Criteria and Trade-offs in PID Design

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Abstract: Control design is a rich problem which requires that many issues such as load disturbances and set-point responses, model uncertainty, and measurement noise are taken into account. These issues are discussed for design of PI and PID controllers. The purpose is to give insight into the different criteria and their trade-offs, not to give specific tuning methods.

Keywords: PID control, optimization, robustness, design, tuning, measurement noise.

1. INTRODUCTION

A rational way to design a controller is to derive a process model and a collection of requirements. Constrained optimization can then be applied to make a trade-off between often conflicting requirements. Tuning of PID controllers is typically not done in this way since the large number of PID loops encountered limits the effort that can be devoted to a single loop. Tuning of PID controllers have instead focused on development of simple tuning rules based on process models characterised by a few parameters.

Requirements typically include specification on load disturbance attenuation, robustness to process uncertainty, measurement noise and set-point response. Load disturbance response is a primary concern in process control where steady-state regulation is a key issue, see Shinskey (1996), while set-point response is a major concern in motion control. Set-point responses can, however, be treated separately by using a control architecture having two degrees of freedom, which is simply done by set-point weighting in PID control. The set-point response will not be treated in this paper.

Control performance can be characterized by the integrated error and the integrated absolute error

$$IE = \int_0^\infty e(t)dt, \qquad IAE = \int_0^\infty |e(t)|dt, \tag{1}$$

where e is the control error due to a unit step load disturbance. These are good measures of load disturbance attenuation for controllers with integral action. For systems that are well damped, the two criteria are approximately the same. The integrated error is also equal to the inverse of the controller integral gain, $IE = 1/k_i$.

Robustness to process uncertainty can be captured by the maximum sensitivites M_s and M_t;

$$M_s = \max_\omega \left|\frac{1}{1+G_l(i\omega)}\right|, \quad M_t = \max_\omega \left|\frac{G_l(i\omega)}{1+G_l(i\omega)}\right|, \tag{2}$$

where $G_l(s) = P(s)C(s)$ is the loop transfer function, and $P(s)$ and $C(s)$ are the process and controller transfer function, respectively.

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The control actions generated by measurement noise should not be large. The fluctuations in the control signal can be computed from the transfer functions of the process and the controller together with a characterization of the measurement noise, for example its spectral density. Such detailed information is rarely available for PID control and we will therefore use simpler measures. The largest high-frequency gain of the combination of the controller and the noise filter is a possible measure. Filtering of the measured signal is essential.With a second-order filter, which is advisable, we have

$$C(s) = k_p + \frac{k_i}{s} + k_d s, \quad G_f(s) = \frac{1}{1+sT_f+s^2T_f^2/2}, \tag{3}$$

where G_f is the filter transfer function. For P and PI controllers, the high-frequency gain is essentially determined by proportional gain k_p and filter-time constant T_f. For controllers with derivative action, derivative gain k_d and filter-time constant T_f determine the high-frequency gain. Detailed discussions of the effect of measurement noise are given in Åström and Hägglund (2005), Garpinger (2009), Larsson and Hägglund (2011), and Kristiansson and Lennartson (2006).

2. TUNING METHODS

There are few PID controller tuning methods that take the major requirements on load disturbance attenuation, robustness, and measurement noise sensitivity into account.

The Ziegler-Nichols methods focused on attenuation of load disturbances. Robustness to process uncertainty and measurement noise were not considered and the tuning rules give controllers with poor robustness. Shinskey improved upon the rules by optimizing IAE, and he also discussed robustness in Shinskey (1990, 1996). In the AMIGO tuning rules, Åström and Hägglund (2005), IE was minimized subject to a robustness constraint on the combined sensitivity M but measurement noise was not considered.

Lambda tuning or internal model control, Dahlin (1968); Higham (1968); Rivera et al. (1986), is a simple design method which is commonly used in the process industry. The method has the closed-loop time constant T_{cl} as a tuning parameter which admits a compromise between performance and robustness. Skogestad (2003) introduced modifications of the lambda tuning method called SIMC that improves performance especially for lag-dominant processes. In Skogestad and Grimholt (2012) the methods were further improved for delay-dominant processes.

