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Panel Cointegration Tests of the Fisher Hypothesis*

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Abstract

Recent empirical studies suggest that the Fisher hypothesis, stating that inflation and nominal interest rates should cointegrate with a unit parameter on inflation, does not hold, a finding at odds with many theoretical models. This paper argues that these results can be explained in part by the low power inherent in univariate cointegration tests and that the use of panel data should generate more powerful tests. In doing so, we propose two new panel cointegration tests, which are shown by simulation to be more powerful than other existing tests. Applying these tests to a panel of monthly data covering the period 1980:1 to 1999:12 on 14 OECD countries, we find evidence supportive of the Fisher hypothesis.

JEL Classification: C12; C15; C32; C33; E40.

Keywords: Fisher Hypothesis; Residual-Based Panel Cointegration Test; Monte Carlo Simulation.

1 Introduction

The ex ante real interest rate affects all intertemporal investment and savings decisions in the economy. As such, the ex ante real rate is a key variable in understanding the dynamics of asset prices over time. Its long-run behavior is often analyzed through the Fisher identity, which defines the ex ante real rate as the difference between the nominal rate and expected inflation. Beginning with Mishkin (1992), research usually suggests that both realized inflation and nominal interest rates are nonstationary, and hence are affected by permanent shocks. Thus, for the ex ante real rate to be affected by only transitory disturbances, these findings imply that any permanent shocks to either the nominal

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rate or expected inflation must cancel out through the Fisher relationship. Since permanent shocks to rationally expected inflation match the permanent shocks to realized inflation, this suggests that ex ante real rates will be subject to permanent shocks unless inflation and nominal rates move one-for-one in the long run. Thus, if inflation and nominal rates are nonstationary processes and the Fisher hypothesis holds in the long run, then these series should be cointegrated with a unit parameter on inflation. In this case, the series move one-for-one in the long run such that their permanent disturbances cancel out leaving the real rate stationary.

Despite the general acceptance of the Fisher hypothesis among economic theoreticians, a stable long-run one-for-one relationship between inflation and nominal interest rates has proven extremely difficult to establish empirically. In fact, most time series evidence based on data for the United States tend to favor a rejection of the hypothesis. A number of studies, including those of Mishkin (1992), Crowder and Hoffman (1992), and Evans and Lewis (1995), observe cointegration between inflation and nominal interest rates but with the estimated parameter on inflation being significantly different from one suggesting that ex ante real rates are subject to permanent shocks. Other studies, such as those of Rose (1988), MacDonald and Murphy (1989), Bonham (1991), and King and Watson (1997), fail to find cointegration in the first place in which case the Fisher hypothesis may be rejected out of hand. Findings of this sort are puzzling since they seem to directly contradict the first-order condition of standard intertemporal models insofar these models predict that consumption growth rates should also be affected by permanent shocks, a hypothesis typically rejected by the data. Furthermore, assuming that inflation is primarily driven be monetary growth, superneutrality fails as changes in the rate of monetary growth affects inflationary expectations and subsequently real rates.

There are at least two limitations to the existing literature. One limitation is the failure to account for the low power inherent in conventional cointegration tests against highly autoregressive alternatives in small samples. In spite of this, the low power of commonly applied tests continues to be one of the most widely held explanations of the apparent failure of the Fisher hypothesis to materialize. Another limitation of the earlier literature is that it is almost exclusively concerned with data on the United States and only a few attempts have been made based on international data. Three such studies are those of Ghazali and Ramlee (2003), Koustas and Serletis (1999), and Strauss and Terrell (1995). Ghazali and Ramlee (2003) employs monthly data from 1974:1 to 1996:6 on the G7 countries and cannot reject the null hypothesis of no cointegration between the inflation and nominal interest rates. Similarly, using quarterly data covering the period 1957:1 to 1995:2 for 11 OECD countries, Koustas and Serletis (1999) find no evidence of cointegration for any of the countries except Japan. Strauss and Terrell (1995) employs quarterly data between 1973:1 and 1989:4. Among the six OECD countries considered, the null of no cointegration

can only be rejected for Japan. All three studies therefore reject the Fisher hypothesis, which its indicative of its poor support internationally.

Given these apparent weaknesses in the earlier literature, it is surprising that so little attention has been paid to panel data. Tests based on panel data are distinct in that they bring more information to bare on the Fisher hypothesis through the increased number of observations that derives from adding individual time series. To correct for these shortcomings, in this paper we investigate the Fisher hypothesis using a panel of monthly data covering the period 1980:1 to 1999:12 on 14 OECD countries. In doing so, we propose two new residual-based tests for the null hypothesis of no cointegration. The tests are based on the Durbin-Hausman principle whereby two estimators of a unit root in the residuals of an estimated regression are compared. Both estimators are consistent under the null hypothesis but only one retains the property of consistency under the alternative. Using sequential limit arguments, it is shown that the test statistics are free of nuisance parameters and that they have a limiting normal distribution under the null hypothesis. Results from a small Monte Carlo study suggest that the proposed tests have greater power than other popular residual-based tests is samples comparable with ours. In our empirical analysis, contrary to much of the earlier literature, we find evidence in favor of the Fisher hypothesis.

The paper proceeds as follows. Section 2 provides a brief presentation of the Fisher hypothesis. Section 3 introduces the Durbin-Hausman test statistics, whereas Sections 4 and 5 are concerned with their asymptotic and finite sample properties. Sections 6 and 7 then present our empirical results. Section 8 concludes the paper. For notational convenience, the Bownian motion $B_i(r)$ defined on the unit interval $r \in [0,1]$ will be written as only B_i and integrals such as $\int_0^1 W_i(r)\mathrm{d}r$ will be written $\int_0^1 W_i$ and $\int_0^1 W_i(r)\mathrm{d}W_i(r)$ as $\int_0^1 W_i\mathrm{d}W_i$. We will use $\Rightarrow$ to signify weak convergence, $\xrightarrow{p}$ to signify convergence in probability and $[z]$ to signify the largest integer less than z.

2 The Fisher hypothesis

The Fisher hypothesis states that in long-run equilibrium, nominal rates should adjust perfectly to changes in expected inflation leaving the expected ex ante real interest rate unaffected. Formally, ignoring tax effects, the Fisher identity can be stated as

$$r_{it} = E(p_{it}) + E(q_{it}), \tag{1}$$

where r_{it} is the nominal interest rate observed at time t for country i, $E(p_{it})$ is the expected rate of inflation based on the currently available information set, and $E(q_{it})$ is the corresponding ex ante real interest rate. Under rational expectations, the realized rate of inflation may be written as follows

$$p_{it} = E(p_{it}) + u_{it}, \tag{2}$$

where u_{it} is a mean zero stationary forecast error that is orthogonal to any information known at time t. Equations (1) and (2) imply that the following relation between r_{it}, p_{it} and $E(q_{it})$ must hold by identity

$$r_{it} - p_{it} = E(q_{it}) - u_{it}. \tag{3}$$

In this expression, only the inflation and nominal interest rates are observable. The difference between these variables is identically q_{it}, the ex post real interest rate, comprised of the ex ante real rate and the forecast error. Because the inflation and nominal interest rates are unit root processes, we can use panel cointegration techniques to infer whether the ex post real interest rate contains shocks with the same degree of persistence as those variables. In particular, the relationship between the unit root components of these variables may be examined through the following regression

$$r_{it} = \alpha_i + \beta_i p_{it} + e_{it}. \tag{4}$$

The regression is said to be cointegrated if the error e_{it} is stationary, while it is spurious if e_{it} is nonstationary. The Fisher hypothesis posits the ex post real interest rate a stationary variable. This suggests that inflation and the nominal rate should cointegrate with a unit slope on inflation. To get an intuition on this, notice that (3) and (4) imply that the ex post real interest rate can be written in the following fashion

$$q_{it} = \alpha_i - (1 - \beta_i)p_{it} + e_{it} - u_{it}. \tag{5}$$

This expression is very instructive when deriving testable long-run predictions of the Fisher hypothesis. Given our assumption of rational expectations, the forecast error of inflation must be unforecastable conditional on any information known at time t suggesting that u_{it} must be a stationary variable. Hence, q_{it} can only be nonstationary if $E(q_{it})$ is nonstationary. In this model therefore, the problem of testing the Fisher hypothesis is equivalent to testing whether the ex ante real rate is stationary or not. The expressions in (3) and (5) suggest that this variable can be written as

$$E(q_{it}) = \alpha_i - (1 - \beta_i)p_{it} + e_{it}. \tag{6}$$

Suppose that the inflation and nominal interest rates are cointegrated. In this case, e_{it} is stationary and the integratedness of the ex ante rate therefore only depend on the integratedness of $(1 - \beta_i)p_{it}$. Towards this end, consider first the implications of letting $\beta_i = 1$. In this case, $(1 - \beta_i)p_{it}$ vanishes so variations in the ex ante real rate only reflects temporary deviations from its mean value, which is given by α_i. Apparently, since the nominal interest rate moves one-for-one with the rate of inflation in the long run, their unit root components cancel out leaving the ex ante real rate unaffected in which case the full Fisher effect is said to hold. By contrast, if we let $\beta_i \neq 1$, then $(1 - \beta_i)p_{it}$ will not vanish

suggesting that the ex ante real rate must contain the same unit root component as inflation and that it will be nonstationary. Of cause, r_{it} and p_{it} may still be cointegrated even though the ex ante real interest rate is nonstationary. But since $\beta_i \neq 1$, there is said to be only a partial Fisher effect. If inflation and nominal interest do not cointegrate, then e_{it} is nonstationary so (4) becomes spurious and there is no Fisher effect. It follows that cointegration is a necessary condition for the Fisher hypothesis to hold in the long run.

3 The Durbin-Hausman tests

The previous section suggests that the testing for panel cointegration is key in inferring the long-run Fisher hypothesis. In this section, therefore, we propose two new tests for panel cointegration, which are shown through simulations to be more powerful than other existing tests. The data generating process (DGP) may be characterized in terms of the vector $z_{it} = (r_{it}, p_{it})'$ as follows

$$z_{it} = z_{it-1} + v_{it}. \tag{7}$$

To be able to derive the tests, we make the following assumptions regarding the cross-sectional and temporal properties of v_{it}.

Assumption 1. (Error process.) (i) The process v_{it} is i.i.d. cross-sectionally; (ii) The partial sum process $S_{iT} = \sum_{t=1}^{[Tr]} v_{it}$ satisfies the invariance principle $T^{-1/2} S_{iT} \Rightarrow B_i \equiv L_i W_i$ as $T \longrightarrow \infty$ with N held fixed, where B_i is a vector Brownian motion with covariance matrix $\Omega_i = L_i' L_i$.

