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PLENARY LECTURE AT THE 8TH IFAC WORLD CONGRESS
AUGUST 24-28, 1981

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PLENARY LECTURE AT THE 8th IFAC WORLD CONGRESS, AUGUST 24-28, 1981

Abstract

The purpose of the travel was a formal invitation to present a plenary lecture at the 8th IFAC World Congress, August 24-28, 1981 to be held in Tokyo, Japan.

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INNEHÅLLSFÖRTECKNING

1. INLEDNING

IFAC (International Federation of Automatic Control) är den internationella sammanslutningen för reglertekniker. Kongressen arrangeras vart tredje år. Den är mycket omfattande och har ambitionen att täcka alla aspekter av reglertekniken.

2. INTRYCK

Kongressen var mycket välorganiserad. Alla föredrag fanns tillgängliga i preprints (13 kg). Proceedings är under utgivning. En intressant nyhet var ett datorsystem där deltagarnas intresseprofiler hade lagrats. Adaptiv reglering låg högst upp på listan. Jag följde föredragen i special sessionerna om adaptiv reglering. Dessutom besökte jag sessioner om robotics, computer aided design, large scale system, education, and picture processing. Jämsides med det formella programmet visades också filmer om robotforskning i Japan.

Specialsessioner på denna typ av kongresser är tyvärr sällan aktuella på grund av den långa administrativa fördröjningen. Det stora värdet ligger i stället på de informella kontakter som man kan knyta. Jag hade goda tillfällen att förnya många gamla kontakter, liksom att knyta nya kontakter. Detta kommer att påverka inriktningen av den framtida forskningen på institutionen.

3. PLENARFÖRELÄSNINGEN

THEORY AND APPLICATIONS OF ADAPTIVE CONTROL

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Abstract. Progress in theory and applications of adaptive control are reviewed. Different approaches to adaptive control are discussed with particular emphasis on model reference adaptive systems and self-tuning regulators. Techniques for analysing adaptive systems are discussed. This includes stability and convergence analysis, averaging methods, and stochastic control theory. Issues of importance for applications are covered. This includes parameterization, tuning, and tracking, as well as different ways of using apaptive control. An overview of applications is given. This includes feasibility studies as well as products based on adaptive techniques.

Keywords. Adaptive control; model reference; self-tuning regulators, gain scheduling, stability analysis, stochastic control theory; dual control; autotuning.

1. INTRODUCTION

Research on adaptive control was very active in the early fifties. It was motivated by design of autopilots for high performance aircrafts. The work was characterized by a lot of enthusiasm, bad hardware, and nonexisting theory. A presentation of the results are found in Gregory (1959). Interest in the area diminished due to lack of fundamental insight and disaster in a flight test.

In the sixties there were many contributions to control theory, which were fundamental for the development of adaptive control. State space and stability theory were developed. There were also important results in stochastic control theory. Dynamic programming, introduced by Bellman (1961) and dual control theory introduced by Feldbaum (1965), increased the understanding of adaptive processes. Fundamental contributions were also made by Tsypkin (1973), who showed that many of the schemes for learning and adaptive control could be described in a common framework and that certain recursive equations of the stochastic approximation type played a fundamental role. There were also major developments in system identification and in parameter estimation.

The interest in adaptive control was renewed in the seventies. The progress in control theory during the sixties contributed to an improved understanding of adaptive control. The rapid and revolutionary progress in microelectronics has made it possible to implement adaptive regulators simply and cheaply. There is now a vigorous development of the field both at universities and in industry.

This paper gives an overview of theory and applications of adaptive control. Particular emphasis is given to those techniques which are used in current applications.

2. APPROACHES TO ADAPTIVE CONTROL

Three schemes for parameter adaptive control: gain scheduling, model reference control and self-tuning regulators are described in a common framework. The starting point is an ordinary feedback control loop with a process and a regulator with adjustable parameters. The key problem is to find a convenient way of changing the regulator parameters in response to changes in process and disturbance dynamics. The schemes differ only in the way the parameters of the regulator are adjusted.

Gain scheduling

It is sometimes possible to find auxiliary process variables, which correlate well with the changes in process dynamics. It is then possible to eliminate the influences of parameter variations by changing the parameters of the regulator as functions of the auxiliary variables. See Fig. 1. This approach is called gain scheduling, because the system was originally used to accomodate changes in process gain only.

Gain scheduling is an open loop scheme comparable to feedforward compensation. There is no feedback to compensate for an incorrect schedule. It has the advantage that the parameters can be changed very quickly in response to process changes.

