Control System Synthesis - The PhD Course 1995

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Control System Synthesis
The PhD Course 1995

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Dec 1995

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Sponsoring organisation

Title and subtitle
Control System Synthesis – The PhD Course 1995

Abstract

This is the documentation for the PhD course 1995. It contains course program, lecture slides, exercises and handin problems. Most of the figures in the slides were added after the fourslides were produced. Hence they are not included in this documentation.

Key words

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title
0280–5316

ISBN

Language
English

Number of pages

Security classification

Recipient's notes

Summary

The course 1995 consisted of 15 lectures and 8 exercise sessions. We used the book "Multivariable Feedback Design" by J. Maciejowski (Addison-Wesley, 1989). Part of "Linear controller design, limits of performance", by Boyd and Barrat (Prentice Hall, 1991) was also used together with notes and articles. The course was followed by 14 PhD-students including 3 students from the department of Industrial Electrical Engineering and Automation. Lectures 2-3 were given by Karl Johan Åström, Lecture 4 by the participants, Lecture 6 by Mikael Johansson, Lecture 8 by Tore Hägglund, Lecture 11 by Björn Wittenmark, part of Lecture 12 by Anders Rantzer and Lecture 15 by Sven Erik Mattson. There was also one laboration, in fuzzy control. This was developed by Mikael Johansson and Johan Eker. Thank you all for your help!

Material that was used:

- Maciejowski "Multivariable Feedback Design", Ch 1-7
- Boyd-Barrat "Linear Controller Design", Ch 1,4-5.
- Green-Limebeer, Ch 9 on Model Reduction
- The μ-toolbox manual
- A benchmark example by Landau
- Rolling mill example by Pedersen
- Bristols AC-paper on RGA
- Note on aeroplane dynamics
- Gunther Stein's Bode lecture noters
- Freudenberg-Loozc Trans AC 1985
- Kwakernaak "Symmetries in Control System Design", ECC 1995
- Doyle, QFT and Robust Control
- TFRT-7454 A collection of Matlab Routines for Control Analysis and Synthesis, by Kjell Gustafsson, Mats Lilja and Michael Lundh.
- TFRT-5477 A Quantitative Feedback Theory Toolbox for Matlab 4.1, Michael Lekman.
- "Guaranteed Margins for LQG Regulators", J Doyle, Trans AC.

Included in this documentation are

- Course WWW home page
- Lecture slides 1-15 (however without most figures)
- lab-pm

Lund, Dec 1995

Bo Bernhardsson

Control System Synthesis 1995

This material is collected at `http://www.control.lth.se/~bob`

Schedule

Lectures mondays 13.15. (Exercises thursdays 10.15.) New time for exercises: Fridays 13.15-15.

Extra lecture : **Tisd 14/11 kl 15.15 - 17.00, M:B**, SattLine -- Lars Pernebo & Staffan Andersson, Alfa Laval Automation Presentation av SattLine inkl interaktiv videokanonsdemonstration

Organization

Lectures will be held by Bo Bernhardsson and guests. The course starts Monday September 4, 13.15 and ends in December.

Prerequisities:

Regler AK and Computer Controlled Systems.

Literature

- Maciejowski, *Multivariable feedback design,* Addison-Wesley 1989, ISBN 0-201-18243-2.
- Boyd, Barratt, *Linear controller design, limits of performance*, Prentice Hall 1991.

More info about Maeciejowski's book (gohper) for example an errata list (errata.ps)

Additional Reading

- D'Azzo, J., C. Houpis, *Linear control system analysis and design, 3rd ed.*, McGraw Hill 1988, Ch 21.
- Anderson, Moore, *Optimal Control, Linear quadratic methods, 2nd ed*, Prentice Hall 1990, Ch 8.
- Astrom and Hagglund, *PID Controllers: Theory, Desing and Tuning, 2nd ed*, Instrument Society of America, 1995

- Doyle et al, μ-toolbox
- Doyle, Francis, Tannenbaum, *Feedback control design*, MacMillan 1992.
- G F Franklin, J D Powell, A Emami-Naeini, *Feedback control of Dynamical Systems, 2nd ed*, Addison-Wesley 1991.
- B Friedland, *Control system design*, McGraw-Hill 1987.
- Morari, Zafirou, *Robust process control*, Prentice Hall 1989.

Project, exam

The project consists of controller design on an interesting process chosen by you. The projects should be presented in January. There will be a written exam.

Preliminary Contents

- Course overview, the synthesis problem, the check list, tools, pole placement, benchmark problems.
- Control paradigms, feedback/feedforward, mode switches (KJ)
- AK+-design, root locus, nichols etc (participants)
- FK+-design, ch10-12, ch15 (participants)
- QFT, Limits of performance, Ch 1
- Model-based vs non-model based, fuzzy
- Multivariable issues (Nyquist, zeros, robustness...), Ch 2-3
- PID-design, (Tore)
- MIMO, Nyquist-like techniques, Ch 4
- The servo problem (Björn)
- LQG-LTR, Ch 5
- Structured singular values, gain scheduling
- H_∞, $\mu $-methods, L_1 Ch 6
- Parametric optimization, Ch 7
- Control Design example, Wind Power Plant Design (Sven Erik)

Lectures

- Lecture 1-Intro
- Lecture 2-KJ1
- Lecture 3-KJ2
- Lecture 4-AK/FK
- Lecture 5-QFT
- Lecture 6-Fuzzy
- Lecture 7-MIMO1

- Lecture 8-PID/Tore
- Lecture 9-MIMO Nyquist + LQG1
- Lecture 10-LQG/LTR
- Lecture 11-Tracking problems
- Lecture 12-Robust Control 1
- Lecture 13-Hinfty-mu
- Lecture 14-Design by optimization
- Lecture 15-Example: Wind Power Plant Control

Exercises

- Exercise 1
- Exercise 2
- Exercise 3
- Exercise 4--Fuzzylab
- Exercise 5
- Exercise 6
- Exercise 7
- Exercise 8

Take Home Problems

Cooperation is allowed, unless otherwise stated

- Problem 1-Landau
- Problem 2-AK/FK
- Problem 3-Landau/QFT
- Problem 4-AIRC
- Problem 5-Labprocess

Presentations

Everybody should do a short presentation on lecture 3 and 4. The presentation should be prepared to September 25. The goal is to recapture well known material. Work in groups of 2 or 3. Choose between

- AK, Nichols Plots with lead/lag design
- AK, Root Locus with examples (e.g. some lab-processes)
- AK, "Centrifugalregulatorn"
- FK Ch. 10-12

- FK Ch. 7 + 15

Laboration

A laboration on fuzzy control and the fuzzy toolbox in matlab Should be made. A fuzzy controller is designed in matlab and down-loaded to the Palsjo real time system. More information is available in the lab-pm. This laboration was developed by Mikael Johansson and Johan Eker.

Examples (and matlab code)

- Lateral Dynamics of Aeroplane
- Landau's Flexible Transmission
- Thickness Control of a Rolling Mill
- The ACC 1990 Benchmark(two masses and a spring)
- Vertical Aeroplane Dynamics, AIRC, Mac. 4.4, 4.8 and 5.8
- Aircraft with wind turbulence
- Turbo-Generator, Mac. p 406
- LQG-examples, Lecture 9

Matlab-code

The department's Matlab boxes are available via anonymous ftp: qft.tar.Z and boxes_matlab4.tar.Z You might also want the functions spread.m and circle.m Matlab-code for L1-design is availabe. Ask akesson or me.

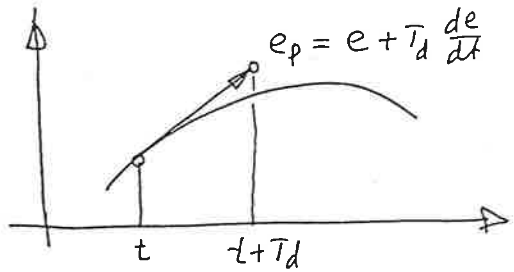
PID Control

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(s)ds + T_d\frac{de(t)}{dt}\right)$$

$$e(t) = y_{sp}(t) - y(t)$$

Integral action: zero steady state error

Derivative action: prediction



Modified linear behaviour

$$U(s) = K\left[bY_{sp}(s) - Y(s) + \frac{1}{sT_i}E(s) + \frac{sT_d}{1+sT_d/N}(-Y(s))\right]$$

The Tuning Problem

Specifications:

Load disturbance attenuation

$$\text{IE} = \int_0^\infty e(t)\,dt = \frac{1}{k_i} = \frac{T_i}{k}$$

$$\text{IAE} = \int_0^\infty |e(t)|dt$$

Measurement noise k_{hf}

Sensitivity M_s

Set-point following

Augmentation of PID

- Time delays
  - Smith predictor
  - Proc w. integration
- Oscillatory modes
  - Special filters

The PIP controller

The PID controller:

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int e(t)dt + T_d\frac{de(t)}{dt}\right)$$

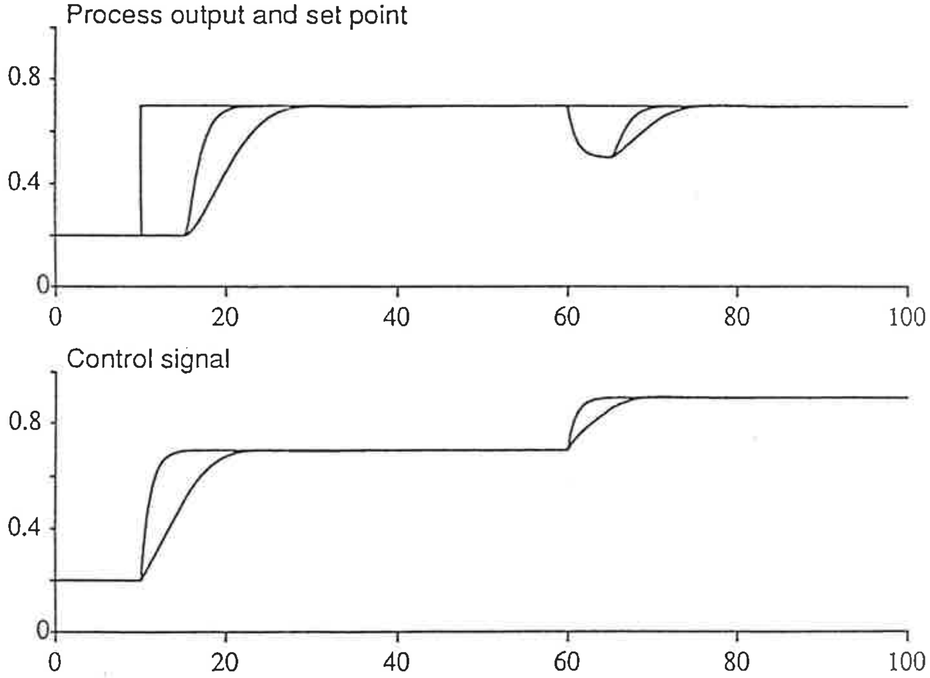
The PIP controller

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int e(t)dt\right) - \frac{1}{T_i}\int\Big(u(t) - u(t-L)\Big)dt$$

Prediction performed by a low-pass filtering of u instead of a high-pass filtering of y

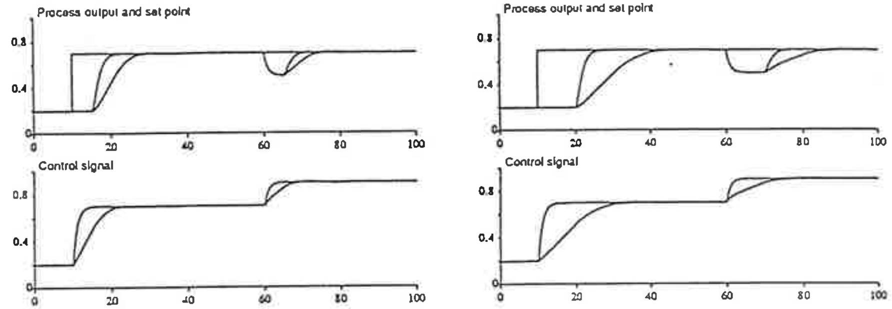
Only 3 parameters to set: K, T_i, L

Simulation results

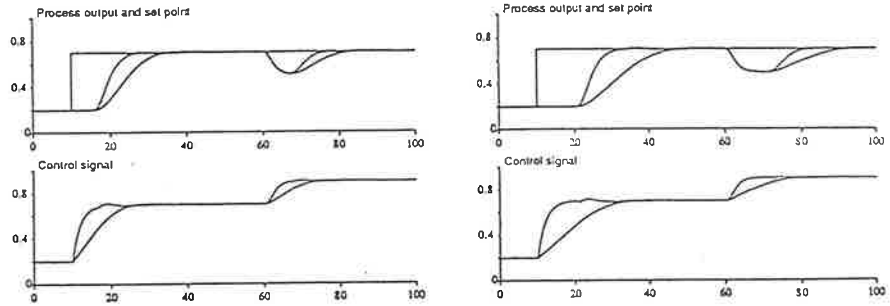


PI and PIP control of G_1

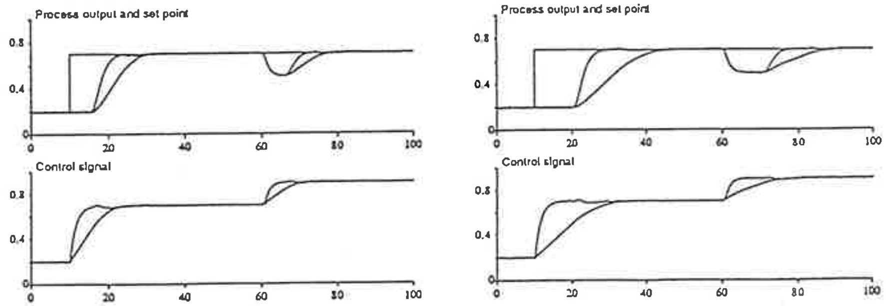
Simulation results



PI and PIP control of G_1 and G_2



PI and PIP control of G_3 and G_4



PI and PIP control of G_5 and G_6

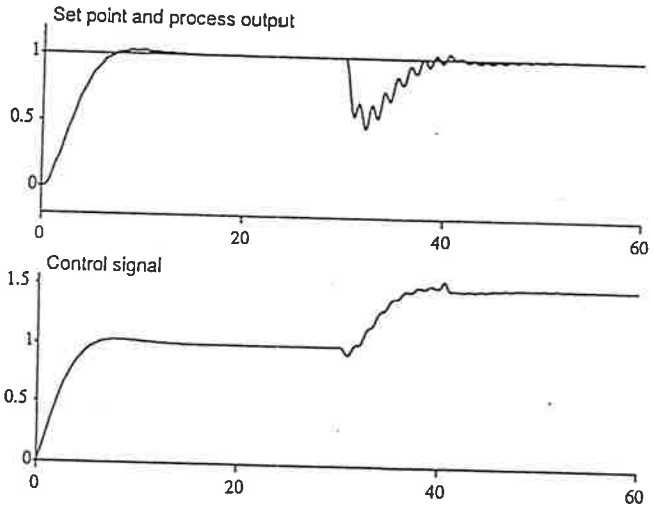


Figure 3.41 Response of the closed loop system to set point and load disturbances. The controller parameters are $K = -0.25$ $T_i = -1$ and $b = 0$

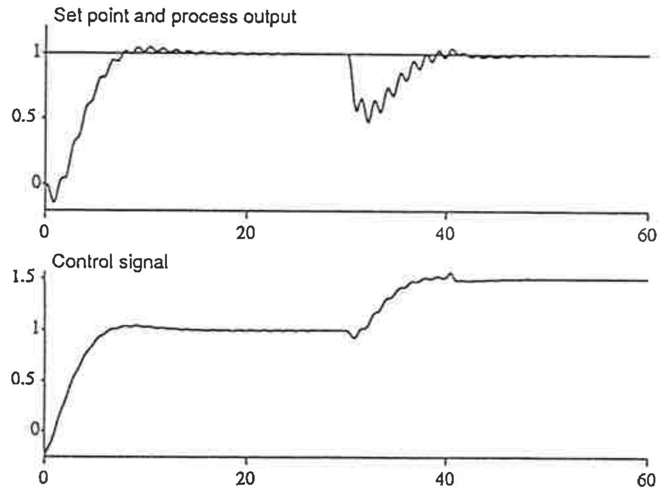


Figure 3.42 Response of the closed loop system to set point and load disturbances. The controller parameters are $K = -0.25$ $T_i = -1$ and $b = 1$

SELECTORS

Systems like this are commonly used but not well understood theoretically

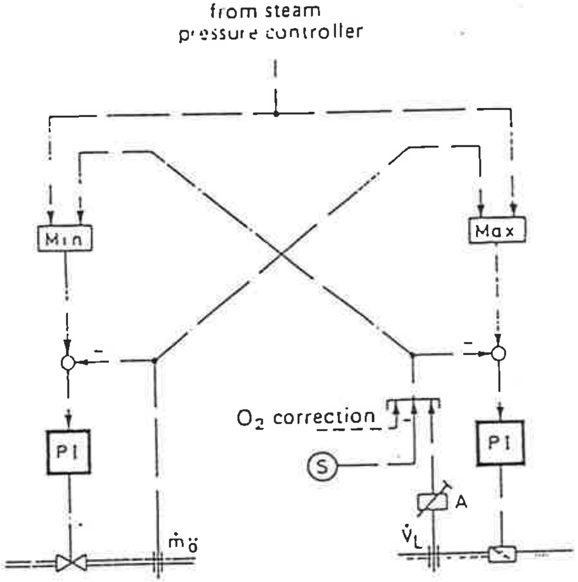


Fig. 55.1 Control scheme for oil firing.

MERGER OF PROGRAMMABLE LOGIC CONTROLLERS (PLC) AND CONTINUOUS CONTROL (DDC) POSE SIMILAR PROBLEMS

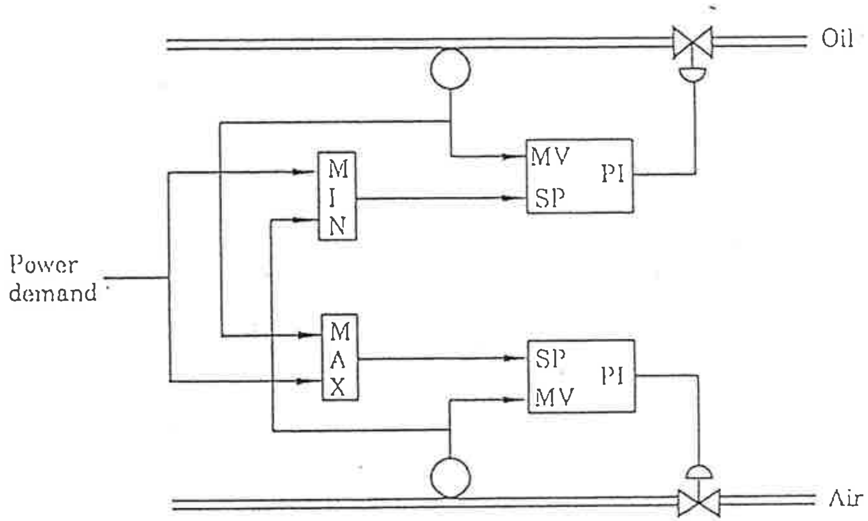
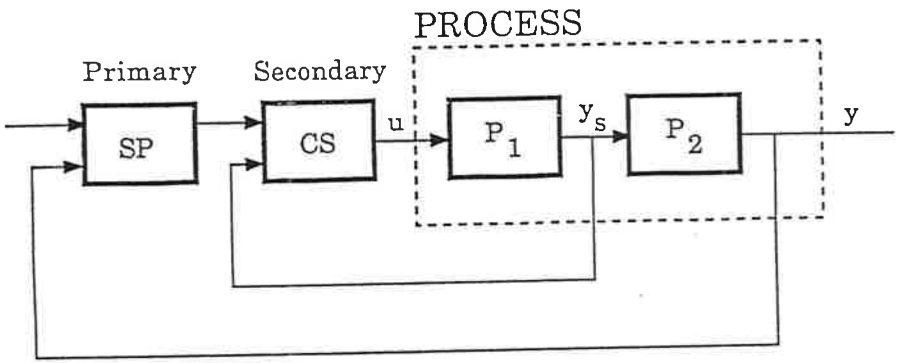


Figure 6.19 Air-fuel controller based on selectors. Compare with the ratio controller for the same system in Figure 6.16.

Cascade Control



An extra measurement compare with state feedback

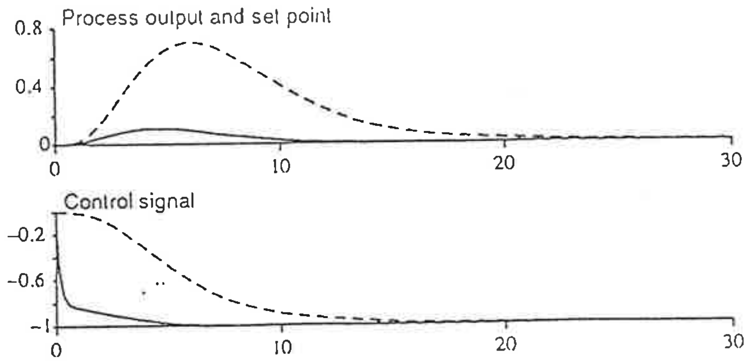
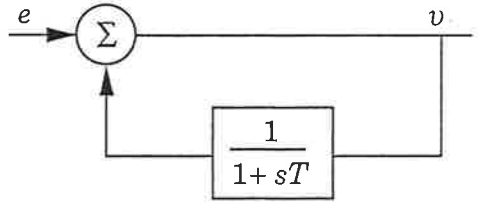
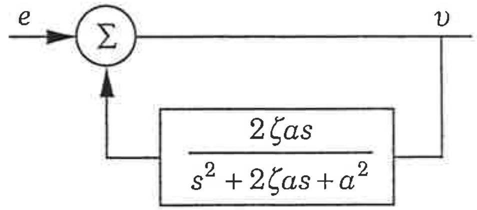


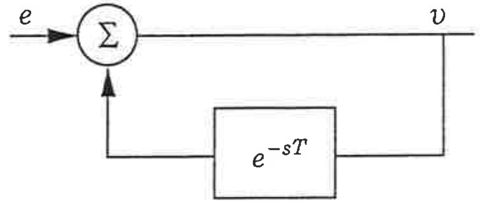
Figure 6.2 Responses to a load disturbance for a system with (full line) and without (dashed line) cascade control.

Paradigms for Disturbance Rejection

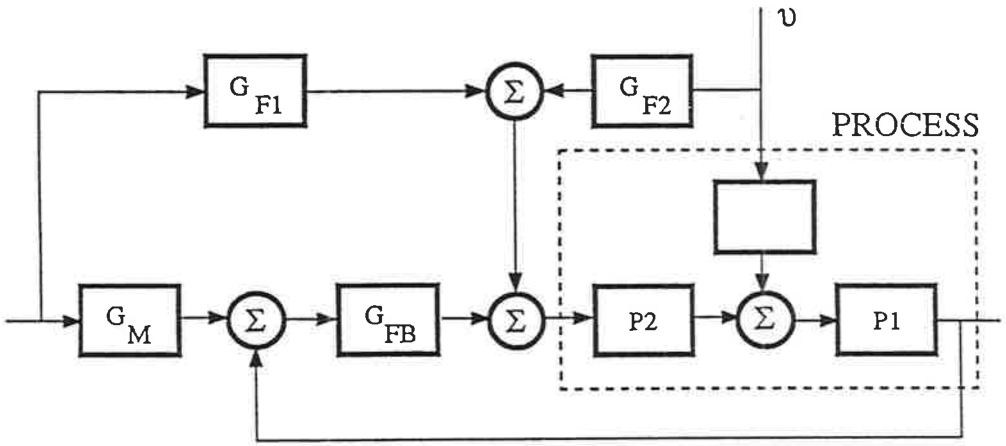
- Constant disturbances
- Sinusoidal disturbances
- Periodic disturbances







FEEDFORWARD



υ = MEASURABLE DISTURBANCE

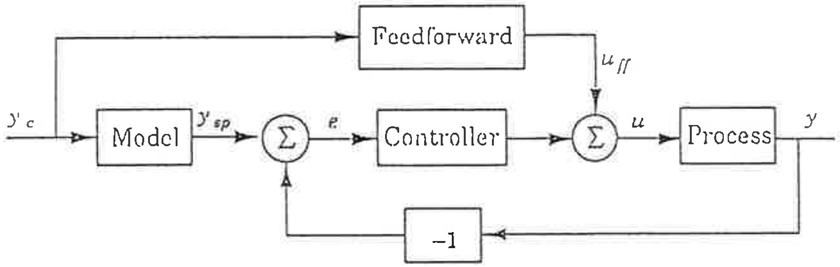


Figure 6.9 Block diagram of a system which combines model following and feedforward from the command signal.

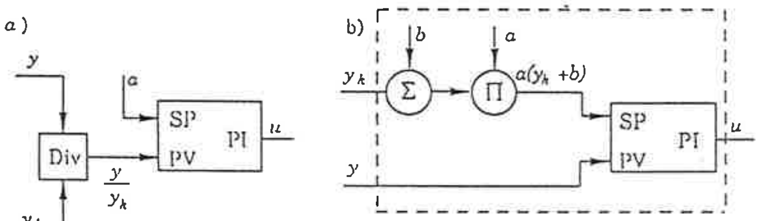


Figure 6.15 Block diagram of a ratio controller.

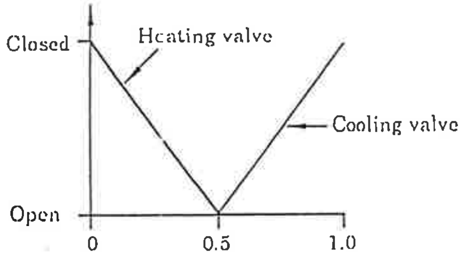
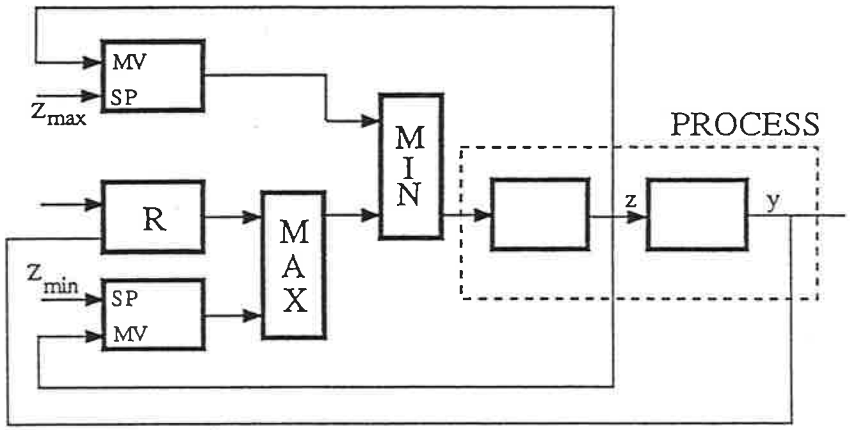


Figure 6.17 Illustration of the concept of split range control.

SELECTOR CONTROL



✌ LIMITS ON MEASURED SIGNALS

Control System Design

The Synthesis Problem

Bad models – Modeling is expensive

No specifications – Don't know what is possible or important

Can change/rebuild the process

$\boxed{\text{Design is ITERATIVE}}$

The Checklist

What are the requirements of "a good controller"?

Controller Design

No single method covers all aspects of controller design

Focus on different goals

PID

Lead/Lag

Pole Placement

Horowitz QFT

Adaptive Control

LQG

H_∞

l_1

Model Predictive Control

Fuzzy Control

Neuron-net based

Optimization Based

etc

Project

Make a good design on an interesting process.

Hard deadline Dec 31

Seminars in January.

Control Paradigms

K. J. Åström

Department of Automatic Control
Lund Institute of Technology
Lund Sweden

1. Introduction

2. Bottom Up

3. Top Down

4. Conclusions

CHEMICAL PROCESS CONTROL
CPCIV

Proceedings of the Fourth International Conference on Chemical Process Control

Padre Island, Texas
February 17-22, 1991

PRESENT STATUS AND FUTURE NEEDS: THE VIEW FROM JAPANESE INDUSTRY

Shigehiko Yamamoto
Application Engineering Department
Yokogawa Electric Corporation
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Tokyo, 163 Japan

Iori Hashimoto
Department of Chemical Engineering
Kyoto University
Sakyo-ku
Kyoto, 606 Japan

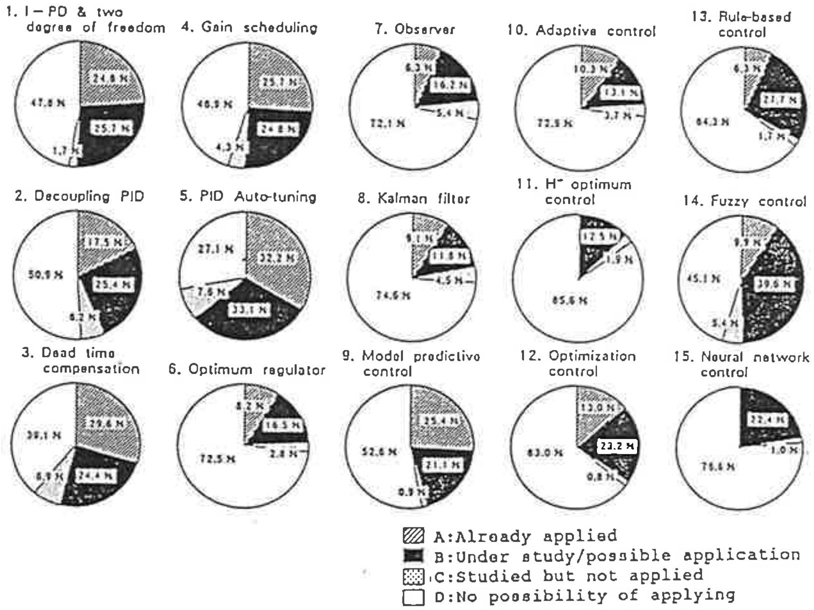
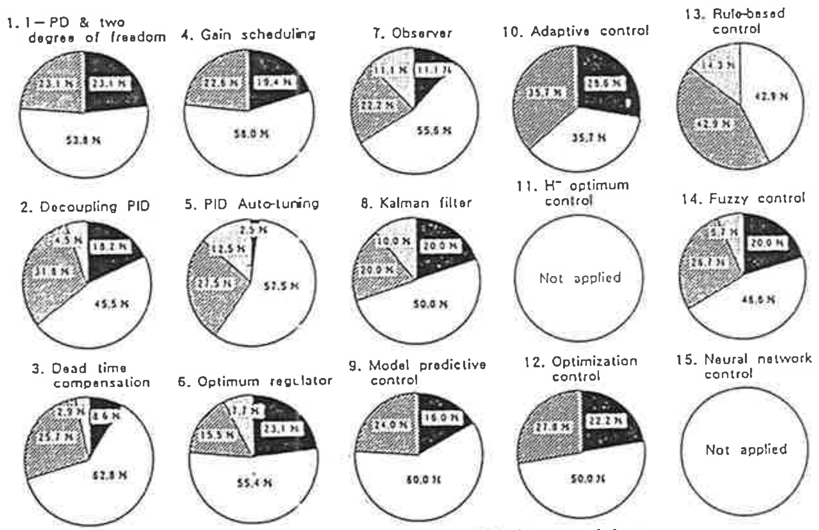


Fig.6 Application of control methods [By courtesy of the JEMIMA]

Evaluation of Results

Evaluations of application results are shown in Fig. 7



Very satisfactory
Satisfactory
Rather unsatisfactory
Unsatisfactory

Fig.7 Evaluation of results

Process Control Performance is Not as Good as You Think

D. Ender Techmation

- More than 30 % of the installed controllers operate in manual
- More than 30% of the loops actually increase variability
- About 25% of the loops used default settings
- 30% of the loops have equipment problems

Introduction

- Paradigm, pattern

Gr: To show side by side

- If the only tool you have got is a hammer, everything looks like a nail.

- If you have many tools you need a tollbox and skills to use them.

1. Introduction

2. Bottom Up

Feedback
- PID
- Windup
- Smith predictor
- Oscillatory systems
- Disturbance rejection
- Cascade

Feedforward

Nonlinear schemes
- Ratio
- Split range
- Selectors

3. Top Down

4. Conclusions

Control System Design

- Specifications
  - Load disturbances
  - Measurement noise
  - Model uncertainty
  - Command signals
- Methods
- Structures
- Tools

Bottom Up

PID
- Introduction
- Tuning
- Extensions
  - Time delays
  - Oscillatory modes

Cascade

Feedforward

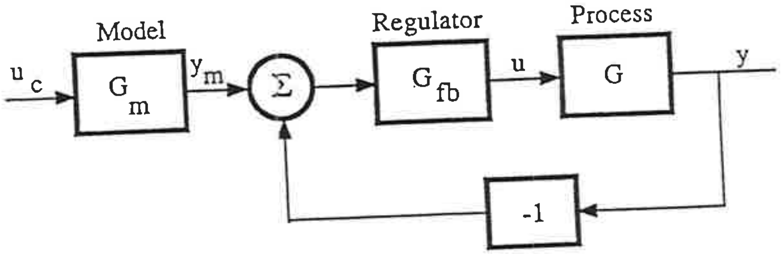
Ratio

Split Range

Selectors

Conclusions

HIGH GAIN FEEDBACK CONTROL



WARNING!

G. K. McMillan InTech Jan 1986: **Advanced Control Algorithms: Beware of False Prophesies**

1. Did the algorithm add an appreciable amount of dead time to the control loop? If so, forget it.

2. Did the algorithm perform well for unmeasured load disturbances? If not, forget it.

3. Was derivative action used in the conventional algorithm it was compared against? If not, your comparison was unfair; add the derivative mode and try again,

4. Was the PID controller tuned with a reputable method such as the Ziegler Nichols closed loop approach? If not, the comparison was unfair; tune the PID controller and try again.

1. Introduction
2. Bottom Up

3. Top Down

- State feedback
  - Observers
  - Disturbance modeling
  - A complete system
- Internal model control
  - Cancellation of poles
  - Relations to SFB
- Predictive control
  - Minimum variance
  - Model predictive control
- Adaptation
  - Tuning
  - Gain scheduling
  - MRAS STR
- Feedback linearization

4. Conclusions

Top Down

State Feedback
- Observers
- Explicit disturbance models
- Controller structure
- Windup

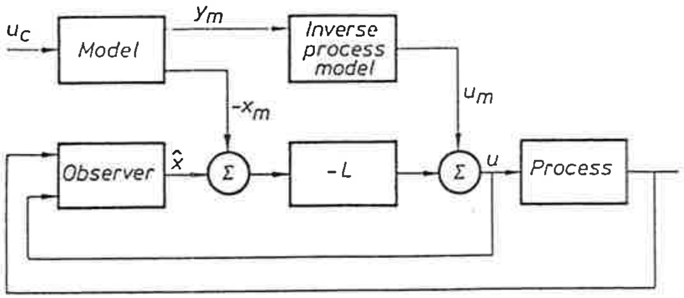
Model predictive Control
- Internal Model Control
- Pole cancellation
- MPC MVC GPC
- Sociology!

Tuning and Adaptation

Nonlinear Techniques

State space design

- Model all disturbances and command signals
- Controller structure



Disturbance models appear in the observer

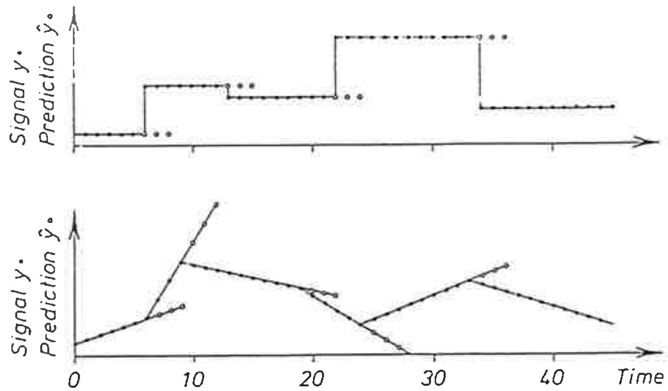
- Minimal realization

Disturbance Modeling

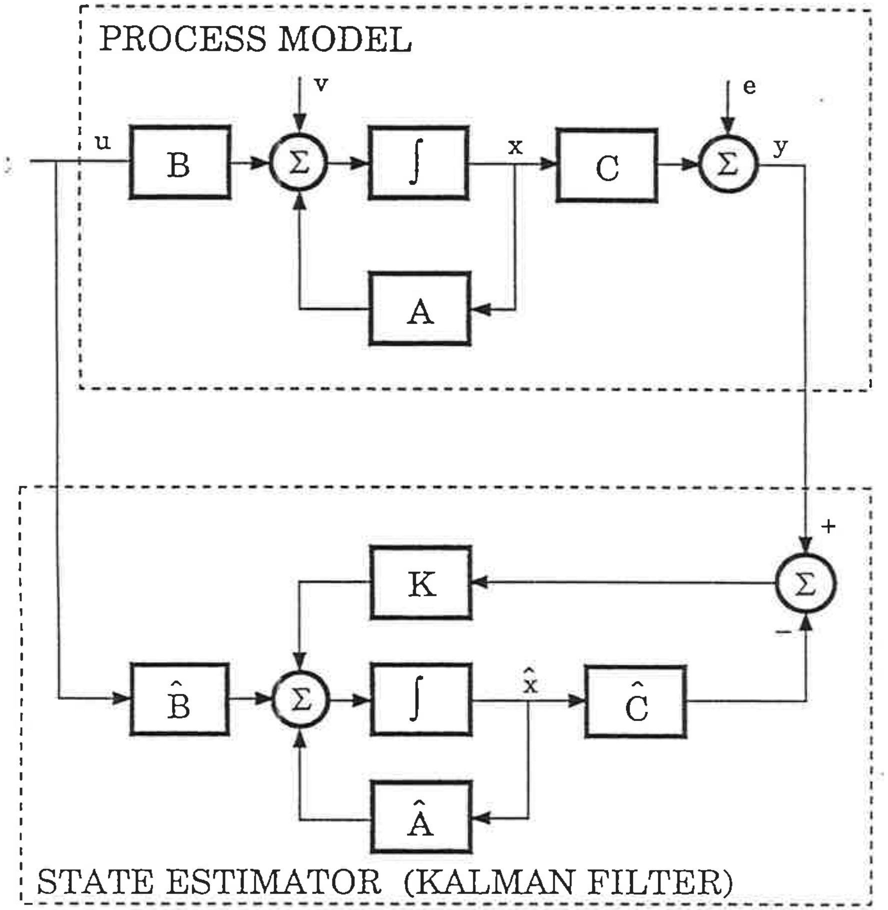
- The classics

step	$pv = 0$
ramp	$p^2 v = 0$
sinusoid	$(p^2+\omega^2)v=0$

- Piece wise deterministic
  $Av = 0$ a.e.

