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A Radial Basis Function Method for Approximating the Optimal Event-Based Sampling Policy

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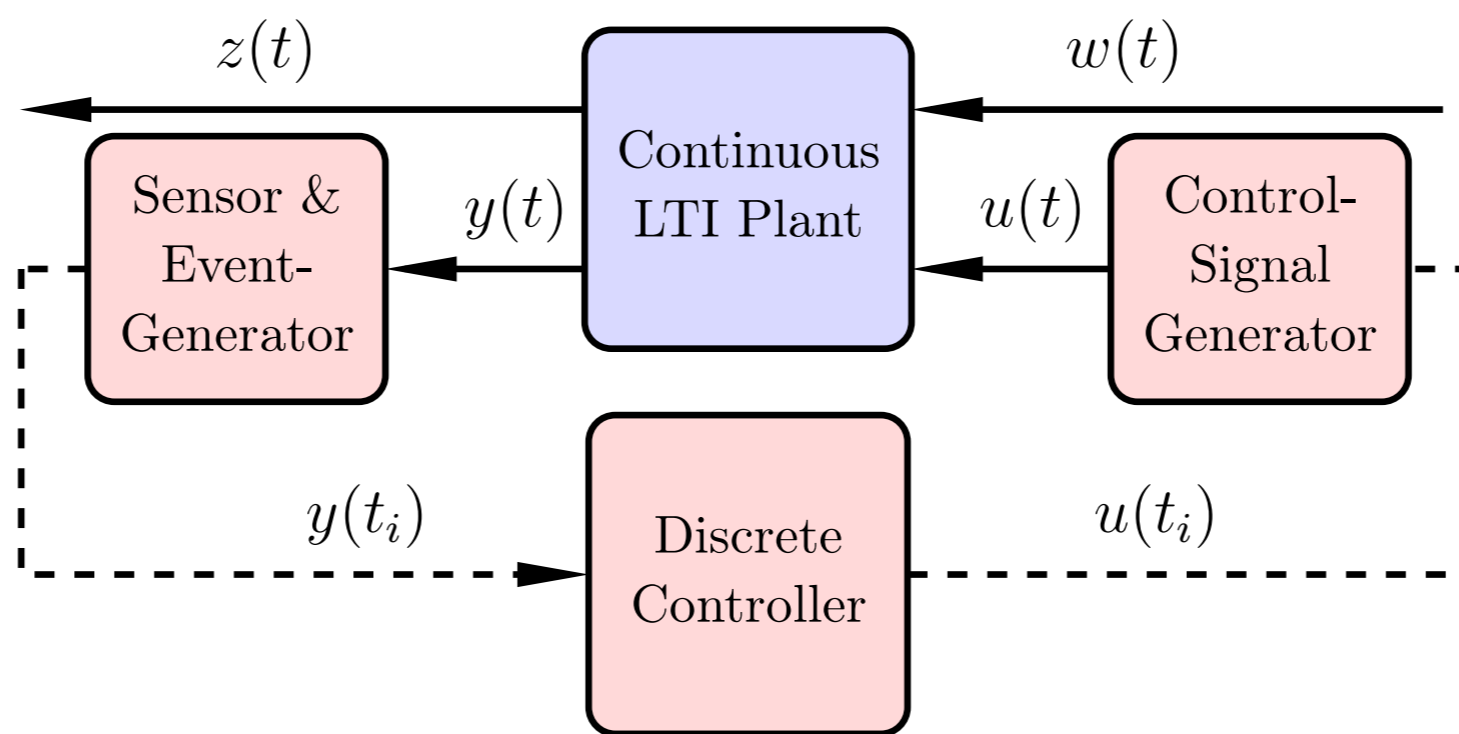


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INTRODUCTION

In networked control systems it is desirable to have efficient wireless communication (saving energy and bandwidth) while still ensuring good control performance. By abandoning periodic sampling, communication can be made more efficient by sampling and updating the control signal only "when required" based on the system's behaviour. This is the concept of *event-based control*.

In this work we consider the classic LQG problem with an added penalty ρ on the average sampling rate f .