There is a controversy in nomenclature whether gain scheduling should be considered as an adaptive scheme or not because the parameters are changed in open loop. Irrespective of this discussion, gain scheduling is a very useful technique to reduce the effects of parameter variations.

Model reference adaptive systems MRAS

Another way to adjust the parameters of the regulator is illustrated in Fig. 2. This scheme was originally developed for the servo problem. The specifications are given in terms of a reference model, which tells how the process output ideally should respond to the command signal. Notice that the reference model is part of the control system. The regulator can be thought of as consisting of two loops. The inner loop is an ordinary control loop composed of the process and the regulator. The parameters of the regulator are adjusted by the outer loop, in such a way that the error e between the model output y_m and the process output y becomes small. The outer loop thus also looks like a regulator loop. The key problem is to determine the adjustment mechanism so that a stable system, which brings the error to zero, is obtained. This problem is nontrivial. It is easy to show that it can not be solved with a simple linear feedback from the error to the controller parameters.

The following parameter adjustment mechanism, called the 'MIT-rule', was

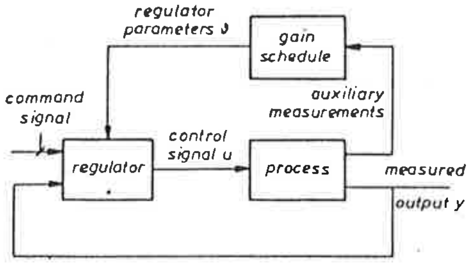

Fig. 1. Block diagram of a system with gain scheduling.

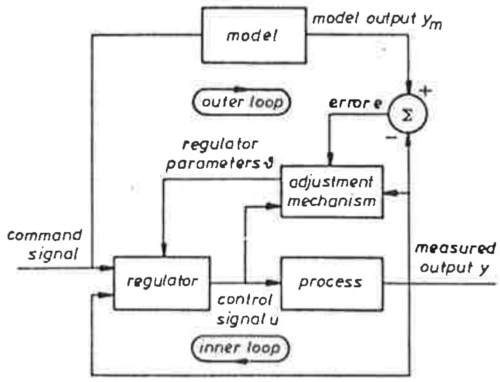

Fig. 2. Block diagram of model reference adaptive system (MRAS).

used in the original MRAS:

$$\frac{d\vartheta}{dt} = - k\ e\ \mathrm{grad}_{\vartheta}\ e. \qquad (1)$$

The number k is a parameter, which determines the adjustment rate, e is the model error, and the components of the vector ϑ are the adjustable parameters. Equation (1) represents an adjustment mechanism, which is composed of three parts: a linear filter for computing the sensitivity derivatives from process inputs and outputs, a multiplier, and an integrator. This configuration is typical for many adaptive systems.

The MIT-rule will adapt slowly but otherwise perform well, if the parameter k is small. The allowable size depends on the magnitude of the reference signal. Consequently it is not possible to give fixed limits, which guarantee stability. The MIT-rule can thus give an unstable closed loop system. Modified adjustment rules can be obtained using stability theory. These rules are similar to the MIT-rule. The sensitivity derivatives in (1) will be replaced by other functions. This is discussed further in Section 3.

The MRAS was originally proposed by Whitaker (1958). Further work was done by Parks (1966), Monopoli

(1974), and Landau (1974). The book Landau (1979) gives a comprehensive treatment of work up to 1977. It also includes many references. Recent contributions are discussed in Section 4.

Self-tuning_regulators_STR

A third way of adjusting the parameters is to use the self-tuning regulator. Such a system is shown in Fig. 3. The regulator can be thought of as composed of two loops. The inner loop consists of the process and an ordinary linear feedback regulator. The parameters of the regulator are adjusted by the outer loop, which is composed of a recursive parameter estimator and a design calculation.

Notice that the self-tuner in Fig. 3 includes an on-line solution to a design problem for a system with known parameters. This is called the underlying_design_problem.

The self-tuning regulator is very flexible with respect to the design method. Virtually any design technique can be accomodated. So far self-tuners based on phase and amplitude margins, pole-placement, minimum variance control, and linear quadratic gaussian control have been considered. Many different parameter estimation schemes may be used, for example stochastic approximation, least squares, extended and generalized least squares, instrumental variables, extended Kalman filtering and the maximum likelihood method. See Åström (1980) and Kurz et al (1980).