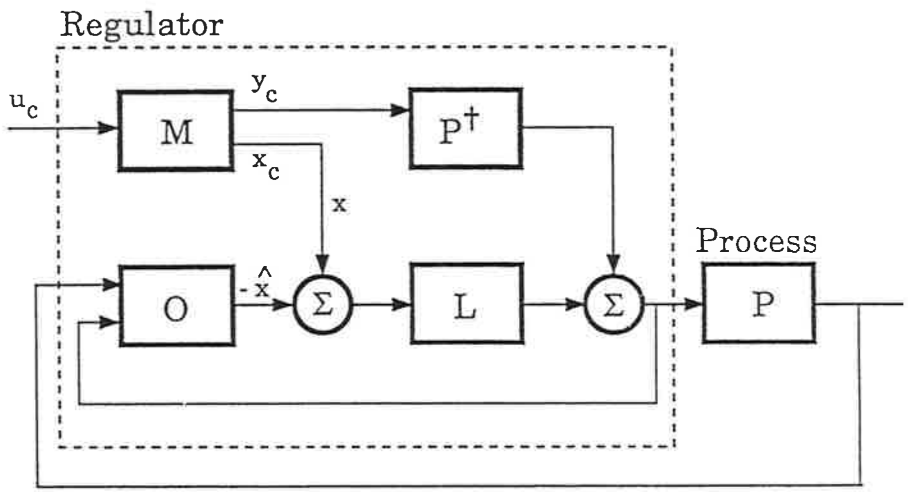


- Stochastic
  Singular or purely deterministic
  Av = white



- MATH MODEL + MEASUREMENTS
  ➠ INDIRECT MEASUREMENT
- SENSOR FUSION

REGULATOR STRUCTURE



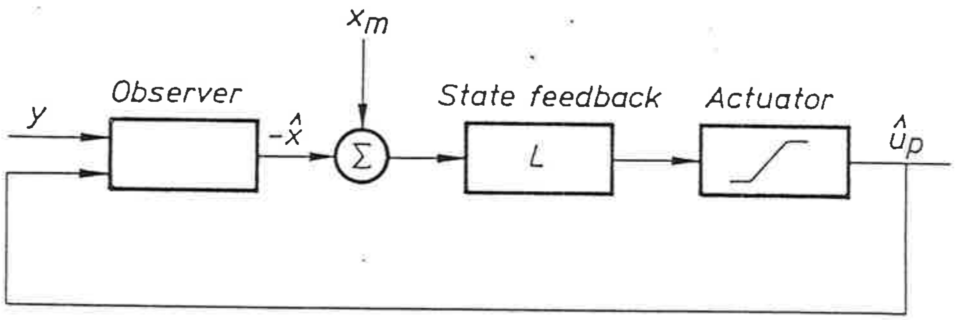
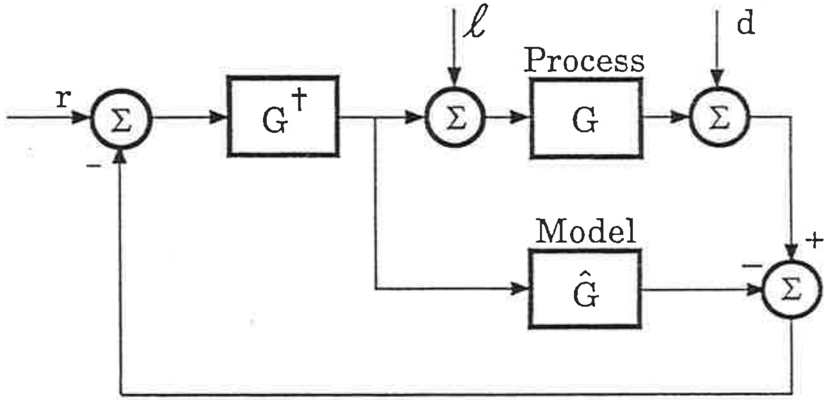
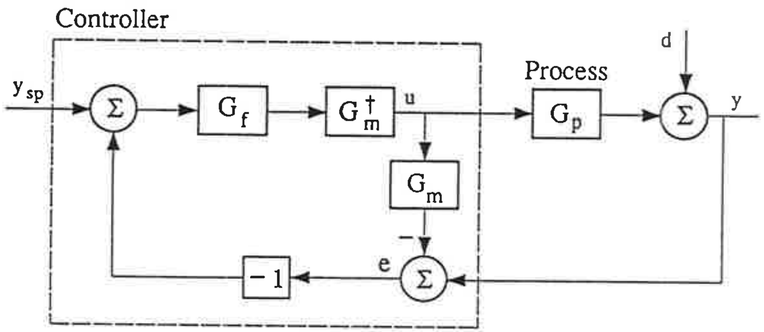


Figure 15.4 Regulator based on an observer and state feedback with antiwindup compensation.

INTERNAL MODEL PRINCIPLE



Internal Model Control (IMC)



Key idea:

- Ideally $e = d$ irrespectively of u.
- Choosing $G_m^{\dagger} = G_m^{-1}, G_f = 1$ gives perfect cancellation of d.
- G_m^{-1} not realizable.
- Controller transfer function

$$G_c = \frac{G_f G_m^{\dagger}}{1 - G_f G_m^{\dagger} G_m}$$

Notice cancellation of process poles.

Questions

- IMC is so beatifully simple
- Is it a general structure?
- Are there some snags?

$S = A_+ S_1$

$y = \frac{AR}{AR+BS} d$

$y = \frac{BR}{AR+BS} l = \frac{BR}{A_+(R+BS_1)} l$

Example PI Control

Process dynamics

$$G_p(s) = \frac{K_p}{1+sT}\, e^{-sL}$$

Approximate inverse

$$G_m^\dagger(s) = \frac{1+sT}{K_p}$$

Controller transfer function

$$G_c = \frac{G_f G_m^\dagger}{1 - G_f G_m^\dagger G_m}$$

Filter

$$G_f(s) = \frac{1}{1+sT_f}$$

Approximation 1

$$e^{-sL} \approx 1 - sL$$

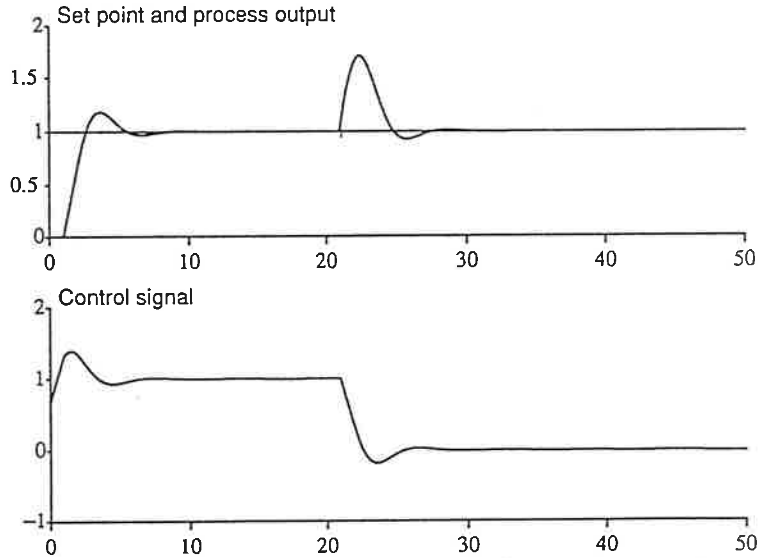
gives

$$G_c(s) = \frac{1+sT}{K_p s(L+T_f)}$$

$L \to 0$
$T_f \to 0$ $\Rightarrow G_c \to \infty$

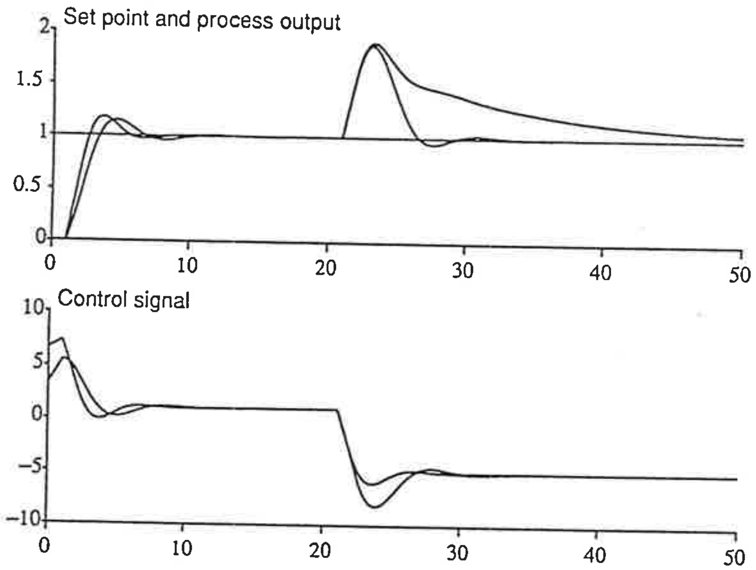
Example

$$G_p(s) = \frac{e^{-s}}{s+1} \qquad G_c = \frac{2}{3}\left(1+\frac{1}{s}\right) = \frac{2(s+1)}{3s}$$



Example

$$G_p = \frac{e^{-s}}{10s + 1} \qquad G_c = \frac{2(10s + 1)}{3s}$$



Remark

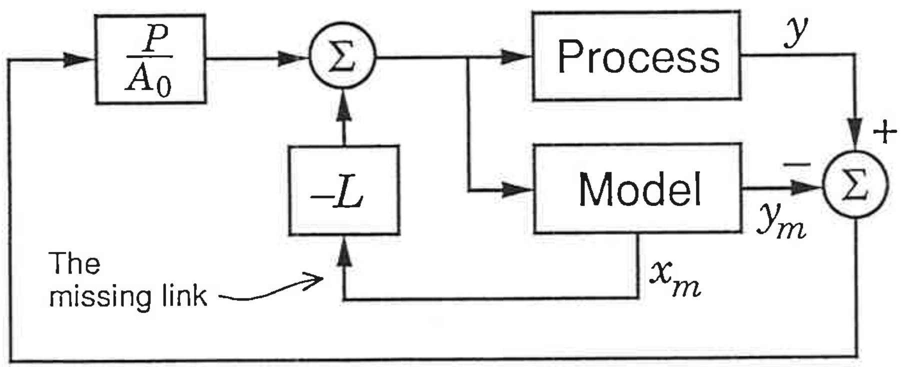
- Beware of cancellations
- Never cancel slow process poles

Relation between SFB & IMC

$$
\begin{aligned}
\frac{dx_m}{dt} &= Ax_m + Bu, \quad y_m = Cx_m \\
\frac{d\hat{x}}{dt} &= A\hat{x} + Bu + K(y - C\hat{x})
\end{aligned}
$$

Introduce $z = \hat{x} - x_m$

$$
\begin{aligned}
\frac{dz}{dt} &= Az + K(y - Cz - Cx_m) \\
&= (A - KC)z + K(y - y_m) \\
u &= -L\hat{x} = -Lx_m - Lz
\end{aligned}
$$



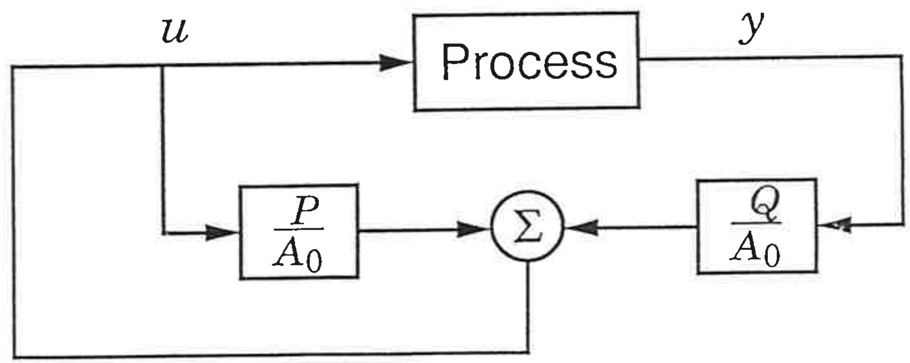
Relation between SFB & IMC

Control law

$$
\begin{aligned}
u &= -L\hat{x} \\
\frac{d\hat{x}}{dt} &= A\hat{x} + Bu + K(y - C\hat{x}) \\
&= (A - KC)\hat{x} + Bu + Ky
\end{aligned}
$$

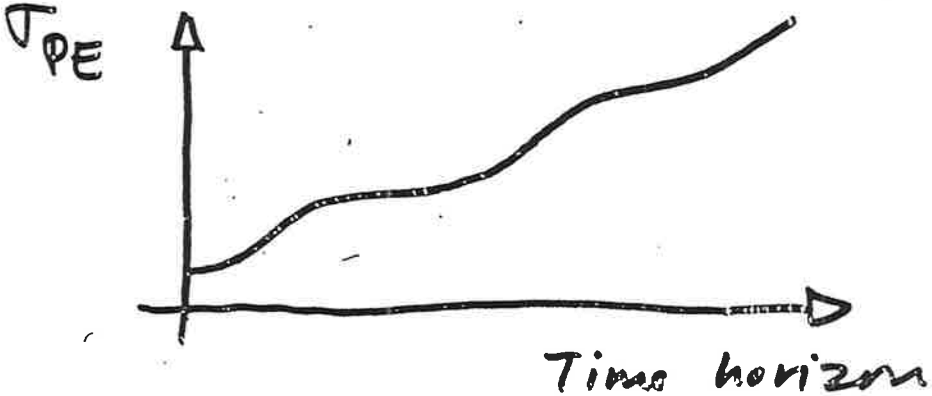
Introduce

$$
\begin{aligned}
\frac{d\xi}{dt} &= (A - KC)\xi + Bu \\
\frac{d\zeta}{dt} &= (A - KC)\zeta + Ky \\
u &= -L\hat{x} = -L\xi - L\zeta
\end{aligned}
$$

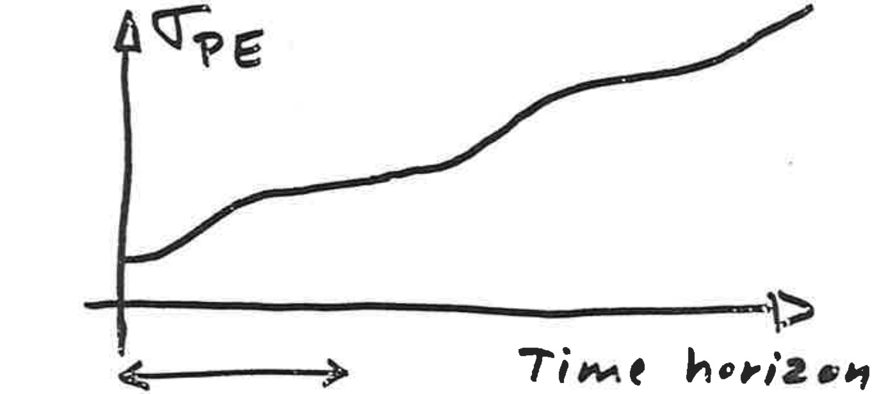


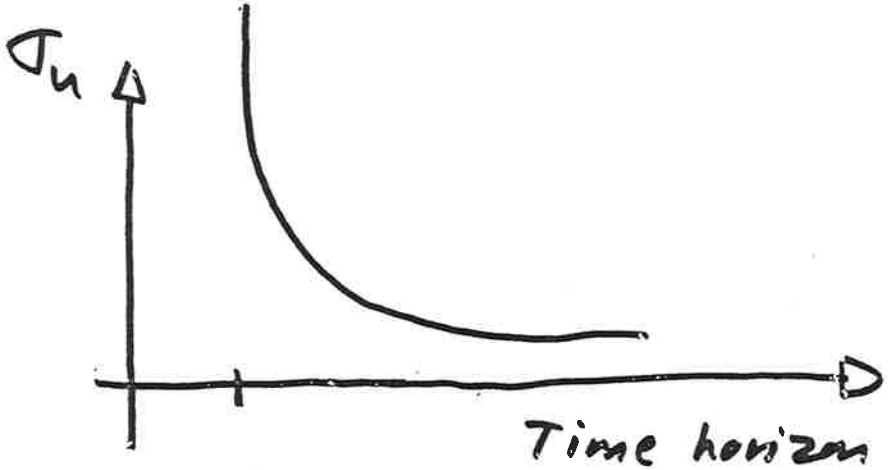
Minimum Variance Control

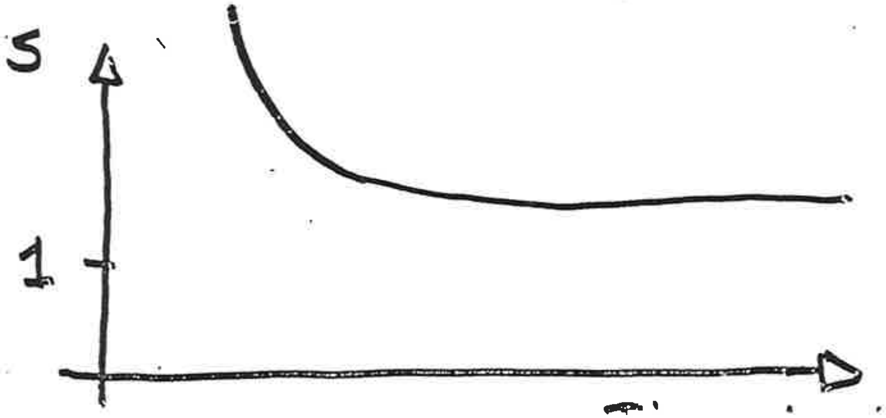
- Find a controller that is tuned to process variations
- Two factors
  - Process dynamics
  - Disturbance characteristics



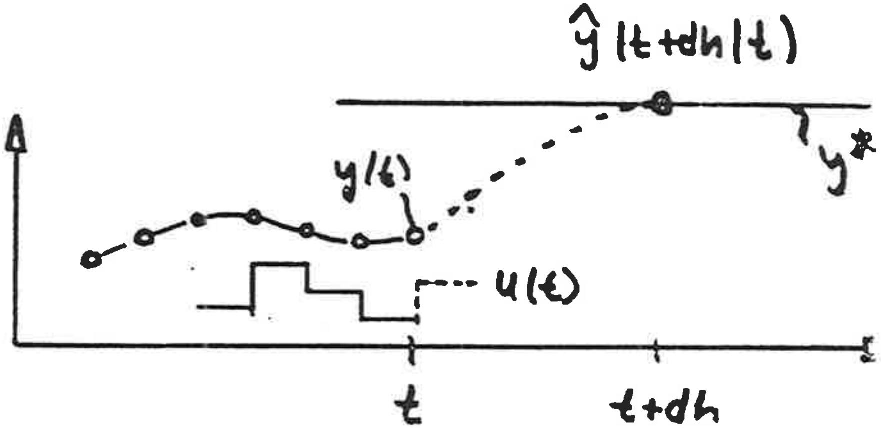
Trade-offs







MINIMUM VARIANCE CONTROL



FIND u(t) SO THAT THE PREDICTED VALUE $\hat{y}(t+dh|t)$ EQUALS THE DESIRED VALUE y^*

- SIMPLE ALGORITHM
- PROPERTIES DEPEND STRONGLY ON d & h!

  $d \cdot h > T_0$

  SMALL dh LARGE GAIN

MODEL PREDICTIVE CONTROL

Richalet et al 1979 (Idcom)

Mehra and Rouhani 1980
(Model Algorithmic Control)

Cutler and Rademacher 1979
Prett and Gillette 1979
(Dynamic Matrix Control)

Garcia 1984
(Quadratic Dynamic Matrix Control)

Mosca 1982 (Musmar)

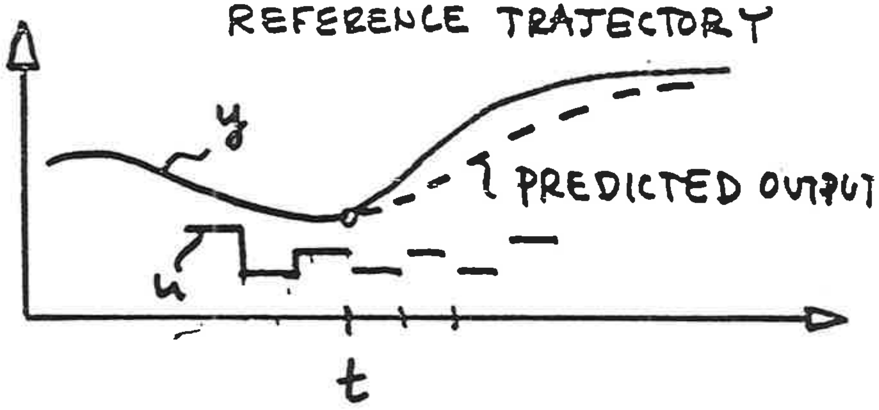
de Keyser 1985

Peterka 1984

Ydstie 1982

Clarke 1985

PREDICTIVE CONTROL



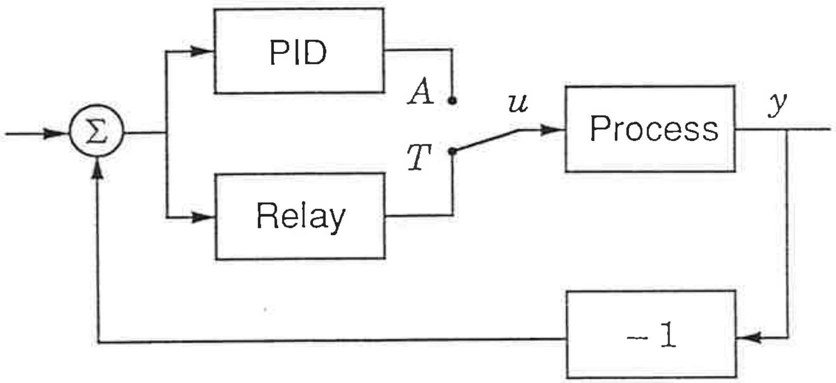
DETERMINE $\{u(s);\ t \le s \le t+T\}$
TO MINIMIZE THE CRITERION

$$J = \int_t^{t+T} F(r(s), \hat{y}(s|t), u(s))\,ds$$

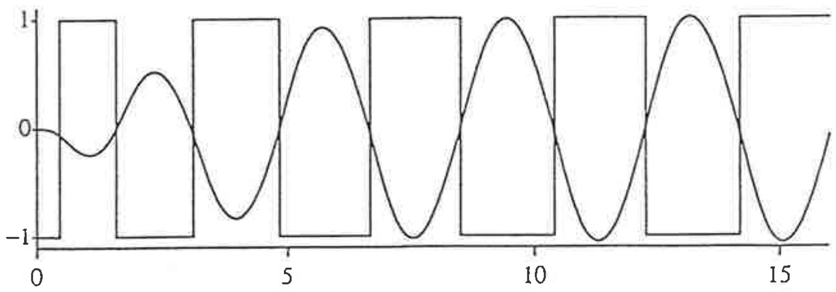
POSSIBLY WITH CONSTRAINTS
APPLY $u(t)$. REPEAT THE
PROCEDURE FOR EACH t.

Relay Auto-tuning

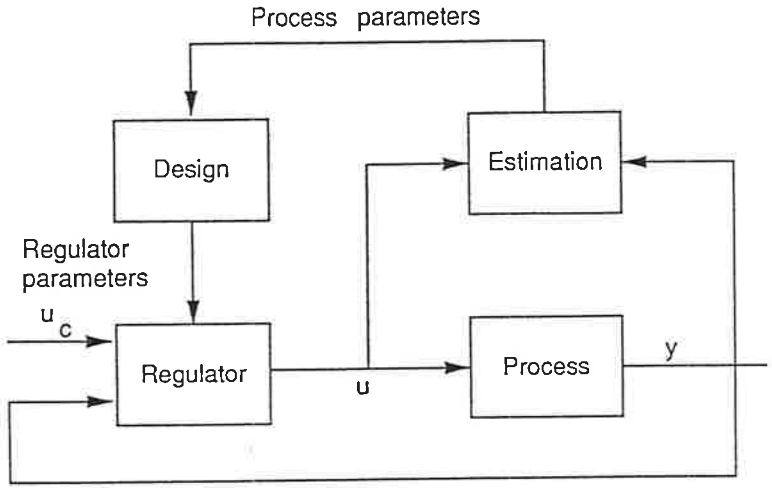
* Generate an oscillation by relay feedback



* Determine ultimate gain and ultimate period



The Self-Tuning Regulator STR



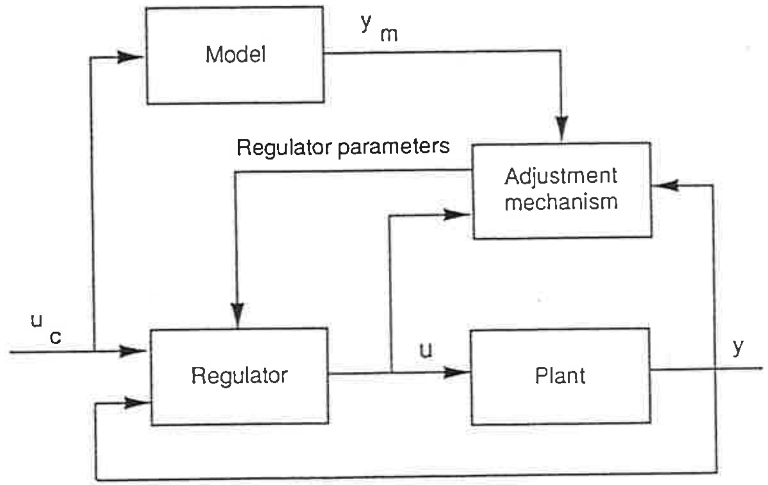
Estimation Methods

- Gradient methods
- Least squares

Design Methods

- PID
- Pole placement
- LQG
- Special methods

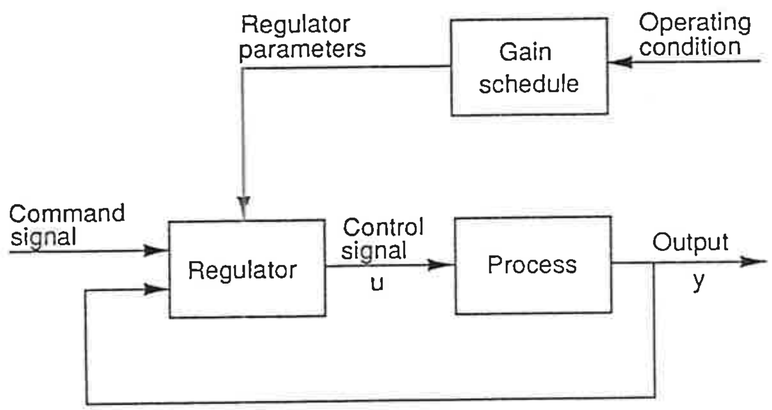
Model Reference Adaptive System MRAS



The MIT rule

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta}$$

Gain Scheduling



Example of scheduling variables

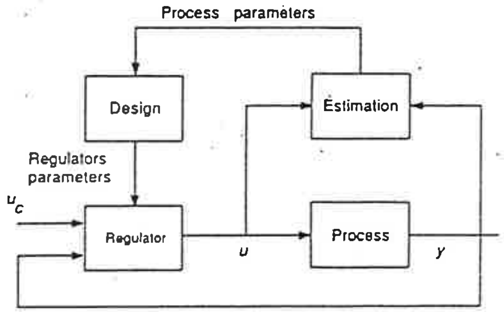
Production level

Machine speed

Mach number

Dynamic pressure

Adaptive Techniques



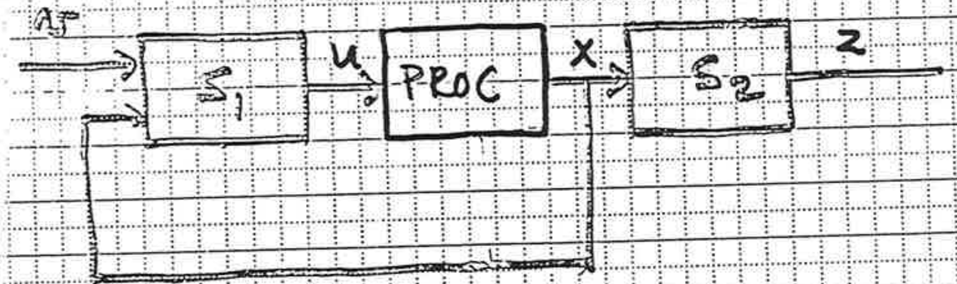
Many possible choices of controller structure, and methods for parameter estimation and control design.

Different functions:

auto-tuning
gain scheduling
continuous adaptation (FB & FF)

FEEDBACK LINEARIZATION

- Find nonlinear systems S_1 & S_2 so that signal transmission $v \rightarrow z$ becomes linear



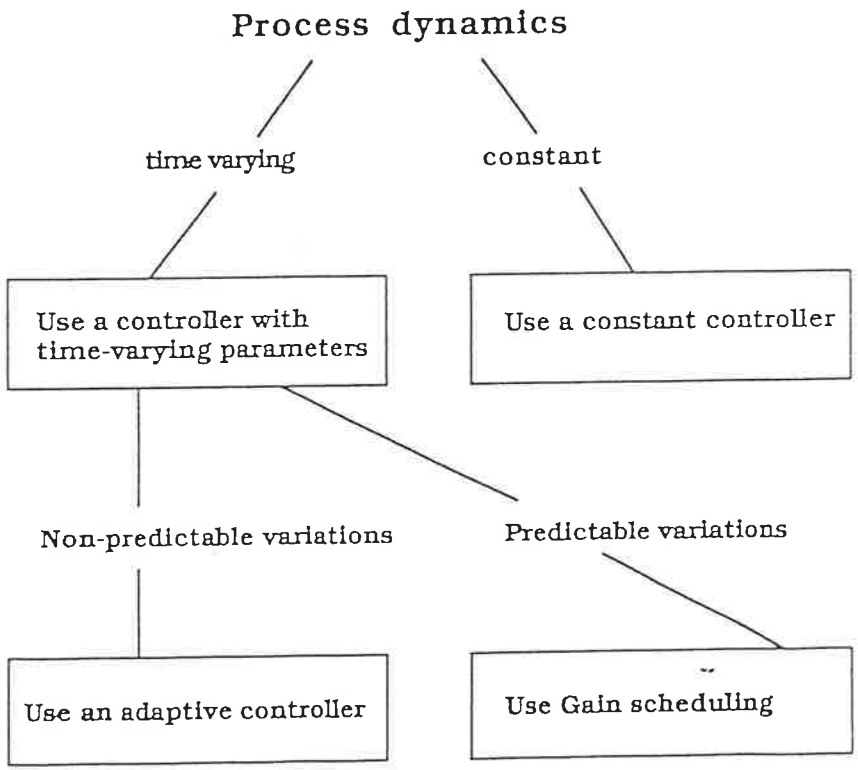
- Find linear control law

$$v = L(z_m - z)$$

- Transform back to u & x!

What kind of regulator?

Depends on process dynamics and specifications



1. Introduction

2. Bottom Up

3. Top Down

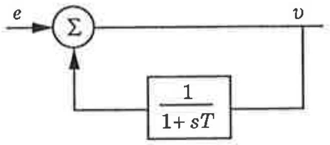
4. Conclusions

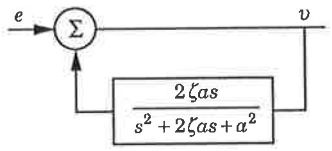
Paradigms

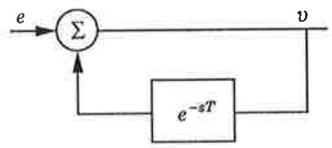
- Feedback
- Cascade
- Disturbance rejection
- Feedforward
- State feedback
- Observers
- Internal model control
- Predictive control
- Tuning
- Gainscheduling
- Adaptation
- Ratio
- Split range
- Selectors
- Feedback linearization

Paradigms for Disturbance Rejection

- Constant disturbances
- Sinusoidal disturbances
- Periodic disturbances

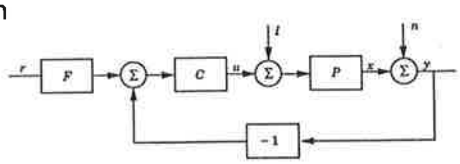






The Primary System Transfer Functions

System



Three inputs r l and n and three interesting signals u, x and y.

Nine transfer functions!

$$G_{ur} = \frac{CF}{1+PC} \qquad G_{xr} = \frac{PCF}{1+PC} \qquad G_{yr} = G_{xr}$$
$$G_{ul} = -\frac{PC}{1+PC} \qquad G_{xl} = \frac{P}{1+PC} \qquad G_{yl} = G_{xl}$$
$$G_{un} = -\frac{C}{1+PC} \qquad G_{xn} = -\frac{PC}{1+PC} \qquad G_{yn} = \frac{1}{1+PC}$$

Only 6 are different!

Several different versions!

Plant Uncertainty

$$G = \frac{PCF}{1+PC}$$

Small variations in P

$$dG = \frac{CFdP}{1+PC} - \frac{PCFCdP}{(1+PC)^2}$$

Hence

$$\frac{dG}{G} = \frac{dP}{P} - \frac{CdP}{1+PC} = \frac{1}{1+PC}\frac{dP}{P}$$
$$\frac{d\log G}{d\log P} = S$$

Stability robustness: How much can P be perturbed without violating stability?

$$|C\Delta P| < |1+PC|$$

Interpretation

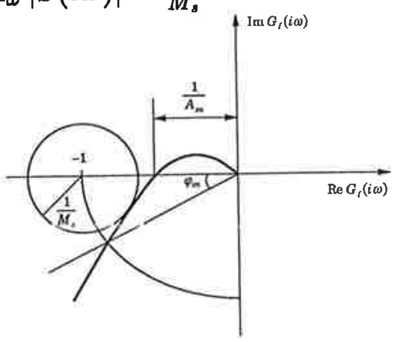
$$|\frac{\Delta P}{P}| < |\frac{1+PC}{PC}| = |\frac{1}{T}|$$
$$|\Delta P| < |\frac{FG_{ol}}{G_{cl}}|$$

The Sensitivity Functions

$$S = \frac{1}{1+PC}$$
$$T = 1 - S = \frac{PC}{1+PC}$$

Interpretations

- $S = \frac{Y(s)}{N(s)} = \frac{Y_{cl}(s)}{Y_{ol}(s)}$
- $S = \frac{\partial \log T}{\partial \log P}$
- $\min_\omega |S(i\omega)| = \frac{1}{M_s}$



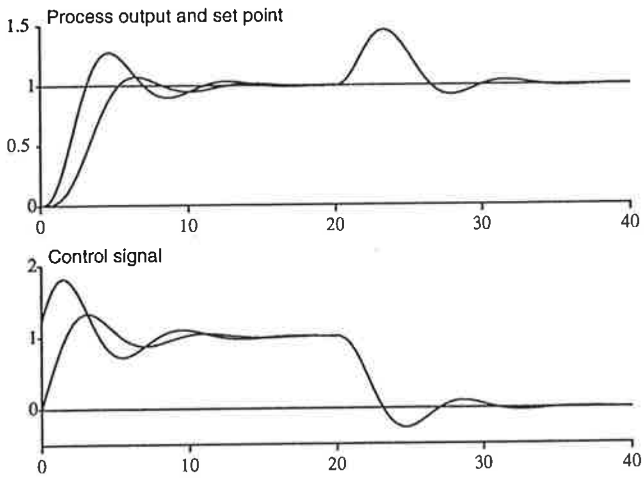
- Bodes integral

How to Illustrate Design

What should we show?

A suggestion:

- Unit step in r ($PCF/(1+PC)$ & $CF/(1+PC)$) followed by unit step in l ($P/(1+PC)$ & $1/(1+PC)$)
- Unit negative step in n ($PC/(1+PC)$ & $C/(1+PC)$)



Design of Feedforward

- System inverses
- Approximate (pseudo) inverses
- Wiener Theory
- Linear systems Blue Book page 241!

$$G(s) = G^+(s)G^-(s)$$

$$G^\dagger = (G^+(s)G^-(-s))^{-1}$$

Many extensions!

