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A STATE SPACE MODEL OF A MULTIMACHINE POWER SYSTEM.

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S. Lindahl

ABSTRACT.

The equations for a multimachine power system are derived. The model includes hydro turbines as well as steam turbines and boilers. The nonlinear equations are derived from basic physical laws. They are linearized to obtain a linear state space model, which is valid for small perturbations about an operating point. A method of obtaining the equations on standard state space form is proposed. Except matrix multiplication the method only requires the inversion of one n×n matrix (complex) and n 3×3 matrices (real), where n is the number of generating plants.

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1. INTRODUCTION.

A power system consists of several plants, a large distribution network and a variety of consumers. To analyze the performance of the system, power engineers have developed computer programs for simulation of multimachine power systems [1], [3], [8], [9]. Methods of analyzing the stability of the linearized model, describing small perturbations about an operating point, has also been developed [2], [4], [5].

Methods of improving the performance of a power system have also been proposed, but these methods often assume that:

- a single generator is connected to an infinite bus,
- the mechanical input to the generator is constant.

Under the above assumptions the voltage regulator becomes a single-input single-output system, and the classical methods can be applied to design the voltage regulator. If we remove one of the above assumptions, the model becomes multivariable and classical control theory does not provide a systematic method of designing the regulators. Of course, simulation can be used to find suitable tuning of the regulators. One drawback of such simulations is the amount of computing time required.

Modern control theory enables us to handle multivariable systems, described by a set of first order linear differential equations

$$\dot{x} = Ax + Bu \qquad (1.1)$$

where x is the state vector, u the control vector and

A, B the coefficient matrices. To apply linear-quadratic control theory we assume that the performance can be described by

$$V = \int_0^\infty \left[x^T(s)Q_1x(s) + u^T(s)Q_2u(s)\right]ds \tag{1.2}$$

where Q_1 is a symmetric nonnegative definite matrix, and Q_2 is a symmetric positive definite matrix. The problem is to find a control u(t), such that the loss function V is minimized. The solution to the problem is given by the linear time-invariant feed-back.

$$u(t) = -Lx(t) \tag{1.3}$$

where

$$L = Q_2^{-1}B^TS \tag{1.4}$$

The matrix S is the symmetric nonnegative definite solution of the stationary Riccati equation

$$A^TS + SA + Q_1 - SBQ_2^{-1}B^TS = 0 \tag{1.5}$$

The control signals are linear combinations of all state variables. To implement such a controller it is necessary to transmit the whole state vector to every plant and this may not be realistic. Since we obtain a solution with all possible feedbacks we have a yard-stick to evaluate the importance of feeding certain variables from one station to another. We also have the tools to analyze various suboptimal strategies. In any case it is necessary to simulate the nonlinear equations, describing the system, and using the actual control law. If the chosen control law works it is immaterial that we have found it by applying linear-quadratic control theory. This approach is feasible only if the total amount of computing time is less than the computing time required for straightforward simulation.

In this report we derive the equations for a multimachine power system with hydro turbines as well as steam turbines and boilers as prime movers. We also propose a method of building up the system matrices A and B in (1.1).

In Section 2 we describe the power system configuration. The basic equations for the synchronous machine are derived in Section 3. The synchronous machines are connected to the transmission network and in Section 4 we consider the transmission network. In Section 5 we present the nonlinear equations for the prime movers. Finally we describe the method of obtaining the system matrices.

2. DESCRIPTION OF THE POWER SYSTEM.

The basic system studied is shown in Fig. 2.1.

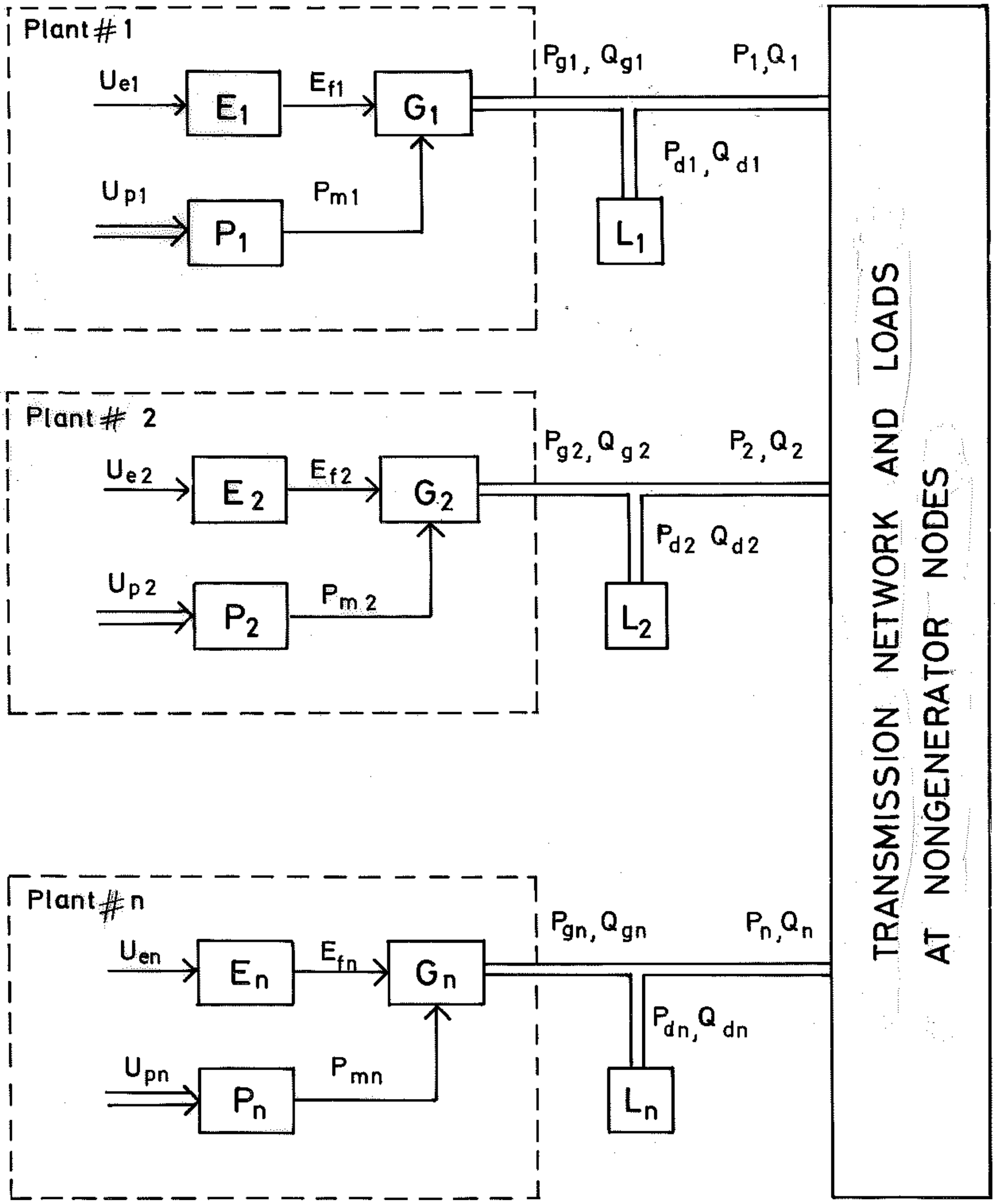

Fig. 2.1 - Schematic diagram of a power system consisting of n generating plants. Each plant is composed of a prime mover (P_i), a synchronous machine (G_i) and an excitation system (E_i).

The system consists of a linear multiport lumped parameter electrical transmission network, n generating plants and local loads at the generator nodes. The loads at nongenerator nodes are already included in the transmission network.

At every generator node active and reactive power, denoted P_i and Q_i respectively, is fed into the transmission network. The active and reactive power demand at the generator nodes is denoted by P_{di} and Q_{di} in Fig. 2.1. The sum of injected power (P_i,Q_i) and local demand (P_{di},Q_{di}) is equal to the generated power (P_{gi}, Q_{gi}).

The generating plants consist of a synchronous machine (G_i), an excitation system (E_i) and a prime mover (P_i). The inputs to the synchronous machine are the field voltage (E_{fi}) and the mechanical power (P_{mi}). The input signal to the excitation system is denoted by U_{ei} in Fig. 2.1.

The input signal (s) to the prime mover is denoted by U_{pn}.

The network is treated as if it was in steady-state operating condition. The alternating node voltages and current are represented by the complex quantities $\tilde{V}$ and $\tilde{I}$ respectively. The transmission network is assumed to be completely described by the complex nodal admittance matrix Y. The nodal admittance equation can be written

$$\tilde{I} = \tilde{Y} \cdot \tilde{V} \qquad (2.1)$$

where all nonsynchronous loads are represented by constant admittances and incorporated into $\tilde{Y}$ by eliminating all nongenerator nodes.

The n complex equations (2.1) are separated into 2n real equations as proposed in [10].

$$\begin{bmatrix} i_{D1} \\ i_{D2} \\ \vdots \\ i_{Dn} \\ \hline i_{Q1} \\ i_{Q2} \\ \vdots \\ i_{Qn} \end{bmatrix} = \left[\begin{array}{cccc|cccc} g_{11} & g_{12} & \cdots & g_{1n} & -b_{11} & -b_{12} & \cdots & -b_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} & -b_{21} & -b_{22} & \cdots & -b_{2n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} & -b_{n1} & -b_{n2} & \cdots & -b_{nn} \\ \hline b_{11} & b_{12} & \cdots & b_{1n} & g_{11} & g_{12} & \cdots & g_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} & g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} & g_{n1} & g_{n2} & \cdots & g_{nn} \end{array}\right] \begin{bmatrix} v_{D1} \\ v_{D2} \\ \vdots \\ v_{Dn} \\ \hline v_{Q1} \\ v_{Q2} \\ \vdots \\ v_{Qn} \end{bmatrix} \tag{2.2}$$

This equation is written symbolically as

$$\begin{bmatrix} I_D \\ I_Q \end{bmatrix} = \begin{bmatrix} G_N & -B_N \\ B_N & G_N \end{bmatrix} \begin{bmatrix} V_D \\ V_Q \end{bmatrix} \tag{2.3}$$

The synchronous machines are described by the set of Park's equations [6], [7] given in Section 3. The excitation systems are modelled by first order dynamics.

Each synchronous machine is connected to either a hydro turbine or a boiler and steam turbine. The equations for the prime movers are derived in Section 5.

The following variables are used as state variables:

- Rotor angle
- Rotor angular velocity

- Flux linkage of field winding
- Flux linkage of d-axis winding
- Flux linkage of q-axis winding
- Excitation voltage
- Water speed (hydro plants)
- Steam pressure (steam plants)

In comparison with the other variables the flux linkages of d- and q-axis winding (ψ_d and ψ_q respectively) changes very rapidly and the differential equations for ψ_d and ψ_q are often approximated by algebraic equations. In this case ψ_d and ψ_q are not contained in the state vector but can be expressed as a linear combination of the state variables.