The regulator shown in Fig. 3 can also be derived from the MRAS approach if the parameter estimation is done by updating a reference model. The scheme is then called an indirect_MRAS, because the regulator parameters are updated indirectly via the design calculation. See Narendra and Valavani (1979).

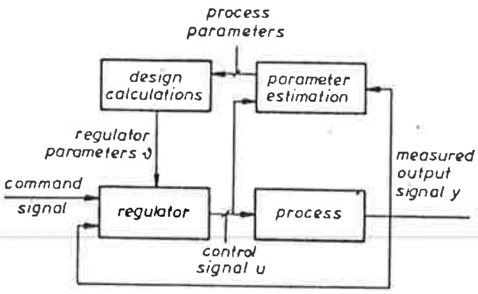

Fig. 3. Block diagram of a self-tuning regulator (STR).

The self-tuning regulator was originally proposed by Kalman, who built a special purpose computer to implement the regulator. The self-tuning regulator has recently received considerable attention, because it is flexible and easy to understand. See Peterka (1970), Åström and Wittenmark (1973), Kurz et al (1980), Clarke and Gawthrop (1975, 1979), Wellstead et al (1979), Åström et al (1977), Åström and Wittenmark (1980).

Implicit_and_explicit_self-tuners

The self-tuner shown in Fig. 3 is called an explicit_STR or an STR based on estimation of an explicit process model. It is sometimes possible to reparameterize the process so that it can be expressed in terms of the regulator parameters. This gives a significant simplification of the algorithm, because the design calculations are eliminated. Such a self-tuner is called an implicit_STR, because it is based on estimation of an implicit process model. The algorithm is closely related to the direct MRAS. The relations between MRAS and STR are further discussed in Egardt (1979,1980), Landau (1979), and Åström (1980).

3. AN EXAMPLE

The different approaches to adaptive control are illustrated by an example. Some formalism needed to discuss theory are also introduced.

The_underlying_design_method_for known_systems

A pole-placement design method is taken as the starting point. This will include many of the proposed schemes. Consider a single-input single-output system

$$Ay = Bu, \qquad (2)$$

where u is the control signal and y the output signal. The symbols A and B denote polynomials in the forward shift operator. Assume that it is desired to find a regulator such that the transfer function from command signal to output signal is given by

$$G_m = Q/P, \qquad (3)$$

where Q and P are polynomials in the forward shift operator. The solution to the design problem is well known. See e.g. Åström (1979). The regulator is given by

$$Ru = Ty_c - Sy, \quad (4)$$

where y_c is the command signal and R, S and T are polynomials. To determine the regulator the polynomial B is factored into a stable polynomial B^+ and an unstable polynomial B^-. The design problem has a solution if A and B are coprime, if B^- divides Q, and if

$$\deg P - \deg Q > \deg A - \deg B.$$

Let T_1 be the observer polynomial. Solve the diophantine equation

$$AR_1 + B^- S = PT_1 \quad (5)$$

with respect to R_1 and S. The desired feedback is given by (4) with

$$R = R_1 B^+ \quad \text{and} \quad T = T_1 Q/B^-. \quad (6)$$

The equation (5) is obtained from the requirement that the system (2) with the feedback (4) has the transfer function (3).

An explicit self-tuner

The explicit self-tuner can be expressed as follows.

ALGORITHM 1

Step 1: Estimate the coefficients of the polynomials A and B in (2).

Step 2: Substitute A and B by the estimates obtained in step 1 and solve the equation (5) for R_1 and S.

Step 3: Calculate the control signal from (4).

The steps 1, 2, and 3 are repeated at each sampling period.

□

An implicit self-tuner

An implicit self-tuner may be derived as follows. It follows from (5) that

$$PT_1 y = AR_1 y + B^- Sy = BR_1 u + B^- Sy$$
$$= B^- [Ru + Sy], \quad (7)$$

where the second equality follows from (2) and the third from (6).

Notice that equation (7) can be interpreted as a process model, which is parameterized in B^- R and S. An estimation of the parameters of the model (7) gives the regulator parameters directly. Notice also that the model (7) is linear in the parameters only if $B^- = 1$. The implicit algorithm can be expressed as follows.

ALGORITHM 2

Step 1: Estimate the coefficients of the polynomials R, S and B^- in (7).

Step 2: Calculate the control signal from (4), where R and S are substituted by the estimates obtained in Step 1.

