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Some Rate $\frac{1}{2}$ and $\frac{1}{3}$ Binary Convolutional Codes with an Optimum Distance Profile

ROLF JOHANNESSON, MEMBER, IEEE

Abstract—A tabulation of binary systematic convolutional codes with an optimum distance profile for rates $\frac{1}{2}$ and $\frac{1}{3}$ is given. A number of short rate $\frac{1}{2}$ binary nonsystematic convolutional codes are listed. These latter codes are simultaneously optimal for the following distance measures: distance profile, minimum distance, and free distance; they appear attractive for use with Viterbi decoders. Comparisons with previously known codes are made.

Recently [1] we introduced a new distance measure for fixed convolutional encoders (FCE's), viz., the distance profile, $d = [d_0, d_1, \ldots, d_M]$, where $d_i$ is the $i$th order column distance [2] and $M$ is the code memory. When comparing codes of the same rate and memory, we say that a distance profile $d$ is superior to a distance profile $d'$ if $d_i > d'_i$ for the smallest index $i$, $0 \leq i \leq M$, where $d_i \neq d'_i$. The code with the larger $d$ will generally require less computation with sequential decoding than will the other code [1], [3]. Extensive lists of rate $R = \frac{1}{3}$ FCE's of various types, viz., general nonsystematic codes, quick-look in (QLI) [4] codes, and systematic codes, with an optimum distance profile (ODP codes), i.e., with a distance profile equal to or superior to that of any other code, have been published [1], [5]. Most of these codes have also minimum distance $d_M$ and free distance $d_n$ equal to or superior to those of any other previously published code of the same rate, memory, and type. In this correspondence, we report an extension of our previous work to rates less than one-half and present ODP FCEs of rates $\frac{1}{2}$ and $\frac{1}{3}$.

In Tables I and II, we list rate $\frac{1}{2}$ ODP systematic convolutional codes, for $1 \leq M \leq 23$. The generators are written in an octal form according to the convention introduced in [1]. For each value of $M$, we give both the code with the fewest weight of $d_M$ paths and, if not the same, the code with the largest $d_n$, (ties were resolved by using the number of low-weight $d_n$ paths as a further optimality criterion). The codes in Table I are also optimum minimum distance (OMD) codes [6]. The consistent excellence as regards $d_M$ of the rate $\frac{1}{2}$ systematic ODP codes can be seen from Fig. 1 in which we have plotted $d_M$ for these codes; $d_M$ for the codes of Bussgang ($M \leq 6$) [6], Lin–Lyne ($7 \leq M \leq 17$) [7], and Costello ($18 \leq M \leq 23$) [2]; and, for comparison, the Gilbert lower bound [6], [2] on $d_M$. For some memories $M$, the ODP codes have a minimum distance $d_M$ superior to that of any other known code of the same rate and memory.

In Table III, we list rate $\frac{1}{3}$ ODP general nonsystematic convolutional codes which are also optimum free distance (OFD) codes. Ties were resolved first according to low-weight $d_n$ paths and then according to low-weight $d_M$ paths. We have plotted $d_n$ for these remarkable codes, which appear attractive for use with Viterbi decoders, and for the ODP systematic codes in Fig. 1. We note that for all rates $R = 1/n$, we have $2^M$ nonsystematic ODP codes equivalent to each systematic ODP code [6]. Our empirical data suggest that the number of systematic ODP codes is independent of the rate. Since the number of potential nonsystematic OFD codes, viz., $2^{2M}$, increases exponentially with $n$, we conclude that a reduction of the rate makes the ODP property more restrictive. Hence, it is even more surprising that it can be obtained