Most design methods do not take measurement noise into account. It is often suggested to choose the filter-time constant as a fraction of the derivative time, i.e. $T_f = T_d/N$. This simple solution has severe drawbacks as was pointed out in Isaksson and Graebe (2002).

Åström and Hägglund (2005) and Skogestad (2006) suggested methods to detune the AMIGO method and the SIMC method, respectively, to make the designs less noise sensitive. Methods where both the controller parameters and the filter-time constant are determined are more complicated than the previous ones. Examples of such methods are given in Kristiansson and Lennartson (2006), Garpinger (2009), Sekara and Matausek (2009), and Larsson and Hägglund (2011).

3. PI CONTROL

We will now investigate PI control of some representative processes. The criterion IE is convenient to use because it relates directly to the controller parameters. Relations between IAE, M_s and M_t and the controller parameters are more complicated. They can, however, be represented in trade-off plots, which give level curves for the IAE, and the sensitivities M_s, M_t in the $k_p - k_i$ plane. The level curves for the sensitivities denote controller parameters such that M_s and M_t are less than the indicated values.

Processes with positive impulse responses can conveniently be characterized by the normalized dead time $\tau = L/(L+T)$, where T is the apparent lag and L the apparent dead time of the process, Åström and Hägglund (2005).

Trade-off plots for processes with delay dominated (τ close to 1), balanced (intermediate values of τ), and lag dominated dynamics (small values of τ), are shown in Fig. 1. The level curves for IE are horisontal lines $IE = 1/k_i$. The level curves of IAE and IE are almost identical in the lower parts of the graphs, where the IAE level curves are horisontal. IAE is greater than IE outside these regions where the controller parameters give closed-loop systems with overshoot in response to load disturbances. The smallest values of IAE are denoted by dots.

The robustness regions are loci for constant M_s, M_t, they are parabola-shaped curves. High robustness (low values of M_s and M_t) are obtained for small values of the controller gains. Controllers that minimize IE subject to the robustness constraint correspond to the maxima of the robustness curves. The loci of controller gains that minimize IAE for a given robustness are indicated by a dashed line in the graphs.

In the regions where the level curves of IAE are horisontal we have $IAE = IE = 1/k_i$. The performance is given by the integral gain k_i and robustness by the proportional gain k_p. The trade-off plots show that in the region to the right of the dashed line where the level curves of IAE are horisontal it is possible to decrease k_p with maintained performance and improved robustness. Notice that there is a value of the proportional gain that maximizes robustness.