Assumption 1 provides us with the basic conditions for developing the Durbin-Hausman tests. Assumption 1 (i) states that the individuals are i.i.d. over the cross-sectional dimension. This condition is convenient as it will allow us to apply standard central limit theory in a relatively straightforward manner. For many empirical applications, however, the i.i.d. assumption may quite restrictive and Section 7 therefore suggests alternative tests based on the bootstrap principle. Notwithstanding, for the present we shall require Assumption 1 (i) to hold. Assumption 1 (ii) imposes a restriction on the temporal dependence of v_{it}. This restriction is generally considered to be quite weak and include, for example, the entire class of all stationary autoregressive moving average processes. In particular, it ensures that the covariance matrix of B_i, equally the long-run covariance matrix of v_{it}, exist and that it may be written as

$$\Omega_i \equiv \lim_{T \longrightarrow \infty} T^{-1} E(S_{iT} S_{iT}') = \begin{pmatrix} \omega_{i11}^2 & \omega_{i21} \\ \omega_{i21} & \Omega_{i22} \end{pmatrix}.$$

In keeping with the previous cointegration literature, we place no restrictions on Ω_i. Notably, the fact that Ω_i is permitted to vary between the individuals of the panel reflects that we are in effect allowing for a completely heterogeneous

long-run covariance structure. Moreover, we make no assumptions regarding the endogeneity structure of the regressor, which is captured by the off-diagonal element ω_{i21} of Ω_i.

In this section, we are concerned with the problem of testing the hypothesis of no cointegration in panel data. To this end, consider fitting the regression in (4) using OLS. This regression may be written as

$$r_{it} = \hat{\alpha}_i + \hat{\beta}_i p_{it} + \hat{e}_{it}. \tag{8}$$

The residual series $\hat{e}_{it}$ is stationary when r_{it} and p_{it} are cointegrated and it has a unit root when they are not. Thus, testing the null hypothesis of no cointegration is equivalent to testing the regression residuals for a unit root using the following autoregression

$$\hat{e}_{it} = \rho_i \hat{e}_{it-1} + u_{it}. \tag{9}$$

In what follows, we shall propose two test statistics that are based on the value taken by the autoregressive parameter ρ_i. The first statistic is restricted in the sense that it is constructed under the maintained assumption that the autoregressive parameter takes on a common value $\rho_i = \rho$ for all individuals i. The second statistics is unrestricted and does not require ρ_i to be equal for all i. A consequence of this distinction arises in the formulation of the alternative hypothesis of the test. For the restricted statistic, the null and alternative hypotheses is formulated as $H_0 : \rho_i = 1$ for all i versus $H_1 : \rho_i = \rho < 1$ for all i. Hence, in this case, we are in effect presuming a common value for the autoregressive parameter both under the null and alternative hypotheses. A rejection of the null should therefore be taken as evidence in favor of cointegration for all the individuals in the panel. By contrast, for the unrestricted statistic, H_0 is tested versus the alternative that $H_1 : \rho_i < 1$ for $i = 1, ..., N_1$ and $\rho_i = 1$ for $i = N_1 + 1, ..., N_2$. Thus, in this case, we are not presuming a common value for the autoregressive parameter and, as a consequence, a rejection of the null cannot be taken to suggest that the entire panel is cointegrated. Instead, a rejection should be interpreted as providing evidence in favor of rejecting the null hypothesis for a nonzero fraction of the panel.

Let $\tilde{e}_{it} = (\hat{e}_{it}, \hat{e}_{it-1}, \Delta\hat{e}_{it})'$, $E_i = \sum_{t=1}^{T} \tilde{e}_{it}\tilde{e}'_{it}$ and $E = \sum_{i=1}^{N} \hat{\omega}_{i1.2}^{-2} E_i$, where $\hat{\omega}_{i1.2}^2 = \hat{\omega}_{i11}^2 - \hat{\omega}_{i21}^2 \hat{\Omega}_{i22}^{-1}$ is any consistent estimator of $\omega_{i1.2}^2 \equiv \omega_{i11}^2 - \omega_{i21}^2 \Omega_{i22}^{-1}$. The Durbin-Hausman statistics of H_0 versus H_1 is composed of two estimators of ρ_i, which have different probability limits under the alternative hypothesis but share the same property of consistency under the null. As shown by Choi (1992, 1994), the pseudo instrumental variables (IV) estimators $\tilde{\rho}_i = E_{i12}^{-1} E_{i11}$ and $\tilde{\rho} = E_{12}^{-1} E_{11}$ are consistent under the null hypothesis but are inconsistent under the alternative. On the other hand, the OLS estimators $\hat{\rho}_i = E_{i22}^{-1} E_{i12}$ and $\hat{\rho} = E_{22}^{-1} E_{12}$ are consistent both under the null and alternative hypotheses. Hence, the pseudo IV and OLS estimators may be used to construct the Durbin-Hausman statistics.

Definition 1. (The Durbin-Hausman test statistics.) The statistics are defined as follows

$$DH_R \equiv \hat{\sigma}^2 \hat{\gamma}_0^{-2} (\tilde{\rho} - \hat{\rho})^2 E_{22} \quad \text{and} \quad DH_U \equiv \sum_{i=1}^{N} \hat{\sigma}_i^2 \hat{\gamma}_{i0}^{-2} (\tilde{\rho}_i - \hat{\rho}_i)^2 E_{i22},$$

where $\hat{\sigma}_i^2 = \sum_{k=-M}^{M} \omega(k/M) \hat{\gamma}_{ik}$ and $\hat{\Omega}_i = \sum_{k=-M}^{M} \omega(k/M) \tilde{\gamma}_{ik}$ with $\hat{\gamma}_{ik}$ and $\tilde{\gamma}_{ik}$ being the k order autocovariances of the OLS estimates $\hat{u}_{it}$ and $\hat{v}_{it}$ of u_{it} and $v_{it} = z_{it} - z'_{it-1}\delta_i$, and $\omega(j/M)$ is the Bartlett window $1 - j/(1+M)$. The quantities $\hat{\sigma}^2$ and $\hat{\gamma}_0$ are the cross-sectional averages of $\hat{\omega}_{i1.2}^{-2} \hat{\sigma}_i^2$ and $\hat{\omega}_{i1.2}^{-2} \hat{\gamma}_{i0}$.

For consistency of $\hat{\sigma}_i^2$ and $\hat{\Omega}_i$, it is necessary that M does not increase too fast relative to T. Sufficient conditions are given by $M \longrightarrow \infty$ and $M = O(T^{1/3})$ as $T \longrightarrow \infty$. Also, in view of DH_R, note that, although the autoregressive parameters are presumed equal, both the variances and the cointegration vectors themselves are allowed to vary between the individuals of the panel. Thus, the statistic only pools the information regarding the possible existence of a cointegration relationship as indicated by the stationarity properties of the estimated residuals. The weighting terms $\hat{\omega}_{i1.2}^2$ also deserves a special comment. Asymptotically, the distribution of the test is invariant with respect to $\hat{\omega}_{i1.2}^2$, which suggests that we may construct a cumputually simpler unweighted statistic that is asymptotically equivalent to DH_R. In small samples, however, our Monte Carlo experiments indicate that the weighted statistic performs better and that the $\hat{\omega}_{i1.2}^2$ terms therefore should be included in order to ensure that the small-sample distribution of the statistic is free of the nuisance parameters associated with the serial correlation properties of the data.

The restricted statistic is constructed by summing the separate terms over the cross-section prior to multiplying them together. In contrast, the unrestricted statistic is constructed by first multiplying the various terms and then summing over the N dimension. This makes the construction of DH_U particularly simple. In fact, closer inspection reveals that DH_U is nothing but the sum of N ratios corresponding to the conventional time series statistics studied in Choi (1994). Note also the multiplicative form of the endogeneity and serial correlation corrections employed by both statistics. This makes them computually convenient in comparison to the semiparametric versions of the Dickey-Fuller test statistics proposed by Pedroni (1999, 2004), where the corrections enters both multiplicatively and additively.

4 Asymptotic distribution

In this section, we characterize the asymptotic distribution of the test statistics proposed in Section 3. For this purpose, we shall invoke the sequential limit theory developed by Phillips and Moon (1999). In particular, it will be shown

that both statistics require standardization based on the first two moments of the following vector Brownian motion functional

$$K_i \equiv \left(V_i' V_i, \int_0^1 Q_i^2, V_i' V_i \left(\int_0^1 Q_i^2 \right)^{-1} \right)',$$

where

$$\begin{aligned} Q_i &= \bar{W}_{i1} - \left(\int_0^1 \bar{W}_{i1} \bar{W}_{i2} \right) \left(\int_0^1 \bar{W}_{i2}^2 \right)^{-1} \bar{W}_{i2}, \\ F_i &= \int_0^1 \bar{W}_i \bar{W}_i' = \begin{pmatrix} f_{i11} & f_{i21} \\ f_{i21} & F_{i22} \end{pmatrix} \text{ and } V_i = \left(1, -f_{i21} F_{i22}^{-1}\right). \end{aligned}$$

Notice that Q_i may be interpreted as the residual from a continuous time regression of $\bar{W}_{i1}$ on $\bar{W}_{i2}$. It is the limiting representation of the residual $\hat{e}_{it}$ obtained from (8). Therefore, to account for the fact that (8) is fitted with an individual specific constant term, this suggests that Q_i should be based on the demeaned standard Brownian motion $\bar{W}_i = W_i - \int_0^1 W_i$ rather than W_i. In deriving the asymptotic theory, it is convenient to define Θ and Σ as, respectively, the mean and the covariance of K_i. It is also convenient to define the vector $\phi \equiv (\Theta_2^{-1}, -\Theta_1 \Theta_2^{-2})'$ and to let $\tilde{\Sigma}$ denote the upper left 2×2 submatrix of Σ. Making use of these notations, we are now ready to state our first main result.

Theorem 1. *(Asymptotic distribution.) Under Assumption 1 and 2, and the null hypothesis of no cointegration, as $T \longrightarrow \infty$ prior to N*

$$N^{-1/2} DH_R - N^{1/2} \Theta_1 \Theta_2^{-1} \Rightarrow N(0, \phi' \tilde{\Sigma} \phi), \tag{10}$$

$$N^{-1/2} DH_U - N^{1/2} \Theta_3 \Rightarrow N(0, \Sigma_{33}). \tag{11}$$

The proof of Theorem 1 is outlined in the appendix. The proof of (10) proceeds by showing that the intermediate limiting distribution of DH_R can be written entirely in terms of the elements of the vector Brownian motion functional K_i. Therefore, by virtue of cross-sectional independence, the limiting distribution of the test statistic can be described in terms of differentiable functions of i.i.d. vector sequences to which the Delta method is applicable. Hence, by subsequently passing $N \longrightarrow \infty$, we obtain a limiting normal distribution for the test statistic, which depend only on the first two moments of K_i. The DH_U statistic also attains a limiting normal distribution under the null hypothesis. In this case, however, asymptotic results follow directly by the application of the Lindberg-Lévy central limit theorem to an average of N i.i.d. random variables.