The following variables are used as input variables:

- Excitation input
- Gate opening (hydro plants)
- Steam valve setting (steam plants)
- Fuel flow (steam plants)

3. SYNCHRONOUS MACHINE AND EXCITER.

In this section we rederive Park's equations [6], [7] for the synchronous machine. Often these equations are rederived under the assumption that the machine is in steady-state, but used for the machine in transient state. Our task is to find a set of equations valid for transient as well as steady-state conditions, and this is one reason to rederive Park's equations. The material in this section is mainly based on [12].

The first step in this process is to transform the original 3-phase machine to a 2-phase machine with the same magnetomotive force (mmf). Then we transform the 2-phase machine to the dq-machine applying a second linear transformation which removes the time-varying inductances of the 2-phase machine.

3.1. The Ideal Synchronous Machine.

The windings of a 3-phase 2-pole synchronous machine are shown in Fig. 3.1. On the stator there are the three distributed a-c windings r, s and t, one in each phase. They are symbolized by the correspondingly labeled concentrated coils. The magnetic axes of the phase windings coincide with the coil axes.

The d-c field winding, f, is on the rotor. The effect of the damper windings is included in a general damping term as described in Section 3.5.

The rotor has two axes of symmetry, the polar, or direct axis d and the interpolar, or quadrature, axis q. The magnetic flux paths have different permeances in the two directions of axis. The d and q

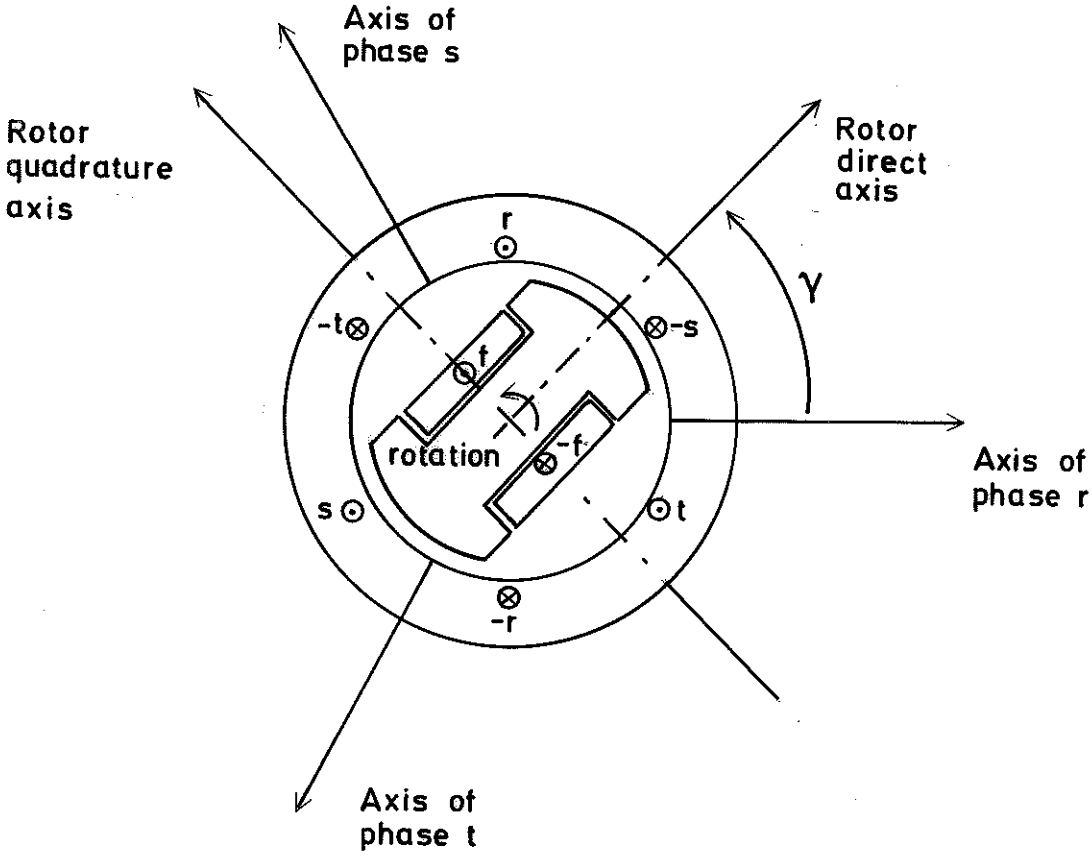

Fig. 3.1 - An idealized synchronous machine.

axes revolve with the rotor, while the magnetic axes of the three stator phases remain fixed.

In deriving the basic equations required for modelling of a synchronous machine it is assumed that:

A 3.1) The stator windings are sinusoidally distributed around the air-gap as far as the mutual effects between them and the rotor are concerned.

A 3.2) The stator winding self-and mutual-inductances vary sinusoidally as the rotor revolves, and are of the form $a+b\cdot\cos 2\gamma$ and $c+b\cos(2\gamma-2\pi/3)$ respectively, where a,b,c and d are constants.

A 3.3) Saturation and hysteresis are negligible.

The circuits r, s, t and f have their own resistance and their own self-inductance and mutual inductance with respect to every other circuit. The script letter ℓ with appropriate subscripts is used to denote these inductances for any value of γ. In terms of the self and mutual inductances ℓ, the flux linkages are

$$\begin{bmatrix}\psi_r\\ \psi_s\\ \psi_t\\ \psi_f\end{bmatrix} = \begin{bmatrix}\ell_{rr} & \ell_{rs} & \ell_{rt} & \ell_{rf}\\ \ell_{sr} & \ell_{ss} & \ell_{st} & \ell_{sf}\\ \ell_{tr} & \ell_{ts} & \ell_{tt} & \ell_{tf}\\ \ell_{fr} & \ell_{fs} & \ell_{ft} & \ell_{ff}\end{bmatrix}\begin{bmatrix}i_r\\ i_s\\ i_t\\ i_f\end{bmatrix} \tag{3.1}$$

or symbolically

$$\Psi_{rstf} = L_{rstf}I_{rstf} \tag{3.2}$$

In (3.1) all inductances except ℓ_{ff} are functions of γ and thus time-varying. We observe that assumption (A 3.3) is necessary for (3.1).

The following expressions for induced emf are valid for the 3-phase machine:

$$v_r = r_a i_r + d\psi_r/dt \tag{3.3}$$

$$v_s = r_a i_s + d\psi_s/dt \tag{3.4}$$

$$v_t = r_a i_t + d\psi_t/dt \tag{3.5}$$

$$v_f = r_f i_f + d\psi_f/dt \qquad (3.6)$$

or symbolically

$$V_{rstf} = R_{rstf} I_{rstf} + p\Psi_{rstf} \qquad (3.7)$$

where

$px = dx/dt$

r_a - armature resistance

r_f - field resistance

3.2. Transformation from a 3-phase Machine to a 2-phase Machine.

In this section we transform the 3-phase machine to a 2-phase machine with the same mmf distribution. We do not change the geometry of the iron circuits but permit the number of effective turns to change. For the transformation from 3-phase to 2-phase we require that:

R 3.1) The instantaneous value of the mmfs must be equal.

R 3.2) The currents, voltages and flux linkages must be transformed with the same matrix.

R 3.3) The instantaneous power ($I^T V$) must be invariant.

Denote the effective number of turns/phase with N_3 and N_2 and divide the mmf into components on the

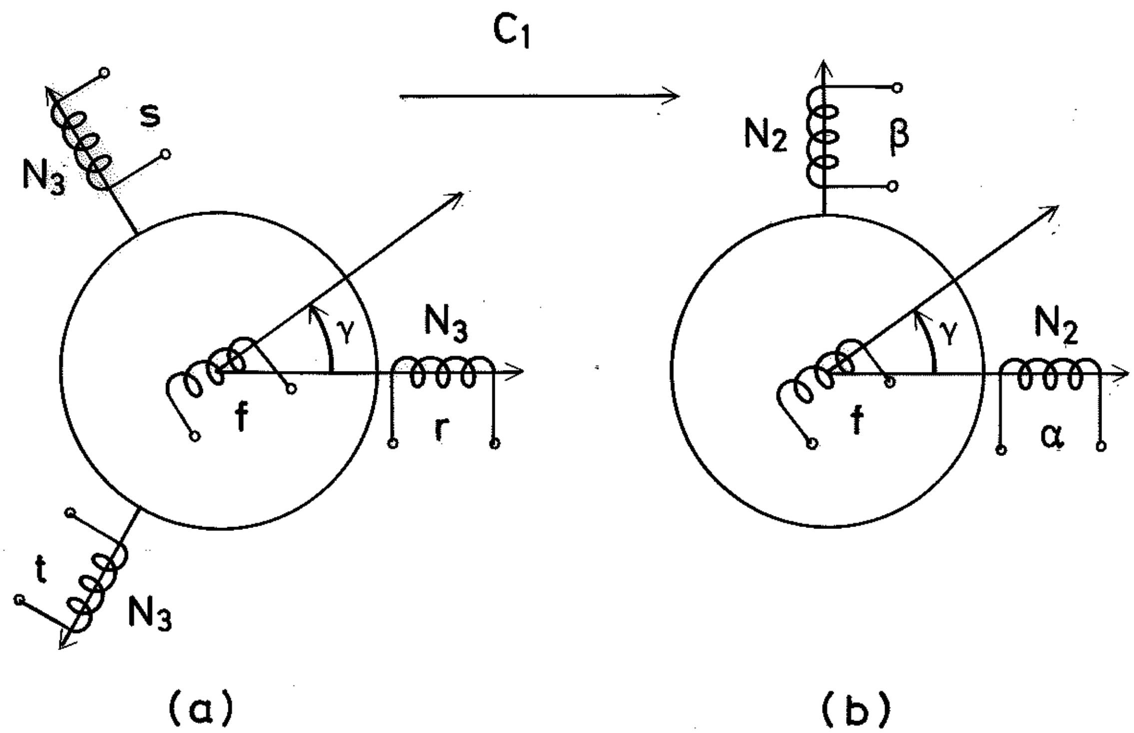

Fig. 3.2 - Transformation from 3-phase to 2-phase. The original 3-phase machine (a). The resulting 2-phase machine with the same mmf distribution (b).

α- and β-axes. R 3.1) now gives:

$$N_2 i_\alpha = N_3(i_r - i_s/2 - i_t/2) \quad (3.8)$$

$$N_2 i_\beta = N_3(\sqrt{3}\, i_s/2 - \sqrt{3}\, i_t/2) \quad (3.9)$$

$$N_o i_o = N_3(i_r + i_s + i_t) \quad (3.10)$$

$$i_f = i_f \quad (3.11)$$

The current i_o does not produce any field in the air-gap and is associated with the stator leakage inductance.

