = unemployment rate	
G_t = GNP gap	

*Other than lagged variables.

(ΔX), current price change (ΔP), and anticipated price change (ΔP^A). The price anticipations term is included to capture the Fisher effect.⁹ The anticipated price function is defined in equation (6). Anticipated price change (ΔP^A) in the current period is assumed to depend on past price changes (ΔP).

Equation (7) is the unemployment rate equation and is a transformation of the GNP gap (G), as defined in equation (8), into a measure of unemployment relative to the labor force. This transformation is based on "Okun's Law."¹⁰

Workings of the Model

The workings of the model are summarized by a flow diagram (Exhibit II). Only variables in the current period are shown in the diagram; lagged variables, with the exception of past changes in prices, are omitted. The relationship that determines total spending is the fundamental one among those that determine the endogenous variables of the model. Total spending is determined by monetary actions and fiscal actions (Federal spending financed by taxes or borrowing from the public), though no direct information is provided as to *how* such actions affect spending.

The change in total spending is combined with potential (full employment) output to provide a measure of demand pressure. Anticipated price change, which depends on past price changes, is combined with demand pressure to determine the change in the price level.

The total spending identity enables the change in output to be determined, given the change in total spending

⁹For a detailed study of interest rates and the Fisher effect, see William P. Yohe and Denis S. Karnosky, "Interest Rates and Price Level Changes, 1952-69," this *Review* (December 1969), pp. 18-38.

¹⁰Arthur M. Okun, "Potential GNP: Its Measurement and Significance," 1962 *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*, pp. 98-104. Okun's Law relates the GNP gap to the unemployment rate as follows:

$$X^P_t - X_t = .03(U_t - 4)X_t.$$

The number .03 is a productivity factor and 4 is defined as the unemployment rate consistent with full resource utilization.

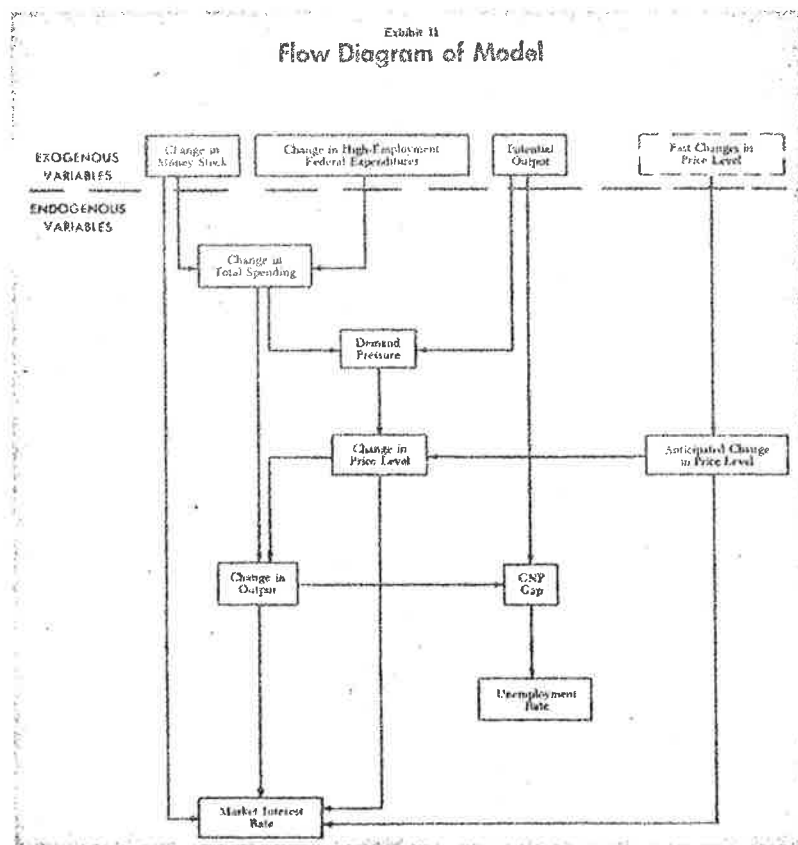
and the change in prices. This method of determining the change in total spending and its division between output change and price change differs from most econometric models. A standard practice in econometric model building is to determine output and prices separately, then combine them to determine total spending.

The change in output, the change in prices and in anticipated prices, along with the change in the money stock, determine market interest rates. The flow diagram shows that the market interest rate does not exercise a direct role in the model in the determination of spending, output, and prices.

To determine the unemployment rate, the change in output is first combined with potential output to determine the GNP gap relative to potential output. The GNP gap is then transformed into the unemployment rate.

Summary

The model has been presented in general form to show the basic linkages postulated among money, Federal expenditures, prices, and output. The purpose of the following statistical section is to estimate



the response of output and prices to monetary and fiscal actions, not to test a hypothesized structure. The focus is on the response in the short run — periods of two or three years — but the long-run properties of the model also are examined.

Estimation of the Model

The general form of the model indicates those variables that are included in each equation. Estimation requires selection of the algebraic form of the equations and the techniques to be used in estimation.

Each of the equations of the model is estimated by ordinary least squares. Lag structures, with one exception, are estimated by the Almon lag technique. The reported relationships reflect considerable experimentation with the number of lags and the degree of the polynomial.¹¹ The sample period starts with 1953 for the spending equation and with 1955 for all the others. The data are quarterly and, with the exception of interest rates, are seasonally adjusted.

Criteria used in the selection of the equations were minimizing the standard error of estimate and eliminating serial correlation in the estimated residuals. In addition, the signs and statistical significance of the estimated coefficients received consideration, along with the pattern of the lag distribution. Since these criteria frequently could not be satisfied simultaneously, an element of subjectivity was present in selecting the "best" equation.

Total Spending

The change in total spending is specified as a function of current and past changes in the money stock (demand deposits and currency held by the nonbank public) and in high-employment Federal expenditures (expenditures on goods and services plus transfer payments adjusted to remove the influence of variations in economic activity on unemployment benefit payments). The choice of the particular equation (Table I) is based on previous work by Andersen and Jordan.¹² Implicit in this choice is the assumption that the change in the money stock is an exogenous variable. A more complete model would specify a mechanism whereby the money stock is determined by actions of the monetary authorities, the public, and the banking system.

¹¹For discussion of the use and interpretation of the Almon lag technique, see Keran, p. 10.

¹²Leonall C. Andersen and Jerry L. Jordan, "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization," this *Review* (November 1968), pp. 11-24. See also Keran, pp. 5-24.

Table I

TOTAL SPENDING EQUATION

Sample Period: I/1953 - IV/1969

Constraints: 4th Degree Polynomial
($e_{-1} = e_{-2} = 0$; $m_5 = e_5 = 0$)

$$\Delta Y_t = 2.67 + \sum_{i=0}^4 m_i \Delta M_{t-i} + \sum_{i=0}^4 e_i \Delta E_{t-i}$$

(3.46)

$m_0 = 1.22$ (2.73)	$e_0 = .56$ (2.57)	$R^2 = .66$
$m_1 = 1.80$ (7.34)	$e_1 = .45$ (3.43)	S.E. = 3.84
$m_2 = 1.62$ (4.25)	$e_2 = .01$ (.05)	D-W = 1.75
$m_3 = .87$ (3.65)	$e_3 = -.43$ (-3.18)	
$m_4 = .06$ (.12)	$e_4 = -.54$ (-2.47)	
$\Sigma m_i = 5.57$ (8.06)	$\Sigma e_i = .05$ (.17)	

Symbols are defined as:

ΔY_t = dollar change in total spending (GNP in current prices) in quarter t

ΔM_{t-i} = dollar change in money stock in quarter $t-i$

ΔE_{t-i} = dollar change in high-employment Federal expenditures in quarter $t-i$

Note: "t" statistics appear with each regression coefficient, enclosed by parentheses. R^2 is the percent of variation in the dependent variable which is explained by variations in the independent variables. S.E. is the standard error of the estimate. D-W is the Durbin-Watson statistic.

The pattern of the coefficients indicates a large and rapid influence of monetary actions on total spending relative to that of fiscal actions.¹³ Changes in high-employment expenditures, with the money stock held constant, first have a positive influence on total spending, but the influence becomes significantly negative after three quarters. Fiscal actions, unaccompanied by changes in money, have little net effect on GNP over five quarters.¹⁴ For short periods, and for extended periods in which the rate of change of Federal expenditures is either accelerating or decelerating, fiscal effects are significant. The estimated coefficients for changes in money and changes in Federal expenditures are in general agreement with the monetarist view of the response of total spending to these two variables.

The specification of the total spending equation, as shown in Table I, has been criticized as being incom-

¹³Andersen and Jordan tried several measures of fiscal actions in their basic equation. The best results were obtained by using only high-employment expenditures, rather than the high-employment surplus or both high-employment expenditures and receipts. They justify their choice by appealing to the notion that financing expenditures by borrowing from the public and taxes have essentially the same impact on total GNP. For some results that contradict those of Andersen and Jordan, see E. C. Corrigan, "The Measurement and Relative Importance of Fiscal Policy," forthcoming in *Federal Reserve Bank of New York Monthly Review*. It should be repeated that, *a priori*, specification of the total spending equation was sufficiently general as to be consistent with a number of theories of GNP determination.

¹⁴Andersen and Jordan, p. 18, indicate that these results are consistent with a "crowding-out" theory of effects of government spending.

plete in that it allegedly ignores the effects of interest rates on velocity.¹⁵ However, since the spending equation is a reduced form, such effects are embodied in the coefficients of money.¹⁶

Prices and Output

The total spending equation is the cornerstone of the model, providing its monetarist character. The focus of this paper, however, is on determining the division of the change in total spending between price and output changes. Price changes are estimated as a function of (1) current and past demand pressure, and (2) anticipated price change.

Demand pressure — As a measure of demand pressure on prices, the change in total spending is related to the potential change in output (GNP in constant prices).¹⁷ These two variables, when combined, provide a measure of the economy's demand for goods and services relative to its capacity to supply goods and services. The change in prices is specified as a positively related linear function of this measure of demand pressure (see Appendix A).

Demand pressure, D_t , is defined as:

$$D_t = \Delta Y_t - (X_t^f - X_{t-1}),$$

where ΔY_t is the change in total spending in quarter t ; X_t^f is potential (full employment) GNP in 1958 prices in quarter t ; and X_{t-1} is real GNP in the previous quarter.¹⁸ Given the GNP gap, defined as $X_t^f - X_{t-1}$, the larger is the change in total spending

(ΔY_t), the greater is the spillover into higher prices. Given ΔY_t , the larger is $X_t^f - X_{t-1}$, the greater is the expansion of output and the less the spillover into higher prices.

In addition to current values, past values of the demand pressure variable are included in the price equation. The purpose of including past values is to allow for lags in the determination of prices in response to changing demand. Furthermore, the impact of changing demand through changing input prices and costs of production is given a chance to operate by including lagged values for the demand pressure variable.

