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MODELING AND IDENTIFICATION OF NUCLEAR POWER REACTOR DYNAMICS FROM MULTIVARIABLE EXPERIMENTS

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ABSTRACT

The purpose of this work is to find linear stochastic point models of low order of the Halden Boiling Water Reactor (HBWR). The models should be used for control of core pressure and nuclear power. Another goal is to test different identification methods on a large system like a nuclear reactor.

The plant is a multivariable system and three inputs are considered essential, viz. a consumers steam valve, a subcooling flow valve and reactivity from control rods. The models are assumed linear, where some parameters depend on the operating level.

A number of identification experiments have been performed on the HBWR in cooperation with the OECD Halden Reactor Project in Norway. After a series of single input experiments several multivariable runs have been performed. In the latter case two or three inputs were used independently in pseudo random binary sequences, while the nuclear power and pressure controls were removed.

1. INTRODUCTION

The increased need of complex dynamical models and of improved model building techniques for nuclear reactors is becoming mandatory. The reasons are, that the plant units are becoming larger and that the control demands are increasing as well. It is predicted, that more complex control schemes will be needed in the near future.

What are the special model building and identification problems in connection with a nuclear reactor? The wide span of time constants is a major problem. Typically the time constants of a plant may vary from less than a second, like neutron kinetics, up to several days, like poisoning and burn-up effects.

In large nuclear reactors the spatial effects become significant. Nonlinear phenomena occur frequently and the systems are typically multivariable.

The purpose of this paper is to apply some different model building and identification techniques on a special nuclear reactor, the Halden BWR (HBWR) in order to find space independent, multivariable linear stochastic models. Multivariable experiments have been performed, on the plant, in most cases under constant operating conditions. One experiment, performed under variable operating conditions, has been analyzed in order to test the feasibility of tracking time variable parameters in a multivariable reactor system. In this case an Extended Kalman filter has been used.

A major question is, how complicated a model must be in order to be accurate enough for control purposes. If the operating conditions do not vary too much, if the signals are small, then a time-invariant linear model seems to fit very well. The noise description, however, is crucial. In the paper two approaches to describe the multivariable system have been made, i.e. vector difference equations and state equations.

Since there is not a one-to-one relationship between the two representations it is not possible to compare the parameters. Other comparisons, however, such as calculations of the model outputs and residuals, have been made.

For the present model purposes the time constants from some second up to some minutes were considered interesting. Then the models could be used for control of e.g. nuclear power or vessel pressure. For technical reasons the sampling time could not be made smaller than 2 seconds. Therefore the neutron kinetics seems to be prompt. The dynamics will include heat diffusion in fuel, moderator, and coolant channels as well as reactivity coupling between the core and the heat removal circuits. On the other hand, xenon reactivity feedback is too slow to be detected from a two hours experiment. The xenon transients are also so small in the HBWR, that they can be excluded from the model. Moreover, if xenon transients need to be considered in a reactor, it is relatively easy to get a good xenon model without any dynamical experiments.

The disturbances have been modeled from the experiments. From earlier experiences in the Halden Project it is known that the measurement noise level on the actual outputs is very low. The process noise, however, is more complex.

Several papers on identification and modeling of nuclear reactors have been published earlier. The Maximum Likelihood (ML) method was compared with other methods for a reactivity-nuclear power model by Gustavsson (1972). Previously single input experiments have been performed at the HBWR and some ML identification models have been reported by Olsson (1973). Bjørlo et.al. (1970) have derived linear multivariable models of the HBWR. Habegger et. al. (1969) used an Extended Kalman filter to track parameters in a nuclear system. Moore et. al. (1972) have used a combination of a least squares and a ML approach to get an adaptive control scheme of a model of a pressurized water reactor.

In the present paper already known identification methods have been applied to a reactor system. The main contribution is supposed to be the comparison of different approaches to get suitable structures of a stochastic multivariable system. The models have also been compared to real data. The parameter tracking was performed in a multivariable system with measurements from real multivariable experiments. To the author's knowledge this type of results has not been published earlier for nuclear reactor applications.

The rest of the paper is organized as follows. A brief review of the experimental conditions and a short technical description of the plant are made in section 2.

Maximum likelihood (ML) identifications have been used to derive multiple-input single-output (MISO) models in section 3. In section 4 it is shown that the model error and the one-step prediction of the models can be decreased significantly if couplings between the outputs are taken into account.

In section 5 a multivariable state model of order six has been identified. It is partly compared to the previous models and partly used as initial model for an Extended Kalman filter. The filter is tracking time variable parameters during an experiment, where the operational conditions are changing.

2. EXPERIMENTAL CONDITIONS

The HBWR is a small well instrumented experimental nuclear reactor. Therefore it can be used for advanced dynamical experiments.

The reactor is a heavy water boiling reactor. In the actual experiment the nuclear power did not exceed 11 MW. A mixture of heavy water and steam leaves the coolant channels at the top of the core. The steam of the primary circuit is condensed in two steam transformers. Light water steam is then produced in the secondary circuit. The light water steam in the secondary circuit produces light water steam in the tertiary circuit by means of a steam generator. The steam there can be distributed to the consumers through a control valve (u_1).