Under balanced 3-phase conditions i_o is zero. Equations (3.8) to (3.11) can be written in matrix form:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_o \\ i_f \end{bmatrix} = \begin{bmatrix} K & -K/2 & -K/2 & 0 \\ 0 & \sqrt{3}K/2 & -\sqrt{3}K/2 & 0 \\ K_1 & K_1 & K_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_r \\ i_s \\ i_t \\ i_f \end{bmatrix} \qquad (3.12)$$

where $K = N_3/N_2$ and $K_1 = N_3/N_o$.

Equation (3.12) can be written in symbolical form as:

$$I_{\alpha\beta of} = C_1 I_{rstf} \qquad (3.13)$$

Requirement R 3.2) now gives:

$$V_{\alpha\beta of} = C_1 V_{rstf} \qquad (3.14)$$

Requirement R 3.3) and equations (3.13) and (3.14) further yield:

$$P_{\alpha\beta of} = I^T_{\alpha\beta of} V_{\alpha\beta of} = I^T_{rstf} C_1^T C_1 V_{rstf} =$$

$$= P_{rstf} = I^T_{rstf} V_{rstf}$$

Hence

$$C_1^T C_1 = I \qquad (3.15)$$

Condition (3.15) implies

$$K = \sqrt{2/3}$$

$$K_1 = 1/\sqrt{3}$$

Summing up we find that the linear transformation C_1 is given by

$$I_{\alpha\beta of} = C_1 I_{rstf} \tag{3.16}$$

$$V_{\alpha\beta of} = C_1 V_{rstf} \tag{3.17}$$

$$\Psi_{\alpha\beta of} = C_1 \Psi_{rstf} \tag{3.18}$$

$$C_1 = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 & 0 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3/2} \end{bmatrix} \tag{3.19}$$

For the 2-phase machine we want to retain the structure of the equations for flux linkages and write

$$\Psi_{\alpha\beta of} = L_{\alpha\beta of} I_{\alpha\beta of} \tag{3.20}$$

Substituting (3.18), (3.2) and (3.16) into (3.20) and multiplication with C_1^T from the left yields:

$$L_{rstf} = C_1^T L_{\alpha\beta of} C_1 \tag{3.21}$$

The elements of the inductance matrix $L_{\alpha\beta of}$ will in general depend on γ. For induced voltages in the 3-phase we have (3.7)

$$V_{rstf} = R_{rstf} I_{rstf} + p\Psi_{rstf} \qquad (3.7)$$

Substitution of (3.7) into (3.17) gives

$$V_{\alpha\beta of} = C_1 R_{rstf} I_{rstf} + C_1 p \Psi_{rstf} \qquad (3.22)$$

Using (3.16) and (3.18) to eliminate I_{rstf} and Ψ_{rstf} in (3.22) we obtain:

$$V_{\alpha\beta of} = C_1 R_{rstf} C_1^T I_{\alpha\beta of} + C_1 p C_1^T \Psi_{\alpha\beta of}$$

Observing that C_1 does not depend on t we find

$$V_{\alpha\beta of} = R_{\alpha\beta of} I_{\alpha\beta of} + p\Psi_{\alpha\beta of} \qquad (3.23)$$

where

$$R_{\alpha\beta of} = C_1 R_{rstf} C_1^T \qquad (3.24)$$

From (3.3) to (3.6) we have

$$R_{rstf} = \begin{bmatrix} r_a & 0 & 0 & 0 \\ 0 & r_a & 0 & 0 \\ 0 & 0 & r_a & 0 \\ 0 & 0 & 0 & r_f \end{bmatrix} \qquad (3.25)$$

Observing that C_1 and R_{rstf} comutate we finally have

$$R_{\alpha\beta of} = R_{rstf} \qquad (3.26)$$

3.3. Transformation from a 2-phase Machine to a dq Machine.

In this section we are concerned with the transformation from a 2-phase machine with fixed coils to a dq-machine with moving coils.

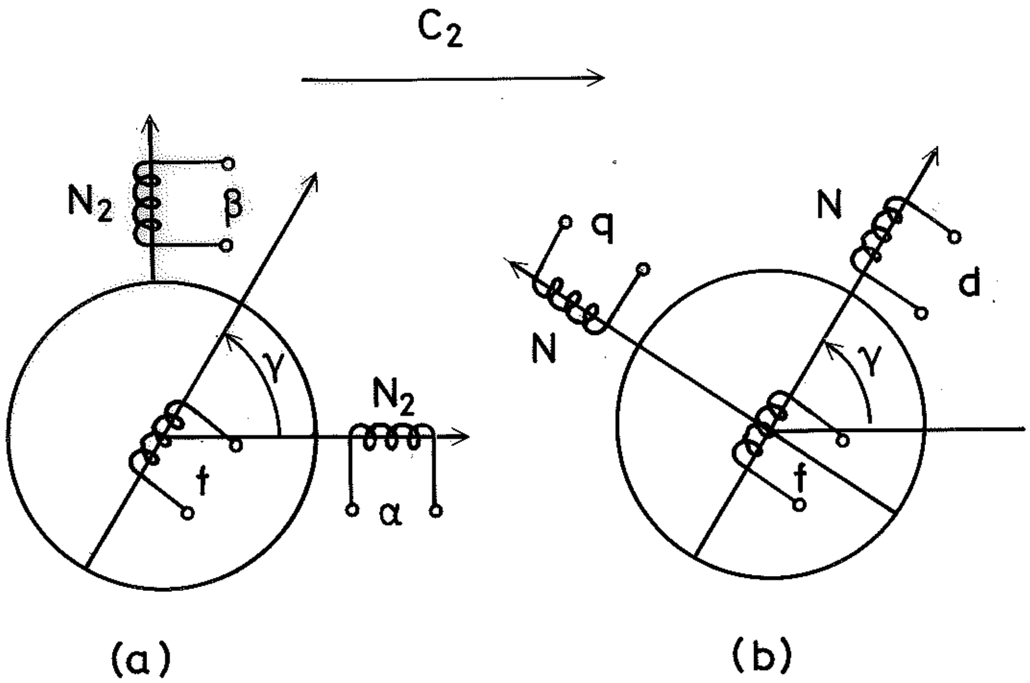

Fig. 3.3 - Transformation from a 2-phase machine to a dq-machine. Original 2-phase machine (a). Resulting dq-machine (b).

The same requirements are made on this transformation as in the previous section.

Denote the effective number of turns/phase with N_2 and N and divide the mmf into two components on the d-and q-axes respectively. Requirement R 3.1) now gives:

$$Ni_d = N_2(i_\alpha \cos\gamma + i_\beta \sin\gamma)$$

$$Ni_q = N_2(-i_\alpha \sin\gamma + i_\beta \cos\gamma)$$

or in matrix form

$$\begin{bmatrix} i_d \\ i_q \\ i_o \\ i_f \end{bmatrix} = \begin{bmatrix} N_2/N \cos\gamma & N_2/N \sin\gamma & 0 & 0 \\ -N_2/N \sin\gamma & N_2/N \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_o \\ i_f \end{bmatrix} \qquad (3.27)$$

Requirements R 3.2) and R 3.3) imply, after similar algebra as in the previous section, that:

$$C_2^T C_2 = I \qquad (3.28)$$

Condition (3.28) implies

$$N_2 = N$$

Summing up we find that the linear transformation C_2 is given by

$$I_{dqof} = C_2 I_{\alpha\beta of} \qquad (3.29)$$

$$V_{dqof} = C_2 V_{\alpha\beta of} \qquad (3.30)$$

$$\Psi_{dqof} = C_2 \Psi_{\alpha\beta of} \qquad (3.31)$$

where

$$C_2 = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.32}$$

For the dq-machine we write the flux linkage equation

$$\Psi_{dqof} = L_{dqof} I_{dqof} \tag{3.33}$$

and postulate that L_{dqof} shall be independent of γ. The value of the self-inductance of the d-axis winding can be different from the value of the self-inductance of the q-axis winding.

We write the inductance matrix L_{dqof} as

$$L_{dqof} = \begin{bmatrix} \ell+\ell_2 & 0 & 0 & \ell_1 \\ 0 & \ell-\ell_2 & 0 & 0 \\ 0 & 0 & \ell_o & 0 \\ \ell_1 & 0 & 0 & \ell_f \end{bmatrix} \tag{3.34}$$

where ℓ can be interpreted as the mean value of the self-inductance of an armature winding. The inductance ℓ_2 can be interpreted as a variation in self-inductance of an armature winding. The self-inductance of a stator winding has its maximum value when the polar axis of the rotor coincides with the magnetic axis of the stator winding. The minimum value is taken on when the interpolar axis coincides with the magnetic axis of the stator winding. The d- and q-axis windings are orthogonal, which motivates that both $[L_{dqof}]_{12}$ and $[L_{dqof}]_{21}$ are zero. The stator leakage inductance is not coupled with any other in-

ductance, which motivates the off-diagonal zeroes in the third row and the third column. The q-axis and field windings are orthogonal, which motivates that both $[L_{dqof}]_{24}$ and $[L_{dqof}]_{42}$ are zero.
Substituting (3.20) and (3.29) into (3.33) and multiplication with C_2^T from the left yields:

$$L_{\alpha\beta of} = C_2^T L_{dqof} C_2 \tag{3.35}$$

which after substitution of (3.32) into (3.35) gives:

$$L_{\alpha\beta of} = \begin{bmatrix} \ell+\ell_2 \cos 2\gamma & \ell_2 \sin 2\gamma & 0 & \ell_1 \cos\gamma \\ \ell_2 \sin 2\gamma & \ell-\ell_2 \cos 2\gamma & 0 & \ell_1 \sin\gamma \\ 0 & 0 & \ell_o & 0 \\ \ell_1 \cos\gamma & \ell_1 \sin\gamma & 0 & \ell_f \end{bmatrix} \tag{3.36}$$

To derive an expression for L_{rstf} we use equation (3.21)

$$L_{rstf} = C_1^T L_{\alpha\beta of} C_1 \tag{3.21}$$

Substitution of (3.36) into (3.21) now yields:

$$L_{rstf} = \begin{bmatrix} \ell_{11} & \ell_{12} & \ell_{13} & \ell_{14} \\ \ell_{21} & \ell_{22} & \ell_{23} & \ell_{24} \\ \ell_{31} & \ell_{32} & \ell_{33} & \ell_{34} \\ \ell_{41} & \ell_{42} & \ell_{43} & \ell_{44} \end{bmatrix} \tag{3.37}$$

where

$$\ell_{11} = \ell_0/2 + \ell + \ell_2 \cos 2\gamma$$

$$\ell_{12} = \ell_{21} = - \ell/2 + \ell_2 \cos(2\gamma-2\pi/3)$$

$$\ell_{13} = \ell_{31} = - \ell/2 + \ell_2 \cos(2\gamma+2\pi/3)$$

$$\ell_{14} = \ell_{41} = \sqrt{3/2}\ \ell_1 \cos \gamma$$

$$\ell_{22} = \ell_0/2 + \ell + \ell_2 \cos(2\gamma+2\pi/3)$$

$$\ell_{23} = \ell_{32} = - \ell/2 + \ell_2 \cos 2\gamma$$

$$\ell_{24} = \ell_{42} = \sqrt{3/2}\ \ell_1 \cos(\gamma-2\pi/3)$$

$$\ell_{33} = \ell_0/2 + \ell + \ell_2 \cos(2\gamma-2\pi/3)$$

$$\ell_{34} = \ell_{43} = \sqrt{3/2}\ \ell_1 \cos(\gamma+2\pi/3)$$

$$\ell_{44} = 3/2 \cdot \ell_f$$

We now observe that assumption A 3.1) and assumption A 3.2) allow L_{dqof} to be a constant matrix.

