Some rate 1/3 and 1/4 binary convolutional codes with an optimum distance profile

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Some Rate ⅓ and ¼ Binary Convolutional Codes with an Optimum Distance Profile

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***Abstract*—A tabulation of binary systematic convolutional codes with an optimum distance profile for rates ⅓ and ¼ is given. A number of short rate ⅓ binary nonsystematic convolutional codes are listed. These latter codes are simultaneously optimal for the following distance measures: distance profile, minimum distance, and free distance; they appear attractive for use with Viterbi decoders. Comparisons with previously known codes are made.**

Recently [1] we introduced a new distance measure for fixed convolutional encoders (FCE's), viz., the *distance profile,* $\boldsymbol{d} = [d_0,d_1,\cdots,d_M]$, where d_j is the jth order *column distance* [2] and M is the code *memory*. When comparing codes of the same rate and memory, we say that a distance profile $\boldsymbol{d}$ is superior to a distance profile $\boldsymbol{d}'$ if $d_i > d_i'$ for the smallest index i, $0 \leq i \leq M$, where $d_i \neq d_i'$. The code with the larger $\boldsymbol{d}$ will generally require less computation with sequential decoding than will the other code [1], [3]. Extensive lists of rate $R = \frac{1}{2}$ FCE's of various types, viz., general nonsystematic codes, quick-look in (QLI) [4] codes, and systematic codes, with an *optimum distance profile* (ODP codes), i.e., with a distance profile equal to or superior to that of any other code, have been published [1], [5]. Most of these codes have also *minimum distance* d_M and *free distance* d_∞ equal to or superior to those of any other previously published code of the same rate, memory, and type. In this correspondence, we report an extension of our previous work to rates less than one-half and present ODP FCEs of rates ⅓ and ¼.

In Tables I and II, we list rate ⅓ ODP systematic convolutional codes, for $1 \leq M \leq 23$. The generators are written in an octal form according to the convention introduced in [1]. For each value of M, we give both the code with the fewest number of weight d_M paths and, if not the same, the code with the largest d_∞ (ties were resolved by using the number of low-weight d_∞ paths as a further optimality criterion). The codes in Table I are also *optimum minimum distance* (OMD) codes [6]. The consistent excellence as regards d_M of the rate ⅓ systematic ODP codes can be seen from Fig. 1 in which we have plotted d_M for these codes; d_M M for the codes of Bussgang ($M \leq 6$) [6], Lin–Lyne ($7 \leq M \leq 17$) [7], and Costello ($18 \leq M \leq 23$) [2]; and, for comparison, the Gilbert lower bound [6], [2] on d_M. For some memories M, the ODP codes have a minimum distance d_M superior to that of any other known code of the same rate and memory.

In Table III, we list rate ⅓ ODP general nonsystematic convolutional codes which are also *optimum free distance* (OFD) codes. Ties were resolved first according to low-weight d_∞ paths and then according to low-weight d_M paths. We have plotted d_∞ for these remarkable codes, which appear attractive for use with Viterbi decoders, and for the ODP systematic codes in Fig. 1. We note that for all rates $R = 1/n$, we have 2^M nonsystematic ODP codes equivalent to each systematic ODP code [6]. Our empirical data suggest that the number of systematic ODP codes is independent of the rate. Since the number of potential nonsystematic OFD codes, viz., 2^{nM}, increases exponentially with n, we conclude that a reduction of the rate makes the ODP property more restrictive. Hence, it is even more surprising that it can be obtained

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TABLE I
ODP SYSTEMATIC CONVOLUTIONAL CODES WITH RATE ⅓ THAT ARE ALSO OMD CODES

M	$G^{(2)}$	$G^{(3)}$		d_M	#paths	d_∞	#paths
1	6	6		4	1	5	1
2	5	7	B	5	1	6	1
3	64	74	B	6	1	8	2
4	56	72		7	2	9	1
5	57	73		8	3	10	1
6	564	754		9	5	12	4

Note: B denotes that this generator was previously found by Bussgang.