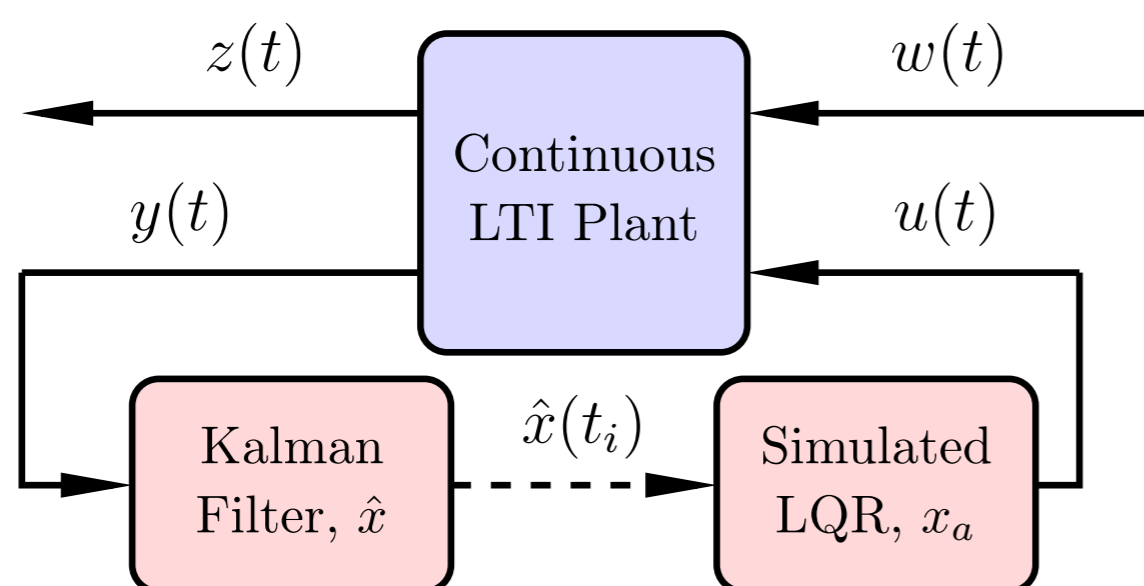
$$\text{Minimize } J = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\int_0^T z(t)^\top z(t) dt \right] + \rho f$$

With the optimal controller structure known [1], the challenge is to find the optimal sampling policy. We extend the work in [2], and make the following contributions:

- Deriving an RBF method to approximate the value function and optimal sampling policy.
- Proving *guaranteed existence and uniqueness* of optimal RBF weights.
- *Numerical validation* of the proposed method.

AN EQUIVALENT SETUP

The optimal structure has the equivalent representation [1]:



Sampling sets $x_a(t_i) = \hat{x}(t_i)$, and resets error $\tilde{x} = \hat{x} - x_a$ to zero.

$$\text{Error: } \begin{cases} d\tilde{x} = A\tilde{x}dt + d\epsilon, \\ \tilde{x}(t_i) = 0, \end{cases} \quad \text{Noise: } \begin{cases} \mathbb{E}[d\epsilon] = 0, \\ \mathbb{E}[d\epsilon d\epsilon^\top] = R dt. \end{cases}$$

With optimal cont. time LQG cost γ_0 , the sampled system cost is

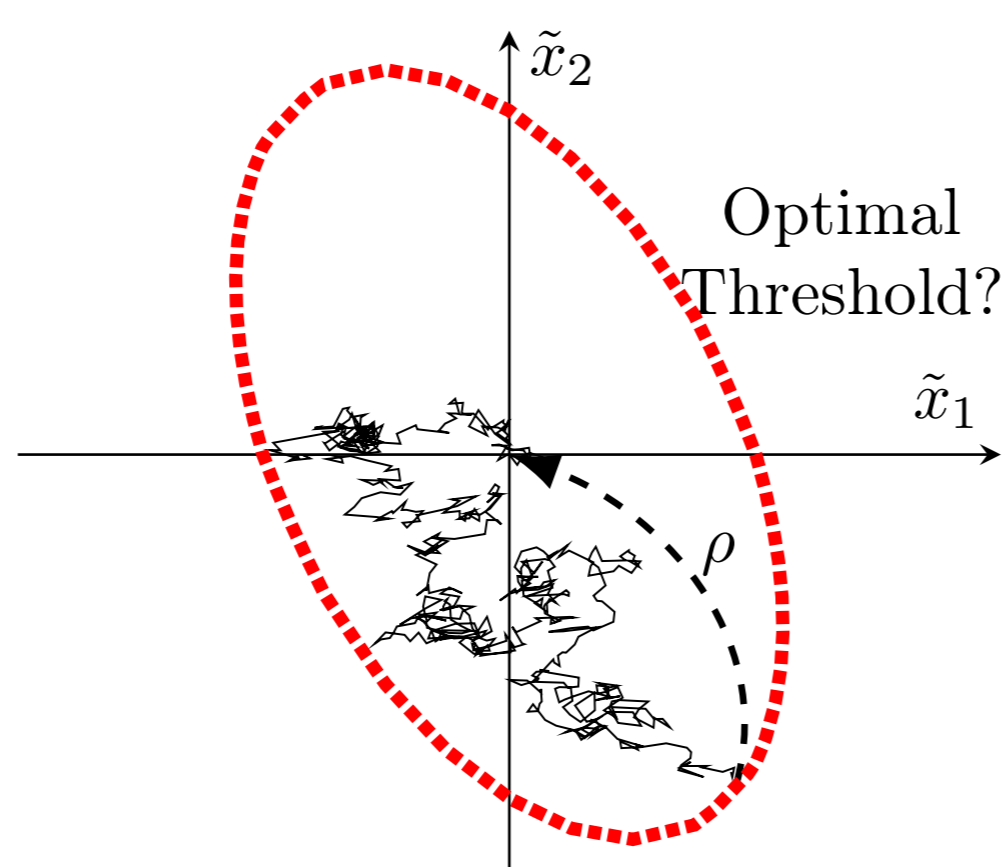
$$J = \gamma_0 + \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\int_0^T \tilde{x}(t)^\top Q \tilde{x}(t) dt \right] + \rho f.$$