To derive an expression for induced voltages in the dq machine we use equation (3.23)

$$V_{\alpha\beta of} = R_{\alpha\beta of} I_{\alpha\beta of} + p\Psi_{\alpha\beta of} \tag{3.23´}$$

A substitution of (3.29) and (3.31) into (3.23´) yields after multiplication with C_2 from the left

$$V_{dqof} = C_2 R_{\alpha\beta of} C_2^T I_{dqof} + C_2 p(C_2^T \Psi_{dqof}) \tag{3.38}$$

Taking derivative of the second term in (3.38) gives

$$V_{dqof} = R_{dqof} I_{dqof} + C_2 p(C_2^T)\Psi_{dqof} + p\Psi_{dqof} \qquad (3.39)$$

where R_{dqof} is given by

$$R_{dqof} = C_2 R_{\alpha\beta of} C_2^T \qquad (3.40)$$

Using (3.26) we find

$$R_{dqof} = C_2 R_{rstf} C_2^T \qquad (3.41)$$

Observing that C_2 and R_{rstf} commutate we have the following expression for R_{dqof}

$$R_{dqof} = R_{rstf} \qquad (3.42)$$

Introducing

$$W = C_2 p(C_2^T) = \begin{bmatrix} 0 & -\frac{d\gamma}{dt} & 0 & 0 \\ \frac{d\gamma}{dt} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3.43)$$

(3.39) can be written

$$V_{dqof} = R_{dqof} I_{dqof} + W\Psi_{dqof} + p\Psi_{dqof} \qquad (3.44)$$

The first term in equation (3.44) represents voltage drop across the armature resistance. The second term represents the speed voltages and the third term represents transformer voltages.

3.4. The Air-Gap Torque.

To derive an expression for the air-gap torque we apply the principle of conservation of energy, which can be formulated

$$\frac{dE}{dt} = \frac{dE_e}{dt} + \frac{dE_m}{dt} = P_m + P_n - P_{\ell r} - P_{\ell m} - P_{\ell d} \qquad (3.45)$$

where

E_e = energy stored in the magnetic circuits

E_m = energy stored in the rotating masses

P_m = power delivered from the prime mover

P_n = power delivered from the network

$P_{\ell r}$ = power losses in the resistances

$P_{\ell m}$ = mechanical power losses

$P_{\ell d}$ = power losses in the damping winding

The energy stored in the magnetic circuits can be written

$$E_e = \frac{1}{2} I^T_{dqof} L_{dqof} I_{dqof} \qquad (3.46)$$

The energy stored in the rotating masses is given by

$$E_m = \frac{1}{2} J\omega^2 \qquad (3.47)$$

where

J = moment of inertia of the combined turbine generator

ω = angular velocity of rotor

The electrical power delivered from the network to the generator is given by the expression

$$P_n = I^T_{dqof} V_{dqof} \tag{3.48}$$

Substituting (3.44) into (3.48) gives

$$P_n = I^T_{dqof} R_{dqof} I_{dqof} + I^T_{dqof} W\psi_{dqof} + $$
$$+ I^T_{dqof} p\Psi_{dqof} \tag{3.49}$$

The power losses in the resistances is given by

$$P_{\ell r} = I^T_{dqof} R_{dqof} I_{dqof} \tag{3.50}$$

Finally we assume that the power losses in the damping windings can be written

$$P_{\ell d} = D_1 \omega(\omega - \omega_0) \tag{3.51}$$

and the mechanical power losses can be written

$$P_{\ell m} = D_2 \omega^2 \tag{3.52}$$

Taking derivatives of (3.47) and (3.48) now gives

$$\frac{dE_m}{dt} = J\omega \frac{d\omega}{dt} \tag{3.53}$$

$$\frac{dE_e}{dt} = I_{dqof}^T \frac{d}{dt} (L_{dqof} I_{dqof}) = I_{dqof}^T p\psi_{dqof} \qquad (3.54)$$

Substitution of (3.49) to (3.54) into (3.45) and rearranging the terms yields

$$J \frac{d\omega}{dt} = P_m/\omega - M_e - M_d \qquad (3.55)$$

where the air-gap torque M_e is given by

$$M_e = - I_{dqof}^T W\psi_{dqof}/\omega = \psi_q i_d - \psi_d i_q \qquad (3.56)$$

The damping torque M_d is given by

$$M_d = D_1(\omega-\omega_0) + D_2\omega \qquad (3.57)$$

Equations (3.55), (3.56) and (3.57) will be used in the following sections.

3.5. Linearized Equations for the Synchronous Generator.

The nonlinear equations for the synchronous machine will now be linearized. To avoid a lot of negative numerical values of the generator currents we will also change the sign conventions for i_d, i_q and i_o. Motor references were previously used for all circuits. We will now use generator references for d- and q-axis windings as well as for the zero sequence winding.

Fig. 3.4 - Sign conventions. Motor references (a) and generator references (b).

Motor references means that applied voltage and current into the winding are positive.

Generator references, on the other hand, means that generated voltage and current out of the winding are positive.

In Section 3.3 we derived expressions for the flux linkages

$$\Psi_{dq0f} = L_{dq0f} I_{dq0f} \qquad (3.33')$$

$$L_{dq0f} = \begin{vmatrix} \ell+\ell_2 & 0 & 0 & \ell_1 \\ 0 & \ell-\ell_2 & 0 & 0 \\ 0 & 0 & \ell_0 & 0 \\ \ell_1 & 0 & 0 & \ell_f \end{vmatrix} \qquad (3.34')$$

After change of sign conventions we can write

$$\psi_f = L_f i_f - L_{af} i_d \qquad (3.58)$$

$$\psi_d = L_{af} i_f - L_d i_d \tag{3.59}$$

$$\psi_q = - L_q i_q \tag{3.60}$$

where we have introduced

$$L_f = \ell_f$$

$$L_{af} = \ell_1$$

$$L_d = \ell + \ell_2$$

$$L_q = \ell - \ell_2$$

As the equations (3.58) to (3.60) already are linear they are immediately valid for small deviations and we have

$$\begin{bmatrix} \delta\psi_f \\ \delta\psi_d \\ \delta\psi_q \end{bmatrix} = \begin{bmatrix} L_f & -L_{af} & 0 \\ L_{af} & -L_d & 0 \\ 0 & 0 & -L_q \end{bmatrix} \begin{bmatrix} \delta i_f \\ \delta i_d \\ \delta i_q \end{bmatrix} \tag{3.61}$$

where $\delta x = x - x_o$. Equation (3.61) can be written symbolically as

$$\delta\Psi = L\delta I \tag{3.62}$$

which after multiplication with ω_o yields

$$\delta\omega_o\Psi = X\delta I \tag{3.63}$$

where

$$X = \omega_o L \tag{3.64}$$

In Section 3.3 we also derived expressions for induced voltages in the dq machine.

$$V_{dqof} = R_{dqof} I_{dqof} + W\Psi_{dqof} + p\Psi_{dqof} \qquad (3.44')$$

$$W = \begin{bmatrix} 0 & -\frac{d\gamma}{dt} & 0 & 0 \\ \frac{d\gamma}{dt} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3.43')$$

After change of sign conventions (3.43´) and (3.44´) can be written in component form

$$v_f = p\psi_f + r_f i_f \qquad (3.65)$$

$$v_d = p\psi_d - r_a i_d - \omega\psi_q \qquad (3.66)$$

$$v_q = p\psi_q - r_a i_q + \omega\psi_d \qquad (3.67)$$

After linearization (3.65) to (3.67) become

$$\delta v_f = p\delta\psi_f + r_f \delta i_f \qquad (3.68)$$

$$\delta v_d = p\delta\psi_d - r_a \delta i_d - \omega_o \delta\psi_q - \psi_q \delta\omega \qquad (3.69)$$

$$\delta v_q = p\delta\psi_q - r_a \delta i_q + \omega_o \delta\psi_d + \psi_d \delta\omega \qquad (3.70)$$

where the angular velocity $\omega = d\gamma/dt$.

In Section 3.4 we derived the following expression for the air-gap torque

$$M_e = \psi_q i_d - \psi_d i_q \qquad (3.56')$$

After change of sign conventions (3.56´) can be written

$$M_e = \psi_d i_q - \psi_q i_d \tag{3.71}$$

which after linearization becomes

$$\delta M_e = i_q \delta\psi_d - i_d \delta\psi_q - \psi_q \delta i_d + \psi_d \delta i_q \tag{3.72}$$

3.6. Basic Equations for the Exciter.

The exciter system of each generator is assumed to be described by a first order linear system with time-constant T_e.

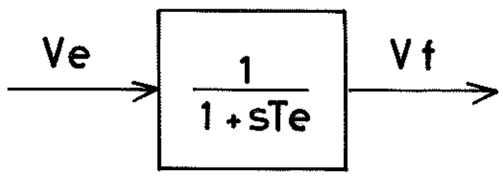

Fig. 3.5 - Block diagram for the exciter.

The differential equation describing the exciter is obtained from Fig. 3.5

$$pv_f = (- v_f + v_e)/T_e \tag{3.73}$$

Since equation (3.73) already is linear it is also valid for small deviations from an equilibrium point and we have

$$p\delta v_f = (- \delta v_f + \delta v_e)/T_e \tag{3.74}$$

4. TRANSMISSION NETWORK.

In the previous sections we derived equations for the individual generators and used the polar and the interpolar axes of the rotor as reference frames for the electric quantities. These axes do not in general coincide with corresponding axes of another generator. In this section we choose a common frame of references for the electric quantities. We also derive a transformation from rotor-based to network-based quantities.

4.1. Selection of Angular References.

The equations for each generator are expressed with reference to pairs of axes (d,q) which rotate in synchronism with the rotor of the generators. On the other hand, the equations for the connecting network refer to axes (D,Q) rotating at constant speed (ω_o). In steady-state these axes rotate at the same speed.

The angular displacements, defined in Fig. 4.1, can be obtained from the solution of the load-flow problem.

