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Borisson, Ulf

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U. BORISSON

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Department of Automatic Control
Lund Institute of Technology

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U. Borisson (Sweden)

ABSTRACT

Control of a class of multivariable systems described by linear vector difference equations with constant but unknown parameters is discussed. A multivariable minimum variance strategy is first presented. This gives a generalization of the minimum variance strategy for single-input single-output systems. A multivariable self-tuning regulator based on the minimum variance strategy is then proposed. It uses a recursive least squares estimator and a linear controller obtained using the current estimates. The asymptotic properties of the algorithm are discussed. If the estimated parameters converge, the resulting controller will under certain conditions give the minimum variance strategy. The analysis also gives insight into the case when several simple self-tuning regulators are operating in cascade mode.

1. INTRODUCTION

Many methods for the design of regulators for linear multivariable systems are based on the availability of a model of the process and its environment. However, in practical applications the process dynamics are often unknown, and some model building or identification technique must be used before the regulator can be designed. An algorithm will now be described, which simplifies this procedure for a class of multivariable systems.

The controlled process is assumed to be described by a linear vector difference equation including a moving average of white noise. The process is required to have as many inputs as outputs, to have a minimum phase property and to have an impulse response starting with a nonsingular matrix. It will be shown how a generalization of the minimum variance strategy for single-input single-output systems given by Åström [1] can be obtained in the multivariable case.

The proposed self-tuning regulator uses a recursive parameter estimator, which is based on the least squares method, and a linear controller whose parameters are given by the current estimates. By choosing a special model structure for the identification, the parameters of the controller are obtained directly from the estimator without any further computations. The idea of self-tuning control has been discussed by several authors, mainly for the control of single-input single-output systems. See for example Kalman [2], Peterka [3], Åström and Wittenmark [4] and Clarke and Gawthrop [5]. Applications to multivariable systems with the linear controller computed from a Riccati equation have been reported by Peterka and Åström [6].

2. MULTIVARIABLE MINIMUM VARIANCE CONTROL

A minimum variance strategy for multivariable systems will now be discussed.

Process model

Let q^{-1} denote the backward shift operator and consider the process given by

$$A(q^{-1})y(t) = B(q^{-1})u(t-k-1) + C(q^{-1})e(t) \tag{2.1}$$

where y is the output vector, u the input vector and $\{e(t)\}$ a sequence of independent, equally distributed, random vectors with zero mean value and covariance $E[e(t)e^T(t)] = R$. The vectors y, u and e are all of dimension p. The polynomial matrices A, B and C are all of dimension $p \times p$, and they are given by

$$A(z) = I + A_1z + \dots + A_nz^n$$

$$B(z) = B_o + B_1z + \dots + B_{n-1}z^{n-1}, \quad B_o \text{ nonsingular}$$

$$C(z) = I + C_1z + \dots + C_nz^n$$

where det B(z) and det C(z) have all their zeros strictly outside the unit disc. The requirement on det C(z) can be considered as a weak condition. The requirement on det B(z) is not necessary for the derivation of the minimum variance strategy. However, the requirement is necessary in practical cases in order to get a stable closed loop system, see Borisson [7].

The assumption that the matrix B_o is nonsingular implies that a nonsingular transformation of the following type can be introduced

$$\bar{u}(t) = B_o u(t) \tag{2.2}$$

Thus the process can be considered to have essentially one control variable for each loop ($\bar{u}_i(t)$ i=1,..., p), which influences the corresponding output before the other control variables. Furthermore, the outputs in the different loops will be influenced with the same delay.

Preliminaries

Introduce the identity

$$C(z) = A(z)F(z) + z^{k+1}\, G(z)$$

where

$$F(z) = I + F_1z + \dots + F_kz^k$$

$$G(z) = G_o + G_1z + \dots + G_{n-1}z^{n-1}$$

Since A(0) is nonsingular, the polynomial matrices F(z) and G(z) are unique. Introduce also $\tilde{F}(z)$ and $\tilde{G}(z)$ given by

$$\tilde{F}(z)G(z) = \tilde{G}(z)F(z), \quad \det \tilde{F}(z) = \det F(z), \quad \tilde{F}(0) = I$$

The polynomial matrices $\tilde{F}(z)$ and $\tilde{G}(z)$ always exist but they are not unique, see e.g. Wolovich [8].

Criterion

Let Q be a positively semidefinite matrix and consider the criterion

$$\min_{u(t)} \; E[y^T(t+k+1)Qy(t+k+1)] \tag{2.3}$$

where k is the time delay in the process model (2.1). The minimum is taken with respect to the admissible control strategies defined below.

Admissible control strategy

At time t the measurements y(t), y(t-1),... and the past control actions u(t-1), u(t-2),... are known. An admissible control strategy is such that u(t) is a function of y(t), y(t-1),... and u(t-1), u(t-2),... .