Heavy water can also be withdrawn from the core tank into a subcooling circuit. The subcooling flow can be controlled by a valve (u_3). Generally if the subcooling decreases the reactor becomes less stable.

The reactor can also be controlled by means of absorbtion rods in the core (u_2). More detailed description of the reactor can be found in Jamne et. al. (1967).

In the present experiments two or three inputs have disturbed the plant independently of each other. The signals have been of the type pseudo random binary sequences (PRBS).

The shortest pulse length have been chosen to 12 seconds, while the sampling interval was 2 seconds. The input sequence period was more than 2000 sec. The longest input pulse was 196 seconds. A number of rules of thumb on input characteristics have been developed, see e.g. Briggs et. al. (1967). From these rules one can conclude that time constants in the range from 6 - 12 seconds to 1 - 7 minutes can be found by using such an input.

During the experiments 32 different signals were recorded. Only four of them were used for identification purposes. The other signals were recorded for checking purposes.

At each sampling interval the time delay from the first to the last recording is 40 ms. This delay is neglected in comparison with the sampling time.

The actuators dynamics are included in the models, because the computer commands are defined as the inputs.

The input valve amplitude is defined in % opening. The reactivity from the rod movements is defined in "steps", where one step is a reactivity corresponding to 7 - 10 pcm, depending on the rod position. In order to get larger reactivity amplitudes two or three rods were moved simultaneously.

In order to limit the material no experiment with the subcooling valve u_3 as input is presented in this paper.

Four output variables as function of two input variables have been analysed. The signals are

u_1 = consumers steam valve (VB 282) of the tertiary circuit

u_2 = reactivity in "steps" from the rods

y_1 = nuclear power (MW)

y_2 = normalized primary (vessel) pressure $\frac{\delta p_1}{p_1}$

y_3 = normalized secondary pressure $\frac{\delta p_2}{p_2}$

y_4 = normalized tertiary pressure $\frac{\delta p_3}{p_3}$

The operational conditions of the experiments are summarized in the following table

Exp.	Inputs u_1(%)	u_2(steps)	Reactor power (MW)	Subcooling power (MW)
1	-	3 (rod 13,17,18)	9.95	1.85
2	± 2.5	3 "	10	1.95
3	± 3	2 (rod 20,21)	10	1.95
4	± 2.5	2 (")	10	1.95→1.35

3. MULTIPLE INPUT SINGLE OUTPUT (MISO) MODELS

In this section some Maximum Likelihood (ML) identification results with one or two inputs and one output are displayed. The four outputs, mentioned earlier have been examined. The data has been corrected by subtraction of mean values, but no drift needed to be eliminated.

The plant dynamics is represented by the structure introduced by Åström et. al. (1966)

$$(1+a_1q^{-1}+\ldots+a_nq^{-n})y(t)=\sum_{i=1}^{p}(b_{i0}+b_{i1}q^{-1}+\ldots+b_{in}q^{-n})$$

$$u_i(t)+\lambda(1+c_1q^{-1}+\ldots+c_nq^{-n})e(t) \qquad (1)$$

or

$$A^*(q^{-1}) = \sum_{1}^{p} B_i^*(q^{-1})u_i(t) + \lambda C^*(q^{-1})e(t) \quad (2)$$

where q is the shift operator and p the number of inputs. The disturbance e(t) is assumed to be a sequence of independent random variables N(0 1). The ML-method is described elsewhere, and the survey by Åström-Eykhoff (1971) gives numerous references.

Special experiences of SISO ML identifications of the Halden Boiling Water Reactor have been reported in Olsson (1973). The extension to the MISO case was straight-forward.

The order decisions are based on a lot of tests, including earlier experiences. The loss function changes have been statistically tested. Parameter variances, continuous models, poles and zeroes, Bode plots, residual tests and simulations have been performed to get relevant models. The orders of the MISO models havebeen typically three or four.

The MIMO model for the nuclear power and the three pressures have been identified primarily from experiment 2. For comparison purposes SISO models are identified from exp. 1 as well.

The number of sampling points varies between 1000 and 2000, i.e. an experiment length between 33 and 66 minutes.