TABLE II
ODP SYSTEMATIC CONVOLUTIONAL CODES WITH RATE ⅓

M	$G^{(2)}$	$G^{(3)}$	d_M	#paths	d_∞	#paths
7	626	736	10	7	12	1
8	531	676	11	15	13	1
9	5314	6760	11	3	14	4
	5314	6764	11	5	15	3
10	5312	6766	12	7	16	4
11	5312	6766	13	15	16	4
	5317	6767	13	18	16	1
12	51444	73254	13	3	16	1
	65304	71274	13	4	17	1
13	51444	73256	14	6	17	1
	65306	71276	14	7	18	1
14	65376	71261	15	19	18	1
	65305	71273	15	21	19	2
15	653764	712614	15	4	20	2
16	514112	732374	16	9	20	1
17	653761	712611	16	1	22	4
	653767	712611	16	2	22	2
18	6537614	7126104	17	4	23	1
	6530574	7127304	17	6	24	9
19	5141132	7323756	18	14	24	2
20	5312071	6766735	18	1	25	1
	6530547	7127375	18	5	26	4
21	65376164	71261060	19	7	26	7
	65305474	71273750	19	8	26	3
22	51445036	73251266	20	22	26	1
23	53176335	67677341	20	3	26	3
	65305477	71273753	20	7	28	3

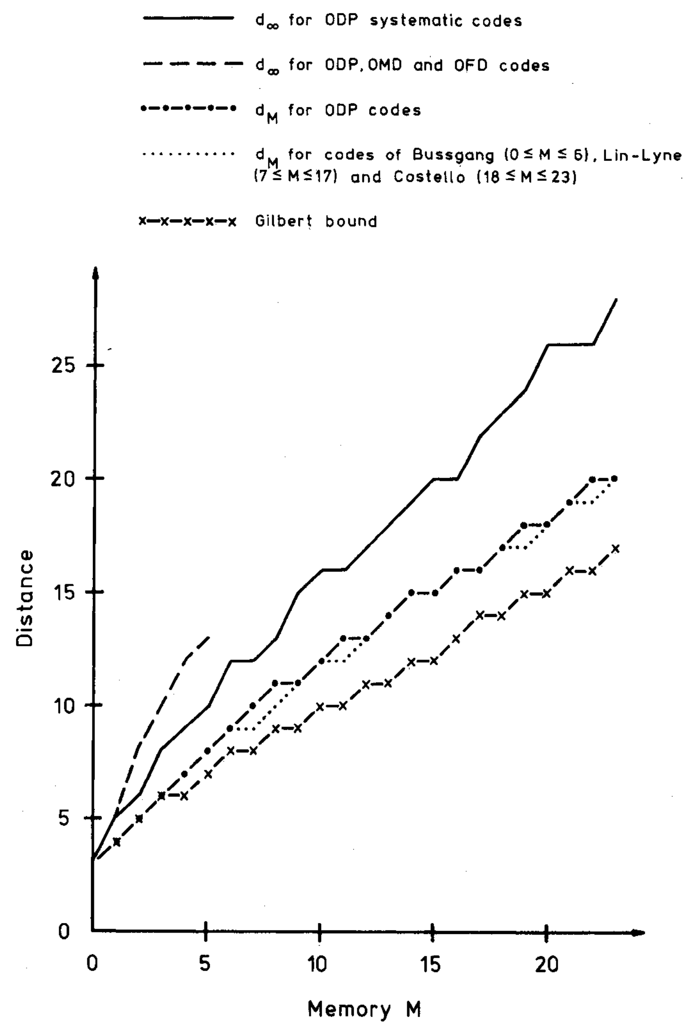

Fig. 1. Minimum distance d_M and free distance d_∞ for some rate $\frac{1}{3}$ convolutional codes.