Theorem 1 indicates that each of the standardized statistics converges to a normal distribution whose moments depend on various terms that are derived from the underlying vector Brownian motion functional K_i. Although the stated results are for the speciel case when (8) is fitted with a constant term and a

single regressor, they are readily extendable to accommodate other deterministic specifications as well as multiple regressors. In particular, if the test statistics are based on (8) with no deterministic terms, then the limiting distributions of DH_U and DH_R still have the same form as in (10) and (11) but now with moments that are based on the standard Brownian motion W_i rather than $W_i - \int_0^1 W_i$. Analogously, if the regression involves fitted constant and trend terms, then the limiting distributions in (10) and (11) retain their stated forms but involve moments of the demeaned and detrended standard Brownian motion $W_i + (6r-4)\int_0^1 W_i + (6-12r)\int_0^1 rW_i$. If we have multiple regressors, then W_{i2} becomes a K dimensional vector Brownian motion and the formulaes should be adjusted accordingly.

Approximations of the moments may be obtained by means of Monte Carlo simulations. In this paper, we simulate both finite and asymptotic moments. In the former case, the simulations are carried out by repeated application of the test statistics to the DGP described in Section 3. In the latter case, the moments are obtained on the basis of $10,000$ draws of $K+1$ independent scaled random walks of length $T = 1,000$. Using these random walks as simulated Brownian motions, we construct approximations of the functional K_i and then compute approximate asymptotic moments. The simulated moments are reported for up to six regressors in Table 1 of the appendix. For DH_R statistic, our simulation results indicate that the asymptotic results is borne out well in small samples with the asymptotic moment approximations being close to their finite sample counterparts. For the DH_U statistic, however, the results suggest that the finite sample moments sometimes can be far away from their theoretical values. To account for such discrepancies, we estimate response surface regressions.

The estimation proceeds as follows. For each combination of K, T and N, we generate $1,000$ test statistics according to the DGP given by (7). For the DH_R statistic, $T \in \{50, 60, 70, 80, 100, 200\}$ and $N \in \{5, 10, 15, 20\}$, whereas, for the DH_U statistic, $T \in \{50, 60, 70, 80, 100, 200, 500, 1000\}$ and $N = 1$. Most samples are relatively small as these seem to provide more information about the shape of the response surfaces. A few large values of T are also included, however, to ensure that the estimates of the asymptotic moments are sufficiently accurate. Pending on the deterministic component of (8), moments for three different model specifications are extracted and stored, one with no deterministic component, one with a constant, and one with a constant and a linear time trend. We then perform 50 replications of each experiment, which means that the total number of observations available for each regression for the DH_R and DH_R statistics are 1200 and 400, respectively. Although the fit of the regressions generally were quite good, the results suggest that the estimates associated with powers of T greater than unity have a tendency of becoming explosive in cases where the dependent variable takes on relatively large values. To avoid this, we fit the regressions with the inverse of the simulated moments as the dependent variable.

Since the regressions are heteroskedastic by construction, we use the generalized method of moments estimator discussed in MacKinnon (1996). The estimated response surface parameters for each of the experiments are reported in Table 2. For brevity, the table only report the estimated response surfaces for up to three regressors. To test the null hypothesis of no cointegration based on the estimated response surfaces from Table 2, one must first obtain the appropriate moments. This is done by calculating the fitted value of the dependent variable, which is then inverted to get the moment. For example, for the DH_U statistic in the model with no deterministic component with $K = 1$ and $T = 50$, the approximate expected value is 18.0420, which is computed as the inverse of the fitted value $0.0730 - 0.8755/50 - 0.1595/50^2$. Based on the calculated moments, one then computes the value of the relevant standardized test statistic so that it is in the form of (10) or (11). Because both statistics diverges to positive infinity under the alternative hypothesis, the computed value should be compared with the right tail of the normal distribution. If the computed value is greater than the appropriate right tail critical value, we reject the null hypothesis.

5 Monte Carlo simulations

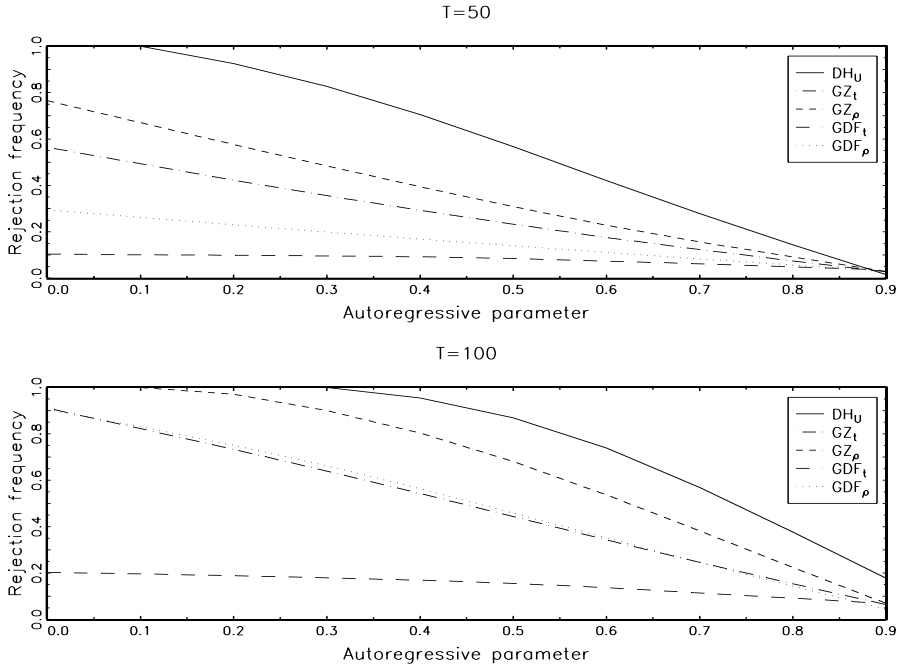
In this section, we compare and evaluate the small-sample properties of the Durbin-Hausman test statistics relative to that of eight other residual-based tests for cointegration recently proposed by Pedroni (1999, 2004). For this purpose, a small set of Monte Carlo experiment were conducted with the DGP tailored to reflect the most relevant features for the long-run Fisher hypothesis. In particular, we assume that the regression is fitted with a constant term only and that there is a single regressor in which case the DGP may be written as

$$\begin{aligned} r_{it} &= \alpha_i + \beta_i p_{it} + e_{it}, \\ e_{it} &= \rho_i e_{it-1} + u_{it} + \theta u_{it-1}, \end{aligned}$$

where $(u_{it}, \Delta p_{it})' \sim N(0, V)$ and V is a symmetric matrix with $V_{11} = V_{22} = 1$ and $V_{12} = V_{21}$. For each experiment, we generate $1,000$ panels with $N \in \{10, 20\}$ individual and $T \in \{50, 100\} + 50$ time series observations. The first 50 observations for each cross-section is then disregarded in order to attenuate the effect of the initial value. The DGP is parameterized as follows. The autoregressive parameter ρ_i determines whether the null hypothesis it true or not. Under the null hypothesis, we set $\rho_i = 1$ for all i, while, under the alternative hypothesis, $\rho_i < 1$. Specifically, for the restricted test, $\rho_i = \rho$ for all i, whereas, for the unrestricted test, the fraction of spurious individuals is set equal to 0.1. The regression parameters α_i and β_i are both allowed to vary and are drawn from $U(0.4, 1.2)$. The remaining parameters θ, δ and V_{12} introduce nuisance in the DGP. First, a nonzero value on θ imply that e_{it} will have a first order moving average component. Second, the degree of exogeneity in the DGP is

governed by V_{12}. The regressor is strictly exogenous if $V_{12} = 0$, while it is weakly exogenous if $V_{12} = 0.4$. All computations was performed in GAUSS.

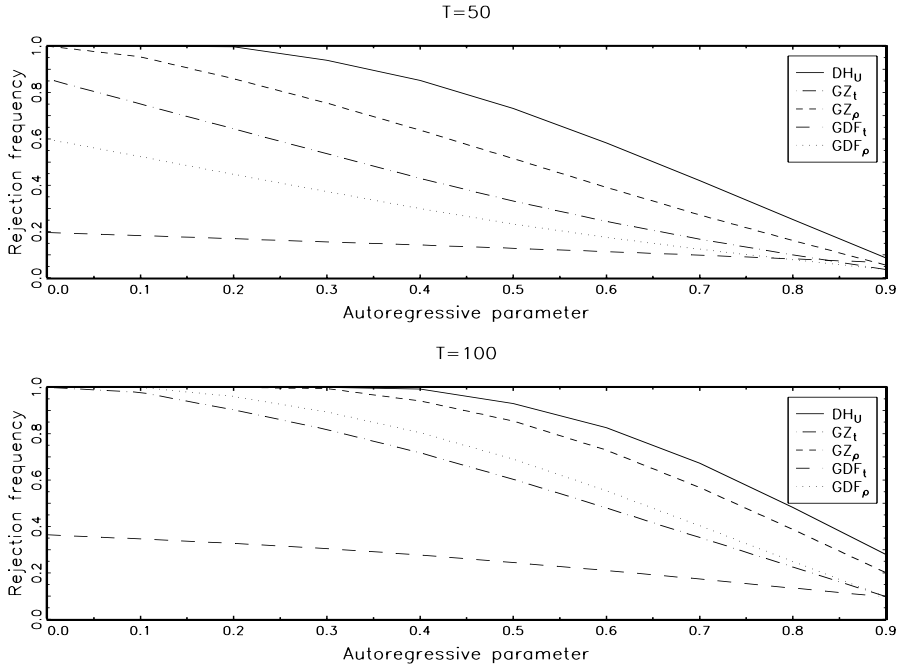
Figure 1: Size-adjusted power for the restricted tests when $N = 10$.



The tests are constructed using the moments based on the response surface estimates presented in Table 2. The performance of the Durbin-Hausman statistics are compared to eight of the statistics developed in Pedroni (1999, 2004). To this effect, we shall use GZ_t and GZ_ρ to denote the semiparametric group mean t and ρ statistics proposed in Pedroni (2004). The corresponding panel statistics are denoted PZ_t and PZ_ρ, respectively. The augmented Dickey-Fuller versions of these test statistics are presented in Pedroni (1999). They are denoted by GDF_t, GDF_ρ, PDF_t and PDF_ρ. As with DH_R and DH_U, the Panel and Group Mean statistics differ mainly due to the fact that, while the former presumes a common value of the autoregressive parameter ρ_i under the alternative, the latter does not. Therefore, the most relevant comparisons here are between DH_R and the panel statistics, and between DH_U and the group mean statistics. For brevity, we present only the size-adjusted power of the tests and the empirical size on the five percent level.