Anticipated Price Change — The other independent variable included in the price equation is a measure of anticipated price change (ΔP_t^A). The purpose of including this variable as a factor influencing current changes in the price level is to allow anticipations of future price movements to influence the decisions of market participants. Since such a variable is not

Table II

ANTICIPATED PRICE DEFINITION

(p_i from Long-Term Interest Rate Equation)

$$\Delta P_t^A = Y_{t-2} \left\{ \left[\sum_{i=1}^{17} p_i \frac{\dot{P}_{t-i}}{U_{t-i/4}} \right] .01 + 1 \right\}^{\frac{1}{4}} - 1$$

$p_1 = .02$	$p_7 = .08$	$p_{13} = .06$
$p_2 = .03$	$p_8 = .08$	$p_{14} = .06$
$p_3 = .04$	$p_9 = .08$	$p_{15} = .04$
$p_4 = .06$	$p_{10} = .08$	$p_{16} = .03$
$p_5 = .06$	$p_{11} = .08$	$p_{17} = .02$
$p_6 = .07$	$p_{12} = .07$	$\Sigma p_i = .96$

Symbols are defined as:

ΔP_t^A = anticipated price change (scaled in dollar units) in quarter t

\dot{P}_{t-1} = annual rate of change in GNP deflator (1958 = 100) in quarter $t-1$

$U_{t-1/4}$ = index of unemployment as a per cent of labor force (base = 4.0) in quarter $t-1$

Y_{t-2} = total spending (GNP in current prices) in quarter $t-2$

¹⁵See Paul S. Anderson, "Monetary Velocity in Empirical Analysis," in *Controlling Monetary Aggregates*, Proceedings of the Monetary Conference held on Nantucket Island (June 1969), pp. 37-51, and the discussion of that paper by Leonall C. Andersen, pp. 52-55. See also Henry A. Latané, "A Note on Monetary Policy, Interest Rates and Income Velocity," *Southern Economic Journal* (January 1970), pp. 328-30.

¹⁶See A. A. Walters, "Monetary Multipliers in the U. K.: 1880-1962," *Oxford Economic Papers* (November 1966):

¹⁷This measure was apparently first used by Ray Fair of Princeton University. See his "The Determination of Aggregate Price Changes," forthcoming in the *Journal of Political Economy*. For a similar specification of a price equation, see Milton Friedman, "A Theoretical Framework for Monetary Analysis," also forthcoming in the *Journal of Political Economy*. See also a paper by William Conditine of Stanford University, "Public Policy and the Current Inflation," prepared as a part of a summer intern program at the U.S. Treasury Department (September 5, 1969).

¹⁸The series on potential output is based on that used by the Council of Economic Advisers. Currently, potential output is estimated to be rising at a 4.3 per cent annual rate. For alternative estimates of potential output, see Fair, "The Determination of Aggregate Price Changes."

observable, it has to be constructed. This is accomplished by assuming that anticipations about future price changes are formed on the basis of past price experience.

The measure of price anticipations used in this study is a by-product of estimating long-term market interest rates.¹⁹ Yoho and Karnosky showed that long-term market interest rates respond to price anticipations of borrowers and lenders, since commit-

¹⁹For other ways of handling expectations, see Appendix C on alternative price equations.

ments to borrow and lend funds require an assessment of anticipated changes in the price level for the period of the loan. The problem consists of isolating this price effect on market interest rates from factors influencing the real rate.

In the process of constructing a measure of anticipated price change, past changes in prices are adjusted by a summary measure of current economic conditions. Since price changes tend to lag changes in total spending, the degree of resource utilization as measured by the unemployment rate is used as a leading indicator of future price movements.²⁰ For example, if unemployment is rising relative to the labor force, decision-making economic units would tend to discount current inflation in forming anticipations about future price movements. Reflecting this consideration, the price change in each quarter is divided by an index of the unemployment rate applicable to that quarter. Thus the measure of price anticipations would be less for a given inflation rate accompanied by high or rising unemployment than when unemployment is low or falling.

The specific definition of price anticipations is shown in Table II. The weights and the length of the lag period were obtained from the estimated long-term interest rate equation.²¹

Estimated price equation—The estimated price equation is shown in Table III, where ΔP_t is defined as the dollar change in total spending due to price changes in quarter t . The influence on prices of the demand pressure variable, D_{t-1} , is significant and positive for five quarters but very small thereafter.²² The pattern of influence is one of steady decay, with 70 per cent of the total effect of demand pressure taking place in the first three quarters and 95 per cent in the first five quarters.

Anticipated price change, represented by ΔP^A_t , is a significant determinant of current price change.

²⁰For purposes of exposition the unemployment rate was not included in the definition of anticipated price change in Exhibits I and II.

²¹The price expectations variable as shown in Table II is scaled in dollar units. This transformation is made because prices are estimated as the dollar change in total spending due to price changes.

²²When the price equation is estimated with the components of D_{t-1} separated, the coefficients for the ΔY_t portion are not statistically significant at the five per cent level, implying that the gap portion, $(X^p_t - X_{t-1})$, explains most of the changes in ΔP_t . However, there may be collinearity problems which influence the estimated coefficients. Furthermore, the D_{t-1} form is used because, theoretically, it is a measure of excess demand (see Appendix A).

Table III

PRICE EQUATION

Sample Period: I/1955 - IV/1969

Constraints: 2nd Degree Polynomial

$$(d_{-1} \neq 0; d_6 = 0)$$

$$\Delta P_t = 2.70 + \sum_{i=0}^5 d_i D_{t-i} + .86 \Delta P^A_t \quad R^2 = .87$$

(7.07) (8.55)

$d_0 = .02$ (2.63)	$d_4 = .01$ (1.86)
$d_1 = .02$ (6.33)	$d_5 = .$ (1.38)
$d_2 = .02$ (6.63)	$\Sigma d_i = .09$ (9.18)
$d_3 = .01$ (2.93)	

S.E. = 1.07
D-W = 1.41

Symbols are defined as:

ΔP_t = dollar change in total spending (GNP in current prices) due to price change in quarter t

$D_t = \Delta Y_t - (X^p_t - X_{t-1})$

ΔY_t = dollar change in total spending (GNP in current prices) in quarter t

X^p_t = potential output in quarter t

X_{t-1} = output (GNP in 1958 prices) in quarter $t-1$

ΔP^A_t = anticipated price change (scaled in dollar units) in quarter t

Note: "t" statistics appear with each regression coefficient, enclosed by parentheses. R^2 is the percent of variation in the dependent variable which is explained by variations in the independent variables. S.E. is the standard error of the estimate. D-W is the Durbin-Watson statistic.

*Less than .01

Though significant, the measure of the impact of this variable should not be taken too literally, because its construction indicates that it cannot really be viewed independently of the demand pressure variable.²³ The influence of these two variables should perhaps be viewed in combination, rather than as independent and separate influences.²⁴

Determination of output—Given ΔY_t as determined by the total spending equation, and ΔP_t from the price equation, the dollar change in total spending due to output changes, defined as ΔX_t , can be derived from the following identity:

$$\Delta Y_t = \Delta P_t + \Delta X_t + (P_t - P_{t-1})(X_t - X_{t-1}).$$

The cross product term is assumed equal to zero.²⁵ Thus,

$$\Delta X_t = \Delta Y_t - \Delta P_t.$$

²³From the standpoint of the model as a unit, price anticipations are important only in determining the division of total spending between prices and output, not the level (or change) of spending itself. To allow for the possible direct influence of price expectations on total spending, the spending equation was estimated with the price anticipations variable. The coefficient of the price anticipations variable was not significant for this specification.

²⁴There is, however, some evidence that the price anticipations variable may be interpreted as an independent and separate influence. When the price equation is estimated without ΔP^A_t , the sum of the coefficients on D_{t-1} is only slightly more than shown in Table III and the standard error is increased considerably.

²⁵The value of this cross-product term was calculated from 1953 to the present and provides ample justification for

Unemployment Rate

The unemployment rate plays a role in the model, representing the means by which past prices are adjusted to take into account varying economic conditions in the formation of anticipated price changes. To estimate the unemployment rate, the unemployment rate is regressed on current and lagged values of the GNP gap, expressed as a per cent of potential GNP. This equation is estimated by unconstrained ordinary least squares, and is shown in Table IV.

Table IV

UNEMPLOYMENT RATE EQUATION

Sample Period: I/1955 - IV/1969

$$U_t = 3.90 + .04G_t + .28G_{t-1}$$

(72.50) (1.10) (6.80)

$R^2 = .92$
S.E. = .30
D-W = .60

Symbols are defined as:

U_t = unemployment as a per cent of labor force in quarter t

$$G_t = \frac{XF_t - X_t}{XF_t} \cdot 100$$

XF_t = potential output in quarter t

X_t = output (GNP in 1958 prices) in quarter t

Note: "t" statistics appear with each regression coefficient, enclosed by parentheses. R^2 is the percent of variation in the dependent variable which is explained by variations in the independent variables. S.E. is the standard error of the estimate. D-W in the Durbin-Watson statistic.

Interest Rates

Interest rates do not function explicitly in the model as a part of a transmission mechanism running from changes in the money stock to output and prices. The estimated long-term interest rate equation plays a vital role in the model, however, providing the information to calculate the measure of price anticipations.

Market interest rates are specified to depend on current and past rates of change of output (\dot{x}), the current rate of change in the money stock (\dot{M}), and current and past rates of change in prices (\dot{P}) adjusted by an index of the unemployment rate. This specification draws on Sargent's work, which was explored further by Yohe and Karnosky.²⁶

Table V

LONG-TERM INTEREST RATE EQUATION

Sample Period: I/1955 - IV/1969

Constraints: 2nd Degree Polynomial
($\alpha_{-1} \neq 0, p_{-1} \neq 0; x_{17} = p_{17} = 0$)

$$R^L_t = 1.28 - .06\dot{M}_t + 1.42Z_t + \sum_{i=0}^{16} x_i \dot{x}_{t-i} +$$

(5.20) (-3.53) (11.01)

$R^2 = .92$
S.E. = .28
D-W = .69

$x_0 = .02$ (3.85)	$x_8 = .01$ (2.61)	$x_{12} = .01$ (1.38)
$x_1 = .02$ (4.35)	$x_7 = .01$ (2.27)	$x_{13} = .01$ (1.28)
$x_2 = .02$ (4.44)	$x_6 = .01$ (2.01)	$x_{14} = *$ (1.20)
$x_3 = .02$ (4.08)	$x_5 = .01$ (1.80)	$x_{15} = *$ (1.13)
$x_4 = .02$ (3.54)	$x_{10} = .01$ (1.64)	$x_{16} = *$ (1.07)
$x_5 = .02$ (3.03)	$x_{11} = .01$ (1.50)	$\Sigma x_i = .20$ (2.88)

$p_0 = .02$ (1.23)	$p_6 = .08$ (17.13)	$p_{12} = .06$ (9.29)
$p_1 = .03$ (3.05)	$p_7 = .08$ (14.49)	$p_{13} = .06$ (8.89)
$p_2 = .04$ (5.96)	$p_8 = .08$ (12.64)	$p_{14} = .04$ (8.57)
$p_3 = .06$ (10.82)	$p_9 = .08$ (11.37)	$p_{15} = .03$ (8.30)
$p_4 = .06$ (17.34)	$p_{10} = .08$ (10.47)	$p_{16} = .02$ (8.07)
$p_5 = .07$ (19.66)	$p_{11} = .07$ (9.81)	$\Sigma p_i = .96$ (19.04)

Symbols are defined as:

R^L_t = Moody's seasoned corporate Aaa bond rate in quarter t

\dot{M}_t = annual rate of change in money stock in quarter t

Z_t = dummy variable in quarter t (0 for I/1955 - IV/1960 and 1 for I/1961 - IV/1969)

\dot{x}_{t-i} = annual rate of change in output (GNP in 1958 prices) in quarter $t-i$

\dot{P}_{t-i} = annual rate of change in GNP deflator (1958 = 100) in quarter $t-i$

$U_{t-1}/4$ = index of unemployment as a per cent of labor force (base = 4.0) in quarter $t-1$

Note: "t" statistics appear with each regression coefficient, enclosed by parentheses. R^2 is the percent of variation in the dependent variable which is explained by variations in the independent variables. S.E. is the standard error of the estimate. D-W is the Durbin-Watson statistic.

*Less than .01

Long-term interest rate - The long-term market rate (R^L_t) is measured by the rate on seasoned corporate Aaa bonds. Changes in output and prices (adjusted for unemployment) lagged for 17 quarters provide the most satisfactory results. The estimated equation for the long-term rate is shown in Table V.