THE OPTIMAL SAMPLING PROBLEM

We search for a threshold in the \tilde{x} -space, from which we reset \tilde{x} to zero and accrue the cost ρ . This is an *optimal stopping problem*.

The optimal policy is given by the *relative value function* $V(\tilde{x})$ of the problem.

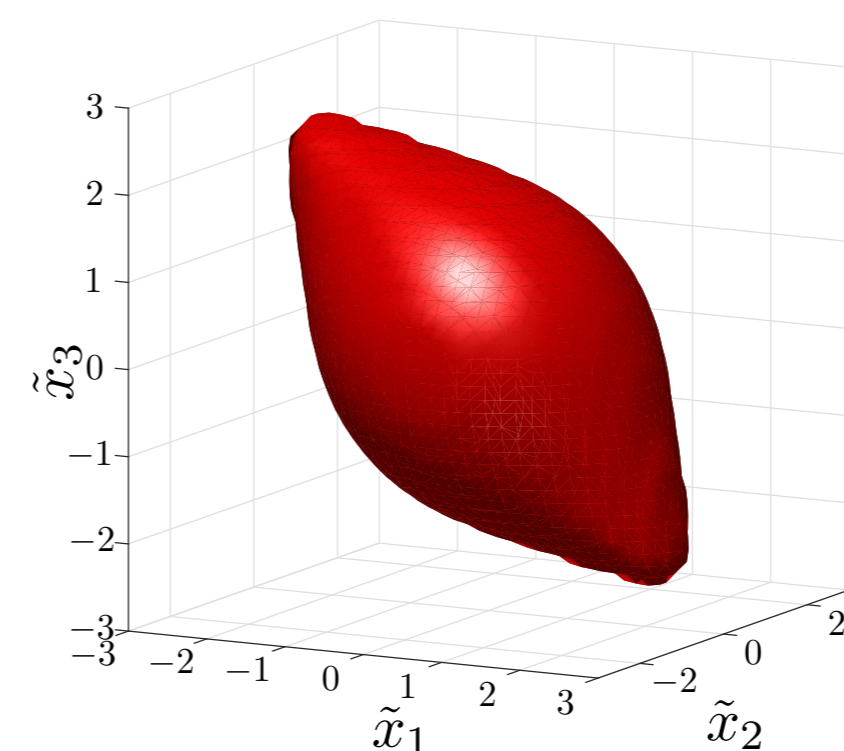
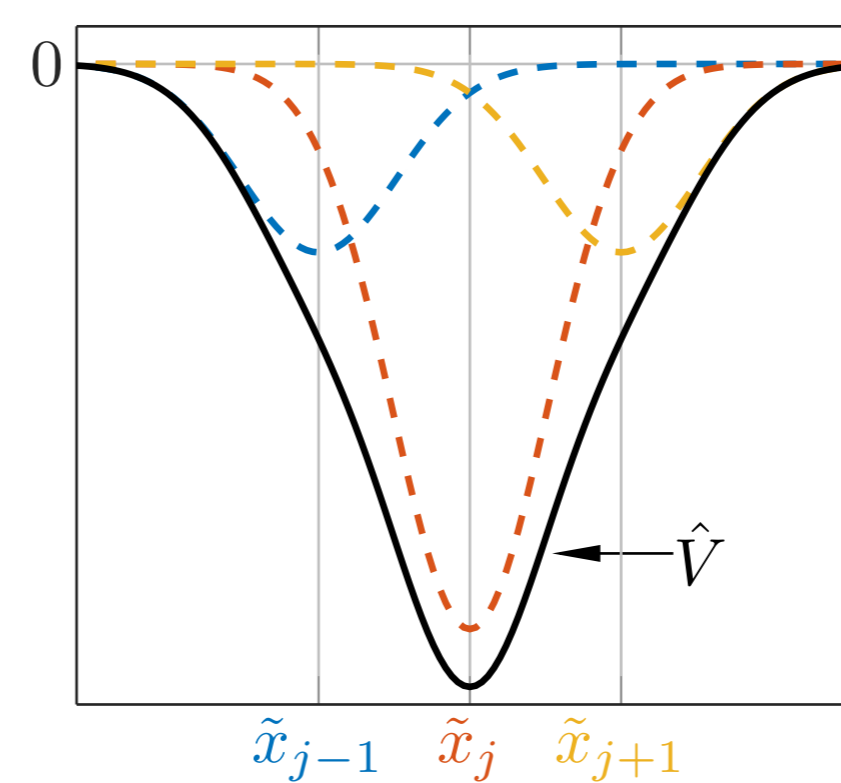
Optimal Policy:
Sample when $V(\tilde{x}) = 0$.



