Production Control of a Pulp and Paper Mill

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1970

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Pettersson, B. (1970). *Production Control of a Pulp and Paper Mill*. [Licentiate Thesis, Department of Automatic Control]. Department of Automatic Control, Lund Institute of Technology.

Total number of authors:
1

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PRODUCTION CONTROL OF A PULP AND PAPER MILL

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THESIS FOR THE DEGREE OF TEKNOLOGIE LICENTIAT

REPORT 7007 SEPTEMBER 1970
LUND INSTITUTE OF TECHNOLOGY
DIVISION OF AUTOMATIC CONTROL

PRODUCTION CONTROL OF A PULP AND PAPER MILL

Bengt Pettersson

ABSTRACT

A summary of a study devoted to computerized scheduling of a complex integrated pulp and paper mill is presented.

A mathematical model of the mill was built and the scheduling problem was formulated as an optimal control problem for a multivariable deterministic system. The mill is described by a linear, first-order, vector-valued differential equation. The control system consists of 10 state variables and 9 control variables. A method of solution based upon the Pontryagin maximum principle was developed. The maximum principle leads to a boundary value problem for which no straightforward solution is available. A major part of the work was the development of a feasible computational procedure. The minimization of the Hamiltonian leads to a linear programming problem with 50 rows and 40 columns. The numerical algorithm developed is feasible for a process computer. The off-line execution time is about 20 minutes on an IBM 1800.

The production control system was implemented in practice at the Gruvön mill of Billeruds AB in November 1969. It has been in continuous operation since then. Experience from the first six months of operation is described.

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ACKNOWLEDGEMENTS

The work summarized in this thesis was carried out as part of a process computer project at the Gruvön mill of Billeruds AB.

The author would like to express his gratitude to Mr T Bergek, Executive Vice President of Billerud, who initiated the work and supported the project, and to Professor K J Åström, whose guidance made the problem possible to solve. Thanks are also due to colleagues at Billerud for advice and help, and to Billeruds AB for permission to publish the results of the work.

1. INTRODUCTION

During the last decade, control theory has developed considerably. New mathematical concepts and methods have appeared, the development of computer technology has made it feasible to solve complex problems and the appearance of process computers has facilitated the practical implementation of advanced control strategies.

In this work, optimal control theory is applied to an industrial planning problem. The work has been carried out as part of a process control computer project at one of the largest Swedish pulp and paper mills, the Gruvön mill of Billeruds AB, and the methods developed have been practically implemented at the mill.

The production control system developed is essentially a system for production scheduling of the sub-processes of the mill. A mathematical model of the mill has been built and the scheduling problem has been formulated as an optimal control problem for a multivariable deterministic system. The optimization problem has been attacked by means of the Pontryagin maximum principle. The minimization of the Hamiltonian leads to a linear programming problem. The solution method can be characterized as a successive solution of a number of small linear programming problems (about 50 rows and 40 columns), defined and linked together by means of the maximum principle. It can also be interpreted as a decomposition algorithm of a large linear programming problem. The numerical calculations are carried out on a process computer, an IBM 1800, connected to the plant.

The thesis consists of the paper

- Production control of a complex integrated pulp and paper mill [19],

the report

- Mathematical methods of a pulp and paper mill scheduling problem [21],

and

- this summary.

Material on the Gruvön scheduling problem has also been published in [3, 4, 20].

The first publication of the thesis is a technically oriented description of the problem. A mathematical model of the mill is presented and its accuracy is discussed. Planning objectives are defined and the mathematical formulation of the problem is discussed briefly. The result of a test example is given.

The second publication deals with the mathematical formulation and solution of the problem. Optimality criteria are discussed and formulated mathematically. The maximum principle is applied to the problem and the solution method of the problem is presented in detail, using a simple 2-dimensional model as an illustration. Several test examples are given.

This report is intended to be an introduction to the problem and a summary of the results. The practical implementation is also reported.

In ch. 2, the Gruvön mill is presented. In ch. 3, the background and purpose of the scheduling system is discussed and related to the control hierarchy of the mill. The mathematical model of the mill is presented in ch. 4, and in ch. 5 the formulation and solution of the problem is given. The practical implementation of the system and experience of system performance, based upon six months of operation, are reported in ch. 6. In order to illustrate the actual performance, a planning example, taken from routine planning at the mill, is presented in ch. 7. Some possible extensions and improvements are discussed in ch. 8.

For readers not familiar with the pulp and paper industry, a brief description of the processes of a sulphate pulp and paper mill is given in an appendix.

2. THE GRUVÖN PRODUCTION SYSTEM

In Fig. 1, a block diagram of the Gruvön mill is shown. The mill consists of a sulphate pulp mill (eight batch digesters and one continuous digester), a bleach plant (75,000 tons/year), a kraft paper mill (150,000 tons/year) with five paper machines, an NSSC† pulp mill and a paper machine for corrugating medium†† (130,000 tons/year). The chemical recovery system consists of an evaporation plant, two Tomlinson recovery boilers, two parallel causticizing lines and a lime kiln. The NSSC digester is supplied with cooking liquor from a cooking liquor preparation plant. The steam flow from the recovery boilers and from a combined bark and oil burning boiler is fed to turbines before distribution to the different processes. Most of the production units are separated by storage tanks.

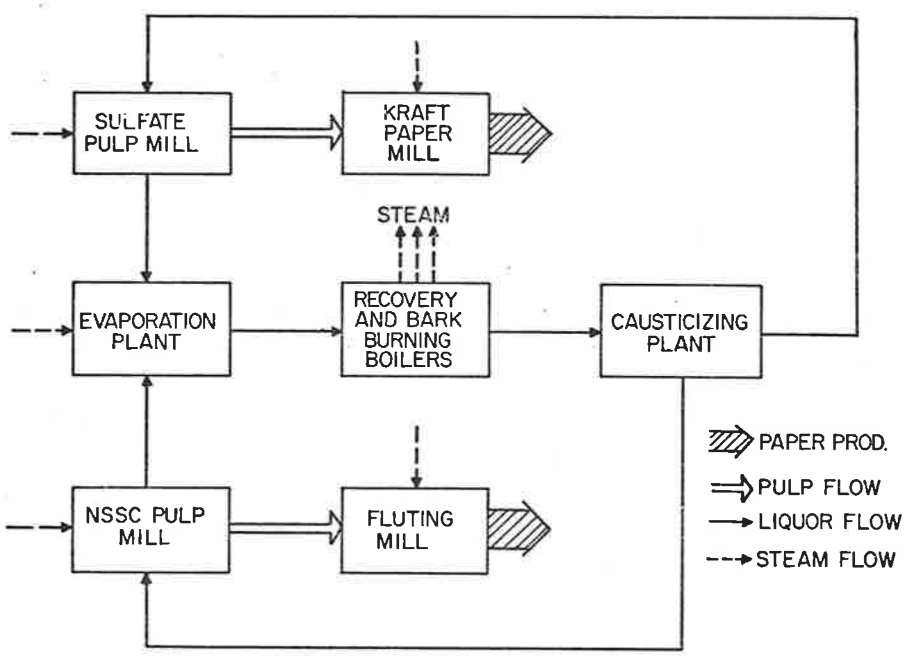

Fig. 1. Block diagram of the Gruvön mill.

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† NSSC = Neutral Sulphite Semi-Chemical

†† The wavy layer of corrugated paper-board (also called fluting)

The processes of the two pulp and paper mills are strongly interconnected by the following:

- cross recovery of chemicals
- common steam system
- possible use of kraft fibres in the corrugating medium
- possible use of green liquor to the NSSC digester

Because of these interconnections, the different parts of the mill greatly influence each other [8, 19]. Careful planning of the production of the processes and systematic utilization of storage capacities are thus necessary. The computerized production control system has been developed in order to help the plant managers in this respect.

3. PRODUCTION CONTROL AS PART OF AN INTEGRATED CONTROL SYSTEM

In 1964, Billerud installed its first process computer, an IBM 1710, at the Gruvön mill. This installation was devoted to process control and supervision of a kraft paper machine and to production planning of the kraft paper mill [2, 12]. In 1968, the second computer, an IBM 1800, was installed in connection with the start-up of the fluting production line. The 1800 system covers closed loop control of the NSSC digester and the fluting machine, operator guide control of parts of the chemical recovery system, production scheduling (production control) of the whole mill and reporting of production and quality data for several processes of the mill [2].

A hierarchical control philosophy as been applied to the Gruvön mill. The control system is divided into three levels with the frequency of action as a basis [1] (cf Fig. 2):

- production planning (frequency: weeks and days)
- production control (days and hours)
- process control (minutes and seconds)

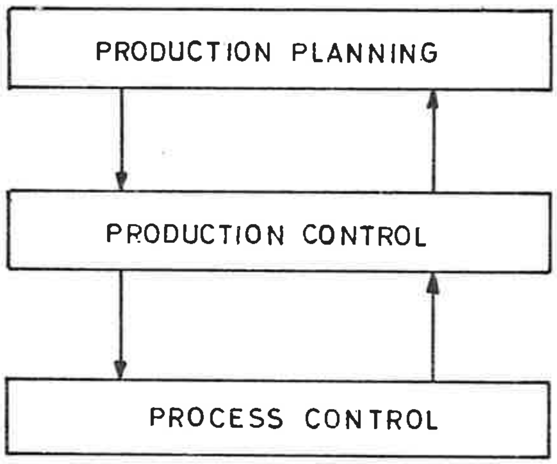

Fig. 2. The control hierarchy of the Gruvön mill is divided into three levels with the frequency of action as a basis.

Production planning involves allocation and sequencing of orders to the paper machines. Based upon this planning, the production of the pulp mills and the recovery system is scheduled (production control). Process control has the highest frequency. Its purpose is to control and supervise individual processes and control loops.

The purpose of production control is to help the plant managers to co-ordinate the production of the different processes of the mill. Such co-ordination has of course always been done and is thus nothing new. However, the production systems tend to be more and more complex and consequently more difficult to survey. Simultaneously, the production capacities of the processes are increasing and one hour's shut-down means an increasing loss of revenue. This was envisaged by the management of the company [8] and the production control project was started in order to investigate the possibilities of utilizing computer technology when co-ordinating the production of the mill.