- Nonminimum phase and delays
- Feedforward design and feedforward compensation
- Nonlinear systems
- Computed torque

Problem 1 - PI Control

Consider a process with the transfer function

$$G(s) = \frac{b^2}{(s+a)(s^2+b^2)}$$

where $a > 0$ Let the system be controlled with a controller having the transfer function

$$C(s) = k\frac{s+c}{s}$$

Show that both k and c must be negative in order to have a stable closed loop system. Also demonstrate that the two-degree-of freedom controller

$$U(s) = -kY(s) + \frac{ck}{s}(Y_{sp}(s) - Y(s))$$

i.e. a PI controller with set-point weighting, gives a better set point response than a controller with error feedback.

Problem 2 - Disturbance Rejection

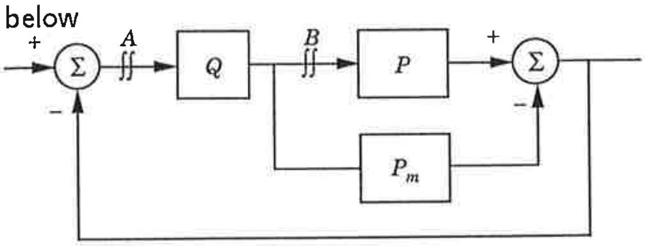
Consider a system with transfer function

$$G(s) = \frac{1}{(s+1)^3}$$

with a proportional controller. Assume that the disturbance $l = \sin \omega t$ acts on the process input. Investigate the error obtained with proportional control with $k = 1.22$ and PI control with $k = 1.22$ and $T_i = 1.78$. Construct a controller so that the error due to the sinusiodal disturbance is less than 0.001! In the problem you can set $\omega = 2\pi/T$ with $T = 30$. If you have time you can also investigate how small T can be. You can also compare with the performance of a PI controller. It is useful to plot frequency responses and to do a few simulations to check your thinking.

Problem 3 - IMC

Consider the system with IMC control shown below



Determine the loop transfer functions when the loop is cut at A and B. Investigate the limits of these transfer functions when $P_m \to P$ and $Q \to P^{-1}$.

Problem 4 - Disturbance Rejection

Can you obtain the classical disturbance rejection schemes through a state space approach?

Solution to PI Problem

The characteristic equation of the closed loop is

$b^2(a+k)$

$$s^4 + as^3 + b^2s^2 + ab^2(1+k)s + b^2kc = 0$$

Routh Hurwitz criterion gives

$$\begin{aligned} &1+k > 0 \\ &kc > 0 \\ &k < 0 \\ &k\left(1+k-\frac{ac}{b^2}\right) > 0 \end{aligned}$$

These inequalities imply that $k < 0$ and $c < 0$ and moreover that

$$a|c| < b^2$$

and

$$-1 - \frac{ac}{b^2} < k < 0$$

There is also a nice root locus argument

Project Proposal

Classic disturbance rejection. Little written background material exist. There are many possibilities. A good start is to investigation some specific schemes to obtaind design guidlines. Extended project: Paradigms for disturbance rejection.

Feedforward design. Develop design schemes. Read Wieners and Newton-Gould-Kaiser. I have good examples. Can be extended.

The Oscillation Predictor. This is my own idea. Make an extension analogous to the Smith Predictor that works for PI control. This is my own idea. I have some background material.

PIP for systems with integration. The Smith predictor does not work for processes with integration. I have an extension that works IEEE-AC-94. Specialize this result to PIP. Extended project: The Smith Predictor Revisited. Tore will be involved too.

Department of Automatic Control
Lund Institute of Technology

Fundamental Limitations of Control System Performance

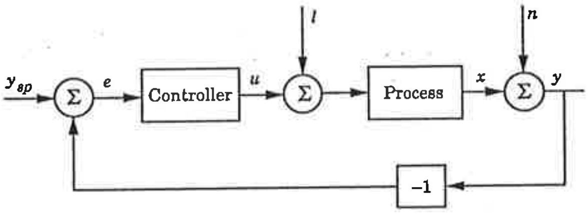
K. J. Åström

1. Introduction
2. Bode's relations
3. Minimum phase systems
4. Dynamics limitations
5. Conclusion

Introduction

- Why look at this?
- Control systems design
- Autonomous control
- Loop assessment
- Bode and Shannon
- Information theory emphasized limits control theory has not
- The Bode Lecture - Bode's integral
- This Lecture - Bode's relations

A Classic Compensation Problem



Key issues

- Noise
- Actuator saturation
- Dynamics limitations
  - RHP zeros
  - RHP poles
  - Time delays

How to capture some of this in a simple way?

Bode's Relations

Consider a transfer function $G(s)$ with no RHP poles and zeros which satisfies some regularity conditions, then

$$\arg G(i\omega_c) = \frac{2\omega_c}{\pi}\int_0^\infty \frac{\log|G(i\omega)| - \log|G(i\omega_c)|}{\omega^2 - \omega_c^2} d\omega$$

If the amplitude curve in the Bode diagram is a straight line the formula reduces to

$$\varphi = n\frac{\pi}{2}$$

A a minimum phase system can always be compensated so that this relation holds with an arbitrary accuracy.

- Implications of the inequalities.
- Bode's Ideal Cut-off characteristics
- Robust designs
- Can we do better by nonlinear compensation?
- The Clegg integrator

Minimum Phase Systems

Lead Compensation

$$G(s) = \frac{s+a}{s/N+a}$$

$$\max_\omega \arg G(i\omega) = \arctan \frac{N-1}{2\sqrt{N}}$$

$$\max_\omega |G(i\omega)| = N$$

$$N_n = \left(1 + 2\tan^2\frac{\varphi}{n} + 2\tan\frac{\varphi}{n}\sqrt{1+\tan^2\frac{\varphi}{n}}\right)^n$$

What happens for large N?

$$N_\infty = e^{2\varphi}$$

Gain required to obtain given lead

$n =$	2	4	6	8	∞
90	34	25	24	24	23
180	–	1150	730	630	540
225	–	14000	4800	3300	2600

The Design Inequality

Factor transfer function as

$$G(s) = G_{mp}(s)G_{nmp}(s)$$

Assume that the minimum phase part is perfectly compensated so that

$$\varphi_{mp} = n_{gc}\frac{\pi}{2}$$

The phase margin condition

$$\varphi_m - n_{gc}\frac{\pi}{2} + \arg \Delta G(i\omega_{gc}) + \arg G_{nmp}(i\omega_{gc}) \geq -\pi + \varphi_m$$

gives an inequality for the crossover frequency

System with one RHP Zero

$$G_{nmp}(s) = \frac{a-s}{a+s}$$

The condition

$$\arg G(i\omega_{gc}) \geq -\pi + \varphi_m$$

gives

$$n_{gc}\frac{\pi}{2} - 2a \quad \tan\frac{\omega_{gc}}{a} \geq -\pi + \varphi_m$$

Hence

$$\omega_{gc} \leq \arctan(\frac{\pi}{2} - \frac{\varphi_m}{2} + n_{gc}\frac{\pi}{4})$$

Example Specifications

$$\varphi_m = \pi/4, \quad n_{gc} = -1/2$$

give

$$\omega_{gc} < a$$

System with Dead Time

$$G_{nmp}(s) = e^{-sL}$$

The condition

$$\arg G(i\omega_{gc}) \geq -\pi + \varphi_m$$

gives

$$n_{gc}\frac{\pi}{2} - 2\omega_{gc}L \geq -\pi + \varphi_m$$

Hence

$$\omega_{gc}L \leq \frac{\pi}{2} - \frac{\varphi_m}{2} + n_{gc}\frac{\pi}{4}$$

Example The specifications

$$\varphi_m = \frac{\pi}{4}, \quad n_{gc} = -\frac{1}{2}$$

give

$$\omega_{gc}L \leq \frac{\pi}{2}$$

System with one RHP Pole

One encirclement of the critical point is required

$$G_{nmp}(s) = \frac{s+b}{s-b}$$

The condition

$$\arg G(i\omega_{gc}) \geq -\pi + \varphi_m$$

gives

$$n_{gc}\frac{\pi}{2} - 2\arctan\frac{b}{\omega_{gc}} \geq -\pi + \varphi_m$$

Hence

$$\omega_{gc} \geq \frac{b}{\tan(\pi/2 - \varphi_m/2 + n_{gc}\pi/4)}$$

Example The specifications

$$\varphi_m = \frac{\pi}{4}, \quad n_{gc} = -\frac{1}{2}$$

give

$$\omega_{gc} \geq b$$

A RHP Pole-Zero Pair

$$G_{nmp}(s) = \frac{(a-s)(s+b)}{(a+s)(s-b)}$$

We have

$$\arg G_{nmp}(i\omega) = -2\arctan\frac{\omega}{a} - 2\arctan\frac{b}{\omega}$$
$$= -2\arctan\frac{\omega/a + b/\omega}{1 - b/a}$$

The condition $\arg G(i\omega_{gc}) \geq -\pi + \varphi_m$ gives

$$\frac{\omega_{gc}}{a} + \frac{b}{\omega_{gc}} \leq (1 - \frac{b}{a})\tan(\frac{\pi}{2} - \frac{\varphi_m}{2} + n_{gc}\frac{\varphi}{4})$$

Minimizing LHS with respect to ω_{gc} gives

$$2\sqrt{\frac{b}{a}} \leq \tan(\frac{\pi}{2} - \frac{\varphi_m}{2} + n_{gc}\frac{\varphi}{4})$$

A phase margin of $\varphi_m = \pi/4$ requires

$$a > 5.83b$$

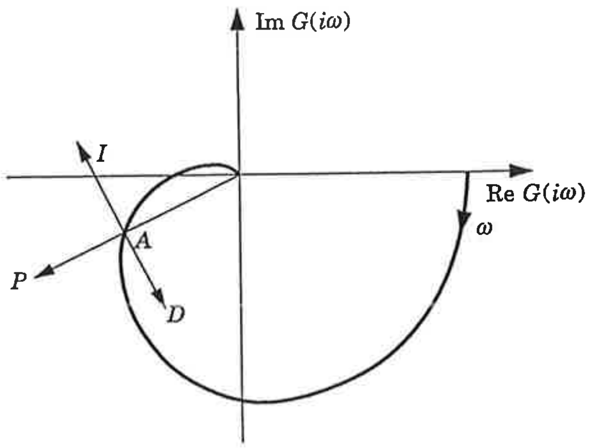
Example X-29

$$G_{nmp}(s) = \frac{s-26}{s-6}$$

With $a = 4.33b$, we cannot achieve $\phi_m > 45°$

Impact on Current Projects

- Automatic Tuning
- Autonomous Control
- Control loop assessment



Conclusions

- Do not forget history
- Important to stress fundamental limitations
- The start of all design work
- Process design and controller design
- Even more important with increasing use of CACE. The X-29 lesson!!
- This version is teachable in a basic course
- Autotuning and autonomous control
- A key reason for introducing Bode diagrams
- Nonlinear systems – An interesting research problem

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Lecture 5

- Quantitative Feedback Theory (QFT)
- Limits of Performance
  - Bode's relations
  - Bode's integral formulae for S and T
  - Zames-Francis

Literature

- Maciejowski Ch 1 + 203-207
- Kwakernaak ECC 95
- QFT Toolbox, TFRT-5477
- Bode Lecture
- Doyle, QFT etc, AC 1986
- Dazzo - Houpis

QFT-Paradigms

Focus: Plant uncertainty

2 degree of freedom structure

Translate specs to sepcs on $L = PC$

Design $C(i\omega)$ so $L(i\omega)$ is put where plant uncertainty is not dangerous

Tradition: Use Nichols chart

Check other specs afterwards (or during)

"Use common sense"

2 Degree of Freedom

Often: first design C, then F

$$\begin{pmatrix} y \\ u \end{pmatrix} = \begin{pmatrix} S & R \\ U & T \end{pmatrix} \begin{pmatrix} d \\ v \end{pmatrix}$$

With $L = PC$ (loop gain)

$$S = \frac{1}{1+L} \quad \text{sensitivity, } \frac{\partial T}{T} = S\frac{\partial P}{P}$$
$$T = \frac{L}{1+L} \quad \text{complementary sensitivity, } m \longrightarrow y$$
$$U = \frac{C}{1+L} \quad m \rightarrow u$$
$$R = \frac{P}{1+L}$$

Note: U, R not given by L alone

Robustness – Small gain theorem

$P = P_0(1+\Delta)$ stable if Δ stable and

$$|\Delta(i\omega)| < |T_0(i\omega)|^{-1} = \frac{1}{|G_m|}\frac{|G_{ff}|}{|G_{fb}|}, \qquad \forall\omega$$

Explanation in figure:

Additive: $P = P_0 + \Delta$? (Answer: U)

Robustness – Circle Criteria

L avoids circle, then robust against static nonlinearities before plant

Typical S and T curves

Bode's "ideal loop transfer"

S and T in Nichols

Bode: $\int \log |S(i\omega)| d\omega = c \geq 0$ if roll off > 1.

Nyquist Criterium in Nichols

Carl Bildt-rule: Go left to right

Example $G(s) = (s-1)/s^3$

First Order Systems, Lead/Lag Nets

Sometimes necessary to use nonminphase or unstable controller

Second Order Systems

Bode: Can not twist amplitude and phase independently

Example, PID with complex zeros

Use PID-structure that allows complex zeros

QFT

Several toolboxes in Matlab.

Example $P(s) = \frac{Ka}{s^2+as}$, $K \in [1, 10], a \in [1, 10]$

Specification

$$a(i\omega) \le |G_{cl}(i\omega)| \le b(i\omega)$$
$$\partial|T(i\omega)| \le |b(i\omega)| - |a(i\omega)|$$

(Only looks on gain!)

```
frqdspec; specmatrix=getspec
```

Templates $G(i\omega, \theta)$

```
tf2tmpl(nummatrix,denmatrix,omegavect)
```

Generate Bounds

Can be based on

- $\partial T(i\omega)$ suff. small each ω
- S small for ω up to bandwidth
- S and T no excessive peaks for any ω
- Common sense
- Stability

```
bnd=tmpl2bnd(tmpl,specmatrix);
bndpl(bnd)
```

Design Compensator

```
compensator; [cnum,dnum]=getcompensator
prefilter(cnum,cden,...,specmatrix)
[fnum,fden]=getprefilter
```

Kidron's Design on Landau

Bounds on $S(i\omega)$.

Implicit bound on bandwidth. $w_B > 10$rad/s

Stability, common sense

Bounds

First Design, PI

PI+2nd order net

PI+4th order net

Time domain specifications

How translate time domain specifications to bounds in Nichols chart?

Questionable, but works sometimes

Varying number of unstable poles

What if number of unstable poles to $G(s, \theta)$ varies?

Example $P(s) = \frac{-k}{s(s-a)}$, $a \in [-1, 1], k \in [10, 20]$

Templates:

Template split!

Hankel Transforms

f analytic in RHPL.

Cauchy gives

$$0 = \frac{1}{2\pi i}\int_C \frac{f(s)}{s - i\omega_c}ds = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{f(i\omega)}{\omega - \omega_c}d\omega + \frac{f(i\omega_c)}{2} + \int_{C_R}$$

Hence, if $f(s) \to F$ on C_R then

$$u(i\omega_c) + iv(i\omega_c) = \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{u(i\omega) + iv(i\omega)}{\omega - \omega_c}d\omega - F$$

Hankel transforms, if $F = 0$:

$$v(i\omega_c) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{u(i\omega)}{\omega - \omega_c}d\omega$$
$$u(i\omega_c) = \frac{-1}{\pi}\int_{-\infty}^{\infty} \frac{v(i\omega)}{\omega - \omega_c}d\omega$$

Bode's Relations

(Dispersion relations in physics)

Put $f(s) = \log(G(s))$, where

- $G(s)$ analytic in RHPL
- $G(s) \neq 0$ in RHPL
- $|G(s)| < (1 + |s|)^N$ in RHPL
- $|\arg G(s)|/s \to 0$ on C_R

Allowed $G(s)$: $-1, (1+s)^n,\ e^{(s-1)/(s+1)}, e^{-\sqrt{s}T}$

Not allowed: e^{-sT}, $(1-s)/(1+s)$

$$\arg\ G(i\omega_c) - \arg\ G(0) = \int_{-\infty}^{\infty} \frac{d\log|G(i\tilde{\omega})|}{d\tilde{\omega}} W(\tilde{\omega})d\tilde{\omega}$$

Bode's Relations, 2nd form

If $f(\bar{s}) = \bar{f}(s)$ and $f(s)/s \to 0$ on C_R then one also has (check)

$$v(i\omega_c) = \frac{2\omega_c}{\pi}\int_0^{\infty} \frac{u(i\omega)}{\omega^2 - \omega_c^2}d\omega$$

Since $\int_0^\infty \frac{1}{\omega^2 - \omega_c^2}d\omega = 0$ this can be written

$$\begin{aligned} v(i\omega_c) &= \frac{2\omega_c}{\pi}\int_0^{\infty} \frac{u(i\omega) - u(i\omega_c)}{\omega^2 - \omega_c^2}d\omega \\ &= \frac{1}{\pi}\int_0^{\infty} \frac{du(i\omega)}{d\omega}\log\left|\frac{\omega + \omega_c}{\omega - \omega_c}\right| d\omega \\ &= \int_{-\infty}^{\infty} \frac{du(i\tilde{\omega})}{d\tilde{\omega}} W(\tilde{\omega})d\tilde{\omega} \end{aligned}$$

$W(x) = \frac{1}{\pi}\log(\coth(\frac{|x|}{2}))$, total weight $\frac{\pi}{2}$

$\tilde{\omega} = \log(\omega/\omega_c)$, logarithmic frequency

Rolloff n gives phase $-n \cdot 90°$

The weighting function

Peak at ω_c. Zero at low and high frequencies.

92 % weight in $\omega/\omega_c \in [0.1, 10]$

Nonminimum phase

If $G(s)$ has zeros and poles in RHPL

and arg $G(s)/s \to T$ on C_R then

$$\arg\, G(i\omega_c) - \arg\, G(0) = \int_0^\infty \frac{d \log |G(i\tilde{\omega})|}{d\tilde{\omega}} W(\tilde{\omega}) d\tilde{\omega}$$
$$+T + \Phi_{\text{zeros}} + \Phi_{\text{poles}}$$

where $\Phi_{\text{zeros}}(\omega_c) \leq 0$, $\Phi_{\text{poles}}(\omega_c) \geq 0$, $\omega_c \geq 0$.

Proof:
$$G(s) = B_p(s) B_z(s) e^{sT} \tilde{G}(s)$$
where $|B_p(i\omega)| = |B_z(i\omega)| = 1$.

Ex. $B_p(s) = (s+a)/(s-a)$.

Bode's Integral Formula

$1 + L(s)$ no RHPL zeros, $sL(s) \to k$

$$\int_0^\infty \log \left| \frac{1}{1 + L(i\omega)} \right| d\omega = \pi \sum \text{Re } p_i - \frac{\pi}{2} \lim_{s \to \infty} sL(s)$$

Proof

$$0 = \int_D \log(1 + L(s)) ds$$

(any logarithm to $1 + L(s)$ in D)

$$\int_{AB} \log(1+L(s-i\epsilon)) - \log(1+L(s+i\epsilon)) ds \to 2\pi i m_i \text{Re } p_i$$

(Use $\log(f(s_2)) - \log(f(s_1)) = \int f'/f\, ds$)

$$0 = 2i \int_0^\infty \log\left(1 + L(i\omega)\right) d\omega +$$
$$2\pi i \sum \text{Re } p_i - \pi i \lim_{s \in C_R} s \log(1 + L(s))$$

Bode's Integral Formula for $T(1/j\omega)$

$$T = \frac{L}{1+L} = \frac{1}{1 + \frac{1}{L}}$$

But $1/L(s)$ no roll-off so use

$$T(1/s) = \frac{1}{1 + \frac{1}{L(1/s)}}$$

Hence if $L(s)$ has at least one integrator

$$\int_0^\infty \log |T(1/i\omega)|\, d\omega = \pi \sum \text{Re } \frac{1}{z_i} - \frac{\pi}{2} e_1^{-1}$$

where e_1= ramp error $= \lim_{s \to 0} sL(s)$

Contour lines of Re $(1/z)$ = const

What if $G(0) = 0$?

Zames-Francis Formula

$$\int_0^\infty \log|S(i\omega)|dW_z(\omega) = \frac{1}{2}\log|S(z)| + \log|B_p^{-1}(z)|$$

where

$$W_z(\omega) = \frac{1}{\pi}(\phi(z,\omega) + \phi(\bar{z},\omega))$$

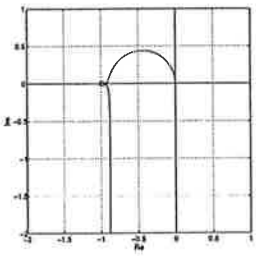
(Bode I.F. follows by Re $z \to \infty$)

A nasty example

Pole in $a > 0$, zero $b > 0$

$$|S(i\omega)|_\infty \geq \frac{b+a}{|b-a|}$$

$$G(s) = \frac{s-5}{(s-4)(s+1)}.$$



Proof of Zames-Francis

Poisson's integral formula

$$u(z) = \frac{\text{Re } z}{\pi}\int_{-\infty}^{\infty} \frac{u(i\omega)}{|i\omega - z|^2}d\omega$$

Proof

$$\begin{aligned} 2\pi i f(z) &= \int \frac{f(w)}{w-z} - \frac{f(w)}{w+\bar{z}}dw \\ &= \int \frac{f(w)(z+\bar{z})}{(w-z)(w+\bar{z})}dw \end{aligned}$$

Use on $\log|\tilde{S}|$ where $S(s) = B_p(s)\tilde{S}(s)$ and $|B_p(i\omega)| = 1$. Gives Zames-Francis

Cyber-Nichols

Fuzzy Systems, Nonlinear Maps and Control

"Bringing it all back home"

Mikael Johansson
Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

Outline

1. Introduction
2. Fuzzy Logic
3. Fuzzy Systems
4. Nonlinear Maps
5. Fuzzy Models
6. Fuzzy Systems for Control
7. Conclusions

Motivation – Why Fuzzy Control ?

Incorporate heuristics into control strategy.

- Example: Model operator's actions.

Define nonlinearities in an intuitive way.

- Rules and interpolation.

Rule Based Systems – The Basics

Terminology:

$$\underbrace{\text{IF } \overbrace{x \text{ is } A_1}^{\text{proposition}}}_{\text{premise}} \; \underbrace{\text{THEN } y \text{ is } B_1}_{\text{consequent}}$$

Propositions are characterized by sets.

Sets are specified by charecteristic functions

$$\mu_A(x) = \begin{cases} 1, & x \in A \\ 0, & \text{otherwise} \end{cases}$$

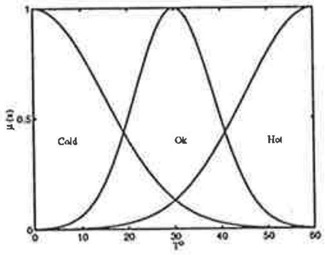
What's New with Fuzzy Sets ?

From true/false logic to "grades of truth"

$$\mu_A : x \rightarrow [0,1]$$

$\mu_A(x)$ expresses to what degree "x is A"

Example: "The water is hot"

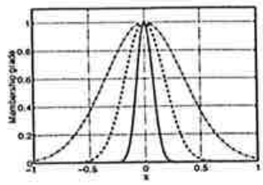


μ_A membership function

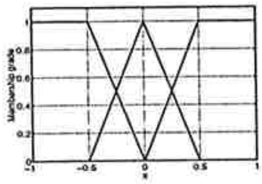
Probabilities ?

Typical Membership Functions

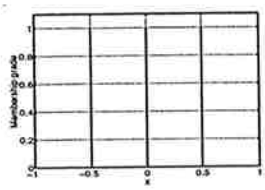
Gaussian:



Triangular/Trapezoidal:



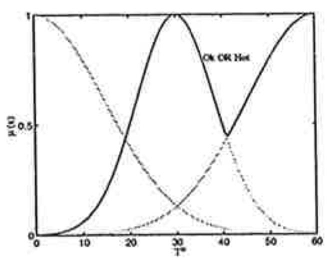
Singleton:



Logic and Fuzzy Logic

Fuzzy Logic

AND	$\mu_{A_1 \cap A_2}(x) = \min(\mu_{A_1}(x), \mu_{A_2}(x))$
OR	$\mu_{A_1 \cup A_2}(x) = \max(\mu_{A_1}(x), \mu_{A_2}(x))$
NOT	$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$



– Generalization of logic.

AND, OR and NOT connects simple propositions into compound propositions:

$$x_1 \text{ is } A_1^{(i)} \text{ AND } x_2 \text{ is } A_2^{(i)} \text{ OR} \ldots$$

Fuzzy Rules

Mamdani-type:

IF $< \mathit{FuzzyProposition} >$ THEN y is $B^{(i)}$

"Everything is fuzzy"

Sugeno-type:

IF $< \mathit{FuzzyProposition} >$ THEN $y = f^{(i)}(x)$

only rule-premises fuzzy logic expressions.

Approximate Reasoning

Modus ponens:

$$\begin{array}{llll} \text{Observation:} & & x \text{ is } A & \\ \text{Knowledge:} & \text{IF} & x \text{ is } A \text{ THEN} & y \text{ is } B \\ \hline \text{Conclusion:} & & & y \text{ is } B \end{array}$$

Generalized modus ponens:

$$\begin{array}{llll} \text{Observation:} & & x \text{ is } A' & \\ \text{Knowledge:} & \text{IF} & x \text{ is } A \text{ THEN} & y \text{ is } B \\ \hline \text{Conclusion:} & & & y \text{ is } B' \end{array}$$

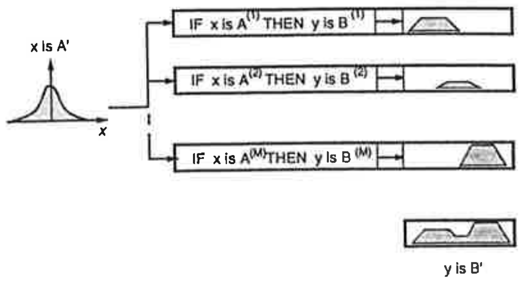
"The more B the B', the more A the A'."

In terms of membership functions:

$$\mu_{B'}(y) = \sup_{x \in X} [\mu_{A'}(x) \cap \mu_A(x) \cap \mu_B(y)]$$

Reasoning with Several Rules

Individual-rule inference ...



... followed by aggregation.

In terms of membership functions:

$$\mu_{B'}(y) = \mu_{B'^{(1)}}(y) \cup \cdots \cup \mu_{B'^{(M)}}(y)$$

Connecting to Physical Systems

Problem: Inputs and outputs numeric values.

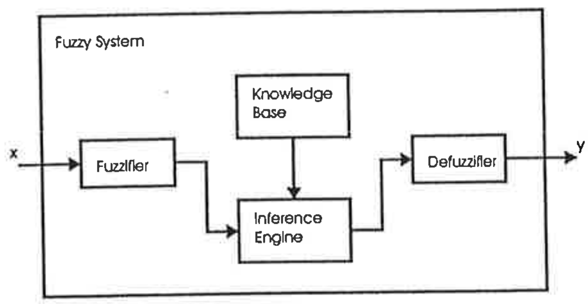
Solution: Add interfaces.

- Fuzzifier : Numbers $\longrightarrow$ fuzzy sets.
- Defuzzifier: Fuzzy sets $\longrightarrow$ numbers.

Fuzzy System:

- Knowledge Base
- Logic and Inference
- Interfaces

Architecture



- Fuzzifier : Numbers $\longrightarrow$ fuzzy sets.
- Defuzzifier: Fuzzy sets $\longrightarrow$ numbers.

Fuzzifiers and Defuzzifiers

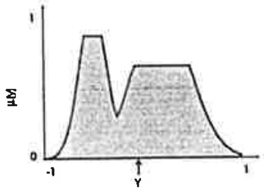
Fuzzifier : Number x' $\longrightarrow$ fuzzy set A'.

- Common choice: Singleton fuzzifier

$$\mu_{A'}(x) = \begin{cases} 1, & x = x' \\ 0, & \text{otherwise} \end{cases}$$

Defuzzifier: Fuzzy set B' $\longrightarrow$ number y'.

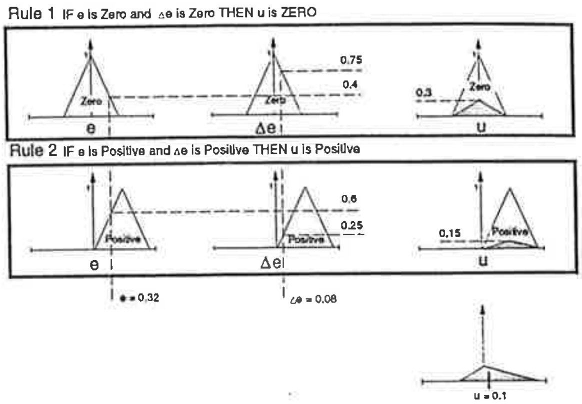
- Common choice: Center of Gravity



$$y' = \frac{\int_Y w \cdot \mu_{B'}(w)\,\mathrm{dw}}{\int_Y \mu_{B'}(w)\,\mathrm{dw}}$$

Inference in Mamdani-Type Systems

Simplified inference for singleton fuzzifier:



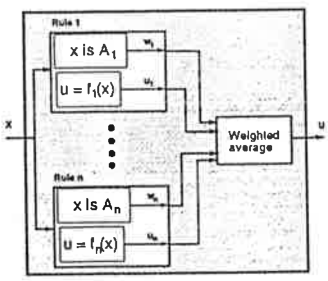
1. Evaluate rule-premises
2. Infer rule consequents
3. Aggregate individual rules' outputs
4. Defuzzify

Inference in Sugeno-Type Systems

Sugeno-type rules

$$\text{IF } x \text{ is } A^{(i)} \text{ THEN } y = f^{(i)}(x)$$

Simple Inference:

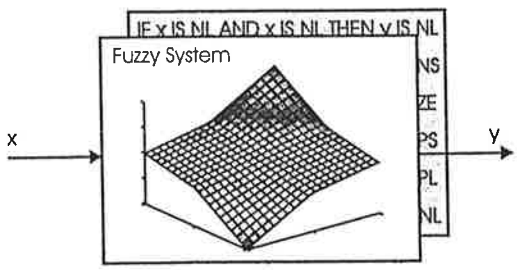


1. Evaluate rule-premises
2. Evaluate output functions
3. Output is a weighted average:

$$y = \frac{\sum_{i=1}^{M} \mu_{A^{(i)}}(x) f^{(i)}(x)}{\sum_{i=1}^{M} \mu_{A^{(i)}}(x)}$$

An External View

Externally: A nonlinear map



Fuzzy systems and nonlinear maps:

- Rule $\leftrightarrow$ Value
- Linear function $\mathbb{R}^2 \longrightarrow \mathbb{R}$: 2 rules.

A Fuzzy PD Rule Base

Rules:

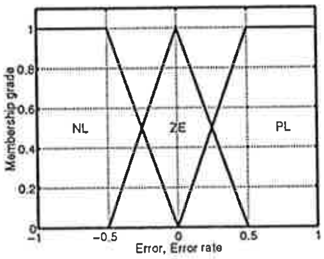
IF e is PL AND $\dot{e}$ is NL THEN u is NL

$\vdots$

Illustrated in a rule table:

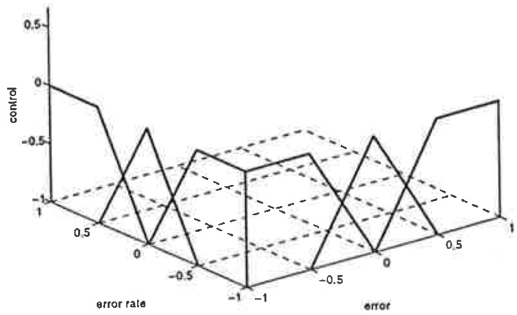
		e: NL	ZE	PL
	PL	ZE	PL	PL
$\dot{e}$	ZE	NL	ZE	PL
	NL	NL	NL	ZE

Typical membership functions:



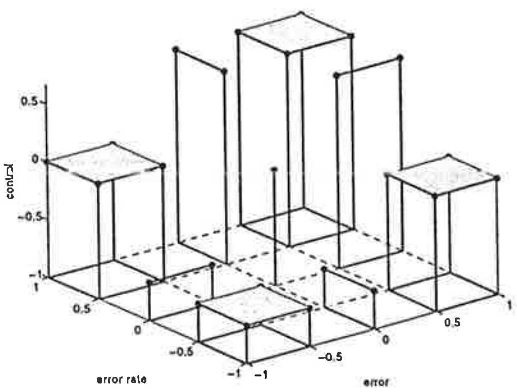
A Table Look-up Analogy

- Rule premises partition controller state space into a set of intervals



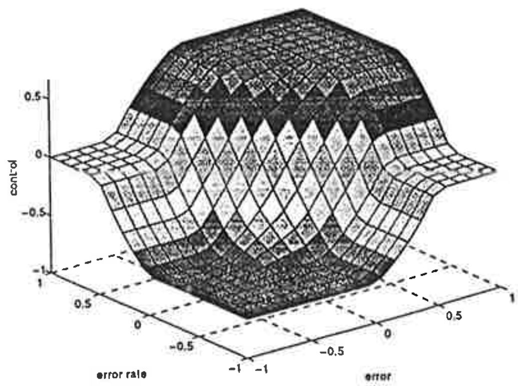
A Table Look-up Analogy

- Rule consequents specify nonlinearity at interval endpoints



A Table Look-up Analogy

- Reasoning process performs interpolation (also influenced by fuzzifier/defuzzifier)

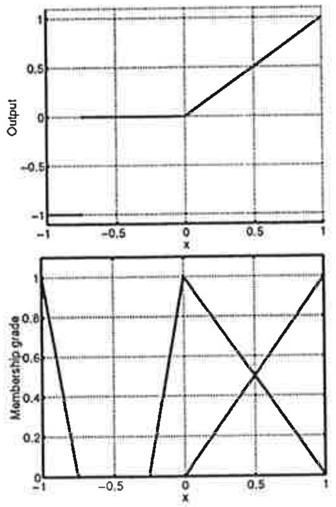


Why Overlapping Fuzzy Sets ?

Insight:

- Several active rules: interpolation.
- One valid rule: constant output.
- No valid rule : zero output.

Example:



Nonlinear Maps

- Very Common
- Difficult to represent

$$\mathbb{R}^n \rightarrow \mathbb{R}$$

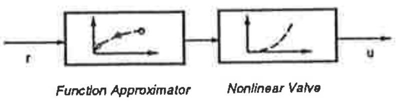
  – Homogenous discretization N

  $\Rightarrow N^n$ parameters.

- Several approaches:
  - Table Look-up
  - Splines
  - Fuzzy Systems
  - Neural Nets
  - Wavelets

Function Approximation in Control

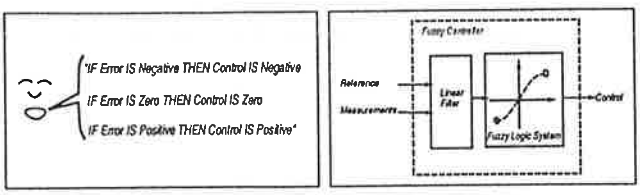
Compensation of static nonlinearities:



Nonlinear system identification:

$$x_{k+1} = f(x_k, u_k) + g(x_k, u_k) + e_k$$

Rule based controller design:



Fuzzy Systems and Nonlinear Maps

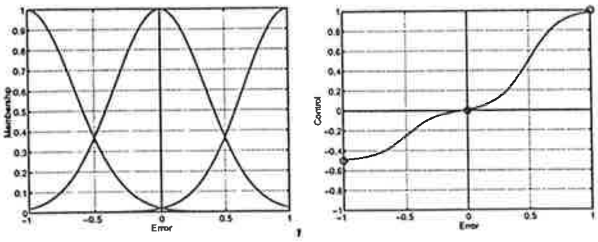
- Two representations
  - Nonlinear Function
  - Fuzzy system
- Closed forms sometimes possible

$$y = \sum_{i=1}^{M} \overbrace{g_i(x)}^{\text{IF-part}} \underbrace{w_i}_{\text{THEN-part}}$$

- One-to-One
- Simple implementation gives restrictions (Inference parameters, Interfaces, Rule format, etc.)

Fuzzy System Nonlinearities I

Gaussian Membership Functions:



Formula

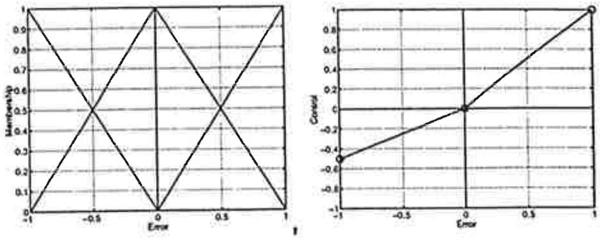
$$\hat{f}(x) = \sum_{i=1}^{M} \frac{\mu_i(x;\theta)}{\sum_{i=1}^{M}\mu_i(x;\theta)} w_i = \sum_{i=1}^{M} g_i(x) w_i$$

Remarks:

- Global formula.
- Continuously differentiable.
- $\hat{f}(x) \in \overline{\mathrm{Co}}(w_i)$.
- Radial Basis Functions.