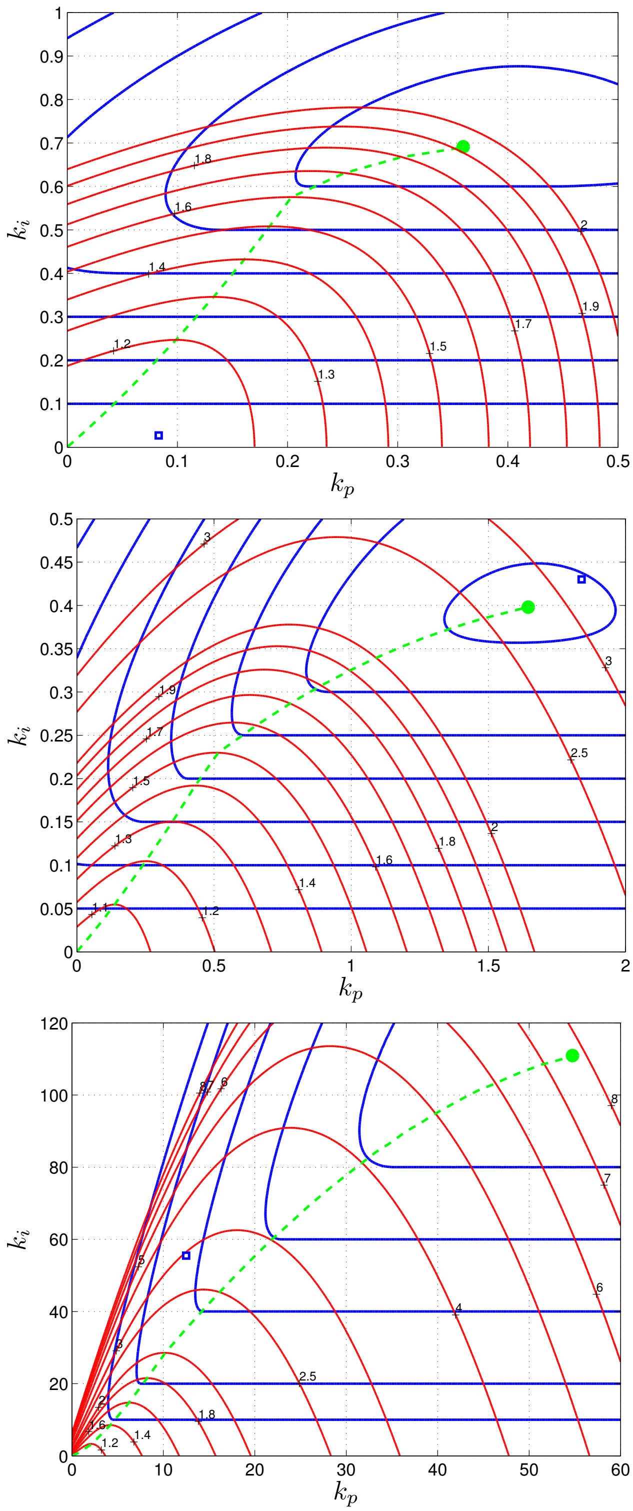

Fig. 1. Trade-off plots for PI control of the delay-dominant process $P_1(s) = e^{-s}/(1+0.05s)^2$ (top graph), the balanced process $P_2(s) = 1/(s+1)^4$ (middle graph), and the lag-dominant process $P_3(s) = 1/((s+1)(0.1s+1)(0.01s+1)(0.001s+1))$ (bottom graph). The dashed lines are the loci of controller gains that minimize IAE for a given robustness. Controllers tuned by the Ziegler-Nichols rules are marked by squares.

The sensitivity to measurement noise is not captured directly in the trade-off plots. However, if the effect of the low-pass filter is neglected, the high-frequency gain is determined by k_p. Requirements on noise sensitivity thus requires that proportional gain has to be less than a specified value.

The trade-off plots show that the absolute minimum of IAE gives controllers with robustness $M_s, M_t = 1.9$, 2.75 and 7.7, respectively. The absolute minimum thus gives controller with very poor robustness for systems with balanced or lag-dominated dynamics.

3.1 Delay-Dominated Dynamics

Consider the system with the transfer function

$$P_1(s) = \frac{1}{(1+0.05s)^2} e^{-s}. \tag{4}$$

A FOTD approximation gives $K = 1$, $T = 0.1$, $L = 1.0$, and $\tau = 0.92$ indicating that the dynamics is delay dominated. The trade-off plot is shown in the top graph in Fig. 1.

The level curves of IAE shows that the absolute minimum $IAE = 1.49$ occurs for $k_p = 0.36$ and $k_i = 0.69$. The sensitivities are $M_s, M_t = 1.9$. The controller that minimizes IAE subject to $M_s, M_t = 1.4$ has $IAE = 2.32$, the parameters are $k_p = 0.16$ and $k_i = 0.44$.

The level curves of IAE are almost horisontal in a large region. Performance is then given by the integral gain k_i, and the IAE values are close to $IE = 1/k_i$. In this region robustness is determined by the proportional gain k_p. In particular minimization of IAE and IE give the same controller parameters if robustness is restricted to $M_s, M_t < 1.6$, The figure shows that there is a significant freedom in choosing controller gain. For example, if $k_i = 0.2$ proportional gains between 0 and 0.26 give M_s and M_t smaller than 1.4.

Measurement noise is of little concern because the controller gains are small.

3.2 Process with Balanced Dynamics

Consider a system with the transfer function

$$P_2(s) = \frac{1}{(s+1)^4}. \tag{5}$$

A FOTD approximation gives $K = 1$, $T = 2.9$, $L = 1.4$, and $\tau = 0.33$ and the process has balanced dynamics. The trade-off plot is shown in the middle graph in Fig. 1. The absolute minimum of IAE is 2.8, which is achieved for $k_p = 1.64$ and $k_i = 0.4$. Performance changes little with the controller parameters close to the minimum, the level curve closest to the minimum has $IAE = 2.86$. The controller that minimizes IAE has poor robustness, $M_s, M_t = 2.75$. Assuming that we require $M_s, M_t = 1.4$, the integrated absolute error increases to $IAE = 5.2$. Controllers that minimize IAE or IE subject to the robustness constraints have the gains $k_p = 0.43$ and $k_i = 0.19$. If the robustness requirement is relaxed to $M_s, M_t = 1.6$, the smallest $IAE = 3.8$, is obtained for $k_p = 0.65$ and $k_i = 0.26$. The controller that minimizes IE has the gains $k_p = 0.62$ and $k_i = 0.29$.