The results reflect, in a general way, the view stressed by monetarists that a change in the rate of monetary expansion influences market interest rates in three stages.²⁷ First, the liquidity effect of an increase in the rate of change of the money stock

the assumption that it be equated to zero for purposes of the model here. Also note that ΔP_t is defined in dollar units, that is, as $(P_t - P_{t-1})X_{t-1}$, not $(P_t - P_{t-1})$. ΔX_t is defined analogously.

²⁶Thomas Sargent, "Commodity Price Expectations and the Interest Rate," *Quarterly Journal of Economics* (February 1969), pp. 127-40, and Yohe and Karnosky, pp. 31-34, 38. The estimated interest rate equations also contain a dummy variable (0 for 1955-60 and 1 for 1961-69). The significance of this dummy variable indicates a shift of structure within the sample period. Questions can be raised

about this procedure, but it is felt that a price expectations variable should not be constructed on the basis of a sample period containing only an expansion like 1961-69. Including the dummy variable leaves unexplained that factor (or factors) which changed the relationship, but it does provide a way of estimating a set of coefficients on prices that is based on a sample period reflecting varying economic circumstances.

²⁷See Friedman, "The Role of Monetary Policy," p. 6.

on market interest rates is negative. Second, an increase in the rate of monetary expansion influences the rate of change in output, which in turn has a positive influence on market interest rates. Finally, an increase in money growth influences the rate of change in prices, which has a positive effect on market interest rates.

Short-term interest rate — The short-term interest rate (R^s_t) which is estimated is the four- to six-month commercial paper rate. The equation is shown in Table VI. Price changes are found to enter significantly for a shorter lag period than in the long-term rate equation. Also, the short-term rate, as measured by the four- to six-month commercial paper rate, is much more sensitive to changes in output and the money stock than is the long-term rate as measured by the rate on seasoned corporate Aaa bonds.

Time Response to Monetary Actions:

A Summary

The pattern of the coefficients in the equations provides information about the time response of total spending, output, and prices to monetary and fiscal actions. The equations indicate that monetary actions generally affect total spending with a two- to three-quarter lag. A change in the rate of growth of total spending is accompanied by a simultaneous change in the rate of growth of output, and it is not until three quarters later that the response of prices to a change in demand pressure builds to 70 per cent of the total. The response of prices to a change in total spending is yet slower when there are anticipations of a high rate of inflation.

The spending equation (Table I) indicates that about half of the total response to a change in monetary growth occurs in the first two quarters, and about 80 per cent in the first three quarters.

The pattern of coefficients in the price equation (Table III) indicates that the effect of a change in total spending is reflected first in output and later in prices. Operating through the demand pressure variable, about a fourth of the response of prices to a change in total spending is in the first quarter, which is about two quarters after the change in monetary actions. Over 70 per cent of the price response is in the first three quarters, and 95 per cent in the first five quarters. The response of the price level to changes in total spending is also influenced by anticipated prices. The greater the anticipated rise in prices, the longer delayed is the response of the price level to a decline in the rate of change in total spending.

Table VI

SHORT-TERM INTEREST RATE EQUATION

Sample Period: I/1955 - IV/1969

Constraints: 2nd Degree Polynomial

($x_{-1} \neq 0$, $p_{-1} \neq 0$, $x_{11} = p_{11} = 0$)

$$R^s_t = -1.13 - .17\dot{M}_t + .92Z_t + \sum_{i=0}^{10} x_i \dot{X}_{t-i} + \sum_{i=0}^{10} p_i \left(\frac{P_{t-i}}{U_{t-i/4}} \right)$$

$R^2 = .90$
 $S.E. = .47$
 $D-W = .69$

$x_0 = .08$ (8.10)	$p_0 = -.04$ (-.91)
$x_1 = .08$ (8.99)	$p_1 = .04$ (1.53)
$x_2 = .08$ (8.97)	$p_2 = .09$ (8.26)
$x_3 = .08$ (8.45)	$p_3 = .14$ (18.52)
$x_4 = .08$ (7.83)	$p_4 = .17$ (13.17)
$x_5 = .07$ (7.25)	$p_5 = .18$ (10.29)
$x_6 = .07$ (6.76)	$p_6 = .19$ (8.92)
$x_7 = .06$ (6.36)	$p_7 = .18$ (8.14)
$x_8 = .05$ (6.02)	$p_8 = .15$ (7.64)
$x_9 = .03$ (5.75)	$p_9 = .11$ (7.30)
$x_{10} = .02$ (5.52)	$p_{10} = .06$ (7.04)
$x_{11} = .71$ (8.24)	$p_{11} = 1.27$ (16.89)

Symbols are defined as:

R^s_t = four- to six-month commercial paper rate in quarter t

\dot{M}_t = annual rate of change in money stock in quarter t

Z_t = dummy variable in quarter t (0 for I/1955 - IV/1960 and 1 for I/1961 - IV/1969)

\dot{X}_{t-i} = annual rate of change in output (GNP in 1958 prices) in quarter $t-i$

\dot{P}_{t-i} = annual rate of change in GNP deflator (1958 = 100) in quarter $t-i$

$U_{t-i/4}$ = index of unemployment as a per cent of labor force (base = 4.0) in quarter $t-i$

Note: "t" statistics appear with each regression coefficient, enclosed by parentheses. R^2 is the percent of variation in the dependent variable which is explained by variations in the independent variables. S.E. is the standard error of the estimate. D-W is the Durbin-Watson statistic.

Tests of the Model's Performance

The equations of the model are to be viewed as a unit, and the specification of the model is such that given the change in money (ΔM), and the change in high-employment expenditures (ΔE), the model can be solved in the following sequence: for the change in total spending (ΔY), the change in the price level (ΔP), the change in real output (ΔX), the unemployment rate (U), and the long- and short-term interest rates (R^l and R^s).

The explanatory power of each of the equations shown in Tables I-VI may be acceptable by conventional standards, but this provides no guarantee that the model will perform satisfactorily as a unit. There are interdependencies in the model that have to be taken into account when evaluating the complete

model. Of interest in evaluating the model as a unit is the implied pattern for the endogenous variables when only an initial set of lagged endogenous variables and the time paths of the exogenous policy variables (money stock and high-employment Federal expenditures) are assumed known. To conduct such a test, several dynamic simulation experiments were performed. These simulations take the form of *ex post* dynamic simulations and an *ex ante* dynamic simulation.²⁸

Ex Post Dynamic Simulations

An *ex post* dynamic simulation is confined to the sample period from which the estimated relationships are derived. Actual values for all current and lagged exogenous variables are used, but only initial actual values for the lagged endogenous variables are used. The model generates solution values for the endogenous variables in the first simulation period, which are then used to generate solution values for the second period, and so on for each succeeding period.²⁹ A comparison of these calculated time paths for the endogenous variables with their actual time paths enables one to formulate some judgment as to how well the model performs as an interdependent unit in tracking the movements of certain strategic economic variables.

Ex post dynamic simulations were conducted for several subperiods within the sample period (1955-69). The results for the entire sample period are summarized in Chart I on the next page. When simulations are conducted for subperiods within the 1955-69 period, the pattern of movement as shown for the whole period simulation tends to hold, but the levels are closer to the actual values at the beginning of each subperiod.

Chart I indicates that the model tends to track the movement of the endogenous vari-

ables quite well during the 1955-69 period. Since criteria for judging the performance of the model in such a simulation have not been developed, any conclusions are necessarily subjective.³⁰ The tendency for the model to avoid diverging sharply from the actual path for extended periods is an especially important feature. Such a feature provides some basis for trusting the tracking ability of the model over several quarters, even if on a quarter-by-quarter basis it may appear to be off the mark.

To gain additional information about the predictive performance of the model, a comparison is made with an *ex post* simulation from another model. Results of an *ex post* simulation for 1963 and 1964 have been published for the Wharton model. The results for the model are compared with those of the Wharton model in Table VII.

The period 1963-64 includes the 1964 tax cut, which, according to the Wharton model, is considered an important factor influencing economic developments in 1964. However, the St. Louis model, which does not emphasize such fiscal actions, did about as well, on average, for the years 1963 and 1964 (see Table VII). The main difference to be remembered in evaluating these simulations is that the St. Louis model contains three primary exogenous variables, while the Wharton model contains forty-three.

³⁰See Robert H. Rasche and Harold T. Shapiro, "The F.R.B. - M.I.T. Econometric Model: Its Special Features," *American Economic Review*, Papers and Proceedings (May 1968), p. 142.

Table VII

ALTERNATIVE EX POST SIMULATIONS: ACTUAL MINUS PREDICTED¹ Comparison of Wharton and St. Louis Models for 1963-64

	Nominal GNP ²		Real GNP ²		Price Level ³		Unemployment Rate ⁴	
	Wharton	St. Louis	Wharton	St. Louis	Wharton	St. Louis	Wharton	St. Louis
1963: I	-4.6	0.4	-3.9	-0.4	-0.1	0.2	0.9	0.3
II	-0.2	0.3	0.4	-0.7	-0.1	0.2	0.7	0.1
III	1.3	1.5	2.5	0.6	-0.3	0.1	0.9	-0.1
IV	0.9	2.1	2.2	-0.4	-0.2	0.5	1.2	0.2
1964: I	0.9	1.7	2.7	-1.4	-0.3	0.6	1.4	0.2
II	1.1	0.1	2.3	-3.2	-0.3	0.6	1.4	0.2
III	1.5	1.7	4.0	-2.7	-0.5	0.8	1.6	0.2
IV	0	-1.7	2.2	-6.8	-0.4	0.9	1.2	0.2
Average Error	0.11	0.76	1.55	-1.88	-0.28	0.49	1.16	0.16
Root Mean Squared Error	2.00	1.49	2.92	3.09	0.33	0.60	1.28	0.21

¹Sample period: * Exogenous variables:
Wharton: 1948-1964 Wharton: 45
St. Louis: 1955-1969 St. Louis: 3

²Billions of dollars.

³Computed from the level of implicit price deflator.

⁴Per cent.