Solution

In Borisson [7] it is shown that the strategy

$$\tilde{G}(q^{-1})y(t) + \tilde{F}(q^{-1})B(q^{-1})u(t) = 0 \tag{2.4}$$

minimizes the criterion (2.3) asymptotically. It will be called the minimum variance strategy. The asymptotic control error with this strategy is

$$y(t) = F(q^{-1})e(t).$$

When initial effects have settled, the minimum variance strategy (2.4) also minimizes the criterion

$$\min_{u(0),\ \dots,\ u(N-1)} \; E\,\frac{1}{N}\sum_{t=0}^{N-1} y^T(t+k+1)Qy(t+k+1)$$

where the class of admissible control strategies is the same as above. This is also shown in Borisson [7].

With the restrictions introduced on the class of systems, the strategy (2.4) can thus be described by a rational transfer function matrix, which does not depend on the weighting matrix Q of the loss function or on the covariance matrix R of the random vectors {e(t)}. These properties of the minimum variance strategy are fundamental for the multivariable self-tuning regulator described in the next section.

3. A SELF-TUNING MINIMUM VARIANCE REGULATOR

Let the process to be controlled be given by (2.1). At each sampling interval the self-tuning algorithm performs a least squares identification based on a model given below. The obtained parameters are then used to compute the control signal.

Estimation

The algorithm estimates recursively the parameters of the model

$$y(t)+A(q^{-1})y(t-k-1) = B(q^{-1})u(t-k-1)+\varepsilon(t) \tag{3.1}$$

in such a way that the error $\varepsilon(t)$ is as small as possible in the sense of least squares. In (3.1) k is the time delay of the process model (2.1), and $A(z)$ and $B(z)$ are $p \times p$ -dimensional polynomial matrices given by

$$A(z) = A_o + A_1 z + \ldots + A_{n_A} z^{n_A}$$

$$B(z) = B_o + B_1 z + \ldots + B_{n_B} z^{n_B} \tag{3.2}$$

Control

At each time t the control is computed from

$$B(q^{-1})u(t) = A(q^{-1})y(t)$$

The parameters of the controller are thus equal to the estimated parameters.

Introduce the matrix Θ given by

$$\Theta = [\theta_1 \theta_2 \ldots \theta_p] = [A_0 A_1 \ldots A_{n_A} B_0 B_1 \ldots B_{n_B}]^T \tag{3.3}$$

In the algorithm the least squares estimation of Θ is done by estimating one vector θ_i at a time. The following recursive equations can be used, see Åström and Eykhoff [9].

$$\begin{cases} \theta_i(t) = \theta_i(t-1)+K(t-1)[y_i(t)-\Phi(t-k-1)\theta_i(t-1)] \\ K(t-1) = P(t-1)\Phi^T(t-k-1)[1+\Phi(t-k-1)P(t-1)\Phi^T(t-k-1)]^{-1} \\ P(t) = P(t-1)-K(t-1)[1+\Phi(t-k-1)P(t-1)\Phi^T(t-k-1)]K^T(t-1) \end{cases}$$

where

$$\Phi(t-k-1) = [-y^T(t-k-1)\ldots -y^T(t-k-1-n_A)u^T(t-k-1)\ldots \ldots u^T(t-k-1-n_B)]$$

Observe that the estimation is divided into p steps and that the initial values of P(t) are assumed to be the same for these steps. The corresponding gain vectors K(t-1) will then also be the same for all parameter vectors θ_i. In this way significant savings in the computations are obtained.

The matrix B_o in (3.2) can either be estimated as in (3.3) or be set equal to a constant nonsingular matrix. In the former case it must be required that the estimated matrix is nonsingular at each step of time. In the latter case the constant value of B_o must be chosen in such a way that it does not prevent the estimated parameters from converging, see Borisson [7].

Remark. By making a transformation of the type (2.2) so that B_o is a unit matrix, it follows that the algorithm can be interpreted as an interconnection of p simple self-tuning regulators. For example, the regulator in the first loop controls the output y_1 using the control variable u_1. The signals $y_2(t-i),\ldots,y_p(t-i)$ and $u_2(t-1-i),\ldots,u_p(t-1-i)$, $i\geq 0$, can be considered as feed-forward signals.

4. ANALYSIS

Convergence of a general algorithm of the type discussed here has been investigated in the work by Ljung [10]. There it has been shown that this type of algorithms does not converge always. However, the algorithm will stabilize a minimum phase system under weak conditions, even if the estimated parameters do not converge. See Ljung and Wittenmark [11].

In Borisson [7] the asymptotic properties of the multivariable self-tuning regulator are analysed. It is shown that the minimum variance strategy is always a possible resulting strategy of the algorithm. Conditions which guarantee that the algorithm gives the minimum variance strategy will now be discussed.

In the sequel all norms and related equalities are in the sense of mean square.