The accepted MISO model orders from exp. 2 and corresponding one-step prediction errors are,

nuclear power - 3, λ = .282-1 (MW)
primary pressure - 4, λ = .714-4
secondary pressure- 3, λ = .310-3
tertiary pressure - 2, λ = .174-3

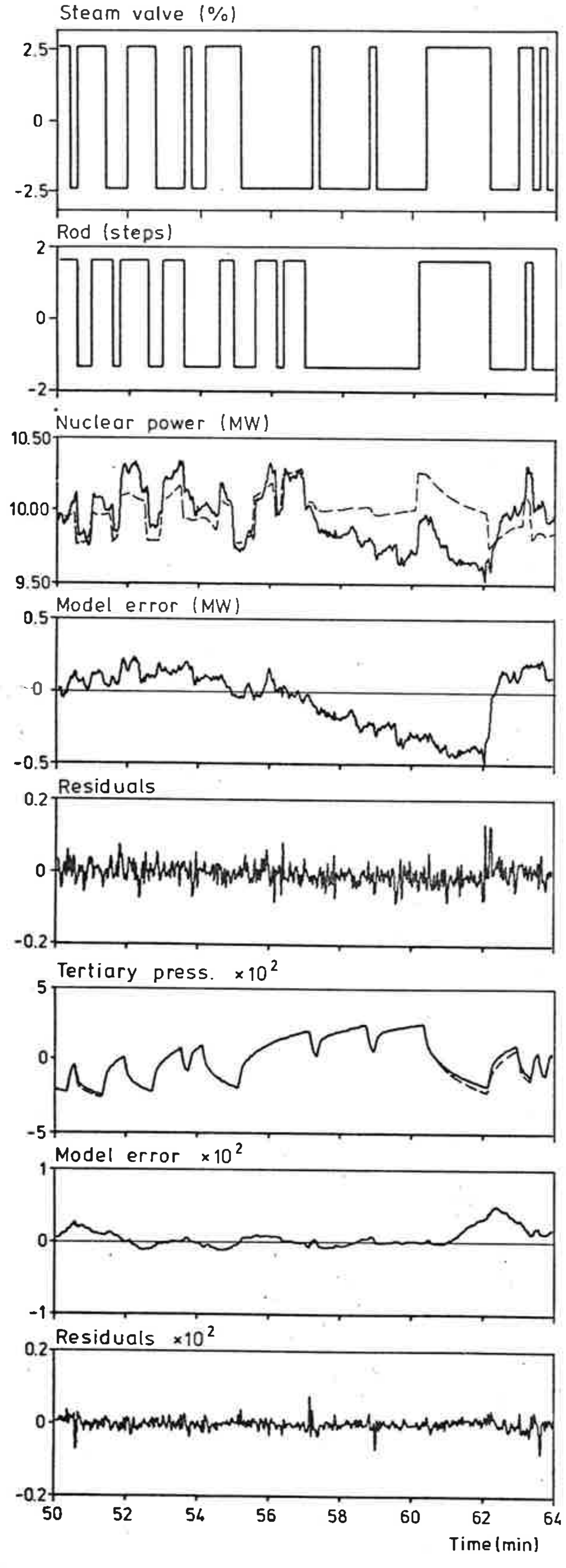
The values of the A^* and B^* parameters differ little in experiment 1 and 2. The C^* parameter differences are somewhat larger, i.e. more than one standard deviation.

In all models there is at least one pole quite close to the unit circle, representing long time constants. In the nuclear power model this pole is almost cancelled by a zero of the reactivity B^*-polynomial. Therefore the model reflects the fast response from reactivity to nuclear power.

The neutron kinetics is prompt, compared to the sampling time. Therefore the nuclear power model contains a direct coupling term from reactivity input (u_2) to output, i.e. $b_{20}\neq 0$ (1). The vessel pressure (y_2) is coupled to the reactivity through the heat diffusion from fuel elements and moderator. Therefore this model does not contain a "direct term" b_{20}. The pressures of the heat removal circuit are very little influenced by the reactivity. In the tertiary circuit it is completely insignificant, and corresponding B^* polynomial is zero.

When the steam valve is opened, steam is removed from the tertiary circuit and the pressure (y_4) drops so fast that the parameter b_{10}(1), connecting input directly to output, gives a most significant improvement of the model. The pressure drop propagates to the secondary circuit (y_3) and to the vessel (y_2) through the heat exchangers. Also the nuclear power is affected. The correlation between u_1 and the different outputs is reflected in the B^*-coefficients corresponding to u_1. These coefficients are relatively large for the tertiary circuit and relatively small for the core variables y_2 and y_1.

In fig 1 a section of experiment 2 has been plotted, where the curves show up some interesting irregularities. The nuclear power in fig 1A has a strong negative drift between 56 and 62 minutes. At about t=62 it grows rapidly. The model, however, does not follow, neither the slow drift nor the rapid change. The model error also changes rapidly and the residuals have a large pulse at this point. The reason for the change at t=62 is, that a manual control rod had to be moved manually for security reasons to keep the power within permitted limits. Of course this additional input could have been added to the reactivity



input, but it is not included into the model in order to show, how the ML method can detect abnormal behaviour during the experiment. A plot of the tertiary pressure (y_4) is also shown to demonstrate the model accuracy, Fig 1 B.

4. A VECTOR DIFFERENCE EQUATION APPROACH

In the preceeding MISO models the couplings between the outputs have been neglected. If the couplings should be significant, then such a model could not be used for multivariable control.

In order to find better relationships between the inputs and outputs a vector difference equation approach has been tried. Certain assumptions about the noise have been made. Then this approach is worked out in order to examine the model accuracy and compare it to other models.