Fuzzy System Nonlinearities II

Triangular Membership Functions:



Formula:

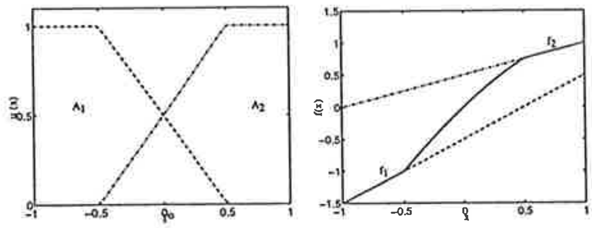
$$\hat{f}(x) = \sum_{i=1}^{M} \mu_i(x) w_i = \sum_{i=1}^{M} g_i(x) w_i$$

Remarks:

- "Linear B-Splines"
- Piecewise multilinear
- Can be made exactly linear

Fuzzy System Nonlinearities III

Sugeno-Type Models, Linear Consequents:



Formula:

$$\hat{f}(x) = \sum_{i=1}^{M} \frac{\mu_i(x;\theta)}{\sum_{i=1}^{M}\mu_i(x;\theta)} \left(L^{(i)}\right)^T x = \sum_{i=1}^{M} g_i(x) \left(L^{(i)}\right)^T x$$

Remarks:

- Gain-scheduling: $\hat{f}(x) = L^T(x)x$
- Can be made exactly linear
- $\hat{f}(x) \in \overline{\mathrm{Co}}\left(\left(L^{(i)}\right)^T x\right)$

Fuzzy Systems are Universal Approximators

AN APPROXIMATION THEOREM

Let

$$f : U \subseteq \mathrm{IR}^n \to \mathrm{IR}$$

be a continuous function defined on a compact set U. Then, for each $\epsilon > 0$ there is a fuzzy system $\hat{f}_\epsilon(x)$ such that

$$\sup_{x \in U} |f(x) - \hat{f}_\epsilon(x)| \leq \epsilon$$

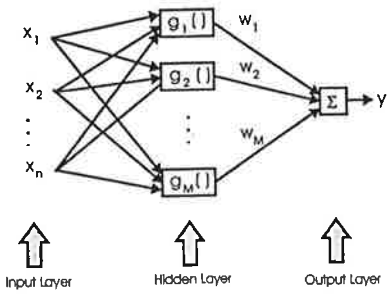
Valid for Mamdani and Sugeno fuzzy systems.

Relation to Neural Nets

Evaluation of a fuzzy system mapping

$$f(x) = \sum_{i=1}^{M} g_i(x) w_i$$

can be recast as a "feedforward" net



Basis for "neuro-fuzzy" systems.

Multi-layers, basis functions.

System Modeling using Fuzzy Systems

Modeling – A Rough Outline

1. Determine relevant process variables
2. Formulate heuristic knowledge as rules
3. Transform rules into nonlinear formula
4. Adjust parameters to fit data

(5.) Transform back to rules

Many important issues left out.

Parameter Identification

Fit fuzzy model to N measurements (x_k, y_k).

Fix $g_i(x;\theta)$, adjust w_i ($w_i \leftrightarrow$ consequents).

Writing

$$\hat{f}(x) = \sum_{i=1}^{M} g_i(x;\theta) w_i = \phi^T(x) w$$

we have

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} \phi^T(x_1) \\ \phi^T(x_2) \\ \vdots \\ \phi^T(x_N) \end{bmatrix} w = \Phi w$$

Optimal parameters in LS sense:

$$w^* = \Phi^+ Y$$

Stability of Fuzzy Models

Let

$$\dot{x} = A(i)x$$

denote a family of linear systems where

$$A(i) \in \overline{\mathrm{Co}}\,(A^{(i)}),\ \ i = 1, 2, \ldots, M$$

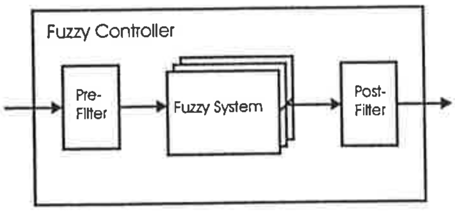
Then, the system is stable if there exists a common Lyapunov function P, i.e.

$$PA^{(i)} + \left(A^{(i)}\right)^T P \leq -\epsilon I$$

Searching for a P matrix is an LMI-problem.

Feels conservative ...

Fuzzy Controller Structure



"A fuzzy controller is a controller that contains a nonlinear mapping that can be interpreted as a set of fuzzy logic based rules."

Pre- and post-filtering

- Signal conditioning
- Dynamic filtering
- Coordinate transforms

⋮

Example: Fuzzy PID

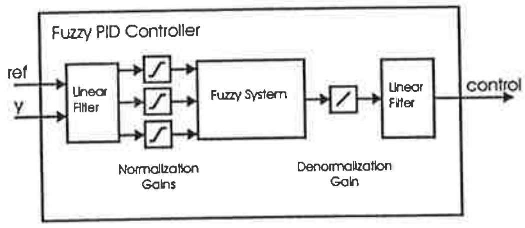
Linear PID on velocity form:

$$\frac{du}{dt} = K\left(\frac{de(t)}{dt} + \frac{1}{T_i}e(t) + T_d\frac{d^2e}{dt^2}\right)$$

Linear mapping replaced by fuzzy rules

IF $\dot{e}$ is NL AND e is ... THEN $\dot{u}$ is NL

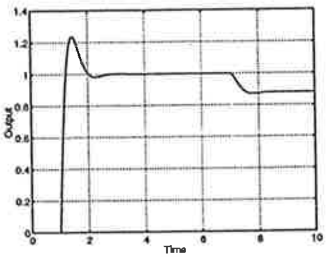
Structure:



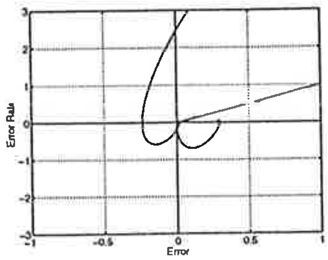
Linear filters also important.

Fuzzy Control – Qualitative Analysis

A time response ...



... illustrated in the phase plane ...



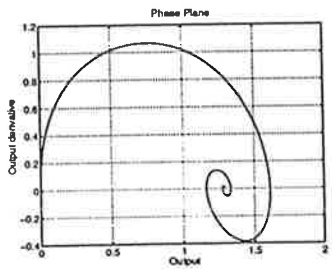
... gives hints for operating regimes.

Fuzzy Control – Qualitative Design

Consider the system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}f_1(x_1,x_2)\\ f_2(x_1,x_2)\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

"Crafting" a nonlinear compensator

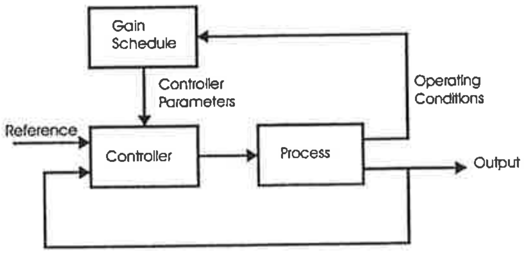


Design by simulation ?

Fuzzy Gain Scheduling

Operating condition p schedules parameters:

$$\text{IF } p_1 \text{ is } A_1^{(l)} \text{ AND } \ldots \text{ THEN } y = L^{(l)}x$$



In many cases,

$$\dim(p) \ll \dim(x)$$

and parameter reduction is probable.

A Nonlinear Pole Placement

Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots = \\ \dot{x}_n &= f(x) + g(x)u \end{aligned}$$

The control

$$u = \frac{1}{g(x)}\left(-f(x) - L^T x + l_r r\right)$$

gives the linear closed loop system

$$\dot{x} = \begin{bmatrix} 0 & I_{n-1} \\ & -L^T \end{bmatrix} x + \begin{bmatrix} 0 \\ l_r \end{bmatrix} r$$

What if $f(x)$ and $g(x) \neq 0$ are poorly known ?

Model Based Fuzzy Control

Approximate $f(x)$ and $g(x)$ by fuzzy systems:

$$\begin{aligned} f(x) &\approx \hat{f}(x) + \epsilon_f(x) \\ g(x) &\approx \hat{g}(x) + \epsilon_g(x) \end{aligned}$$

use approximate control

$$\hat{u} = \frac{1}{\hat{g}(x)}\left(-\hat{f}(x) + L^T x + l_r r\right)$$

What about approximation errors ϵ_f and ϵ_g?

Adaptive Fuzzy Control

On-line adjustments of model parameters.

Assume $g(x)$ known, re-write $\hat{f}(x)$

$$\hat{f}(x) = w^T g(x)$$

Use control

$$u = \frac{1}{g(x)}\left(-\hat{f}(x) + L^T x + l_r r\right)$$

Parameter update law from Lyapunov function

$$V(x; w) = x^T P x + \frac{1}{\gamma}(w - w^*)^T (w - w^*)$$

and w^* dentoes "optimal" parameters.

Not fully satisfactory.

Summing up ...

Fuzzy Logic, Systems and Control

- Incorporate heuristics into control
- Extension of logic and reasoning
- Function synthezis using rules
- Related to Feed-forward Neural Nets
- Externally: a Nonlinear Map
- Function Approximation
- Approximation-based control schemes

Laboratory Exercise

Design a fuzzy servo controller for a DC motor.

Tools:

- Fuzzy Logic Toolbox (Matlab)
- Automatic Code Generation
- Pålsjö – Real Time Implemetation

Next Thursday.

Lecture 7

- MIMO Control
- Distillation column example
- MIMO tools
  - Poles, zeros
  - Nyquist criterium
  - MFD
  - Norms, singular values

Literature

- Maciejowski Ch 2-3.9, pp. 37-102

MIMO issues

Several sensors

Several actuators

Communication between different parts

Greater requirements on safety-net, startup, integrity to errros etc

Typical control problems

Suggest control structure

Add sensors, add actuators, change process structure?

Disturbance rejection

Robustness

Tracking

Decoupling

Reliability

MIMO Synthesis

What models should one use?

How should one formulate specifications?

How should one do design?

Theory and tools for

- linear models, linear control
- continuous variables
- (small examples)

Challenge:

- nonlinear systems, control
- hybrid control
- more complex systems

Synthesis SISO-MIMO

Loop gain $L = PC$

$$y = T(y_c - n) + Sd + \dots$$

where $T = L(I+L)^{-1}$ and $S = (I+L)^{-1}$

For frequencies where loop gain L is large, (hence $I + L$ large) we have S small. Nice.

What does L "large" mean in MIMO?

Stability and the point -1

Multivariable Nyquist

det $(P(i\omega)C(i\omega))$ should encircle -1 the correct number of times:

Sum of encirclements of $\lambda_i(P(i\omega)C(i\omega))$ should equal number of unstable poles

Characteristic loci : $\lambda_i(P(i\omega)C(i\omega))$

Gain-phase relationship, MIMO

$$\sum_i \arg\, \lambda_i(G(i\tilde{\omega})) = \sum_i \int_{-\infty}^{\infty} \log|\lambda_i(G(i\tilde{\omega}))| W(\tilde{\omega})\, d\tilde{\omega}$$

Mixes the char. loci (?)

Inherent problem with Nyquist techniques

Why?

Use singular values or other norms

A high purity distillation column

Ref: Morari Robust Process Control

$$P(s) = \frac{1}{75s+1}\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}$$

$$Y = T(s)Y_r = PC(I+PC)^{-1}Y_r$$

Desired : $T \approx I$, good robustness, etc

A Simple Design

Dynamic decoupling

$$C(s) = \frac{0.7}{s}(P(s))^{-1}$$

$$L(s) = P(s)C(s) = \frac{0.7}{s}I$$

This gives closed loop

$$T = L(I+L)^{-1} = \frac{1}{s/0.7+1}I$$

Decoupled first order systems

Time constants 1.4s

Evaluation, simulation

Step responses

Nice step responses, decoupled

Nyquist Curves

Good robustness (?)

Evaluation on real plant

Enormous interactions, 500% overshoot

Why?

Distillation column, explanation

Assume some actuator imperfections

$$P_0(s) = P(s)\begin{pmatrix} 1.2 & 0 \\ 0 & 0.8 \end{pmatrix} = \frac{1}{75s+1}\begin{pmatrix} 1.054 & -0.691 \\ 1.298 & -0.877 \end{pmatrix}$$

Gives

$$P_0(s)C(s) = \frac{0.7}{s}\begin{pmatrix} 14.83 & -11.06 \\ 17.29 & -12.83 \end{pmatrix}$$

Change by more than 1000%

Directional gains

One large gain input direction and one small.

Physical explanation:

Problem with Nyquist techniques

$L(i\omega) = P(i\omega)C(i\omega)$ CAN give totally irrelevant indications of sensitivity and robustness

Distillation column

Ex 2

$$\begin{aligned} sy_1 &= u_1 + bu_2 \\ sy_2 &= \epsilon u_1 + u_2 \end{aligned}$$

Char. eq. with $u = -y$: $(s+1)^2 - b\epsilon$

Unstable if $\epsilon b > 1$, very sensitive if b large

Not seen in char. loci

Nyquist plot for $\epsilon = 0$.

Use singular values instead of $\lambda_i(i\omega)$

MIMO tools – Ch 2

Theorem (Smith-Mc Millan Form)
$P(s)$ rational matrix. Can find row and column operations $U(s), V(s)$ so

$$P(s) = U(s)\text{diag}\left\{\frac{\epsilon_1(s)}{\psi_1(s)}, \frac{\epsilon_2(s)}{\psi_2(s)}, \ldots, \frac{\epsilon_r(s)}{\psi_r(s)}, 0, \ldots, 0\right\} V(s)$$

det $U(s)$ and det $V(s)$*unimodular*
$\{\epsilon_i(s), \psi(s)\}$ monic, coprime for each i
$\epsilon_i(s) \mid \epsilon_{i+1}(s)$
$\psi_{i+1}(s) \mid \psi_i(s)$

Poles: $\psi_i(s) = 0$

Transmissions zeros: $\epsilon_i(s) = 0$

Mc-Millan degree= $\sum_i \deg \psi_i(s)$

Example 3

$$\begin{pmatrix} \frac{1}{s+1} & \frac{2}{s+3} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{pmatrix} = U(s) \begin{pmatrix} \frac{1}{(s+1)(s+3)} & 0 \\ 0 & \frac{s-1}{s+1} \end{pmatrix} V(s)$$

Hence: three poles, one (non-min phase) zero, third order system

Example 4

$$\begin{pmatrix} \frac{s-1}{(s+1)^2} & \frac{5s+1}{(s+1)^2} \\ \frac{-1}{(s+1)^2} & \frac{s-1}{(s+1)^2} \end{pmatrix} = U(s) \begin{pmatrix} \frac{1}{(s+1)^2} & 0 \\ 0 & \frac{s+2}{s+1} \end{pmatrix} V(s)$$

Hence: three poles, one zero, third order system

Matrix Fraction Descriptions(MFD)

$$G(s) = \underbrace{\bar{A}^{-1}(s)\bar{B}(s)}_{\text{left-fraction}} = \underbrace{B(s)A^{-1}(s)}_{\text{right-fraction}}$$

$A(s), B(s)$ not unique:

$$B(s)A(s)^{-1} = (B(s)X(s))(A(s)X(s))^{-1}$$

Common factors

$A(s), B(s)$ "right coprime" iff

$$\begin{pmatrix} A(s) \\ B(s) \end{pmatrix} = \begin{pmatrix} A_0(s) \\ B_0(s) \end{pmatrix} U(s) \quad \Rightarrow \quad U(s) \text{ unimodular}$$

$\bar{A}(s), \bar{B}(s)$ "left coprime" similarly

Polynomial design

Right-fraction plant:

$$Y = B\underbrace{A^{-1}U}_{\xi}$$

Left-fraction controller

$$RU = -SY + TY_r$$

$$\begin{aligned} RA\xi = RU &= -SB\xi + TY_r \\ Y &= B(RA + SB)^{-1}TY_r \end{aligned}$$

Not easy to choose R, S, T

Internal Stability

Definition The feedback system is internally stable iff the transfer functions from $e_1(s), e_2(s)$ to $u_1(s), u_2(s)$ are all asymptotically stable

Remark : Enough to look on $u_1(s) \to e_2(s)$ if $C(s)$ stable.

Multivariable Nyquist

Assumption: No "right-half plane cancellations" in forming $L(s) = P(s)C(s)$

Feedback loop is stable iff

$$\det\,(I + P(s)C(s)) = 0$$

has no roots in the RHPL. But

$$\det\,(I + P(s)C(s)) = \prod(1 + \lambda_i(P(s)C(s)))$$

Therefore count the number of anti-clockwise encirclements of

$$\lambda_i\,(P(s)C(s))$$

around -1. Should equal the number of unstable poles of $P(s)C(s)$.

Why no contradiction with diagonal structure?

Example 5

Mac. page 61

$$G(s) = \frac{0.8}{(s+1)(s+2)} \begin{pmatrix} s-1 & s \\ -6 & s-2 \end{pmatrix}$$

Stable if $k \in [-1.89, 1.25)$ or $k \in (2.5, \infty]$

Inverse Nyquist criterium

Count the number of anti-clockwise encirclements of

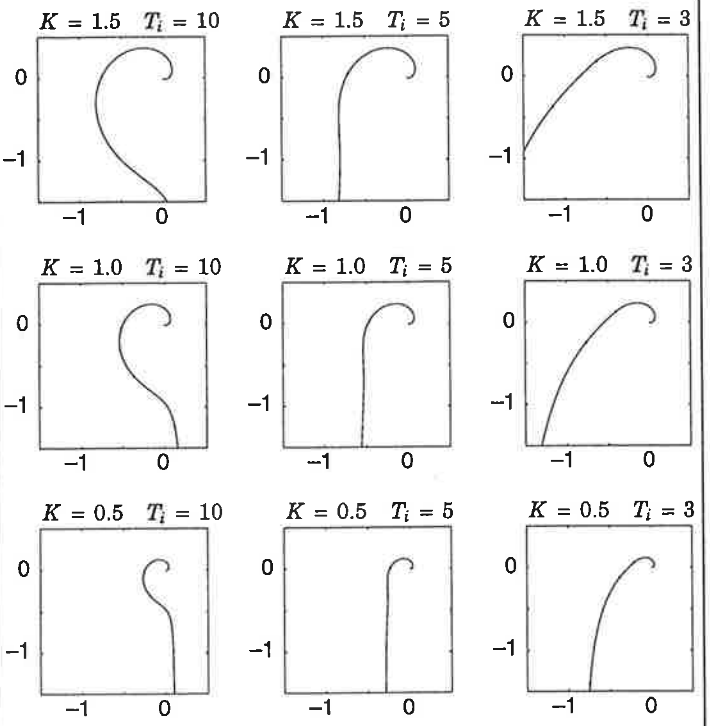
$$\lambda_i^{-1}\,(P(s)C(s))$$

around -1. Should equal the number of RHPL transmission zeros of $P(s)C(s)$.

(Do not forget the large and small semicircles)

Sometimes easier to apply

Tuning maps

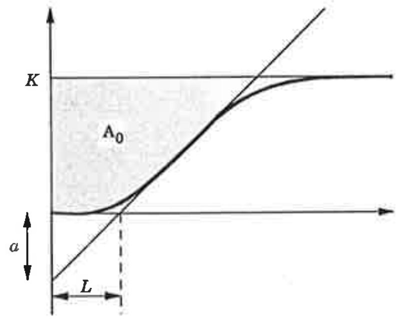


Specifications

- Load disturbance attenuation
- Set-point following
- Measurement noise (K_{hf})
- Sensitivity, Robustness

Desirable to have a design variable

Ziegler-Nichols' step response method



Design criterion: Decay ratio 0.25

Two parameters: a and L

Controller	K	T_i	T_d
P	$1/a$		
PI	$0.9/a$	$3L$	
PID	$1.2/a$	$2L$	$0.5L$

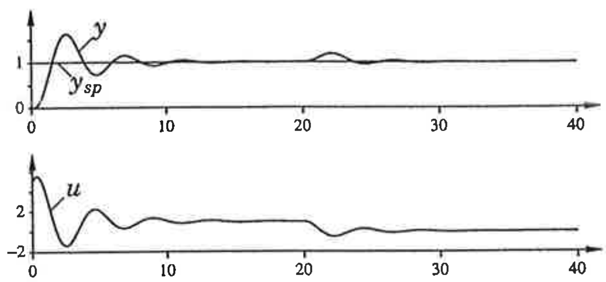
Example: ZN step response method

Process:

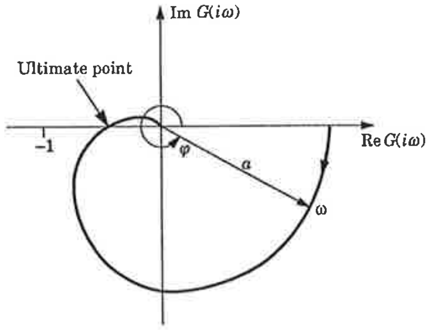
$$G(s) = \frac{1}{(s+1)^3}$$

Controller:

$$K = 5.50$$
$$T_i = 1.61$$
$$T_d = 0.403$$



Ziegler-Nichols' frequency response method



Two parameters: K_u and T_u

Design criterion: Decay ratio 0.25

Controller	K	T_i	T_d
P	$0.5K_u$		
PI	$0.4K_u$	$0.8T_u$	
PID	$0.6K_u$	$0.5T_u$	$0.125T_u$

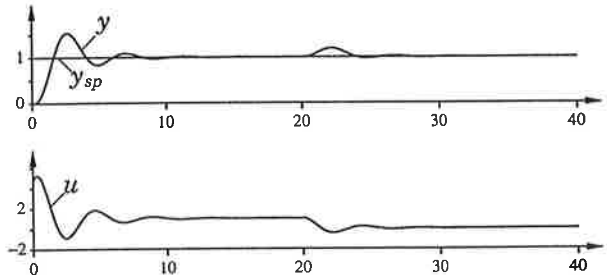
Example: ZN frequency response method

Process:

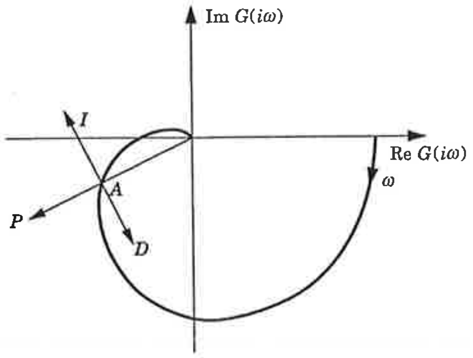
$$G(s) = \frac{1}{(s+1)^3}$$

Controller:

$$K = 4.8$$
$$T_i = 1.81$$
$$T_d = 0.44$$



Interpretation - Nyquist diagram



Interpretation - Nyquist diagram

P:

$$G_c(i\omega_u) = K_u 0.5$$

PI:

$$G_c(i\omega_u) = K_u(0.4 - 0.08i)$$

Phase decreases 11.2°

PID:

$$G_c(i\omega_u) = K\left(1 + i\left(\omega_u T_d - \frac{1}{\omega_u T_i}\right)\right)$$
$$\approx K_u(0.6 + 0.28i)$$

Phase advance 25°

Modified Ziegler-Nichols method

Move the point

$$A = G_p(i\omega_0) = r_a e^{i(\pi+\phi_a)}$$

to the new point

$$B = G_\ell(i\omega_0) = r_b e^{i(\pi+\phi_b)}$$

using a controller with

$$G_c(i\omega_0) = r_c e^{i\phi_c}$$

Solution:

$$r_c = \frac{r_b}{r_a}$$
$$\phi_c = \phi_b - \phi_a$$

Modified Ziegler-Nichols method

PI:

$$K = \frac{r_b \cos(\phi_b - \phi_a)}{r_a}$$
$$T_i = \frac{1}{\omega_0 \tan(\phi_a - \phi_b)}$$

PID:

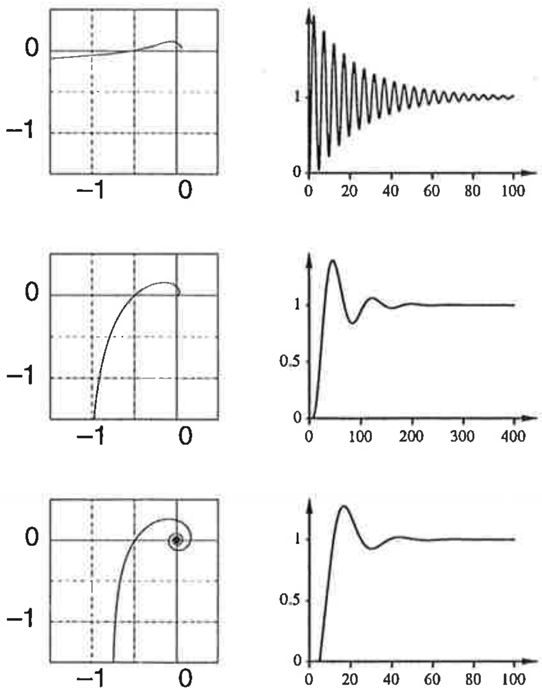
With a fixed relation between T_i and T_d

$$T_d = \alpha T_i$$

we get

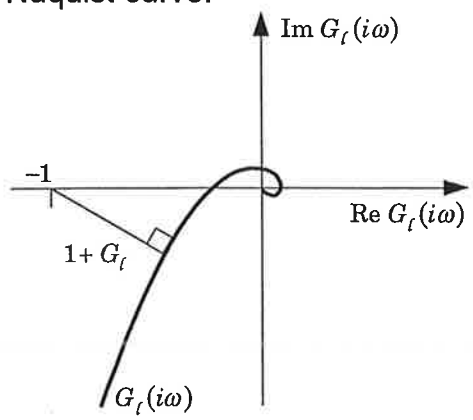
$$K = \frac{r_b \cos(\phi_b - \phi_a)}{r_a}$$
$$T_i = \frac{1}{2\alpha\omega_0}\left(\tan(\phi_b - \phi_a) + \sqrt{4\alpha + \tan^2(\phi_b - \phi_a)}\right)$$
$$T_d = \alpha T_i$$

Problems with determining only one point



Loop shaping

Use the third parameter to adjust the slope of the Nuquist curve:



Example: Loop shaping

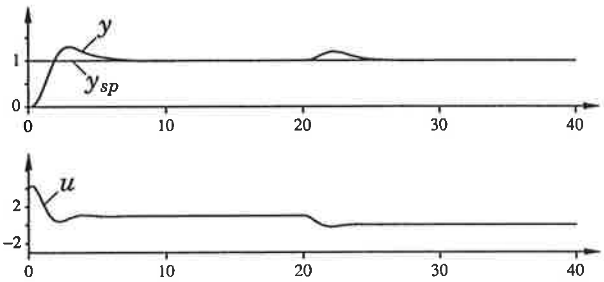
Process:

$$G(s) = \frac{1}{(s+1)^3}$$

$$r_b = \frac{1}{\sqrt{2}} \qquad \phi_b = 45°$$

Controller:

$$K = 4$$
$$T_i = 1.9$$
$$T_d = 0.75$$



Analytical tuning methods

Specify the closed loop transfer function

$$G_0 = \frac{G_p G_c}{1 + G_p G_c}$$

Solving this equation for G_c we get

$$G_c = \frac{1}{G_p} \cdot \frac{G_0}{1 - G_0}$$

Example: IMC

Warning: Pole-zero cancellation!

Modulus and Symmetrical Optimum

Idea:

Make the transfer function between r and y as close to one as possible for low frequencies.

Ensure that $G(0) = 1$ and make $d^n|G(i\omega)|/d\omega^n = 0$ at $\omega = 0$ for as many n as possible.

Modulus optimum

Consider

$$G(s) = \frac{a_2}{s^2 + a_1 s + a_2}$$

$$|G(i\omega)|^2 = \frac{a_2^2}{a_2^2 + \omega^2(a_1^2 - 2a_2) + \omega^4}$$

Choosing $a_1 = \sqrt{2a_2}$ gives

$$|G(i\omega)|^2 = \frac{a_2^2}{a_2^2 + \omega^4}$$

The first three derivatives of $|G(i\omega)|$ will vanish at the origin.

$$G(s) = \frac{\omega_0^2}{s^2 + \sqrt{2}\omega_0 s + \omega_0^2}$$

$$G_\ell(s) = \frac{G(s)}{1 - G(s)} = \frac{\omega_0^2}{s(s + \sqrt{2}\omega_0)}$$

Modulus Optimum design: Try to obtain G_ℓ

Symmetrical optimum

Consider

$$G(s) = \frac{a_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

If $a_1^2 = 2a_2$ and $a_2^2 = 2a_1 a_3$, five derivatives of $|G(i\omega)|$ will vanish at $\omega = 0$.

$$G(s) = \frac{\omega_0^3}{(s+\omega_0)(s^2+\omega_0 s+\omega_0^2)}$$

With error feedback:

$$G_\ell(s) = \frac{\omega_0^3}{s(s^2+2\omega_0 s+2\omega_0^2)}$$

With $b = 0$:

$$G_\ell = \frac{\omega_0^2(2s+\omega_0)}{s^2(s+2\omega_0)}$$

Symmetrical Optimum design: Try to obtain G_ℓ

Pole Placement

PI control of first-order process:

Process:

$$G_p(s) = \frac{K_p}{1+sT}$$

Desired characteristic polynomial:

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

Solution:

$$K = \frac{2\zeta\omega_0 T - 1}{K_p}$$
$$T_i = \frac{2\zeta\omega_0 T - 1}{\omega_0^2 T}$$

Pole Placement

PID control of second-order process:

Process:

$$G_p = \frac{K_p}{(1+sT_1)(1+sT_2)}$$

Desired characteristic polynomial:

$$(s+\alpha\omega_0)(s^2+2\zeta\omega_0 s+\omega_0^2) = 0$$

Solution:

$$K = \frac{T_1T_2\omega_0^2(1+2\alpha\zeta)-1}{K_p}$$
$$T_i = \frac{T_1T_2\omega_0^2(1+2\alpha\zeta)-1}{T_1T_2\alpha\omega_0^3}$$
$$T_d = \frac{T_1T_2\omega_0(\alpha+2\zeta)-T_1-T_2}{T_1T_2\omega_0^2(1+2\alpha\zeta)-1}$$

Model reduction

Poles and zeros that are much slower than ω_0 are approximated with integrators.

Poles and zeros close to ω_0 are retained.

Poles and zeros that are much faster than ω_0 are neglected or approximated by a fast pole or zero.

Approximation of fast modes

Consider

$$G(s) = \frac{K(1+sT_1)(1+sT_2)}{(1+sT_3)(1+sT_4)(1+sT_5)(1+sT_6)} e^{-sL}$$

where

$$T = T_3 + T_4 + T_5 + T_6 - T_1 - T_2 - L > 0$$

It is assumed that $L \ll T$.

The transfer function G can be approximated by

$$G(s) = \frac{K}{1+sT}$$

Approximation of fast and slow modes

Consider

$$G(s) = \frac{K(1+sT_1)(1+sT_2)}{(1+sT_3)(1+sT_4)(1+sT_5)(1+sT_6)} e^{-sL}$$

where

$$T_3 > T_4 > T_5 > T_6$$
$$T_5 > \max\,(T_1, T_2, L)$$

Assumption:

$$\frac{1}{T_4} < \omega_0 < \frac{1}{T_5}$$

Approximations:

$$\frac{1}{1+sT_3} \approx \frac{1}{sT_3}$$
$$T = T_6 - T_1 - T_2 - L$$

If T is positive:

$$G(s) = \frac{K}{sT_3(1+sT_4)(1+sT_5)(1+sT)}$$

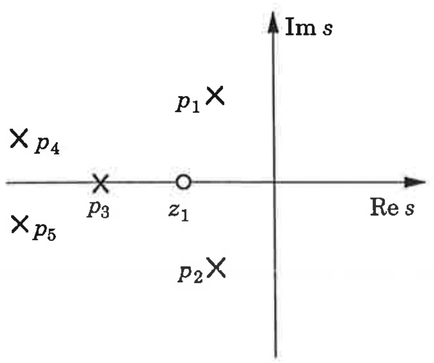
If T is negative:

$$G(s) = \frac{K(1+sT)}{sT_3(1+sT_4)(1+sT_5)}$$

Dominant Pole Design

Place the dominant poles.

Ensure that they are dominant.



