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NONPERTURBATIVE TREATMENT OF THE P_3 DEGREE OF FREEDOM OF THE NUCLEAR POTENTIAL AT EQUILIBRIUM AND FISSION DISTORTIONS.

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The computation of the nuclear potential-energy surface has been made more realistic by the introduction of the Strutinsky normalization procedure [1] relative to earlier computational methods [2]. In some previous publications [3] we have employed this procedure to describe the surface of the nuclear potential energy with respect to P_2 and P_4 distortions. The nuclear ground state energy is obtained as the lowest minimum of the nuclear energy surface. For actinide nuclei the path to fission is usually found to be associated with two barrier peaks [1,3]. The secondary minimum in between the two peaks (see figs. 1-3) is associated with the fission isomeric state. Such an isomeric state is encountered empirically for several nuclei between ^{235}U and ^{246}Cm. Theoretical results on the two-peak barrier structure are given in refs. [1,3,4].

These barrier calculations employ the Strutinsky method [1] whereby the sum of the single-particle energies, based on the single-particle potential described below, is renormalized so as to reproduce *on the average* the liquid-drop behaviour with respect to distortions. The average is there to be taken according to a well-defined recipe [1] over sufficiently many single-particle levels.

In this way a reasonable barrier height and thickness is obtained. If the barrier shapes computed are combined with the inertial penetration parameters based on the adiabatic approximation of the microscopic model [5], reasonable half lives for the heavier of the actinide nuclei [3] are obtained.

It thus appears that fission barrier heights and half lives are mainly associated with the P_2 and P_4 degrees of freedom. Here we want to emphasize the great importance of the inclusion of the P_4 degree of freedom [3], which on the whole serves to make the barrier finite for actinide elements without the necessity of resorting to improper expansions of Coulomb and surface energies. The P_4 distortion is also found to *lower* the ground state minimum by about 2 MeV for elements near 90Th.

However, as is well known, nuclei between Th and Cf exhibit strong asymmetry in the mass distribution of the fission fragments. Between A =230 to A =250 the ratio [6] of the mass of the heavy fragment to that of the light fragment varies from 1.5 to 1.2. As suggested by several other authors* [7] it appears highly possible that this effect in part reflects an instability already of the static potential-energy surface with respect to fission distortions. To investigate this possibility the P_3 degree of freedom is incorporated into the nuclear field by the employment of the following potential

$$V=\tfrac{1}{2}\hbar\omega_0(\epsilon,\epsilon_4,\epsilon_3)\rho^2\left(1-\tfrac{2}{3}\epsilon P_2+2\epsilon_4 P_4+2\epsilon_3 P_3\right)+ + \kappa\hbar\omega_0\left[2.\boldsymbol{l}_t\cdot\boldsymbol{s}+\mu\left(\boldsymbol{l}_t^2-\langle \boldsymbol{l}_t^2\rangle_N\right)\right],$$

where κ and μ are constants given in ref. 3. The oscillator parameter ω_0 has such a dependence on ϵ, ϵ_4 and ϵ_3 as to assure the conservation of the volume enclosed by equipotential surfaces of the entire ρ^2 part of the potential. In this way the observance of the incompressibility condition is approximately assured. An improved volume conservation is introduced by the application of the Strutinsky method. We have also assumed different oscillator constants for neutrons and protons and with a Z and N dependence so as to warrant approximately the same root mean square radii for the neutron and proton densities [3].

Single-particle levels are now computed in

*Such a degree of freedom for heavy nuclei was first considered by R. Christy, communication to A. Bohr (1954).