V satisfies the free boundary PDE:

$$\min \{ \tilde{x}^\top Q \tilde{x} + \tilde{x}^\top A^\top \nabla V + \frac{1}{2} \text{Tr}(R \nabla^2 V) - J, \rho + V(0) - V(\tilde{x}) \} = 0.$$

RBF SOLUTION



3D example: $\hat{V}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) = 0$

1. RBF-Approximation:

$$\hat{V} = \sum_{i=1}^n \alpha_i \phi(\tilde{x} - \tilde{x}_i)$$

2. Rescale PDE & insert \hat{V} :

$$\min \{ \Gamma \alpha + \beta, -\Phi \alpha \} = 0, \\ \alpha = [\alpha_1, \dots, \alpha_j, \dots, \alpha_n]^\top, \\ \Phi_{i,j} = \phi(\tilde{x}_i - \tilde{x}_j).$$

3. Reformulate as LCP:

$$z^\top (Mz + \beta) = 0, \\ z \geq 0, \quad (Mz + \beta) \geq 0$$

4. Solve QP:

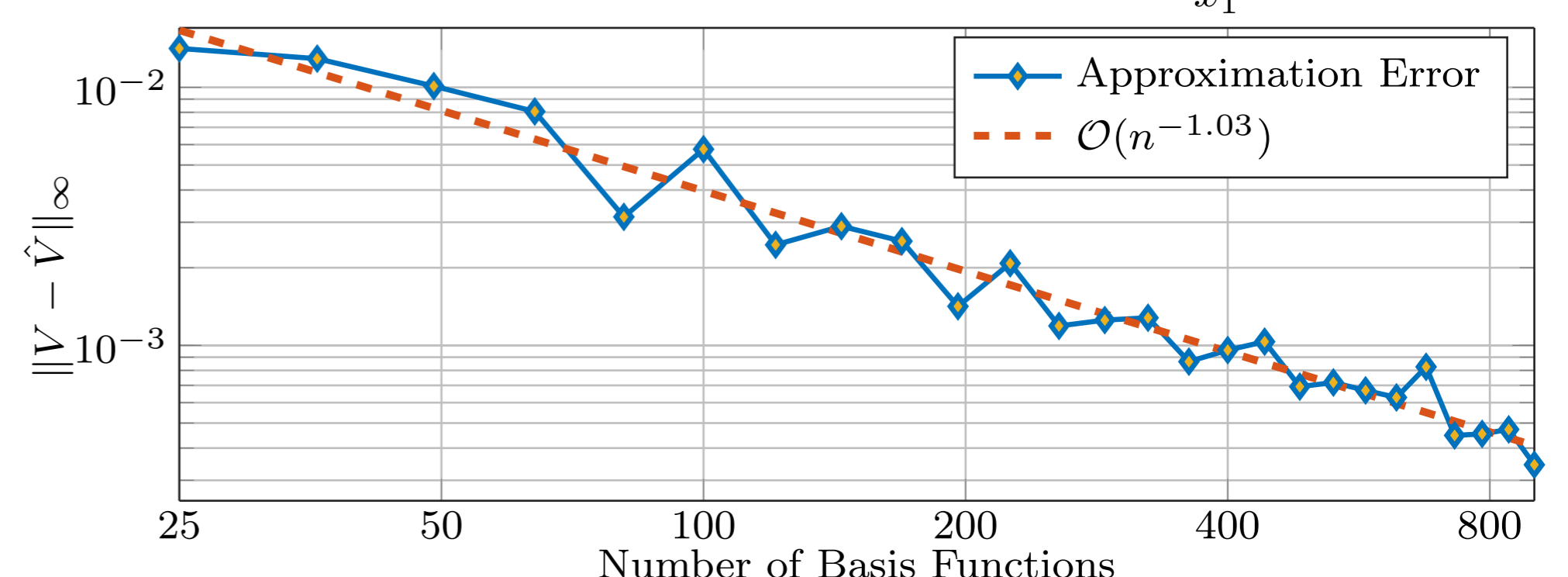
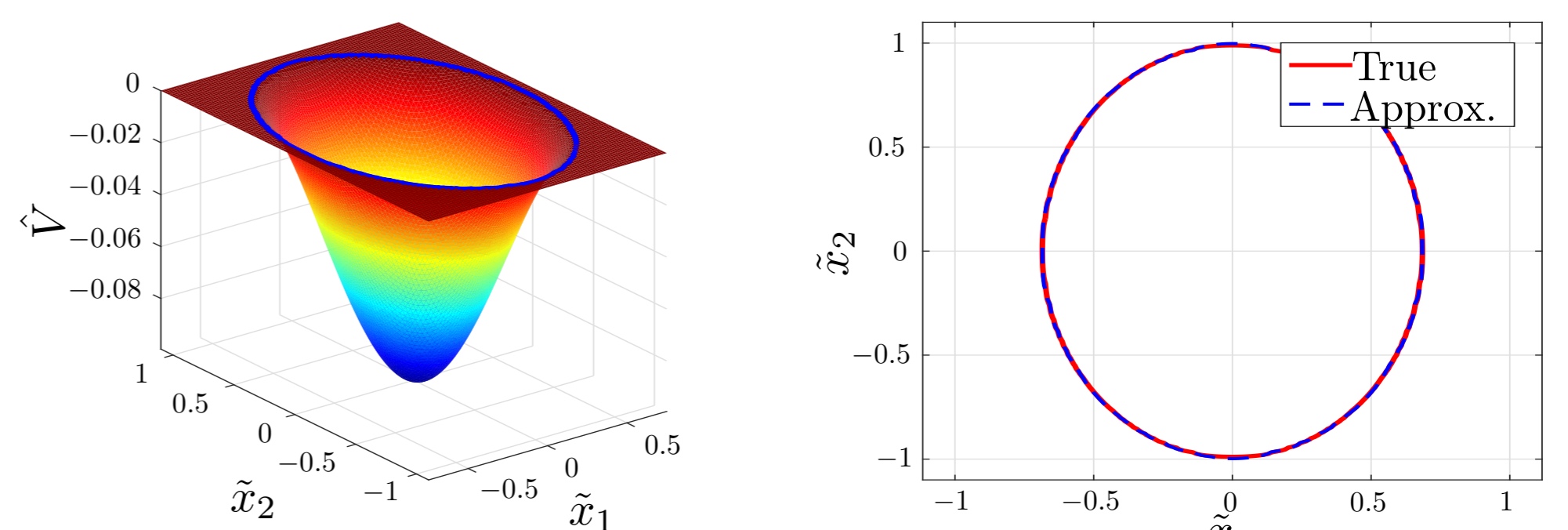
$$\min_z z^\top (Mz + \beta), \\ \text{s.t. } z \geq 0 \quad (Mz + \beta) \geq 0$$

5. RBF Weights:

$$\alpha = -\Phi^{-1} z$$

VALIDATION

$$\text{Analytic solution for } A = 0 : V(\tilde{x}) = -\frac{1}{4} \max(2\sqrt{\rho} - \tilde{x}^\top P \tilde{x}, 0)^2$$



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- [1] L. Mirkin. "Intermittent Redesign of Analog Controllers via the Youla Parameter". *IEEE Trans. Automatic Control*, vol 62, no. 4, 2017.
- [2] M. Thelander Andrén, B. Bernhardsson, A. Cervin, K. Soltesz. "On Event-Based Sampling for LQG-Optimal Control". In: *56th IEEE Conference on Decision and Control*, 2017.