Correlation analysis has been used to achieve a preliminary structure of the causal relations between the variables. After that, simplifying assumptions of the noise have been made. The consequence is that the vector difference equation can be identified row by row. Then identifiability problems are avoided. If there are strong couplings, then, however, the result might be too an inaccurate model.

Correlation analysis.

Pairs of input and output sequences have been studied to find out the causality relations. The impulse response has been estimated, using a fast fourier transform algorithm. The "input" time series $\{x_i(k)\}$ was tested against an "output" time series $\{x_j(k)\}$. Generally 2000 data points were used. Both input and output are filtered in the same filter so that the input becomes white. Then the crosscorrelation will be an estimate of an impulse response from a white noise input.

From the correlation analysis structures of the system have been found.

Nuclear power y_1 $= f_1(u_1, u_2^0, y_2)$

Primary pressure y_2 $= f_2(u_1, u_2, y_1, y_3)$

Secondary pressure y_3 $= f_3(u_1^0, y_2, y_4)$

Tertiary pressure y_4 $= f_4(u_1^0, y_2, y_3)$

The symbols just mean, that the time function on the left depends on the time functions on the right. The superscript "zero" is assigned, where a prompt relation exist.

Fig. 1 Comparisons between the real outputs and ML model outputs from a part of exp 2. In (A) the nuclear power is a function of u_1 and u_2. In (B) the tertiary pressure is a function of u_1. The dotted lines are the outputs y_d from the deterministic part of the models. The error is defined

$y - y_d$. The residuals are defined from equation (2) and should be independent $N(0,\lambda^2)$

Assumptions of the noise.

The structure of the system is assumed

$$[I+A_1 q^{-1}+\ldots+A_n q^{-n}]y(t) = [B_0+B_1 q^{-1}+\ldots+B_n q^{-n}]u(t)$$

$$+ [I+C_1 q^{-1}+\ldots+C_n q^{-n}]e(t) \qquad (3)$$

where all capital letters assign constant matrices, while y, u and e are vectors. The noise e is assumed to be a sequence of independent stochastic variables $N(0,R)$.

A likelihood function is defined by L,

$$-2 \ln L(\theta,R)=\sum_{i=1}^{N} \varepsilon^T(i)R^{-1}\varepsilon(i)+N \ln(\det R)+\text{const} \qquad (4)$$

where θ denotes the unknown parameters and the residuals ε are the solution of the vector difference equation

$$\varepsilon(t)=[I+C_1 q^{-1}+\ldots+C_n q^{-n}]^{-1}\{[I+A_1 q^{-1}+\ldots]y(t) -$$

$$- [B_0+B_1 q^{-1}+\ldots]u(t)\} \qquad (5)$$

The function ln L can be written as a sum of n functions if R is assumed diagonal

$$R = \text{diag}(\lambda_1^2,\ldots, \lambda_n^2)$$

If all matrices C_i are made diagonal, the components of ε can be calculated independent of each other. Thus it is assumed, that every row of the equation is disturbed by a separate noise source independent of other noise sources.

The structure of the model is thus given by (2) where the inputs consist of all signals, that can influence a certain output.

The parameter estimates are not surely unbiased, consistent and with minimum variance, because the conditions, derived by Åström et.al. (1965) are not satisfied. It will be shown, however, that these models are better than the former MISO models, because of the introduction of new causality relations.

Identification results.

Some specific examples will be discussed to illustrate the model improvements. With the nuclear power (y_1) as function of u_1 and u_2 no improvement could be achieved with more than 16 parameters of the structure from eq. (2) (third order, λ = .282 - 1). The primary pressure (y_2) was introduced as a new input and improved the model considerably. Already for 13 parameters (second order) λ = .282 - 1 was achieved. Significant improvements were found until 19 parameters, (third order) when λ = .277 - 1. No significant contribution could be found from y_3 and y_4, thus verifying the correlation analysis.

In general also the parameter accuracy was improved when y_2 was introduced as input, especially in A and C . The auto covariance function of the residuals was also more like white noise than for the model $y_1 = f(u_1, u_2)$.

The plot of the model output fig 2A shows an interesting behaviour compared to the previous model output (fig 1A). The latter model can follow the drift of the nuclear power between 56 and 62 minutes much better. During this time the valve u_1 was negative almost all the time which forced the pressures to grow very much and consequently the nuclear power dropped. Thus the drift of the nuclear power is noticed through the primary pressure "input" y_2. The rapid change of the model error at $t \approx 62$ is therefore much smaller and the error mean is closer to zero compared to the former model. The difference between the residuals of the two models is not so large, but large enough to be a significant improvement of the loss function.

One variable from the heat removal circuits will also be discussed, the tertiary pressure (y_4). For y_4 it was found, that the secondary pressure (y_3) and the steam valve (u_1) are significant inputs. The best model with only u_1 as input contains 9 parameters (second order, λ = .174-3). If y_3 were added as an input a second order model with 12 parameters was accepted (λ = .156-3) and the improvement was significant.