The peaks of the curves for constant sensitivity correspond to the parameter values which minimizes IE. The figure shows that minimization of IE and IAE give the same results for M_s and M_t lower than 1.5. For larger values of M_s and M_t, minimization of IAE gives higher values of k_p and lower values of k_i. The plot shows the trade-off between performance and robustness. IAE is decreased from 10 to 5.2 when the sensitivities are increased from 1.2 to 1.4, indicating that there is an incentive to do frequent tuning or adaptation.

To minimize the effects of measurement noise the proportional gain should be as small as possible. From a robustness point of view, it is desirable to choose those PI parameters where the the sensitivities are small. It is interesting to note, that for reasonable values of M_s and M_t, i.e. $1.2 \leq M_s, M_t \leq 2$, these two requirements give the same choice of controller parameters, namely those where gain k_p is minimized.

3.3 Process with Lag-dominated Dynamics

Consider a system with the transfer function

$$P_3(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)}. \tag{6}$$

A FOTD approximation gives $K = 1$, $T = 1.0$, $L = 0.075$, and $\tau = 0.067$. The dynamics is thus lag dominated. The trade-off plot is shown in the lower graph in Fig. 1.

The unconstrained minimum $IAE = 0.0102$ is obtained for $k_p = 54.7$ and $k_i = 110.9$. The sensitivities are $M_s = M_t = 7.68$. Minimization of IAE thus gives a closed loop system with very poor robustness. The controller also has very high gains. With stricter robustness requirements the smallest IAE occurs at the boundary of the robustness region. Minimization of IE and IAE give the same results if the maximum sensitivities are less than $M_s, M_t = 1.95$. The range of sensitivities where IE and IAE give the same result are larger than in the two previous cases.

For lag-dominant processes, it is necessary to take measurement noise into account and noise filtering is essential. Assume for example that the robustness requirement is $M_s, M_t = 2$. The controller that minimizes IE has $k_p = 10$. Measurement noise of 1% of the signal span then results in control signal variations of 10% of the signal span. Since the gain is high, measurement noise may generate too large control actions. They can be reduced by filtering or by requiring lower controller gain k_p. A natural way to do this is to choose the largest proportional gain k_p that is acceptable from the view point of measurement noise and to pick the integral gain k_i from the dashed line.

The trade-off plot shows that k_p can be reduced while keeping the robustness constraint $M_s, M_t = 2$. However, this means that controller parameters are chosen in a region where the IAE level curves are almost vertical. Hence, the same performance can be obtained with a higher robustness by reducing integral gain k_i.

3.4 Tuning rules

The trade-off plots can be used to explore tuning rules for PI controller design. The controller parameters given by the Ziegler-Nichols step response method are shown by points marked by squares in Fig. 1. They show that Ziegler-Nichols tuning gives a controller that is close to the one that gives the absolute minimum of IAE for the process with balanced dynamics. This is not surprising since the systems explored by Ziegler and Nichols were primarily balanced and they focused on load disturbance response. The Ziegler-Nichols tuning rule gives poor robustness for balanced ($M_s = 3.2$) and lag-dominated ($M_s = 2$) systems. For delay dominated systems the rules give systems with very low integral gain. The response to load disturbances will therefore be very sluggish.