4. MATHEMATICAL MODEL OF THE MILL

In order to attack the production co-ordination problem quantitatively, a mathematical model of the mill has been developed [19, 25]. The model is illustrated in Fig. 3. It consists of

- 3 paper machines, producing bleached kraft paper, unbleached kraft paper and corrugating medium (fluting) respectively. In reality these three model machines correspond to six actual paper machines. The paper production is assumed to be known as a function of time during a period of 2-3 days (the planning period).
- 9 process units, the productions of which are to be calculated as a function of time during the planning period,
- 10 storage tanks, the contents of which are assumed to be known at the beginning of the planning period.

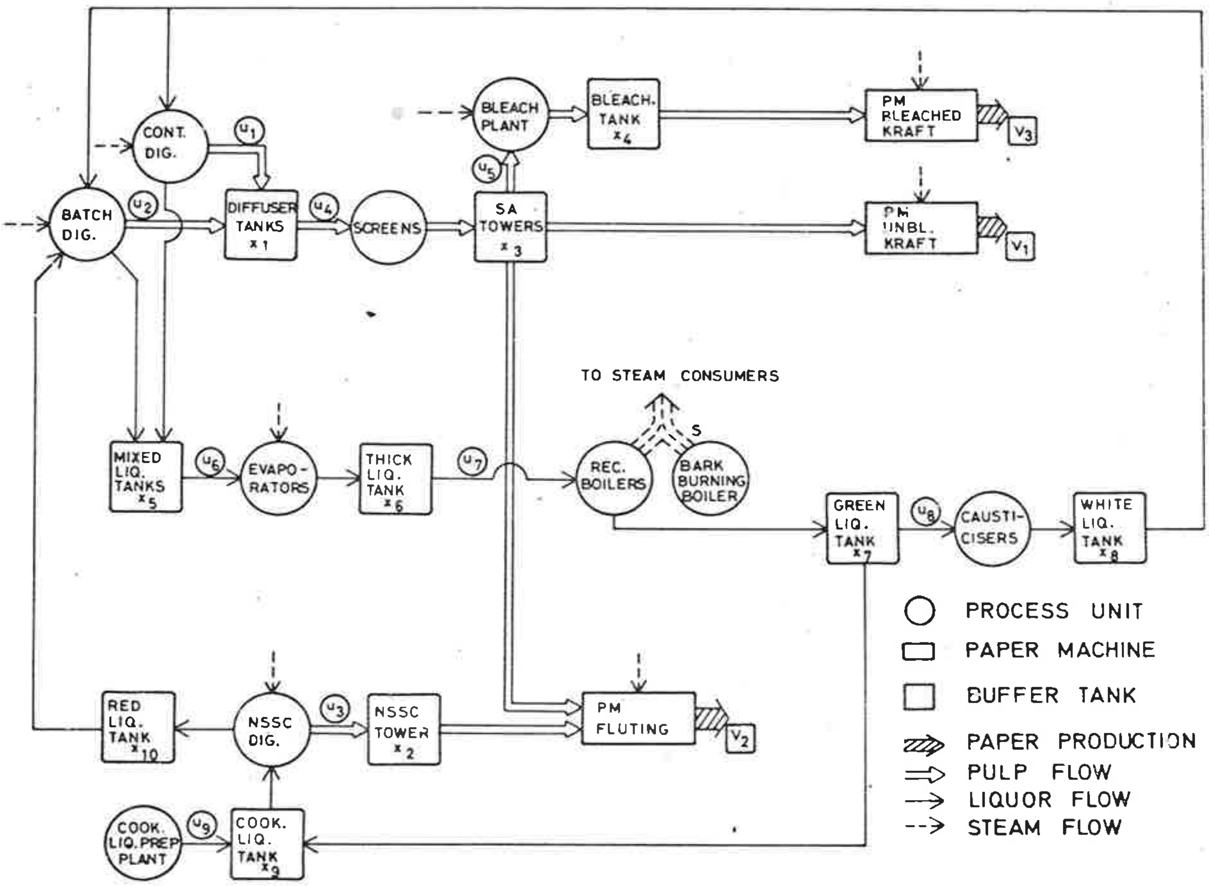

Fig. 3. Model of the Gruvön mill. The model consists of 3 paper machines, 9 process units and 10 storage tanks.

The model units are interconnected by flows of pulp, liquors and steam. No steam buffer exists. The ratio between flows around a process is assumed to be constant. The dynamics of individual processes are neglected.

We will now describe the model as per Fig. 3 mathematically. The storage tank levels $x_1,\ldots,x_{10}$ are introduced as <u>state</u> of the system. The productions $u_1,\ldots,u_9$ of the process units are chosen as <u>control variables</u>. The given paper production $v_1,\ldots,v_3$ is regarded as a <u>disturbance</u> of the system. To describe the steam balance, an extra variable, S, corresponding to steam production of the bark and oil burning boiler, is introduced.

In vector notation, the relations between the variables of the system are described by

$$\frac{dx(t)}{dt} = B \cdot u(t) + C \cdot v(t) \qquad (1)$$

$$S(t) = D \cdot u(t) + E \cdot v(t) \qquad (2)$$

B, C, D and E are time-independent coefficient matrices, describing relationships between flows of the system.

The variables of the model are constrained by capacity limits, described by

$$x_i^{min} \leq x_i(t) \leq x_i^{max} \qquad i = 1,\ldots,10 \qquad (3)$$

$$u_i^{min} \leq u_i(t) \leq u_i^{max} \qquad i = 1,\ldots,9 \qquad (4)$$

$$S^{min} \leq S(t) \leq S^{max} \qquad (5)$$

The construction of the model has been achieved by a combination of analysis, simulation and experimental verification. The first version of the model was based on a knowledge of the physical relationships [22] . The quantitative relations thus obtained were refined by measurements performed during one month on the actual plant. Simulations, based upon these measurements, were then performed in order to check the dynamic behaviour of the model. The results of one of the simulations are shown in Figure 4. The simulations and the experimental verification of the model are further commented upon in [19] . Although the model is simple, the simulations showed that it describes the complex plant in a satisfactory manner.

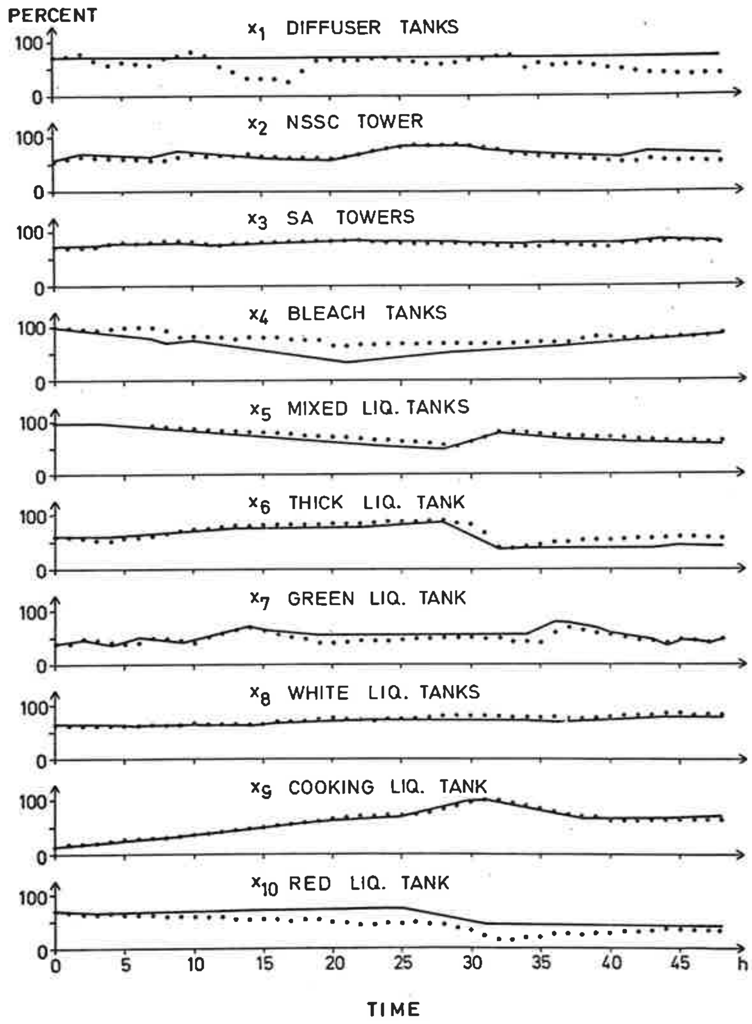

Fig. 4. Simulations based upon measurements. Ordinates: tank levels in percent of tank capacity. The dots show the measured tank levels, the curves the simulated ones. The level of x_1 was put constant in the simulation, cf. [19] .

5. FORMULATION AND SOLUTION OF THE CO-ORDINATION PROBLEM

5.1. Problem formulation

The production co-ordination problem has been formulated in the following manner:

Given

- planned paper production during a period of 2-3 days (the planning period)
- tank levels at the beginning of the planning period
- desired tank levels at the end of the planning period
- planned maintenance shut-downs during the planning period,

calculate

- production schedules for the process units of the model, minimizing the total number of production rate changes,

with due allowance for

- capacity restrictions of processes
- capacity restrictions of tanks
- steam balance of the system (production and consumption of steam must always balance).

The mathematical formulation of the system performance criterion defined above is not obvious. We have found [21] that the functional

$$J(u) = \int_0^T \| u(t) - a(t) \| dt$$

where $\|y\|$ is the norm

$$\|y\| = \sum_i |y_i|$$

will give production schedules with few production rate changes, provided the vector a(t) is suitably chosen. a(t) can be physically interpreted as a desired mean value of the production vector u(t) during the period. An initial value of a(t) can be calculated from the given input data. The choice of loss function has been found to be a reasonable compromise between realism and solvability. It should, however, be emphasized that there might be other alternatives.

Linear programming [13] was our first approach towards the production scheduling problem [20,22] . The magnitude of the LP problem obtained was about 800 rows and 500 columns [21, 23] . The execution time for a typical problem was 40 minutes on an IBM 7044 [23] . The problem size was thus far beyond the computer capacity available (IBM 1800).