Consider first the size of the tests presented in Tables 3 and 4. Since all tests have been constructed using either semiparametric or parametric adjustments to

Figure 2: Size-adjusted power for the restricted tests when $N = 20$.

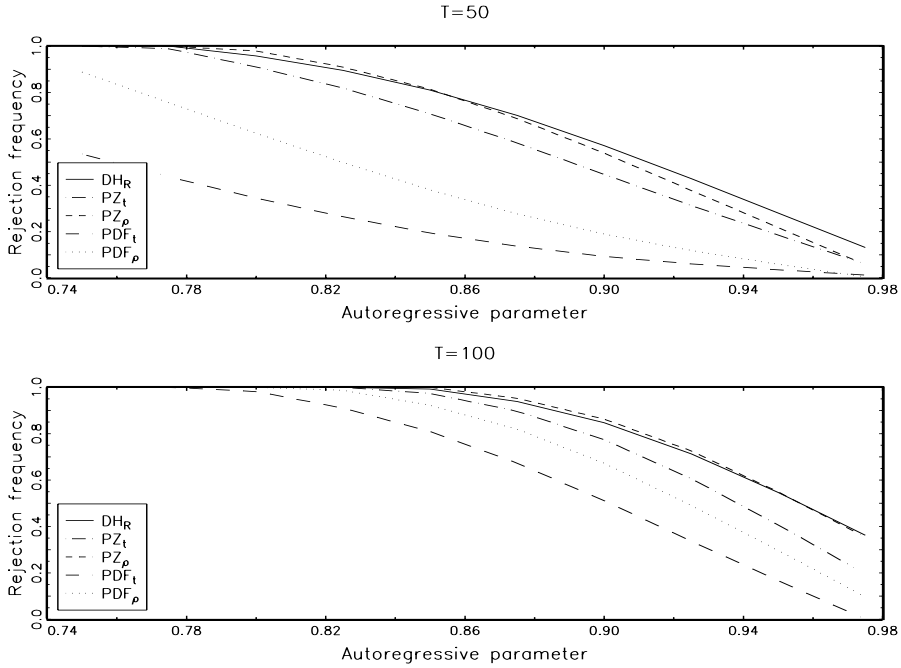


account for the temporal dependence of the data, special attention is paid to the choice of bandwidth or lag length parameter. Three such choices are considered; $M_1 = [4(T/100)^{2/9}]$, $M_2 = [2T^{1/3}]$ and $M_3 = [0.5T^{1/3}]$. Judging from the results presented in the tables, although all three choices generally produce test with good size, it appears as that M_1 tend to work best for the Durbin-Hausman statistics. In fact, when the moving average parameter is nonnegative, size accuracy is almost perfect in all panels. The results are less encouraging when $\theta = -0.4$ in which case both the DH_R and the DH_U statistics suffer from severe size distortions. Similar results are obtained for the other tests. One exception is the ADF type tests, where M_2 tend to lead to a over-rejection of the null hypothesis. Apparently, too generous a lag length causes the number of free parameters to become unwieldy thereby leading to a deterioration in the small-sample performance of the tests.

Next, we continue to the results on the power of the tests presented in Figures 1 through 4.[1] In this case, $\theta = 0$ so there is no moving average component present. Based on the good performance of the tests under the null hypothe-

[1] In Figures 1 through 5, the curves representing the size-adjusted power of the test statistics have been smoothed slightly by means of a least squares spline of neighboring points.

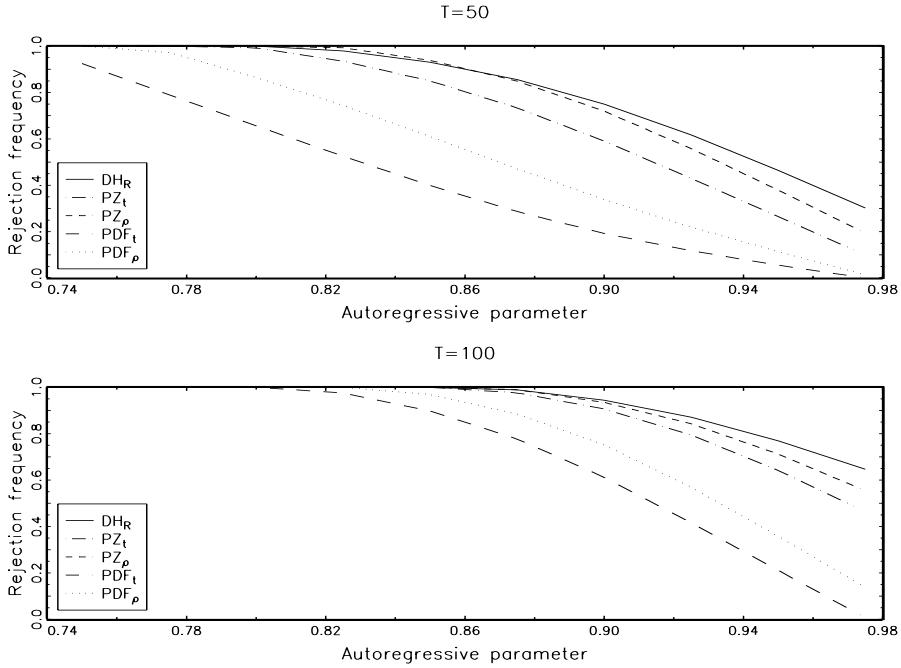
Figure 3: Size-adjusted power for the unrestricted tests when $N = 10$.



sis, the bandwidth parameter is set equal to M_1. All results are adjusted for size so that each test has the same rejection frequency of five percent when the null hypothesis is true. As suggested by the figures, the proposed test statistics are almost uniformly more powerful than the other tests. Notably, the power advantage appear to be much greater for the DH_U statistic than for the DH_R statistic. As expected, we see that the power falls as the autoregressive parameters approach one. Another expected result is that the power increases quickly for both statistics as N and T grows. To emphasize the additional power that comes from using panel data rather than a single time series, Figure 5 plots the raw power of the Durbin-Hausman statistics for some small values on N when $T = 50$. As illustrated by the figure, the power is strictly increasing in N and the advantage relative to the pure time series case when $N = 1$ may be considerable even for as small panels as $N = 5$. Among the remaining test statistics, Figures 1 trough 4 indicate that the semiparametric tests generally perform best and that the DF type tests perform worst. In fact, for the group mean statistics, Figures 3 and 4 suggest that the power can be very poor unless $T > 50$ or $N > 10$, or both.

In summary, we find that the Durbin-Hausman tests show higher size-adjusted

Figure 4: Size-adjusted power for the unrestricted tests when $N = 20$.



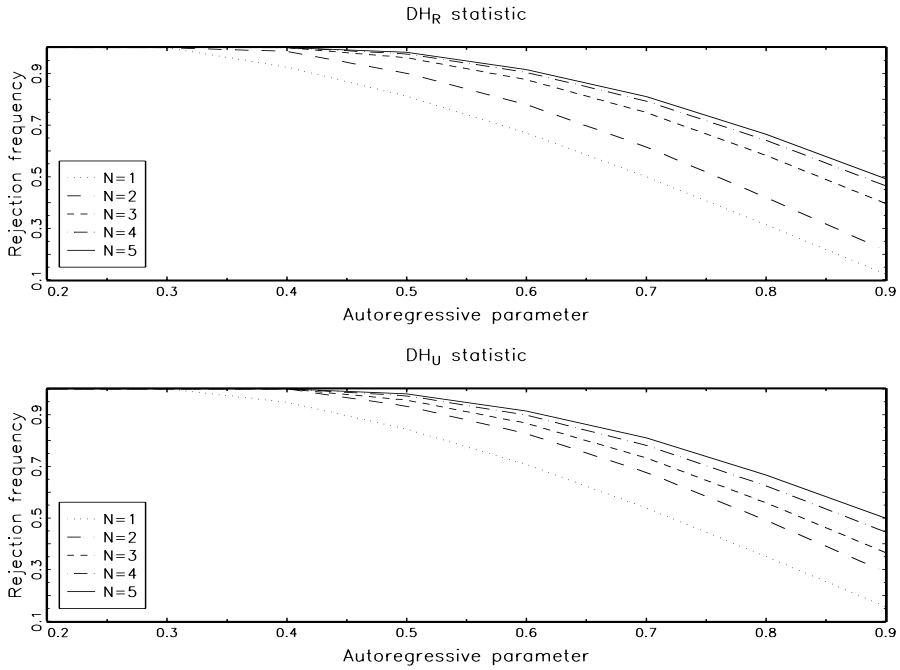
power than the other tests considered and, at the same time, maintain the nominal size well in small samples. Since the power advantage is particularly striking in small panels, this leads us to the conclusion that the proposed tests should be particularly well suited for testing the long-run Fisher hypothesis.

6 Empirical results

In this section, we present the empirical evidence on the Fisher hypothesis. For this purpose, data on 14 OECD countries between 1980:1 and 1999:12 are obtained from the OECD Main Economic Indicators data base. The data is monthly and include for each county a short-term nominal interest rate and the consumer price index. Both variables were converted into annualized values.

We begin the empirical analysis by testing the variables for unit roots. In a recent study, Im *et al.* (2003) develop two panel ADF type test statistics that can be used for testing the null hypothesis of a unit root in the variables when the underlying DGP is heteroskedastic and serially correlated. They are the $Z_{\bar{t}}$ and $\tilde{Z}_{\bar{t}}$ statistics. Both are consistent and attains a limiting normal distribution under the null hypothesis as $T \longrightarrow \infty$ prior to N. Under the

Figure 5: Power for different values of N when $T = 50$.



alternative hypothesis, both statistics diverge to negative infinity, which suggests that the left tail of the normal distribution should be used to reject the null. The statistics differ in that they have different distributional properties for a fixed T in which case the $\tilde{Z}_{\bar{t}}$ statistic is analytically more manageable and is likely to lead to more accurate tests in small samples.

Both statistics are constructed as simple averages of N individual ADF test statistics, which implies that we need to chose the functional form of the individual ADF test regressions in order to implement the tests. To this end, since the series are clearly trending, we are interested in testing the null hypothesis of a unit root against the alternative of trend stationarity. This imply that we should fit the regressions with both a constant and a linear time trend. Different choices of lag length seem to have little or no effect on the test results so we set the lag length equal to M_1 defined in the previous section. In fact, since all results presented herein seem very robust to various choices of lag lengths and bandwidths, we use M_1 throughout.