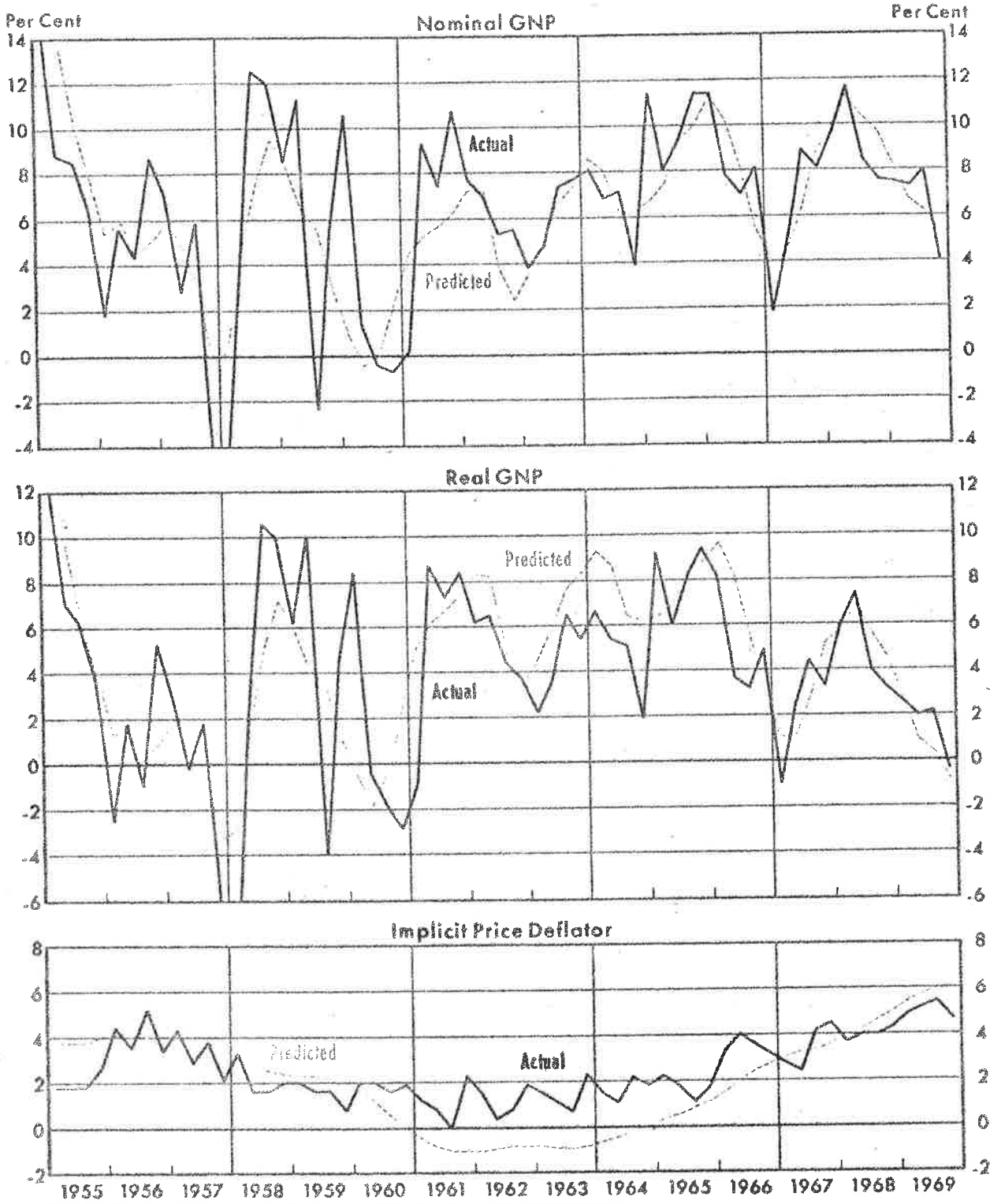
Sources: M. K. Evans and L. R. Klein, *The Wharton Econometric Forecasting Model*, 2nd, Enlarged Edition (Philadelphia: University of Pennsylvania, 1968); and Federal Reserve Bank of St. Louis.

²⁸For a discussion of the different ways of assessing the tracking ability of econometric models, see Carl F. Christ, "Econometric Models of the Financial Sector," forthcoming in the *Journal of Money, Credit, and Banking*. For a discussion of simulation procedures and results with an income-expenditure model, see Evans and Klein, pp. 50-69.

²⁹See de Leeuw and Gramlich, "The Channels of Monetary Policy . . ." p. 485.

Chart I
Results of Ex Post Dynamic Simulation

Annual Rates of Change



Note: Predicted values based on estimate of model for sample period ending IV/1969.

The comparison is not meant to imply that the St. Louis model is superior. Rather, the suggestion is offered that a small model constructed within a monetarist framework may yield as much information about the key aggregates as a large structural model. In summary, small monetarist models may be useful as a guide in the formulation of stabilization policy.

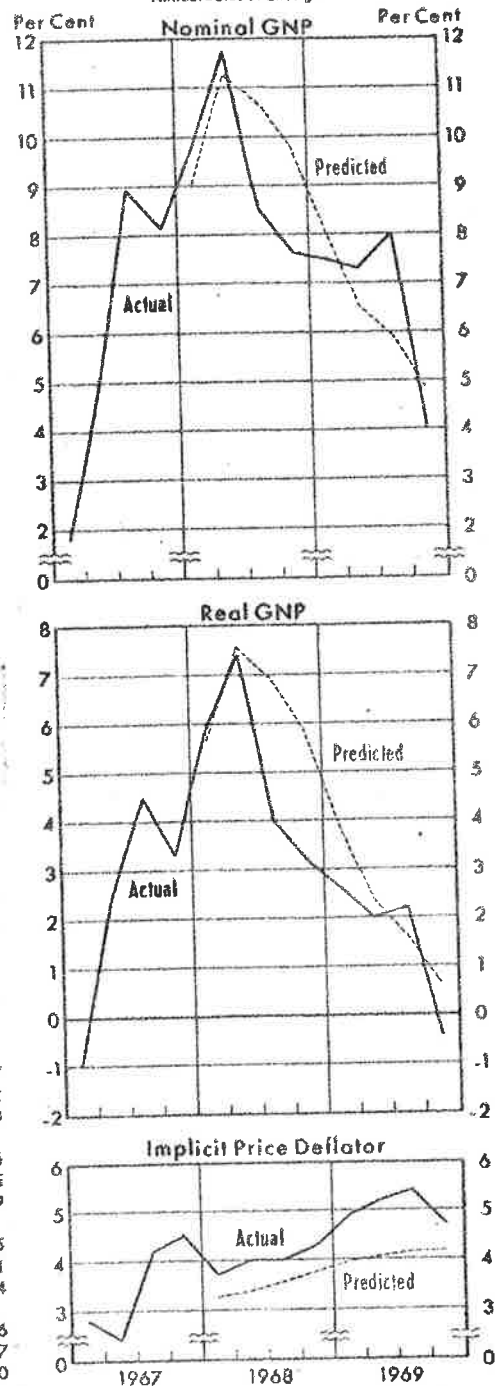
Ex Ante Dynamic Simulations

An *ex ante* dynamic simulation is like an *ex post* dynamic simulation, except that it extends beyond the sample period. To conduct such a simulation for this model, it was necessary to re-estimate the model for a subperiod within the full sample period. All equations of the model were re-estimated with data through 1967. The period of the *ex ante* dynamic simulation is 1968 and 1969. The results are summarized in Chart II and in Tables VIII and IX.

The success of the *ex ante* dynamic simulation can be assessed by comparing it with the tracking record of the *ex post* simulation for the same period. A comparison of the errors associated with the *ex ante* simulation with those of the *ex post* simulation (where the errors in both cases are computed with reference to actual values) suggests that any structural shifts that occurred in the 1968-69 period were not of such a magnitude that the *ex ante* tracking ability of the model was significantly different from that of *ex post* simulation.

Any conclusions about the tracking ability of the model are necessarily tentative, because they are based on only one *ex ante* dynamic simulation experiment. Nevertheless, these results provide a tentative basis for confidence in the tracking ability of the model in es-

Chart II
Results of
Ex Ante Dynamic Simulation
Annual Rates of Change



Note: Predicted values based on estimate of model for sample period ending IV/1967.

Table VIII

MODEL SIMULATIONS

	1968				1969			
	I	II	III	IV	I	II	III	IV
GNP Level (Billions of dollars)								
Actual	835.3	858.7	876.4	892.5	908.7	924.8	942.8	952.2
Ex Ante ¹	834.1	856.7	878.9	899.9	917.6	932.3	945.9	957.2
Ex Post ²	834.6	856.7	877.7	897.8	914.9	929.4	943.4	955.1
Annual Rate of Change in Y								
Actual	9.7	11.7	8.5	7.6	7.5	7.3	8.0	4.0
Ex Ante	9.1	11.3	10.8	9.9	8.1	6.6	6.0	4.9
Ex Post	9.4	11.0	10.2	9.5	7.8	6.5	6.1	5.1
Annual Rate of Change in X								
Actual	5.9	7.4	4.0	3.2	2.6	2.0	2.2	-0.4
Ex Ante	5.7	7.6	7.0	5.9	4.0	2.4	1.7	0.7
Ex Post	5.4	6.7	5.7	4.8	3.0	1.6	1.1	0.0
Annual Rate of Change in P								
Actual	3.7	4.0	4.0	4.3	4.9	5.2	5.4	4.7
Ex Ante	3.3	3.4	3.6	3.8	4.0	4.1	4.2	4.2
Ex Post	3.8	4.0	4.3	4.5	4.7	4.9	5.0	5.0
Unemployment Rate (per cent)								
Actual	3.7	3.6	3.6	3.4	3.4	3.5	3.6	3.6
Ex Ante	3.9	3.8	3.5	3.3	3.2	3.2	3.3	3.5
Ex Post	3.9	3.7	3.5	3.4	3.4	3.5	3.7	3.9
Corporate Aaa Rate (per cent)								
Actual	6.1	6.3	6.1	6.2	6.7	6.9	7.1	7.5
Ex Ante	5.8	5.9	6.0	6.4	6.5	6.7	7.0	7.1
Ex Post	5.9	6.0	6.1	6.5	6.7	6.9	7.2	7.4
Commercial Paper Rate (per cent)								
Actual	5.6	6.1	6.0	6.0	6.7	7.5	8.5	8.6
Ex Ante	5.1	5.1	5.1	5.6	5.7	6.0	6.6	6.7
Ex Post	5.8	5.8	5.8	6.5	6.6	7.0	7.8	8.0

Key to Abbreviations:
Y = Nominal GNP
X = Real GNP
P = GNP price deflator

¹Simulation based on equations estimated through IV/1967.
²Simulation based on equations estimated through IV/1968.

Table IX
EX ANTE VS. EX POST SIMULATION:
ACTUAL MINUS PREDICTED
I/1968 - IV/1969
(Root Mean Squared Errors)

	Ex Post	Ex Ante
GNP ¹	1.26	1.45
GNP in 1958 Prices ¹	0.99	1.58
GNP Deflator ¹	0.25	0.76
Unemployment Rate	0.14	0.20
Corporate Aaa Rate	0.17	0.26
Commercial Paper Rate	0.44	1.26

¹Computed from actual minus predicted annual rates of change.

timating the economic response to monetary and fiscal actions. Unfortunately, it is difficult to conduct additional tests of this type for other subperiods in the sample, because degrees of freedom are severely reduced when the sample period is shortened further.

Using the Model for Current Analysis

The model is used in this section to simulate the effects of possible future rates of monetary expansion on spending, output, prices, unemployment and interest rates, given the economic circumstances of late 1969 and early 1970. Simulation of these alternative courses of monetary action suggests how the model may provide information which will be helpful to policymakers.

Simulations of the model are conducted only for alternative rates of monetary expansion. This is done because of the nature of the results for the spending equation. The net effect of a change in Federal expenditures on total spending (GNP) over a five-quarter period is very small if unaccompanied by monetary expansion.

Short-Run Projections

The results of simulating the model for alternative growth rates of money, and for the growth of Federal expenditures as projected in the fiscal 1971 budget, are shown in Table X. These simulation results reflect the accelerating inflation of the past several years and the fiscal and monetary restraint in force throughout 1969 and early 1970. These projections assume that empirical relationships based on past experience will continue to hold in the near future.

Rates of change in the money stock were computed from the first quarter of 1970. Three alternative rates are shown in Table X. The "no-change case" corresponds to the course of monetary actions in the second half of 1969. The "three per cent case" corresponds to the trend rate of increase in money from 1961 to 1965. Finally, the "six per cent case" represents monetary actions similar to those of 1967 and 1968.