Simulations have also been performed [20, 24] . This method led to short execution times on the computer but the manual work was so much more. Thus, simulation is not really useful during operating conditions, but it was found to be a very valuable means of getting insight into the problem.

Dynamic programming [7] was judged to be unrealistic due to the dimensionality of the problem [5] . An approach based upon the Pontryagin maximum principle and discussed in the next section turned out, however, to be successful. A survey of different formulations of the problem and a discussion of the computational merits of the different approaches are given in [5] , where it is also indicated that an approach through optimal control theory might indeed be feasible. This has been verified in later works [25] .

5.2. Mathematical preliminaries.

The idea of solution is based upon the continuous form of the Pontryagin maximum principle [6, 26] . The minimization problem obtained cannot be solved in a straightforward manner and we do not even know if a solution exists. However, based upon the continuous formulation, the problem was discretized and a feasible numerical algorithm was developed.

In order to motivate the numerical algorithm described in the next section we will first formulate the problem in continuous form according to the maximum principle. We will emphasize, however, that the treatment is quite formal since we do not know if a solution exists and since the domains and functions involved are not sufficiently smooth [26] .

The system, illustrated in Fig. 3, is described by

$$\frac{dx}{dt} = B \cdot u(t) + C \cdot v(t) \tag{1}$$

where

$x(t)$ is an n-vector of state variables (n=10)

$u(t)$ is an m-vector of control variables (m=9)

$v(t)$ is a given vector function (the planned paper production)

t is the time

B and C are time-independent matrices.

The control space as well as the state space are constrained:

$$u(t) \in \Omega_u \subset E^m, \quad x(t) \in \Omega_x \subset E^n$$

Ω_u is a convex hyperpolyhedron, described by

$$u_i^{min} \leq u_i(t) \leq u_i^{max} \qquad i = 1,\ldots,m \tag{4}$$

$$S^{min} \leq S(t) = D \cdot u(t) + E \cdot v(t) \leq S^{max} \qquad (2), (5)$$

where D and E are time-independent row vectors. Ω_x is a hyper-parallelepiped, described by

$$x_i^{min} \leq x_i(t) \leq x_i^{max} \qquad i = 1,\ldots,n \tag{3}$$

The initial state x(0) and the final state x(T) are assumed to be known. Consider the system during the fixed period $0 \leq t \leq T$.

Our problem is to find a control strategy u(t), $0 \leq t \leq T$, $u(t) \in \Omega_u$, $x(t) \in \Omega_x$, minimizing the performance functional

$$J(u) = \int_0^T \| u(t) - a(t) \| \, dt$$

where $\| y \|$ is the norm

$$\| y \| = \sum_i | y_i |$$

and a(t) is a given vector function.
Assume that a solution exists and that the functions and domains involved are sufficiently smooth. The maximum principle now states that a necessary condition for minimum of the performance functional J(u) is that the Hamiltonian function

$$H(x,u,p,t) = \| u(t) - a(t) \| + \langle p(t), B \cdot u(t) + C \cdot v(t) \rangle$$

is minimized as a function of u. $\langle a,b \rangle$ denotes the scalar product of the vectors a and b, and p(t) is a new vector function, satisfying the ordinary differential equation

$$\frac{dp(t)}{dt} = - \frac{\partial H(x,u,p,t)}{\partial x} \tag{6}$$

if the optimal trajectory of the system lies in the interior of Ω_x, and the ordinary differential equation

$$\frac{dp(t)}{dt} = - \frac{\partial H(x,u,p,t)}{\partial x} + \lambda(t) \cdot \frac{\partial \varphi(x,u)}{\partial x} \qquad (7)$$

if the optimal trajectory lies on the boundary of Ω_x. The boundary condition of equations (6) and (7) is

$x(T)$ given

In eq. (7), $\lambda(t)$ are certain Lagrange multipliers [26]. φ is defined by

$$\varphi(x,u) = \langle \text{grad } g(x),\ B \cdot u(t) + C \cdot v(t) \rangle$$

where

$$g(x) = 0$$

describes the boundary of Ω_x.

Since $v(t)$, $0 \leq t \leq T$, is known, a necessary condition for minimum of $J(u)$ is that

$$\| u(t) - a(t) \| + \langle p(t),\ B \cdot u(t) \rangle$$

is minimized as a function of u.

In [21], the following statements are proved:

- $p(t)$ is piecewise constant
- when a storage tank reaches a limit, the corresponding element of $p(t)$ makes a jump to a new constant value, while the other components remain unchanged.

These simplifications of the problem are due to the simple structure of the system, described by eq. (1), and to the simplicity of the state space constraints (eq. (3)).

Thus, the following function is to be minimized:

$$\| u(t) - a(t) \| + p_o^* \cdot B \cdot u(t)$$

where p_o^* is the transpose of the piecewise constant vector function $p(t)$ and B is the system matrix.

There is no direct way of solving this minimization problem and consequently some iterative technique must be used. An immediate approach is the following: guess the p-vector, solve the minimization problem and iterate until the desired boundary value $x(T)$ is reached. The search for p-vectors can be considerably simplified by introducing a new vector A, related to p by

$$A = p_o^* \cdot B$$

It can be shown [21] that only a finite number of A-values influence the optimal solution. These values are:

A-value	Physical interpretation
$\vert A_i \vert < 1$,	implying that the production of process u_i is kept at the level a_i, if possible
$A_i < -1$,	implying that the production of process u_i is increased
$A_i > 1$,	implying that the production of process u_i is reduced
	u_j, $j \neq i$, is not affected

Instead of iterating over the p-values, we can iterate over A. Despite this considerable simplification, the iteration method described is impossible in practice since it leads to excessive computations [20, 21] . As will be described in the next section, another iteration technique has been derived leading to numerically feasible computations.

5.3. Numerical algorithm

The numerical algorithm developed uses iteration over the a-vector and utilizes the physical interpretation of the A-vector given above. The optimization problem is discretized by dividing the planning period into a number of time intervals. Each time interval gives rise to a small linear programming problem (about 50 rows and 40 columns). The LP problems are solved successively. The algorithm is not proved to be convergent. It generates, however, in a systematic way solutions that are quite satisfactory from a practical point of view. Besides, the demand for core storage and computational speed does not exceed the capacity of a process computer.

A detailed description of the solution method developed is given in [21] . A discussion can also be found in [3] . The solution technique can be summarized in the following steps:

1) An initial value of the vector a is calculated from input data. In order to illustrate the calculation, consider the sub-system of Fig. 5, consisting of one process and one tank.

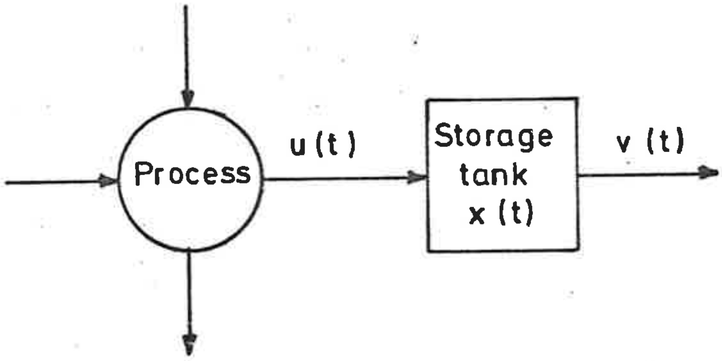

Fig. 5. A sub-system of the model. An initial value of the vector a can be calculated from a material balance over the tank.

Assume that the output v(t) of the tank is known, together with initial and final tank levels x(0) and x(T). A material balance over the tank gives:

$$x(T) = x(0) + \int_0^T u(t)dt - \int_0^T v(t)dt$$

Hence

$$\overline{u} = \frac{1}{T} \cdot \int_0^T u(t)dt$$

can be calculated. Generalizing to the system of Fig. 3, values $\overline{u_i}$ can be calculated, starting from the known outputs v_1, v_2 and v_3 of the system and calculating "backwards" through the system. We have found that

$$a_i = \overline{u_i}$$

is a good initial value of the vector a.

2) The planning period is divided into a number of time intervals, not necessarily of equal length. During each interval, the production of processes and paper machines are assumed to be constant.

3) For time interval No. 1, the following non-linear optimization problem is defined: Minimize

$$\sum_{i=1}^{m} \left\{ |u_i(1) - a_i| + A_i(1) \cdot u_i(1) \right\}$$

subject to capacity restrictions according to (2) - (5). Suitable values of A_i are chosen in accordance with the physical interpretation of the A-vector given above. Usually, all A_i are put equal to zero. However, if a shut-down of process No. i is shceduled for the interval, choose $A_i \gg 1$, etc

The non-linear optimization problem now defined is solved as a linear programming problem by introducing auxiliary variables in the standard manner. The size of the LP problem is about 50 rows and 40 columns.

4) The calculations give optimal values of $u_i(1)$. The system eq. (1) is then integrated for the first time interval. Using tank levels at the end of this interval as new initial tank levels, new values of a_i are calculated. The LP problem for interval No. 2 is then defined and solved, etc.

5) When reaching t = T, the end of the planning period, a production schedule for all processes of the mill has been obtained.

The numerical algorithm can thus be described as a successive solution of a number of small linear programming problems, defined and linked together by means of the maximum principle. It can also be interpreted as a decomposition algorithm of a large linear programming problem.

Computer programs, written in Basic FORTRAN IV, have been developed in order to carry out the calculations [18]. The total program size, including programs for input and output, corresponds to about 30.000 16-bit words on the IBM 1800. The total execution time on the computer (cycle time 4 μs, software floating-point arithmetic) lies between 1/2 and 2 hours, depending on the problem size and the load of priority programs.

6. PRACTICAL IMPLEMENTATION

The computerized scheduling system has been implemented at the Gruvön mill of Billeruds AB. It was started up in November 1969 and has since then been in permanent operation [x)]. The production control system was positively accepted by the users and it has proved to be a useful tool in co-ordinating and controlling the production of the different sub-processes of the mill [4].