The appropriate moments needed to construct the $Z_{\bar{t}}$ and $\tilde{Z}_{\bar{t}}$ statistics for the model with a time trend are not available, and must therefore be obtained

by means of Monte Carlo simulation.[2] As described in Section 4, this calls for an evaluation of the intermediate limiting distribution of the tests, which is the conventional Dickey-Fuller distribution defined in e.g. Im *et al.* (2003). For this purpose, we make $10,000$ draws of a single random walk of length $T = 1,000$, which is then used to compute the moments. The simulated expectation and variance are -2.2208 and 0.5785, respectively. The lower tenth and fifth percentiles of the Dickey-Fuller distribution is also simulated in order to implement the tests on an individual basis. The simulated ten and five percent critical values are -3.1476 and -3.4798, respectively.

The individual $Z_{\bar{t}}$ and $\tilde{Z}_{\bar{t}}$ statistics, abbreviated t_{iT} and $\tilde{t}_{iT}$, are presented in Table 5. We see that the null hypothesis of a unit root in the variables cannot be rejected on the five percent level for any of the 14 countries.[3] These results suggest that we also should be able to reject the stationarity hypothesis for the panel as a whole. Indeed, the calculated values on the $Z_{\bar{t}}$ and $\tilde{Z}_{\bar{t}}$ statistics for the nominal interest rate are -0.7805 and -0.6429, respectively. The corresponding values for inflation are -0.9421 and -0.7938. Hence, the null cannot be rejected individually nor for the panel as a whole on any conventional significance level. We therefore conclude that the variables are nonstationary.

As argued in Section 2, in the presence of unit roots, the long-run Fisher hypothesis necessitates that inflation and the interest rate be cointegrated. Therefore, we now proceed by testing the variables for cointegration. Results for each individual country are presented in Table 4. The DHS statistic is the Durbin-Hausman test developed by Choi (1994), while the ADF statistic appears in Phillips and Ouliaris (1990). Consistent with the specification of the cointegrated regression derived in Section 2, the tests are based on a regression fitted with a constant term only.

The results reported in Table 5 suggest that the null hypothesis of no cointegration can be rejected on the 10 percent level for at least 10 countries, Austria, Belgium, Canada, France, Germany, Italy, Spain, Switzerland, United Kingdom, and United States. For the remaining countries, except possibly for Finland, we end up marginally accepting the null hypothesis on the 10 percent level. It is well known, however, that univariate tests of this sort may have low power in small samples when the variables are nearly spurious. For this reason, we now employ the panel Durbin-Hausman statistics developed in Section 3 and 4. As in Section 5, we compute the statistics based on the moments calculated from the response surface estimates reported in Table 2. The calculated values on DH_R and DH_U are 4.4785 and 4.6465, respectively. Thus, compared with the right tail of the normal distribution, we reject the null on all conventional levels of significance. Consequently, since the variables appear to be cointegrated, we conclude that there is at least a partial Fisher effect present.

[2] Im *et al.* (2003) tabulate both finite and asymptotic moments for the $Z_{\bar{t}}$ and $\tilde{Z}_{\bar{t}}$ statistics when the data is generated while allowing for a nonzero mean.

[3] On the 10 percent level, we marginally reject the null of a unit root in the rate of inflation for the United States using the t_{iT} statistic.

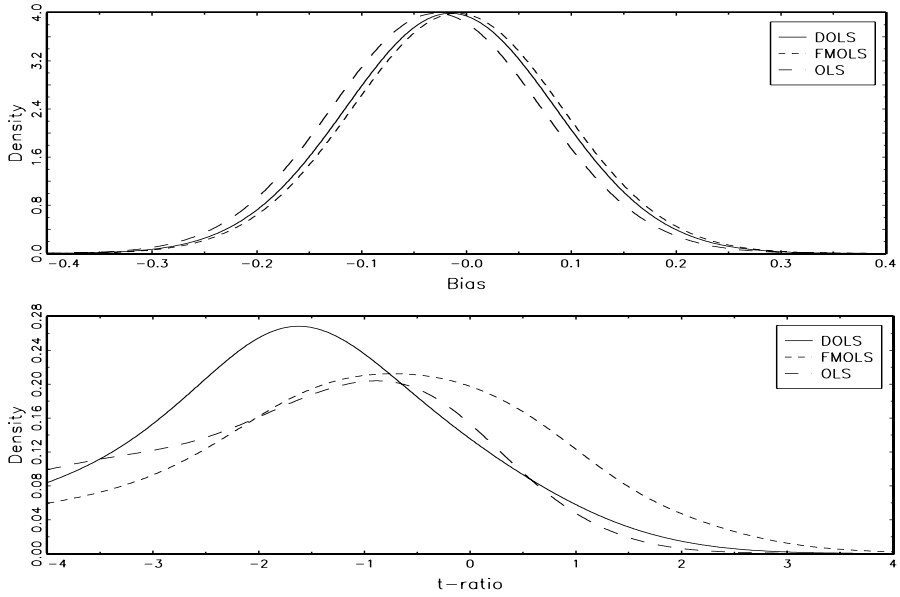
As pointed out in Section 2, the OLS estimator will be consistent under fairly general conditions when applied to the cointegrated regression in (9). However, the nonzero correlation between the regression error and the first differentiated regressor induces nuisance parameters in the asymptotic distribution of the OLS estimator, which then falls outside the local asymptotic mixture of normals family. Moreover, the use of overlapping data, where the horizon of the inflation and interest rates are longer than the monthly observation interval, may induce serial correlation in the equilibrium errors. To account for both of these features, we employ the dynamic OLS (DOLS) estimator of Stock and Watson (1993), and Saikkonen (1991) and the fully modified OLS (FMOLS) estimator of Phillips and Hansen (1990). These estimators are asymptotically equivalent and fully efficient in the presence of serially correlated errors and endogenous regressors. The difference between them lies in the methods undertaken in order to ensure efficiency of the cointegration parameters. Specifically, while the DOLS employs a parametric correction whereby lags and leads of the first differentiated regressor are introduced, the FMOLS adjusts for the temporal dependencies of the data by directly estimating the various nuisance parameters semiparametrically. To this effect, we use two lags and leads of the first difference of inflation to construct the DOLS, whereas the FMOLS is based on the Bartlett kernel.

The results from the estimated cointegration parameters are reported in Table 6. We see that the estimated individual slope parameters generally lie close to their hypothesized value of one. The range of the estimated slopes are 0.6134 to 1.3310 for the DOLS, 0.4184 to 1.3394 for the FMOLS, and 0.4548 to 1.2499 for the OLS. Based on the asymptotic normal distribution, we can rarely reject the null hypothesis of a unit slope parameter on the one percent level. The pooled estimates are reported in the last row of the table. Consistent with the individual county regressions, we see that the pooled slopes are close to unity and that the null hypothesis of a unit slope cannot be rejected on the five percent level using the normal distribution for any of the estimators. These estimates should, however, be interpreted with caution as the poolability restriction do not seem to be supported by the data.[4]

Consistent with the results of e.g. Mishkin (1992), Crowder and Hoffman (1992), and Evans and Lewis (1995), we observe some countries where the estimated slope is significantly less than unity. One interpretation of this finding is that the ex ante real interest rate is subject to permanent shocks and that these shocks are negatively correlated with the permanent shocks to inflation, which is inconsistent with the Fisher hypothesis. To appreciate this, note from (5) that the unit root component of the ex post real rate can be written as $-(1-\beta_i)p_{it}$. Thus, a unit positive permanent shock to inflation translate into a permanent shock in the ex post real rate of magnitude $-(1-\beta_i)$. Since $\beta_i < 1$ in this case,

[4]The calculated Wald test statistics for a homogenous slope parameter are 87.0098 for the DOLS, 42.9126 for the FMOLS and 87.2586 for the OLS estimator. Under the null hypothesis of a common slope, these statistics have a limiting chi-squared distribution with 13 degrees of freedom in which case the five percent critical value is 22.3621.

Figure 6: Kernel densities of the slope parameters and their t-ratios.



a positive shock to inflation correspond to a negative shock to the ex post real rate. Another implication of (5) is that the ex post real rate is equal to the ex ante real rate less a stationary error term. It follows that the ex ante rate is nonstationary and that its long-run trajectory is opposite to that of inflation.

Contrary to this interpretation, Crowder and Hoffman (1996), and Caporale and Pittis (2004) argues that a less than unit slope may not reflect the actual DGP but rather a downward endogeneity bias on the part of the estimators employed. To investigate this possibility, we engage in a small Monte Carlo study where the DGP is chosen to mimic the process generating the observed data. To this effect, recall that e_{it} is the regression error in (4). The DGP may be described by the error vector $v_{it} = (e_{it}, \Delta p_{it})'$, which is assumed to evolve according to the first order vector autoregression $v_{it} = v_{it-1}\delta + u_{it}$, where $u_{it} \sim N(0, \Sigma)$. The DGP is first calibrated using the observed OECD data to obtain values for δ and Σ. These are then used to generate simulated data along the lines described in Section 5.[5]

Figure 6 present the kernel densities of the estimated slope parameters and their t-ratios.[6] There are two important results. First, the bias distributions are

[5]The data is generated for $N = 1$ and $T = 200$ observations with $\alpha_i = \beta_i = 1$. The number of replications is $10,000$. The DGP is parameterized by $\delta_{11} = 0.92$, $\delta_{12} = 0$, $\delta_{21} = -0.16$, $\delta_{22} = 0.13$, $\Sigma_{11} = 0.8$, $\Sigma_{12} = \Sigma_{21} = -0.1$ and $\Sigma_{11} = 0.14$.

[6]The densities for the slope parameters and their t-ratios are estimated using a Gaussian

peaked to the left of zero suggesting that the estimators are biased downwards with the OLS estimator being the most biased. Second, the distributions of the t-ratios are highly non-central and shifted to the left. Notably, the mean values of the t-ratios are -1.2736 for the DOLS, -0.5468 for the FMOLS and -1.5202 for the OLS, which is indicative of large size distortions. Indeed, the probability of rejecting a true null hypothesis of $\beta_i = 1$ using a double-sided test against the normal distribution on the nominal five percent level is 0.342 for the DOLS, 0.198 for the FMOLS and 0.386 for the OLS. Hence, inference based on the normal distribution is likely to be highly deceptive. To account for this, we obtain the five percent critical values form the empirical distribution, which should enable valid inference. The left tail critical values for the DOLS, FMOLS and OLS estimators are -3.8392, -4.1619 and -5.5901, respectively. Based on these values, the null of a unit slope cannot be rejected for any of the countries using the t-ratios reported in Table 6.

