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SEARCH FOR THE YRAST TRAPS IN NEUTRON DEFICIENT RARE EARTH NUCLEI

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The yrast energies are calculated for some neutron-deficient spherical or oblate rare earch nuclei in the domain of high angular momentum I. Predictions are made for some yrast traps in the region of $I \lesssim 40$.

The average behaviour of nuclei at large angular momenta seems to be well described by the rotating liquid drop model [1]. However, also shell effects [2–6] prove to be very essential. In particular, they may lead to the existence of local minima ("yrast traps") in the nuclear energy surface resulting in the relatively long lived isomeric states at high angular momenta as suggested earlier by Bohr and Mottelson [2]. Recently, some isomeric states have been seen in the domain of angular momentum $10 \lesssim I \lesssim 20$ [8–10]. An attempt at understanding the nature of these states in terms of the single particle structure in nuclei seems very interesting. It is also a great challenge to search for the yrast traps in the domain of still higher angular momenta.

The aim of this paper is to investigate the detailed structure of the yrast line in case of some neutron deficient rare earth nuclei. The considerations have been limited to the region of nearly spherical or oblate nuclear shapes since the axial symmetry provides more chance for strong shell effects.

The calculation is entirely based on ideas and methods developed in sec. VII of ref. [6]. However, in addition to the modified harmonic oscillator potential we have employed here the Woods-Saxon potential as a basis for the calculation of the single particle orbitals. The reason for this extension of the calculation is connected with analysis of the effective moment of inertia of the nuclear system [6].

The calculations in this paper are performed essentially within the framework of the pure single particle model. Nevertheless, we believe that in addition to the single particle nature of the effect, also the short range pairing correlations acting between the nucleons may play a certain role, especially in the domain of not too high intermediate angular momenta. Thus we have also attempted a calculation including the pairing force. However, our treatment has been limited to the BCS method (plus eventually the correction for the particle-number fluctuation) which is known not to work very reliably in the domain of vanishing pairing (this is just the case at intermediate I). Therefore, we consider our treatment of pairing as a very preliminary and include it only for the sake of illustration for the size of the effect.

The method of calculation in case of no pairing has been described in ref. [6]. Here, we only recall the main points. For each angular momentum I we search for the lowest energy (yrast) state. Some of these states may be calculated from the minimization of the expectation value of the single particle Hamiltonian h^0 with the subsidiary condition of the fixed sum Σm_i, where m_i denotes the x-component of the particle angluar momentum. This is equivalent to minimizing an auxiliary Hamiltonian

$$h^\omega = h^0 - \hbar\omega j_x$$

where the Lagrange multiplier $\hbar\omega$ is determined from the condition

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$$\sum_i \langle \chi_i^\omega | j_x | \chi_i^\omega \rangle = \sum_i m_i = I .$$

Here $|\chi_i^\omega\rangle$ are eigenstates of h^ω, x is the symmetry axis of the Hamiltonian h^0 and m_i is the eigenvalue of j_x (j_x commutes with h^0). The Hamiltonian h^0 contains the axially symmetric modified harmonic oscillator or Woods-Saxon potential characterized by the deformation parameter ϵ or β_2 respectively.

It appears that the states obtained from this approach correspond only to certain selected values of I [6]. We call them "optimal". They may be simply obtained from the plot of a "sloping Fermi surface" which is a straight line in the (m_i, e_i) plane [6, 11] where e_i denotes the eigenvalue of h^0. The slope may be interpreted as the angular velocity of rotation ω.

The yrast states lying between the optimal states may be calculated as one, or many particle-hole states with respect to the neighbouring optimal states.

All the above procedure which is based on summation of the particle energies e_i is then used for the calculation of the shell correction in the spirit of the Strutinsky approach and , finally, superimposed on the picture of the rotating liquid drop.

The calculation of pairing correlations has been also included in one case as an illustration. This has been done in a very simple way by using the BCS function for all the single particle pairs except for those that are employed to create the $I \neq 0$ state (i.e. those pairs for which the $m_i > 0$ state is populated while the m_i state is not). The corresponding energy of the system is then given by

$$\mathcal{E} = \sum_{i\text{-blocked}} e_i + \sum_{i\text{-unblocked}} 2v_i^2 e_i - \Delta^2/G - G\sum_i v_i^4 ,$$

where v_i and Δ, the usual parameters of the BCS formalism are calculated for the pairing forces acting in the unblocked states only. In addition we have also included the correction related to the particle number fluctuation which modifies the usual BCS approach. This correction has been calculated by methods described e.g. in ref. [12].

Such a simple blocking treatment of high angular momentum states is only possible when the nuclear potential is axially symmetric and oblate. The rotation of a more general triaxial nuclear shape would require the application of a much more involved Hartree-Fock-Bogolyubov approach. Such a method is being developed by Bengtsson and Frauendorf [13] (see also [14]).