The production control programs are used by the planning personnel at the mill in the following way. At a daily meeting, the paper production is planned and maintenance shut-downs are fixed for the current planning period (usually 2 days). Based upon this information, data cards are punched and the production control programs are initiated. Other inputs to the programs are current tank levels, division of the planning period into time intervals and desired tank levels at the end of the planning period. Current tank levels are automatically fed into the computer memory by means of analog inputs. Desired final tank levels have been fixed to standardized values (cf. the planning example). Division of the planning period into time intervals is made by the user by means of simple rules [17].

The programs are run as a background job on the IBM 1800 computer. After finishing the calculations, the computer prints the calculated schedule on a line printer. The machine lists contain the planned paper production (input data), the calculated production schedule and a prognosis of the tank level variations, provided that the schedule is followed. The lists are immediately distributed to plant superintendants and shift foremen.

Some experience of the actual performance of the system, based upon 6 months of operation, will now be described.

Compared with the model of Fig. 3, the model of the mill has been further simplified. Since tanks x_1 (diffuser tanks) and x_7 (green liquor tank) are small, these tanks have been skipped together with processes u_3 (screens) and u_8 (causticizers). Hence, the production of these processes are put proportional to the calculated production of the sulphate digesters and recovery boilers respectively. It is interesting

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[x)] This report was written in May 1970.

to note (cf the planning example) that this very simple model describes the complex plant in a quite satisfactory manner. It is believed, however, that the model cannot be further simplified without seriously risk the performance of the system.

Considerable work has been devoted to keeping the execution times within reasonable limits. The total execution time now varies between 30 minutes and 2 hours, depending on the load of priority programs and the complexity of the planning problem. As a comparison, it can be mentioned that the execution time for a typical planning problem on an IBM 360/40 is about 2 minutes and on a CDC 3600 about 15 seconds. However, during the autumn of 1970 the process computer capacity of the Gruvön mill will be increased [4]. It is believed that the execution time will then be reduced to 5-10 minutes.

Reliability of input data is crucial for the performance of the scheduling system. The most important input is planned paper production including planned maintenance shut-downs. The paper production is stated as planned gross production for the 6 individual paper machines. However, only net paper production is of interest to the pulp demand since the machine broke system is considered to be an internal circulation of the paper machine [19]. Thus, the gross productions have to be multiplied by "efficiency factors", individual for each machine and compensating for average trim losses, wet and dry broke, sheet breaks, etc. Such factors have been calculated, based upon production statistics. The factors are updated monthly.

Thus, predicted net paper production and planned maintenance shutdowns are used as inputs to the production optimization programs. This means that small disturbances to paper production, distributed over the planning period, are taken into account. If major production disturbances occur (e.g. an unplanned wire change), the scheduling breaks down (cf the planning example). On such occasions, a new production scheduling should ideally be performed as soon as the length of the unplanned shutdown can be estimated. However, neither manpower resources nor available computer capacity permits such replanning at the mill.

When planning the production control system, the steam balance of the mill was believed to be a critical restriction [19]. During the implementation of the system, the steam production capacity of the mill was increased and the steam system is no longer a major problem. Instead, sulphate pulp capacity is now a bottleneck of the system, due to expanded paper production (cf the planning example). However, an expansion of the pulp capacity is under construction and within a year there will be overcapacity of pulp. This situation is believed to be typical: the production system is never stationary. However, the scheduling system developed is very flexible and changes in the production system can easily be taken into account by changing vectors and matrices. This is further commented upon in ch. 8.

7. A PLANNING EXAMPLE

In order to illustrate the actual performance of the system, a planning example is presented in this section. The example is taken from routine planning, performed in accordance with the description in the previous chapter. The planning example is illustrated in Fig. 6.

The planning period was 50 hours. Planned (net) production is shown by the coarse lines of Fig. 6A. In this planning, a wire change on the largest kraft paper machine during 20 h $\leq$ t $\leq$ 29 h was included. Maintenance shut-downs were scheduled for the continuous sulphate digester during 20 h $\leq$ t $\leq$ 28 h, evaporators during 23 h $\leq$ t $\leq$ 26 h and one of the recovery boilers during 23 h $\leq$ t $\leq$ 28 h. The following (standardized) final tank levels were used: 80 % in the tank for unbleached pulp, 70 % in the tank for bleached pulp, 20 % in the red liquor tank, 50 % in the remaining tanks. Tank levels at the beginning of the planning period are automatically read into the computer when the scheduling programs are executed.

In Fig. 6B, the calculated production schedule (coarse lines) and in Fig. 6C the tank level prognosis (coarse lines) are shown. In Fig. 6B, the dashed lines mark the production capacities of the processes and the horisontal arrows of Fig. 6C illustrate the desired final tank levels.

From Fig. 6B it can be seen that the schedule only contains production changes caused by the planned maintenance shut-downs of the processes. The desired tank levels are predicted to be reached with the exception of the tank for unbleached pulp (Fig. 6C). The reason for this is that the production of the sulphate pulp digesters cannot be further increased (Fig. 6B).

Thin lines of Fig. 6A and 6B illustrate how the production units of the mill were actually run during the planned period. The follow-up ceases at t = 48 h, when the next planning period begins. The data collection has been performed by means of the reporting routines of the 1800 system [2]. Manual data collection was thus necessary only for the batch digesters and the kraft paper machines.

The first two figures of Fig. 6 A show that the kraft paper machines were run in accordance with the planning. The disturbances are small and the net production is obtained in average. The calculated schedule has been followed in the sulphate pulp mill and in the recovery system (Fig. 6B). However, disturbances have occurred in the continuous digester at

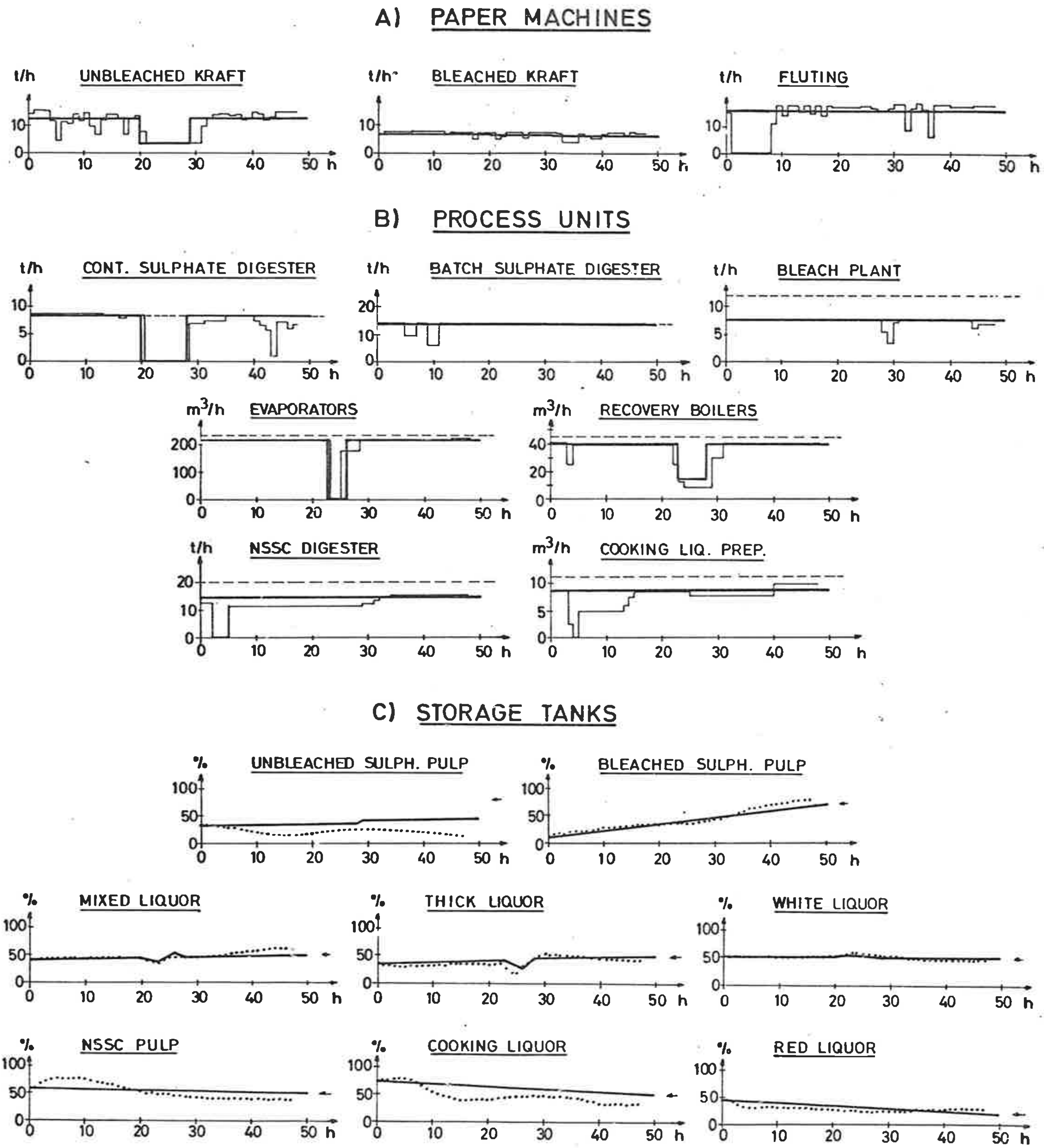

Fig. 6. A planning example, taken from routine planning at the Gruvön mill.
In figures A, coarse lines mark planned paper production, in figures B the calculated production schedule and in figures C predicted tank level variations. Thin lines in figures A and B and dotted lines in figures C mark the actual running of the mill. Dashed lines in figures B mark maximal production capacity. Desired final tank levels are marked by horisontal arrows in figures C.

$t \approx 45$ h and in the batch digesters at $t \approx 10$ h, causing a certain loss of pulp production. Since scheduled pulp production equals maximal production, there was no possibility of compensating for the production losses. As a consequence, the actual final level of unbleached pulp is lower than the predicted one. Other tanks in the sulphate pulp mill and in the recovery system show good agreement between prognosis and reality.