In summary, consistent with the results of Crowder and Hoffman (1996), and Caporale and Pittis (2004), the evidence of this section suggests that we cannot reject the full Fisher effect for any of the countries or for the panel as a whole. To test the robustness of this conclusion, we reestimated the empirical model based on both annual and quarterly OECD data. Some additional estimates were also obtained using the yield on long-term government bonds as interest rate. For brevity, however, we do not report these results but we briefly describe them. Regardless of sample frequency, we still find cointegration between the inflation and nominal interest rates. The full Fisher effect is also supported using the empirical critical values. The results obtained using long-term interest rates are qualitatively similar. An additional, and perhaps even more important, caveat is that all forms of cross-sectional dependency thus far has been disregarded. To this end, the next section proposes bootstrapped cointegration tests that are robust to general forms of cross-sectional dependencies.

7 Bootstrap tests

Recall that the properties of the Durbin-Hausman test statistics rely on the assumption of cross-sectional independence. When this assumption is violated, the statistics suffer from nuisance parameter dependencies in which case their asymptotic distributions are unknown. In our case, there are at least two reasons for believing that the data may not be i.i.d. cross-sectionally. First, inflation rates may be correlated across countries because of common oil price shocks. Second, nominal interest rates may be correlated across countries due to the strong links between financial markets. Although the first type of dependence may be accommodated by using data that has been demeaned with respect to a common time effect, the size of such tests may be quite unreliable once we allow for more general types of correlation structures. One possible response to

kernel with bandwidth parameter 0.1 and 0.6, respectively.

this is to follow Maddala and Wu (1999) in employing the bootstrap approach, which enables us to make accurate inference using the empirical distributions of the test statistics.

The problem is how to generate the bootstrap distributions. To this end, we propose generating the data nonparametrically using the sampling scheme S_2 described in Li and Maddala (1996, 1997). Specifically, if the null hypothesis is specified as $\rho_i = 1$ for all i, then this scheme suggests that the sample should be generated while imposing a unit root in the equilibrium errors. Thus, the bootstrap sample e_{it}^* is generated as

$$e_{it}^* = e_{it-1}^* + u_{it}^*,$$

where u_{it}^* is the bootstrap sample from the centered residual $\tilde{u}_{it} = \hat{u}_{it} - T^{-1}\sum_{t=1}^{T}\hat{u}_{it}$ obtained by performing OLS on (9). Because the errors are cross-sectionally correlated, however, we cannot resample $\tilde{u}_{it}$ directly. Instead, we resample $\hat{v}_t = (\tilde{u}_t', \Delta p_t')'$, where $\tilde{u}_t = (\tilde{u}_{1t}, ..., \tilde{u}_{Nt})'$ and $\Delta p_t = (\Delta p_{1t}, ..., \Delta p_{Nt})'$. By resampling the data in this way, we can preserve the cross-sectional correlation structure of v_t. Notably, by resampling $\hat{v}_t$ rather than $\tilde{u}_t$, we can preserve any endogenous effects that may run across the individual regressions of the system. Next, we generate the bootstrap sample r_{it}^* as

$$r_{it}^* = \hat{\alpha}_i + \hat{\beta}_i p_{it}^* + e_{it}^*,$$

where $\hat{\alpha}_i$ and $\hat{\beta}_i$ are the parameters from (8). For initiation of e_{it}^* and p_{it}^*, we use the value zero. Once the sample r_{it}^* and p_{it}^* has been obtained, the bootstrapped test statistics may be readily computed. This procedure is then repeated a large number of times, which gives us the empirical distributions of the statistics.

The five percent critical values for the DH_R and DH_U statistics obtained using $1,000$ bootstrap replications are 19.922 and 4.435, respectively. Thus, based on the DH_U statistic, we reject the null hypothesis of no cointegration on the five percent level using the bootstrapped distribution. In contrast, we cannot reject the null on the five percent level using the DH_R statistic. This is not unexpected, however, given the results on the individual cointegration tests, which suggest that the homogenous alternative may be too restrictive in this case. Generally speaking, these empirical findings seem to confirm our simulation results, which suggest that the DH_R statistic suffers from substantial size distortions when the errors are cross-sectionally correlated. The DH_U statistic also suffers from size distortions but not nearly as severe as those for the DH_R statistic. In fact, the DH_U statistic appears to be rather robust against small to moderate degrees of cross-section dependence. The bootstrap approach eliminates these distortions and leads to tests with good size properties, although both tests tend to be somewhat under-sized.

8 Conclusions

Recent empirical studies suggest that the Fisher hypothesis, stating that inflation and nominal interest rates should cointegrate with a unit parameter on inflation, do not hold, a finding at odds with many theoretical models. This paper argues that these results can be explained in part by the low power inherent in univariate cointegration tests and that the use of panel data should generate more powerful tests. Therefore, in this paper we investigate the Fisher hypothesis using a panel of monthly data covering the period 1980:1 to 1999:12 on 14 OECD countries. In doing so, we propose two new residual-based tests for the null hypothesis of no cointegration. The tests are based on the Durbin-Hausman principle whereby two estimators of a unit root in the residuals of a cointegrated regression are compared. Both estimators are consistent under the null hypothesis but only one retains the property of consistency under the alternative. Using sequential limit arguments, it is shown that the test statistics are free of nuisance parameters and that they have a limiting normal distribution under the null hypothesis. Results from a small Monte Carlo study suggest that the proposed tests have greater power than other popular residual-based tests is samples comparable with ours. In our empirical analysis, contrary to much of the earlier literature, we find evidence in favor of the Fisher hypothesis.

Appendix A Mathematical proofs

In this appendix, we derive the limiting distributions of the Durbin-Hausman test statistics under the null hypothesis of no cointegration. Unless otherwise stated, the limit arguments are taken passing $T \longrightarrow \infty$ with N held fixed. For illustrative purposes, we shall focus on the simple case when the regression (11) is fitted without any deterministic components.

Proof of Theorem 1. To prove the limiting distribution for the restricted Durbun-Hausman test statistic, we make the assumption that $\rho_i = \rho$ for all i. Thus, since $\rho = 1$ under the null hypothesis, we may write

$$T\hat{\rho} = \left(T^{-2}E_{22}\right)^{-1} T^{-1}E_{12} = T + \left(T^{-2}E_{22}\right)^{-1} T^{-1}E_{23}, \tag{A1}$$

$$T\tilde{\rho} = \left(T^{-2}E_{12}\right)^{-1} T^{-1}E_{22} = T + \left(T^{-2}E_{12}\right)^{-1} T^{-1}E_{13}. \tag{A2}$$

The estimated OLS regression (11) with no deterministic component may be expressed as $\hat{\lambda}_i' z_{it} = \hat{e}_{it}$, where $\hat{\lambda}_i = (1, -\hat{\beta}_i)'$. Furthermore, by Theorem 2 of Phillips (1986), we obtain the limit of $\hat{\lambda}_i$ as

$$\hat{\lambda}_i \Rightarrow \lambda_i = (1, -a_{i21}A_{i22}^{-1})',$$

where

$$A_i = \int_0^1 B_i B_i' = \begin{pmatrix} a_{i11} & a_{i21} \\ a_{i21} & A_{i22} \end{pmatrix}.$$

Combining the results, it follows that

$$T^{-2}E_{i11} \Rightarrow \lambda_i' A_i \lambda_i = \omega_{i1.2}^2 \int_0^1 Q_i^2, \tag{A3}$$

Moreover, by Theorem 2.6 of Phillips (1988), we obtain

$$T^{-1}E_{i23} = \hat{\lambda}_i' \left(T^{-1} \sum_{t=2}^{T} v_{it} z_{it-1} \right) \hat{\lambda}_i \Rightarrow \lambda_i' \left(\int_0^1 B_i \mathrm{d}B_i + \Gamma_i \right) \lambda_i. \tag{A4}$$

From (A3) and (A4), we deduce

$$T^{-2}E_{i12} = T^{-2}E_{i11} + T^{-2}E_{i23} = T^{-2}E_{i11} + o_p(1) \Rightarrow \lambda_i' A_i \lambda_i, \tag{A5}$$

$$\begin{aligned} T^{-1}E_{i13} &= T^{-1}E_{i23} + T^{-1}E_{i11} \\ &= \hat{\lambda}_i' \left(T^{-1} \sum_{t=2}^{T} v_{it} z_{it-1} + T^{-1} \sum_{t=2}^{T} v_{it} v_{it}' \right) \hat{\lambda}_i \\ &\Rightarrow \lambda_i' \left(\int_0^1 B_i \mathrm{d}B_i + \Sigma_i + \Gamma_i \right) \lambda_i. \end{aligned} \tag{A6}$$

Combining (A3) through (A6), since $\hat{\omega}_{i1.2}^2$ is consistent for $\omega_{i1.2}^2$ under Assumption 2 (see, e.g. Phillips and Ouliaris, 1990), we obtain the limit of $T(\hat{\rho}-1)$ and $T(\tilde{\rho}-1)$ as follows

$$\begin{aligned} T(\hat{\rho}-1) &\Rightarrow \left(\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}\lambda_i' A_i \lambda_i\right)^{-1}\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}\lambda_i'\left(\int_0^1 B_i \mathrm{d}B_i + \Gamma_i\right)\lambda_i, \\ T(\tilde{\rho}-1) &\Rightarrow \left(\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}\lambda_i' A_i \lambda_i\right)^{-1}\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}\lambda_i'\left(\int_0^1 B_i \mathrm{d}B_i + \Sigma_i + \Gamma_i\right)\lambda_i. \end{aligned}$$

It follows that

$$\begin{aligned} T(\tilde{\rho}-\hat{\rho}) &= \left(T^{-2}\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}E_{i12}\right)^{-1} T^{-1}\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}E_{i13} \\ &- \left(T^{-2}\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}E_{i22}\right)^{-1} T^{-1}\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}E_{i23} \\ &\Rightarrow \left(\sum_{i=1}^{N}\omega_{i1.2}^{-2}\lambda_i' A_i \lambda_i\right)^{-1}\sum_{i=1}^{N}\omega_{i1.2}^{-2}\lambda_i'\Sigma_i\lambda_i \\ &= \left(\sum_{i=1}^{N}\int_0^1 Q_i^2\right)^{-1}\sum_{i=1}^{N}\omega_{i1.2}^{-2}\lambda_i'\Sigma_i\lambda_i. \end{aligned} \tag{A7}$$