TABLE III
NONSYSTEMATIC CODES WITH RATE $\frac{1}{3}$ THAT ARE SIMULTANEOUSLY ODP AND OFD

M	$G^{(1)}$	$G^{(2)}$	$G^{(3)}$	d_M	# paths	d_∞	# paths
1	4	6	6	4	1	5	1
2	5	7	7	5	2	8	2
3	54	64	74	6	1	10	3
4	52	66	76	7	3	12	5
5	47	53	75	8	3	13	1
6	-	-	-*	9	-	15	-
7	516	552	656**	10	9	16	4

* No code which is simultaneously ODP and OFD exists at M=6. The values d_M and d_∞ given are those for an ODP and an OFD code respectively.

** The search for the code with the smallest number of weight $d_\infty = 16$ paths was not exhaustive and hence a slightly better code might exist.

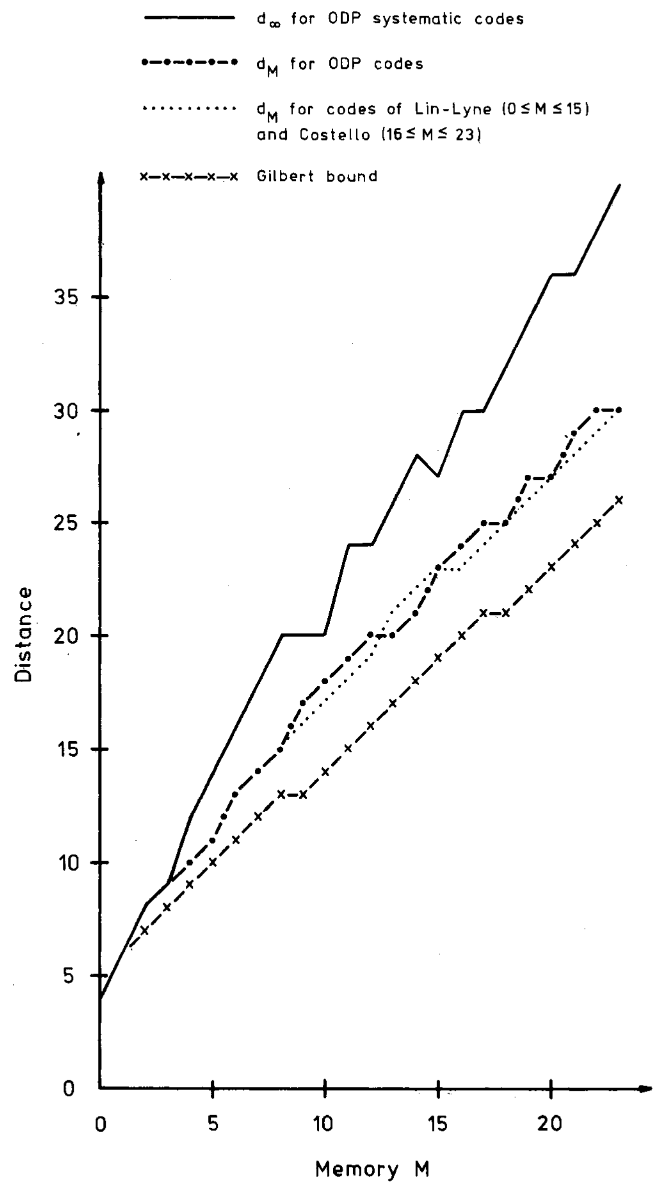

Fig. 2. Minimum distance d_M and free distance d_∞ for some rate $\frac{1}{4}$ convolutional codes.