The control law will not be causal if the leading coefficient of the estimate of the polynomial R is zero. Minor modifications are required to avoid this difficulty. See Åström and Wittenmark (1973), Goodwin et al (1980), Goodwin and Sin (1980).

The steps 1 and 2 are repeated each sampling period.

□

Parameter estimation

The parameter estimators for models like (2) and (4), which are linear in the parameters, are all very similar. Consider for example estimation of the coefficients of the polynomials R and S in (4). Several estimations methods can be described by

$$\theta(t) = \theta(t-1)+a(t)M(t)\varphi(t)\varepsilon(t), \quad (8)$$

where

$$\varepsilon = PT_1 y - \hat{R}u - \hat{S}y = PT_1 y - \varphi^T \theta. \quad (9)$$

The elements of the vector φ are delayed values of the input u and the output y, and θ is a vector of the unknown parameters. The variable M depends on the particular estimation technique. It is a constant in MRAS. In stochastic approximation it is the scalar

$$M(t) = t\,[\Sigma\varphi^T(k)\varphi(k)]^{-1}. \quad (10)$$

In the least squares method it is the matrix

$$M(t) = t\,[\Sigma\varphi(k)\varphi^T(k)]^{-1}. \quad (11)$$

4. THEORY

The closed loop systems obtained with adaptive control are nonlinear. This makes analysis difficult, particularly if there are random disturbances. Progress in theory has therefore been slow and painstaking. Current theory gives insight into some special problems. Much work still remains before a reasonably complete theory is available.

Analysis of stability, convergence, and performance are key problems. Since parameter estimation is an essential part of the systems, it is also of interest to know how the parameter estimates behave. It would also be desirable to have theory, which tells if control structures like those in Section 2 are reasonable, or if there are better ways to do adaptive control.

Stability

Stability analysis has not been applied to systems with gain scheduling. This is surprising, because such systems are simpler than MRAS and STR.

The stability theories of Lyapunov and Popov have been extensively applied to adaptive control. The major developments of MRAS were all inspired by the desire to construct adjustment mechanisms, which would give stable solutions. Parks (1966) applied Lyapunov theory to the general MRAS problem for systems with state feedback and also output feedback for systems, whose transfer functions are strictly positive real. Landau (1979) applied hyperstability to a wide variety of MRAS configurations. The key observation in all these works is that the closed loop system can be represented as shown in Fig. 4.

The system can thus be viewed as composed of a linear system and a nonlinear passive system. If the linear system is strictly positive real, it follows from the passivity theorem that the error e goes to zero. See e.g. Desoer and Vidyasagar (1975).

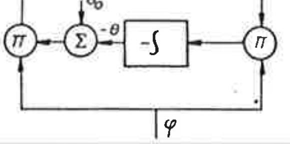

Fig. 4. Block diagram representation of a MRAS. e is the (filtered) model error, φ is a vector of regression variables, θ is the adjustable parameters and θ_0 their true values.

To obtain the desired representation it is necessary to parameterize the model so that it is <u>linear in the parameters</u>. This requirement strongly limits the algorithms that can be considered.

Problems with output feedback poses additional problems, because it is not possible to obtain the desired representation by filtering the model error. Monopoli (1974) showed that it is necessary to augment the error by adding additional signals. For systems with output feedback the variable e in Fig. 4 should thus be the augmented error.

There are some important details in the stability proofs based on Fig. 4. To ensure stability it must be shown that the vector φ is bounded. This is easy for systems which only has a variable gain, because φ is simply the command signal. The components of the vector φ are, however, in general functions of the process inputs and outputs. It is then a nontrivial problem to ensure that φ is bounded. It should also be noticed that it follows from the passivity theorem that ε goes to zero. The parameter error will not go to zero unless the matrix $\Sigma\varphi\varphi^T/t$ is always larger than a positive definite matrix.

For the case of output feedback there is an additional difficulty because the signal ε is the augmented error. It thus remains to show that the model error also goes to zero.

Several of these difficulties remained unnoticed for several years. The difficulties were pointed out in Morgan and Narendra (1977), Feuer and Morse (1978). Complete stability proofs were given recently by Egardt (1979), Fuchs (1979), Goodwin et al (1980), de Larminat (1979), Morse (1980), and Narendra et al (1980). The following result is due to Goodwin et al (1980).

THEOREM 1

Let the system (2) be controlled by the adaptive Algorithm 2 with $PT_1 = z^m$, $\bar{B} = 1$ and modified stochastic approximation estimation. Assume that

(A1) the pole excess d = deg A − − deg B is known

(A2) the estimated model is at least of the same order as the process
(A3) the polynomial B has all zeros inside the unit disc.