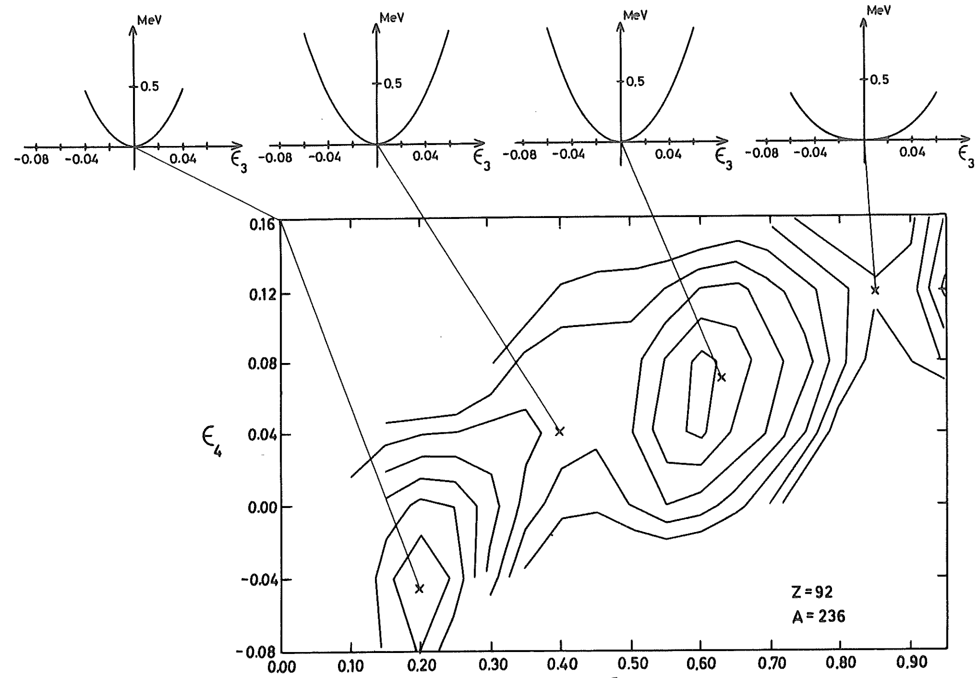

Fig. 1. Total energy for ^{236}U calculated by the Strutinsky method in coordinates ϵ, ϵ_4 and ϵ_3. The plot in coordinates ϵ and ϵ_4 is given at the bottom of the figure. In four points as indicated in the figure the separate ϵ_3-dependence is studied.

the above potential for sets of ϵ, ϵ_4 and ϵ_3. Energies are added with due regard for pairing, where the pairing matrix element is assumed isospin dependent and proportional to the nuclear surface area as discussed in ref. 3. A shell and pairing energy is obtained as described in the reference cited. Finally the Coulomb and surface energy terms are included with constants assumed in accordance with Myers and Swiatecki [8]. A total investigation of the entire energy surface is excluded for reasons of computational time limitations. Thus we have selected what we consider important points in the (ϵ, ϵ_4) plane along the static fission "paths" (see figs. 1-3). For these particular ϵ - and ϵ_4-values we have studied the stability with respect to P_3 distortions.

In table 1 we list the stiffness parameter $\partial^2 E/\partial\epsilon_3^2$ at $\epsilon_3 = 0$ based on these points for (ϵ, ϵ_4)-values at the ground state minimum, at the first barrier peak, at the secondary minimum, at the second barrier peak and just beyond the second peak. For comparison we list the corresponding stiffness values for the liquid-drop model. As immediately observed from figs. 1-3 and table 1, the stiffness decreases with increasing distortion* while instability is not quite reached even at the second barrier peak. The inclusion of couplings between shells beyond those of $N \to N+1$ have no sizable effects at the second barrier peak when the Strutinsky normalization method is applied. Without this normalization method the stiffness is found to be highly dependent on the number of shells included.

The increased tendency to development of a P_3 distortion at large elongations is thus made apparent already from the calculation of the static-potential-energy surface. The weak remaining stability in the ϵ_3 direction of the nuclear energy surface may be overestimated by the surface energy term introduced. Alternatively the weak remainder may be overcome by dynamic effects corresponding to the nonisotropy of the inertia. A quantitive discussion will have to await the outcome of a dynamic barrier penetration calculation.

It should be noted that there is a center-of-

* The softening of the resistance to ϵ_3 distortions with increasing quadrupole deformation was observed by Johansson [7] in his early calculations.