However, it was not possible to follow the schedule in the fluting mill. The reason is seen in Fig. 6A, showing a long unplanned shut-down of the fluting machine. As a consequence, the production of NSSC digester and cooking liquor preparation plant had to be reduced. Towards the end of the planning period, when the production conditions of the fluting mill had been stabilized, the agreement between schedule and reality is good.

As can be seen from the figures, the calculated schedule is followed whenever possible. The model describes the complex plant in a quite satisfactory manner. Small disturbances have little influence on the scheduling. Major disturbances, such as unplanned shut-downs for several hours, does not happen very often. When they do happen, however, the scheduling breaks down.

8. RELATED WORK, EXTENSIONS AND IMPROVEMENTS

In ch. 3, a hierarchical approach to the control of a complex plant was presented. This multi-level philosophy has been discussed by several authors [9, 11, 16]. For the higher levels and for large-scale systems with many interacting processes, however, the dimensionality in the optimization of the system performance is a great problem. A decentralization algorithm has been suggested by Lasdon and Schoeffler [16] in order to solve this problem. In their notation, the solution method developed in this thesis can be considered as an optimization of a number of sub-systems, where each sub-system is equivalent to the mill system during one time interval.

The performance of the mill is measured by a scalar-valued criterion. For complex systems, however, the demand for scalar-valued criteria is a serious weakness of current theories of optimal control. This was illustrated during an earlier stage of the development of our system, when the wish for specified final tank levels was included in the performance criterion [21, 23]. It proved to be very difficult to quantitatively weigh a production rate change against a certain divergence from the desired final tank level in one single scalar function. The possibility of formulating non-scalar-valued criteria has been discussed by Zadeh [28]. However, no worked-out methods for vector-valued optimization are yet available.

Two papers dealing with pulp mill scheduling have been published recently [10, 27]. The main difference between these approaches and ours is the dimensionality of the problems. The system of Brewster and Al-Shaikh [10] consists of one process and two tanks and the scheduling problem is solved analytically. Svensson's system [27] consists principally of one process and one tank. However, the power generation is an important restriction of his system. The scheduling problem is solved by one-dimensional search.

The model and the solution method presented in this thesis are quite flexible. It is an easy matter to add more tanks and processes by changing system matrices and vectors. It should be noted, however, that the problem size increases rapidly with the number of processes and tanks of the model. If the sum of processes and tanks is N, the demand for core memory is roughly proportional to N^2 and the execution time to N^3.

The work was initiated by a practical planning problem at a specific pulp and paper mill. The model and the solution method are, however, believed to be applicable to other production systems, consisting of production units separated by storage capacities.

As has been emphasized [10], scheduling and tank sizing are dual problems that should be solved simultaneously at the design stage. Thus, plant modeling, based upon eq. (1) and (2), has been used as a tool for designing storage capacities for future expansion of the Gruvön mill.

In the system developed, the division of the planning period into a number of time intervals must be done by the user. The choice of time intervals can, however, influence the total number of production rate changes during the planning period [21]. Thus, the quantization of the planning period should ideally be iterated on and chosen by the system itself.

Even for a fixed quantization of the planning period, the optimal solution is not unique. For a specific planning example, using a simple 2-dimensional model, it has been shown [21] that there were an infinite number of solutions with the same minimum number of production changes. This fact gives us the possibility of defining sub-criteria. Some possible sub-criteria are discussed in [21]. Since computational time has been critical, no attempts have been made to improve the solutions by an iteration over planning period quantization or by defining sub-criteria.

A general investigation of the mathematical properties of the optimization problem defined in this thesis might be fruitful. Particularly, more close attention should be devoted to the consequences of different mathematical formulations of the performance criterion.

The productions of the paper machines are used as an input to the system developed. Ideally, planning of paper production and maintenance planning should also be included when trying to optimize the performance of the whole mill. However, when including the paper machines in the optimization, sales planning and shipping of finished products should also be accounted for, implying that we are entering the marketing field. The potential benefits of integrating process and business control systems have been pointed out by several authors [11, 14, 15]. A generalization in this direction of the control system described in this work should thus be a fruitful task for future work.

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APPENDIX. The processes of a sulphate pulp and paper mill

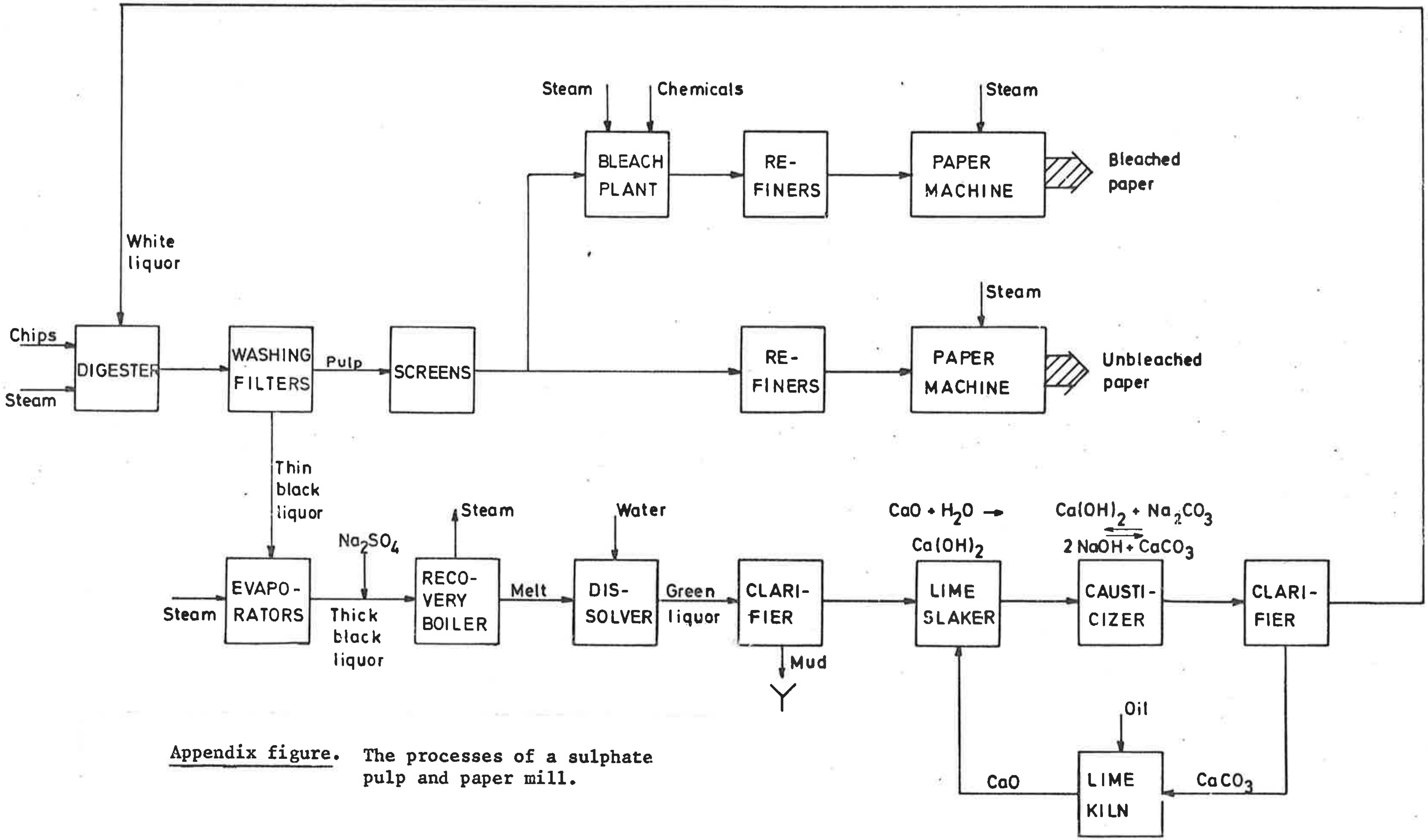
For readers not familiar with the pulp and paper industry, a brief description of the processes of a sulphate pulp and paper mill is given in this appendix.

In the appendix figure, a simplified flow diagram of a sulphate pulp and paper mill is shown. Wood chips and chemicals (primarily NaOH and Na_2S), "white liquor", are heated for 1-2 hours at about 170°C in the digester. Washing filters separate the cellulose fibres and the liquor, now containing inactive chemicals and lignin ("thin black liquor"). After screening, bleaching and refining, the fibre suspension is fed to paper machines. When entering the paper machine, the suspension holds less than 1 % fibres. The fibre suspension is distributed over an endless rotating wire-net, where most of the water is drained. After pressing and drying on steam-heated cylinders, the finished paper contains about 5 % water.

The inactive chemicals are reactivated in the recovery system. The weight fraction of solids in the thin black liquor is increased by the evaporators, giving "thick black liquor". The thick liquor is burnt in recovery boilers, having two missions: the heat content of the lignin is taken care of and the sulphur of the liquor is reduced (activated). The melt from the recovery boiler is dissolved and clarified (a sedimentation process), giving "green liquor". Green liquor and burnt lime (CaO) is fed to slakers, where the burnt lime is slaked. In causticizers, the slaked lime ($Ca(OH)_2$) reacts with Na_2CO_3 from the green liquor, giving NaOH (an active chemical) and $CaCO_3$. In a sedimentation process, the lime sludge ($CaCO_3$) is separated from the liquor, now called "white liquor" and ready for use in the digesters again. The $CaCO_3$ is converted to burnt lime in a lime kiln.

The inevitable loss of chemicals is compensated for by feeding Na_2SO_4 into the system. This has given rise to the name sulphate pulp.

Most of the processes are separated by storage tanks in order to make different departments more independent of each other.



Appendix figure. The processes of a sulphate pulp and paper mill.