The correlation analysis showed that y_4 should be a function also of y_2. The ML identifications, however, did not prove any significant contributions from y_2. The reason is that also the "inputs" y_2 and y_3 are strongly correlated, so all causality relations from y_2 and y_3 to y_4 can be explained by y_3 alone.
The plots of fig 1B and 2B should be compared. The model error improvement is much more drastic, than the residual improvement might indicate.

Simulation of the vector difference equation (3)

The assumption of the noise now will be tested by simulation of the whole vector difference equation (VDE). When each row of the VDE was simulated separately, then all inputs had their experiment value. As "inputs" was defined steam

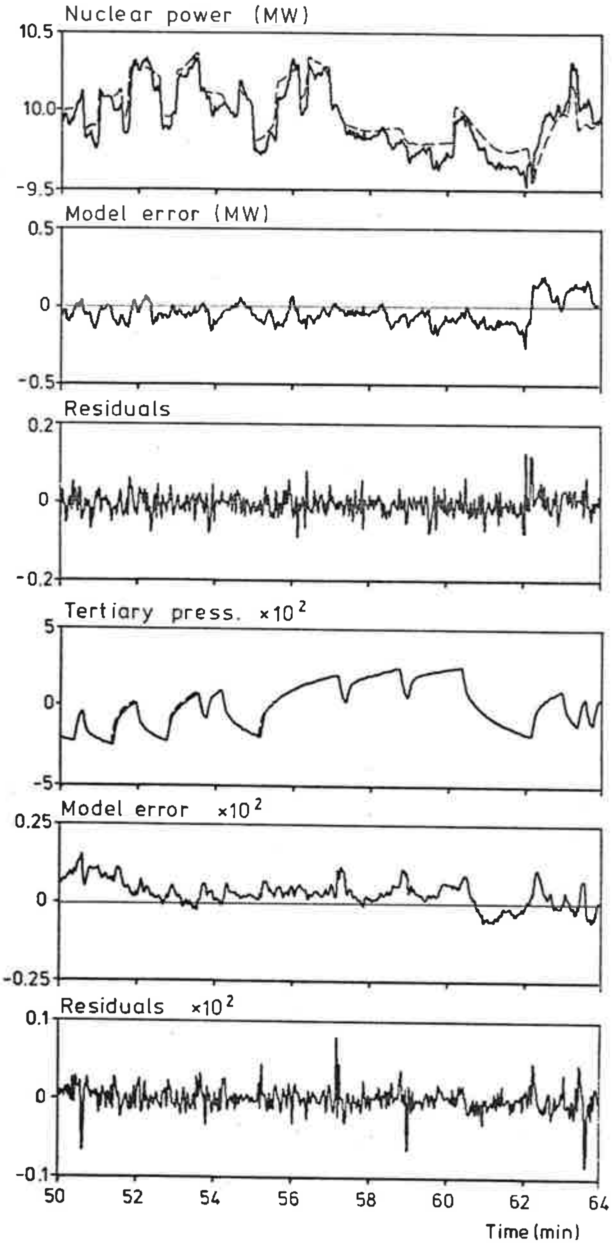


Fig 2. Comparisons between the real outputs and ML model outputs from a part of exp. 2. In (A) the nuclear power is a function of u_1, u_2 and y_2. In (B) the tertiary pressure is a function of u_1 and y_3. The dotted lines are the model outputs.

valve, rods as well as other measured variables. When the whole VDE is simulated, then only the steam valve and rods are given as inputs. The output error then naturally becomes larger. On the other hand, the model errors are smaller compared to the MISO models, presented in section 3, because new causality relations are described.

Fig 3 shows how two of the four outputs behave during the same part of exp. 2, as was discussed before. The nuclear power (3A) has a smaller error (in mean square) than the MISO model of section 3 (fig. 1A). The error is larger than the error indicated in fig. 2A. For example, in fig 2A the experimental valve of the primary pressure y_2 was used as an additional input, while the calculated value of y_2 was used in fig. 3A.

The tertiary pressure (y_4) model error is of the same order as for the MISO model of section 3 (cf. fig. 1B). Especially slow variations occur in the model errors of the VDE.