Table X
SIMULATION OF ALTERNATIVE RATES OF MONETARY EXPANSION

Projected Rate of Change in M ¹	Actual		Projected						
	IV/1969	I/1970	II/1970	III/1970	IV/1970	I/1971	II/1971	III/1971	IV/1971
0 Per Cent									
Annual Rate of change in Y	4.0 (5.1) ²	3.5	3.1	1.0	0.3	0.7	0.9	2.1	0.9
X	-0.4 (0.1)	-1.3	-1.6	-3.3	-3.6	-2.7	-2.0	-0.4	-1.0
P	4.7 (5.0)	4.9	4.7	4.4	4.0	3.5	3.0	2.5	1.9
Unemployment Rate	3.6 (4.0)	4.3	4.8	5.2	5.8	6.4	6.9	7.4	7.7
Corporate Aaa Rate	7.5 (7.4)	7.4	7.5	7.4	7.4	7.3	7.1	6.9	6.7
Commercial Paper Rate	8.6 (8.0)	7.6	7.6	7.1	6.5	5.7	5.0	4.4	3.6
3 Per Cent									
Annual Rate of change in Y	4.0 (5.1) ²	3.5	3.8	2.9	3.1	4.1	4.4	5.5	4.3
X	-0.4 (0.1)	-1.3	-0.8	-1.5	-1.0	0.3	0.8	2.2	1.4
P	4.7 (5.0)	4.9	4.7	4.5	4.2	3.8	3.5	3.2	2.9
Unemployment Rate	3.6 (4.0)	4.3	4.8	5.2	5.6	6.0	6.3	6.5	6.7
Corporate Aaa Rate	7.5 (7.4)	7.4	7.3	7.3	7.3	7.3	7.2	7.1	6.9
Commercial Paper Rate	8.6 (8.0)	7.6	7.2	6.8	6.4	5.9	5.5	5.0	4.5
6 Per Cent									
Annual Rate of change in Y	4.0 (5.1) ²	3.5	4.6	4.8	6.0	7.6	7.8	8.9	7.7
X	-0.4 (0.1)	-1.3	-0.1	0.2	1.6	3.3	3.7	4.8	3.8
P	4.7 (5.0)	4.9	4.7	4.5	4.3	4.2	4.0	3.9	3.8
Unemployment Rate	3.6 (4.0)	4.3	4.8	5.1	5.4	5.6	5.6	5.7	5.7
Corporate Aaa Rate	7.5 (7.4)	7.4	7.2	7.2	7.2	7.3	7.3	7.3	7.2
Commercial Paper Rate	8.6 (8.0)	7.6	6.7	6.5	6.3	6.1	5.9	5.7	5.5

Key to Abbreviations:
Y = Nominal GNP P = GNP Price Deflator
X = Real GNP M = Money Stock

¹Rates of change in money projected from I/1970. High-employment Federal expenditures projected on basis of fiscal 1971 budget, as released in January 1970.

²Model estimates.

No-change case — A course of no change in the money stock from the first quarter of 1970 would lead to further reduction in the rate of increase of total spending in 1970 and 1971 (Table X). A slowing of total spending along with upward pressures on prices from the past inflation (anticipated price effect) would lead to continued declines in output through 1971. Such a restrictive course of monetary actions would slow the rate of price increase to a four per cent rate by late 1970 and to a two per cent rate by late 1971. The decline in output would be accompanied by a rise in the unemployment rate to over seven per cent by late 1971.

The effects of such restrictive monetary actions on interest rates would be to keep the long-term interest rate at recent levels through 1970, mainly because of the effects of past inflation. By early 1971, the slower advance of prices in 1970 and the slowing of output growth would lead to declines in the long-term rate. The short-term interest rate, on the other hand, would hold at recent levels only temporarily, partly because of continued restrictive monetary actions. Short-term rates would drop sharply by the second half of 1970, reflecting primarily the slowing of output growth. Since the price lags are shorter for the short-term rate, the effects of past inflation are not so pervasive as for the long-term rate.

Three per cent case — Growth of the money stock at a three per cent annual rate is presented to illustrate the effects of a moderate expansion of money. This rate corresponds to the trend rate of increase in money from 1961 to 1965. In the current economic situation, a three per cent rate of expansion in money would represent a moderate easing of monetary policy from its restrictive influence of late 1969 and early 1970.

The effect of such expansion would be to maintain growth in total spending at a rate about the same as in the fourth quarter of 1969. Given the influence of past inflation on prices, output would decline slightly through 1970, but would resume its increase by 1971. The effect on prices in 1970 would be little different from the no-change case, but by late 1971 the difference would be marked. In the three per cent case prices would still be rising at a three per cent rate by late 1971 compared with a two per cent rate for the no-change case. Moderate expansion of the money stock leads to a rise in the unemployment rate through 1970 and 1971. In general, for this model, the unemployment rate rises as long as output grows at less than a four per cent rate.

The long-term interest rate would remain at recent levels throughout 1970, and not until early 1971 would the effect of slower price increases and output growth be enough to offset the effects of past inflation. The short-term interest rate would fall more quickly than the long-term rate but would not fall as much by late 1971 as in the no-change case. Such a pattern for the short-term rate illustrates the short- and longer-run influence of quickened monetary expansion.

Six per cent case — A six per cent annual rate of increase in money is shown to illustrate the effects of a sudden shift to a very rapid rate of monetary expansion in the second quarter of 1970. Such increase in money would be about the same as during 1967 and 1968.

A major effect of shifting to rapid monetary expansion would be to advance the rate of total spending growth. By late 1971, total spending would be increasing at an eight per cent rate with such monetary actions. The rate of price increase would fall somewhat, however, because of past restrictive monetary actions. But the gain in price performance would be small, because by late 1971 prices would still be increasing at a four per cent rate. The effects of past monetary and fiscal actions, along with past inflation, would lead to a decline in output through mid-1970. From then through 1971, output growth would increase.

Despite a shift to a very rapid rate of monetary growth, unemployment would rise until mid-1971. This increase in unemployment would follow because of the continued influence of past monetary and fiscal actions. By late 1971, the recovery in output growth would be pushing the unemployment rate back down.

A shift to rapid monetary expansion has a pronounced effect on market interest rates. The long-term rate would stay at recent levels through 1971, because the influence of past prices (anticipated price effect) would not be offset by a sustained reduction in output growth. The short-term rate would fall, in response to the temporary reduction in output growth, but the decline by late 1971 would be less than for either the no-change or three per cent case.

Implications of the Model for the Long Run

Short-run prospects for economic variables tend to dominate policymakers' decisions. However, the longer-run consequences of alternative monetary policies should also be given consideration. This model

is incomplete for long-run analysis; nevertheless, it yields results that are of interest and may not be too far removed from results that might evolve from a more complete specification.³¹

When simulations are conducted for long periods into the future (thirty years), the model demonstrates properties consistent with those expounded by the classical economists. Over the long run, monetary actions have no effect on real magnitudes; the rate of growth of output, the unemployment rate, and the real rate of interest all tend to move toward some equilibrium rate, regardless of which rate of money growth is maintained. The effects of alternative rates of monetary expansion are on nominal magnitudes, namely, total spending, prices, and market interest rates.

Based on the assumptions of the model, a six per cent rate of growth in money along with a six per cent growth rate in Federal expenditures, for example, would lead ultimately to about a six per cent rate of growth in total spending, a four per cent rate of growth in output, a two per cent rate of increase in prices, and market interest rates about two percentage points in excess of the real rate. Alternatively, a two per cent growth rate in money would result approximately in a two per cent growth in total spending, a four per cent rate of growth in output, a two per cent rate of decline in prices, and market interest rates about two percentage points below the real rate. Over the long run, the model indicates that high employment and price stability are compatible.

Summary

The main purpose of this study has been to quantify the effects of monetary and fiscal actions

³¹The shortcomings of the model for the long-run analysis are quite evident. There are no assumptions specified as to labor force growth and productivity. Furthermore, there is no investment function and, therefore, the capital stock is not an endogenous variable. All long-run assumptions are embodied in assumptions about the growth rate of potential output. With these assumptions, policy actions cannot affect the economy's long-run growth rate.

within a small-model framework and thereby offer an alternative to existing large-scale econometric models. Such a model has been formulated and the effects of monetary and fiscal actions on spending, output, prices, employment, and interest rates have been estimated.

The model developed in this article is primarily "monetarist" in character. The estimated equations indicate that monetary actions, as measured by changes in the money stock, play a strategic role. Fiscal actions, as measured by high-employment Federal expenditures, have some short-run effects, but for periods of a year or more the net effect on spending, output, and prices is near zero. Simulations of alternative rates of monetary expansion produce short-run and long-run responses which are consistent with the general monetarist view of the economy.

One of the chief advantages of this model is that it depends primarily on information about only two variables—the money stock and high-employment expenditures.³² Considerable insight can be gained about the pattern of expected movements of certain strategic economic variables by considering alternative courses of monetary and fiscal actions. However, since the model is limited to only monetary and fiscal influences, to the exclusion of other independent forces, it is not suitable for exact forecasting.³³ Its primary purpose is to measure the general pattern of influence of monetary and fiscal actions on several strategic economic variables. Since the economy is viewed as being basically stable, other factors influencing total spending, output, and prices are not considered to be of great importance in estimating the response to monetary and fiscal actions.

³²This feature has led John Deaver to conjecture that the standard error of forecast in the Andersen-Jordan model may be far lower than that of the FRB-MIT model. See his "Monetary Model Building," *Business Economics*, (September 1969), p. 30.

³³See Andersen and Jordan, pp. 15, 23, 24, and Leonall C. Andersen, "Money in Economic Forecasting," *Business Economics*, (September 1969), p. 17.

The Appendices to this article begin on the next page.

APPENDIX A

EXPLANATION OF THE PRICE EQUATION

The price equation (omitting timescripts and lags) is

$$\Delta P = f(D, \Delta P^A),$$

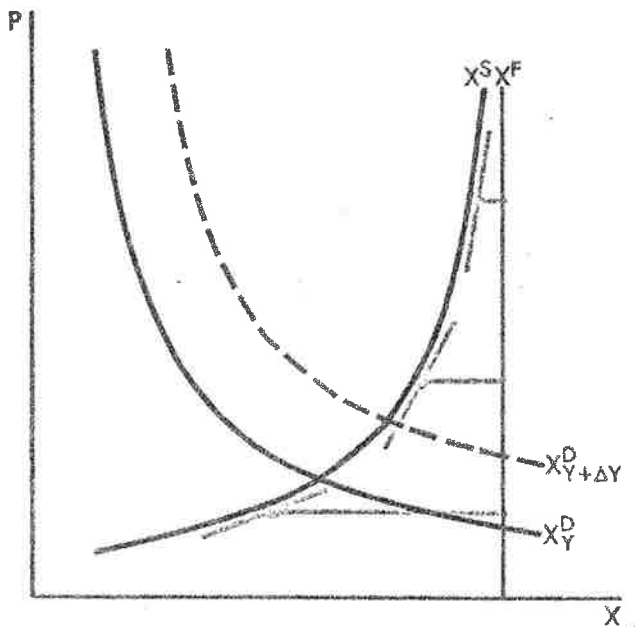
where D , demand pressure, is defined as

$$D = \Delta Y - (X^F - X).$$

ΔY is the change in total spending, $(X^F - X)$ is the GNP gap, that is, the difference between potential and actual output, and ΔP^A is anticipated price change. This specification of the price equation is based on standard theory of macroeconomic equilibrium.

Macroeconomic equilibrium can be depicted graphically as in Figure I. The solid downward-sloping line, X_Y^D , is the total spending line, which represents the combinations of prices and output consistent with a particular level of total spending, Y . This total spending line can be interpreted as total demand for output.

Figure I
Macroeconomic Equilibrium
(Determination of Output and Prices)



The upward-sloping line, labeled X^S , is the total supply line. This line corresponds to that combination of prices and output which maximizes profits of firms, given the prices of factors of production, the degree of competition among firms and the stock of human and physical capital (defined to embody the state of technology).

The intersection of total supply and total demand determines the levels of output and prices. The equilibrium price level is that level which equates the amount of output supplied with the amount demanded.

The focus of the model is on the change in prices and the change in output. In terms of Figure I, changes in prices and output are brought about by shifts in demand and/or supply. Since X_Y^D is drawn for a level of total spending, a shift of that line upward and to the right to $X_{Y+\Delta Y}^D$ represents an increase in total spending. If the total supply line remains fixed, the effect of ΔY on prices depends on (1) the magnitude of ΔY , and on (2) the slope of the total supply line, X^S .