BENGT PETTERSSON

Production Control of a Complex Integrated Pulp and Paper Mill

The mill described comprises a sulfate pulp mill, a kraft paper mill, a mill for semi-chemical neutral sulfite (NSSC) pulp, and a paper machine for corrugating medium. The chemicals from the pulp mills are cross-recovered. The ratio between the NSSC and sulfate pulp productions is as high as 2:3. The steam capacity of the system is limited. Coordinating the production of the different processes is therefore a difficult problem. A method has been developed to help plant managers decide upon good production schemes. A mathematical model, describing the plant, has been built and verified by measurements carried out during normal operating conditions. The production-coordination problem is formulated as an optimization problem and solved by a technique utilizing optimal control theory. Production schemes covering all the plant processes for a period of 2–3 days are calculated on the basis of planned paper production which is assumed to be known. The calculations are carried out on an IBM 1800 process computer connected to the plant. Despite the complexity of the plant, the model describes it in a proper manner. The solution technique has been found to give good production schemes.

Keywords: Mills · Coordination · Optimization · Production control · Scheduling · Computers · Linear programming · Mathematical models · Optimal control theory*

NOMENCLATURE

$x(t)$ = state vector, describing levels of the buffer tanks

Components:

- x_1 = diffuser tanks
- x_2 = NSSC pulp tower
- x_3 = sulfate pulp (SA) towers
- x_4 = bleached pulp tanks
- x_5 = mixed liquor tanks
- x_6 = thick liquor tank
- x_7 = green liquor tank
- x_8 = white liquor tanks
- x_9 = NSSC cooking liquor tank
- x_{10} = red liquor tank

$u(t)$ = control vector, describing production of the processes

Components:

- u_1 = sulfate pulp from continuous digester
- u_2 = sulfate pulp from batch digesters
- u_3 = NSSC pulp from NSSC digester
- u_4 = sulfate pulp to screens
- u_5 = screened pulp to bleach plant
- u_6 = mixed liquor to evaporators
- u_7 = thick liquor to recovery boilers
- u_8 = green liquor to lime slakers
- u_9 = cooking liquor from cooking liquor preparation plant

$v(t)$ = disturbance vector, describing planned paper production

Components:

- v_1 = unbleached kraft paper production
- v_2 = production of corrugating medium
- v_3 = bleached kraft paper production

S = steam from bark and oil burning boiler

T = length of planning period

$x(O)$ = levels of tanks at the beginning of planning period

$x(T)$ = levels of tanks at the end of planning period

B, C, D, E = coefficient matrices, describing relations between flows

b = element of the B matrix, describing the ratio between thick and thin liquor flows

BENGT PETTERSSON, Billeruds AB, Säffle, Sweden

IN 1964, Billerud installed its first process computer, an IBM 1710, at the Gruvön mill. The computer is used for process control and supervision of a kraft paper machine and for production planning for the entire kraft paper mill (*1*). In June 1968, a new production line, incorporating an NSSC pulp mill and a paper machine for corrugating medium, was started up. The new line was designed for computer control, based on an IBM 1800 system. Operator guide control for certain processes in the chemical recovery cycle was also included (*2*). A new application, called production control, was also planned. It is essentially a production-scheduling system which will coordinate the different subprocesses of the plant and which is the subject of this paper.

THE COORDINATION PROBLEM

The Gruvön mill consists of a sulfate pulp mill (eight batch digesters and one continuous digester), a bleach plant (75,000 tons/year), a kraft paper mill (150,000 tons/year) with five paper machines, an NSSC pulp mill, and a paper machine for corrugating medium (130,000 tons/year). The chemical recovery system consists of an evaporation plant, two Tomlinson recovery boilers, two parallel causticizing lines, and a lime kiln. The NSSC digester is supplied with cooking liquor from a liquor preparation plant. The steam flow from the Tomlinson boilers and from a combined bark and oil burning boiler is fed to turbines before distribution to the different processes.

The pulp and paper mills are interconnected by the following: cross recovery of chemicals; common steam system; possible use of kraft fibers in the corrugating medium; possible use of green liquor to the NSSC digester. This is illustrated in the block diagram, Fig. 1.

The interconnection introduces difficulties in coordinating the production of the entire plant. Before the new production line existed, for example, it was natural to cut down the production of the sulfate digesters, the evaporators, and the recovery boiler while changing a wire on a large kraft paper machine. This is no longer advantageous and perhaps even impossible since the liquor from the NSSC digester must be taken care of and the steam supply to the new pulp and paper mill must be guaranteed. Thus, the use of the storage tanks has to be planned systematically. Furthermore, since no steam storage of any significance exists, the production scheduling must guarantee a balance between production and consumption of steam. Another difficulty arises from the steam system: since the steam capacity is somewhat low, there is a risk of a steam shortage.

Because of these difficulties in coordinating the production of the different processes, the production-control application has been developed for the purpose of helping the plant managers decide upon good production schemes, with due allowance for maintenance shutdowns, limited buffer capacity, and steam restrictions.

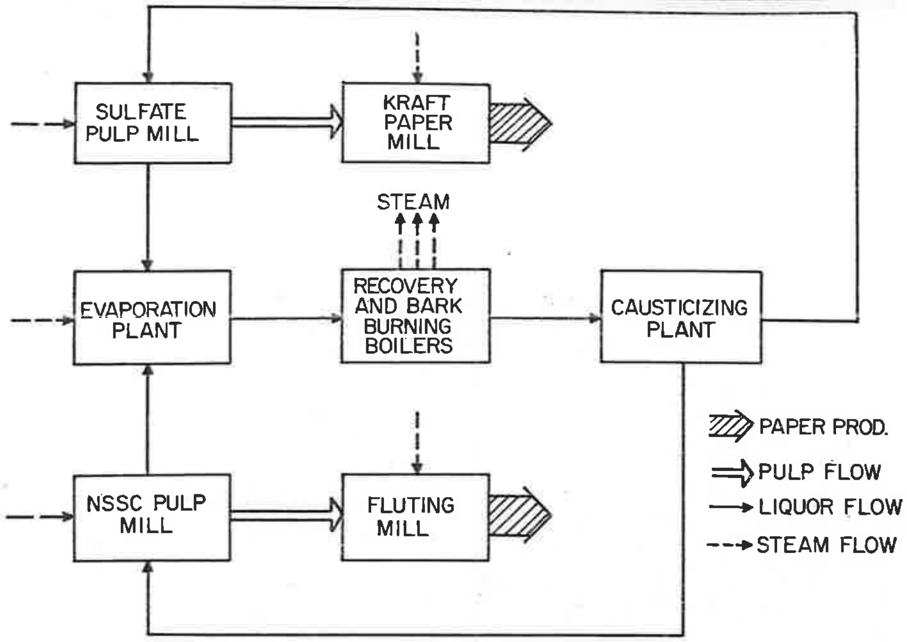


Fig. 1. Block diagram of the Gruvön plant. The chemicals of two pulp mills are cross-recovered.

PLANNING OBJECTIVES

The planning must obviously fulfill the following requirements: pulp production must satisfy the demand from the planned paper production; the storage tanks of the system must not be empty or overflowing; production and consumption of steam must balance.

To be able to choose between schedules, all satisfying the restrictions, criteria of a "good" solution must be stated. Usually, the objective of a problem like this is to minimize cost or maximize profit. The most economic way of running a mill is to maintain a high and uniform production for all processes. This objective can be expressed in economic terms. However, we have chosen to formulate the planning objectives in the following, synonymous, way: Calculate production schedules, involving

(1) few changes in the production rates of the processes
(2) possibility of indirect storage of steam
(3) acceptable tank levels at the end of the planning period.

Since any change in production rate introduces a disturbance in the system, it is desirable to keep each process production as even as possible. As mentioned above, the steam supply of the system is barely sufficient, and there is no real possibility of storing steam directly. It would, therefore, be desirable to store steam indirectly. This can be done by filling up the pulp buffers and the thick liquor tank during a period of low steam consumption, for example during a wire change. After this period, the high levels of these tanks will enable us to run the paper machines harder because there will be less demand for steam to digesters and evaporators. Condition (3) implies that tank levels at the end of the planning period must be at a good position before the next period commences. This means that final tank levels should usually be about 50%.

MODELING OF THE PLANT

Since the Gruvön plant is very complex, modeling is a difficult task. Apart from smaller tanks giving some freedom in running the processes, there are more than 20 tanks which can be used for storage. To include all tanks in the model would be impossible. In order to simplify the problem, our first model contained about 15 tanks and somewhat fewer processes. Attempts to take the dynamics of the alkali and sulfur balance of the system into consideration were also made. When formulating this problem mathematically, the magnitude of the problem turned out to be far beyond the capacity of our 1800 computer. Consequently, the complexity of the model had to be further reduced. Too great a simplification (for example, a model such as that in Fig. 1), however, does not give a satisfactory description of the plant.

Our present model is illustrated in Fig. 2. It consists of:

3 paper machines, producing bleached kraft paper, unbleached kraft paper, and corrugating medium, respectively. In reality, these three model machines correspond to six actual paper machines. The paper production is assumed to be known as a function of time during a period of 2–3 days (the planning period). The machine broke system is disregarded and considered as an internal circulation of the paper machine.

9 process units, where a process is understood to mean some arrangement in which a qualitative or quantitative change of flows is made. The ratio between the flows around a process is assumed to be constant.

10 buffer tanks, the contents of which are assumed to be known at the beginning of the planning period.

Chemicals are disregarded in the model, partly because of the wish for a simple model and partly because of a lack of information about mixing conditions in the buffer tanks. Thus, concentration of chemicals, pulp yield, weight fraction of solids, etc., are assumed to be constant.

The dynamics of individual processes are disregarded since the response times are often small (less than half an hour) when compared with the length of the planning period. This is not true of all the model processes, however, but the approximation has nevertheless been acceptable. As an example, it may be mentioned that the response time of the digesters is in the order of 2 hr.

Small tanks with a storage capacity for full production of about 1 hr or less have been disregarded and other tanks have been combined, thus reducing the number of tanks in the model.