The signals u and y are then bounded and y goes to the command signal as time goes to infinity.

□

The proof is not based on hyperstability theory. It is an analysis based on the particular structure of the problem. Notice that the theorem does not say that the parameter estimates converge.

Theorem 1 is important, because it is a simple and rigorous stability proof for a reasonable adaptive problem. The assumptions required are however very restrictive.

The assumption A1 means for discrete systems that the time delay is known with a precision, which corresponds to a sampling period. This is not unreasonable. For continuous time systems the assumption means that the slope of the high frequency asymptote of the Bode diagram is known. If this is the case, it is possible to design a robust high gain regulator for the problem. See Horowitz (1963).

Assumption A2 is very restrictive, since it implies that the estimated model must be at least as complex as the true system, which may be nonlinear with distributed parameters. Almost all control systems are in fact designed based on strongly simplified models. High frequency dynamics is often neglected in the simplified models. It is therefore very important that a design method can cope with model uncertainty at high frequencies. Compare Horowitz (1963).

Assumption A3 is also crucial. It arises from the necessity to have a model, which is linear in the parameters. It follows from (8) that this is possible only if $B^-=1$. In other words the underlying design method is based on cancellation of all process zeros. Such a design will not work even for systems with known constant parameters if the system is nonminimum phase.

Also notice that the theorem applies only to the tuning case, i.e. when the estimator gain goes to zero as time increases. It is a nontrivial extension to consider tracking as is discussed in Section 4.

Theorem 1 also requires that there are no disturbances. Similar results for bounded disturbances are given by Egardt (1980 a,b). To obtain stability under disturbances the estimation algorithm is, however, modified. A saturation, which limits the parameter estimates, is introduced. Alternatively a dead zone is introduced, which keeps the estimates constant when the residuals are small. It is not known whether these assumptions are technicalities or necessities. Egardt also gives results for continuous time systems.

Convergence analysis

The essential problems of convergence analysis are to investigate if the parameter estimates converge and to determine the convergence rate.

For explicit algorithms the problem is equivalent to analysing the convergence of the recursive parameter estimator. This problem is dealt with extensively in identification theory. There are complications in the adaptive case, since the process input is generated by feedback.

The excitation of the process depends on the process disturbances. When developing the theory it is commonly assumed that the system is driven by random disturbances. It is then possible to use ergodic theory and martingale theory.

A very general proof for convergence of the least squares algorithm was given by Sternby (1977) by applying a martingale convergence theorem. An extension of this result can be applied to show convergence of adaptive systems. See Sternby (1981).

A convergence theorem for the simple self-tuner based on modified stochastic approximation estimation, eq. (8) with $a(t) = a_0/t$, and minimum variance control, i.e. Algorithm 2 with $P T_1 = z^m$, $B^- = 1$ and $d=1$ was given by Goodwin et al (1981). They investigated a system described by the model

$$Ay = Bu + Ce, \tag{12}$$

where e is white noise.

Under assumptions A1, A2, and A3 of Theorem 1, with a pole excess equal to one, it was shown that the input and the output are mean square bounded, and that the variance of the output will converge to the minimum variance if the function

$$G(z) = C(z) - a_0/2 \tag{13}$$

is strictly positive real.

Various extensions to larger pole excess and different modifications of the least squares estimation have been given. See Goodwin and Sin (1980), where convergence proofs for many variations of the algorithm are given. A convergence proof for the general Algorithm 2 with least squares estimation is, however, still not available.

The method of averaging

The algorithms in Section 2 are motivated by the assumption that the parameters change slower than the other variables in the system. It is then natural to try to describe the paramters approximatively by approximating $P\varphi\varepsilon$ in (8) by its average. For estimators, whose gain does not go to zero, the estimates may then be approximated by the solution to the difference equation, obtained by taking averages of (8). A better approximation is obtained by adding a stochastic term, which approximates the fluctuations around the mean value. Such approximations have been investigated in great detail by Kushner (1977).

For the tuning problem the gain a of the estimator (8) will go to zero. To apply the method of averages, it is useful to transform the time scale. Consider for example the stochastic approximation method. Introduce the transformed time defined by

$$\tau = m_o \sum a(k), \qquad (14)$$

where m_o is the limit of (10).