\begin{table}[h]
\centering
\caption{ODP Systematic Convolutional Codes with Rate $\frac{1}{2}$ That Are Also OMD Codes}
\begin{tabular}{|c|c|c|c|c|}
\hline
M & $d^{(2)}_M$ & $d^{(3)}_M$ & $d_M$ & $\#\text{paths}$ & $d_n$ & $\#\text{paths}$ \\
\hline
1 & 0 & 0 & 1 & 1 & 1 \\
2 & 0 & 0 & 1 & 1 & 1 \\
3 & 0 & 0 & 1 & 1 & 1 \\
4 & 0 & 0 & 1 & 1 & 1 \\
5 & 0 & 0 & 1 & 1 & 1 \\
6 & 0 & 0 & 1 & 1 & 1 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{ODP Systematic Convolutional Codes with Rate $\frac{1}{3}$}
\begin{tabular}{|c|c|c|c|c|}
\hline
M & $d^{(2)}_M$ & $d^{(3)}_M$ & $d_M$ & $\#\text{paths}$ & $d_n$ & $\#\text{paths}$ \\
\hline
7 & 0 & 0 & 1 & 1 & 1 \\
8 & 0 & 0 & 1 & 1 & 1 \\
9 & 0 & 0 & 1 & 1 & 1 \\
10 & 0 & 0 & 1 & 1 & 1 \\
11 & 0 & 0 & 1 & 1 & 1 \\
12 & 0 & 0 & 1 & 1 & 1 \\
13 & 0 & 0 & 1 & 1 & 1 \\
14 & 0 & 0 & 1 & 1 & 1 \\
15 & 0 & 0 & 1 & 1 & 1 \\
16 & 0 & 0 & 1 & 1 & 1 \\
17 & 0 & 0 & 1 & 1 & 1 \\
18 & 0 & 0 & 1 & 1 & 1 \\
19 & 0 & 0 & 1 & 1 & 1 \\
20 & 0 & 0 & 1 & 1 & 1 \\
21 & 0 & 0 & 1 & 1 & 1 \\
22 & 0 & 0 & 1 & 1 & 1 \\
23 & 0 & 0 & 1 & 1 & 1 \\
\hline
\end{tabular}
\end{table}
Fig. 1. Minimum distance $d_M$ and free distance $d_\omega$ for some rate $\frac{3}{4}$ convolutional codes.

**TABLE III**

<table>
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<tr>
<th>M</th>
<th>$G(1)$</th>
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* No code which is simultaneously ODP and OFD exists at $M=6$. The values $d_M$ and $d_\omega$ given are those for an ODP and an OFD code respectively.

** The search for the code with the smallest number of weight $d_\omega = 16$ paths was not exhaustive and hence a slightly better code might exist.
In the preface of this book, the authors state that, “it is our feeling that many of the techniques and algorithms currently employed by engineers and scientists in carrying out signal-processing tasks are widespread and basic enough to warrant introduction into the undergraduate engineering curriculum.” Indeed, this textbook has fulfilled that purpose in an excellent manner. Despite the existence of various fairly good books at the senior–first year graduate level dealing with the topics covered here, there is no single book quite like this one. This book is written in a casual manner that invites the reader to learn something about certain statistical concepts and applications in engineering systems.

Chapter 1 contains a brief introduction and gives the motivation for studying signal analysis. A few simple illustrative examples are given and a slightly more detailed discussion on the air traffic radar and control system is presented.

Chapter 2, Discrete-Time Signals, consists of a review of basic material on Fourier series, Fourier transforms, and linear system analysis. In this chapter, the term “discrete Fourier transform,” indicates only that the signal sample times are discrete. When the frequency values of a “discrete Fourier transform” are also discrete, it is then called a “finite Fourier transform.” Unfortunately, this latter term is called a discrete Fourier transform (DFT) in most literatures and books on digital signal processing. The treatment of the fast Fourier transform (FFT) is disappointingly brief for a modern book on signal processing.

Chapter 3, Random Discrete-Time Signals, consists of a review of probability, correlations, and spectral densities. Most discussions here are so brief that these materials are meaningful only at the level of this book. This chapter may be convenient for analysts, but not necessarily intellectually satisfying to a student, nor necessarily helpful to those who may have to deal with real-life random data. Thus a chapter devoted to the evaluation of approximate signal statistics, spectra, and correlations are meaningful at the level of this book.

Chapter 4, Spectral Analysis of Random Signals, deals with estimation and shaping of pseudorandom noise is interesting; this topic indicates only that the signal sample times are discrete. When in Fig. 2 we have plotted the free distance for rate \( \frac{1}{2} \) codes that it can be obtained for rate \( \frac{1}{2} \) codes [1]. It should also be mentioned that we have not found any code of rate \( \frac{1}{4} \) which is simultaneously ODP and OFD.

In Table IV, we list rate \( \frac{1}{4} \) ODP systematic convolutional codes for \( 1 \leq M \leq 23 \), and in Fig. 2 we have plotted the free distance for these codes; \( d_M \) for the codes of Lin-Lyne (\( M < 15 \)) [7] and Costello (\( 16 \leq M < 23 \)) [2]; and, for comparison, the Gilbert lower bound [6], [2] on \( d_M \). We note that the ODP codes as regards \( d_M \) are as good as or superior to previously known codes, except for \( M = 13 \) and 14.

In a forthcoming paper [8], a tabulation of ODP PCEs of rate \( \frac{1}{2} \) will be given.

REFERENCES


**Table IV**

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