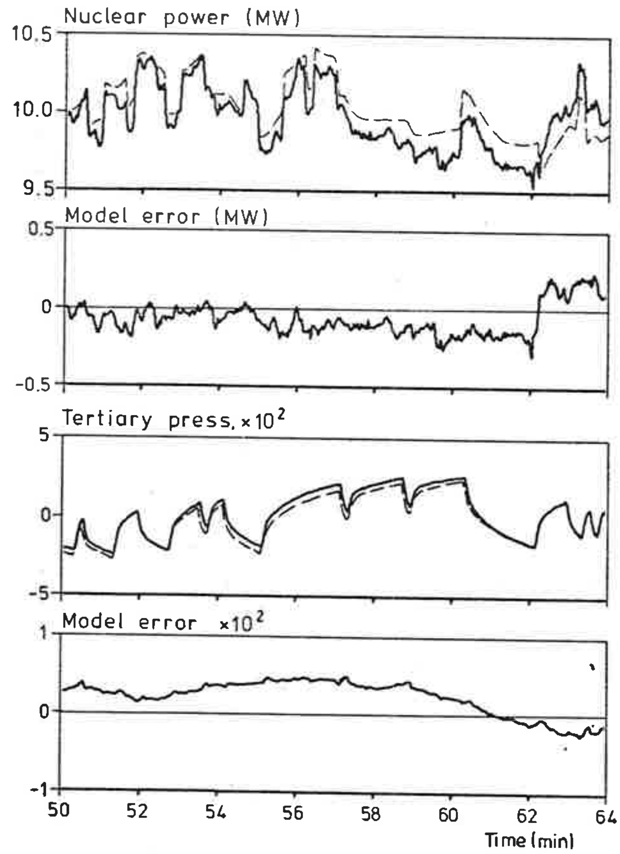


Fig. 3. Simulation of deterministic part of the vector difference equation. Two outputs are shown, nuclear power (A) and tertiary pressure (B). The dotted lines are the model outputs. Only a fraction of experiment 2 is shown.

5. STATE MODELS

From a control point of view one is interested to get an accurate model of the plant, and still limit the number of state variables.

In the previously derived vector difference equation the parameters of the model had no special physical interpretation. A multivariable state model also can describe the system, and then it is easier to give physical interpretations of the parameters. This model also can be used for linear-quadratic-gaussian control. Such a state model is derived here from identification experiments. The purpose is to find the lowest possible model order, useful for control.

As already mentioned, Bjørlo et. al. (1970) have made a linear state model of the HBWR. This model has been developed further within the OECD Halden Reactor Project. It contains nine physical state variables and six additional states.

In this section the choice of state variables will be discussed first. Then the general dynamical relations are considered and a suggestion for a suitable structure of the plant dynamics is made. Finally the deterministic and stochastic parameters of this structure are identified.

The neutron density is changing too fast to be detected in a dynamic model with the actual sampling interval. Therefore the neutron density is not used as a state variable. The delayed neutrons c (one group) will represent one state variable. The average fuel temperature is coupled to the kinetics as well as to the heat content. The nuclear power depends strongly on the void fraction. The void fraction varies from reactor bottom to top. It is very difficult to measure the void content and of course even more difficult to get a relevant average value. The boiling boundary also affects the power very much. The void content and boiling boundary in turn depend on the vessel pressure, water temperature, and the subcooled water temperature. The pressure can be measured with good accuracy and the temperatures can be estimated much easier than the void.

The pressures of the secondary and tertiary heat removal circuits represent the states of these circuits.

In the first approximation seven states were assumed,

x_1 primary (vessel) pressure

x_2 secondary "

x_3 tertiary "

x_4 delayed neutrons

x_5 fuel average temperature

x_6 water temperature

x_7 subcooled water temperature

When only u_1 and u_2 were used as inputs the subcooled water temperature was neglected. The addition of the corresponding elements of the A matrix did not give noticeable changes of the loss function.

The structure of the system now will be discussed.

The kinetics can be described by a model with one group of delayed neutrons,

$$\frac{dn^*}{dt} = \frac{\delta k-\beta}{\ell} \quad n^* + \lambda \cdot c$$

$$\frac{dc}{dt} = \frac{\beta}{\ell} \quad n^* - \lambda \cdot c$$

where n^* is nuclear power (neutron density), c is concentration of the delayed neutrons, β the delayed neutron fraction, λ a weighted average value of the decay constants of the precursors of the six groups of delayed neutrons, ℓ the neutron generation time and $\delta k = k_{eff} - 1 \approx$ reactivity. The reactivity includes the feedback effects from fuel and water temperatures, void, vessel pressure and the control rods.

A prompt jump approximation now is made, i.e. dn^*/dt is put to zero. The feedback reactivity δk is independent of c, but is a function of other state variables. The differential equation in c can be linearized to the following structure,

$$\frac{dc}{dt} = \frac{dx_4}{dt} = a_{41}x_1 + a_{42}x_2 + a_{45}x_5 + a_{46}x_6 + b_{42}u_2$$

Notice that $a_{44}=0$.
The fuel temperature, Θ_F, is supposed to have the general structure

$$\frac{d\Theta_F}{dt} = \alpha_1\Theta_F + \alpha_2 \cdot n^*$$

where n^* is the neutron density. Because of the prompt jump approximation n^* can be written as a function of δk, or

$$\frac{dx_5}{dt} = a_{51}x_1+a_{52}x_2+a_{54}x_4+a_{55}x_5+a_{56}x_6+ b_{52}u_2$$