The purpose of the model is to estimate the response of spending, output, and prices to monetary and fiscal actions, not to test a hypothesized structure. Consequently, rather than attempt to determine the shape of the total supply line empirically, its variable slope is proxied by the difference between potential output and actual output. As drawn in Figure I, there is a one-to-one relationship between $X^F - X^S$ and the slope of X^S . Assuming that this relationship is approximately linear within the range of experience since 1955, and that the observed values fall on the supply line, the effect of a variable slope for X^S can be approximated by $X^F - X$. In this way the term $[\Delta Y - (X^F - X)]$ brings together both the magnitude of demand shift and the slope of the supply line.

The other term in the price equation, anticipated price change, ΔP^A , is considered as a separate influence on prices. In terms of Figure I, the anticipated price term is a shift parameter for the total supply line (an increase in ΔP^A shifts X^S upward and to the left). Including it in this way allows for the influence of past prices on current pricing policies of firms and factors of production.

APPENDIX B

GRAPHICAL ILLUSTRATION OF THE MODEL

The workings of the model can be demonstrated with graphical techniques. Figure II is a representation of the core of the model, showing the determination of changes in spending, output, and prices.

Panel A of Figure II is a graphical representation of the total spending equation with ΔM on the horizontal axis and ΔY on the vertical axis. Changes in ΔE shift the total spending line.

Panel B shows prices (ΔP) as a function of ΔY . A short-run price line (ΔP_1) is drawn consistent with empirical results showing that ΔP is not very sensitive to ΔY in the short run. Important determinants of the position of the short-run price line are the size of the GNP gap and anticipated price changes. The long-run price line ($\Delta P(LR)$) is drawn to show the relationship between ΔP and ΔY when the GNP gap is zero and anticipated prices are equal to actual prices. Its slope (45 degrees

from its origin on the ΔY axis) is based on the monetarist view that in the long run, ΔM influences only ΔP .

Panel C expresses the total spending identity in graphical terms. Total spending is divided between output and prices; to reflect this, the line in panel C is drawn as a 45 degree line with its position determined by the magnitude of total spending (ΔY). There is a family of 45 degree lines, one for each possible ΔY . Also included in panel C is a horizontal line representing the long-run growth rate of output. It is shown as a horizontal line to indicate that long-run output growth is exogenously determined by resource growth and technology.

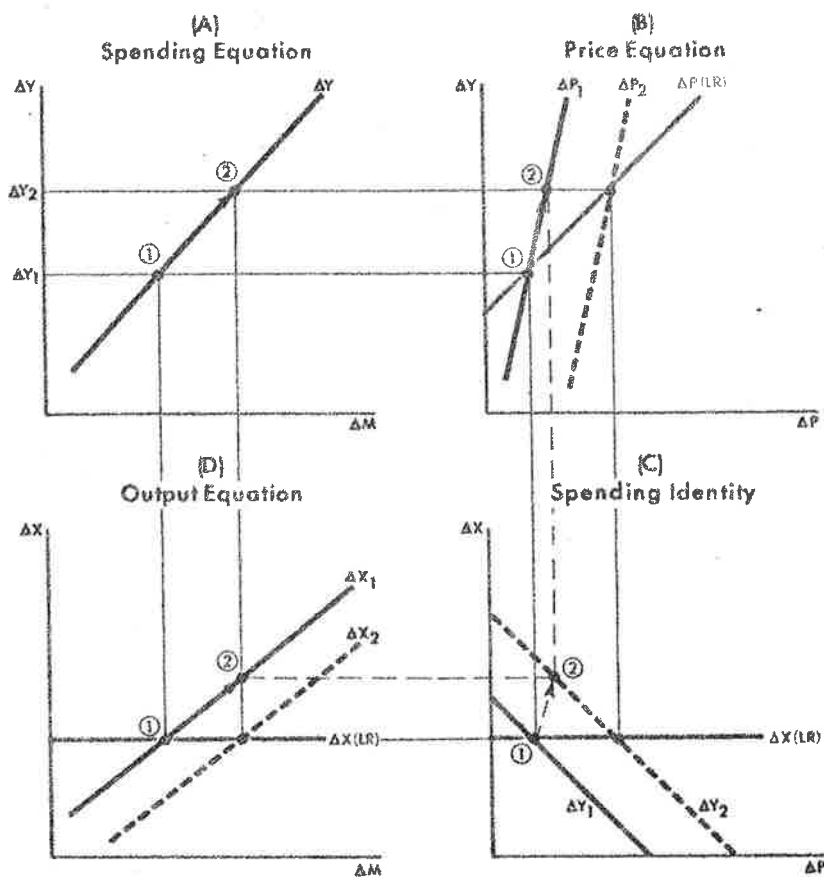
In panel D, the ΔX_1 line shows the relationship between money (ΔM) and output (ΔX) as derived from the other three panels. The equation for this line is not shown in Exhibit I in the text, but it can be derived from the other equations of the model.

Figure II is drawn to represent an initial equilibrium for a given ΔM , which has associated with it the short-run price and output lines, ΔP_1 and ΔX_1 . The effect of a change in ΔM , given ΔE , is shown as a movement along the spending line in panel A from ① to ②. Given the initial price line, ΔP_1 , and the changed ΔY , the effect on prices and output is shown in panels B, C and D as a movement from ① to ②.

This case illustrates the impact of a change in ΔM in the short run. For longer periods, anticipated price changes and the GNP gap will also change; they become endogenous variables in a long-run model. To illustrate the effects for the long run, the long-run price line, $\Delta P(LR)$, in panel B, is relevant. The interpretation of the long-run price line is that changes in ΔM are reflected only in ΔP , with ΔX determined by considerations of resource growth and technology. The horizontal line in panels C and D is the long-run relation between prices and output.

In the short-run, the solution of the model need not lie on the long-run price line in panel B (or the long-run output line in panels C and D). However, a succession of short-runs (shown as a shift of the ΔP and ΔX lines to ΔP_2 and ΔX_2) will tend to move equilibrium toward the long-run price and output lines, as anticipated prices adjust to actual prices and the GNP gap goes to zero.

Figure II
Model in Graphical Form



APPENDIX C

ALTERNATIVE PRICE EQUATIONS

The model consists, for the most part, of equations which have been estimated in previous studies. The purpose of this paper is to combine the equations in a way which represents the general monetarist view.

The primary distinguishing feature of this model, other than the reduced-form total spending equation, is the inclusion of a price anticipations variable in the price equation. Two alternative methods of introducing price anticipations were considered. One alternative bypasses the precise form of the price anticipations function and uses the long-term market interest rate (yield on corporate Aaa bonds) as an independent variable in the price equation. The other alternative bypasses both price expectations and interest rates, and introduces changes in money as an independent variable in the price equation. Such a specification allows monetary actions to serve as a proxy for anticipated prices.

Table XI

ALTERNATIVE PRICE EQUATIONS

(Dependent Variable — ΔP_t)

Sample Period: I/1955 - IV/1969

Independent Variable

	ΣD_{t-1}	ΔP_t	$(R_{t-1}^L X_{t-1}) .01$	$\Sigma \Delta M_{t-1}$	Con- stant	R ²	S.E.	D-W
Price Anticipations Specification ¹	.09 (9.18)	.86 (8.55)	.	.	2.70 (7.07)	.87	1.07	1.41
Interest Rate Specification ²	.09 (8.72)	.	.11 (8.73)	.	1.93 (4.10)	.88	1.04	1.49
Money Stock Specification ³	.06 (4.61)	.	.	2.61 (7.82)	2.11 (4.22)	.86	1.11	1.37

¹ Where i goes from 0 to 5² Where i goes from 0 to 10³ Where i goes from 0 to 9⁴ Not estimated for this equation

Note: Regression coefficients are the top figures; their "t" statistics appear below each coefficient, enclosed by parentheses. R² is the per cent of variation in the dependent variable which is explained by variations in the independent variable. S.E. is the standard error of the estimate. D-W in the Durbin-Watson statistic.

same as for the price anticipations version of the equation. However, the length of the lag structure is longer, indicating that the response of prices to changes in demand pressure may be slower than in the basic equation. But this need not imply that prices are slower to respond to monetary actions, since the magnitude of the interest rate contribution to price change is smaller than with the price anticipations specification.

A Market Interest Rate in the Price Equation

The first alternative replaces the price anticipations variable with the long-term market interest rate.¹ The rationale is that the process of price anticipations formation is so complex that it defies measurement. However, there seems to be agreement that the level of market interest rates reflects anticipated price changes, however formed. Thus the market interest rate can be used as a proxy for price anticipations.

Since interest rates reflect factors other than price anticipations, including the interest rate does not provide a clean measure of price anticipations. Using the market interest rate allows those factors influencing the real rate of interest to enter indirectly as an influence on prices. In general, however, it has been argued that the real rate of interest is very stable.

Following this reasoning, the price equation was estimated by including the long-term interest rate. The results are shown in Table XI. The coefficient of the interest rate variable is significant at the five per cent level for this specification, and the sum of the coefficients for the demand pressure variable is roughly the

Money in the Price Equation

Several observers have been critical of price equations that do not include monetary variables directly. As shown in the text, excluding monetary variables from the price equation does not necessarily imply a non-monetary theory of inflation.² Such a conclusion cannot be derived by examining the price equation alone, but requires an examination of the whole model, and the linkages between money and prices in particular.

The second alternative that is considered is based on the central proposition of the quantity theory—that changes in money are ultimately reflected in changes in the price level. Accordingly, current and past changes in money are used as a proxy to measure anticipated movements in prices.³ Though this rationale for including money is somewhat narrower than that proposed by some monetary economists, the direct and indirect effects of money are being measured once it is included in the price equation.

¹The suggestion for using the interest rate in the price equation came from the Money and Banking Workshop at the University of Chicago.

²See Fand, "Some Issues in Monetary Economics," pp. 20-23.

³This suggestion was made by Professors David Fand and Allan Meltzer.

The price equation incorporating current and lagged values of changes in money is shown in Table XI. Except for the current quarter, the coefficients are significant for nine lagged quarters. The effect of including changes in money lowers the sum of the coefficients on the demand pressure variable, however. The overall explanatory power of the equation is about the same as for the price anticipations model.

Comparison of Ex Post Simulations

To compare the price equation in the text with the two alternatives in this Appendix, the model was simulated with each of the three different specifications from 1965 through 1969. The period starting in 1965 is used because the relative tracking ability of the models during a period of accelerating inflation is especially relevant in assessing the current economic situation. Since the price equation is the only part that varies from one model to the next, only the results for the rate of change of prices are reported. (see Table XII).

The price anticipations specification has the smallest average absolute error and the smallest root mean squared error for the period. During the last two years of the period, 1968 and 1969, each of the alternative specifications tends to underestimate price changes. However, for 1968 and 1969, the price anticipations specification again has both the smallest average absolute error and root mean squared error.

Conclusions

An examination of the model reflecting three different specifications for the price equation indicates that none of the specifications is clearly superior as judged by conventional criteria. A policymaker might well consider the results provided by each of the three.