Using the method of averages Ljung (1977a) showed that the estimates will approximatively be described by the solutions to the ordinary differential equation

$$\frac{d\theta}{d\tau} = f(\theta), \qquad (15)$$

where

$$f(\theta) = E\varphi\varepsilon$$

and the mean value is calculated under the assumption that the parameter θ is constant.

Ljung also showed that the estimates will converge to the solution of (15) as time increases.

The method of averages is useful, because it makes it possible to investigate convergence rates for special problems. It also makes it possible to determine if equilibrium values for the parameters are stable.

A drawback with the method of averages is that it is based on the assumption that the signals are bounded. To use the method boundedness must be determined by other techniques.

Under the assumption of bounded signals Ljung (1977b) showed that the self-tuner, based on least squares estimation and minimum variance control, converges to the minimum variance solution under assumptions A1, A2 and A3 of Theorem 1, if the function

$$H(z) = C(z) - 1/2 \qquad (16)$$

is strictly positive real.

Using the method of averages, Holst (1979) showed that the algorithm is locally stable if the function C(z) is positive at the zeros of B(z). If C is negative at a zero of B, Ljung and Wittenmark (1974) have constructed examples, which show that (15) is unstable and that the parameter estimates do not converge. In this example the equation (15) has a limit cycle. Because of the transformation (14) the estimates will oscillate with ever increasing period.

Since the convergence of the parameters depend on the polynomial C, it is clear that convergence can be lost by changes of the disturbances.

Stochastic control theory

Regulator structures like MRAS and STR are based on heuristic arguments. It would be appealing to obtain the regulators from a unified theoretical framework. This can be done using nonlinear stochastic control theory. The system and its environment are then described by a stochastic model. The criterion is formulated as to minimize the expected value of a loss function, which is a scalar function of states and controls.

The problem of finding a control, which minimizes the expected loss function, is difficult. Conditions for existence of optimal controls are not known. Under the assumption that a solution exists, a functional equation for the optimal loss function can be derived using dynamic programming. This equation, which is called the Bellman equation, can be solved numerically only in very simple cases. The structure of the optimal regulator obtained is shown in Fig. 5. The controller can be thought of as composed of two parts: an estimator and a feedback regulator. The estimator generates the conditional probability distribution of the state from the measurements. This distribution is called the hyperstate of the problem. The feedback regulator is a nonlinear function, which maps the hyperstate into the space of control variables.

The structural simplicity of the solution is obtained at the prize of introducing the hyperstate, which is a quantity of very high dimension. Notice that the structure of the regulator is similar to the STR in Fig. 3. There is, however, no distinction between parameters and other state variables.

The optimal control law has an interesting property. The control will not only try to drive the output to its desired value. When the parameters are uncertain, the regulator also introduces perturbations, which will improve the estimates and the future controls. The optimal control gives the correct balance between maintaining small control and estimation errors. This property is called dual control. See Feldbaum (1965), Jacobs and Patchell (1972), and Bar-Shalom and Tse (1974). Optimal stochastic control theory also offers other possibilities to obtain sophisticated adaptive algorithms. See Saridis (1977).

EXAMPLE

A simple example is used for illustration. Consider a system described by

$$y(t+1) = y(t) + bu(t) + e(t),$$

where u is the control, y the output, e white noise, and b is a constant parameter or a Wiener process. Let the criterion be to minimize the mean square deviation of the output y.

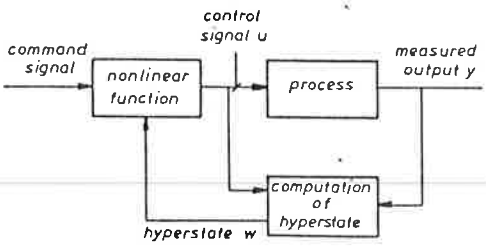

Fig. 5. Block diagram of an adaptive regulator obtained from stochastic control theory.

If the parameter b has a gaussian prior distribution, it follows that the conditional distribution of b, given inputs and outputs up to time t, is gaussian with mean $\hat{b}(t)$ and standard deviation $\sigma(t)$. The hyperstate can then be characterized by the triple $(y(t), \hat{b}(t), \sigma(t))$. The euations for updating the hyperstate are the same as the ordinary Kalman filtering equations. See Åström (1970). The dual control law can be simplified, because the scaled control variable $v = u\hat{b}/y$ depends only on the variables y and $\hat{b}/\sigma$. A graphical representation of the control law is given in Fig. 6.