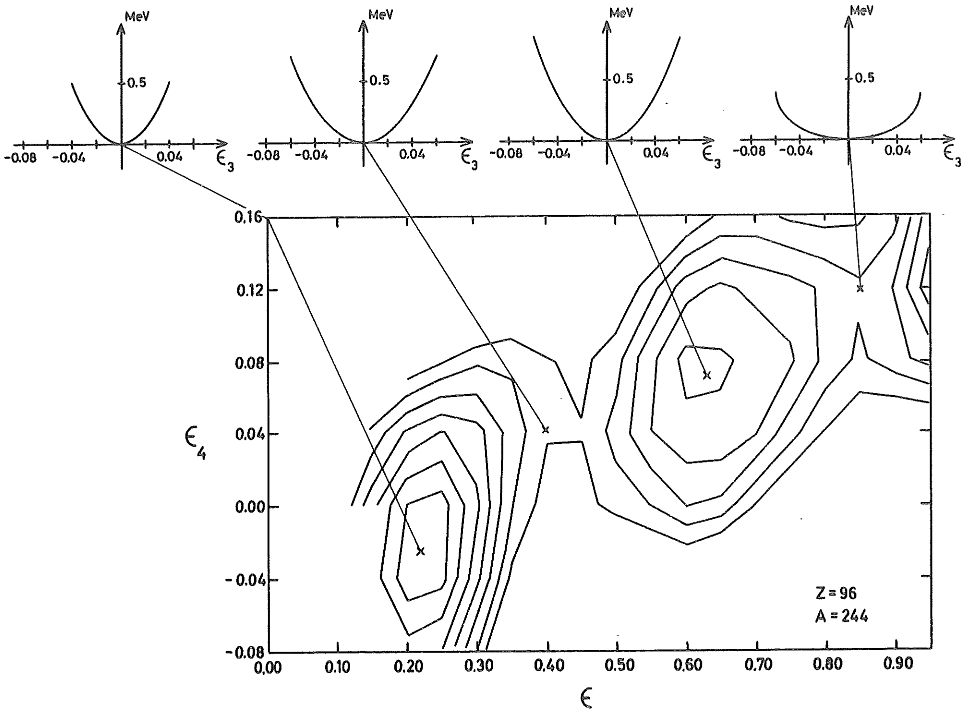

Fig. 2. Same as fig. 1 for ^{244}Cm

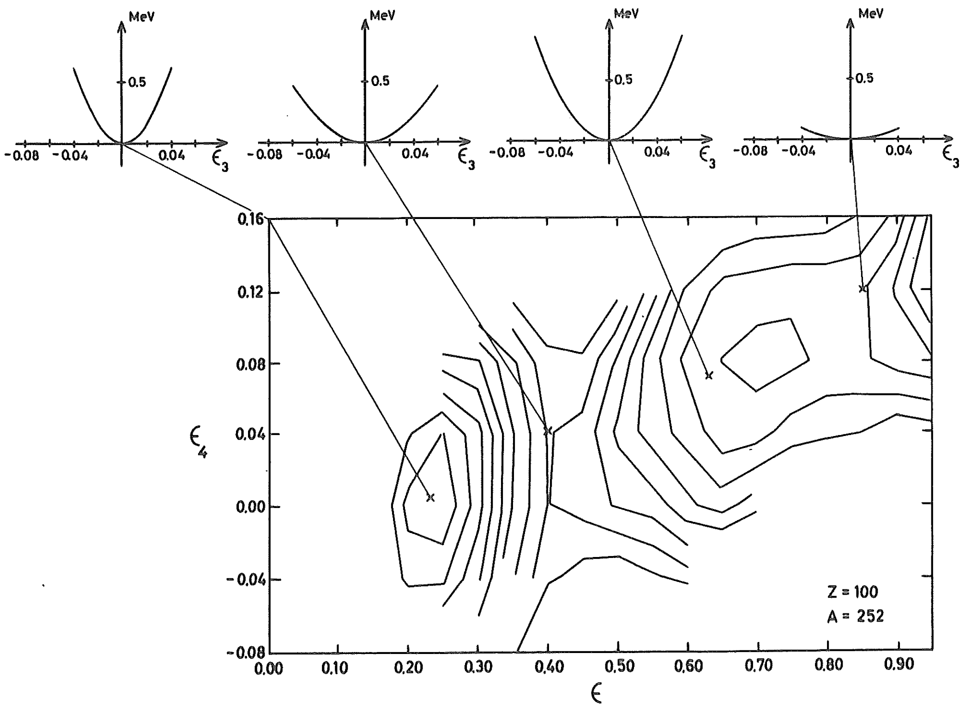

Fig. 3. Same as fig. 1 for ^{252}Fm.

Table 1
Octupole stiffness parameters $\partial^2 E/\partial\epsilon_3^2$ in units of MeV for a few nuclei at the equilibrium distortion, at first barrier peak, at secondary minimum, at second barrier peak and just beyond the second peak. The distortions are listed in columns 1-2. The nuclei are identified in columns 3-4. Column 5 lists the stiffness calculated from the Strutinsky-normalized energy (denoted E'' (STRUT)) and column 6 the same quantity based on the liquid-drop theory (denoted E'' (LIQ)). Stiffness parameter in brackets represent interpolated values and are somewhat uncertain.

EPS2	EPS4	Z	A	E'' (STRUT)	E'' (LIQ)
0.190	-0.051	92	234	(580)	(560)
0.198	-0.045	92	236	610	570
0.205	-0.038	92	238	(570)	(570)
0.212	-0.036	94	240	(610)	(560)
0.218	-0.032	96	242	(660)	(540)
0.230	0.007	98	252	(720)	(500)
0.234	0.005	100	252	790	490
		92	234	560	490
		92	236	530	490
		92	238	500	490
0.40	0.04	94	240	490	480
		96	242	450	460
		98	252	310	460
		100	252	260	450
		92	234	560	480
		92	236	520	480
		92	238	490	480
0.63	0.07	94	240	500	470
		96	242	510	440
		98	252	500	440
		100	252	480	410
		92	234	200	460
		92	236	180	460
		92	238	160	480
0.85	0.12	94	240	160	440
		96	242	150	410
		98	252	160	400
		100	252	130	360
		92	234	370	520
		92	236	360	530
		92	238	330	530
0.95	0.13	94	240	340	500
		96	242	360	470
		98	252	160	450
		100	252	240	410

mass shift due to the P_3-term for which no correction is made. This cannot affect the conclusions as to the signs of the curvature term, however. On the other hand, the inclusion of the P_5 degree of freedom is found to be of great importance in the liquid-drop theory in reducing the liquid-drop stiffness at large distortions. An investigation of this term remains to be carried out also in our case.

The dynamic treatment may give the quantitative explanation also for the problem of the A-dependence of the mass asymmetry. However, an interesting qualitative explanation suggests itself at this point. For actinides with mass less than 240 the second barrier peak is the highest one while beyond 245 the second peak diminishes rapidly in magnitude relative to the first one. As the near-instability to P_3-distortion develops first in connection with the second peak, it is expected that the tendency to P_3 shape deformations is of most importance for the lighter actinides, for which the second peak dominates. For the heavier ones the second barrier is dynamically much less influential.

An investigation parallel to this has been carried out in Copenhagen by C. Pauli and V. Strutinsky [10]. These authors find a similar result to ours if a term $\rho^2 P_3$ is added while instability occurs for a $\rho^3 P_3$ term. This may very likely be due to the fact that the latter kind of term cannot be properly included in the volume conservation condition.

After this investigation was completed it came to the author's attention that calculations similar to ours have been carried out simultaneously with ours by Krappe and Wille [11], who also obtained results very similar to the ones presented here. These authors also show that the difference between the barrier obtained on the basis of a Woods-Saxon potential and that obtained from a deformed oscillator is almost negligible even at large distortions.

A valuable discussion with Dr. W. J. Swiatecki is highly appreciated.

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