When simulations are performed for thirty-year periods beginning in 1970, the price anticipations version (as presented in the text) approaches closest a long-run classical solution. For the other two specifications the unemployment rate does not stabilize at the same

Table XII

GNP PRICE DEFLATOR ALTERNATIVE EX POST SIMULATIONS: ACTUAL MINUS PREDICTED (Compounded Annual Rates of Change)				
	Price Anticipations Specification	Interest Rate Specification	Money Stock Specification	
1965	I	0.6	0.3	-0.5
	II	0.2	-0.2	-1.0
	III	-0.7	-1.0	-1.7
	IV	-0.2	-0.4	-1.0
1966	I	1.0	1.0	0.4
	II	1.4	1.5	1.0
	III	0.8	0.8	0.6
	IV	0.2	0.1	0.3
1967	I	-0.3	-0.3	-0.2
	II	-0.8	-0.8	-0.8
	III	1.0	1.1	0.9
	IV	1.3	1.4	1.1
1968	I	0.4	0.4	0.3
	II	0.5	0.7	0.6
	III	0.2	0.7	0.4
	IV	0.2	0.9	0.3
1969	I	0.6	1.4	0.7
	II	0.7	1.6	0.8
	III	0.8	1.7	1.0
	IV	0.1	0.9	0.5
Average Absolute Error				
	1965-69	0.60	0.86	0.71
	1965-67	0.71	0.74	0.79
	1968-69	0.44	1.04	0.58
Root Mean Squared Error				
	1965-69	0.50	0.96	0.63
	1965-67	0.67	0.76	0.79
	1968-69	0.25	1.27	0.39

level for alternative growth rates of money. These two alternatives yield the same equilibrium growth rates of output for alternative growth rates of money, but since this rate is approached asymptotically, unemployment stabilizes at a different rate for each alternative growth rate of money.⁴

⁴Supplementary materials relating primarily to the long-run simulations are available on request.

"QUARTERLY ECONOMIC SUMMARY," a new release of this bank, will replace the quarterly "Triangles of U.S. Economic Data" next month, and is available to the public without charge. Based on the approach to stabilization analysis presented in this article, "Quarterly Economic Summary" will contain an outlook for Total Spending, Real Product, and Prices. It will include an explanation of the analysis, as well as charts and rates-of-change tables for national income accounts data and related series. Persons who have been receiving "Triangles of U.S. Economic Data" will automatically receive the new release. For subscriptions, write: Research Department, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, Missouri 63166.

MONEY STOCK SPECIFICATION
OF THE PRICE EQUATION

Sample Period: I/1955 - IV/1969

Constraints: 2nd Degree Polynomial ($d_{-1} \neq 0$, $v_{-1} \neq 0$, $d_9 = v_9 = 0$)

$$\Delta P_t = 2.110277 + \sum_{i=0}^8 d_i D_{t-i} + \sum_{i=0}^8 v_i \Delta M_{t-i}$$

$R^2 = .863$
 $S.E. = 1.109$
 $D-W = 1.372$

$d_0 = -.024316$ (-2.913530)	$v_0 = .063407$ (.805144)
$d_1 = -.009318$ (-1.793838)	$v_1 = .209061$ (4.076842)
$d_2 = .002605$ (.920162)	$v_2 = .316541$ (7.296062)
$d_3 = .011455$ (6.515344)	$v_3 = .385845$ (7.767497)
$d_4 = .017231$ (7.939602)	$v_4 = .416975$ (7.273062)
$d_5 = .019933$ (7.299374)	$v_5 = .409930$ (6.798819)
$d_6 = .019561$ (6.745534)	$v_6 = .364710$ (6.445428)
$d_7 = .016114$ (6.414681)	$v_7 = .281315$ (6.185521)
$d_8 = .009594$ (6.191319)	$v_8 = .159745$ (5.989865)
$\Sigma d_i = .062859$ (4.611571)	$\Sigma v_i = 2.607526$ (7.823640)

Table XI Coefficients of the price equation. From: L.C. Andersen and K.M. Carlson, Supplementary Materials accompanying "A Monetarist Model for Economic Stabilization", Federal Reserve Bank of St. Louis REVIEW, April 1970.

Program Listings

APPENDIX II

PROGRAM USEC

SIMULATES THE ECONOMY OF USA ACCORDING TO
THE SAINT LOUIS MODEL

1973-05-04

REFERENCE: L.C. ANDERSON, K.M. CARLSON, "A MONETARIST
MODEL FOR ECONOMIC STABILIZATION"

THE RESULTS OF THE SIMULATION ARE STORED ON THE DISK IN A
FILE OF STANDARD TYPE. THIS FILE CAN BE PLOTTED AFTER THE
SIMULATION, E.G. USING THE PLOTTING FACILITIES OF IDPAC.

ILP AND IDK INDICATE THE TYPE OF DATA OUTPUT:
ILP=0 GIVES NO OUTPUT ON LINE PRINTER
IDK=0 GIVES NO OUTPUT ON DISK

SUBROUTINES REQUIRED:
INDAT/READIN
OUTDAT
COEF
SIMUS

DIMENSION FNAME(2),KOD(10),DATA(20)

COMMON DY1,DM(9),RM,RM0,DE(5),DPT,P,P0,D(9),U(17),G(2),
1RLT,RMDOT,XDOT(17),PDOT(17),RST,XFT,X(2),DXT,
2X0,RMDOT0,XDOT0,PDOT0,U0,DM0,DE0,ADXF,QDXF,XFT0,
3CDY,CHY(5),CE(5),CDP,CD(9),CMP(9),CU1,CU2,CU3,
4CRL1,CRL2,CRL3,CXL(17),CPL(17),
5CRS1,CRS2,CRS3,CXS(17),CPS(17),NSAMP,IDK,FNAM,ILP,
6ADM,QDM,ADE,QDE,ADYT,ADPT,ADXT,DYT1,DPT1,DXT1,
7DMPRO,DEPRO,MSAMP,ADM1,ADE1

DATA FNAME(2)/4H BIN/

NCOL=12

CALL COEF

CALL INDAT

CALL OUTDAT(1)

GIVE INITIAL VALUES TO THE VARIABLES

DO 6 I=1,17
U(I)=U0
XDOT(I)=XDOT0
PDOT(I)=PDOT0
RMDOT=RMDOT0
X(1)=X0
X(2)=X0
P=P0
RM=RM0
XFT=XFT0
DM(1)=DM0
DE(1)=DE0

IF(IDK) 10,20,10
FNAME(1)=FNAM

```

CALL ENTER(1,FNAME)
KOD(1)=NSAMP
KOD(2)=NCOL
WRITE(1) (KOD(I),I=1,10)

C
C
C
20 SIMULATION LOOP
C
CALL SIMUS

C
IF(JSAMP-1) 30,40,30
30 ADYT=((DYI/DYT1)**4-1.)*100.
ADPT=((DPT/DPT1)**4-1.)*100.
ADXT=((DXT/DXT1)**4-1.)*100.

C
GO TO 50

C
40 ADYT=0.
ADPT=0.
ADXT=0.

C
50 DYT1=DYT
DPT1=DPT
DXT1=DXT

C
C
C
STORE DATA

DATA(1)=DYT
DATA(2)=DPT
DATA(3)=U(1)
DATA(4)=RLT
DATA(5)=RST
DATA(6)=X(1)
DATA(7)=G(1)
DATA(8)=ADYT
DATA(9)=ADPT
DATA(10)=ADXT
DATA(11)=D(1)
DATA(12)=XFT

C
C
C
WRITE ON LINE PRINTER

C
IF(ILP) 100,110,100
100 CALL OUTDAT(2)
C
WRITE ON DISK
C
110 IF(IDK),120,130,120
120 WRITE(1) (DATA(I),I=1,NCOL)
C
130 CALL RMOVE(DM(8),-1,DM(9),-1,8)
CALL RMOVE(DE(4),-1,DE(5),-1,4)
CALL RMOVE(D(8),-1,D(9),-1,8)
XFT=(1.+QDXF/100.)*XFT
IF(ISAMP-MSAMP) 150,140,150
140 DM(1)=(1.+DMPRO/100.)*DM(1)
DE(1)=(1.+DEPRO/100.)*DE(1)
QDM=100.*(SQRT(SQRT(1.+ADM1/100.))-1.)
QDE=100.*(SQRT(SQRT(1.+ADE1/100.))-1.)
GO TO 500
150 DM(1)=(1.+QDM/100.)*DM(1)
DE(1)=(1.+QDE/100.)*DE(1)
500 CONTINUE

```

C
C
C

CLOSE DISK FILE

510
520IF (IDK) 510,520,510
CALL CLOSE(1)
CONTINUE
GO TO 1
END

SUBROUTINE INDAT

```
C
C INPUT OF DATA TO USEC
```

```
C
C 1973-05-04
```

```
C
C SUBROUTINES REQUIRED:
C   READIN
```

```
C
COMMON DYT,DM(9),RM,RNO,DE(5),DPT,P,PO,D(9),U(17),G(2),
1RLT,RMDOT,XDOT(17),PDOT(17),RST,XFT,X(2),DXT,
2X0,RMDOT0,XDOT0,PDOT0,U0,DM0,DE0,ADXF,QDXF,XFT0,
3CDY,CHY(5),CE(5),CDP,CD(9),CMP(9),CU1,CU2,CU3,
4CRL1,CRL2,CRL3,CXL(17),CPL(17),
5CRS1,CRS2,CRS3,CXS(17),CPS(17),NSAMP,IDK,FNAM,ILP,
6ADM,QDM,ADE,QDE,ADYT,ADP1,ADXT,DYI1,DPT1,DXT1,
7DMPRO,DEPRO,MSAMP,ADM1,ADE1
```

```
C
C LUP=10
```

```
C
C WRITE(LUP,1000)
```

```
1000 FORMAT('1TYPE INITIAL VALUES'/ ' DM(2)-DM(9)')
```

```
DO 100 I=2,9
```

```
CALL READIN(IND,VAL,IVAL)
```

```
DM(I)=VAL
```

```
C
C WRITE(LUP,1010)
```

```
1010 FORMAT(' DE(2)-DE(5)')
```

```
DO 110 I=2,5
```

```
CALL READIN(IND,VAL,IVAL)
```

```
DE(I)=VAL
```

```
C
C WRITE(LUP,1020)
```

```
1020 FORMAT(' D(2)-D(9)')
```

```
DO 120 I=2,9
```

```
CALL READIN(IND,VAL,IVAL)
```

```
D(I)=VAL
```

```
C
C WRITE(LUP,1030)
```

```
1030 FORMAT(' G(1),RMDOT0,XDOT0,PDOT0,U0')
```

```
CALL READIN(IND,VAL,IVAL)
```

```
G(1)=VAL
```

```
CALL READIN(IND,RMDOT0,IVAL)
```

```
CALL READIN(IND,XDOT0,IVAL)
```

```
CALL READIN(IND,PDOT0,IVAL)
```

```
CALL READIN(IND,U0,IVAL)
```

```
C
C WRITE(LUP,1040)
```

```
1040 FORMAT(' DM0,DE0,XFT0,X0')
```

```
CALL READIN(IND,DM0,IVAL)
```

```
CALL READIN(IND,DE0,IVAL)
```

```
CALL READIN(IND,XFT0,IVAL)
```

```
CALL READIN(IND,X0,IVAL)
```

```
C
C WRITE(LUP,1050)
```

```
1050 FORMAT(' NSAMP,ILP,IDK')
```

```
CALL READIN(IND,VAL,NSAMP)
```

```
CALL READIN(IND,VAL,ILP)
```

```
CALL READIN(IND,VAL,IDK)
```

```
C
C IF(IDK) 125,130,125
```

```
125 WRITE(LUP,1060)
```

```
1060 FORMAT(' FNAM')
```

```
CALL READIN(IND,FNAM,I VAL)
C
130 WRITE(LUP,1070)
1070 FORMAT(' RM0,P0,ADXF')
CALL READIN(IND,RM0,I VAL)
CALL READIN(IND,P0,I VAL)
CALL READIN(IND,ADXF,I VAL)
C
WRITE(LUP,1080)
1080 FORMAT(' ADM,ADE,DMPRO,DEPRO,ADM1,ADE1,MSAMP')
CALL READIN(IND,ADM,I VAL)
CALL READIN(IND,ADE,I VAL)
CALL READIN(IND,DMPRO,I VAL)
CALL READIN(IND,DEPRO,I VAL)
CALL READIN(IND,ADM1,I VAL)
CALL READIN(IND,ADE1,I VAL)
CALL READIN(IND,VAL,MSAMP)
C
C COMPUTE THE RATE OF CHANGE IN XFT PER QUARTER
C
QDXF=100.*(SQRT(SQRT(1.+ADXF/100.))-1.)
QDM=100.*(SQRT(SQRT(1.+ADM/100.))-1.)
QDE=100.*(SQRT(SQRT(1.+ADE/100.))-1.)
C
RETURN
END
```

```

SUBROUTINE READIN(IND,VAL,IVAL)
C
C READS IN DATA
C
C 1973-05-04
C
C IND-INDICATES TYPE OF INDATA
C 1-HOLLERITH STRING IN VAL
C 3,4-FLOATING POINT IN VAL AND INTEGER IN IVAL
C 7-CARRIAGE RETURN FOUND
C
C SUBROUTINES REQUIRED:
C RLINE
C
C DIMENSION BUFF(16)
C
C LTTIN=8
C LTTOUT=9
C IR=2
C
100 CALL RLINE(LTTOUT,LTTIN,BUFF,IR)
C ICHAR=1
C CALL RIFF(ICCHAR,VAL,IVAL,IND)
C IF(IND-2) 110,100,110
110 IF(IND-6) 120,100,120
120 IF(IND-8) 130,100,130
130 RETURN
C END