Dominant Pole Design

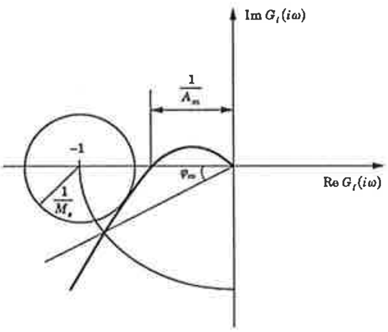
- Load disturbance attenuation

$$IE = \int_0^\infty e(t)\,dt = \frac{T_i}{K}$$

- Set-point following

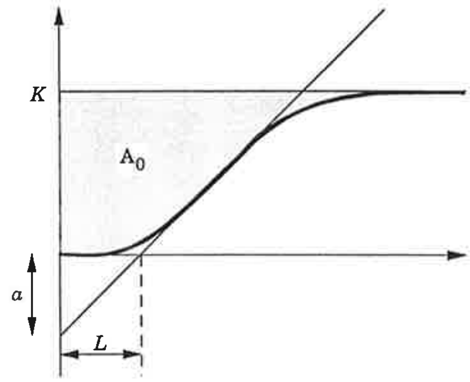
$$u = K\left(by_{sp} - y + \frac{1}{T_i}\int edt - T_d\frac{dy}{dt}\right)$$

- Sensitivity M_s



Ziegler-Nichols' methods

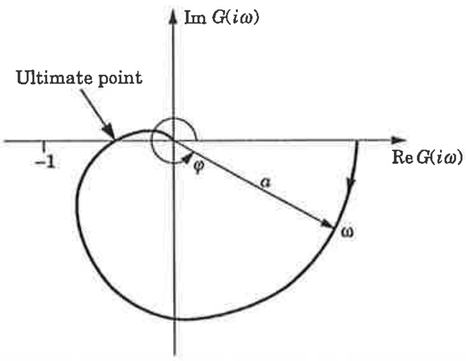
The Step Response Method



Two parameters: a and L

Ziegler-Nichols' methods

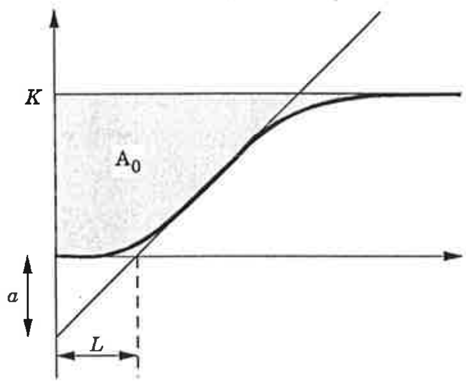
The Frequency Response Method



Two parameters: K_u and T_u

KT–Tuning

The Step Response Method

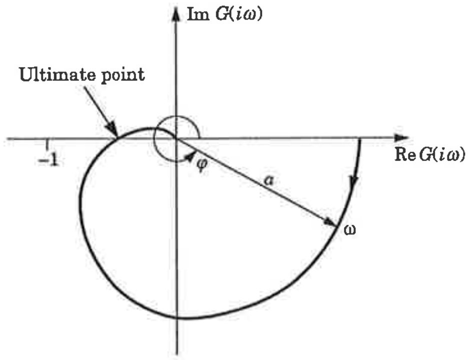


Three parameters: a, L, and K_p

Normalized dead-time: $\tau = \frac{L}{L+T} = \frac{a}{a+K_p}$

KT–Tuning

The Frequency Response Method



Three parameters: K_u, T_u, and K_p

Gain ratio: $\kappa = |\frac{G(i\omega_u)}{G(0)}| = \frac{1}{K_p K_u}$

KT–Tuning

The Test batch

$$G_1(s) = \frac{e^{-s}}{(1+sT)^2}$$

$$G_2(s) = \frac{1}{(s+1)^n}$$

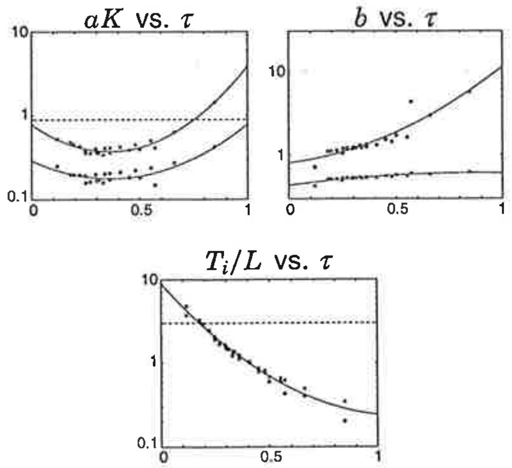
$$G_3(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)}$$

$$G_4(s) = \frac{1-\alpha s}{(s+1)^3}$$

Not included: $G(s) = \frac{e^{-s}}{1+sT}$

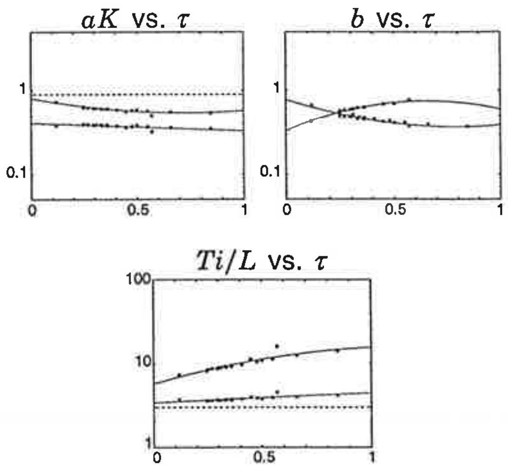
Also integrating processes

PI – Stable Processes



- Compare with Ziegler-Nichols
- We need three parameters

PI – Integrating Processes

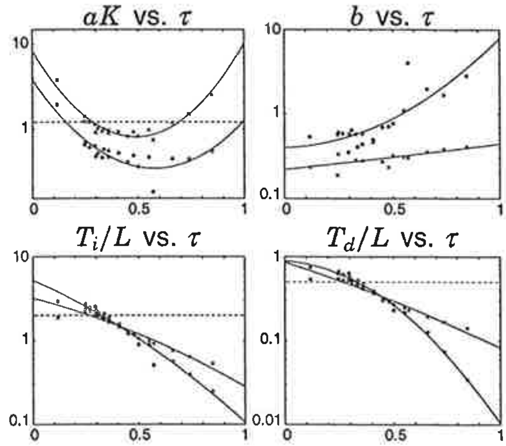


KT–Tuning

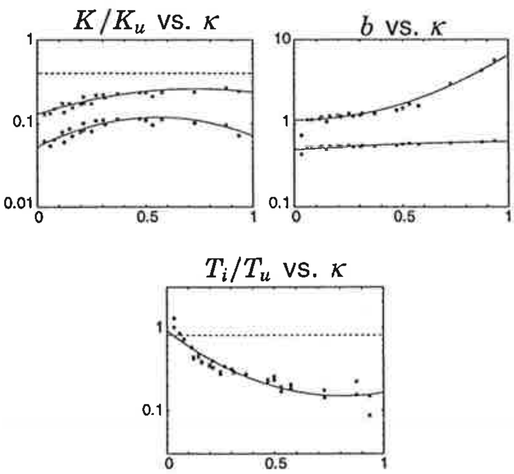
$$f(\tau) = a_0 e^{a_1\tau + a_2\tau^2}$$

	$M_s = 1.4$			$M_s = 2.0$		
	a_0	a_1	a_2	a_0	a_1	a_2
aK	0.29	-2.7	3.7	0.78	-4.1	5.7
T_i/L	8.9	-6.6	3.0	8.9	-6.6	3.0
b	0.81	0.73	1.9	0.44	0.78	-0.45

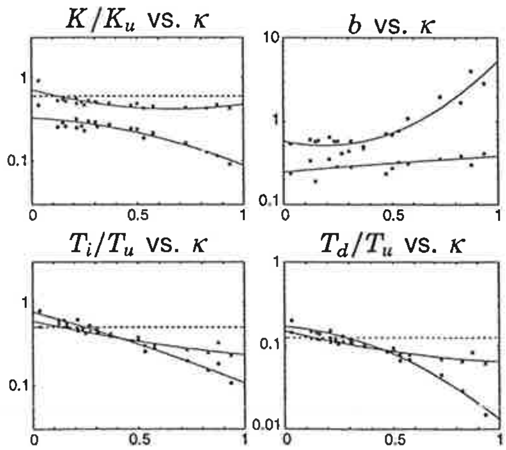
PID – Stable Processes



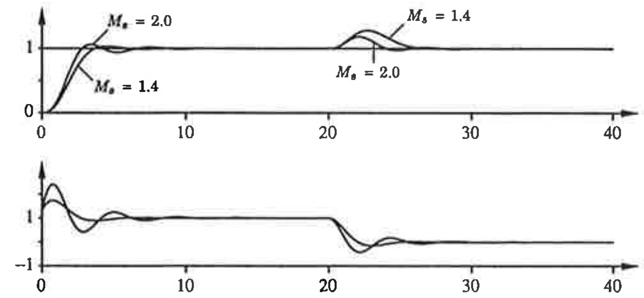
PI – Stable Processes



PID – Stable Processes

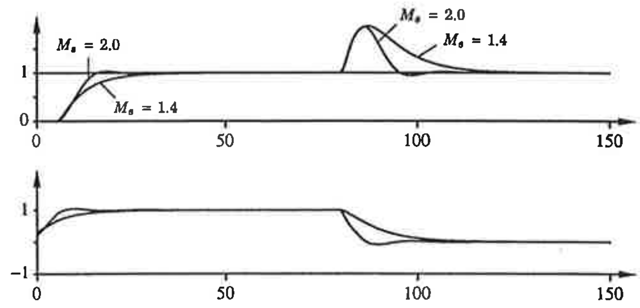


Example 1



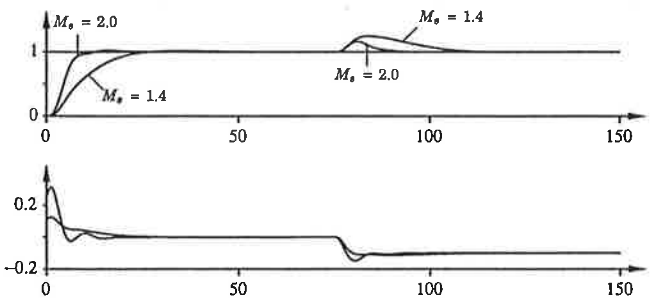
$$G(s) = \frac{1}{(s+1)^3}$$

Example 2



$$G(s) = \frac{e^{-5s}}{(s+1)^3}$$

Example 3



$$G(s) = \frac{1}{s(s+1)^3}$$

Conclusions

Dominant Pole Design

- Gives good control
- Requires $G(s)$

With two parameters

- We have to compromise
- We can do better than Ziegler-Nichols

With three parameters

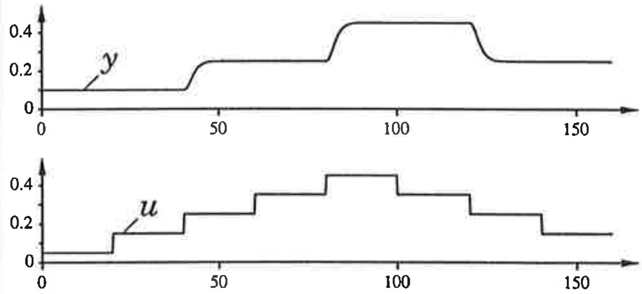
- KT-tuning is almost as good as Dominant Pole Design

Before you start tuning ...

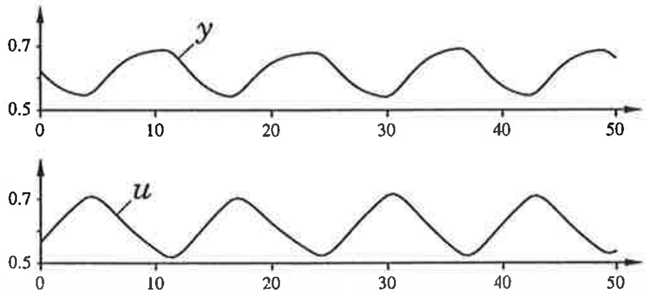
investigate the process!

- Are there any scaling factors?
- Are they constant?
- Are there any filters?
- Cascade control – watch out for windup!
- Controller series or parallel?
- Friction or hysteresis?

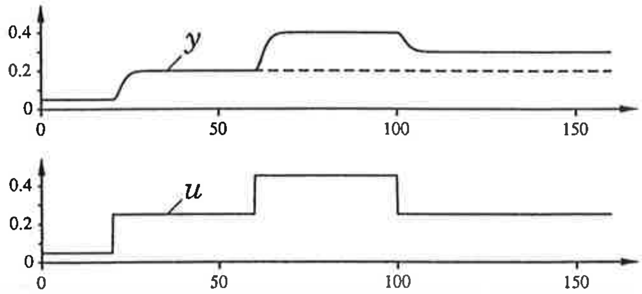
Friktionskontroll



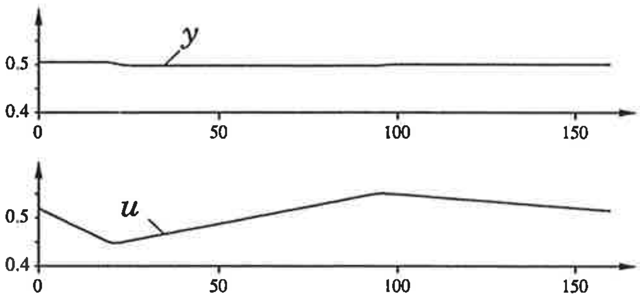
Stick-slip motion



Hystereskontroll



Reglering med hysteres



Nyquist-array methods

1970s methods

DNA (direct Nyquist array):

Make $G(s)K_0(s)$ column dominant for interesting frequencies

Design diagonal K_1 so GK_0K_1 has nice Geshgorin bands

INA (inverse Nyquist array):

Make $K_0^{-1}(s)G^{-1}(s)$ row dominant

Design diagonal K_1 so $K_1^{-1}K_0^{-1}G^{-1}$ has nice "Ostrowski bands"

Claim: Easier to predict influence of interaction on closed loop with INA (?)

Illustrative example, INA

3 slides showing a succesful (?) INA design

4.8 AIRC example revisited

Fig 4.23: Not column dominant

Fig 4.25 $G(s)K_b(s)$ column dominant

Fig 4.26-4.31 Design diagonal PI-controllers $K_c(s)$, 3 SISO designs

Fig 4.32 Nice char. loci $\lambda_i(G(i\omega)K(i\omega))$

Fig 4.33 Nice closed loop singular values

Fig 4.34 Nice step response

Fig 4.36-37 Problems with input disturbance

Controller cancels badly damped process poles

Several plots are missing (control signal, ...)

4.10 Relative Gain Array (RGA)

Measure of ineraction, Bristol 1966

g_{ij} open loop transfer function

h_{ij} transfer function when $y_k \equiv 0$, $k \neq i$

$$\gamma_{ij}(s) := \frac{g_{ij}(s)}{h_{ij}(s)}$$

Easy to prove that (exercise)

$$\Gamma = G(s) \times (G^{-1})^T$$

($\times$ stands for elementvise mult.)

Used to choose pairings of inputs and outputs for diagonal control

Normally used only at $s = 0$.

RGA

Nice if $\Gamma = I$

Several problems if Γ has large or negative elements

Large RGA $\Rightarrow$ large condition number

$$\kappa(G) \geq 2\|\Gamma\|_\infty - 1$$

If $\gamma_{jj}(G) < 0$ then for any diagonal $K(s)$ either a) closed loop is unstable, b) loop j is unstable if all other loops open or c) closed loop is unstable if loop j is removed

See Robust Process Control, Morari for more details

Linear Quadratic Control, LQG

See LQG course for details

- Introduction
- The H_2 norm
- Formula for the optimal LQG controller
- Software
- Example

Ch. 5
lqgbox in Matlab

History

Computers

50-60s: Use optimization to find "optimal controller"

Newton, Gould, Kaiser (1957):

In place of a relatively simple statement of the allowable error, the analytical design procedure employs a more or less elaborate performance index. The objective of the performance index is to encompass in a single number a quality measure for the performance of the system.

Optimization based approach

"Optimal" controller

Absolute scale of merit

Limits of performance

"Euphoria" in late 60s

Classical article: "Good, Bad, Optimal"

LQG Theory

Wiener–Kolmogorov

Kalman–Bucy

Wonham, Willems, Anderson, Åström, Kucera, and MANY others

Still many research papers each year

Why so Popular?

The first "automized" design method

Space program, Aircraft design

Good models

Stabilizing

LQ-control $u = -Lx$ gives

- $[1/2, \infty]$-gain margin
- $60\,\mathrm{deg}$ phase margin

Robustness

LQ-control robust

$\hat{x}$ Kalman filter robust (dual)

Output feedback ($u = -L\hat{x}$) NOT necessarily robust.

Attention turned to "robust" control

Use frequency weights

Check robustness

Norms of Systems

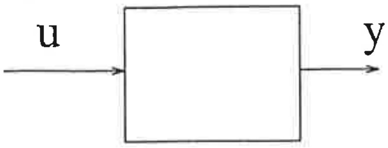
$$
\begin{aligned}
Y &= G(s)U \\
y &= g \star u \\
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{aligned}
$$

The L_2-norm (LQG-norm):

$$
\begin{aligned}
\|G\|_2^2 &= \sum_i \sum_j \int_{-\infty}^{\infty} |g_{ij}(t)|^2 dt = \\
&= \sum_i \sum_j \int_{-\infty}^{\infty} |G_{ij}(j\omega)|^2 d\omega / 2\pi = \\
&= \int_{-\infty}^{\infty} \text{trace } G^*(j\omega) G(j\omega) d\omega / 2\pi =
\end{aligned}
$$

H_2: As L_2 but with $G(s)$ stable also.

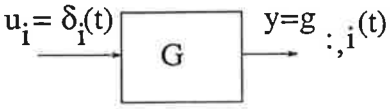
Interpretation of the H_2-norm



u: stationary white noise, mean zero

$$E(u(\tau_1)u(\tau_2)^T) = \delta(\tau_1 - \tau_2)I$$

then $E(y^T y) = \|G\|_2^2$



$$\|G\|_2^2 = \sum_{i=1}^{m} \|G\delta_i\|_2^2$$

"Energy in impulse response"

Proof

$$E(y^T y) = E(\text{tr } yy^T) =$$

$$= \text{tr} \int_{-\infty}^{\infty} g(t-\tau_1)u(\tau_1)d\tau_1 \int_{-\infty}^{\infty} u(\tau_2)^T g^T(t-\tau_2)d\tau_2$$

$$= \text{tr} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t-\tau_1)u(\tau_1)u^T(\tau_2)g^T(t-\tau_2)d\tau_1 d\tau_2$$

$$= \text{tr} \int_{-\infty}^{\infty} g(t-\tau_1)g^T(t-\tau_1)d\tau_1$$

$$= \|G\|_2^2$$

Alternative

$$E(\text{tr } yy^T) = \text{tr} \int S_y(\omega)d\omega/2\pi =$$

$$= \int \text{tr } G^*(j\omega)S_u(\omega)G(j\omega)d\omega/2\pi$$

"Variance of output with white noise in"

How to compute the H_2 norm

1) Residue calculus

$$\|G\|_2^2 = \sum_{i,j} \frac{1}{2\pi i} \oint G_{ij}(-s)^T G_{ij}(s)ds$$

2) Recursive formulas ala Åström-Jury-Schur

3) If $G(s) = C(sI - A)^{-1}B$ then

$$\|G\|_2^2 = \text{trace } (CPC^T) = \text{trace } B^T SB$$

where P is the unique solution to the Lyapunov equation

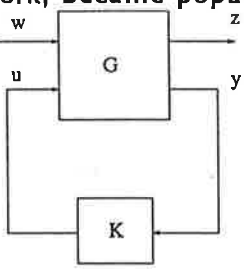
$$AP + PA^T + BB^T = 0$$

and S solves

$$SA + A^T S + C^T C = 0$$

The Standard Problem

Unified framework, became popular in 80s



$u =$ Control Inputs

$y =$ Measured Outputs

$$w = \text{Exogenous Inputs} = \begin{cases} \text{Fixed commands} \\ \text{Unknown commands} \\ \text{Disturbances} \\ \text{Noise} \\ \vdots \end{cases}$$

$$z = \text{Regulated Outputs} = \begin{cases} \text{Tracking Errors} \\ \text{Control Inputs} \\ \text{Measured Outputs} \\ \text{States} \\ \vdots \end{cases}$$

The H_2 Problem

Closed Loop

$$u = K(s)y$$
$$z = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}w = T_{zw}w$$

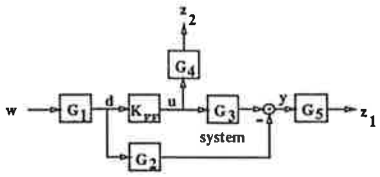
The H_2 problem:

Find $K(s)$ such that the closed loop is stable and

$$\min_{K(s)} \|T_{zw}\|_2$$

is obtained.

Example, Optimal Feedforward



Output

$$y = G_3u - G_2d$$

d is a measurable signal $d = G_1w$

Feedforward regulator

$$u = K_{FF}\, d$$

Minimize a mean square of filtered outputs and filtered control signals:

$$\min \; E(z_1^Tz_1 + z_2^Tz_2)$$

$$G = \begin{pmatrix} \begin{pmatrix} -G_5G_2G_1 \\ 0 \end{pmatrix} & \begin{pmatrix} G_5G_3 \\ G_4 \end{pmatrix} \\ G_1 & 0 \end{pmatrix}$$

The Optimal Controller

Let the system be given by

$$\dot{x} = Ax + B_1w + B_2u$$
$$z = C_1x + \qquad D_{12}u$$
$$y = C_2x + D_{21}w + D_{22}u$$

under some technical conditions the optimal controller is of order n and is given by

$$u = -L\hat{x}$$
$$\dot{\hat{x}} = A\hat{x} + B_2u + K(y - C\hat{x} - D_{22}u)$$
$$L = (D_{12}^TD_{12})^{-1}(D_{12}^TC_1 + B_2^TS)$$
$$K = (B_1D_{21}^T + PC_2^T)(D_{21}D_{21}^T)^{-1}$$

where $P \geq 0$ and $S \geq 0$ satisfy

$$0 = SA + A^TS + C_1^TC_1 - L^TD_{12}^TD_{12}L$$
$$0 = AP + PA^T + B_1B_1^T - KD_{21}D_{21}^TK^T$$

$A - B_2L \quad A - KC_2$ stable

"Technical Conditions"

1) $[A, B_2]$ stabilizable

2) $[C_2, A]$ detectable

3) "No zeros on imaginary axis" $u \to z$

$$\text{rank} \begin{pmatrix} j\omega I - A & -B_2 \\ C_1 & D_{12} \end{pmatrix} = n + m \qquad \forall\omega$$

and D_{12} have full column rank.

4) "No zeros on imaginary axis" $w \to y$

$$\text{rank} \begin{pmatrix} j\omega I - A & -B_1 \\ C_2 & D_{21} \end{pmatrix} = n + p \qquad \forall\omega$$

and D_{21} have full row rank.

Software

Read about the LQGBOX in TFRT-7575

```
[K,P] = lqec(A,C,R1,R2,R12)

[L,S] = lqrc(A,B,Q1,Q2,Q12)

lr = refc(A,B,C,D,L)

[Ac,By,Byr,Cc,Dy,Dyr] = lqgc(A,B,C,D,L,lr,K)
```

lqed, lqrd, refd, lqgd in discrete time

Works reasonably well

$$\begin{pmatrix} Q_1 & Q_{12} \\ Q_{12}^T & Q_{22} \end{pmatrix} = \begin{pmatrix} C_1^T \\ D_{12}^T \end{pmatrix} \begin{pmatrix} C_1 & D_{12} \end{pmatrix}$$
$$\begin{pmatrix} R_1 & R_{12} \\ R_{12}^T & R_{22} \end{pmatrix} = \begin{pmatrix} B_1 \\ D_{21} \end{pmatrix} \begin{pmatrix} B_1^T & D_{21}^T \end{pmatrix}$$

Closed Loop

Loop Gain: $L(sI-A)^{-1}B$
Return Difference: $I + L(sI-A)^{-1}B$

Return Difference Formula

From Riccati equation:

$$M^T(-s)M(s) = \\ (I + L(-sI-A^T)^{-1}B)^T D^T D(I + L(sI-A)^{-1}B)$$

where $M(s) = D + C(sI-A)^{-1}B$

If no crossterms:

If $C^TC = Q_1$, $C^TD = 0$ and $D^TD = Q_2$

$$Q_2 + B^T(-sI-A^T)^{-1}Q_1(sI-A)^{-1}B = \\ (I + L(-sI-A^T)^{-1}B)^T Q_2(I + L(sI-A)^{-1}B)$$

$$(I + L(-sI-A^T)^{-1}B)^T Q_2(I + L(sI-A)^{-1}B) \geq Q_2$$

Scalar Case

$$q_2 \geq q_2|1 + L(sI-A)^{-1}B|^2$$

therefore

$$|1 + L(sI-A)^{-1}B| \geq 1$$

Gain Margin $[1/2, \infty]$

Phase Margin 60 degrees.

Not simultaneously. No cross-terms. All states measurable.

Gain Margin, MIMO

With

$$S = (1 + L(sI - A)^{-1}B)^{-1}$$
$$\bar{\sigma}(Q_2^{1/2} S Q_2^{-1/2}) \le 1$$

If Q_2 diagonal this gives nice MIMO gain/phase margins, see LQG course.

Robustness against nonlinearities

Circle Criterion

Stability with any nonlinear time-varying input gain with slopes in $(1/2, \infty)$.

Scalar Case, no cross terms

Introduce

$$Q_2 = \rho I$$
$$G(s) = C(sI - A)^{-1}B = B(s)/A(s)$$
$$I + H(s) = I + L(sI - A)^{-1}B = P(s)/A(s)$$

Closed loop characteristic equation $P(s) = 0$

$$Q_2 + G^T(-s)G(s) = (I + H^T(-s))Q_2(I + H(s))$$

$$\rho A(-s)A(s) + B(-s)B(s) = \rho P(-s)P(s)$$

Symmetric Root Locus

symlocc, symlocd in matlab

Cheap control $\rho \to 0$

Eigenvalues of closed loop tend to stable zeros of $B(-s)B(s)$ and the rest tend to ∞ as stable roots of

$$s^{2d} = \text{const} \cdot \rho^{-1}$$

Expensive Control $\rho \to \infty$

Eigenvalues of closed loop tend to stable zeros of $A(-s)A(s)$

Example

$$\min u^2, \quad \dot{x} = x + u$$

$A(s) = s + 1$ unstable.

$u = -2x$ gives

$$\dot{x} = -x$$

$P(s) = A(-s) = -s + 1$

High Frequency Behaviour

$$L(j\omega I - A)^{-1}B = LB/\omega = Q_2^{-1}B^TSB/\omega$$

LQ-controller gives loop gain with "roll-off" 1

Same conclusion for

$$L(j\omega I - A + BL)^{-1}B = LB/\omega = Q_2^{-1}B^TSB/\omega$$

Rules of Thumb

$$Q_1 = \text{diag}(\alpha_1, \dots, \alpha_n)$$
$$Q_2 = \text{diag}(\beta_1, \dots, \beta_m)$$

Let $\alpha_i \sim (x_i)^{-2}$ and $\beta_i \sim (u_i)^{-2}$ where x_i and u_i denote allowable sizes on state i and input i

More ideas

Punishing

$$(\dot{x}_i + \alpha x_i)^2$$

"should" give $\dot{x}_i = -\alpha x_i$.

Moving Eigenvalues

Can move one eigenvalue at a time by using

$$Q_1 = qq^T$$

where q is orthogonal to the A-invariant subspace of the rest of the modes

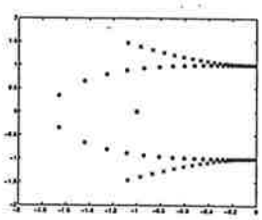
Example

$$G(s) = \frac{1}{(s+1)(s^2+1)}$$

Increase damping without moving pole in $s = -1$.

```
v =
  0.7071              0.7071            0.4082
      0 + 0.7071i         0 - 0.7071i  -0.4082
      0                   0             0.8165
d =
      0 + 1.0000i         0                  0
      0                   0 - 1.0000i        0
      0                   0            -1.0000
```

$$Q_1 = q_i q_i^T, \qquad q_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad q_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$



Example 1, p. 222, Anderson-Moore

6 state model of aircraft subject to wind gust turbulence

Two outputs y_f and y_a forward and aft accelerations

Open loop resonances at 1.5 and 21 rad/s

LQG1

$$\min E[y_f^2 + y_a^2 + 0.2u^2]$$

LQG2

$$\min E[y_f^2 + y_a^2 + 4x_3^2 + 4x_4^2 + u^2]$$

Plot control signal also !

Results, Example1

$\boxed{\text{Would not recommend the LQG2-design}}$

`/home/fulqg/lqg94/matlab/fig822.m`
(Very ugly code)

Example 2, p.232 Anderson-Moore

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u + \begin{pmatrix} 1 \\ 1 \end{pmatrix} v$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \sqrt{\sigma} w$$

$$\min E[x_1^2 + x_2^2 + \rho u^2]$$

What happens as $\rho \to 0$ and $\sigma \to 0$?

Plots of $\rho = \sigma = 1, 10^{-2}, 10^{-4}$.

Result, Example 2

$\boxed{\text{Terrible gain and phase margins}}$

/home/fulqg/lqg94/matlab/doyle.m

Exercises

Mac. 4.7, 4.9

Check the first example on p. 11 where two closed loop eigenvalues are changed without changing the third. What does the closed loop eigenvalues converge to as the control is getting cheaper? Hint: See /home/fulqg/lqg94/matlab/ex.m.

Check the turbulence example from Anderson-Moore p. 222. Compute the eigenvalues of the controller. Is it stable? Code available via WWW

Verify the formula for the H_2 norm given in the lecture

Home Problem

Evaluate the two designs done on the AIRC example in Mac 4.4 and 4.8. Plot for instance singular values for $S(i\omega)$, $T(i\omega)$, $K(i\omega)$, step responses including control signals. Study also the influence of a initial state error in x_5. You can find some code via the home page (AIRC example).

Then use LQG to find a better controller. Try to achieve

- Rise time to 90% in 1s with less than 10% interactions (compare Fig 4.14)
- Smaller control actions than the design in Sec 4.4
- Better response on state error in x_5

Cooperation is allowed

Lecture 10

- More LQG, Ch 5
  - Example, mutools H_2-box
  - Observer design
  - LQG/LTR
  - Example, Doyle/Stein
  - Example, AIRC
- Modeling of uncertainty

Readings: Maciejowski Ch 5 + Ch 3.10

An example

Use mutools H2-box

```
[k,g,norms,kfi,gfi,hamx,hamy] =
h2syn(plant,nmeas,ncon);
```

```
WWW: lqg2.m
```

Violates "technical conditions", why?

Answer

Non-stabilizable, non-detectable modes

Solution: Change $1/s$ and $1/(s^2+1)$ weights

$$\begin{pmatrix} j\omega I - A & -B_2 \\ C_1 & D_{12} \end{pmatrix}$$

looses rank in $s = 0$.

No input noise will lead to Kalman filter with $K_{opt} = 0$, which gives marginally unstable Kalman filter.

Add input noise w_3 to process.

D_{12} not full rank

New punished signal: $z_2 = \rho u$

New system

Short on stochastic differential equations

$$\dot{x} = Ax + v$$
$$y = Cx + e$$

v white noise, $\quad Ev(t)v^T(t-\tau) = R_1\delta(\tau)$

e white noise, $\quad Ee(t)e^T(t-\tau) = R_2\delta(\tau)$

State covariance

$Ex(t)x^T(t) = R(t), \qquad \dot{R} = AR + RA^T + R_1$

Kalman filter

$\tilde{x} = x - \hat{x}, \qquad E\tilde{x}(t)\tilde{x}^T(t) = P(t)$

$\dot{P} = AP + PA^T + R_1 - PC^TR_2^{-1}CP$

"Equivalent" representation of y

$$\dot{\hat{x}} = (A - KC)\hat{x} + K\epsilon$$
$$y = C\hat{x} + \epsilon$$

Reduced order observer

If no measurement noise, $R_2 = 0$

$K \to \infty$,

Can use y directly for some states

Loss of degree of filter, direct term

cf. Linear system course

Example

$$\dot{x}_1 = x_2 + v_1$$
$$\dot{x}_2 = u + v_2$$
$$y = x_1 + e$$

v_1, v_2, and e white noise

Incremental variance ρ^2, 1, and σ^2

Optimal filter as $\sigma \to 0$ is

$$\hat{x}_1 = y$$
$$\hat{x}_2 = \frac{\rho}{\rho s + 1}u + \frac{s}{\rho s + 1}y$$

$\hat{x}_2 \approx \frac{1}{s}u$ if ρ large

$\hat{x}_2 \approx sy$ if ρ small

Influence of an observer

Loop gain at (1):

$$G_1 = L(sI - A)^{-1}B$$

but at (2) (if $D_{22} = 0$)

$$G_2 = L(sI - A + B_2L + KC_2)^{-1}KC_2(sI - A)^{-1}B_2$$

Doyle: You may loose *all robustness*

-Hmm, note what happens if $K \to \infty$

LQG/LTR 1

Loop Transfer Recovery

Want to make G_2 as robust as G_1

References:

- Doyle and Stein, AC79, p. 607-611
- Doyle and Stein, AC81, p. 4-16

First LTR-method: Use fast (in a special way) observer

Sacrifice "noise optimality"

Almost like using an inverse for reconstruction

Not applicable if RHPL Zeros

LQG/LTR 1

First LTR-method: Add fictious input noise :

$$R_1 := R_1 + qB_2B_2^T$$

For square, minimum phase systems this gives $K \to \infty$ and

$$\lim_{q\to\infty} G_{LQG}(s)G(s) = L(sI - A)^{-1}B_2$$

Easy to try this idea, doesn't always lead to good designs

Dont let q go all the way to ∞

Same problem as with all designs with fast observers

LQG/LTR 2

Second LTR-method: Punish more in output direction

$$Q_1 := Q_1 + qC_2^TC_2,$$

(ie use "cheap control")

Makes loop gain approach

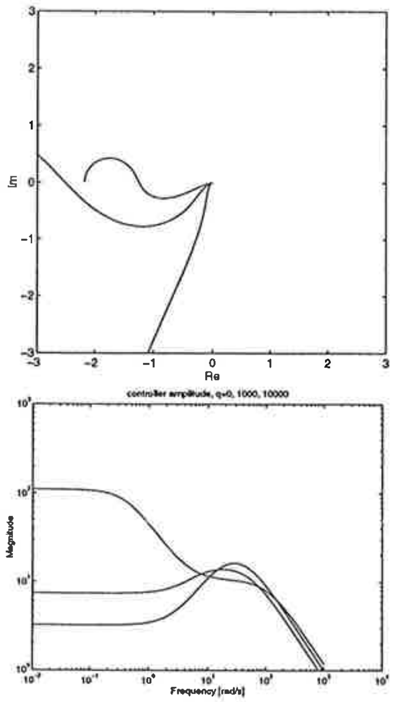
$$\lim_{q\to\infty} G_{LQG}(s)G(s) = C(sI - A)^{-1}K$$

ie the Kalman filter loop gain.

Same problem as with all "cheap control" designs

Doyle-Stein, AC-79

WWW: lqg3.m



LTR polynomial interpretation, SISO

System

$$C(sI - A)^{-1}B = \frac{B(s)}{A(s)}$$

Disturbance influence

$$C(sI - A)^{-1}B_v = \frac{B_v(s)}{A(s)}$$

and $R_1 = B_vB_v^T, \qquad R_2 = 1$

Kalman filter identity

$$\begin{aligned} &1 + C(sI - A)^{-1}R_1(-sI - A^T)^{-1}C^T \\ &\quad = \left[1 + C(sI - A)^{-1}K\right]\left[1 + C(-sI - A)^{-1}K\right]^T \end{aligned}$$

or

$$\begin{aligned} A(s)A(-s) + B_v(s)B_v(-s) &\\ &= [A(s) + K(s)][A(-s) + K(-s)] \\ &= A_o(s)A_o(-s) \end{aligned}$$

LTR polynomial interpration

LTR-modification: $R_1^{mod} = R_1 + q^2 BB^T$ gives

$$C(sI-A)^{-1}R_1^{mod}(-sI-A^T)^{-1}C^T = \frac{B_v(s)B_v(-s)+B(s)q^2B(-s)}{A(s)A(-s)}$$

and

$$\begin{aligned} A(s)A(-s) &+ B_v(s)B_v(-s) + B(s)q^2B(-s) \\ &= \left[A(s)+K^{mod}(s)\right]\left[A(-s)+K^{mod}(-s)\right] \\ &= A_o^{mod}(s)A_o^{mod}(-s) \end{aligned}$$

Looptransfer in LQ

$$L(sI-A)^{-1}B = \frac{L(s)}{A(s)}$$

Looptransfer in LQG

$$C(sI-A)^{-1}BL(sI-A+BL+KC)^{-1}K = \frac{B(s)}{A(s)}\frac{S(s)}{R(s)}.$$

LTR polynomial interpration

Now for very large q

$$A_o^{mod}(s)A_o^{mod}(-s) \approx (-s^2)^n + B(s)q^2B(-s)$$

gives

$$A_o^{mod}(s) \approx B(s)A_k(s), \quad A_k(s)A_k(-s) = b_0^{-2}((-s^2)^k + q^2)$$

where $k = \mathrm{deg}A(s) - \mathrm{deg}B(s)$.

Furthermore, the closed loop denominator is

$$A_c(s)A_o^{mod}(s) = A(s)R(s) + B(s)S(s)$$

and after considerable thought (for fixed s as $q \to \infty$)

$$R(s) \approx B(s)A_k(s), \quad S(s) \approx q\left[A_c(s) - A(s)\right] = qL(s)$$

so the loop transfer is now

$$\frac{B(s)}{A(s)}\frac{S(s)}{R(s)} \approx \frac{L(s)}{A(s)}\frac{q}{A_k(s)}\frac{B(s)}{B(s)} \approx \frac{L(s)}{A(s)}$$

and we have the nice robustness over most frequencies

Integrator 1

CCS 271-273

Extend system with integrators

$$\dot{\bar{x}} = y_m - y$$

$$\min \int x^TQ_1x + u^TQ_2u + \bar{x}^TQ_3\bar{x}$$

gives $\begin{pmatrix} L & \bar{L} \end{pmatrix}$. Kalman filter as before.

$\bar{x}$ noise-free so nonstandard LQG

(D_{21} not full rank).

Integrator 1

Use controller

$$u = -L\hat{x} - \bar{L}\bar{x} + \tilde{u}_c$$

(is this the limit as $\sigma_2 \to 0$?)

Increased order model (A_m in CCS)

Observer order (A_o in CCS) not increased

Integrator, 2

Extend system with fictious bias signals

Non-stabilizable states so nonstandard LQG

Integrator, 2

Use controller (for $D = 0$)

$$\frac{d}{dt}\hat{x} = \begin{pmatrix} A & B_v \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + K(y - \begin{pmatrix} C & 0 \end{pmatrix} \hat{x})$$

$$u = -\begin{pmatrix} L & L_{n+1} \end{pmatrix} \hat{x}$$

where L_{n+1} is chosen to cancel bias at outputs

$$C(A - BL)^{-1}(B_v - BL_{n+1}) = 0 \quad \text{(for } D = 0\text{)}$$

Integrator 2

Controller has integrating action

Proof Controller has A matrix (for $D = 0$)

$$\begin{pmatrix} A - BL - K_1C & B_v - BL_{n+1} \\ -K_2C & 0 \end{pmatrix}$$

which is singular. Hence pole at $s = 0$, i.e. integrator in controller.

Increased observer order (A_o)

Not increased model order (A_m)

Pre-specified factors in $R(s)$

These approaches can be generalized to other pre-specified modes in the controller

Change $1/s$ to a $1/R_1(s)$.

Prespecified factors in $S(s)$

Want pre-specified transmission zero of LQG-controller

Exercise

LQG, AIRC example

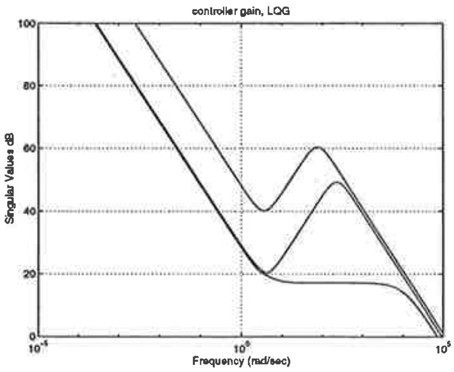
Wanted: bandwidth of 10 rad/s, integral action, well-damped responses

- Start to design Kalman filter, Guess: $R_1 = B_2 B_2^T$, $R_2 = 1$
- Introduce integrators $w = \frac{1}{s+\epsilon}\nu$
- ν colored noise: $W_3 = I + 9xx^T$
- Increase bandwidth, $W_3 := 100W_3$
- Fix $S(i\omega)$ at 5.5 rad/s, see (5.119)
- LTR, cheap control, $\rho = 10^{-8}$, fig 5.15

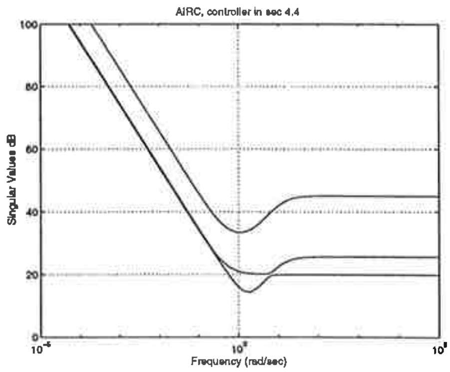
Fig 5.18 shows step responses (where is the control signal?). Compare with Fig 4.14 and 4.34

Matlab code available via WWW

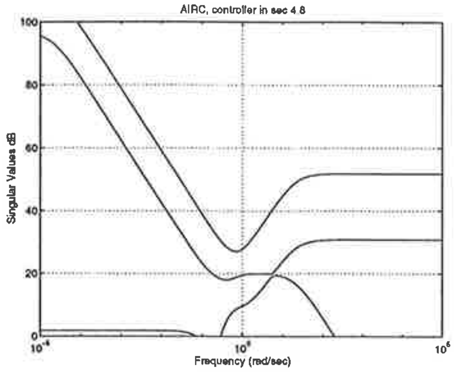
Results, AIRC LQG



Results, AIRC Sec 4.4



Results, AIRC Sec 4.8



Modeling of uncertainty

More info about "uncertainty", more accurate analysis, better control

Use structural information

But how?