Next, consider $\hat{\gamma}_0$ and $\hat{\sigma}^2$. For simplicity, write $R_0 = \sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}\hat{\gamma}_{i0}$ and $R_1 = \sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}\hat{\sigma}_i^2$. From Lemma 2.2 of Phillips and Ouliaris (1990), we have $\lambda_i'\Omega_i\lambda_i = \omega_{i1.2}^2 V_i'V_i$. Thus, using the same arguments as Choi (1994), it is possible to show

$$\hat{\gamma}_0 = N^{-1}R_0 \Rightarrow N^{-1}\sum_{i=1}^{N}\omega_{i1.2}^{-2}\lambda_i'\Sigma_i\lambda_i, \tag{A8}$$

$$\hat{\sigma}^2 = N^{-1}R_1 \Rightarrow N^{-1}\sum_{i=1}^{N}\omega_{i1.2}^{-2}\lambda_i'\Omega_i\lambda_i = N^{-1}\sum_{i=1}^{N}V_i'V_i. \tag{A9}$$

Using the results in (A3), (A7) and (A8), we have

$$\begin{aligned} R_2 &\equiv \left(N^{-1}R_0\right)^2\left(T(\tilde{\rho}-\hat{\rho})\right)^{-2}\left(T^{-2}\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}E_{i22}\right)^{-2} \\ &\Rightarrow N^{-2}\sum_{i=1}^{N}\int_0^1 Q_i^2. \end{aligned} \tag{A10}$$

This, together with (A9), imply

$$DH_R = \hat{\sigma}^2\hat{\gamma}_0^{-2}(\tilde{\rho}-\hat{\rho})^2\sum_{i=1}^{N}\hat{\omega}_{i1.2}^{-2}E_{i22}$$

$$
\begin{aligned}
&= N^{-1}R_1R_2^{-1} \\
\Rightarrow &\left(N^{-1}\sum_{i=1}^{N}V_i'V_i\right)\left(N^{-2}\sum_{i=1}^{N}\int_0^1 Q_i^2\right)^{-1}. \qquad \text{(A11)}
\end{aligned}
$$

This shows that the limiting distribution of DH_R is free of nuisance parameters under the null. Therefore, because the limiting distributions passing $T \longrightarrow \infty$ is i.i.d. over the cross-section, we deduce that $E(K_i) = \Theta$ for all i. The variance of K_i may be decomposed as

$$
\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{12} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{13} & \Sigma_{23} & \Sigma_{33} \end{pmatrix} \text{ and } \tilde{\Sigma} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{pmatrix}.
$$

To derive the limiting distributions of the test statistic, we shall make use of the Delta method, which provides the limiting distribution for continuously differentiable transformations of i.i.d. vector sequences. In so doing, rewrite the statistic as

$$
\begin{aligned}
N^{-1/2}DH_R - N^{1/2}\Theta_1\Theta_2^{-1} &= N^{1/2}\left(N^{-1}R_1 - \Theta_1\right)\left(N^{-1}R_2\right)^{-1} \\
&+ N^{1/2}\Theta_1\left(\left(N^{-1}R_2\right)^{-1} - \Theta_2^{-1}\right). \qquad \text{(A12)}
\end{aligned}
$$

The terms appearing in (A12) with normalizing order N^{-1} converge in probability to the means of the corresponding random variables by virtue of a law of large numbers as $T \longrightarrow \infty$ and then $N \longrightarrow \infty$. Hence, $N^{-1}R_1 \xrightarrow{p} \Theta_1$ and $N^{-1}R_2 \xrightarrow{p} \Theta_2$. Moreover, by direct application of the Lindberg-Lévy central limit theorem, $N^{1/2}\left(N^{-1}R_1 - \Theta_1\right) \Rightarrow N(0, \Sigma_{11})$ as $T \longrightarrow \infty$ prior to N. The remaining expression involves a continuously differentiable transformation of i.i.d. random variables. Thus, by the Delta method, as $T \longrightarrow \infty$ prior to N

$$
N^{1/2}\left(\left(N^{-1}R_2\right)^{-1} - \Theta_2^{-1}\right) \Rightarrow N(0, \Theta_2^{-4}\Sigma_{22}). \qquad \text{(A13)}
$$

This suggests that the limit of $N^{-1/2}DH_R - N^{1/2}\Theta_1\Theta_2^{-1}$ may be written as

$$
N^{-1/2}DH_R - N^{1/2}\Theta_1\Theta_2^{-1} \Rightarrow \Theta_2^{-1}N(0, \Sigma_{11}) - \Theta_1\Theta_2^{-2}N(0, \Sigma_{22}). \qquad \text{(A14)}
$$

It follows that $N^{-1/2}DH_R - N^{1/2}\Theta_1\Theta_2^{-1}$ is mean zero with the variance given by $\Theta_2^{-2}\Sigma_{11} + \Theta_1^2\Theta_2^{-4}\Sigma_{22} - 2\Theta_1\Theta_2^{-3}\Sigma_{12}$. This completes the first part of the proof.

Consider next the limiting distribution of the DH_U statistic. In this case, ρ_i need not take on a common value in which case the OLS and pseudo IV estimators may be rewritten as

$$
\begin{aligned}
T\hat{\rho}_i &= \left(T^{-2}E_{i22}\right)^{-1}T^{-1}E_{i12} = T + \left(T^{-2}E_{i22}\right)^{-1}T^{-1}E_{i23}, \qquad \text{(A15)} \\
T\tilde{\rho}_i &= \left(T^{-2}E_{i12}\right)^{-1}T^{-1}E_{i11} = T + \left(T^{-2}E_{i12}\right)^{-1}T^{-1}E_{i13}. \qquad \text{(A16)}
\end{aligned}
$$

Using (A3) through (A6) it follows that

$$T(\tilde{\rho}_i - \hat{\rho}_i) \Rightarrow \lambda_i' \Sigma_i \lambda_i \left(\lambda_i' A_i \lambda_i\right)^{-1}. \tag{A17}$$

Moreover, from (A8) and (A9), we infer that $\hat{\gamma}_{i0} \Rightarrow \lambda_i' \Sigma_i \lambda_i$ and $\hat{\sigma}_i^2 \Rightarrow \lambda_i' \Omega_i \lambda_i$. Together, these results indicate

$$\begin{aligned} DH_U &= \sum_{i=1}^{N} \hat{\sigma}_i^2 \hat{\gamma}_{i0}^{-2} (\tilde{\rho}_i - \hat{\rho}_i)^2 E_{i22} \\ &= \sum_{i=1}^{N} \hat{\sigma}_i^2 \hat{\gamma}_{i0}^{-2} (T(\tilde{\rho}_i - \hat{\rho}_i))^2 \left(T^{-2} E_{i22}\right) \\ &\Rightarrow \sum_{i=1}^{N} \lambda_i' \Omega_i \lambda_i \left(\lambda_i' A_i \lambda_i\right)^{-1} = \sum_{i=1}^{N} V_i' V_i \left(\int_0^1 Q_i^2\right)^{-1}. \end{aligned} \tag{A18}$$

Next, let $R_i = \sigma_i^2 \hat{\gamma}_{i0}^{-2} \left(T(\tilde{\rho}_i - \hat{\rho}_i)\right)^2 \left(T^{-2} E_{i22}\right)$ and rewrite the statistic as follows

$$N^{-1/2} DH_U - N^{1/2} \Theta_3 = N^{1/2} \left(N^{-1} \sum_{i=1}^{N} R_i - \Theta_{33} \right). \tag{A19}$$

Hence, by the Lindberg-Lévy central limit theorem, $N^{-1/2} DH_U - N^{1/2} \Theta_3 \Rightarrow N(0, \Sigma_{33})$ as $T \longrightarrow \infty$ prior to N. This establishes the second part of the proof. ■

Appendix B Tables

Table 1: Asymptotic moment approximations

		Expected value					
Test	Model	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$
DH_R	1	7.6541	14.6057	22.5867	30.5144	38.7008	46.0467
	2	12.1374	19.1595	26.8663	34.6582	42.2798	50.5897
	3	21.2169	27.9798	35.2532	42.6188	50.7753	58.3518
DH_U	1	13.6819	21.8477	29.8971	37.9786	46.0781	53.9190
	2	18.1627	25.9633	34.0732	42.2808	49.9467	58.0038
	3	27.4615	34.8246	42.4579	50.2396	58.2399	66.0355
		Variance					
Test	Model	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$
DH_R	1	92.0879	232.1380	359.9850	409.8960	458.6499	576.3421
	2	110.1665	201.8621	299.1754	398.8789	479.9372	515.9864
	3	159.3223	241.3131	326.8050	432.0750	477.3330	569.7757
DH_U	1	97.8256	170.6996	235.0440	288.2585	354.9217	420.6487
	2	124.6938	190.4510	260.4231	336.0109	383.1646	448.2359
	3	196.5500	255.9459	329.1439	399.9996	446.7113	519.6418

Notes:

(i) Model 1 refers to the regression with no deterministic terms, Model 2 refers to the regression with a constant term, and Model 3 refers to the regression with a constant and a linear time trend.

(ii) The value K refers to the number of regressors excluding any deterministic constant or trend terms.

Table 2: Response surface estimates of test moments

		Restricted statistic							Unrestricted statistic			
		Expected value										
K	Model	Constant	T^{-1}	T^{-2}	N^{-1}	N^{-2}	$(NT)^{-1}$	R^2	Constant	T^{-1}	T^{-2}	R^2
1	1	0.1319	0.5825	6.3319	−0.1550	0.1969	−1.9325	0.9366	0.0730	−0.8755	−0.1595	0.8607
1	2	0.0825	−0.0745	0.4153	−0.0464	0.0321	−1.2315	0.9082	0.0551	−0.9004	−3.6300	0.9408
1	3	0.0398	−0.2861	−0.7503	−0.0131	0.0093	−0.5083	0.9730	0.0363	−0.8398	−4.9362	0.9655
2	1	0.0679	0.2104	3.4586	−0.0615	0.1056	−1.2077	0.9239	0.0456	−0.7031	−5.1827	0.9472
2	2	0.0522	−0.1035	−0.4471	−0.0222	0.0098	−0.7385	0.9270	0.0383	−0.7327	−7.1360	0.9600
2	3	0.0311	−0.2571	−0.7287	−0.0076	0.0022	−0.4666	0.9799	0.0287	−0.8246	−0.7683	0.9543
3	1	0.0444	0.1050	0.7562	−0.0276	0.0484	−0.8166	0.8967	0.0336	−0.6942	−3.6211	0.9726
3	2	0.0375	−0.1112	−0.9961	−0.0142	0.0159	−0.6743	0.9478	0.0295	−0.7552	−1.8647	0.9621
3	3	0.0253	−0.2596	−0.5317	−0.0049	−0.0008	−0.3941	0.9839	0.0234	−0.7072	−4.9523	0.9495
		Variance										
K	Model	Constant	T^{-1}	T^{-2}	N^{-1}	N^{-2}	$(NT)^{-1}$	R^2	Constant	T^{-1}	T^{-2}	R^2
1	1	0.0113	−0.0659	1.3554	0.0017	−0.0082	−0.3441	0.7179	0.0097	−0.4946	7.2040	0.8637
1	2	0.0094	−0.2097	1.8994	−0.0006	−0.0074	−0.1838	0.7884	0.0076	−0.4930	8.3511	0.9371
1	3	0.0044	−0.1985	2.9244	−0.0013	−0.0013	−0.0257	0.9401	0.0051	−0.4323	9.3951	0.9689
2	1	0.0046	−0.0979	1.8974	0.0108	−0.0236	−0.2525	0.6188	0.0061	−0.4125	7.5931	0.9466
2	2	0.0048	−0.1609	2.1910	0.0046	−0.0097	−0.2230	0.8912	0.0051	−0.3969	8.0195	0.9649
2	3	0.0031	−0.1805	3.4122	0.0006	−0.0037	−0.0433	0.9638	0.0038	−0.3868	10.2825	0.9803
3	1	0.0030	−0.0779	0.8370	0.0075	−0.0145	−0.2058	0.8387	0.0044	−0.3699	8.2495	0.9677
3	2	0.0033	−0.1438	2.1051	0.0034	−0.0075	−0.1401	0.9477	0.0038	−0.3571	8.7167	0.9753
3	3	0.0023	−0.1565	3.2177	0.0004	−0.0015	−0.0371	0.9715	0.0030	−0.3417	9.9159	0.9807

Notes:

(*i*) See Table 1 for an explanation of the models.