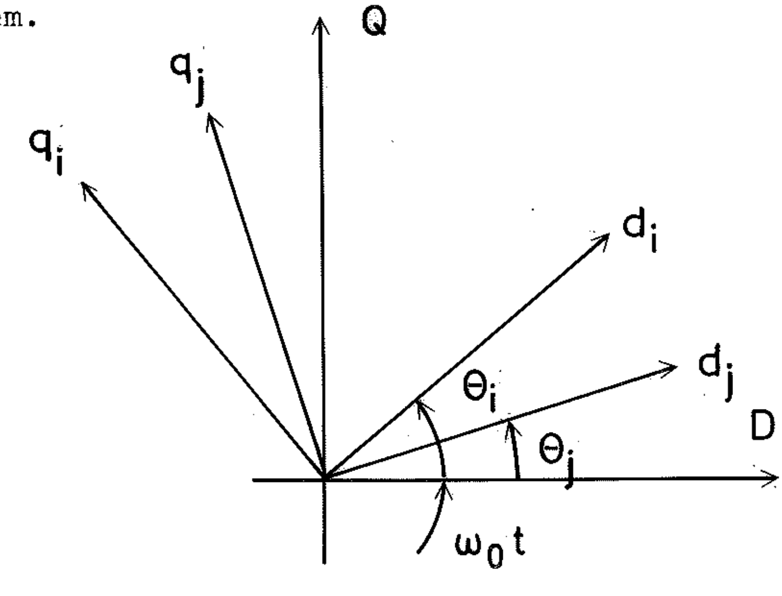

Fig. 4.1 - Angular relationships between network and synchronous machine reference axes.

The choice of common reference frame is not unique. One reasonable choice is that (D,Q) coincide with (d_ℓ, q_ℓ), the reference frame of the largest generator, in steady-state.

Under transient conditions the angles Θ_i will vary as the machine speeds vary. The angles Θ_i are state-variables and i_d and i_q are linear combinations of state-variables but v_d and v_q are needed for the computation of $p\delta\psi_d$ and $p\delta\psi_q$. Therefore it is necessary to have an expression for δv_d and δv_q in $\delta\Theta$, δi_d and δi_q.

4.2. Transformation of Network Equations.

The transformation relating rotor-based voltages to network-based voltages is given in [10]

$$\begin{bmatrix} v_{d1} \\ v_{q1} \\ \vdots \\ v_{dn} \\ v_{qn} \end{bmatrix} = \begin{bmatrix} \cos\Theta_1 & \sin\Theta_1 & \cdots & 0 & 0 \\ -\sin\Theta_1 & \cos\Theta_1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & \cos\Theta_n & \sin\Theta_n \\ 0 & 0 & \cdots & -\sin\Theta_n & \cos\Theta_n \end{bmatrix} \begin{bmatrix} v_{D1} \\ v_{Q1} \\ \vdots \\ v_{Dn} \\ v_{Qn} \end{bmatrix} \tag{4.1}$$

For the present approach we reorder the equations in (4.1) to obtain

$$\begin{bmatrix} v_{d1} \\ \vdots \\ v_{dn} \\ \hline v_{q1} \\ \vdots \\ v_{qn} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \cos\theta_1 & & 0 & \sin\theta_1 & & 0 \\ & \ddots & & & \ddots & \\ 0 & & \cos\theta_n & 0 & & \sin\theta_n \\ \hline -\sin\theta_1 & & 0 & \cos\theta_1 & & 0 \\ & \ddots & & & \ddots & \\ 0 & & -\sin\theta_n & 0 & & \cos\theta_n \end{array}\right] \begin{bmatrix} v_{D1} \\ \vdots \\ v_{Dn} \\ \hline v_{Q1} \\ \vdots \\ v_{Qn} \end{bmatrix} \tag{4.2}$$

Equation (4.2) can be written

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \begin{bmatrix} V_D \\ V_Q \end{bmatrix} \tag{4.3}$$

where

$$V_d = (v_{d1}, v_{d2}, \ldots, v_{dn})^T \tag{4.4}$$

$$V_q = (v_{q1}, v_{q2}, \ldots, v_{qn})^T \tag{4.5}$$

$$V_D = (v_{D1}, v_{D2}, \ldots, v_{Dn})^T \tag{4.6}$$

$$V_Q = (v_{Q1}, v_{Q2}, \ldots, v_{Qn})^T \tag{4.7}$$

$$C = \mathrm{diag}(\cos\theta_1, \cos\theta_2, \ldots, \cos\theta_n) \tag{4.8}$$

$$S = \mathrm{diag}(\sin\theta_1, \sin\theta_2, \ldots, \sin\theta_n) \tag{4.9}$$

We also need a transformation from rotor-based currents to network-based currents. We require that the power I^TV shall be invariant under the transformation. To derive the transformation matrix we rewrite (4.3)

$$V_M = TV_N \tag{4.10}$$

where

$$V_M = (V_d^T, V_q^T)^T$$

$$V_N = (V_D^T, V_Q^T)^T$$

$$T = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}$$

The power invariance requires

$$I_N^T V_N = I_M^T V_M = I_M^T T V_N$$

which implies

$$I_N = T^T I_M \tag{4.11}$$

where

$$I_N = (I_D^T, I_Q^T)^T$$

$$I_M = (I_d^T, I_q^T)^T$$

In Section 2 we stated that the network could be described by the nodal admittance matrix in (2.3)

$$\begin{bmatrix} I_D \\ I_Q \end{bmatrix} = \begin{bmatrix} G_N & -B_N \\ B_N & G_N \end{bmatrix} \begin{bmatrix} V_D \\ V_Q \end{bmatrix} \qquad (2.3')$$

The nodal admittance matrix is always nonsingular, making it possible to write

$$\begin{bmatrix} V_D \\ V_Q \end{bmatrix} = \begin{bmatrix} R_N & -X_N \\ X_N & R_N \end{bmatrix} \begin{bmatrix} I_D \\ I_Q \end{bmatrix} \qquad (4.12)$$

Substitution of (4.11) and (4.12) into (4.3) now yields

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \begin{bmatrix} R_N & -X_N \\ X_N & R_N \end{bmatrix} \begin{bmatrix} C & -S \\ S & C \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix} \qquad (4.13)$$

which can be written

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} R_m & -X_m \\ X_m & R_m \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix} \qquad (4.14)$$

where

$$R_m = CR_NC + SR_NS + SX_NC - CX_NS \qquad (4.15)$$

$$X_m = CX_NC + SX_NS + CR_NS - SR_NC \qquad (4.16)$$

The linearized version of (4.14) can be written

$$\begin{bmatrix} \delta V_d \\ \delta V_q \end{bmatrix} = \begin{bmatrix} R_m & -X_m \\ X_m & R_m \end{bmatrix} \begin{bmatrix} \delta I_d \\ \delta I_q \end{bmatrix} + \begin{bmatrix} E_d \\ E_q \end{bmatrix} \delta\Theta \qquad (4.17)$$

where

$$E_d = \frac{\partial}{\partial \Theta} (R_m I_d - X_m I_q) \tag{4.18}$$

and

$$E_q = \frac{\partial}{\partial \Theta} (X_m I_d + R_m I_q) \tag{4.19}$$

5. PRIME MOVERS.

The fundamental torque balance equation was derived in Section 3.4

$$Jp\omega = P_m/\omega - M_e - M_d \tag{3.55}$$

where

J = moment of inertia of the combined turbine generator

ω = angular velocity of the rotor

P_m = mechanical power delivered from the prime mover

M_e = air-gap torque

M_d = damping torque

Assuming

$$M_d = D_1(\omega-\omega_o) + D_2\omega$$

we find

$$Jp\delta\omega = \delta P_m/\omega_o - \delta M_e - D\delta\omega \tag{5.1}$$

where

$$D = D_1 + D_2 + P_m/\omega_o \tag{5.2}$$

To obtain expressions for δP_m it is necessary to investigate the different types of prime movers.

5.1. Hydro Turbines.

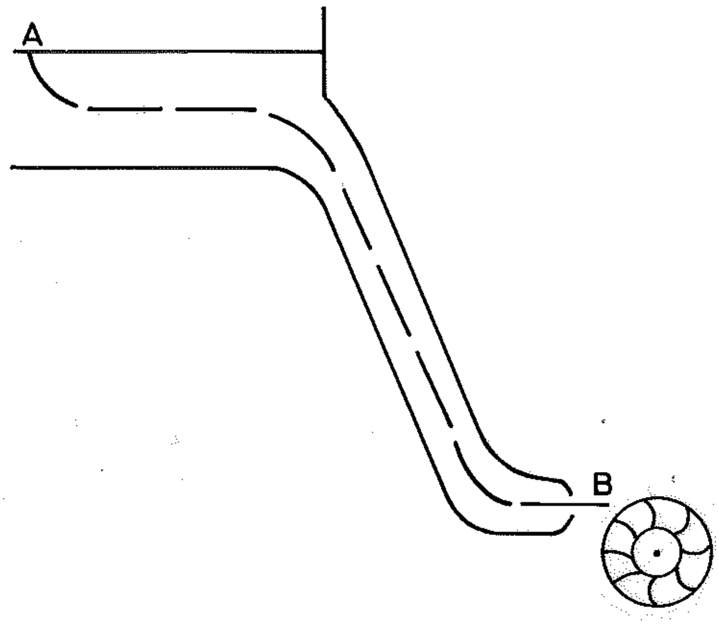

Fig. 5.1 - Simplified diagram of a hydro turbine.

Following [13] we have for the hydro turbine in Fig. 5.1

$$P_m = \frac{1}{2} \rho v_{out}^2 q = \frac{1}{2} \rho a v_{out}^3 \qquad (5.3)$$

which essentially states that all potential energy is converted to kinetic energy and that all kinetic energy is available as output power from the hydro turbine.

In equation (5.3)

P_m - mechanical output power

q - flow of water

ρ - density of water

v_{out} - velocity of water at outlet

a - outlet area

In practice it is observed [15] that the efficiency is depending on the angular velocity of the hydro turbine. This means that (5.3) has to be written

$$P_m = \frac{1}{2}\, \eta(\omega)\rho \cdot a v_{out}^3 \qquad (5.3')$$

where $\eta(\omega)$ is a speed dependent efficiency. Bernoulli's theorem yields further

$$L\dot{v}_t + \frac{1}{2}\, v_{out}^2 - gh = 0 \qquad (5.4)$$

where

v_t - velocity of water in the dash-tube

L - length of the dash-tube

g - constant of gravity

h - water head

The dash-tube is assumed to have the constant area A. The equation of continuity implies

$$a v_{out} = A v_t \qquad (5.5)$$

A substitution of (5.5) into (5.6) yields

$$pv_t = gh/L - (Av_t/a)^2/2L \qquad (5.6)$$

Introducing the maximum steady-state velocity of the water in the dash-tube

$$v_{tmax} = \sqrt{2gh}\; a_{max}/A \tag{5.7}$$

and the state-variable

$$z_p = v_t/v_{tmax} \tag{5.8}$$

we obtain

$$pz_p = \frac{A\sqrt{2gh}}{2La_{max}} (1 - z_p^2 a_{max}^2/a^2) \tag{5.9}$$

Substituting

$$u_t = a/a_{max} \tag{5.10}$$

$$T\omega = La_{max}/A\sqrt{2gh} \tag{5.11}$$

Equations (5.3) and (5.9) are transferred into

$$pz_p = (1 - z_p^2/u_t^2)/T_w \tag{5.12}$$

$$P_m = P_{max}(\omega) z_p^3/u_t^2 \tag{5.13}$$

Linearization of (5.12) and (5.13) finally yields

$$pz_p = (- \delta z_p + \delta u_t)/T_w \tag{5.14}$$

$$\delta P_m = P_{max}(\omega_o)(3\delta z_p - 2\delta u_t) + \frac{\partial P_{max}}{\partial \omega} (\omega_o)\delta\omega \tag{5.15}$$

If we include

$$\frac{\partial P_{max}}{\partial \omega} (\omega_o)\delta\omega$$

in the general damping term in (5.2) we can write (5.14) and (5.15) as

$$p\delta z_p = a\delta z_p + b\delta u_t \tag{5.16}$$

$$\delta P_m = c\delta z_p + d\delta u_t \tag{5.17}$$

5.2. Boilers and Steam Turbines.

Åström and Eklund [14] have shown that a reasonable accurate and low order dynamical model of a boiler and steam turbine unit is given by

$$\frac{dp}{dt} = -\alpha_1(u_1 p^{5/8} - \alpha_5) + \alpha_2 u_2 - \alpha_3 u_3 \tag{5.18}$$

where

p - steam pressure

u_1 - steam valve setting

u_2 - fuel flow

u_3 - feedwater flow

The mechanical output power from the turbine is given by

$$P_m = \alpha_4(u_1 p^{5/8} - \alpha_5) \tag{5.19}$$

The model (5.18) is essentially an energy balance equation and it is assumed that the stored energy in the boiler mainly depends on the steam pressure.