As a result of pairing correlations, the low I points of the yrast line are decreased in such a way that the line becomes more steep in the region below $I \sim 20$. This effect leads to the reduction of the effective moment of inertia below its value characteristic for a rigid body.

Finally working with the finite depth nuclear Woods-Saxon potential we can determine the energy of the highest occupied orbit for each angular momentum I. This leads to the determination of the energy of the orbit that is least bound as function of the angular momentum I. This quantity is simply related to the neutron or proton separation energy for given I. Obviously, it is a decreasing function of I and may become zero at a certain value of I, corresponding to the possibility of a particle emission from the nucleus due to high angular momentum. Calculation along this line will be the subject of a forthcoming publication.

Fig. 1 indicates the way in which the total angular momentum I of the yrast states is formed out of neutron and proton contributions, I_n and I_p, respectively. The figure represents the yrast states for the nucleus $^{150}_{64}$Gd in the (I_n, I_p) plane minimized with respect to deformation. Calculations are performed in two versions corresponding to the Woods-Saxon potential (left) and the modified harmonic oscillator potential (right). The violent fluctuations in the region of low I correspond to the possibility of constructing the $I \neq 0$ state either from the neutron or proton pair. In principle, only the vertical or horizontal parts of the line in fig. 1 correspond to the allowed electromagnetic transitions connecting states along the yrast line. The other transitions would require a simultaneous rearrangement of more than one particle and may thus be substantially delayed.

Fig. 2 illustrates the yrast line in the nucleus $^{150}_{64}$Gd corresponding to the Woods-Saxon potential (upper part) and to the modified harmonic oscillator potential (bottom part). The yrast states calculated without inclusion of the pairing correlations are marked with plus or minus signs indicating the resulting parity of the state. Similarly as in fig. 1, the minimization of the energy with respect to deformation has been performed for each I. The results of the calculation illustrated in fig. 2 indicate a possibility for at least two yrast traps, one at $I = 17$, the other one around $I = 26$

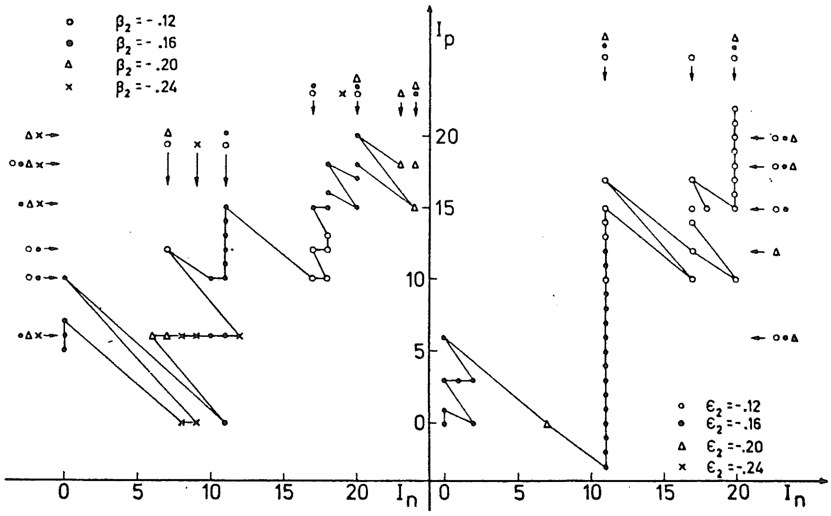

Fig. 1. Yrast states in the (I_n, I_p) plane. Here I_n (I_p) denotes the neutron (proton) contribution to the total angular momentum. Deformations of the yrast states are shown. Optimal states are marked by vertical or horizontal arrows. The left part of the figure corresponds to the Woods-Saxon potential while the right part corresponds to the modified oscillator.

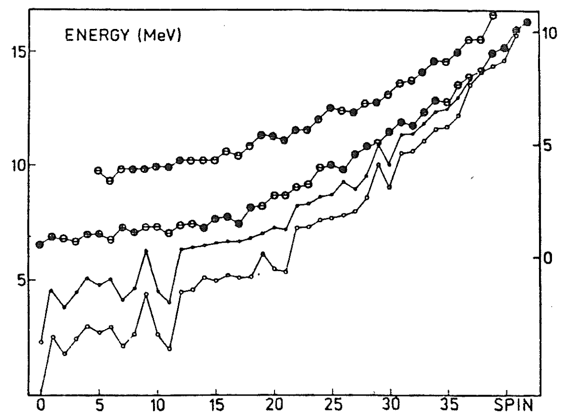

Fig. 2. Calculated yrast line for the nucleus $^{150}_{64}$Gd (energy versus projection of angular momentum plot). The yrast states calculated without inclusion of the pariring correlations are marked with plus or minus signs indicating the resulting parities of the states. Top curve corresponds to the calculation with the Woods-Saxon potential while the second curve corresponds to the m.h.o. (modified harmonic oscillator). The third curve from the top includes the pairing correlations in the BCS approximation (in the m.h.o. case), while the bottom curve includes furthermore the effect of the particle number fluctuation. The left (right) energy scale corresponds to the m.h.o. (Woods-Saxon) potential. *Note*: The peak at spin 9 obtained with included pairing energy is probably unphysical and due to the fact that an insufficient number of basic configurations are considered as input data to the pairing calculations. As the neutron pairing collapses in this region, the order of the states is drastically changed relative to that of the unpaired case.