We will investigate lambda tuning, the SIMC rule, the modified SIMC rule and AMIGO. To do this Fig. 2 shows trade-off plots where the sensitivities are less than 2 with parameters for the tuning rules. The Ziegler-Nichols rule is excluded because the parameters are outside the plots except for delay dominated processes. Both lambda tuning, marked λ, and the SIMC rules, marked S and SM, have a tuning parameter, T_{cl}. It means that these methods are represented by lines in the plots. For the lambda method, the recommended choices $T_{cl} = T$, $2T$ and $3T$ are marked, and the corresponding values for the SIMC rules are $T_{cl} = L$, $2L$ and $3L$. In lambda tuning $T_{cl} = T$ is considered as agressive tuning. Skogestad recommends $T_{cl} = L$ which is designed to give $M_s = 1.6$. The AMIGO method is derived with the goal to minimize IE with the robustness constraint $M = 1.4$.

The trade-off plot for the system with delay-dominated dynamics is shown in the top plot of Fig. 2. Lambda tuning gives closed loop systems with poor robustness with sensitivities larger than 2. The proportional gain is too low and the integral gain too high. The SIMC rule gives a sensitivity close to the design value $M_s = 1.4$, but performance can be increased by increasing the proportional gain as is done by the modified SIMC rule (SM). The AMIGO rule gives a controller close to being IAE optimal with sensitivity $M_s, M_t = 1.46$, i.e. close to the desired. The AMIGO rule does not have a design parameter. Fig. 2 shows that the dashed line indicating the sensitivity constrained IAE controllers is approximately a straight line in the interesting robustness region, which means that the integral time is the same in these controllers, and controllers with different sensitivity can be obtained simply by changing the gain.

The trade-off plot for the system with balanced dynamics is shown in the center plot of Fig. 2. Lambda tuning gives closed loop systems with good performance and robustness. The sensitivities are approximately 1.55, 1.3 and 1.2 for $T_{cl} = T$, $2T$ and $3T$. The SIMC controllers have sensitivities 2, 1.5 and 1.4 and the modified rules have somewhat higher sensitivities. The nominal design has $M_s = 2$ which is significantly larger than the desired value 1.6. The AMIGO rule has sensitivity 1.3 which is slightly lower than the design value 1.4, its integral gain is the same as obtained by constrained IAE optimization but the gain is a bit higher.

The trade-off plot for the system with lag-dominated dynamics is shown in bottom plot of Fig. 2. Lambda tuning gives closed-loop systems with high robustness. The sensitivities are less than 1.1, but the preformance is very poor because the gains are much too low. Both SIMC rules give very similar performance. The sensitivity for