The first term in the right member of (5.18) represents the energy in the steam delivered from the boiler to the turbine. The second term represents energy supplied from the fuel while the last term represents the cooling effect of the feedwater. The output power is proportional to the energy flow from the boiler to the turbine, which implies that the boiler steam-turbine plant has constant efficiency.

In this application we are not allowed to vary u_3, the feedwater flow, independent of the state of the boiler. Instead we assume that the boiler is equipped with a feedwater regulator, which provides the boiler with feedwater flow proportional to the steam flow. The steam flow is given by

$$q \sim u_1\sqrt{p}$$

We also introduce the following normalized variables into (5.18) and (5.19)

$$z_p = p/p_{max} \tag{5.20}$$

$$u_t = u_1/u_{1max} \tag{5.21}$$

$$u_f = u_2/u_{2max} \tag{5.22}$$

and prescribe that the differential equation (5.18) shall have a stationary point at

$$p = p_{max}$$

$$u_1 = u_{1max}$$

$$u_2 = u_{2max}$$

$$u_3 = u_{3max}$$

$$P_b = P_{max}$$

Equations (5.18) and (5.19) can then be written

$$pz_p = \left\{-[(1+\alpha)u_t z_p^{5/8} - \alpha] + (1+\beta)u_f - \beta u_t z_p^{1/2}\right\}/T_b \quad (5.23)$$

$$P_m = P_{max}[(1+\alpha)u_t z_p^{5/8} - \alpha] \quad (5.24)$$

The linearized version of equations (5.23) and (5.24) becomes

$$p\delta z_p = a\delta z_p + b_t\delta u_t + b_f\delta\omega_f \quad (5.25)$$

$$\delta P_m = c\delta z_p + d_t\delta u_t \quad (5.26)$$

6. CONSTRUCTION OF SYSTEM MATRICES.

In the previous sections we derived the equations for one machine at a time but we are now going to derive the differential equations for n interconnected generating plants. To simplify the derivations and to improve the clarity we partition the state and the input vector into subvectors. The system matrices A and B in (1.1) are similarly partitioned into submatrices. In this section we are going to derive the differential equations for the subvectors one by one.

The state vector x is partitioned into seven subvectors in the following manner

$$x^T = (x_1^T, x_2^T, x_3^T, x_4^T, x_5^T, x_6^T, x_7^T) \tag{6.1}$$

where

$$x_1^T = (\delta\Theta_1, \delta\Theta_2, \ldots, \delta\Theta_n) \tag{6.2}$$

$$x_2^T = (\delta\omega_1, \delta\omega_2, \ldots, \delta\omega_n) \tag{6.3}$$

$$x_3^T = (\delta\omega_o\psi_{f1}, \delta\omega_o\psi_{f2}, \ldots, \delta\omega_o\psi_{fn}) \tag{6.4}$$

$$x_4^T = (\delta\omega_o\psi_{d1}, \delta\omega_o\psi_{d2}, \ldots, \delta\omega_o\psi_{dn}) \tag{6.5}$$

$$x_5^T = (\delta\omega_o\psi_{q1}, \delta\omega_o\psi_{q2}, \ldots, \delta\omega_o\psi_{qn}) \tag{6.6}$$

$$x_6^T = (\delta e_{f1}, \delta e_{f2}, \ldots, \delta e_{fn}) \tag{6.7}$$

$$x_7^T = (\delta z_{p1}, \delta z_{p2}, \ldots, \delta z_{pn}) \tag{6.8}$$

where $e_{fi} = x_{afi}v_{fi}/r_{fi}$ and z_{pi} is the prime mover state-variable for the i:th plant (z_h or z_b).

The input vector u is partitioned into 3 subvectors in the following manner

$$u^T = (u_1^T, u_2^T, u_3^T) \tag{6.9}$$

where

$$u_1^T = (\delta u_{e1}, \delta u_{e2}, \ldots, \delta u_{en}) \tag{6.10}$$

$$u_2^T = (\delta u_{t1}, \delta u_{t2}, \ldots, \delta u_{tn}) \tag{6.11}$$

$$u_3^T = (\delta u_{f1}, \delta u_{f2}, \ldots, \delta u_{fn}) \tag{6.12}$$

Here u_{ei} denotes the input signal to exciter i, u_{ti} denotes the first input signal to prime mover No. i (gate opening or steam valve setting) and u_{fi} denotes the second input signal to prime mover No. i (fuel flow for steam plants).

In a similar way the system matrices A and B are partitioned into submatrices and we can write (1.1) as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & A_{67} \\ A_{71} & A_{72} & A_{73} & A_{74} & A_{75} & A_{76} & A_{77} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} +$$

$$+ \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \\ B_{41} & B_{42} & B_{43} \\ B_{51} & B_{52} & B_{53} \\ B_{61} & B_{62} & B_{63} \\ B_{71} & B_{72} & B_{73} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

We are now going to derive expressions for the submatrices one row at a time.

6.1. Differential Equations for Rotor Angles (x_1).

The rotor angle is defined by

$$\Theta_i(t) = \int_0^t \omega_i(s)ds - \omega_o t + \Theta_{io}$$

which immediately gives

$$p\delta\Theta_i = \omega_i(t) - \omega_o = \delta\omega_i \qquad (6.13)$$

or symbolically

$$p\delta\ \Theta = A_{12}\delta\Omega \qquad (6.14)$$

where

$$\delta\ \Theta = (\delta\Theta_1, \delta\Theta_2, \ldots, \delta\Theta_n)^T \qquad (6.15)$$

$$\delta\Omega = (\delta\omega_1, \delta\omega_2, \ldots, \delta\omega_n)^T \tag{6.16}$$

$$A_{12} = I \tag{6.17}$$

6.2. Differential Equations for Rotor Angular Velocities (x_2).

The torque balance equation for the rotor was derived in Section 5.

$$p\delta\omega_i = (\delta P_{mi}/\omega_o - \delta M_{ei} - D_i\delta\omega_i)/J_i \tag{5.1´}$$

where δP_{mi} is the mechanical input power from the prime mover given by (5.17) or (5.26)

$$\delta P_{mi} = c_i\delta z_{pi} + d_i\delta u_{ti} \tag{5.17´}$$

$$\delta P_{mi} = c_i\delta z_{pi} + d_i\delta u_{ti} \tag{5.26´}$$

δM_{ei} is the air-gap torque given by (3.72)

$$\delta M_{ei} = i_{qi}\delta\psi_{di} - i_{di}\delta\psi_{qi} - \psi_{qi}\delta i_{di} + \psi_{di}\delta i_{qi} \tag{3.72´}$$

Collecting the torque balance equations for all rotor we have

$$p\delta\Omega = A_{22}\delta\Omega + G_q\delta I_d + G_d\delta I_q + H_q\delta\omega_o\Psi_d + $$

$$+ H_d\delta\omega_o\Psi_q + A_{27}\delta Z_p + B_{22}\delta U_{p1} \tag{6.18}$$

where

$$\delta I_d = (\delta i_{d1}, \delta i_{d2}, \ldots, \delta i_{dn})^T \qquad (6.19)$$

$$\delta I_q = (\delta i_{q1}, \delta i_{q2}, \ldots, \delta i_{qn})^T \qquad (6.20)$$

$$\delta \omega_o \Psi_d = (\delta \omega_o \psi_{d1}, \delta \omega_o \psi_{d2}, \ldots, \delta \omega_o \psi_{dn})^T \qquad (6.21)$$

$$\delta \omega_o \Psi_q = (\delta \omega_o \psi_{q1}, \delta \omega_o \psi_{q2}, \ldots, \delta \omega_o \psi_{qn})^T \qquad (6.22)$$

$$\delta Z_p = (\delta z_{p1}, \delta z_{p2}, \ldots, \delta z_{pn})^T \qquad (6.23)$$

$$\delta U_t = (\delta u_{t1}, \delta u_{t2}, \ldots, \delta u_{tn})^T \qquad (6.24)$$

$$A_{22} = \mathrm{diag}(-D_1/J_1, -D_2/J_2, \ldots, -D_n/J_n) \qquad (6.25)$$

$$G_q = \mathrm{diag}(-\psi_{q1}/J_1, -\psi_{q2}/J_2, \ldots, -\psi_{qn}/J_n) \qquad (6.26)$$

$$G_d = \mathrm{diag}(\psi_{d1}/J_1, \psi_{d2}/J_2, \ldots, \psi_{dn}/J_n) \qquad (6.27)$$

$$H_q = \mathrm{diag}(i_{q1}/\omega_o J_1, i_{q2}/\omega_o J_2, \ldots, i_{qn}/\omega_o J_n) \qquad (6.28)$$

$$H_d = \mathrm{diag}(-i_{d1}/\omega_o J_1, -i_{d2}/\omega_o J_2, \ldots, -i_{dn}/\omega_o J_n) \qquad (6.29)$$

$$A_{27} = \mathrm{diag}(c_1/\omega_o J_1, c_2/\omega_o J_2, \ldots, c_n/\omega_o J_n) \qquad (6.30)$$

$$B_{22} = \mathrm{diag}(d_1/\omega_o J_1, d_2/\omega_o J_2, \ldots, d_n/\omega_o J_n) \qquad (6.31)$$

To derive an expression for I_d and I_q we use equation (3.63) derived in Section 3.5

$$\delta \omega_o \Psi = X \delta I \qquad (3.63')$$

The matrix X in (3.63´) is always nonsingular for physical reasons.