Linear/nonlinear

Dynamic-static

High-frequency/low-frequency

Time-varying?, slow/fast?

Complex/Real

Much research going on. Much more left to do

Robust control – Adaptive control

Structure

Additive: $G = G_0 + \Delta$

Multiplicative: $G = G_0(1 + \Delta)$

Fractional: $G = (N + \Delta_1)/(D + \Delta_2)$

etc

Note:

Physical parameters are real

The same parameter can occur at several different places in the model

Standard representation

Fig 3.8

$$\Delta(s) = \text{diag}\{\Delta_1(s), \ldots, \Delta_n(s)\}$$

After scaling: $\|\Delta_i\|_\infty \leq 1$

Δ_i can be

- Full complex matrix
- Diagonal real matrix, $\text{diag}\{\delta_1, \ldots, \delta_1\}$
- (diagonal complex matrix) complex matrix

See examples in Fig 3.10, fig 3.12

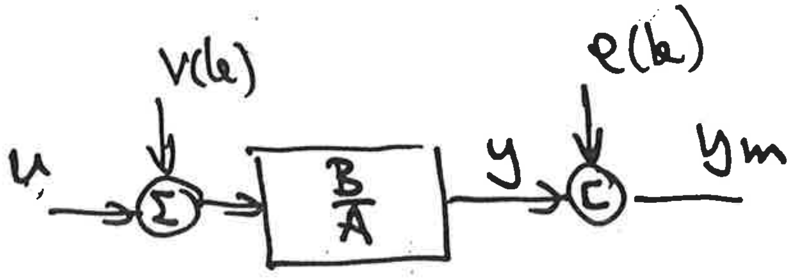
- How to deal with disturbances? (Integrators)
- How to deal with reference signals?
- Unifying approach

PROCESS DESCRIPTIONS

$$\begin{cases} x(k+1) = \Phi x(k) + \Gamma u(k) + v(k) \\ y(k) = C x(k) \end{cases}$$

State feedback $\qquad \Phi - \Gamma L$

Observers $\qquad \Phi - KC$



$$u = -\frac{S}{R} y_m + \frac{T}{R} u_c$$

$$AR + BS = A_o A_m$$

A unifying approach

$$v(k) = \Phi_{xv}\,\xi(k)$$
$$\xi(k+1) = \Phi_v\,\xi(k)$$

$$\begin{bmatrix} x(k+1) \\ \xi(k+1) \end{bmatrix} = \begin{bmatrix} \Phi & \Phi_{xv} \\ 0 & \Phi_v \end{bmatrix} \begin{bmatrix} x(k) \\ \xi(k) \end{bmatrix} + \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} u(k)$$

Generalized feedback

$$u(k) = -Lx(k) - L_v\,\xi(k)$$

Closed loop system

$$\begin{cases} x(k+1) = (\Phi - \Gamma L)\,x(k) + (\Phi_{xv} - \Gamma L_v)\,\xi(k) \\ \xi(k+1) = \Phi_v\,\xi(k) \end{cases}$$

Choose L_v such that $\Phi_{xv} - \Gamma L_v$ small
$\Phi - \Gamma L$ as usual

Problem

$$\begin{bmatrix} \hat{x} \\ \hat{\xi} \end{bmatrix} = \begin{bmatrix} (qI - \Phi + \Gamma L + KC)^{-1} & 0 \\ -\frac{K_v C}{q-1}(\quad)^{-1} & (q-1)^{-1} \end{bmatrix} \begin{bmatrix} K \\ K_v \end{bmatrix} y$$

$$u = -L\hat{x} - \hat{\xi}$$

$$u = -L\underbrace{(\quad)^{-1}K}_{H_x} y - \left(-\frac{K_v C}{q-1}(\quad)^{-1}K + \frac{K_v}{q-1}\right) y$$

$$= -LH_x y + \frac{K_v C}{q-1} H_x y - \frac{K_v}{q-1} y$$

$\Rightarrow$ Integrator in the controller !

Problem: How to interpret this controller?

POLYNOMIAL DESIGN

$$H = \frac{B}{A} \qquad H_m = \frac{B_m}{A_m} \qquad A_o$$

$$AR^0 + BS^0 = A_o A_m B^+ = A_c^0$$

$$u(k) = -\frac{S^0}{R^0} y(k) + \frac{T^0}{R^0} u_c(k)$$

How to get in an integrator?
(2nd Adaptive Control p. 122)

$$R = XR^0 + YB$$
$$S = XS^0 - YA$$

satisfy

$$AR + BS = XA_c^0$$

Want to include A_d ($A_d v = e$)
in the controller

$$R = R' A_d = \underline{X} R^0 + \underline{Y} B$$

Integral action

$X = q + x_0$ Choose x_0

$A_d = q - 1$

$$(q-1)R' = (q + x_0)R^0 + y_0 B$$

$q = 1 \Rightarrow$

$$y_0 = -\frac{(1 + x_0)R^0(1)}{B(1)}$$

$$\Rightarrow \quad R = (q+x_0)R^0 + y_0 B = (q-1)R'$$

$$S = (q+x_0)S^0 - YA$$

HOW TO GENERATE u_{ff}?

$$y_m = H_m(q)\, u_c$$

$$y = H(q)\, u$$

$$u_{ff} = \frac{H_m(q)}{H(q)}\, u_c \Rightarrow y = y_m$$

- H_m stable
- $\deg A_m - \deg B_m \geq \deg A - \deg B$
- Unstable process zeros also in the model

Special case $B_m = B$

$$H = \frac{B}{A} \qquad H_m = \frac{\beta B}{A_m}$$

$$u_{ff} = \beta \frac{A}{A_m} u_c = \beta \Big(1 + \underbrace{\frac{(a_1 - a_1^m) q^{n-1} \ldots (a_n - a_n^m)}{q^n + a_1^m q^{n-1} \ldots a_n^m}}_{u_c}$$

Assume the model in contr. form

$$\Rightarrow u_{ff} = \beta \left(u_c + C_{ff} x_m \right)$$

$$C_{ff} = [\, a_1 - a_1^m \quad \ldots \quad a_n - a_n^m \,]$$

May transform to other representations

REFERENCE SIGNALS SS

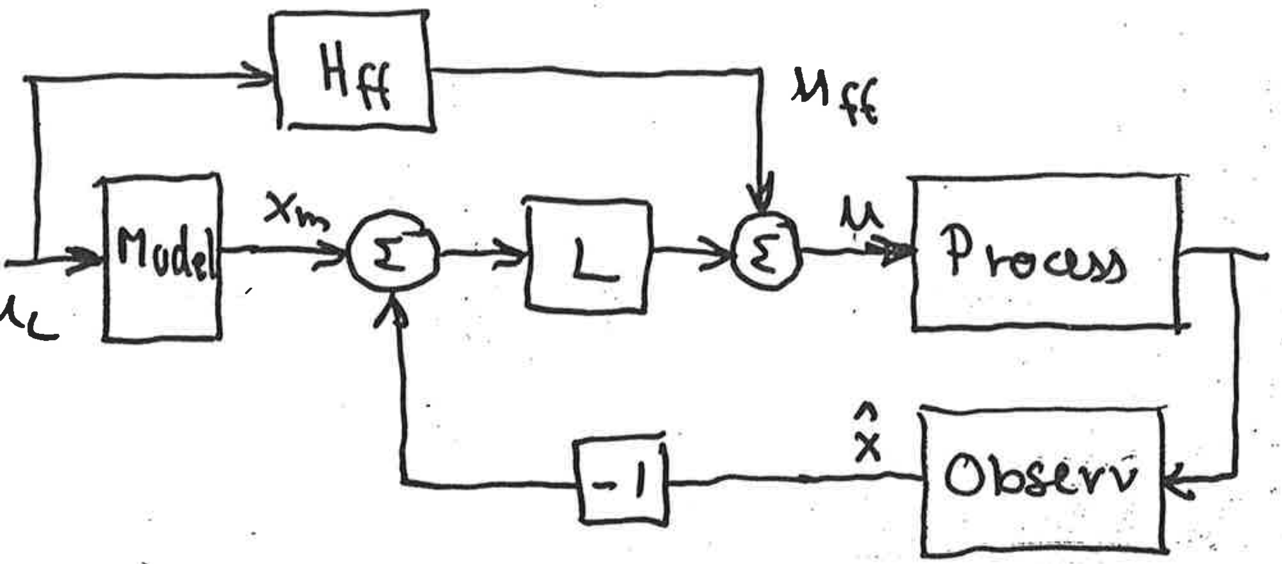
Introduce

$$x_m(k+1) = \Phi_m x_m(k) + \Gamma_m u_c(k)$$

$$y_m(k) = C_m x_m(k)$$

Assume "compatible" states

$$u(k) = \underbrace{L(x_m(k) - \hat{x}(k))}_{\text{feedback}} + \underbrace{u_{ff}(k)}_{\text{feedforward}}$$



Two-degree-of-freedom controller

Separation of servo and regulator problems

The total controller

$$u(k) = L(x_m(k) - \hat{x}(k)) - L_v \hat{\xi} +$$
$$+ \beta(u_c(k) + C_{ff} x_m(k))$$

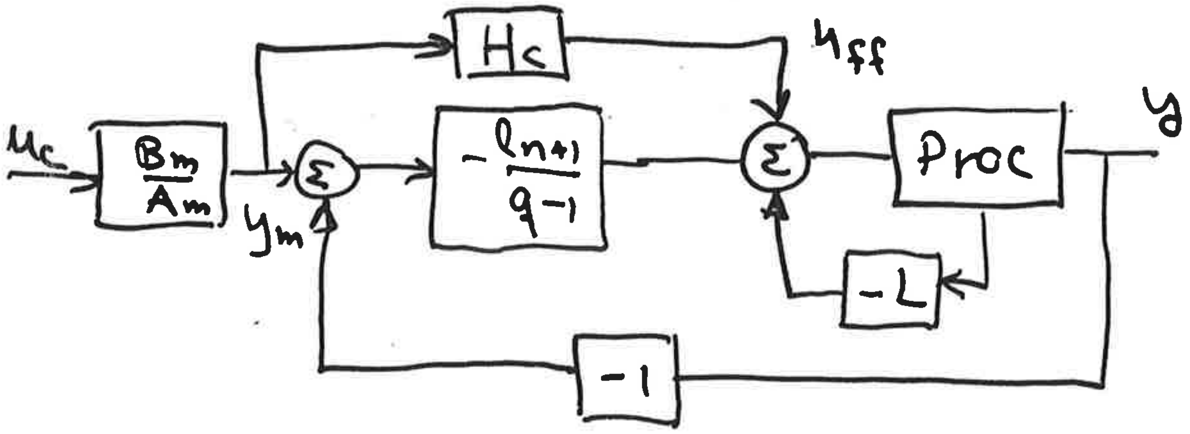
$$\begin{bmatrix} \hat{x}(k+1) \\ \hat{\xi}(k+1) \end{bmatrix} = \begin{bmatrix} \Phi & \Phi_{xv} \\ 0 & \Phi_v \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\xi} \end{bmatrix} + \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} u + \begin{bmatrix} K \\ K_v \end{bmatrix} (y - C\hat{x})$$

$$x_m(k+1) = \Phi_m x_m(k) + \Gamma_m u_c(k)$$

- What happens if there are new zeros? ($B_m \neq B$)
- Not compatible states?

ANOTHER WAY OF INTRODUCING AN INTEGRATOR

CCS 271 − 274



$$x(k+1) = \phi\, x(k) + \Gamma u(k)$$

$$x_{n+1}(k+1) = x_{n+1}(k) + y_m(k) - C x(k)$$

$$y(k) = C\, x(k)$$

Control law

$$u(k) = -L x(k) - l_{n+1} x_{n+1}(k) + u_{ff}(k)$$

$$x_{n+1}(k) = \frac{1}{q-1}(y_m - y)$$

$$x(k+1) = (\phi - \Gamma L) x(k) - \frac{\Gamma l_{n+1}}{q-1}(y_m - y) + \Gamma u_{ff}$$

$$x(k) = (qI - \phi + \Gamma L)^{-1} \Gamma \left(u_{ff} - \frac{\Gamma l_{n+1}}{q-1}(y_m - y)\right)$$

$$y = C x(k)$$

$$y(k) = \underbrace{C(qI-\Phi+\Gamma L)^{-1}\Gamma}_{\frac{B}{A_r}} \left\{ u_{ff} - \frac{l_{n+1}}{q-1}(\underbrace{y_m}_{\frac{B_m}{A_m}u_c} - y) \right\}$$

$$(A_r(q-1) - B l_{n+1}) y = B\left((q-1)u_{ff} - l_{n+1} y_m\right)$$

Choose

$$u_{ff} = H_c y_m = \frac{A_r}{B} y_m$$

$$y = \frac{A_r(q-1) - l_{n+1}B}{A_r(q-1) - l_{n+1}B} \cdot \frac{B_m}{A_m} u_c$$

$$A_r(q-1) - l_{n+1}B = 0$$

Can be given arbitrary roots iff $B(1) \neq 0$ (i.e $(q-1)$ not a factor in B)

EXAMPLE

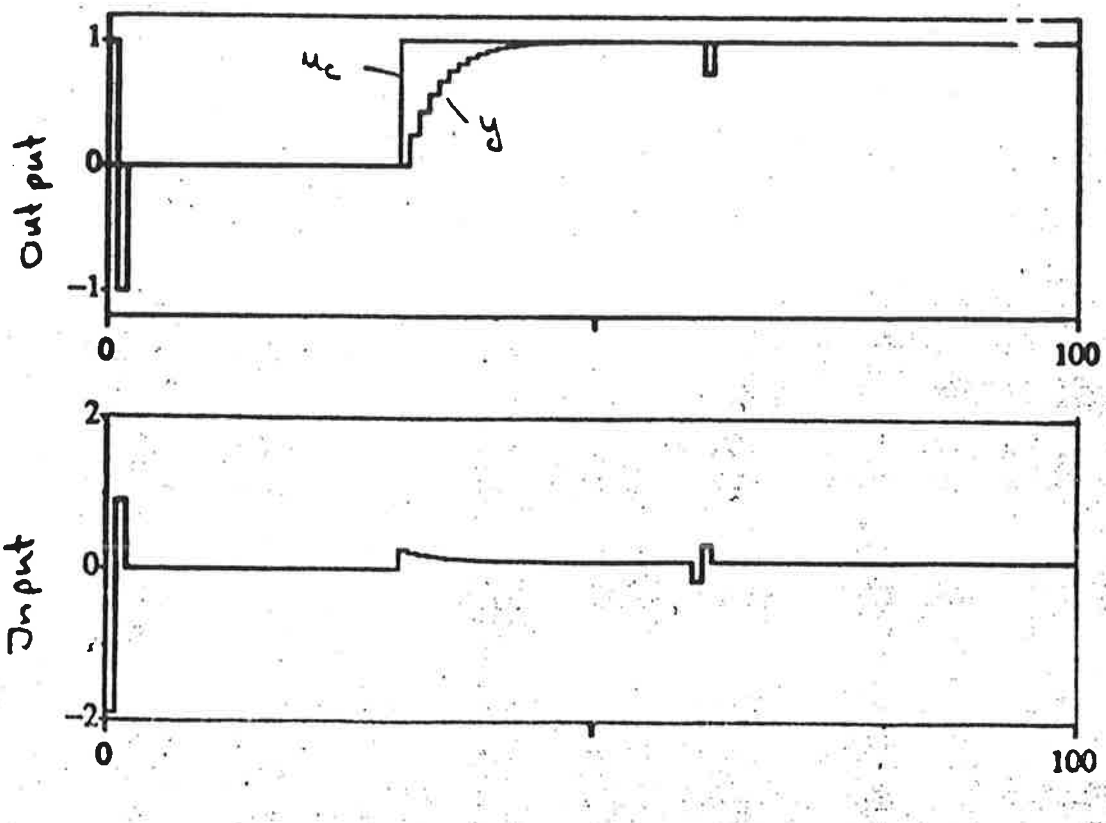
$$x(k+1) = \varphi x(k) + \gamma u(k)$$
$$y(k) = x(k)$$

$$y(k) = \frac{\gamma_m}{q-\varphi_m} \cdot \frac{(q-\varphi+\gamma l)(q-1) - \gamma l_{n+1}}{(q-\varphi+\gamma l)(q-1) - \gamma l_{n+1}} u_c(k)$$

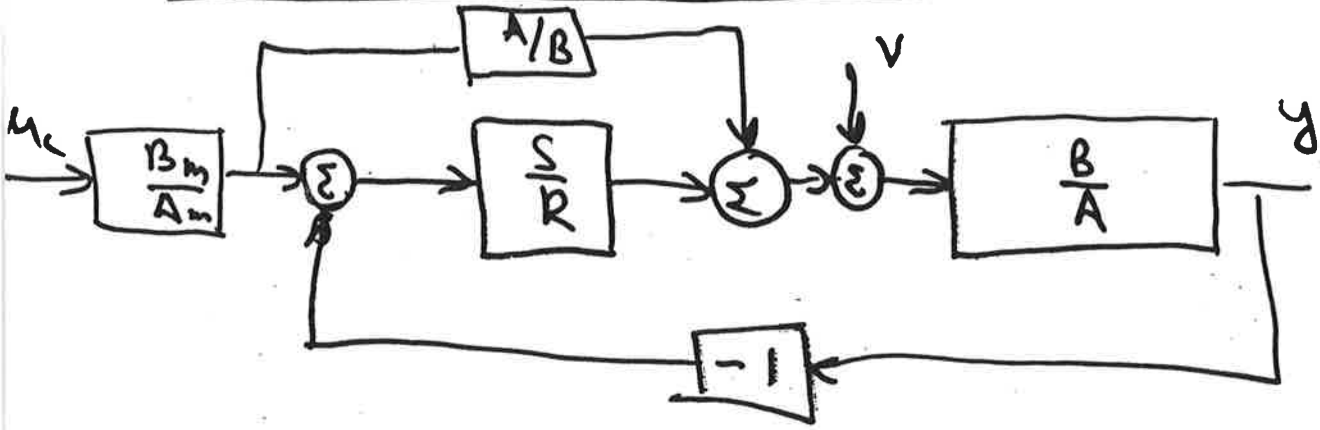
$$l = \frac{1+\varphi}{\gamma}$$

$$l_{n+1} = -1/\gamma \qquad \text{Poles in origin}$$

$$\tilde{u}_c = \frac{\gamma_m (q+1)}{q-\varphi_m} u_c$$



REFERENCE SIGNALS I/O



$$AR + BS = A_o A_r B^+$$

Separation of servo and regulator problem

$$y = \frac{B_m}{A_m} u_c + \frac{BR}{AR + BS} v$$

$$= \frac{B_m}{A_m} u_c + \frac{BR}{A_r} v$$

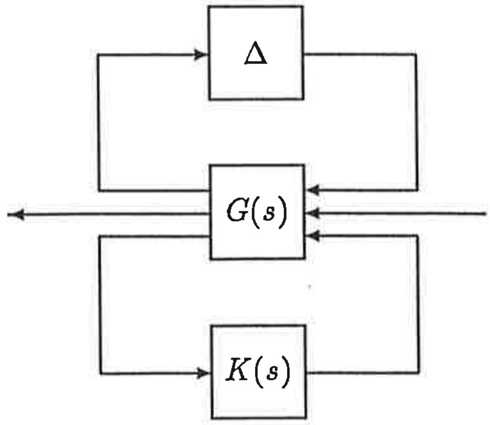
Lecture 12

- Robustness, Introduction
  - Stability Robustness
  - Performance Robustness
- Robustness Optimization (Rantzer)

Readings:

- Maciejowski ,Ch 3.11-12, skip 115, skim 118-123.
- Packard A., "Gain Scheduling via LFT", System and Control Letters, 1994, 79-92.
- Helmersson A., PhD-thesis "Methods for Robust Gain Scheduling" 1995.

The standard problem



Given knowledge of possible Δ construct controller $K(s)$ which minimizes closed loop norm.

Small-gain theorem

The closed loop system is stable for all stable Δ with

$$||\Delta|| ||G|| < 1$$

There is a destabilizing complex matrix Δ with

$$||\Delta|| ||G|| = 1$$

Structured Uncertainty

Let $BD_\delta(m_1, m_2, \ldots, m_n, k_1, k_2, \ldots, k_n)$ denote the set of block diagonal Δ with $m = \sum m_i$ blocks, each block being repeated m_i times and having dimension $k_i \times k_i$.

Example: $BD(1, 5, 1, 1, 1, 7)$

$$\Delta = \begin{pmatrix} \delta_1 & & \\ & \delta_2 I_5 & \\ & & \Delta_3 \end{pmatrix}$$

The first block has $m_1 = 1, k_1 = 1$, the second block(s) has $m_2 = 5, k_2 = 1$ the last block has $m_3 = 1, k_3 = 7$

Make sure you understand how to formulate robustness problems this way. See Example 3.2.

Structured singular values, μ

The system is destabilized iff

$$\det(I - Q_{22}(j\omega)\Delta(j\omega)) = 0$$

for some ω and $\Delta \in BD_1$.

Definition:

$$\mu(M) = \left\{ \min_{\Delta \in BD_\infty} [\sigma_1(\Delta) : \det(I - M\Delta) = 0] \right\}^{-1}$$

($\mu(M)$ is defined as 0 if $\det(I - M\Delta) \neq 0$ for all $\Delta \in BD_\infty$)

More about μ

Large M means that a small Δ can destabilize the system.

$\mu(M) = \sigma_1(M)$ if there is only one full complex block.

Generally $\mu(M) \leq \sigma_1(M)$

μ is not a norm

How to compute μ

Hard to compute exactly

Even harder to find the optimal controller that minimizes μ.

Note that μ only concerns stability

Lower and upper bounds on μ

$$\max_U \rho(UM) = \mu(M) \leq \inf_D \sigma_1(DMD^{-1})$$

where U is any unitary matrix and D is any matrix which commutes with all $\Delta \in BD$ (ie $\Delta D = D\Delta$)

The left hand side is a convex optimization problem.

Numerical solution, μ-box in matlab

(Mac. unnecessarily restricts D to (3.141)?)

Performance Robustness

Previous discussion only concerned stability

Trick: introduce extra Δ_0-block

See Fig 3.18

Theorem 3.7: $\|Q\|_\infty < 1$ for all $\Delta \in BD_1$ same as $\|Q\|_\mu < 1$ for all $(\Delta_0, \Delta) \in \widetilde{BD}_1$

The μ-box in Matlab

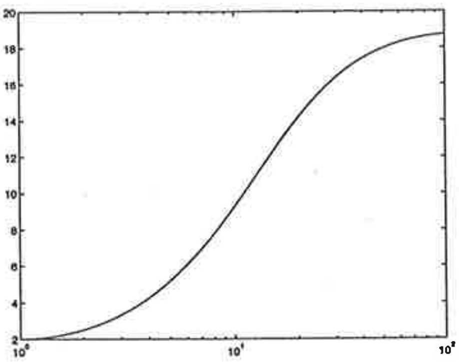
See example 3.5 p. 127

```
A=[0 0 ; 0 0];
B=[10 9 ; 9 8];
C=eye(2);
D=zeros(2);
process=pck(A,B,C,D);

alfa=1;
W1=nd2sys([1 1],[alfa 0],1);
W1=daug(W1,W1);

T=0.001;
W2=nd2sys([1 1],[alfa*T alfa],1);
W2=daug(W2,W2);

KA=[];KB=[];KC=[];KD=[0.118 1 ; 1 -0.118];
controller = pck(KA,KB,KC,KD);
```

```
systemnames = 'process W1 W2 controller';
inputvar = '[pert0(2);pert1(2)]';
outputvar = '[W1; W2]';

input_to_process='[pert1-controller]';
input_to_W1='[process]';
input_to_W2='[-controller]';
input_to_controller='[pert0+process]';
sysoutname = 'ex35';
cleanupsysic = 'yes';
echo off;
sysic;

omega=logspace(0,2,40);
%define uncertainty and performance blocks
blk=[2 2;2 2];

clp_ex35=frsp(ex35,omega);
[bnds1,dvec1,sens1,rp1]=mu(clp_ex35,blk);
vplot('liv,m',bnds1);
```

Warning: Might contain an error

Results ?



Exercises

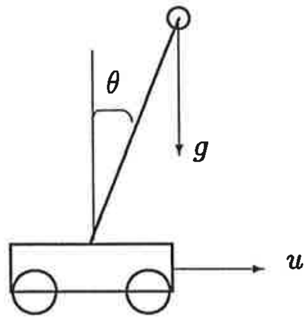
Do the second part of Exercise 3.11 with the help of the μ-toolbox in matlab.

Prove the lower and upper bounds on μ given in the lecture.

Robustness Optimization

- Example: Inverted Pendulum
- H_∞-optimization
- Robust Performance
- Gain Scheduling
- Pendulum Revisited

Example — Inverted Pendulum



$$\frac{d^2\theta}{dt^2} = \sin\theta + u\cos\theta$$

Rotating pendulum

$$\frac{d^2\theta}{dt^2} = \sin\theta(1 + c\dot{\Omega}^2\cos\theta) + u\cos\theta$$

Global Linearization

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}(u + w + e)$$

$$\theta = x_1$$

$$w = \left(\frac{\sin\theta}{\theta} - 1\right)\theta + (\cos\theta - 1)u$$

$e =$ process noise

Karl's Nonlinear Pendulum Observer

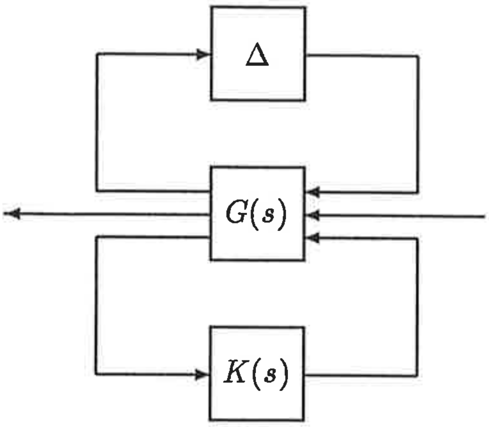
Pendulum equations

$$\frac{d}{dt}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}x_2\\ \sin x_1 + u\cos x_1\end{bmatrix}$$

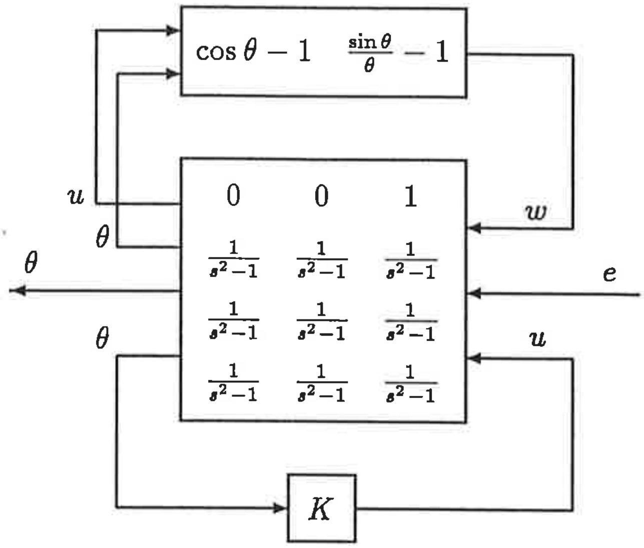
Observer equations

$$\frac{d}{dt}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} = \begin{bmatrix}\hat{x}_2\\ \sin\hat{x}_1 + u\cos\hat{x}_1\end{bmatrix} + \begin{bmatrix}k_1\\ k_2\end{bmatrix}(x_1 - \hat{x}_1 + n)$$

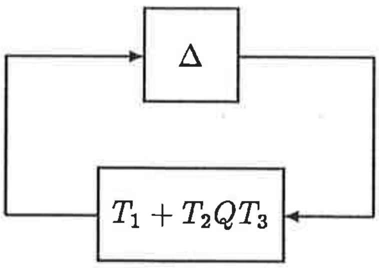
General Synthesis Setup



Pendulum Diagram



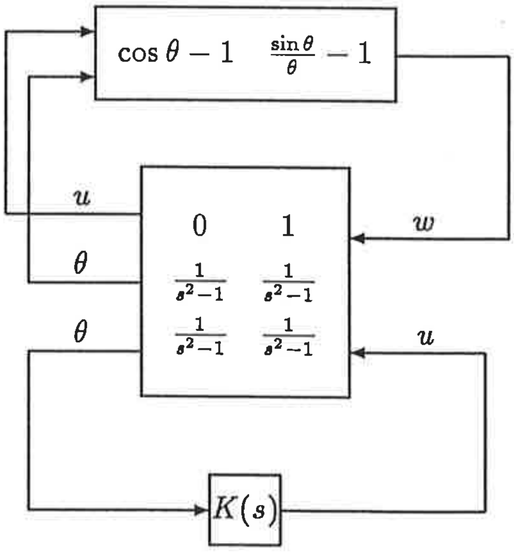
Optimization of Stability Robustness —Unstructured Uncertainty



$$\min_{Q} \|T_1 + T_2 Q T_3\|_\infty$$

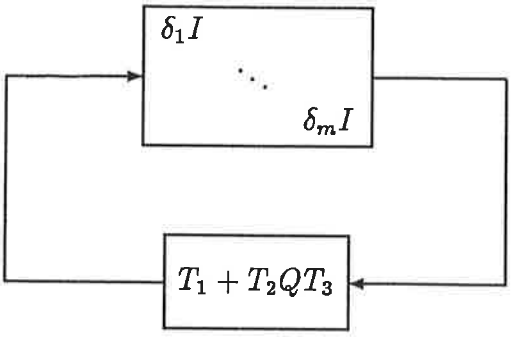
H_∞-optimization!

Robust Pendulum Stability



Minimize gain from w to (θ, u)!

Optimization of Robust Performance



$$D = \begin{bmatrix} D_1 & & 0 \\ & \ddots & \\ 0 & & D_m \end{bmatrix}$$

$$\min_{D,Q} \|D(T_1 + T_2 Q T_3) D^{-1}\|_\infty$$

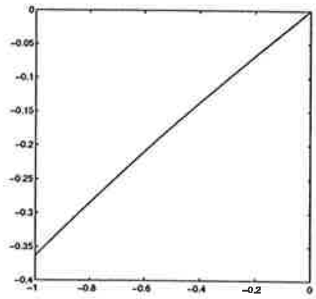
Non-convex!
Hard in general.

Gain Bound on Perturbation

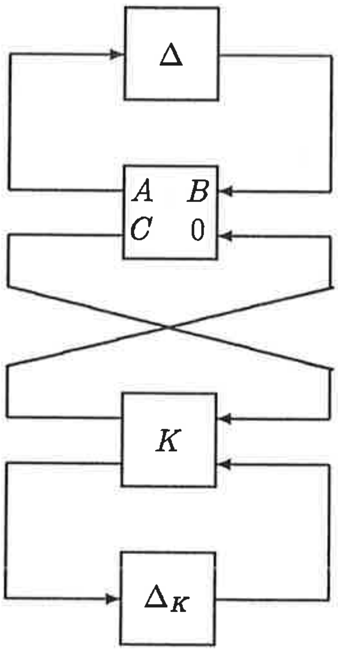
$$|\cos\theta - 1| \le 1$$

$$\left|\frac{\sin\theta}{\theta} - 1\right| \le 1$$

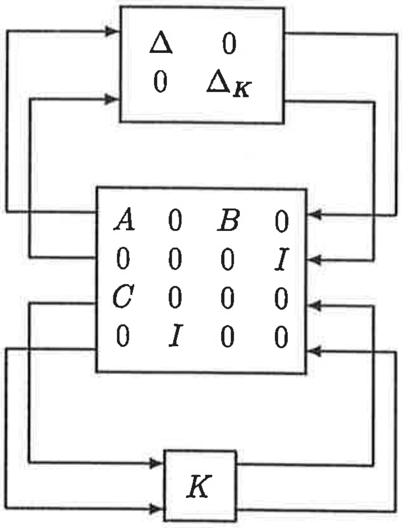
Plot $\frac{\sin\theta}{\theta} - 1$ versus $\cos\theta - 1$:



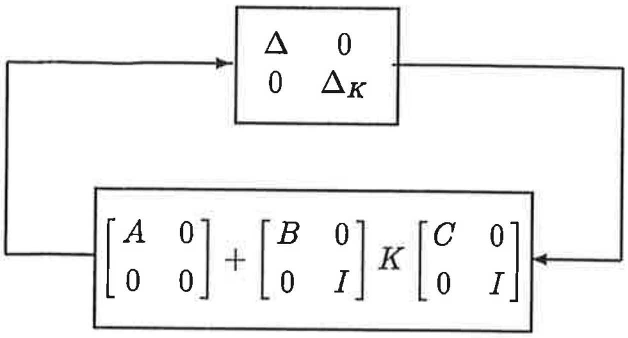
Gain Scheduling Setup



Redrawn Scheduling Setup



Optimization of Gain Schedule



$$\min_{D,K} \left\| D \left(\begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} K \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \right) D^{-1} \right\|_\infty$$

Convex (LMI) optimization if $\Delta_K = \Delta$!

Δ_K larger than Δ does not improve!

Performance block not needed in Δ_K!

Theorem

The robust performance criterion is achievable iff there exist block structured $X > 0$, $Y > 0$, compatible with Δ, such that

$$B_\perp^T \left(A \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} A^T - \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \right) B_\perp < 0$$

$$C_\perp \left(A^T \begin{bmatrix} Y & 0 \\ 0 & I \end{bmatrix} A - \begin{bmatrix} Y & 0 \\ 0 & I \end{bmatrix} \right) C_\perp^T < 0$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0$$

Main Lemma

Consider $Q, U, V, U_\perp, V_\perp$ such that $[U \quad U_\perp]$ and $\begin{bmatrix} V \\ V_\perp \end{bmatrix}$ are invertible, $U^* U_\perp = 0$ and $V V_\perp^* = 0$.

Then

$$Q + UKV + (UKV)^* < 0$$

is solvable for K if and only if

$$U_\perp^* Q U_\perp < 0$$
$$V_\perp Q V_\perp^* < 0$$

Pendulum Model Revisited

$$\begin{bmatrix} w \\ x_1 \\ x_2 \end{bmatrix} = \overbrace{\begin{bmatrix} \cos\theta - 1 & \frac{\sin\theta}{\theta} - 1 & 0 & 0 \\ 0 & 0 & s^{-1} & 0 \\ 0 & 0 & 0 & s^{-1} \end{bmatrix}}^{\Delta} \begin{bmatrix} v_1 \\ v_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ \dot{x}_1 \\ \dot{x}_2 \\ z \\ y \end{bmatrix} = \underbrace{\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \omega_0 & 0 & 0 \\ \omega_0 & \omega_0 & 0 & 0 & \omega_0 \\ 0 & 1 & 0 & -1 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \end{array}\right]}_{\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}} \begin{bmatrix} w \\ x_1 \\ x_2 \\ r \\ u \end{bmatrix}$$

Pendulum Controller

$$\begin{bmatrix} u \\ \hat{v}_1 \\ \hat{v}_2 \\ \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = K \begin{bmatrix} y \\ \hat{w} \\ \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{w} \\ \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} \cos\theta - 1 & \frac{\sin\theta}{\theta} - 1 & 0 & 0 \\ 0 & 0 & s^{-1} & 0 \\ 0 & 0 & 0 & s^{-1} \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix}$$

Conclusions

- H_∞-optimization for stability
- Gain scheduling mathematically tractable
- Copy nonlinearities in controller
- Several open problems:
  Observer interpretation
  Copy saturations, hysteresis, etc.
  Other performance measures

Lecture 13

- Robust Control 2
  - H_∞
  - μ, D(G)K-iteration
- Model Reduction

Readings: Maciejowski, Ch 6
Green,Limebeer copies

The H_∞ norm

$$\|G\|_\infty = \sup_{u \neq 0} \frac{\|Gu\|_2}{\|u\|_2} = \sup_{\|u\|_2 = 1} \|Gu\|_2$$

Lemma

$$\|G\|_\infty = \sup_\omega (\sigma_1(G(j\omega)))\,.$$

How to compute the H_∞ norm

First method: Grid ω.