Some approximations to the optimal control law are also illustrated in Fig. 6. The certainty equivalence control

$$u(t) = -y(t)/\hat{b}$$

is obtained simply by solving the control problem in the case of known parameters and substituting the known parameters with their estimates. The self-tuning regulator can be interpreted as a certainty equivalence control.

The control law

$$u(t) = -\frac{1}{\hat{b}(t)} \cdot \frac{\hat{b}^2(t)}{\hat{b}^2(t)+\sigma^2(t)}\, y(t)$$

is another approximation, which is called cautious control, because it hedges and uses lower gain when the estimates are uncertain.

Notice that the scaled control variable v is equal to one for

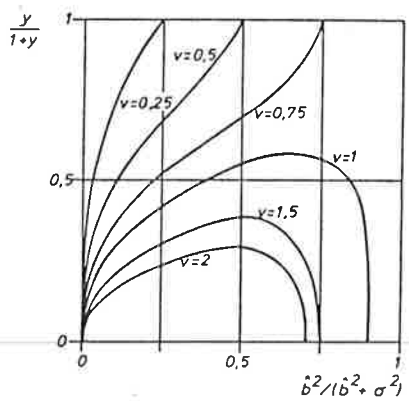

Fig. 6. Optimal dual control law for the integrator plants. The graph shows the level curves for the scaled control variable. $v = u\hat{b}/\sigma$.

certainty equivalence control and that $v = \hat{b}^2/(\hat{b}^2+\sigma^2)$ for cautious control. The different control laws can thus easily be compared in Fig. 6. There are only small differences between all control laws if $\hat{b} > 2\sigma$. Below the curve v=1 the dual control will always give larger control variables than the certainty equivalence control. In this region the certainty equivalence control is a much better approximation than cautious control.

5. APPLICATIONS

There are many ways to use adaptive techniques. A few possibilities are discussed in this section. Since there is no reasonably complete theory for adaptive control, there are many problems, which must be solved intuitively with support of simulation when adaptive control is applied. The situation is not unique for adaptive control; problems of this type also occur when implementing special features in simple regulators. A particular problem, estimator windup, is discussed in some detail. An overview of the status of the applications is also given.

Automatic tuning

Both the MRAS and the STR reduces to constant gain feedback if the estimated parameters are constant. The adaptive loop can thus be used as a tuner for a control loop. In such applications the adaptation loop is simply switched on and run until the performance is satisfactory. The adaptation loop is then disconnected and the system is left running with fixed regulator parameters.

Automatic tuning can be applied to simple PID controllers as well as to more complicated regulators. It is particularly useful when combined with diagnostic tools for checking the performance of the control loops. For minimum variance control the performance evaluation can be done simply by monitoring the covariances of the inputs and the outputs. The tuning can be initiated manually whenever the diagnostics indicate that the loops are out of tune.

Automatic tuning can be incorporated into a DDC-package. One tuning algorithm can then serve many loops. Autotuning can also be included in single loop regulators. For example, it is possible to design regulators where the mode switch has three positions manual, automatic and tuning.

The adaptive control loop may also be used to build a gain schedule. The parameters obtained when the system is running in one operating condition are then stored in a table. The gain schedule is obtained when the process has operated at a range of operating conditions, which covers the operating range.

There are also other ways to combine gain scheduling with adaptation. A gain schedule can be used to quickly get the parameters into the correct region. The adaptive loop can then be used for fine tuning.

Adaptive regulators

The adaptive techniques may of course also be used for genuine adaptive control of systems with timevarying parameters. There are many ways in which this can be done. The operator interface is important, since adaptive regulators also have parameters, which must be chosen.

When applying adaptive techniques it is often desired to have the absolute black box having a blank front panel with no dials. It has been my experience that such regulators can be designed for very specific applications, where the purpose of control can be stated a priori.

In many cases it is, however, not possible to specify the purpose of control a priori. It is at least necessary to tell the regulator what it is expected to do. This can be done by introducing dials that give the desired properties of the closed loop system. Such dials are called performance related. New types of regulators can be designed using this concept. For example, it is possible to have a regulator with one dial, which is labeled the desired closed loop bandwidth. Another possibility would be to have a regulator with a dial, which is labeled with the weighting between state deviation and control action in a LQG problem. A third possibility would be to have a dial labeled phase or amplitude margin.

Parameter tracking

Since the key feature of an adaptive regulator is its ability to track variations in process dynamics, the performance of the parameter estimator is crucial.