The stored energy in the magnetic circuits can be written $I^T X I/\omega_o$ and the stored energy is always positive if any current differs from zero. Hence we can always express δI in terms of $\delta\omega_o\Psi$. Equation (3.63′) can be written

$$\begin{bmatrix} \delta\omega_o\psi_f \\ \delta\omega_o\psi_d \\ \delta\omega_o\psi_q \end{bmatrix} = \begin{bmatrix} X_f & -X_{af} & 0 \\ X_{af} & -X_d & 0 \\ 0 & 0 & X_q \end{bmatrix} \begin{bmatrix} \delta i_f \\ \delta i_d \\ \delta i_q \end{bmatrix} \qquad (6.32)$$

where

$$X_f = \omega_o L_f \qquad (6.33)$$

$$X_{af} = \omega_o L_{af} \qquad (6.34)$$

$$X_d = \omega_o L_d \qquad (6.35)$$

$$X_q = \omega_o L_q \qquad (6.36)$$

Knowing that the inverse of X exists and observing the structure of the matrix we have

$$\begin{bmatrix} \delta i_f \\ \delta i_d \\ \delta i_q \end{bmatrix} = \begin{bmatrix} y_{ff} & y_{fd} & 0 \\ y_{df} & y_{dd} & 0 \\ 0 & 0 & y_{qq} \end{bmatrix} \begin{bmatrix} \delta\omega_o\psi_f \\ \delta\omega_o\psi_d \\ \delta\omega_o\psi_q \end{bmatrix} \qquad (6.37)$$

Now it is possible to form the expressions for δI_f, δI_d and δI_q.

$$\delta I_f = Y_{ff}\delta\omega_o\Psi_f + Y_{fd}\delta\omega_o\Psi_d \qquad (6.38)$$

$$\delta I_d = Y_{df}\delta\omega_o\Psi_f + Y_{dd}\delta\omega_o\Psi_d \qquad (6.39)$$

$$\delta I_q = Y_{qq}\delta\omega_o\Psi_q \qquad (6.40)$$

where

$$\delta I_f = (\delta i_{f1}, \delta i_{f2}, \ldots, \delta i_{fn})^T \tag{6.41}$$

$$\delta I_d = (\delta i_{d1}, \delta i_{d2}, \ldots, \delta i_{dn})^T \tag{6.19´}$$

$$\delta I_q = (\delta i_{q1}, \delta i_{q2}, \ldots, \delta i_{qn})^T \tag{6.20´}$$

$$Y_{ff} = \mathrm{diag}(y_{ff1}, y_{ff2}, \ldots, y_{ffn}) \tag{6.42}$$

$$Y_{fd} = \mathrm{diag}(y_{fd1}, y_{fd2}, \ldots, y_{fdn}) \tag{6.43}$$

$$Y_{df} = \mathrm{diag}(y_{df1}, y_{df2}, \ldots, y_{dfn}) \tag{6.44}$$

$$Y_{dd} = \mathrm{diag}(y_{dd1}, y_{dd2}, \ldots, y_{ddn}) \tag{6.45}$$

$$Y_{qq} = \mathrm{diag}(y_{qq1}, y_{qq2}, \ldots, y_{qqn}) \tag{6.46}$$

$$\delta\omega_o\Psi_f = (\delta\omega_o\psi_{f1}, \delta\omega_o\psi_{f2}, \ldots, \delta\omega_o\psi_{fn})^T \tag{6.47}$$

$$\delta\omega_o\Psi_d = (\delta\omega_o\psi_{d1}, \delta\omega_o\psi_{d2}, \ldots, \delta\omega_o\psi_{dn})^T \tag{6.21´}$$

$$\delta\omega_o\Psi_q = (\delta\omega_o\psi_{q1}, \delta\omega_o\psi_{q2}, \ldots, \delta\omega_o\psi_{qn})^T \tag{6.22´}$$

Substituting (6.39) and (6.40) into (6.18) yields

$$p\delta\Omega = A_{22}\delta\Omega + A_{23}\delta\omega_o\Psi_f + A_{24}\delta\omega_o\Psi_d + $$

$$+ A_{25}\delta\omega_o\Psi_q + A_{27}\delta Z_p + B_{22}\delta U_{p1} \tag{6.48}$$

where

$$A_{23} = G_q \cdot Y_{df} \tag{6.49}$$

$$A_{24} = G_q Y_{dd} + H_q \tag{6.50}$$

$$A_{25} = G_d Y_{qq} + H_d \tag{6.51}$$

6.3. Differential Equations for Field Flux Linkage (x_3).

The differential equation for field flux linkage (3.68) was derived in Section 3.6

$$p\delta\psi_{fi} = \delta v_{fi} - r_{fi}\delta i_{fi} \qquad (3.68')$$

Introducing $e_{fi} = x_{afi}v_{fi}/r_{fi}$ and multiplication with ω_o on both sides of (3.68´) gives

$$= \omega_o r_{fi}/x_{afi}\delta e_{fi} - \omega_o r_{fi}\delta i_{fi} \qquad (6.52)$$

which can be written symbolically

$$p\delta\omega_o\Psi_f = -\omega_o R_f\delta I_f + A_{36}\delta E_f \qquad (6.53)$$

where

$$\delta\omega_o\Psi_f = (\delta\omega_o\psi_{f1}, \delta\omega_o\psi_{f2}, \ldots, \delta\omega_o\psi_{fn})^T \qquad (6.54)$$

$$\delta I_f = (\delta i_{f1}, \delta i_{f2}, \ldots, \delta i_{fn})^T \qquad (6.55)$$

$$\delta E_f = (\delta e_{f1}, \delta e_{f2}, \ldots, \delta e_{fn})^T \qquad (6.56)$$

$$R_f = \mathrm{diag}(r_{f1}, r_{f2}, \ldots, r_{fn}) \qquad (6.57)$$

$$A_{36} = \mathrm{diag}(\omega_o r_{f1}/x_{af1}, \omega_o r_{f2}/x_{af2}, \ldots, \omega_o r_{fn}/x_{afn}) \qquad (6.58)$$

Substitution of (6.38) into (6.53) finally yields

$$p\delta\omega_o\Psi_f = A_{33}\delta\omega_o\Psi_f + A_{34}\delta\omega_o\Psi_d + A_{36}\delta E_f \qquad (6.59)$$

where

$$A_{33} = -\ \omega_o R_f Y_{ff} \tag{6.60}$$

$$A_{34} = -\ \omega_o R_f Y_{fd} \tag{6.61}$$

6.4. Differential Equations for Armature d-Axis Flux Linkage (x_4).

From Section 3.6 we have

$$p\delta\psi_{di} = \delta v_{di} + r_{ai}\delta i_{di} + \omega_o\delta\psi_{qi} + \psi_{qi}\delta\omega_i \tag{3.69'}$$

which after multiplication with ω_o on both sides can be written symbolically

$$p\delta\omega_o\Psi_d = \omega_o\delta V_d + \omega_o R_a\delta I_d + \omega_o\delta\omega_o\Psi_q + \omega_o\Psi_q\delta\Omega \tag{6.62}$$

where

$$\delta\omega_o\Psi_d = (\delta\omega_o\psi_{d1},\ \delta\omega_o\psi_{d2},\ \ldots,\ \delta\omega_o\psi_{dn})^T \tag{6.21'}$$

$$\delta V_d = (\delta v_{d1},\ \delta v_{d2},\ \ldots,\ \delta v_{dn})^T \tag{6.63}$$

$$\delta I_d = (\delta i_{d1},\ \delta i_{d2},\ \ldots,\ \delta i_{dn})^T \tag{6.19'}$$

$$\delta\omega_o\Psi_q = (\delta\omega_o\psi_{q1},\ \delta\omega_o\psi_{q2},\ \ldots,\ \delta\omega_o\psi_{qn})^T \tag{6.22'}$$

$$\delta\Omega = (\delta\omega_1,\ \delta\omega_2,\ \ldots,\ \delta\omega_n)^T \tag{6.16'}$$

$$R_a = \mathrm{diag}(r_{a1},\ r_{a2},\ \ldots,\ r_{an}) \tag{6.64}$$

$$\Psi_q = \mathrm{diag}(\psi_{q1},\ \psi_{q2},\ \ldots,\ \psi_{qn}) \tag{6.65}$$

The influence from the other generators is introduced by the δV_d term. In Section 4.2 we derived expressions for δV_d and δV_q.

$$\begin{vmatrix} \delta V_d \\ \delta V_q \end{vmatrix} = \begin{vmatrix} R_m & -X_m \\ X_m & R_m \end{vmatrix} \begin{vmatrix} \delta I_d \\ \delta I_q \end{vmatrix} + \begin{vmatrix} E_d \\ E_q \end{vmatrix} \delta \Theta \qquad (4.17')$$

After substitution of (4.17´) into (6.62) we have

$$p\delta\omega_o\Psi_d = \omega_o R_m \delta I_d - \omega_o X_m \delta I_q + \omega_o E_d \delta\ \Theta\ +$$

$$+\ \omega_o R_a \delta I_d + \omega_o \delta\omega_o \Psi_q + \omega_o \Psi_q \delta\Omega \qquad (6.66)$$

Combination of (6.39), (6.40) and (6.66) finally yields

$$p\delta\omega_o\Psi_d = A_{41}\delta\ \Theta + A_{42}\delta\Omega + A_{43}\delta\omega_o\Psi_f\ +$$

$$+\ A_{44}\delta\omega_o\Psi_d + A_{45}\delta\omega_o\Psi_q \qquad (6.67)$$

where

$$A_{41} = \omega_o E_d \qquad (6.68)$$

$$A_{42} = \text{diag}(\omega_o\psi_{q1}, \omega_o\psi_{q2}, \ldots, \omega_o\psi_{qn}) \qquad (6.69)$$

$$A_{43} = \omega_o(R_a+R_m)Y_{df} \qquad (6.70)$$

$$A_{44} = \omega_o(R_a+R_m)Y_{dd} \qquad (6.71)$$

$$A_{45} = -\ \omega_o X_m Y_{qq} + \omega_o I \qquad (6.72)$$

6.5. Differential Equations for Armature q-Axis Flux Linkage (x_5).

Similar algebra as in the previous section makes it possible to write

$$p\delta\omega_o\psi_q = A_{51}\delta\ \theta + A_{52}\delta\Omega + A_{53}\delta\omega_o\Psi_f + $$

$$+ A_{54}\delta\omega_o\Psi_d + A_{55}\delta\omega_o\Psi_q \qquad (6.73)$$

where

$$A_{51} = \omega_o E_q \qquad (6.74)$$

$$A_{52} = \mathrm{diag}(-\omega_o\psi_{d1},\ -\omega_o\psi_{d2},\ \ldots,\ -\omega_o\psi_{dn}) \qquad (6.75)$$