The length of the planning period has been set at 2–3 days. A shorter planning period will give small possibilities for systematic use of the buffer capacities. Neither the accuracy of the model nor the

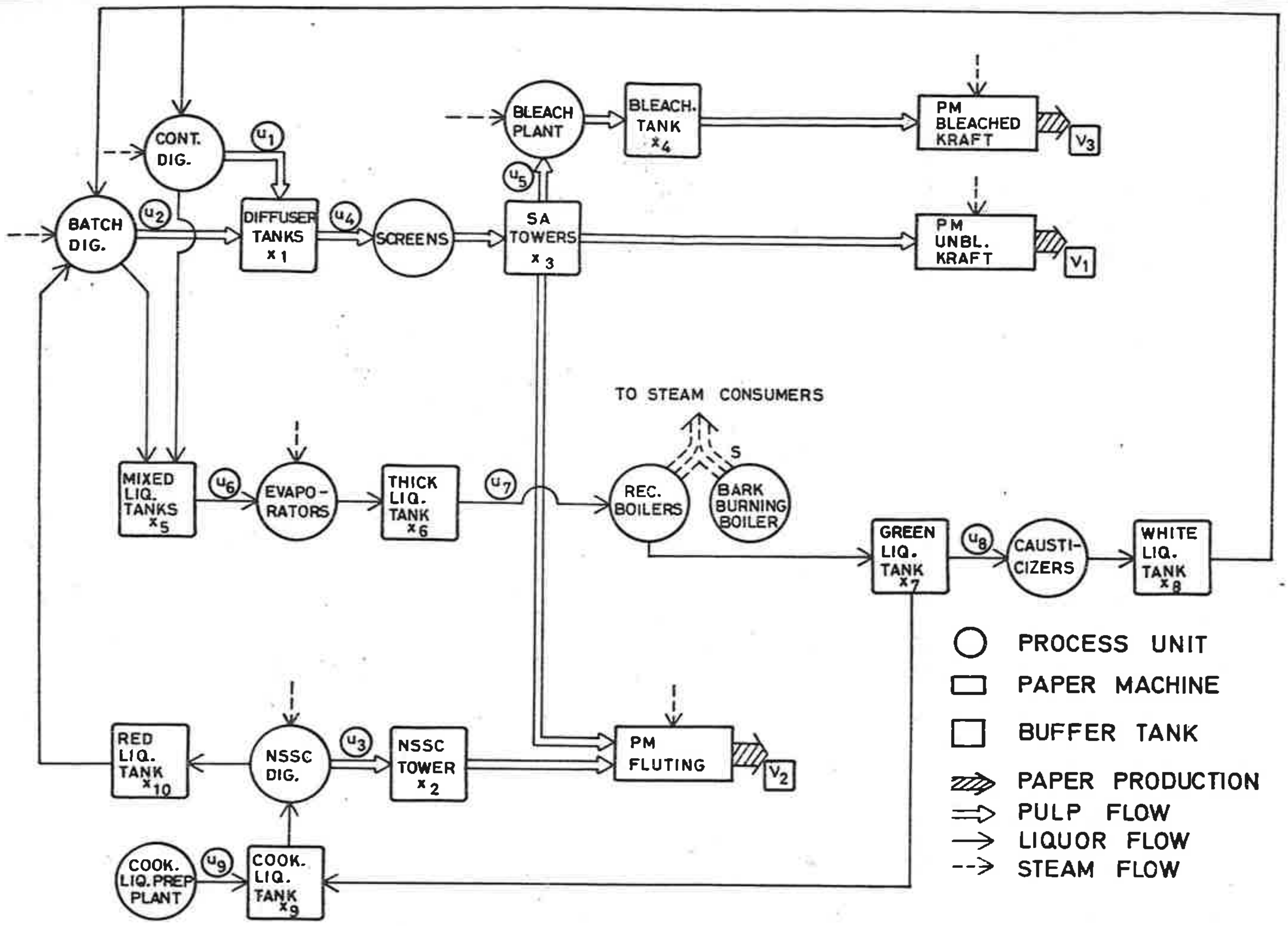


Fig. 2. Model of the plant. The model consists of 3 paper machines, 9 process units, and 10 buffer tanks, interconnected by flows of pulp, liquors, and steam. No steam buffer exists. The ratio between flows around a process is assumed to be constant. The dynamics of individual processes are neglected.

computer capacity permits a substantially longer period.

MATHEMATICAL DESCRIPTION

We will now describe the model shown in Fig. 2 mathematically.

The state of the system is described by the buffer tank levels $x_1, \ldots, x_{10}$ (cf. the Nomenclature). The numbers $x_1, \ldots, x_{10}$ are regarded as components of a vector, the state vector x. The productions of the processes are chosen as control variables $u_1, \ldots, u_9$, components of the control vector u. The given paper production is regarded as a "disturbance" of the system, denoted by the disturbance vector v with the components v_1, v_2, and v_3. To describe the steam balance of the system, an extra variable, S, corresponding to steam production of the bark and oil burning boiler, is introduced.

The relations between the state vector $x(t)$, the control vector $u(t)$, the disturbance vector $v(t)$ and the steam variable $S(t)$ are described by

$$dx(t)/dt = B \cdot u(t) + C \cdot v(t) \qquad (1)$$

$$S(t) = D \cdot u(t) + E \cdot v(t) \qquad (2)$$

The number of components of the column vectors x, u, and v is 10, 9, and 3, respectively. B, C, D, and E are coefficient matrices describing relationships between flows. The matrix sizes are 10×9, 10×3, 1×9, and 1×3, respectively.

The variables of the model are restricted by capacity limits, described by

$$x_i^{\min} \leq x_i(t) \leq x_i^{\max} \quad i = 1, \ldots, 10 \qquad (3)$$

$$u_i^{\min} \leq u_i(t) \leq u_i^{\max} \quad i = 1, \ldots, 9 \qquad (4)$$

$$S^{\min} \leq S(t) \leq S^{\max} \qquad (5)$$

EXPERIMENTAL VERIFICATION OF THE MODEL

The construction of the model has been achieved by a combination of analysis, simulation, and experimental verification. In this section, we will show some experimental results illustrating the accuracy of the model.

To illustrate what degree of accuracy can be expected, let us consider a numerical example with respect to the evaporation plant as described by the model. The level of the thick liquor tank (x_6) is described by (cf. Fig. 2):

$$\frac{dx_6(t)}{dt} = b \cdot u_6(t) - u_7(t) \qquad (6)$$

Here, b is the ratio between the thick and the thin liquor flows and has the magnitude 0.16. Assume that this value is correct and that the model calculations are performed with a b-value that is 5% higher, i.e., $b = 0.168$. A normal value for u_6 is $= 200$ m³/hr. During a period of 48 hr, the false value $b = 0.168$ will result in an error of 77-m³ thick liquor in the thick liquor tank. Since the capacity of the tank is 300 m³, the 5% error in b implies a 25% error in the tank level at the end of the period. Thus, the accuracy demands on the numerical values of the B- and C-matrices are high. Because of this, and as the model parameters can change with time, we are now developing a system for continuous on-line updating of the model parameters.

Simulations based upon measured data have been performed in order to check the model. Measurements of flows and tank levels were performed at Gruvön at normal operating conditions for 4 weeks, using the normal process instrumentation. The sampling interval was 1 hr. The elements of the B- and C-matrices were calculated by forming quotients of flows, simultaneously measured. If flow measurements were missing, elements were calculated from material balances over storage tanks. Measurements of $u_1, \ldots, u_9$ (process production rates) and $v_1, \ldots, v_3$ (paper production) were then used as inputs to the model. The system Eq. 1 was inte-

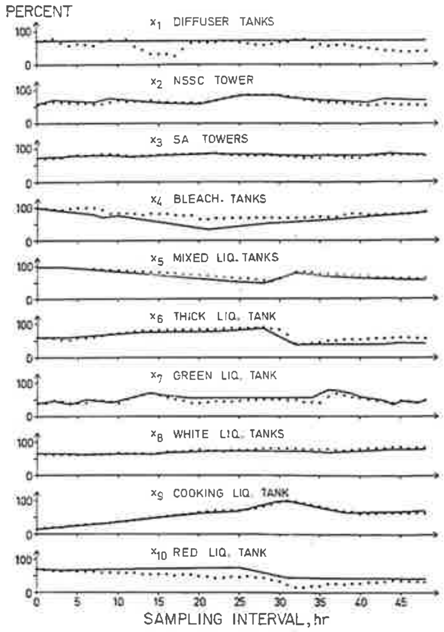


Fig. 3. Simulations based upon measurements. Ordinates: tank levels in percent of tank capacity. The dotted lines show the measured tank levels, the curves the simulated ones.