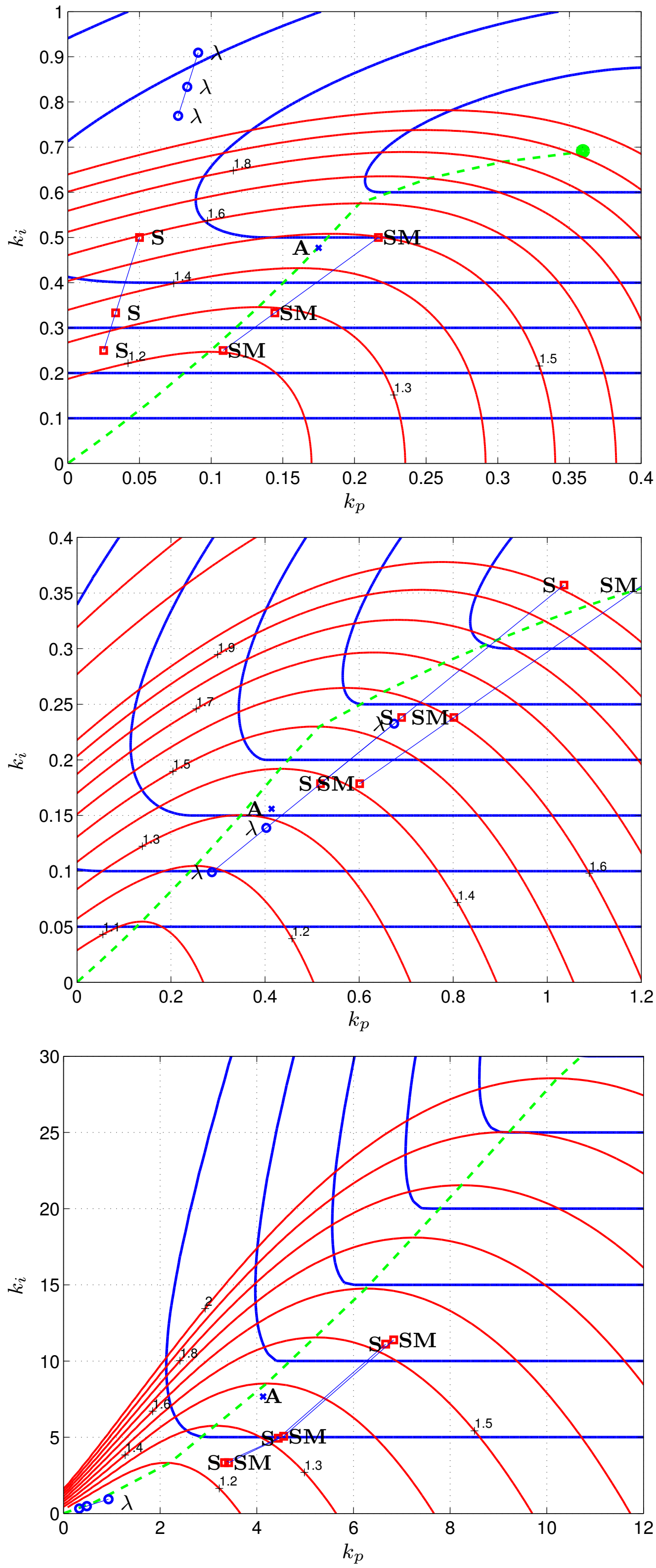

Fig. 2. Scaled trade-off plots with controller parameters obtained with a the tuning rules: lambda tuning (λ), SIMC (S), modified SIMC (SM) and AMIGO (A). The plot shows from top to bottom systems with delay dominated, balanced and lag-dominated dynamics. The dashed lines are the loci of controller gains that minimize IAE for a given robustness.

the nominal design is close to 1.5 instead of the design value 1.6 and the controller gains are higher than the constrained IAE optimal controller. The AMIGO rule has a sensitivity slightly below the design value 1.4 but is close to the constrained IAE optimal controller. Several of the controllers have high proportional gains k_p, which means that they may be too sensitive to measurement noise.

4. PID CONTROL

Since a PID controller has three parameters we will show trade-off plots for fixed values of the derivative gain k_d. Since derivative action has practically no benefit for systems with delay dominated dynamics we will focus on systems with balanced and lag dominated dynamics.

4.1 Balanced Dynamics

Fig. 3 shows trade-off plots for the processes with balanced dynamics with derivative gains k_d = 1, 2, and 3. Comparing with the corresponding plot for PI control, the center plot in Fig. 1, we find that the plots are similar. The gains are larger with derivative action and the sensitivity curves have a peak with discontinous derivative. This is a consequence of the derivative cliff discussed in Åström and Hägglund (2005). The absolute minimum of IAE without robustness contstraint corresponds to controllers with poor robustness, the sensitivities are close to 2.5 in all cases. If we require that the sensitivities are less than 1.4, Fig. 3 shows that the controllers with $k_d = 1, 2$ and 3 have the IAE values 2.5, 2.6 and 2.4. The constrained minimum is $IAE = 2.14$ for $k_p = 1.33$, $k_i = 0.63$, and $k_d = 1.78$. The IAE value can be compared with the corresponding value for PI control $IAE = 4.4$, adding derivative actions thus improves performance by a factor of 2. Fig. 3 also shows that minimization of IE and IAE do not give the same controllers except if the robustness constraint requires very low sensitivities. The dashed line which corresponds to the constrained minimum has a plateau for large values of k_d. The dashed line has a different shape than for PI control. Notice that the dashed line is close to the constraint curve for small values of the sensitivities.

4.2 Lag-dominated Dynamics

The trade-off plot for a system with lag dominated dynamics is shown in Fig. 4, with derivative gains k_d = 3, 4.5, and 6. Comparing with the corresponding plot for PI control, bottom plot in Fig. 1, we find that the gains are significantly larger. The absolute minima of IAE corresponds to controllers with sensitivities above 3.5. Minimizing IAE without a robustness constraint thus gives systems with poor robustness. The level curves for the sensitivities have edges for sensitivities 2 and lower. If we require that the sensitivities are $M_s, M_t = 1.4$ Fig. 4 shows that the controllers with $k_d = 3, 4.5$ and 6 have the IAE values 0.0021, 0.0013, and 0.0016. The constrained minimum is $IAE = 0.0013$ for $k_p = 89.48$, $k_i = 1037.5$, and $k_d = 4.59$. The IAE value can be compared with the corresponding value for PI control $IAE = 0.1175$. Adding derivative actions thus improves performance by a factor of 90. Since the gains are large it is important to consider the effect of measurement noise and it may therefore be essential