TABLE IV
ODP SYSTEMATIC CONVOLUTIONAL CODES WITH RATE 1/4

M	$G^{(2)}$	$G^{(3)}$	$G^{(4)}$	d_M	#paths	d_∞	#paths
1	4	6	6	6	2	6	1
2	5	6	7	8	4	8	1
3	54	64	74	9	4	9	1
4	56	66	74	10	2	12	1
5	51	67	73	11	1	14	1
6	514	674	730	13	7	16	3
	534	634	754	13	8	16	1
7	516	622	744	14	3	15	1
	516	676	732	14	7	18	1
8	535	637	755	15	2	20	2
9	5350	6370	7554	17	21	20	2
10	5350	6370	7556	18	19	20	1
11	5351	6371	7557	19	16	24	1
12	53514	63714	75574	20	18	24	1
13	51056	63116	76472	20	1	26	2
14	51056	63117	76473	21	1	28	2
	51057	63117	76473	21	2	28	1
15	510574	631140	764720	23	14	27	1
16	530036	611516	747332	24	15	30	3
17	530037	611517	747332	25	16	30	1
18	5105444	6311614	7647074	25	1	32	1
19	5105446	6311616	7647072	27	20	34	2
20	5105447	6311617	7647073	27	1	36	2
21	51054474	63116164	76470730	29	22	36	1
22	51563362	62735066	74040356	30	26	38	2
23	51054477	63116167	76470731	30	2	40	1

at no sacrifice in free distance for rate 1/3 codes than it is that it can be obtained for rate 1/2 codes [1]. It should also be mentioned that we have not found any code of rate 1/4 which is simultaneously ODP and OFD.

In Table IV, we list rate 1/4 ODP systematic convolutional codes for $1 \leq M \leq 23$, and in Fig. 2 we have plotted d_M and d_∞ for these codes; d_M for the codes of Lin–Lyne ($M \leq 15$) [7] and Costello ($16 \leq M \leq 23$) [2]; and, for comparison, the Gilbert lower bound [6], [2] on d_M. We note that the ODP codes as regards d_M are as good as or superior to previously known codes, except for M = 13 and 14.

In a forthcoming paper [8], a tabulation of ODP FCEs of rate 2/3 will be given.

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Book Reviews

Signal Processing: Discrete Spectral Analysis, Detection, and Estimation—Mischa Schwartz and Leonard Shaw (New York: McGraw-Hill, 1975, 396 pp., $17.50)

KUNG YAO

In the preface of this book, the authors state that, "it is our feeling that many of the techniques and algorithms currently employed by engineers and scientists in carrying out signal-processing tasks are widespread and basic enough to warrant introduction into the undergraduate engineering curriculum." Indeed, this textbook has fulfilled that purpose in an excellent manner. Despite the existence of various fairly good books at the senior–first year graduate level dealing with the topics covered here, there is no single book quite like this one. This book is written in a casual manner that invites the reader to learn something about certain statistical concepts and applications in engineering systems.

Chapter 1 contains a brief introduction and gives the motivation for studying signal analysis. A few simple illustrative examples are given and a slightly more detailed discussion on the air traffic radar and control system is presented.

Chapter 2, Discrete-Time Signals, consists of a review of basic material on Fourier series, Fourier transforms, and linear system analysis. In this chapter, the term "discrete Fourier transform," indicates only that the signal sample times are discrete. When the frequency values of a "discrete Fourier transform" are also discrete, it is then called a "finite Fourier transform." Unfortunately, this latter term is called a discrete Fourier transform (DFT) in most literatures and books on digital signal processing. The treatment of the fast Fourier transform (FFT) is disappointingly brief for a modern book on signal processing.

Chapter 3, Random Discrete-Time Signals, consists of a review of probability, correlations, and spectral densities. Most discussions here are so brief that these materials are meaningful only to those already familiar with them. The section on the generation and shaping of pseudorandom noise is interesting; this topic should be, but is not generally, covered in most elementary probability books.

Chapter 4, Spectral Analysis of Random Signals, deals with the first of the three topics in the title of the book. Most communication theory, estimation theory, and stochastic processes books assume certain models of random processes. This approach may be convenient for analysts, but not necessarily intellectually satisfying to a student, nor necessarily helpful to those who may have to deal with real-life random data. Thus a chapter devoted to the evaluation of approximate signal statistics, spectra, and correlations are meaningful at the level of this book. This chapter covers sample autocorrelation functions, periodograms, window