$$A_{53} = \omega_o X_m Y_{df} \qquad (6.76)$$

$$A_{54} = \omega_o X_m Y_{dd} - \omega_o I \qquad (6.77)$$

$$A_{55} = \omega_o (R_a+R_m) Y_{qq} \qquad (6.78)$$

6.6. Differential Equations for Excitation Voltages (x_6).

In Section 3.7 it was stated that the exciter system could be described by

$$p\delta v_{fi} = (-\delta v_{fi}+\delta v_{ei})/T_{ei} \qquad (3.74')$$

After multiplication with x_{afi}/r_{fi} on both sides of (3.74') we have

$$p\delta e_{fi} = (-\delta e_{fi}+\delta u_{ei})/T_{ei} \quad (6.79)$$

where

$$u_{ei} = x_{afi}/r_{fi} \quad (6.80)$$

Equation (6.79) can be written symbolically

$$p\delta E_f = A_{66}\delta E_f + B_{61}\delta U_e \quad (6.81)$$

where

$$E_f = (e_{f1}, e_{f2}, \ldots, e_{fn})^T \quad (6.56')$$

$$U_e = (u_{e1}, u_{e2}, \ldots, u_{en})^T \quad (6.82)$$

$$A_{66} = \text{diag}(-1/T_{e1}, -1/T_{e2}, \ldots, -1/T_{en}) \quad (6.83)$$

$$B_{61} = \text{diag}(1/T_{e1}, 1/T_{e2}, \ldots, 1/T_{en}) \quad (6.84)$$

6.7. Differential Equations for Prime Mover State Variables (x_7).

The prime movers were treated in Section 5 where the models

$$p\delta z_{hi} = a_{hi}\delta z_{hi} + b_{hi}\delta u_{hi} \quad (5.16')$$

$$p\delta z_{bi} = a_{bi}\delta z_{bi} + b_{b1i}\delta u_{b1i} + b_{b2i}\delta u_{b2i} \quad (5.25')$$

were obtained for the hydro plants and steam plants respectively. The models may be unified to

$$p\delta Z_p = A_{77}\delta Z_p + B_{72}\delta U_t + B_{73}\delta U_f \qquad (6.85)$$

where

$$\delta Z_p = (\delta z_{p1},\ \delta z_{p2},\ \ldots,\ \delta z_{pn})^T \qquad (6.23')$$

$$\delta U_t = (\delta u_{t1},\ \delta u_{t2},\ \ldots,\ \delta u_{tn})^T \qquad (6.24')$$

$$\delta U_f = (\delta u_{f1},\ \delta u_{f2},\ \ldots,\ \delta u_{fn})^T \qquad (6.86)$$

$$A_{77} = \mathrm{diag}(a_1,\ a_2,\ \ldots,\ a_n) \qquad (6.87)$$

$$B_{72} = \mathrm{diag}(b_{t1},\ b_{t2},\ \ldots,\ b_{tn}) \qquad (6.88)$$

$$B_{73} = \mathrm{diag}(b_{f1},\ b_{f2},\ \ldots,\ b_{fn}) \qquad (6.89)$$

6.8. Structure of the Complete System.

Equations (6.14), (6.48), (6.59), (6.67), (6.73), (6.81) and (6.85) are collected to yield

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \begin{bmatrix} 0 & A_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} & A_{24} & A_{25} & 0 & 0 \\ 0 & 0 & A_{33} & A_{34} & A_{35} & A_{36} & 0 \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & 0 & 0 \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{77} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} +$$

$$+\begin{bmatrix} 0 & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ A_{61} & 0 & 0 \\ 0 & A_{72} & A_{73} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (6.90)$$

Thus we have a state model with 7n state variables, where n is the number of generators. In practice the time derivatives of ψ_d and ψ_q are often very small. In this ψ_d and ψ_q may be eliminated from (6.90) to yield a model with 5n state variables.

To store the entire A and B matrices we need $70n^2$ locations. The storage requirement can be reduced if we store only the nonzero submatrices. Since A_{41}, A_{43}, A_{44}, A_{45}, A_{51}, A_{53}, A_{54} and A_{55} are the only submatrices with elements outside the main diagonal the required storage can be reduced to $8n^2 + 17n$.

The most severe numerical procedure is the inversion of $\tilde{Y}$ to obtain $\tilde{Z}$. The rest of the modelling process only requires the inversion of n 3 by 3 matrices and matrix multiplication.

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APPENDIX: LIST OF SYMBOLS

1. General Control Theory

x	state vector
u	control vector
A,B	system matrices
V	lossfunction
Q_1,Q_2	weighting matrices
L	feedback matrix
S	solution to the Riccati equation

2. General Power System Theory

P_g,P_{gi}	active power generation
Q_g,Q_{gi}	reactive power generation
P_d,P_{di}	active power demand
Q_d,Q_{di}	reactive power demand
P,P_i	active power injection
Q,Q_i	reactive power generation
P_m,P_{mi}	mechanical power
E_f,E_{fi}	excitation voltage or open circuit voltage
u_e,u_{ei}	exciter input
u_p,u_{pi}	prime mover input(s)

$\tilde{I}$	node current (complex)
$\tilde{V}$	node voltage (complex)
$\tilde{Y}$	node admittance matrix (complex)
I_D	real part of $\tilde{I}$
I_Q	imaginary part of $\tilde{I}$
V_D	real part of $\tilde{V}$
V_Q	imaginary part of $\tilde{V}$
G_N	real part of $\tilde{Y}$
B_N	imaginary part of $\tilde{Y}$

3. Synchronous Machine

$\psi_r, \psi_s, \psi_t, \psi_f$	flux linkages, 3-phase machine
i_r, i_s, i_t, i_f	currents, 3-phse machine
v_r, v_s, v_t, v_f	voltages, 3-phse machine
ℓ_{xy}	inductances, 3-phse machine
Ψ_{rstf}	$(\psi_r, \psi_s, \psi_t, \psi_f)^T$
I_{rstf}	$(i_r, i_s, i_t, i_f)^T$
V_{rstf}	$(v_r, v_s, v_t, v_f)^T$
L_{rstf}	$(L_{rstf})_{xy} = \ell_{xy}$
r_a	armature resistance, 3-phase machine
r_f	field resistance, 3-phse machine
R_{rstf}	$\mathrm{diag}(r_a, r_a, r_a, r_f)$
N_3	number of turns/phase, 3-phase machine
N_2	number of turns/phase, 2-phase machine
$\psi_\alpha, \psi_\beta, \psi_o, \psi_f$	flux linkages, 2-phase machine
$i_\alpha, i_\beta, i_o, i_f$	currents, 2-phase machine
$v_\alpha, v_\beta, v_o, v_f$	voltages, 2-phase machine
$\Psi_{\alpha\beta of}$	$(\psi_\alpha, \psi_\beta, \psi_o, \psi_f)^T$
$I_{\alpha\beta of}$	$(i_\alpha, i_\beta, i_o, i_f)^T$
$V_{\alpha\beta of}$	$(v_\alpha, v_\beta, v_o, v_f)^T$
C_1	transformation matrix for flux linkages, currents and voltages from 3-phase to 2-phse machine

P_{rstf}	active pover, 3-phase machine
$P_{\alpha\beta of}$	active power, 2-phase machine
$L_{\alpha\beta of}$	inductance matrix, 2-phse machine
$R_{\alpha\beta of}$	resistance matrix, 2-phase machine
N	number of turns/phase, dq-machine
$\psi_d, \psi_q, \psi_o, \psi_f$	flux linkages, dq-machine
i_d, i_q, i_o, i_f	currents, dq-machine
v_d, v_q, v_o, v_f	voltages, dq-machine
Ψ_{dqof}	$(\psi_d, \psi_q, \psi_o, \psi_f)^T$
I_{dqof}	$(i_d, i_q, i_o, i_f)^T$
V_{dqof}	$(v_d, v_q, v_o, v_f)^T$
P_{dqof}	active power, dq-machine
C_2	transformation matrix for flux linkages, currents and voltages from 2-phase to dq-machine
L_{dqof}	inductance matrix, dq-machine
R_{dqof}	resistance matrix, dq-machine
E	energy
E_e	energy, stored in magnetic circuits
E_m	energy, stored in rotating masses
P_m	mechanical power

P_n	electrical power from network
P_{lr}	power losses in resistances
P_{lm}	power losses, mechanical
P_{ld}	power losses, in damper windings
J	moment of inertia
ω	angular velocity
D_1	damping coefficient,damper windings
D_2	damping coefficient, mechanical damping
M_e	air-gap torque
M_d	damping torque
L_f	self-inductance of field winding
L_{af}	mutual inductance between stator d-axis winding and field winding
L_d	self-inductance of stator d-axis winding
L_q	self-inductance of stator q-axis winding
Ψ	$(\psi_f,\psi_d,\psi_q)^T$
I	$(i_f,i_d,i_q)^T$
L	see (3.61)
W	see (3.43)
T_e	exciter time-constant
u_e	input signal to exciter

4. Transmission Network

v_D, v_{Di}	network terminal voltages expressed with reference to network reference axes
v_Q, v_{Qi}	
V_D	$(v_{D1}, v_{D2}, \ldots, v_{Dn})^T$
V_Q	$(v_{Q1}, v_{Q2}, \ldots, v_{Qn})^T$
θ_i	angular displacement of the (d,q) axes of machine i with respect to (D,Q) axes of the network. (Fig. 4.1)
Θ	$(\theta_1, \theta_2, \ldots, \theta_n)^T$
i_D, i_{Di}	network terminal currents expressed with reference to network reference axes
i_Q, i_{Qi}	
g_{ij}	real component of a network self- or mutual admittance
b_{ij}	imaginary component of a network self- or mutual admittance
R_M	see (4.15)
X_M	see (4.16)
E_D	see (4.18)
E_Q	see (4.19)

5. Hydro Turbines

q	flow of water
ρ	density of water
v_{out}	velocity of water at outlet
a	outlet area
$\eta(\omega)$	speed dependent efficiency
v_t	velocity of water in the dash-tube
L	length of the dashtube
g	constant of gravity
h	water head
A	dash-tube area
v_{tmax}	maximum velocity of water in the dash-tube
z_p	state variable (v_t/v_{tmax})
u_t	control variable (a/a_{max})
a_{max}	maximum outlet area
T_w	time-constant, water-system

6. Boilers and Steam Turbines

p	steam preassure
u_1	steam valve setting
u_2	fuel flow
u_3	feedwater flow
$\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$	parameters, steam plant
q	steam flow
z_p	state variable (p/p_{max})
u_t	normalized steam valve setting
u_f	normalized fuel flow

7. Operators

$p(.)$	differential operator $p(x)=dx/dt$
$\delta(.)$	incremental operator $\delta(x)=x-x_o$
$(.)^T$	transposition of a matrix
$diag(\lambda_1,\lambda_2, \ldots ,\lambda_n)$	$\Lambda=diag(\lambda_1,\lambda_2, \ldots ,\lambda_n)$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & & 0 \\ 0 & \lambda_2 & & 0 \\ & & \ddots & \\ 0 & 0 & & \lambda_n \end{bmatrix}$$