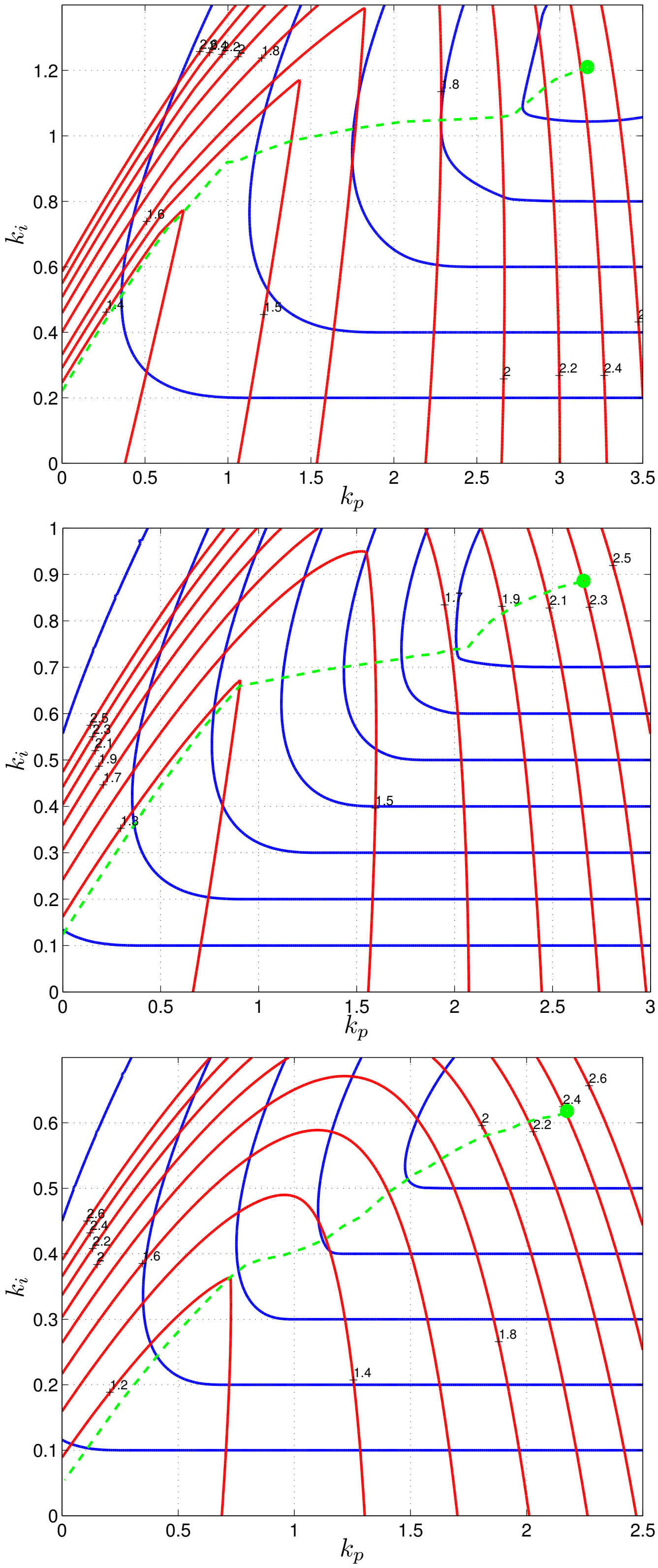

Fig. 3. Trade-off plot for PID control of the process $P_2(s) = 1/(s+1)^4$, and derivative gains $k_d = 1$ (lower graph), $k_d = 2$ (middle graph), and $k_d = 3$ (top graph). The dashed lines are the loci of controller gains that minimize IAE for a given robustness.