or 27. The third candidate, at I = 21, appears only in the Woods-Saxon case. It is interesting to note that all these states are doubly optimal (i.e. optimal both with respect to neutrons and protons). It seems also worth mentioning that the two versions of the calculation produce very similar results, independently of the potential employed.

For the sake of illustration we have also included in fig. 2 the calculation of the correction following from existence of the superfluid pairing correlations. This has been done only in the version of the modified oscillator potential (the two lowest lines at the bottom of fig. 2) in the framework of the BCS method as described above. The inclusion of the pairing correlation causes the energy shift of the order of 4 MeV at I = 0. The further inclusion of the correction for the fluctuation of number of particles causes an additional shift of the order of 2 MeV. With I increasing these shifts decrease systematically and disappear in the region of I above 30. We want to stress once more, however, that we do not expect the BCS method to be a good approximation in the transition region between the superfluid and normal phase. So, we are rather inclined to treat our pairing calculation as a preliminary estimate of the size of the effect.

The yrast traps predicted in some rare earth neutron deficient nuclei are shown in table 1. It appears that there seem to be chances for the existencr of one or two traps in many nucleides. Generally, the Woods-

Table 1

Nucleus	Woods-Saxon			Mod. harmonic oscil.		
1	Ang. mom. 2	Energy 3	Deform. 4	Ang. mom. 5	Energy 6	Deform. 7
^{148}Gd	13^-	0.79	−0.08	10^+	0.735	−0.08
(0–50)	14^-	0.79	−0.08			
	18^+	1.16	−0.12			
^{150}Gd	14^-	1.38	−0.20	14^+	0.788	−0.16
(0–50)	15^-	1.41	−0.24	17^+	0.968	−0.16
	21^+	2.28	−0.16	26^+	3.332	−0.12
	27^+	3.53	−0.12			
^{152}Gd	15^-	1.37	−0.20			
(50–60)	20^+	2.06	−0.28			
	28^+	3.35	−0.28			
^{152}Dy	15^-	1.13	−0.16	11^-	0.696	−0.16
(30–60)	16^-	1.13	−0.16			
	31^-	4.35	−0.12			
experim.	$15<I<18$	5				
^{156}Er	17^+	1.37	−0.16			
(30–70)						
^{158}Er				11^+	0.365	−0.20
(50–80)						
^{158}Yb	15^+	1.14	−0.20	15^-	2.018	−0.16
(50–70)	22^+	2.49	−0.28			
^{160}Yb	13^-	1.53	−0.28	14^-	1.022	−0.20
(50–70)	36^+	4.37	−0.20	30^+	5.152	−0.16
	42^-	6.19	−0.16			
^{162}Yb	12^+	1.03	−0.28	13^-	0.637	−0.20
~ 60	27^+	3.78	−0.28	19^-	1.623	−0.20
	37^+	7.20	−0.16			

Saxon potential seems to produce somewhat more traps relative to the modified harmonic oscillator potential. However, in many cases the two potentials provide us with similar results. It is interesting to note that the isomeric state of spin I lying between 15 and 18 of the energy of the order of 5 MeV has been recently observed in the nucleus $^{152}_{66}$Dy [8]. The resulting range of spin seems to fit very nicely with our predictions. It is worth mentioning that the calculation has been performed without adjusting any parameter. However, the calculated energy of the trap seems to underestimate the observed value. This discrepancy may be connected with the inadequacy of our method in the domain of lowest angular momenta $(I \sim 0)$, and, in particular, with the neglect of pairing.

Our results have been obtained in the framework of the pure single particle approach apart from the effect of pairing. The nucleon-nucleon residual interaction will certainly modify the picture. Investigations along this line lead to interesting conclusions [15,16]. We believe that the inclusion of deformation in our calculation takes into account at least some part of the long range quadrupole interaction. However, the $K = \pm 2$ component of this force has not been included. As suggested by Bohr and Mottelson [17] the inclusion of this component may cause an additional decrease of the $I_1 = I - 2$ state with respect to the yrast state at angular momentum I as a result of the collective vibration (of the γ-type). We have started an analysis along this line [18] and hope that at least some of the traps listed in table 1 survive the inclusion of this correction even when a reasonable strength of the force is employed.

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