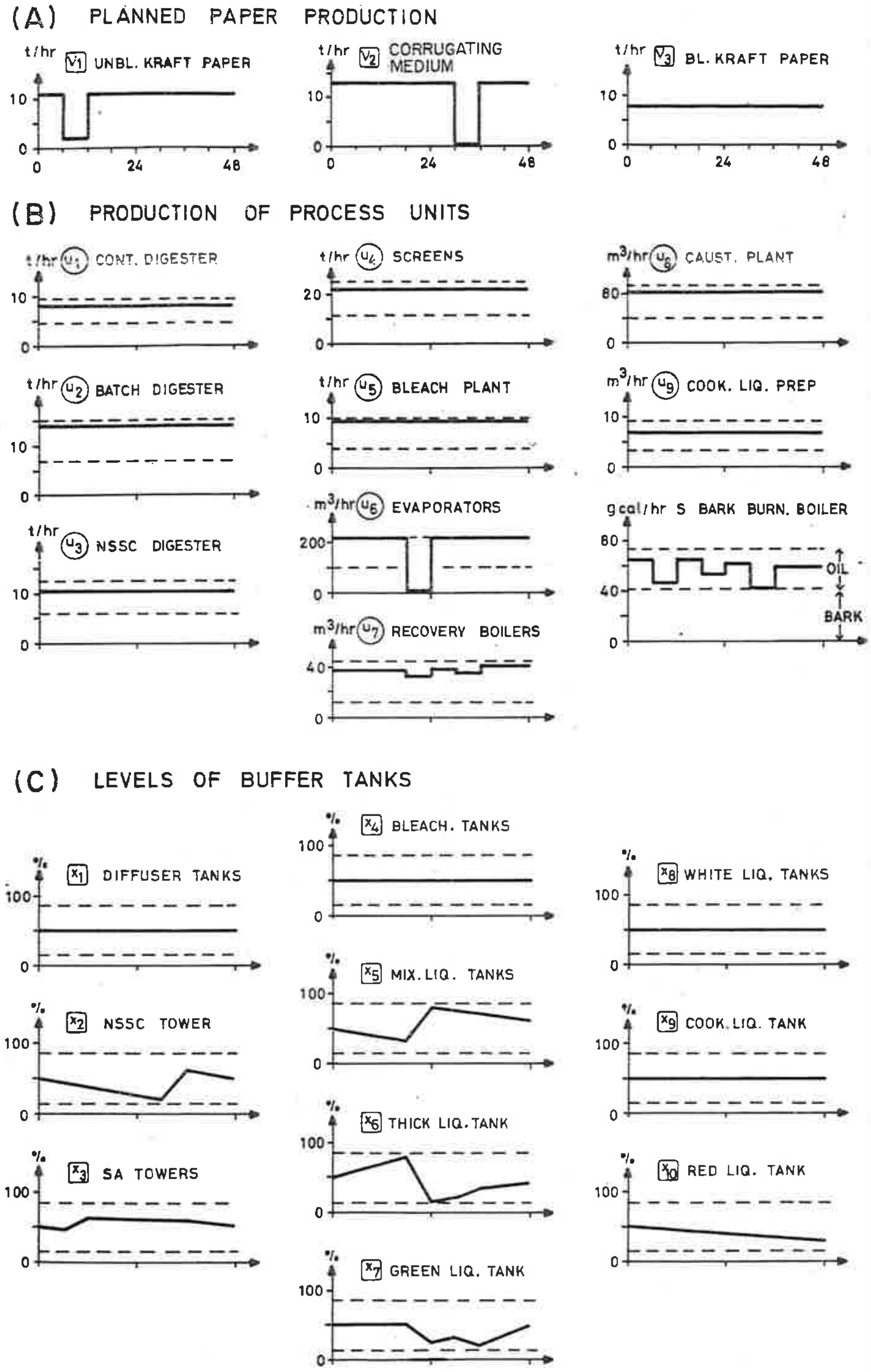


Fig. 4. A planning example. (A) shows the planned paper production; (B) shows the production scheme, as calculated by the optimization technique; and (C) shows the resultant level changes in the tanks. Dotted lines are capacity restrictions.

grated for about 48 hr, with the measured tank levels at the beginning of the period as initial values. Comparison between measured tank levels during the period and tank levels as calculated by the model then gave information about the accuracy of the model.

An example is shown in Fig. 3. Ordinates are the tank levels in percent of their capacity. The dotted lines show the measured tank levels and the curves the simulated ones.

Tank x_1 is rather small (production about 3 hr) and can hardly be used as a buffer in reality (cf. planning example below). For this reason, the level of x_1 has been put constant in the simulation, which explains the discrepancy between measured and simulated values.

The discrepancy in x_4 is due partly to the fact that the machine broke is fed back to one of the tanks corresponding to x_4 and partly to the long time constant of the bleach plant (6–8 hr). These effects have not been included in the present model.

Most of the red liquor is used as dilution liquor in the batch digesters but it can also be used as washing liquor on the washing filters. Thus, the flow of red liquor from the red liquor tank is not necessarily proportional to the production of the batch digesters, which explains the rather poor result when simulating x_{10}.

However, the most important tanks in the system are Nos. 2, 3, 4, 5, 6, 8, and 9. For these tanks, apart from No. 4, the agreement between measured and simulated values is good in this example. Other simulations have shown the same result.

The accuracy illustrated in Fig. 3 is deemed to be sufficient for the production coordination application.

FORMULATION AND SOLUTION OF THE COORDINATION PROBLEM

The production scheduling problem can be formulated in the following way: Determine how to run the processes of the mill during a planning period without violating the capacity restrictions, with as few changes of production rates as possible, and with due regard to the possibility of storing steam indirectly. Further, the tank levels at the end of the planning period must be acceptable. The tank levels at the beginning of the period and the planned paper production during the period are assumed to be known.

In order to solve this problem, we have used simulation and optimization methods (*3*). The simulation technique is not really useful during operating conditions since the manual work required is great. Simulation is, however, a very valuable means of gaining insight into the problem.

Our first optimization attempt was linear programming (*4*). However, the problem size (about 800 rows and 500 columns) was far beyond the capacity of a process computer. Looking into other methods, a formulation based upon the Pontryagin maximum principle (*5, 6*) turned out to be successful. The solution technique developed can be characterized as a successive solution of a number of small linear programming problems defined and linked together by means of the maximum principle.

We will now formulate the scheduling problem as an optimization problem and give an outline of the solution technique developed. The mathematical formulation and solution of the scheduling problem is further discussed in Ref. (*7*).

Mathematically, the mill is described by the model Eqs. 1 and 2. The capacity restrictions of processes and tanks are expressed by Eqs. 3–5. Scheduling objective (1) (few production rate changes) has been expressed by a performance functional. We have found that a loss function of the form

$$J(u) = \int_0^T |u(t) - a(t)| \mathrm{d}t \qquad (7)$$

will give schedules with few production rate changes. Here, $a(t)$ is a vector function that can be physically interpreted as a desired average of the production vector $u(t)$. Objectives (2) and (3) (indirect storage of steam and acceptable final tank levels) are expressed by establishing a suitable boundary value $x(T)$, i.e.,

$$x(T) \text{ fixed} \qquad (8)$$

From a practical point of view, it is not essential to reach the fixed final levels exactly. Besides, a fixed boundary value $x(T)$ implies a risk that the problem will be too rigidly structured. In order to avoid this, the following property of the solution technique has been developed: If the restrictions do not permit the tanks to reach the desired levels, we will obtain levels as close to the fixed ones as possible.

The initial tank levels and the planned paper production are known, viz.:

$$x(0) \text{ given} \qquad (9)$$

$$v(t),\ 0 \leq t \leq T \text{ given} \qquad (10)$$

Hence, we have formulated the production coordination problem as an optimization problem for a vector function $u(t)$, satisfying relationships of Eqs. 1–5 and minimizing that of Eq. 7, given boundary values of Eqs. 8 and 9 and function of Eq. 10.

To solve the optimization problem, the planning period has been divided into a number of intervals (not necessarily of equal length). During each interval, the production of the processes is assumed to be constant. Each time interval gives rise to a small linear programming (LP) problem (about 50 rows and 40 columns). For each LP problem the objective function has the form

$$\sum_i (|u_i - a_i| + A_i u_i)$$

where A_i = parameters, related to the adjoint variables of the Pontryagin theory; a_i = components of the vector a, mentioned above. The solution technique implies an iteration over the a-vector. An initial a-value can be calculated from $v(t)$, $x(0)$, and $x(T)$.

A computer program, written in Basic FORTRAN IV, has been developed to carry out the calculations. The program size is about 15,000 words and the off-line execution time about half an hour on an IBM 1800 (slower version, 4 μsec cycle time).

A PLANNING EXAMPLE

To illustrate the structure of the solutions obtained by the optimization technique, consider the following example.

Let the planning period be 48 hr, divided into 8 intervals of 6 hr each. Let all the tank levels be 50% at the beginning of the period. Assume that we have planned a wire change on the largest paper machine making unbleached paper during the second interval and a wire change on the fluting machine during the sixth interval. Assume further that the evaporation plant must be stopped for cleaning during the fourth interval. We wish to return to 50% tank levels at the end of the period. The tank levels may vary only between the 15 and 85% levels in order to have a safety margin against unforeseen events.

The task is: With this information as a basis, calculate a production scheme for all the processes that satisfies the restrictions and with as few changes of production rates as possible.

The solution calculated by the computer is shown in Fig. 4. The dotted lines are the capacity restrictions. Figure 4A shows the given paper production, 4B the calculated production scheme, and 4C the resultant level changes of the tanks.

Only few changes of production rate during the period were necessary. In principle, the production rate of each process must be changed at $t = 0$. Except for these nine changes, only six production rate changes were made. The shutdown of the evaporators was planned. The production of the recovery boilers was decreased during interval No. 4 to prevent the level of x_6 from falling below its lower limit and during interval No. 6 to obtain steam balance in the system. The variations of the steam consumption are compensated for first by changing the oil feed to the bark and oil burning boiler. As this variable, *S*, reached its lower limit during interval No. 6, steam production of the recovery boilers was also reduced.

In this example, tank x_1 was not used as a buffer (cf. the simulations above). Hence, the screens must be run with a production rate proportional to the sum of the sulfate digesters.

The final tank levels are fixed in this example at 50%. However, if the restrictions do not permit the tanks to reach the desired values, we obtain levels as close to the fixed ones as possible. Thus the upper limitation of the evaporators has caused final levels of x_5 and x_6 that differ from 50%.

The system as described by Eq. 1 is not controllable, which implies that fixed end points for all 10 tanks cannot be handled. This is the reason why x_{10} differs from 50% at the end of the period. However, since in reality all red liquor does not have to pass through the batch digesters, this is not a limitation of the solution.

As can be seen in Fig. 4C, the solution technique has the intuitively correct ability to fill up the thick liquor tank and to empty the mixed liquor tanks before the shutdown of the evaporators. In the same way, the levels of the pulp buffers are reduced before the wire changes.

CONCLUSIONS

Despite the complexity of the plant, the model describes it in a proper manner. The production schemes, obtained by the optimization technique, are satisfactory. The solution technique implies limited demands on core storage capacity and computational speed. The production control application will now be included in the IBM 1800 process computer system at Gruvön.

The model building and simulation technique can also be used as a tool when projecting a new plant, as for example for determining the size of buffer tanks. This possibility will be used in the preplanning of future expansions.

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RECEIVED FOR REVIEW April 14, 1969.
ACCEPTED July 13, 1969.

The author expresses his gratitude to Mr. T. Bergek, Technical Manager of the Billerud Company, who initiated the problem, and to Prof. K. J. Åström of the Lund Institute of Technology, whose guidance concerning the mathematical parts made the problem possible to solve. Thanks are also due to Mr. O. Alsholm, Leader of the Billerud process computer group, to Dr. B. Hammar of the Billerud Research and Development Centre, to Mr. H. Larsen, Resident Manager of the Gruvön mill, for their advice and help, and to Billeruds AB for permission to publish this article.