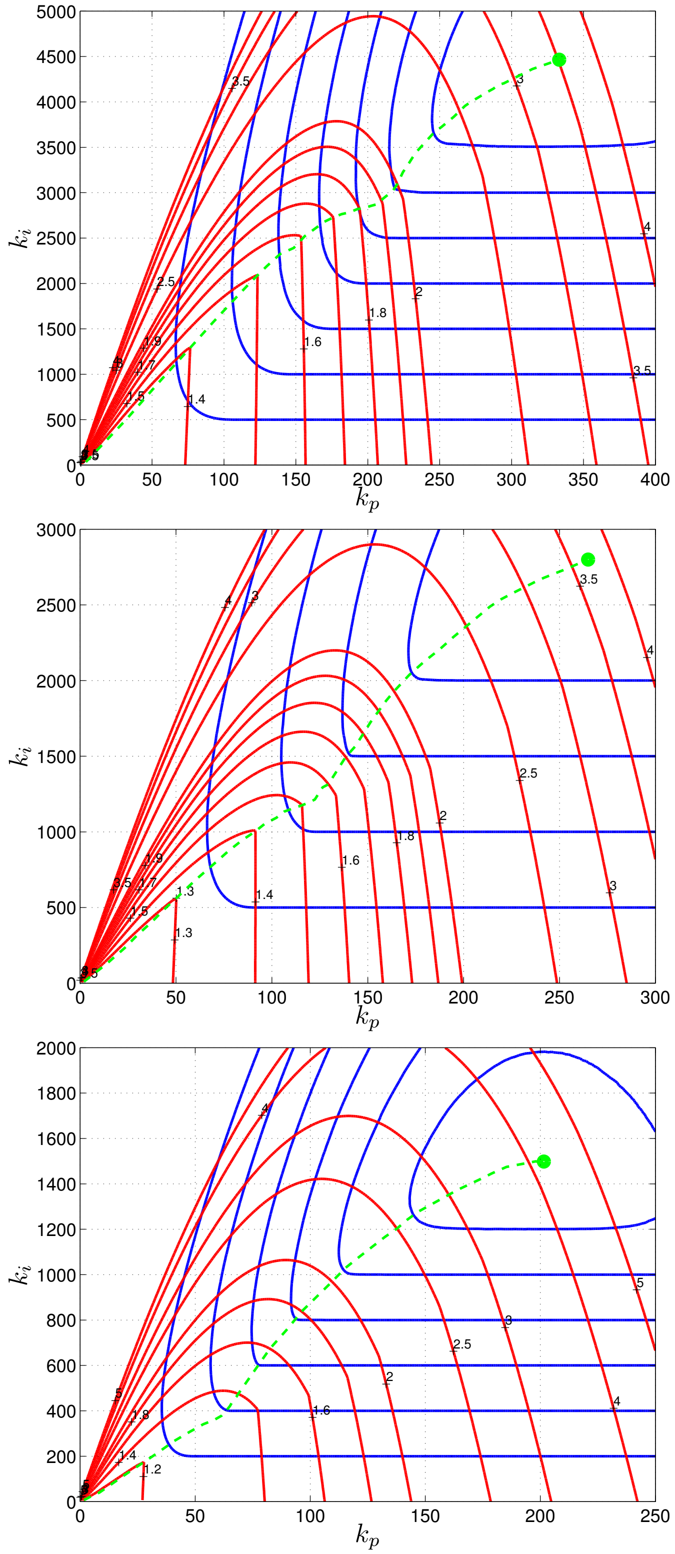

Fig. 4. Trade-off plot for PID control of the process $P_3(s) = 1/((s+1)(0.1s+1)(0.01s+1)(0.001s+1))$, and derivative gains $k_d = 3$ (lower graph), $k_d = 4.5$ (middle graph), and $k_d = 6$ (top graph). The dashed lines are the loci of controller gains that minimize IAE for a given robustness.

to impose constraints on the proportional and derivative gains. The dashed line in the trade-off plots give guidance for detuning.

5. CONCLUSION

The trade-off plots give insight into the design problem. Minimization of the performance criteria IE and IAE without robustness constraint give controllers with poor robustness. The difference between minimizing IE and IAE are small if the robustness requirements are strict but may be significant for sensitivities larger than 1.2. There are significant differences between processes with lag dominant and delay dominant dynamics.

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