The water temperature x_6 is coupled to the fuel temperature x_5. It also is correlated with the vessel pressure. The secondary pressure also may give an influence through the subcoolers, while it is assumed that the tertiary pressure has no significant influence. The water temperature equation then is

$$\frac{dx_6}{dt} = a_{61}x_1 + a_{62}x_2 + a_{65}x_5 + a_{66}x_6$$

The vessel pressure (x_1) equation is probably the most complicated one. The pressure is of course related to the heat inflow from the fuel elements (x_5) and the moderator temperature (x_6). It also is correlated with the void fraction. Here it is supposed that the void content can be expressed in the other core variables x_1, x_5 and x_6. Through the heat exchangers the vessel pressure is coupled to the secondary pressure (x_2). The coupling to the tertiary pressure (x_3) is neglected. The vessel pressure equation becomes,

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + a_{15}x_5 + a_{16}x_6$$

The secondary pressure (x_2) is assumed to be depending only on the vessel (x_1) and the tertiary pressure (x_3) through the heat exchangers,

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + a_{23}x_3$$

The tertiary pressure (x_3) finally is affected by the consumers steam valve (u_1), the secondary pressure (x_2). The coupling to the core is neglected. Thus

$$\frac{dx_3}{dt} = a_{32}x_2 + a_{33}x_3 + b_{31}u_1$$

The structure of the dynamics then can be summarized into

$$\dot{x} = Ax + Bu$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & 0 & 0 & 0 \\ a_{41} & a_{42} & 0 & 0 & a_{45} & a_{46} \\ a_{51} & a_{52} & 0 & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & 0 & 0 & a_{65} & a_{66} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{31} & 0 \\ 0 & b_{42} \\ 0 & b_{52} \\ 0 & 0 \end{bmatrix}$$

For the identification the system is represented in the innovations form, see Åström (1970),

$$x(t+1) = \phi \cdot x(t) + \Gamma \cdot u(t) + K\varepsilon(t)$$

$$y(t) = C \cdot x(t) + D \cdot u(t) + \varepsilon(t)$$

where the sampling rate is normalized to unit time.

The disturbances are defined only in discrete time and are described by the unknown matrix K and the noise vector ε. The noise $\{\varepsilon(t)\}$ is assumed to be a sequence of equally distributed gaussian zero mean variables with covariance R.

The matrix D is introduced to describe dynamics in the heat removal circuits, which is considered prompt compared to the sampling interval. Such results were found in section 3 and 4. The matrices C and D have the structure

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad D = \begin{pmatrix} 0 & 0 \\ d_{21} & 0 \\ d_{31} & 0 \end{pmatrix}$$

The unknown parameter θ is defined in the A, B, D and K matrices. The continuous system matrices then are transformed to sampled form.

$$\phi = e^{A(\theta)} \qquad \Gamma = \left[\int_0^1 e^{A(\theta)s}\, ds\right] B(\theta)$$

The likelihood function is found e.g. in Åström (1971).

To maximize the likelihood function is equivalent to minimize the loss function

$$V = \det \sum_{t=1}^{N} \varepsilon(t)\, \varepsilon^T(t)$$

It is shown by Eaton (1967) that the covariance matrix R can be estimated separately to

$$\hat{R} = \frac{1}{N} \sum_{t=1}^{N} \varepsilon(t)\, \varepsilon^T(t)$$

In order to find the first models both K and D were assumed to be zero. From experiment 2 a sequence of 800 data was used to find a model.

A minimum point was found for

$$V = .58\text{-}6$$

There all parameters had gradients less than .30-6, and trace (R) was found to be

$$\mathrm{tr}(R) = .123\text{-}3$$

This means that the one step predictions errors of the output pressures are $\lambda_1 = .50\text{-}2$, $\lambda_2 = .74\text{-}2$ and $\lambda_3 = .66\text{-}2$. These errors are very unsatisfactory compared to the MISO model errors, see section 3. Moreover, the autocovariance functions are approaching zero too slowly.

The MISO models showed that the noise must be described more elaborately than just as output noise. To get a better noise the matrix K was assumed unknown with all elements zero except the three parameters k_{11}, k_{22}, and k_{33}. Of physical reasons these elements were initially assumed close to one. The loss function now decreased to V=.283-9. The parameters changed very little in the last iterations, and the parameter gradients were .20-7. The standard deviation of the one step prediction errors of the three output pressures were found to be 0.88-3, 0.96-3 and 0.15-2 respectively. The autocovariances were not satisfactory and still were decaying too slowly. It was not tried to refine the model further, because of the slow convergence. Work is presently going on to improve the noise statistics by introducing more K-parameters.

The state model also has been adjusted to experiment 3. The loss function was practically the same as for experiment 2. This model was interesting because the operational conditions are the same during exp. 3 as in the start of exp. 4. In that was initial conditions for the identification of time variable parameters were found. Fig 4 shows a comparison between the experimental and predicted ($K \neq 0$) outputs of experiment 3.

6. RECURSIVE IDENTIFICATION OF TIME VARIABLE PARAMETERS

When the nuclear power or other operational conditions are changing then several model parameters are time-varying. In exp. 4 the subcooling flow was disturbed manually, as indicated in fig.5, from 1.95 MW to 1.4 MW during 15 minutes.