Second, more theoretical, method: Given $G(s) = C(sI - A)^{-1}B$ with A stable and $\gamma > 0$. Compute

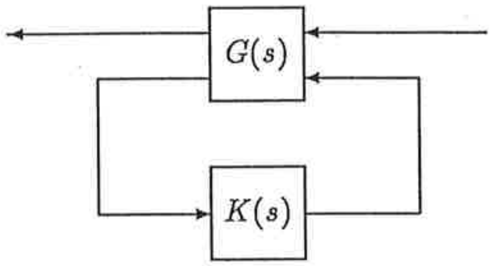
$$G_\gamma = \begin{pmatrix} A & \gamma^{-1}BB^T \\ -\gamma^{-1}C^TC & -A^T \end{pmatrix}$$

$$\|G\|_\infty < \gamma \Leftrightarrow \quad G_\gamma \text{ has no imaginary eigenvalues}$$
$$\Leftrightarrow \exists X > 0 : \begin{pmatrix} A^TX + XA + C^TC & XB \\ B^TX & -\gamma^2 I \end{pmatrix} < 0.$$

Linear Matrix Inequality

mu-box: hinfnorm(sys,ttol)

H_∞ control



Open loop

$$z = G_{11}(s)w + G_{12}(s)u$$
$$y = G_{21}(s)w + G_{22}(s)u$$

$$\dot{x} = Ax + B_1w + B_2u$$
$$z = C_1x + D_{11}w + D_{12}u$$
$$y = C_2x + D_{21}w + D_{22}u.$$

H_∞ control

Closed loop $u = K(s)y$

$$z = T_{zw}(s)w = (G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21})w$$

Find $K(s)$ that minimizes:

$$\min_K \|T_{zw}\|_\infty.$$

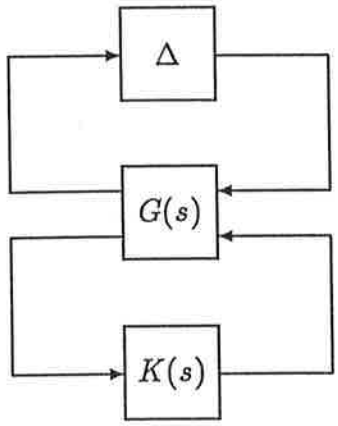
Suboptimal problem: Find $K(s)$ (if possible) so that

$$\|T_{zw}\|_\infty < \gamma$$

Easier

Can then iterate on γ

Small-gain theorem



Closed loop is stable for all stable Δ with

$$\|\Delta\|_\infty \|G\|_\infty < 1$$

Δ full complex matrix

"Technical Conditions"

Same as for LQG

1) $[A, B_2]$ stabilizable

2) $[C_2, A]$ detectable

3) "No zeros on imaginary axis" $u \to z$

$$\text{rank} \begin{pmatrix} j\omega I - A & -B_2 \\ C_1 & D_{12} \end{pmatrix} = n + m \qquad \forall \omega$$

and D_{12} have full column rank.

4) "No zeros on imaginary axis" $w \to y$

$$\text{rank} \begin{pmatrix} j\omega I - A & -B_1 \\ C_2 & D_{21} \end{pmatrix} = n + p \qquad \forall \omega$$

and D_{21} have full row rank.

Solution to H_∞ suboptimal control

Theorem There is a controller $K(s)$ such that $\|T_{zw}\|_\infty < \gamma$ if and only if the Riccati equations

$$0 = XA + A^TX + C_1^TC_1 - X\left(B_2B_2^T - \gamma^{-2}B_1B_1^T\right)X$$
$$0 = AY + YA^T + B_1B_1^T - Y\left(C_2^TC_2 - \gamma^{-2}C_1^TC_1\right)Y$$

have positive definite solutions X and Y such that $\gamma^2Y^{-1} - X > 0$ and such that $A - (B_2B_2^T - \gamma^{-2}B_1B_1^T)X$ and $A^T - (C_2^TC_2 - \gamma^{-2}C_1^TC_1)Y$ are stable.

Central Controller

One such controller ("the central") is then given by

$$
\begin{aligned}
\dot{\hat{x}} &= A\hat{x} + B_2 u + \gamma^{-2} Y C_1^T C_1 \hat{x} + Y C_2^T (y - C_2 \hat{x}) \\
u^* &= -B_2^T X Z \hat{x} \\
Z &= (I - \gamma^{-2} Y X)^{-1}.
\end{aligned}
$$

Equivalent form

$$
\begin{aligned}
\dot{\check{x}} &= A\check{x} + B_2 u + \gamma^{-2} B_1 B_1^T X \check{x} + Z Y C_2^T (y - C_2 \check{x}) \\
u^* &= -B_2^T X \check{x}.
\end{aligned}
$$

The forms are connected through $\check{x} = Z\hat{x}$. If $w = w^*, u = u^*, \check{x}(0) = x(0)$ then $\check{x}(t) = x(t)$ so $\check{x}$ has the interpretation of a state estimate in that case.

Remark: The LQG controller is obtained by letting $\gamma \to \infty$.

Software

Robust control toolbox:

[SS_CP,SS_CL,HINFO,TSS_K]=
HINF(TSS_P,SS_U,VERBOSE)

Mu-box:

```
function [k,g,gfin,ax,ay,hamx,hamy] =
hinfsyn(p,nmeas,ncon,gmin,gmax,tol,
ricmethd,epr,epp) ,
```

```
[k1,g1,gf1]=hinfsyn(himat_ic,2,2,0.8,6,0.05,2);
Test bounds:     0.8000 ;  gamma  <=     6.0000

 gamma    hamx_eig  xinf_eig  hamy_eig   yinf_eig    nrho_xy   p/f
  6.000   2.3e-02   1.2e-07   2.3e-02    0.0e+00     0.0626     p
  3.400   2.3e-02   1.3e-07   2.3e-02    0.0e+00     0.2020     p
  2.100   2.3e-02   1.3e-07   2.3e-02    0.0e+00     0.5798     p
  1.450   2.3e-02   1.4e-07   2.3e-02    0.0e+00     1.4678#    f
  1.750   2.3e-02   1.4e-07   2.3e-02    0.0e+00     0.8961     p
  1.683   2.3e-02   1.4e-07   2.3e-02    0.0e+00     0.9885     p
  1.636   2.3e-02   1.4e-07   2.3e-02   -3.0e-14     1.0619#    f
  1.674   2.3e-02   1.4e-07   2.3e-02    0.0e+00     1.0025#    f
```

Warning

When $\gamma \to \gamma_{opt}$ bad things can happen. Numerical problems, high-gain regulators, controller order reduction.

This can be an indication of a bad problem formulation.

For instance, minimizing $\|S\|_\infty$ usually leads to infinite-gain controllers

Important to have a good Riccati solver

Example, AIRC

Ch 6.8, p 306

$$
\text{Minimize} \quad \left\| \begin{pmatrix} W_1 S \\ W_2 T \end{pmatrix} \right\|_\infty
$$

$$
W_1(s) = \frac{(s+6)^2}{s(s+0.6)} \qquad W_2(s) = \frac{(s+10)(s+50)}{500}
$$

See figure 6.17

$$
\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = \begin{pmatrix} W_1 & -W_1 G \\ 0 & W_2 G \\ I & -G \end{pmatrix}
$$

Make W_2 proper, see (6.218)

Conditions 1 and 2 ok

Make D_{12} full rank, see (6.220)

Cond. 4 not ok. Change poles in zero in G and W_1 (6.222-3)

Results

$\gamma_{opt} = 3.5$, $Y_\infty = 0$

See Figure 6.19

Punish S more, change $W_1 := 4W_1$

See Figure 6.20

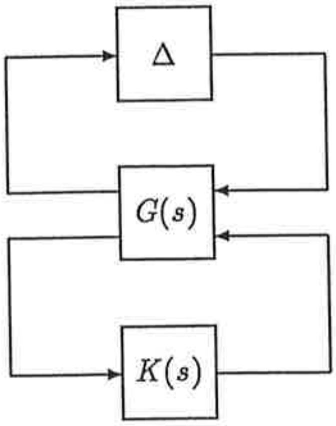
See Fig 6.21 for step responses

Better control signals than LQG design

Controller of degree 17

Notch design

μ-design



Lower and upper bounds on μ:

$$\max_U \rho(UM) = \mu(M) \leq \inf_D \sigma_1(DMD^{-1})$$

DK-iteration

Method: Minimize the upper bound on μ.

Dont know if this gives good μ, but it might

One of the exercises gives an example with arbitrarily bad upper bound.

$$\min_{D,K} \sigma_1(DMD^{-1})$$

1. Fix D and find K using H_∞
2. Fix K and optimize $D(j\omega)$ for each ω
3. Approximate all these $D(j\omega)$ by a dynamical system
4. Include $D(s)$ and $D^{-1}(s)$ into G

Iterate from 1 until convergence

High complexity controller? Do model reduction

Real Parameters

Exists version for real parameters, called DGK or YZK-iteration

Pete Young, Anders Helmersson

Idea: LMI-formulation

$$\begin{pmatrix} I \\ \Delta \end{pmatrix}^* \begin{pmatrix} X & 0 \\ 0 & -X \end{pmatrix} \begin{pmatrix} I \\ \Delta \end{pmatrix} > 0$$

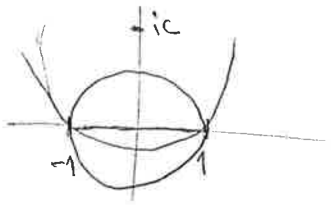
Can be extended to

$$\begin{pmatrix} I \\ \Delta \end{pmatrix}^* \begin{pmatrix} X & Y \\ Y^* & -X \end{pmatrix} \begin{pmatrix} I \\ \Delta \end{pmatrix} > 0$$

if $Y\Delta + \Delta^* Y^* = 0$.

Example $\Delta = \delta I$ real and $Y^* = -Y$

Explanation in figure



$$|ic - \delta|^2 < |ic - 1|^2$$

can be written

$$\begin{pmatrix} I \\ \delta \end{pmatrix}^* \begin{pmatrix} 1 & ic \\ -ic & 1 \end{pmatrix} \begin{pmatrix} I \\ \delta \end{pmatrix} > 0$$

This shows the correspondence with previous discussion

New Mu-box manual has more discussion

Example, μ-design

mudems in Matlab

Model reduction

Make $||\hat{G} - G||$ small

Respect stability, $G(0)$, etc.

balreal	Balanced realization
hankmr	Optimal Hankel norm approximation of a system
sfrwtbal	Frequency weighted balanced realization of a system matrix
sfrwtbld	Stable frequency weighted realization of a system matrix
sncfbal	Balanced realization of coprime factors of a system matrix
srelbal	Stochastic balanced realization of a system matrix
sysbal	Balanced realization of a system matrix

Error bounds

Assume stable system $G(s)$.

Balanced realization "twice the tail"

$$||\widehat{G}(s) - G(s)||_\infty < 2(\sigma_{l+1} + \ldots + \sigma_m)$$

Balanced singular perturbation $\hat{G}(0) = G(0)$

Stochastic Balanced: Relative Error Bound, preserves min-phase

Hankel Model Reduction:

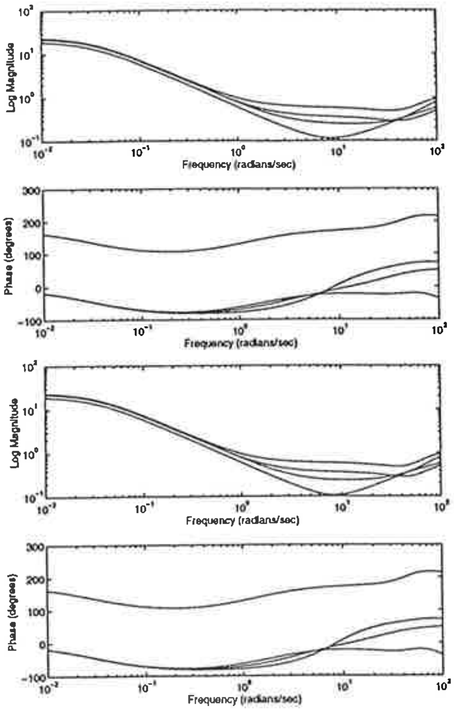
$$||\widehat{G}(s) - G(s)||_\infty < \sigma_{l+1} + \ldots + \sigma_m$$

Example, model reduction

```
>>[kk4,sig]=sysbal(k4);
>> sig
sig =
   1.6999e+01
   1.5399e+01
   2.2202e+00
   2.0056e+00
   4.9289e-01
   2.1384e-01
   1.7140e-01
   9.4974e-02
   2.7353e-02
   2.5791e-02
   2.1279e-02
   1.0651e-02
   4.6374e-03
   3.8297e-03
   2.5818e-03
   2.1740e-03
   4.1439e-05
   4.6069e-06
   2.2060e-10
>>kkk4=hankmr(kk4,sig,10);
```

See copies from Green-Limebeer

Result



Open Problems

Truncate systems with "small" nonlinearities or time-varying parts.

Rantzer-Andersson

Exercises

Ex. 1 Run the demos in

Mubox: mudems

Robust: mudemo, mudemo1, mrdemo,rctdemo

Ex. 2 Check the AIRC H_∞-design done in Ch. 6.8 using the mubox in matlab. Compare the controller amplitude with previous designs. Dont forget to plot the control signals and responses to load disturbances.

Mac 6.5

Mac 6.7

Ex. 3 Show that $\mu = 0$ for the system

$$G = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{pmatrix}$$

where $\Delta = \text{diag}[\delta_1 I_2, \delta_2 I_2, \delta_3]$. It can be shown (LMIs) that the upper bound is $\nu(M) = 2$.

Lecture 14

- Design by optimization
  - Q-parametrization
  - L_1 design
  - Mixed $H_2 - H_\infty$
  - Multi-objective LQG
  - Minimal risk
  - Decentralized control
  - FRLS design

Readings: Maciejowski, Ch 7.1, 7.3-5

Hand-outs on L_1, Q-parametrization and Lilja's thesis

Design by numerical optimization

Newton-Gould-Kaiser (1957) *"Analytic Design of Linear Feedback Control Systems"*

Mayne-Polak

Boyd

Many others

Matlab-toolbox. NAG. Several others

Design by numerical optimization

Choose criterium/criteria

Choose controller structure

Choose optimization method

Optimize controller parameters

Well formulated problem?

Analytical solution?

Avoid local minima

Infinitely many constraints

Idea: run on real system-evaluate response-try better parameters

More information

Optimal control course

Optimization course - math dept.

Mayne-Polaks articles

Boyd-Barrat's book

Controller parametrization

P, PI, PID, PIP, PI^2

RST

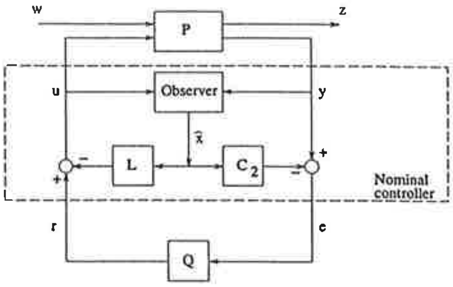
Q-parameterization combined with e.g.

$$Q(s) = \sum_{i=1}^{10} \frac{b_i}{(s+a)^i}$$

or (impulse response)

$$Q(s) = q_0 + q_1 s^{-1} + \ldots + q_N s^{-N}$$

Q-parametrization



$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \\ z \\ e \end{bmatrix} = \begin{bmatrix} A - B_2 L & B_2 L & B_1 & B_2 \\ 0 & A - KC_2 & B_1 - KD_{21} & 0 \\ C_1 - D_{12} L & D_{12} L & D_{11} & D_{12} \\ 0 & C_2 & D_{21} & 0 \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \\ w \\ r \end{bmatrix}.$$

$$\begin{bmatrix} z \\ e \end{bmatrix} = \begin{bmatrix} T_{zw} & T_{zr} \\ T_{ew} & 0 \end{bmatrix} \begin{bmatrix} w \\ r \end{bmatrix}$$

Q-parametrization

$K(s)$ stabilize $P(s)$ iff if it can be written on the form above with $r = Q(s)e$ with $I - D_{22}Q(\infty)$ invertible. We then get

$$G_{zw}(s) = T_{zw}(s) + T_{zr}(s)Q(s)T_{ew}(s)$$

where the stable transfer functions T_{zw}, T_{zr} and T_{ew} are defined indirectly by (1) and (1).

Polynomial version (SISO)

$$\frac{S'}{R'} = \frac{S/A_o - A/A_c \cdot \frac{LA_c}{A_o' A_o A_c'}}{R/A_o + B/A_c \cdot \frac{LA_c}{A_o' A_o A_c'}}$$

See handout

Another parametrization trick

Can rewrite optimization over dynamic controllers of fixed order

$$u = -K(s)y$$

into optimization over static controller

$$u = -Ky$$

where K constant matrix

L_1 optimization

Same set-up as always, w, u, z, y

Find controller that minimizes (induced L_∞)

$$\|G(s)\|_{L_1} = \max_{\|w\|_\infty \leq 1} \|z\|_\infty$$

where

$$\|w\|_\infty = \sup_{t \geq 0} \sup_i |w_i(t)|$$

"Minimize the peak-value of the output when disturbance has peak-value less than 1".

SISO $\quad \|G(s)\|_{L_1} = \int_0^\infty |g(t)|\, dt$

MIMO $\quad \|G(s)\|_{L_1} = \int_0^\infty \max_i \sum_j |g_{ij}(t)|\, dt$

L_1-design

The optimal L_1-controller can be nonlinear. (MIMO example by Stoorvogel 1995)

Suboptimal linear controller can be found via linear programming

See book by M. Dahleh

Matlab-code available (no manual)

An L_1 design

See hand-out

Mixed H_2/H_∞ design

Many different designs with this name

H_2-norm good to measure stochastic performance

H_∞-norm good to guarantee robustness

Combination? How?

Bernstein-Haddad

Doyle, Bernhardsson-Hagander

Khargonekaar, Rigby et al

Mixed H_2/H_∞ design

Rotea et al

$\min_K \|G_{z_0,w_0}\|_2$ under the restriction $\|G_{z_1,w_1}\|_\infty \le \gamma$

Bernstein-Haddad, replace $\|\cdot\|_2$ with upper bound

$$J_{aux}(G,\gamma) = \text{Trace}\left[Q_s \tilde{C}^T \tilde{C}\right]$$

Other setups

$$J_1 = \min_K \max_{\|\Delta\|_\infty \le 1} \mathop{E}_{w_0} \|z\|^2$$
$$J_2 = \min_K \max_{\|w_1\| \le 1} \mathop{E}_{w_0} \|z\|^2$$
$$J_3 = \min_K \max_{w_1} \mathop{E}_{w_0} \|z\|^2 - \gamma^2 \|w_1\|^2 \qquad \text{(min-mix)}$$
$$J_4 = \min_K \max_{w_1 \in BP} \max_{w_0 \in BS} \|z\|^2 - \gamma^2 \|w_1\|^2$$

Min-mix H_2/H_∞

Often results in several coupled Riccati equations

Optimizes upper bounds

Sufficient/necessary ?

Min-mix H_2/H_∞ design, solution

Conditions:

- There exists X such that

$$AX + XA^T + C_1^T C_1 + X(\gamma^{-2}B_1B_1^T - B_2B_2^T)X = 0$$
$$X \ge 0 \text{ and } A_c := A + (\gamma^{-2}B_1B_1^T - B_2B_2^T)X \quad \text{is stable}$$

- There exist L, Y and P which satisfy

$$Y(LD_{20}D_{20}^T + B_0D_{20}^T + PC_2^T + \\ +\gamma^{-2}PXB_1D_{21}^T) + \gamma^{-2}PY(B_1 + LD_{21})D_{21}^T = 0$$
$$YA_{\text{ml}} + A_{\text{ml}}^T Y + Y\tilde{R}Y + F^TF = 0$$
$$Y \ge 0 \text{ and } A_{\text{ml}} + \tilde{R}Y \text{ is stable}$$
$$(A_{\text{ml}} + \tilde{R}Y)P + P(A_{\text{ml}} + \tilde{R}Y)^T + (B_0 + LD_{20})(B_0 + LD_{20})^T = 0$$

where

$$\tilde{R} = \gamma^{-2}(B_1 + LD_{21})(B_1 + LD_{21})^T$$
$$A_{\text{ml}} = A + \gamma^{-2}B_1B_1^TX + L(C_2 + \gamma^{-2}D_{21}B_1^TX)$$
$$F = -B_2^TX$$

When these conditions hold, one such controller is

$$K(s) := \left[\begin{array}{c|c} A_{ml} + B_2F & -L \\ \hline F & 0 \end{array}\right]$$

Multi-objective H_2

Not all LQG-problems can be written in standard form

$$\|T_{z_0w_0}\|_2^2 + \|T_{z_1w_1}\|_2^2$$

Back to LQG if $z_0 = z_1$

$$z = \begin{pmatrix} T_{z_0w_0} & T_{z_1w_1} \end{pmatrix} \begin{pmatrix} w_0 \\ w_1 \end{pmatrix}$$

Similarly if $w_0 = w_1$

Example: $\min_K \begin{pmatrix} SP \\ KS \end{pmatrix}$

Can be solved e.g. with Q-parametrization and completion of squares.

Minimal order of controller unknown ($2n$?)

Minimal risk criterion

z scalar, critical signal

$$\min_{K(s)} \text{Prob} \left(\sup_{0<t<T} |z(t)| > c \right)$$

Nonlinear controllers optimal

Can find suboptimal linear controller by one-dimensional search and Riccati equations

$$E(z^2) + \rho E(\dot{z}^2)$$

Optimal controller close to minimal variance if

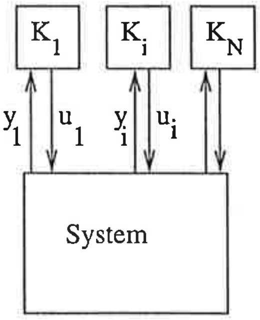
$$E(z^2) << c$$

See Anders Hansson's thesis

Decentralized control

Example: Control of power systems

$$u = \begin{pmatrix} k_{11}(s) & 0 & k_{13}(s) \\ 0 & k_{22}(s) & 0 \\ 0 & k_{32}(s) & k_{33}(s) \end{pmatrix} y$$



Communication

Choose control structure? Combinatorial problem?

For analysis one can always assume diagonal structure $u_i = K_i(s)y_i$ (think)

Decentralized stochastic control

Assume fixed structure

Decentralized LQG : Optimal controller not linear

Witenshausen: Can use control signal for communication

If $u = 0$ is optimal, send instead

$$u = 0.000x_1x_2x_3\ldots$$

where x_1x_2 is a message to other controllers

Hard to find analytical solutions to interesting problems

Can find suboptimal controllers

Strong results upcoming (Johansson 1996)

Decentralized stabilization

Wang-Davison

$$u_i = K_i(s)y_i$$
$$\mathcal{S} = \{\text{diag}(S_1, \ldots, S_N) \mid S_i \in R^{m_i \times p_i}\}$$

Theorem System is stabilizable by decentralized dynamic output feedback iff

$$\bigcap_{S \in \mathcal{S}} \sigma(A + BSC) \subset C_g$$

"If one can move eigenvalues (by decentralized static control) then it can be moved anywhere (by dynamic controllers)"

Lilja design

Find reduced order controllers

Frequency domain least squares

$$
\begin{aligned}
y &= G(s)u \\
R(s)u &= -S(s)y + t_0 A_0(s) r \\
y_m &= G_m(s) r \\
E &= \frac{G_{cl} - G_m(s)}{G_{cl}} = 1 - F_R(s)\frac{R(s)}{t_0} + F_S(s)\frac{S(s)}{t_0}
\end{aligned}
$$

Linear in S/t_0 and R/t_0!

FRLS design

Choose order of $S(s)$ and $R(s)$

$$
\min_{S,R,t_0} \sum_{i=1}^{N} |E(z_i)|^2
$$

where z_i are some interesting frequencies

Lilja's thesis, help frlsbox

```
LSRSTC Fits a continuous time controller to a specified closed
      loop model given a frequency response of the process.

   [R,S,T,THE'ERROR]=LSRSTC(FR,BM,AM,AO,NR,NS,R1,S1,WGT)

      The frequency response FR on the form [w G(iw)]
      is used for least squares fitting of controller parameters
      in a controller structure given by Ru = Tr - Sy.
      The closed loop system (r -¿ y) is specified
      by the transfer function BM(s)/AM(s). For convenience, the
      polynomial BM is multiplied by a constant factor in order to
      get a closed loop stationary gain of 1 (i.e. BM(0)/AM(0)=1).
      Factors to be included in R are specified by the polynomial R1.
      Similarly, S1 pre-specifies factors of S.
      Deg R = NR and deg S = NS. The observer polynomial is given by
      AO (T = const.*AO). Default WGT is unity weighting
      while R1 and S1 both defaults to 1.
      The magnitude of the (weighted) closed loop transfer
      function error is given by THE'ERROR.
```

Example

See handout, appendix in Lilja's thesis

What we haven't talked about

High-level control

Adaptive control

Nonlinear control theory

Numerical algorithms

Discrete-event systems

Hybrid control

Expert systems

Neural networks

Implementation (real-time)

Diagnosis

Manual control

...

Bladvinkelreglering av stora horisontalaxlade vindkraftverk

Sven Erik Mattsson

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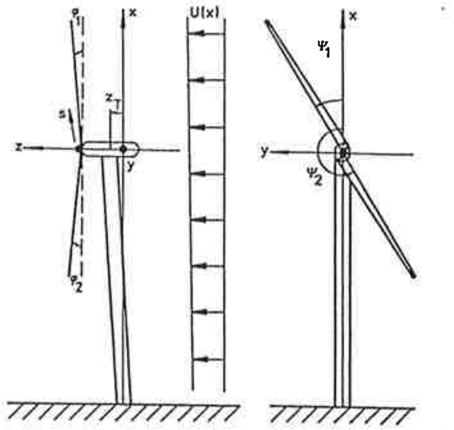
Lunds Tekniska Högskola

Innehåll

1. WTS-3 i Maglarp
2. Reglerkrav
3. Modell
4. Reglerbarhet
5. Observerare, mätbrus
6. Olika väder och driftfall
7. Simuleringar

WTS-3 i Maglarp



- Drift vid vind 7.2–27.2 m/s
- Märkeffekt 3 MW, nås vid 14.2 m/s
- 2 st 39 m långa glasfiberblad
- Roterar 25 varv/minut
- Hydraulisk bladvinkelreglering
- Flerstegs planetväxellåda, 1:60
- Synkrongenerator, 1500 varv/min
- 80 m ståltorn, diam 3.8 m, 4 cm gods

Övergripande reglermål

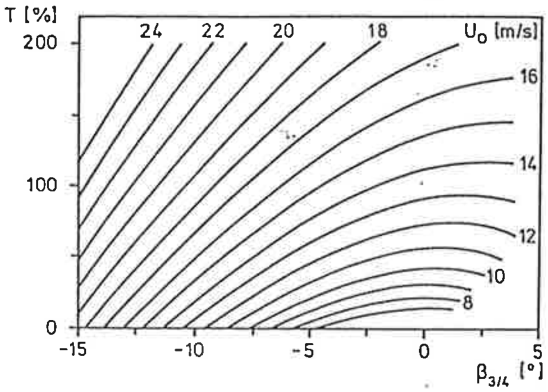
Optimal effektproduktion.

- maximal energikonvertering
  inte reglera nätet
- jämn drift
  - laster i blad
  - utmattning
  - mekniska resonanser
- konstant spänning

Reglermöjligheter

- rotororientering
- bladvinklar
- generatorns magnetisering

Mål med bladvinkelreglering



Under märkeffekt

- Extrahera maximal effekt
- Flackt maximum
- Skatta U_0 statiskt från P_E och β.

Över märkeffekt

- Undvika höga mekaniska laster
- Urkoppling vid 4.2 MW
- Hålla konstant effekt, 3MW

Från aerodynamiskt moment till elektriskt moment

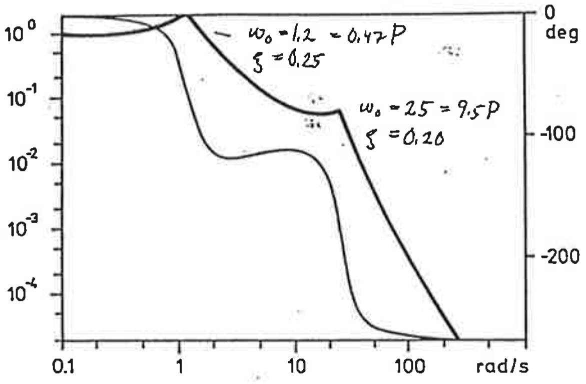
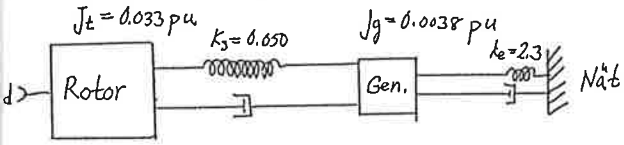


Figure 2.1: Bode plot of the transfer function (2.16) from Δt_a to Δt_e for the WTS-3.



Vilken bandbredd behöver bladvinkelregleringen?

Hur mycket störningar orsakar vinden över en viss frekvens?

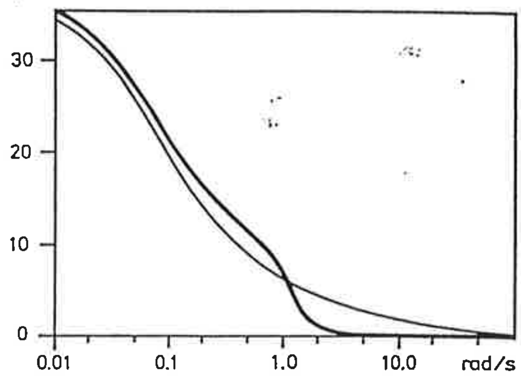
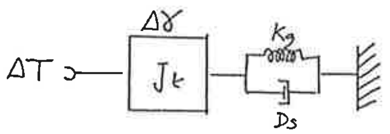


Figure 5.1: Standard deviations $\sigma(\Delta P_E, w; \omega, \infty)$ [% of P_B]. The bold line is for the open loop system. The thin line is for an open loop system with a rigid drive train.

Modell



Rotor: $J_t \Delta \frac{d^2\gamma}{dt^2} + D_s \Delta \frac{d\gamma}{dt} + K_s \Delta\gamma = \Delta T$

Effekt: $\Delta P_E = \dot{\psi}_0 (D_s \Delta \frac{d\gamma}{dt} + K_s \Delta\gamma)$

Varvtal: $\Delta \frac{d\psi}{dt} = \Delta \frac{d\gamma}{dt}$

Bladservo: $\frac{d\beta}{dt} = (\beta_r - \beta)/T_{bs}$

Aerodynamiskt moment:

$$\Delta T = T_\beta \Delta\beta + T_U \Delta U_0 + T_{\dot\psi} \Delta \frac{d\psi}{dt}$$

Designmodell I

$$\Delta \frac{dx}{dt} = A\Delta x + B\Delta\beta_r + B_w w$$

$$\Delta x = \begin{pmatrix} \Delta\beta & \Delta U_0/100 & \Delta\frac{d\gamma}{dt} & \Delta\gamma \end{pmatrix}^T$$

Vindmodell: Gaussiskt vitt brus filtrerat med tidskonstant 20s.

För $\bar{U}_0 = 18m/s$:

$$A = \begin{pmatrix} -2.5 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & 0 \\ 2.0 & 4.7 & -0.74 & -1.5 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$B = (2.5 \quad 0 \quad 0 \quad 0)^T$$

$$B_w = (0 \quad 0.0057 \quad 0 \quad 0)^T$$

$$\sigma_w = 1.8m/s$$

LQ-kriterium

Straffa

1. effektvariationer
2. servorörelser, men ej position

$$J = E\left[\lim_{T\to\infty} \frac{1}{T}\int_0^T q^2 \Delta P_E^2 + q_\beta^2 \frac{d\beta}{dt}^2 dt\right]$$

$$\frac{d\beta}{dt} = (\beta_r - \beta)/T_{bs}$$

$q = 1$ (MW)$^{-1}$

$\Delta\beta_r = -L\Delta x$

Resultat vid tillståndsåterkoppling

$$q_\beta = 3: L_1 = (\ 1.46 \quad 5.85 \quad 3.85 \quad 4.83)$$
$$q_\beta = 5: L_2 = (\ 0.94 \quad 4.60 \quad 2.39 \quad 2.54)$$
$$q_\beta = 10: L_3 = (\ 0.38 \quad 3.26 \quad 1.21 \quad 0.95)$$
$$q_\beta = 15: L_4 = (\ 0.11 \quad 2.62 \quad 0.79 \quad 0.49)$$
$$q_\beta = 30: L_5 = (-0.26 \quad 1.73 \quad 0.34 \quad 0.10)$$

Servot får tidskonstant: $T_{bs}/(1 + l_1)$

Storhet	L_1	L_2	L_3	L_4	L_5
$\sigma(P_E)$ [%]	0.65	0.97	1.7	2.4	4.2
$\sigma(\frac{d\beta}{dt})$ [°/s]	1.7	1.5	1.2	1.1	0.85
$\sigma(\frac{d\gamma}{dt})$ [%]	0.06	0.08	0.13	0.16	0.25
ω_c [rad/s]	3.2	2.7	2.1	1.8	1.3

ω_c = bandbredd i loopsnitt efter servot.

Mätningar och observerare

Kan mäta effekt, P_E och varvtal, $\frac{d\psi}{dt}$.

Svårt att mäta medelvind

- Vindkraftverket stör en lokal mätare.
- Koherenslängd och turbindiameter av samma storlek.
- Korrelationen mellan medelvind U_0 över turbinen och vinden i centrum är ofta under 0.8.

Vi måste rekonstruera vinden från andra mätningar på vindkraftverket.

Modell för design av Kalmanfilter

$$\Delta\frac{dx}{dt} = A_1\Delta x + B_1\Delta\beta_r + B_{w1}w$$

$$\Delta y = (\Delta P_{Em}/10^6 \quad \Delta\dot{\psi}_m)^T$$

$$\Delta x = \left(\Delta U_0/100 \quad \Delta\frac{d\gamma}{dt} \quad \Delta\gamma\right)^T = C_1\Delta x + e$$

e är Gaussiskt vitt brus, okorrelerat med w

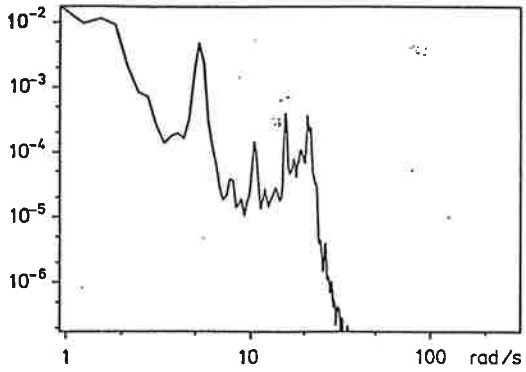


Figure 4.2: Power spectrum for the electrical power P_E [MW] from series 1.

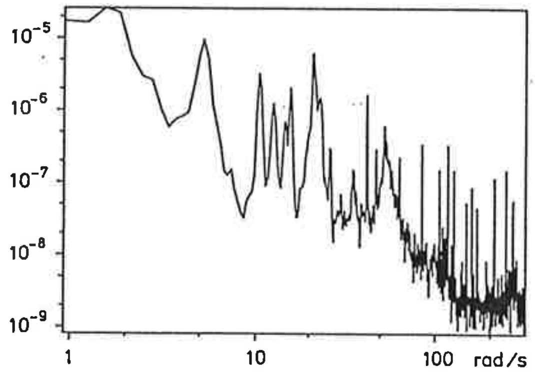


Figure 4.3: Power spectrum for the turbine speed $\dot{\psi}$ [rad/s] from series 1.

Brusmodeller

Vitt brus betyder $R_e[1,1] = 2\pi\Phi_{PEe}$.

Använd spektra från mätningar.

$R_{e1} = \mathrm{diag}\,(2 \cdot 10^4 \quad 1 \cdot 10^6)$

$R_{e2} = \mathrm{diag}\,(2 \cdot 10^4 \quad 3 \cdot 10^6)$

$R_{e3} = \mathrm{diag}\,(2 \cdot 10^4 \quad \infty)$

Filterförstärkningar:

$$K_1 = \begin{pmatrix} 0.21 & 0.24 & 0.06 \\ 4.75 & 4.94 & 0.46 \end{pmatrix}^T$$

$$K_2 = \begin{pmatrix} 0.29 & 0.38 & 0.08 \\ 2.17 & 2.54 & 0.26 \end{pmatrix}^T$$

$$K_3 = (0.40 \quad 0.55 \quad 0.11)^T$$

Regulatorstruktur

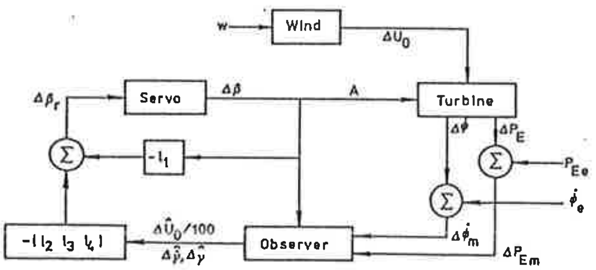


Figure 5.7: Control Configuration.

	L_3	L_3 K_1	L_3 K_2	L_3 K_3
$\sigma(P_E, w)$ [%]	1.7	3.4	3.8	4.2
$\sigma(\frac{d\beta}{dt}, w)$ [°/s]	1.2	1.3	1.3	1.3
$\sigma(\frac{d\beta}{dt}, P_{Ee})$ [°/s]	-	0.48	0.75	1.1
$\sigma(\frac{d\beta}{dt}, \dot{\psi}_e)$ [°/s]	-	0.74	0.67	-
$\sigma(\frac{d\beta}{dt}, \cdot)$ [°/s]	1.2	1.6	1.6	1.7
ω_c [rad/s]	2.1	2.8	2.7	2.6
A_m		3.6	3.3	2.9
ϕ_m [°]		37	35	34
α_{2P}		1.68	1.66	1.59
g_{2P} [°/s/(m/s)]		5.3	4.7	4.1

Hur dämpas effektvariationerna?