```

SUBROUTINE OUTDAT(ITYP)

OUTPUT OF DATA FROM USEC

1973-05-04

ITYP:

1:-PRINT INITIAL DATA

2:-PRINT DATA FROM ONE STEP OF THE SIMULATION

COMMON DYT,DM(9),RM,RM0,DE(5),DPT,P,P0,D(9),U(17),G(2),
 1RLT,RMDOT,XDOT(17),PDOT(17),RST,XFT,X(2),DXT,
 2X0,RMDOT0,XDOT0,PDOT0,U0,DM0,DE0,ADXF,QDXF,XFT0,
 3CDY,CMY(5),CE(5),CDP,CD(9),CMP(9),CU1,CU2,CU3,
 4CRL1,CRL2,CRL3,CXL(17),CPL(17),
 5CRS1,CRS2,CRS3,CXS(17),CPS(17),NSAMP,IDK,FNAM,ILP,
 6ADM,QDM,ADE,QDE,ADYT,ADPT,ADXT,DYT1,DPT1,DXT1,
 7DMPRO,DEPRO,MSAMP,ADM1,ADE1

LP=6

GO TO (100,200),ITYP

PRINT INITIAL DATA

WRITE(LP,1000)

FORMAT(' THE US ECONOMY SIMULATED WITH THE SAINT LOUIS MODEL'
 1//' INITIAL DATA:')

WRITE(LP,1010) (DM(I),I=2,9),(DE(I),I=2,5),(D(I),I=2,9)

FORMAT(' DM(I):',8G13.4,/' DE(I):',4G13.4,/' D(I):',8G13.4)

WRITE(LP,1020) G(1),RMDOT0,XDOT0,PDOT0,U0

FORMAT(' G(1)=',G13.4,5X,'RMDOT0=',G13.4,5X,'XDOT0=',G13.4,
 15X,'PDOT0=',G13.4,5X,'U0=',G13.4)

WRITE(LP,1030) DM0,DE0,XFT0,X0

FORMAT(' DM0=',G13.4,5X,'DE0=',G13.4,5X,'XFT0=',G13.4,5X,'X0=',
 1G13.4)

WRITE(LP,1040) NSAMP,ILP,IDK

FORMAT(' NSAMP=',15,5X,'ILP=',12,5X,'IDK=',12)

IF (IDK) 110,120,110

WRITE(LP,1050) FNAM

FORMAT(' FNAM=',A5)

WRITE(LP,1052) RM0,P0,ADXF,QDXF

FORMAT(' RM0=',G13.4,5X,'P0=',G13.4,5X,'ADXF=',G13.4
 1,5X,'QDXF=',G13.4)

WRITE(LP,1054) ADM,QDM,ADE,QDE

FORMAT(' ADM=',G13.4,'QDM=',G13.4,'ADE=',G13.4,'QDE=',G13.4)

WRITE(LP,1056) DMPRO,DEPRO,ADM1,ADE1,MSAMP

FORMAT(' DMPRO ',G13.4,'DEPRO=',G13.4,'ADM1=',G13.4,'ADE1='
 1,G13.4,'MSAMP=',18)

WRITE(LP,1060)

FORMAT(///' DATA IN THE FOLLOWING ORDER: DYT,DPT,U(1),
 1RLT,RST,X(1),G(1),ADYT,ADPT,ADXT,D(1),XFT')

GO TO 999

PRINT DATA FROM ONE STEP OF THE SIMULATION


```
C
200 WRITE(LP,1200) DYT,DPT,U(1),RLT,RSI,X(1),G(1),ADYT,ADPT,ADXT,
    1 D(1),XFT
1200 FORMAT(1X,5G10.3,G11.4,5G10.3,G11.4)
C
    GO TO 999
C
C
999 RETURN
    END
```

SUBROUTINE COEF

GIVES VALUES TO THE COEFFICIENTS IN THE SAINT LOUIS MODEL

1973-05-04

COMMON DYT,DM(9),RM,RMD,DE(5),DPT,P,PQ,D(9),U(17),G(2),
 1RLT,RMDOT,XDOT(17),PDOT(17),RST,XFT,X(2),DXT,
 2X0,RMDOT0,XDOT0,PDOT0,U0,DM0,DE0,ADXF,QDXF,XFT0,
 3CDY,CMY(5),CE(5),CDP,CD(9),CMP(9),CU1,CU2,CU3,
 4CRL1,CRL2,CRL3,CXL(17),CPL(17),
 5CRS1,CRS2,CRS3,CXS(17),CPS(17),NSAMP,IDX,FNAM,ILP,
 6ADM,QDM,ADE,QDE,ADYT,ADPT,ADXT,DYT1,DPT1,DXT1,
 7DMPRO,DEPRO,MSAMP,ADM1,ADE1

TOTAL SPENDING EQUATION

CDY=2.67
 CMY(1)=1.22
 CMY(2)=1.80
 CMY(3)=1.62
 CMY(4)=.87
 CMY(5)=.06
 CE(1)=.56
 CE(2)=.45
 CE(3)=.01
 CE(4)=-.43
 CE(5)=-.54

PRICE EQUATION

CDP=2.110277
 CD(1)=-.024316
 CD(2)=-.009318
 CD(3)=.002605
 CD(4)=.011455
 CD(5)=.017231
 CD(6)=.019933
 CD(7)=.019561
 CD(8)=.016114
 CD(9)=.009594
 CMP(1)=.063407
 CMP(2)=.209061
 CMP(3)=.316541
 CMP(4)=.385845
 CMP(5)=.416975
 CMP(6)=.409930
 CMP(7)=.364710
 CMP(8)=.281315
 CMP(9)=.159745

UNEMPLOYMENT RATE EQUATION

CU1=3.90
 CU2=.04
 CU3=.28

LONG TERM INTEREST RATE EQUATION

CRL1=1.28
 CRL2=-.06
 CRL3=1.42

5 DO 5 I=1,6
CXL(1)=.02
DO 6 I=7,14
6 CXL(1)=0.01
DO 7 I=15,17
7 CXL(1)=0.
C

CPL(1)=0.02
CPL(2)=.03
CPL(3)=.04
CPL(4)=.06
CPL(5)=.06
CPL(6)=.07
10 DO 10 I=7,11
CPL(1)=.08
CPL(12)=.07
CPL(13)=.06
CPL(14)=.06
CPL(15)=.04
CPL(16)=.03
CPL(17)=.02

C
C
C SHORT TERM INTEREST RATE EQUATION

CRS1=-1.13
CRS2=-.17
CRS3=.92
20 DO 20 I=1,5
CXS(1)=.08
CXS(6)=.07
CXS(7)=.07
CXS(8)=.06
CXS(8)=.05
CXS(10)=.03
CXS(11)=.02

C
CPS(1)=-.04
CPS(2)=.04
CPS(3)=.09
CPS(4)=.14
CPS(5)=.17
CPS(6)=.18
CPS(7)=.19
CPS(8)=.18
CPS(9)=.15
CPS(10)=.11
CPS(11)=.06

C
C
C
RETURN
END

SUBROUTINE SIMUS

SIMULATES ONE STEP OF THE SAINT LOUIS MODEL

1973-05-04

SUBROUTINES REQUIRED: NONE
DIMENSION SL(17)

```
COMMON DYT,DM(9),RM,RM0,DE(5),DPT,P,P0,D(9),U(17),G(2),
1RLT,RMDOT,XDOT(17),PDOT(17),RST,XFT,X(2),DXT,
2X0,RMDOT0,XDOT0,PDOT0,U0,DM0,DE0,ADXF,QDXF,XFT0,
3CDY,CMY(5),CE(5),CDP,CD(9),CMP(9),CU1,CU2,CU3,
4CRL1,CRL2,CRL3,CXL(17),CPL(17),
5CRS1,CRS2,CRS3,CXS(17),CPS(17),NSAMP,IDK,FNAM,ILP,
6ADM,QDM,ADE,QDE,ADYT,ADPT,ADXT,DYT1,DPT1,DXT1,
7DMPRO,DEPRO,MSAMP,ADM1,ADE1
```

TOTAL SPENDING EQUATION

$$DYT = CDY + SCAPRO(CMY(1), 1, DM(1), 1, 5)$$

$$1 + SCAPRO(CE(1), 1, DE(1), 1, 5)$$

DEMAND PRESSURE

$$D(1) = DYT - (XFT - X(1))$$

PRICE EQUATION

$$DPT = CDP + SCAPRO(CD(1), 1, D(1), 1, 9)$$

$$1 + SCAPRO(CMP(1), 1, DM(1), 1, 9)$$

REAL GNP

$$DXT = DYT - DPT$$

$$X(2) = X(1)$$

$$X(1) = X(1) + DXT$$

UNEMPLOYMENT RATE EQUATION

$$G(2) = G(1)$$

$$G(1) = 100. * (XFT - X(1)) / XFT$$

CALL RMOVE(U(16), -1, U(17), -1, 16)

$$U(1) = CU1 + CU2 * G(1) + CU3 * G(2)$$

COMPUTE ANNUAL RATES OF CHANGE

$$RMDOT = 100. * (((RM + DM(1)) / RM) ** 4 - 1.)$$

$$RM = RM + DM(1)$$

$$CALL RMOVE(XDOT(16), -1, XDOT(17), -1, 16)$$

$$XDOT(1) = 100. * ((X(1) / (X(1) - DXT)) ** 4 - 1.)$$

$$CALL RMOVE(PDOT(16), -1, PDOT(17), -1, 16)$$

$$PDOT(1) = 100. * (((P + DPT) / P) ** 4 - 1.)$$

$$P = P + DPT$$

LONG TERM INTEREST RATE EQUATION

DO 100 I=1,17

```
100 SL(1)=PDOT(1)/U(1)*4.  
    RLT=CRL1+CRL2*RMDOT+CRL3+SCAPRO(CXL(1),1,XDOT(1),1,17)  
    1+SCAPRO(CPL(1),1,SL(1),1,17)
```

```
C  
C  
C  
    SHORT TERM INTEREST RATE EQUATION
```

```
    RST=CRS1+CRS2*RMDOT+CRS3+SCAPRO(CXS(1),1,XDOT(1),1,11)+  
    1SCAPRO(CPS(1),1,SL(1),1,11)  
    RETURN  
    END
```