An Fxtended Kalman filter has been used to track the time-varying parameters during the experiment. The algorithm is further described in Olsson-Holst (1973).

In other identifications from experiments at other constant subcooling levels it has been computed, which parameters are changing significantly. Especially the six underlined parameters in the A matrix are changing very much.

It is already well-known that it is difficult to choose the covariance R^{α} of the artificial noise, driving the unknown parameters. No adaptive filter algorithm has been used. . Initially the covariance R^{α} was assumed diagonal. Attempts with different R^{α} were made. As a test quantity to compare the results the sample covariance matrix of the residuals

$$\varepsilon(t) = y(t) - C\hat{x}(t|t-1) - D\cdot u(t)$$

was used. It was found that the diagonal elements of R^{α} should have a value between 10^{-6} and 10^{-7}, i.e. somewhat smaller than those of the process noise covariance.

As it is desirable to minimize the number of unknown parameters the residuals were compared for a different number of parameters. It was found that two time variable parameters are too few, but three might be sufficient.

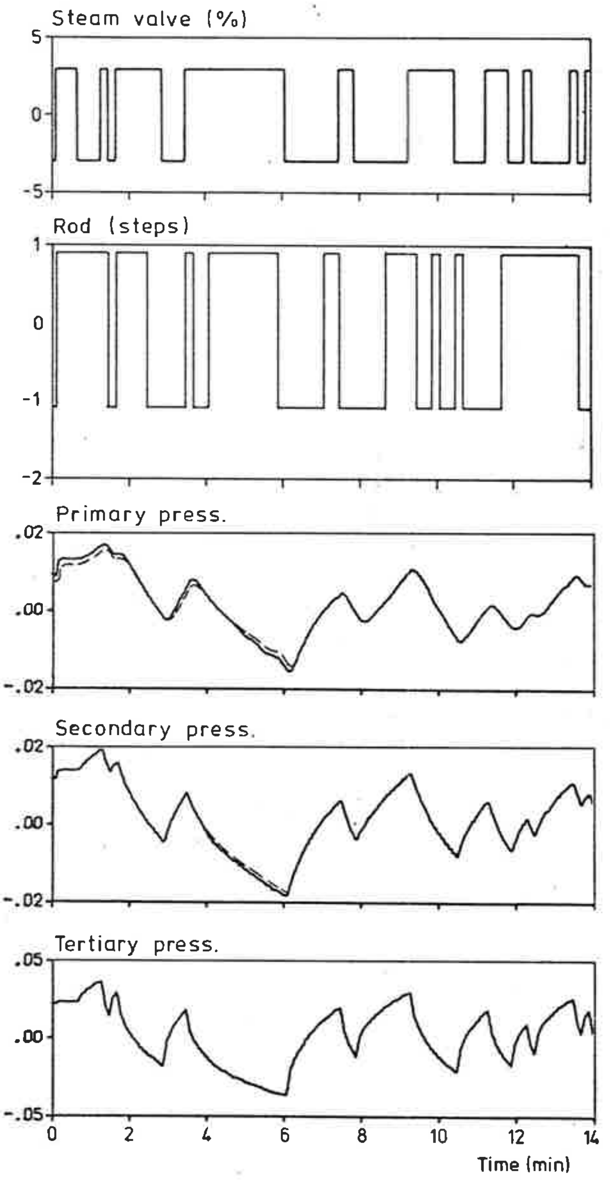


Fig. 4. Real outputs and predicted outputs (dotted lines) of the state model from a part of exp. 3.

In fig. 5 a part of a simulation of six time-varying parameters is shown. As an example a_{41} is displayed.

Much work certainly remains to be done in the area of application of approximative recursive filters, especially the problems to find relevant descriptions of the artificial noise covariance.

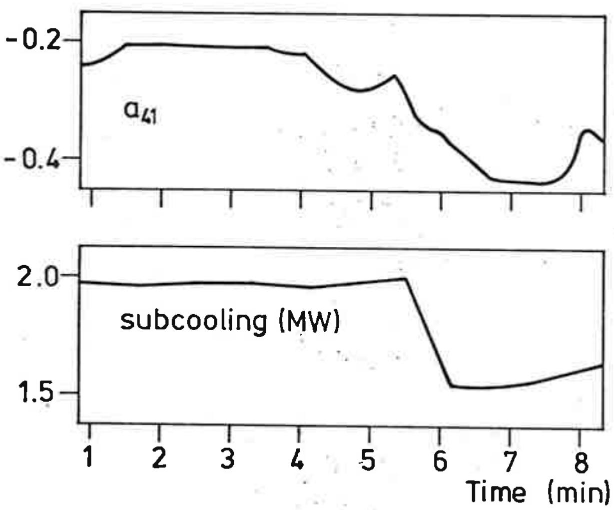


Fig. 5. Estimation of an A-parameter with Extended Kalman filter during changes of the subcooling power. It was assumed six unknown parameters.

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