A fundamental result of system identification theory is that the input signal to the process must be *persistently exciting* or *sufficiently rich* of order n in order to estimate n parameters. In the adaptive systems the input signal is generated by feedback. Under such circumstances there is no guarantee that the process will be properly excited. On the contrary, with good regulation the excitation can be expected to be poor. Consequently there are inherent limitations unless extra perturbation signals are introduced, as is suggested by dual control theory.

To track parameter variations it is necessary to discount old data. This will involve compromizes. If data is discounted too fast the estimates will be uncertain even if the true parameters are constant. If old data is discounted slowly the estimates of constant parameters will be good. The estimator will, however, be unable to track rapid variations.

If there is good excitation of the process, a simple exponential discounting of past data will work very well. In the least squares estimation algorithm this means that the gain a in (8) is constant and that the matrix M in (11) is changed to

$$M(t) = [\Sigma \lambda^{t-k} \varphi(k)\varphi^T(k)]^{-1}, \qquad (17)$$

where $\lambda < 1$ is a *forgetting factor* or a *discounting factor*. If the excitation of the process is poor, it is seen from (17) that the matrix M will grow exponentially. This is called *estimator windup* in analogy with integrator windup in simple regulators. When the gain is sufficiently large the estimator will be unstable. Small residuals will then lead to very large changes in the parameters, and the closed loop system may become unstable. The process will then be well excited, and the parameter estimates will quickly achieve good values. Looking at the process output there will be periods of good regulation followed by bursts.

There are many ways to avoid covariance windup and bursts. Since the problem is fundamentally caused by poor excitation of the process, one possibility is to monitor the excitation condition and to introduce extra perturbations when the excitation is poor. Such a solution is clearly in the spirit suggested by dual control theory.

In some cases it is not feasible to introduce extra perturbations to obtain good excitation. Covariance windup can then be avoided by discounting old data only when there is proper excitation.

There are also several ad hoc procedures proposed to avoid bursts. One possibility is to use variable forgetting factors as proposed by Fortescue et al (1981). It has also been proposed to switch off the parameter updating under certain conditions, or to introduce limitations on the covariances and the estimator gains. See Irving (1979).

Laboratory experiments

Over the past 10 years there have been extensive laboratory experiments with adaptive control mostly in universities but also to an increasing extent in companies. Schemes like MRAS and STR have been explored extensively. The goal of the experiments has been to understand the algorithms and to investigate many of the factors, which are not properly covered by theory.

Industrial feasibility studies

There have been a number of industrial feasibility studies of adaptive control. The following list covers some of the processes that have been studied.

- raw material blending
- cement grinding mills
- rolling mills
- distillation columns
- chemical reactors
- steam generators
- electrical generators
- power systems
- electrical drives
- positioning systems
- papermachines
- pH control
- autopilots for aircrafts and ships
- machine tools
- heat exchangers
- heating and ventilation systems
- glass manufacturing

The recent publications on applications of adaptive control, Narendra and Monopoli (1980) and Unbehauen (1980), contain details and many references.

The feasibility studies have shown that there are indeed cases, where adaptive control is very useful. They have also shown that there are cases where the benefits are marginal. Since adaptive control is more complicated than constant gain feedback, it is always useful to try the simple things first. The feasibility studies have also shown that it is not easy to judge the need for adaptive control from the variations in open loop dynamics.

Algorithms similar to the ones discussed in this paper have also found extensive applications in the communications field. Among the problems considered we can mention adaptive speech coding and adaptive noise cancellation. Adaptive echo cancellation, based on VLSI technology with adjustment of over 100 parameters, is e.g. beginning to be introduced in telephone systems. A survey of these applications are given in Falconer (1980).

Industrial products

Adaptive control is now also finding its way into industrial products. There appears to be many different ways of using adaptive techniques.

Gain scheduling is the predominant technique for design of autopilots for high performance aircrafts. In the industry it is considered as a well established standard technology.

There are also products based on the STR and the MRAS. There are commercial adaptive regulators for motor drives, rolling mills, cement mills, paper machines, and autopilots for ships. There are also general purpose self-tuners incorporated in DDC packages.

6. CONCLUSIONS

The adaptive technique is slowly emerging after 25 years of research and experimentation. Important theoretical results on stability and structure have recently been established. Much theoretical work still remains to be done. The advent of micro processors has been a strong driving force for the applications. Laboratory experiments and industrial feasibility studies have contributed to a better understanding of the practical aspects of adaptive control. There are also a number of adaptive regulators appearing on the market.

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