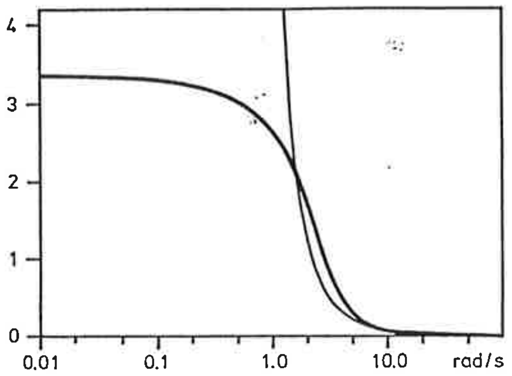


Figure 5.3: Standard deviations $\sigma(P_E, w; \omega, \infty)$ [% of P_B]. The bold line is for the closed loop system when L3 and K1 are used. The thin line is for the open loop system.

Hur påverkar mätbruset servot?

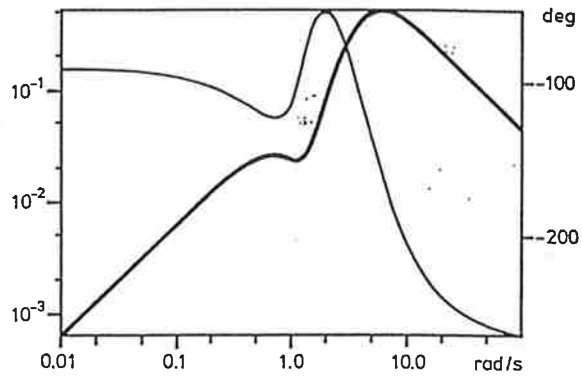


Figure 5.4: Bode plot of the transfer function from P_{Ee} [% of P_B] to $\Delta\dot{\beta}$ [°/s] for the closed loop system when L3 and K1 are used.

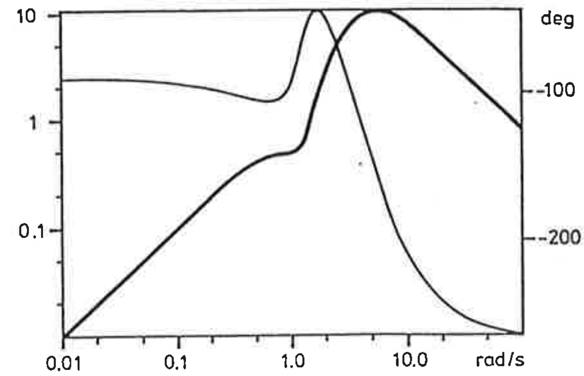


Figure 5.5: Bode plot of the transfer function from $\dot{\psi}_e$ [% of $\dot{\psi}_B$] to $\Delta\dot{\beta}$ [°/s] for the closed loop system when L3 and K1 are used.

2P störningar

Stora störningar vid 2P (5.2 rad/s)

- roterande sampling av vindvirvlar
- tornet blockerar vinden

Antag sinusformad störning vid 2P

- för P_E är amplituden 1-3% av P_B
- motsvarar vindamplitud på 0.3–1.1 m/s

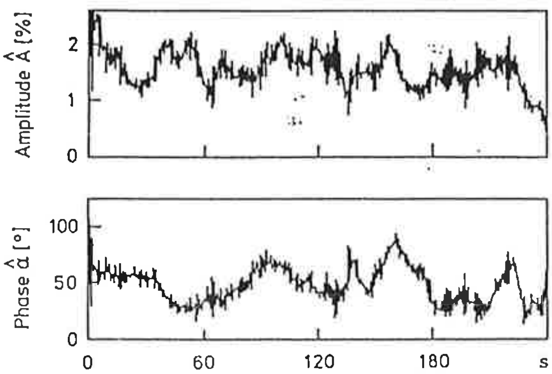


Figure 4.15: Estimates for measurement series 1. The forgetting factor λ was 0.99 and the sampling period h was 0.1 s.

2P-variationerna måste beaktas

Vår design

- förstärker P_E med $\alpha_{2P} = 1.7$
- ger onödiga servorörelser
- förstärkningen från ΔU_0 till $\Delta \dfrac{d\beta}{dt}$, g_{2P} = 5.3°/s/(m/s).
- servot svänger 0.3-1° med hastighet 1.5-6°/s

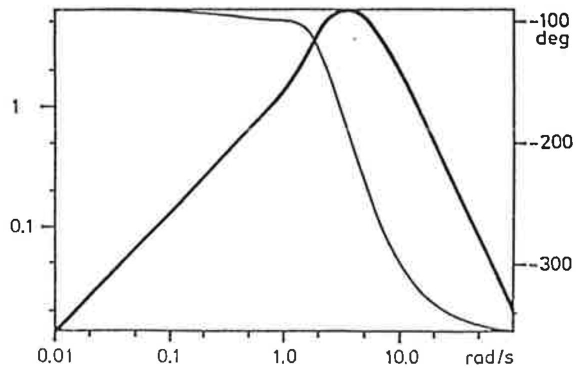


Figure 5.6: Bode plot of the transfer function from ΔU_0 [m/s] to $\Delta\dot{\beta}$ [°/s] for the closed loop system when L3 and K1 are used.

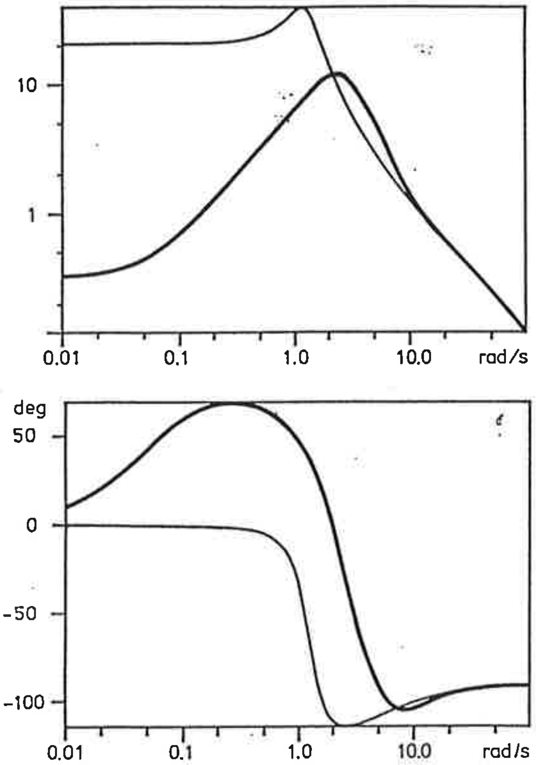
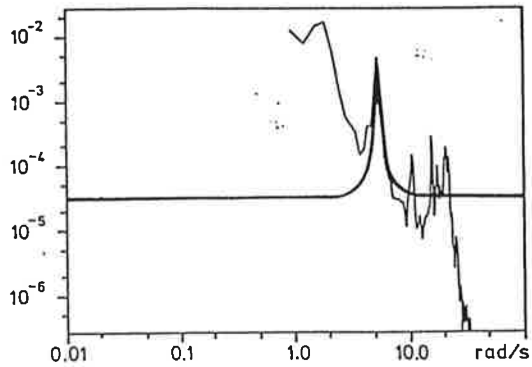


Figure 5.2: Bode plots of the transfer function from ΔU_0 [m/s] to ΔP_E [% of P_B]. The bold lines are for the closed loop system when L3 and K1 are used. The thin lines are for the open loop system.

Vad göra med 2P?

Analys av mätdata visade att störningen inte är en ren sinus.

1. Sätta in notchfilter
   - spektrum är 100 ggr större vid 2P
   - tag $(\zeta_N/\zeta_D)^2 = 0.01$
   - $\zeta_N = 0.03$ och $\zeta_D = 0.3$ ger bra bredd



1. Notchfilter
   - + α_{2P} minskar till 1.07
   - + g_{2P} minskar från 5 till 0.5
   - – $\sigma(P_E)$ ökar med 1%
   - – A_m minskar till 2-2.5
   - – ϕ_m minskar till 27°
2. Frekvensberoende straff på $\dfrac{d\beta}{dt}$
   - ger något bättre resultat
   - kan lägga till fiktivt brus på insignalen för robusthet
3. Se 2P-störningarna som mätbrus; färgat och korrelerat mätbrus
4. LS-skattning är sämre

Tornsvängningar

1. Tornets böjmod har grunfrekensen 0.85P
2. Perfekt effektreglering ger instabilitet.
   - konstant varvtal, konstant moment
   - men trycket på rotorn varierar
   - tornet svänger
   - regulatorn tolkar det som vind-variationer
3. Återkoppla från tornrörelsen.
4. Lägg straff på $\dfrac{dz_T}{dt}$.
5. Orealistiskt att rekonstruera torn-rörelsen från varvtals och effektmät-ningar.

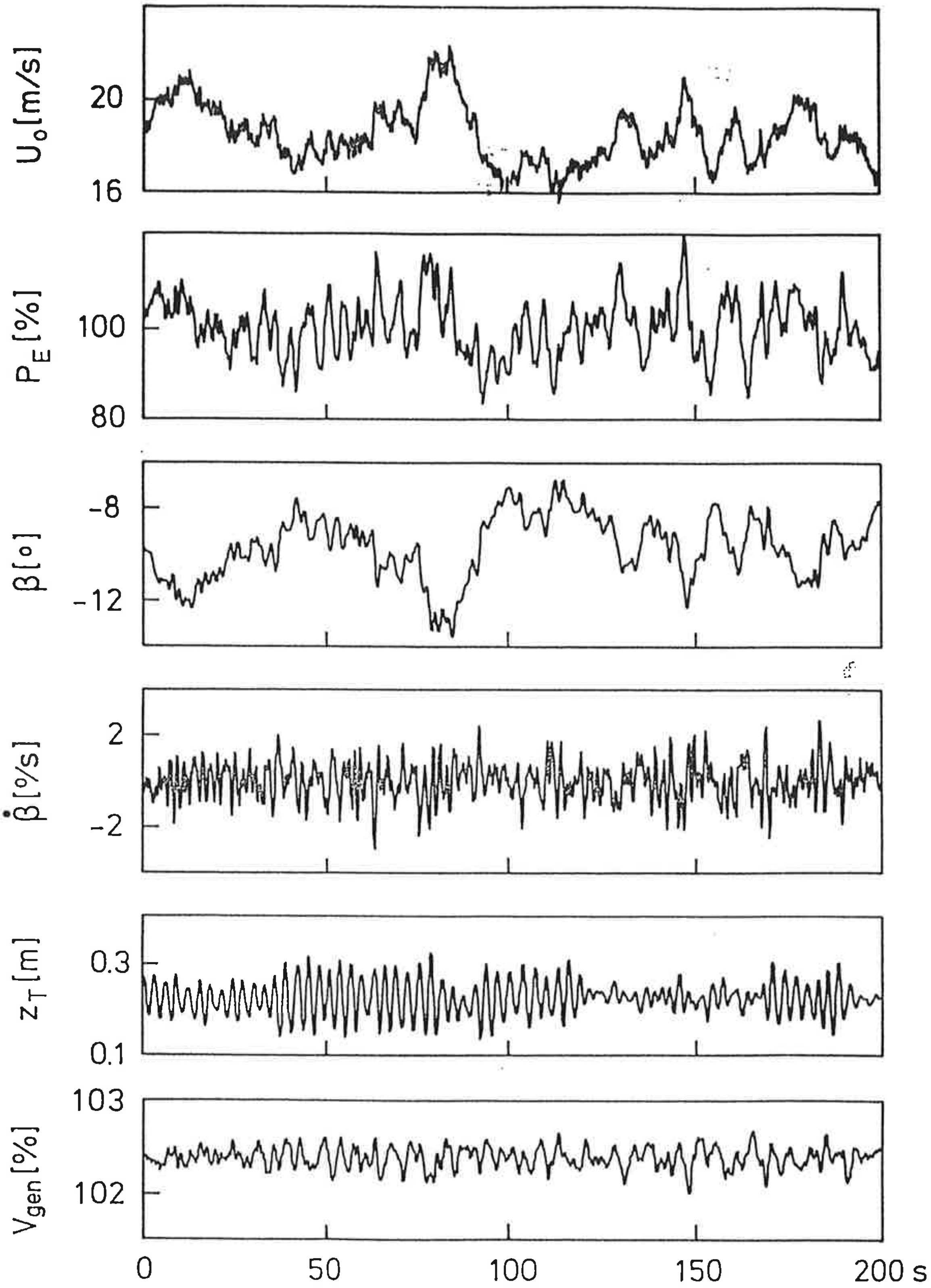


Figure 5.17: Simulated response to turbulent wind around 18 m/s when the controller based on LC and K7T is used.

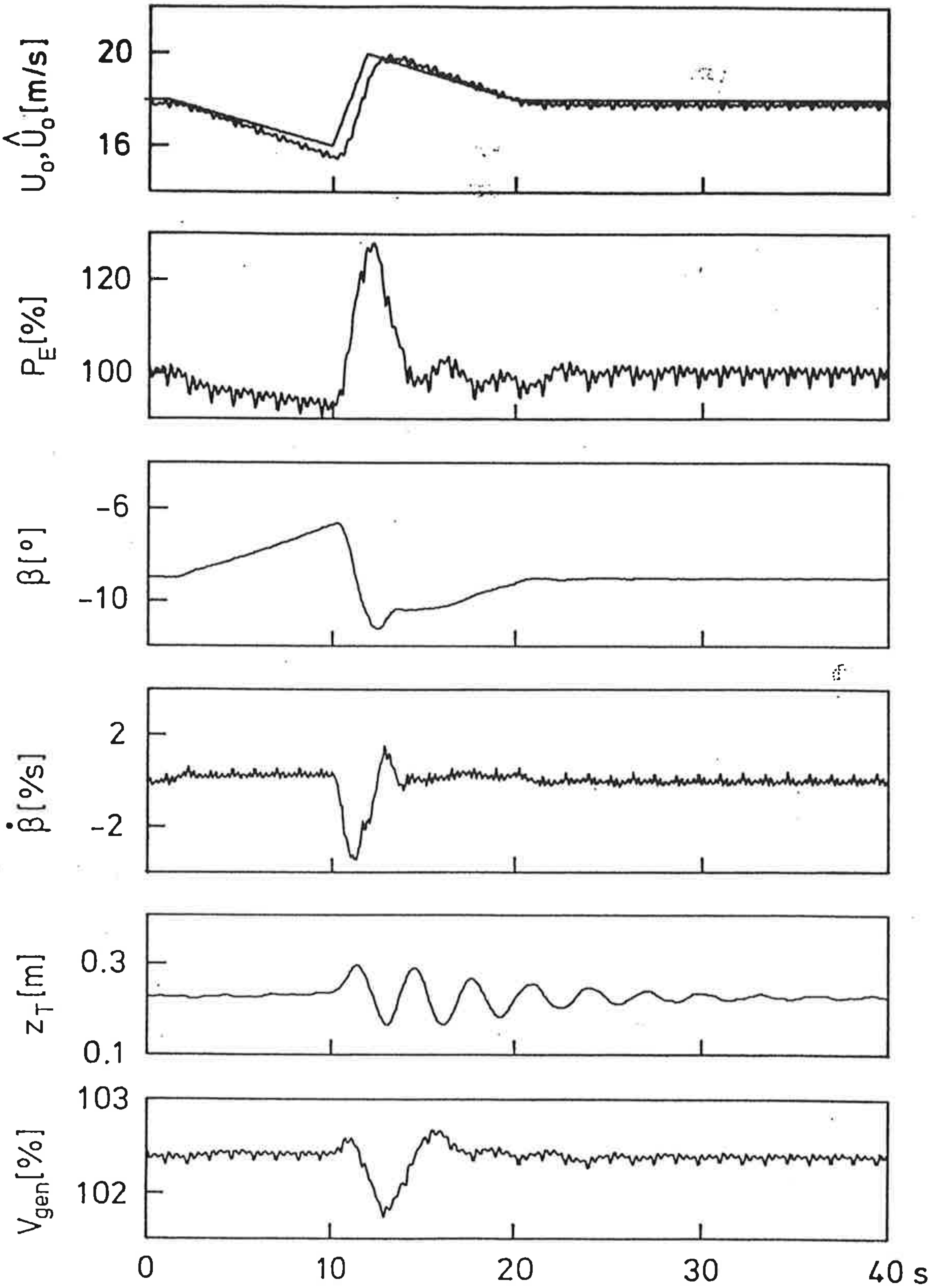


Figure 5.18: Simulated response to a large gust when the controller based on LC and K7T is used.

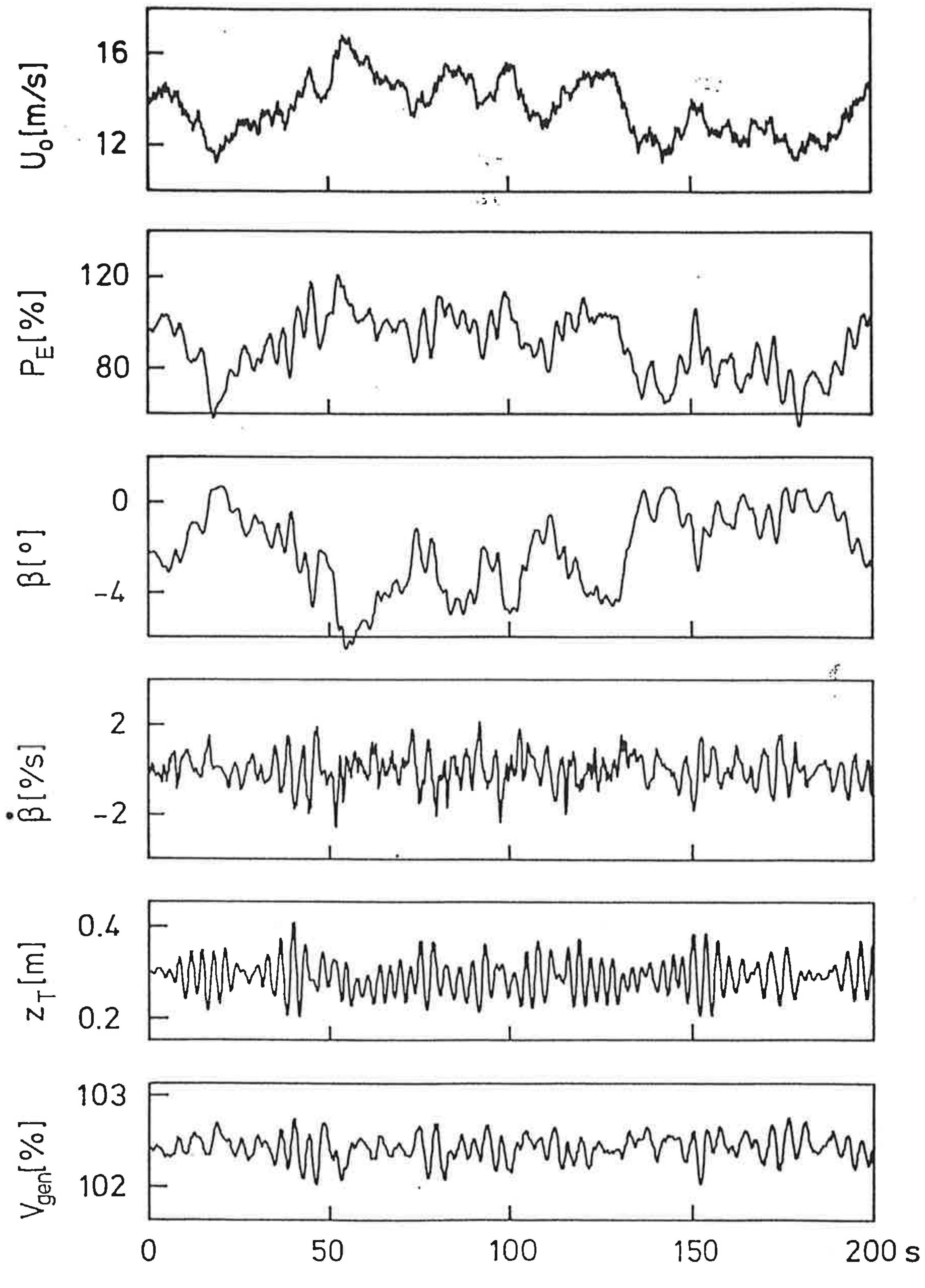


Figure 5.19: Simulated response to turbulent wind around 14 m/s when the controller based on LC and K7T is used.

Fuzzy Control of a DC Servo

– A Laboratory Exercise in Control System Synthesis

Mikael Johansson and Johan Eker

Introduction. The purpose of this lab is to get some practical experience of fuzzy control design. The task is to design a position control system for a simple DC-motor.

1. Laboratory Setup

1.1. Laboratory Equipment

The process is a DC-servo with a flywheel that will be controlled to follow a desired angular position. Ideally, the controller should be designed to give a fast set-point response while having low sensitivity to noise in steady state. All signals are limited to the interval $\pm 10V$.

The fuzzy controller will be designed and evaluated in Matlab/Simulink. The fuzzy system mapping is crafted using Mathwork's Fuzzy Logic Toolbox, and the controller is tested in Simulink. When the simulations indicate a satisfactory design, real-time code is generated automatically.

The code is compiled in the Pålsjö environment, which runs on Sun Workstations connected to VME boards. This enables us to evaluate the fuzzy controllers on the real process.

1.2. A Simulink Model

A Simple Motor Model

From a torque balance for the motor axis, the following model is derived in Lab2, CCS:

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 5.00 \\ 0 & -0.12 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 2.24 \end{pmatrix} (u(t) + v(t)) \tag{1.1}$$

where $x_1(t)$ is the angular position and $x_2(t)$ is the angular velocity. The corresponding input-output description is

$$Y(s) = \frac{11.2}{s\,(s+0.12)} U(s) := P(s)U(s) \tag{1.2}$$

The motor model is implemented in Simulink as illustrated below:

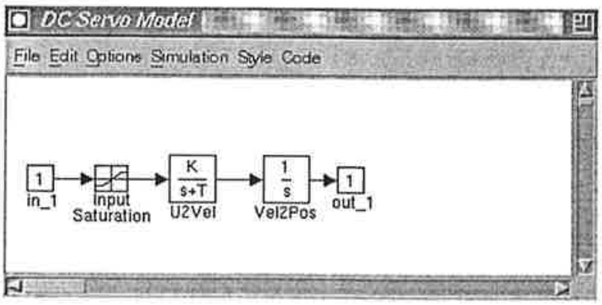


Figure 1.1 Simulink model of the DC-motor.

Control System Simulation

The motor model along with controllers is implemented in the Simulink system

```
>> servosim
```

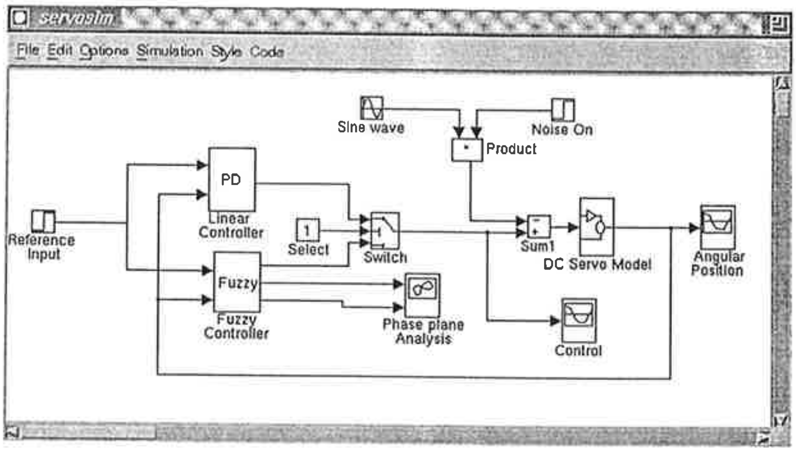


Figure 1.2 The control system simulation model.

Along with the motor model, the Simulink system implements the following features:

- Controllers (linear and fuzzy PD in parallell)
- Signal Generators (reference and disturbance)
- Plotting Tools (process output, control and controller phase plane)

The linear PD controller is included for comparison. What controller should be active is selected using the "select" constant:

1 Activates the linear PD controller

-1 Activates the fuzzy PD controller

Thus, in Figure 1.2 the linear PD controller is currently active.

In the remains of this chapter, the implementation of the linear and fuzzy PD controllers are explained in further detail.

A Linear PD Controller

The linear PD controller is implemented on the form

$$C(s) = k + k_d s \tag{1.3}$$

The proportional gain k and derivative gain k_d can be set by double-clicking on the "Linear PD" block.

The reference signal is not used when forming the error derivative, and in the discrete time implementation, the derivative is approximated by forward differences. The discrete time implementation of the PD controller is illustrated in Figure 1.3.

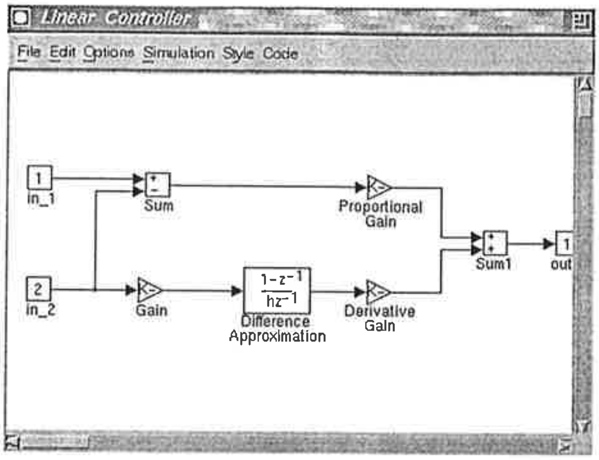


Figure 1.3 Discrete time version of linear PD controller.

A Fuzzy PD Controller

The main purpose of this lab is to design a fuzzy PD controller for the DC-motor. Similarly to the linear PD controller, the fuzzy PD controller can be written in the form

$$u(t) = f\left(e(t), \frac{de(t)}{dt}\right)$$

where $f(\cdot,\cdot)$ denotes a mapping defined by fuzzy logic rules.

Examining the Simulink implementation of the fuzzy controller shown in Figure 1.4, we notice that two new components have been added, the normalization and denormalization blocks.

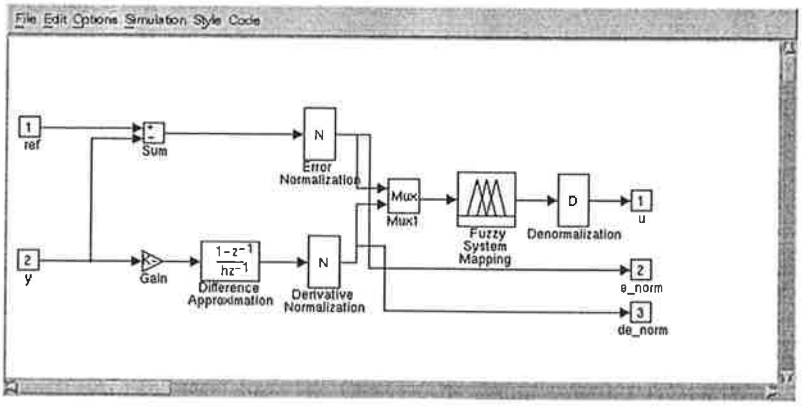


Figure 1.4 Discrete time version of fuzzy PD controller.

Recall that we use the fuzzy rules to "craft" a nonlinear controller mapping. We can only define this mapping on a limited domain of the fuzzy system input space. It is often convenient to define the fuzzy mapping on a normalized domain (often taken to be $[-1,1]^n$), and map the physical domain of the inputs onto this domain.

Defining the fuzzy system mapping on the normalized domain $[-1,1]^n$ means that the fuzzy sets of each input variable should cover the interval $[-1,1]$.

Mapping an actual inputs onto the interval $[-1, 1]$ is accomplished by the introduction of a normalization gain. Normalization gains are implemented as saturated gains. This assures that signals which after scaling fall outside the normalized domain are mapped onto the appropriate end point.

In terms of the fuzzy system mapping, the normalization gains scale the nonlinearity in the input directions:

$$u(t) = f\left(k_e e(t), k_d \frac{de(t)}{dt}\right)$$

The role of the normalization gains is illustrated in Figure 1.5.

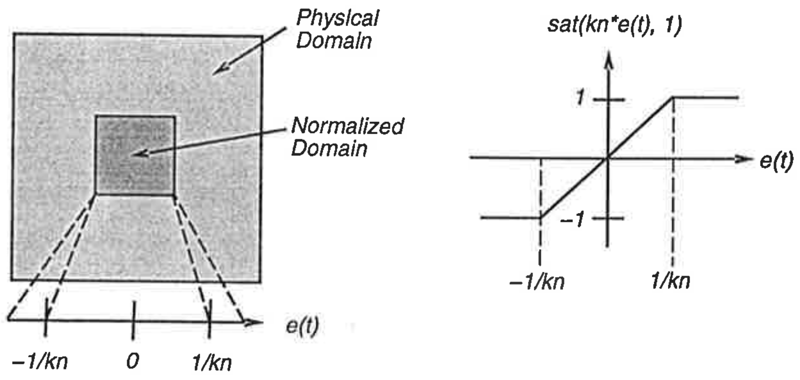


Figure 1.5 Illustration of the input scaling performed by the normalization gains.

The normalization gains define a "window" in the controller input space for which the active part of the nonlinearity is used. For a PD controller, it is a good exercise to think of how this "window" can be illustrated in a step response.

The denormalization gain is a linear gain which scales the fuzzy system mapping globally:

$$u(t) = kf(\cdot, \cdot)$$

2. Exercises

The purpose of the lab is to design a controller giving both

- Fast set-point response
- Low noise sensitivity in steady state.

Linear PD

Design a linear PD controller for the servo.

Fuzzy PD

Design a fuzzy PD controller for the servo. Design a fuzzy PD controller using the Fuzzy Logic Toolbox in Matlab. The toolbox is described briefly in the next chapter.

"Stream-lined" design procedure:

1. Start the Fuzzy Logic Toolbox (`>> fuzzy Fuzzy_PD` in Matlab). This automatically loads the pre-defined file `Fuzzy_PD`.
2. Alter fuzzy set definitions and rules.
3. When satisfied, save your system to a file.
4. Try the fuzzy controller in a simulation (`FuzzySim` in Matlab)
5. Iterate steps 3–5 until you feel satisfied
6. Apply `fis2pal` to your fuzzy system.
7. Try the controller on the real process.

The following "Karnaugh-like" map can be useful for representing the rules:

				e(t)		
		NL	NS	ZE	PS	PL
	PL					
	PS					
$\dot{e}(t)$	ZE					
	NS					
	NL					

3. A Sample Session

3.1. The Main Window – FIS Editor

The Fuzzy Logic Toolbox is started by typing

```
>> fuzzy
```

at the Matlab prompt. This command starts up the FIS-editor (The wonderfully selected acronym FIS is short for Fuzzy Inference System).

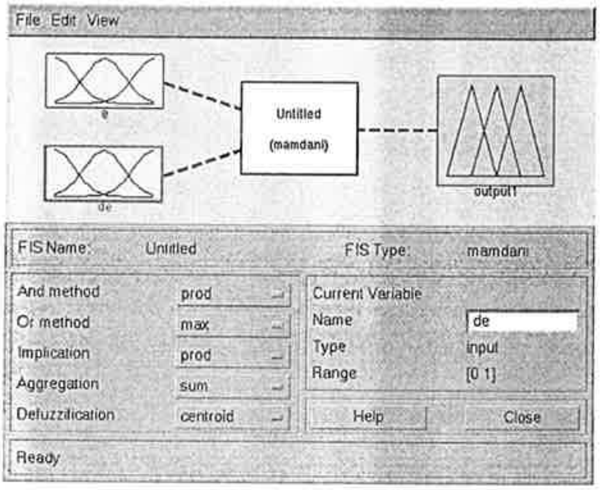


Figure 3.1 The Fuzzy System Editor.

Using the menus at the lower left part of the window, it is possible to specify the inference engine parameters. We recommend you to use

And method	Product
Or method	Max
Implication	Product
Aggregation	Summation
Defuzzification	Centroid

In the upper part of the window, the fuzzy system knowledge base is illustrated. The knowledge base parameters are divided into three classes;

Block	Contains
Input Variable Block	Input variable names, associated membership functions, and their names.
Rulebase Block	Linguistic descriptions of the rules
Output Variable Block	Output variable names, associated membership functions, and their names.

The names of the input and output variables can be altered by activating the block and enter the name in the edit box located in the middle right of the window. A block is activated by clicking once on the block illustration.

A double click on any of the blocks calls an associated editor; double clicking on the variable boxes calls an membership function editor while clicking on the rule base box calls the rule base editor.

3.2. The Membership Function Editor

Double clicking on a variable box calls the membership function editor:

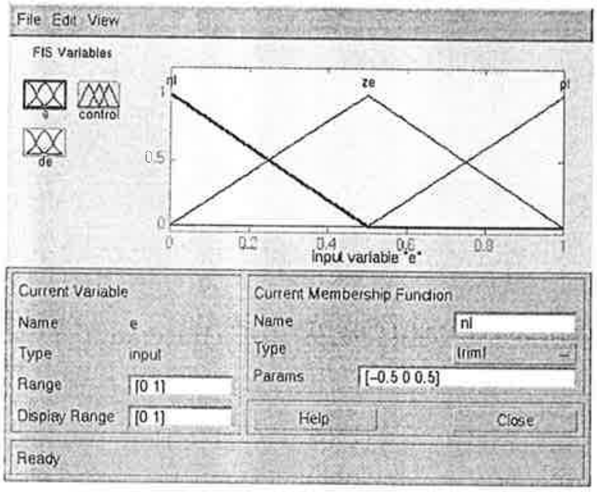


Figure 3.2 The Membership Function Editor

For our purposes, it is convenient to work with membership functions on the normalized domain $[-1,1]$. This can be accomplished by changing the "Range" and "Display Range" from the default $[0,1]$ to $[-1,1]$. These edit boxes are located in the lower left corner of the window.

By clicking on the boxes in the upper left corner, it is possible to change edit-variable. Initially, membership functions are created by selecting "Add MFs..." from the Edit Menu. You can now enter the number of membership functions needed and their shape:

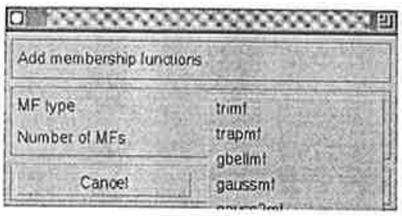


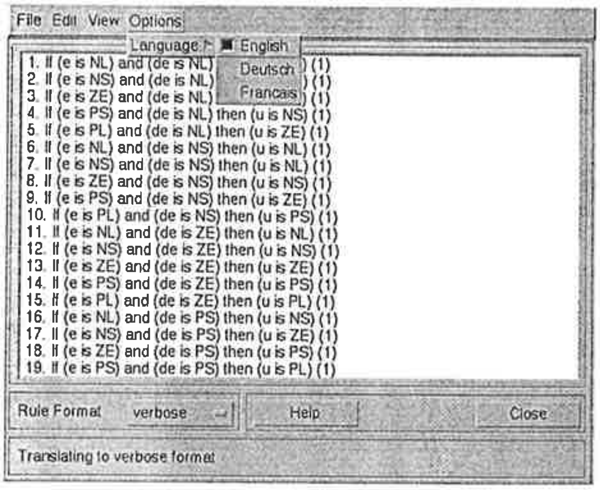
Figure 3.3 Adding membership functions.

Clicking on a membership function activates the attribute editor located in the lower right corner. It is possible to change the name, function class and shape parameters. For triangular membership functions, the parameter vector is on the form

```
[left base, center, right base]
```

3.3. The Rule Editor

The fuzzy-logic based rules are entered in linguistic form, as illustrated in Figure 3.3. The rules can be parsed by pressing `Ctrl+Enter`. The Fuzzy



Logic Toolbox allows for three rule formats; verbose, symbolic and indexed:

Rule Format	Example
Verbose	`If (e is PL) and (de is PL) THEN (u is PL) (1)`
Symbolic	`(e==PL) & (de==PL) => (u=PL) (1)`
Indexed	`5 5, 5 (1):1`

The (1) in the rules are weighting factors. They are included for some obscure histrorical purposes, and altering a rule weight is functionally equivalent to altering the rule's consequent.

Please observe the useful features of the "Options"-menu.

3.4. The Surface Editor

Since we try to design nonlinearities, it is useful to now and then take a look at the fuzzy system mapping. The Fuzzy Logic Toolbox supports this through the Surface Editor:

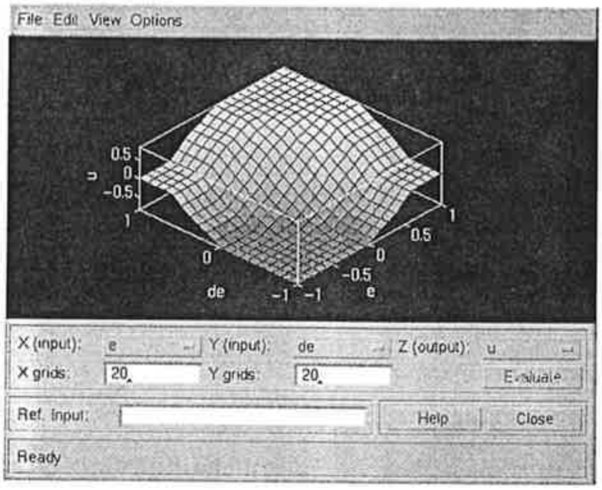


Figure 3.4 The Surface Viewer.