<u>Theorem</u>. Let the sequences $\{u(t)\}$ and $\{y(t)\}$ be uniformly bounded. If the estimated parameters converge, then

$$\lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} y(t+\tau)y^T(t) = 0, \quad \tau = k+1,\dots, k+n_A+1$$

$$\lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} y(t+\tau)u^T(t) = 0, \quad \tau = k+1,\dots, k+n_B+1$$

□

The proof is given in Borisson [7]. Using this theorem it can be shown that the minimum variance strategy is obtained in the case when the disturbances are white noise ($C(z)=I$), provided the number of estimated parameters is large enough. The same result can also be shown for a system with a general C-polynomial matrix, if some additional assumptions involving the Kronecker indices are made. It is mainly required that the obtained regulator has equal observability indices and equal controllability indices. See Borisson [7].

5. SIMULATION

A simulation will now be shown, where the multivariable self-tuning regulator is applied to the control of a head-box of a paper machine. A schematic description is given in figure 1. In Borisson [7] the following head-box model is investigated.

$$y(t) + A_1 y(t-1) = B_o u(t-1) + e(t)$$

where

$$A_1 = \begin{bmatrix} -0.99101 & 8.80512\cdot 10^{-3} \\ -0.80610 & -0.77089 \end{bmatrix} \quad B_o = \begin{bmatrix} 0.89889 & -4.59328\cdot 10^{-3} \\ 19.390 & 0.88052 \end{bmatrix}$$

$$E\,[e(t)e^T(t)] = \begin{bmatrix} 0.02 & 0.35 \\ 0.35 & 7.6 \end{bmatrix}$$

y_1 = stock level
y_2 = total pressure
u_1 = deviation in stock flow from steady state value
u_2 = deviation in air flow from steady state value.

The sampling interval is 1 second. The model is obtained from a continuous time version given in Åström [12]. The self-tuning algorithm estimates recursively the parameters of the following model

$$y(t) + A_o y(t-1) = \mathcal{B}_o u(t-1) + \varepsilon(t)$$

At each step the control is obtained from

$$u(t) = \mathcal{B}_o^{-1} A_o y(t) \tag{5.1}$$

The estimated parameters are shown in figure 2. The regulator parameters given by (5.1) are shown in figure 3. It follows that the resulting regulator tends to the minimum variance regulator. There were no numerical problems with the inversion of the matrix $\mathcal{B}_o$. During the start-up the control was quite acceptable, and after some minutes it was almost optimal, although the estimated parameters had not yet converged.

6. CONCLUSIONS

By restricting the class of systems to include only systems of minimum phase type that have the same number of inputs as outputs and an impulse response starting with a nonsingular matrix, a minimum variance strategy for multivariable systems has been derived. It has then been shown that the simple self-tuning regulator for single--input single-output systems can be extended to the multivariable case with its simplicity retained. The results of the analysis also give insight into the case, when several simple self-tuning regulators are operated in cascade mode.

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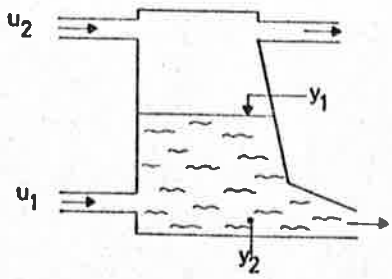

Fig 1. Head-box of a paper machine.

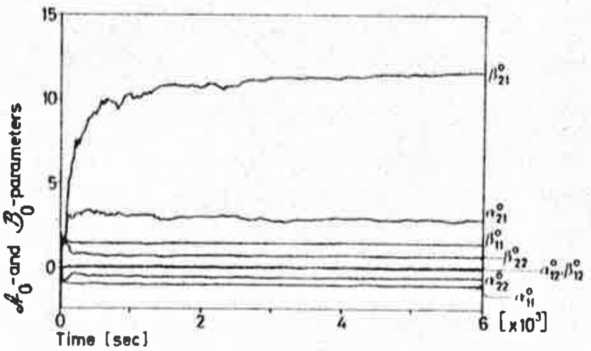

Fig 2. Estimated parameters for the head-box.

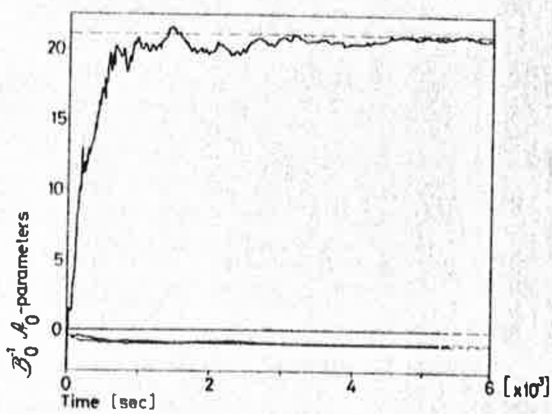

Fig 3. Parameters of the matrix $B_0^{-1}A_0$ for the head-box. They have been computed from the estimated parameters shown in fig 2. The dashed lines show the optimal values.