(*ii*) Usage: The table gives approximate moments for the Durbin-Hausman statistics. For example, to obtain the expectation of the unrestricted statistic in Model 1 with $K = 1$ and $T = 50$, first compute the fitted value $0.0730 - 0.8755/50 - 0.1595/50^2$. The inverse of this value, 18.0420, is the approximate expected value.

Table 3: Empirical size for restricted test

		$N=10$						$N=20$					
		$T=50$			$T=100$			$T=50$			$T=100$		
Test	θ	M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3
DH_R	-0.4	0.324	0.392	0.422	0.134	0.286	0.092	0.300	0.476	0.498	0.060	0.220	0.082
	0.0	0.012	0.012	0.018	0.018	0.014	0.012	0.012	0.012	0.022	0.012	0.001	0.012
	0.4	0.066	0.020	0.050	0.132	0.062	0.130	0.028	0.008	0.040	0.122	0.020	0.150
PZ_ρ	-0.4	0.432	0.460	0.566	0.436	0.438	0.452	0.540	0.524	0.728	0.546	0.574	0.654
	0.0	0.018	0.026	0.004	0.016	0.028	0.004	0.004	0.016	0.000	0.008	0.006	0.002
	0.4	0.056	0.056	0.040	0.072	0.074	0.062	0.028	0.026	0.022	0.080	0.040	0.054
PZ_t	-0.4	0.944	0.900	0.956	0.960	0.936	0.970	0.992	0.988	0.998	1.000	0.988	1.000
	0.0	0.080	0.062	0.084	0.088	0.086	0.080	0.070	0.066	0.046	0.062	0.082	0.078
	0.4	0.002	0.006	0.004	0.006	0.010	0.004	0.000	0.004	0.004	0.002	0.002	0.002
PDF_ρ	-0.4	0.114	0.030	0.888	0.082	0.026	0.342	0.088	0.010	0.962	0.054	0.004	0.420
	0.0	0.054	0.018	0.126	0.098	0.040	0.114	0.042	0.016	0.124	0.046	0.012	0.094
	0.4	0.068	0.014	0.064	0.066	0.032	0.146	0.062	0.010	0.026	0.034	0.010	0.140
PDF_t	-0.4	0.500	0.514	0.852	0.500	0.500	0.650	0.666	0.654	0.974	0.604	0.592	0.844
	0.0	0.090	0.180	0.076	0.114	0.130	0.096	0.084	0.206	0.056	0.094	0.152	0.100
	0.4	0.042	0.118	0.020	0.046	0.094	0.038	0.058	0.098	0.006	0.030	0.068	0.036

Table 4: Empirical size for the unrestricted test

		$N = 10$						$N = 20$					
		$T = 50$			$T = 100$			$T = 50$			$T = 100$		
Test	θ	M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3
DH_U	-0.4	0.982	0.992	0.966	0.988	0.998	0.986	0.998	1.000	1.000	1.000	1.000	1.000
	0.0	0.070	0.058	0.056	0.058	0.088	0.062	0.062	0.056	0.066	0.076	0.110	0.124
	0.4	0.004	0.000	0.000	0.004	0.002	0.012	0.000	0.000	0.002	0.002	0.000	0.002
GZ_ρ	-0.4	0.974	0.994	0.980	0.982	0.996	0.964	1.000	1.000	0.998	1.000	1.000	1.000
	0.0	0.132	0.126	0.102	0.126	0.118	0.096	0.180	0.180	0.142	0.134	0.204	0.102
	0.4	0.018	0.004	0.008	0.028	0.018	0.028	0.012	0.014	0.006	0.024	0.010	0.008
GZ_t	-0.4	0.990	0.988	0.988	0.998	0.998	0.996	1.000	1.000	1.000	1.000	1.000	1.000
	0.0	0.024	0.028	0.018	0.052	0.028	0.044	0.010	0.018	0.026	0.040	0.052	0.040
	0.4	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000
GDF_ρ	-0.4	0.084	0.024	0.940	0.060	0.014	0.358	0.112	0.004	0.998	0.072	0.014	0.494
	0.0	0.056	0.020	0.088	0.068	0.012	0.074	0.058	0.024	0.122	0.054	0.034	0.078
	0.4	0.052	0.038	0.022	0.050	0.034	0.100	0.072	0.012	0.032	0.066	0.024	0.104
GDF_t	-0.4	0.606	0.836	0.910	0.682	0.798	0.792	0.844	0.978	0.994	0.884	0.952	0.960
	0.0	0.098	0.364	0.032	0.108	0.270	0.070	0.096	0.548	0.016	0.130	0.378	0.056
	0.4	0.046	0.232	0.000	0.042	0.150	0.044	0.042	0.358	0.000	0.050	0.218	0.018

Table 5: Individual unit root and cointegration tests

	Interest rate		Inflation		Cointegration	
Country	t_{iT}	$\tilde{t}_{iT}$	t_{iT}	$\tilde{t}_{iT}$	DHS	ADF
Austria	−2.3207	−2.2968	−2.2928	−2.2699	32.0131	−3.5321
Belgium	−2.3279	−2.3038	−1.7246	−1.7165	33.9159	−2.4800
Canada	−2.8373	−2.7909	−2.9627	−2.9094	78.1132	−3.2401
Finland	−1.7035	−1.6959	−2.3305	−2.3063	25.8597	−2.3874
France	−2.6696	−2.6315	−2.1046	−2.0876	28.5929	−3.0958
Germany	−1.8680	−1.8570	−1.9033	−1.8915	33.9345	−3.1960
Italy	−2.7761	−2.7328	−2.1960	−2.1763	22.5989	−3.4771
Japan	−2.8678	−2.8197	−2.9610	−2.9078	27.6394	−2.8370
Netherlands	−2.3747	−2.3489	−2.2890	−2.2663	20.5187	−2.8238
Norway	−2.3431	−2.3185	−2.5300	−2.4981	15.5121	−3.0000
Spain	−2.6006	−2.5657	−2.2742	−2.2520	69.9687	−2.4561
Switzerland	−1.8342	−1.8240	−2.0994	−2.0826	62.3207	−2.5097
United Kingdom	−2.4575	−2.4285	−2.9422	−2.8900	31.3563	−3.3534
United States	−2.3312	−2.3070	−3.1618	−3.0962	30.8219	−4.6986

Notes:

(i) The unit root tests includes a constant and a linear time trend. The cointegration tests are based on a regression with a constant term.

(ii) The ten and five percent critical values for the t_{iT} and $\tilde{t}_{iT}$ test statistics are −3.1476 and −3.4798, respectively. These have been obtained through Monte Carlo simulation.

(iii) The ten and five percent critical values for the DHS statistic are 33.68 and 41.10. These appear in Choi (1994).

(iv) The ten and five percent critical values for the ADF statistic are −3.0657 and −3.3654. These appear in Phillips and Ouliaris (1990).

Table 6: Estimated cointegration parameters

	DOLS				FMOLS				OLS			
Country	α	$t(\alpha=0)$	β	$t(\beta=1)$	α	$t(\alpha=0)$	β	$t(\beta=1)$	α	$t(\alpha=0)$	β	$t(\beta=1)$
Austria	4.7811	11.2519	0.9012	−0.7729	5.3861	15.4078	0.7232	−2.7271	5.6135	15.8032	0.6584	−3.3362
Belgium	4.4128	8.3622	1.2492	1.9663	4.6643	9.1151	1.1851	1.5389	4.8066	9.3441	1.1561	1.2957
Canada	4.4848	7.8630	1.0777	0.6567	4.8941	9.1430	1.0076	0.0734	5.0570	8.7658	1.0104	0.0948
Finland	8.0220	8.7381	0.6495	−1.9337	9.4704	10.9704	0.4184	−3.6704	9.5318	10.8033	0.4548	−3.4068
France	5.1548	8.6750	0.8129	−1.6401	6.1007	11.6995	0.6643	−3.6533	6.2679	11.3671	0.6781	−3.3441
Germany	3.2895	7.7648	1.1068	0.7970	3.2569	8.2615	1.1133	0.9284	3.5021	8.4105	1.0483	0.3765
Italy	5.4345	7.0154	0.9365	−0.6093	8.1670	13.3834	0.6544	−4.8488	8.3329	12.6683	0.6734	−4.2838
Japan	2.9008	5.3928	1.3310	1.2251	2.7730	5.4565	1.3395	1.4181	3.0182	5.7663	1.2499	1.0379
Netherlands	4.3619	8.3559	0.8694	−0.7247	4.5053	8.8624	0.7960	−1.2086	4.5640	8.7915	0.8057	−1.1389
Norway	6.0110	7.0586	0.8335	−1.1084	7.3449	9.0600	0.6280	−2.8683	7.3962	8.9346	0.6503	−2.6710
Spain	5.7284	6.2736	0.9558	−0.3640	7.7738	8.9220	0.7199	−2.5618	7.8573	8.9625	0.6924	−2.8115
Switzerland	2.3809	4.8552	0.6134	−2.6972	2.2192	4.6232	0.6766	−2.3517	2.3941	4.8283	0.6283	−2.6244
United Kingdom	5.6841	6.6683	0.8519	−0.9630	6.0745	7.6940	0.7662	−1.7094	6.2723	7.3190	0.7659	−1.5923
United States	2.8884	3.1371	1.2224	0.9686	3.5737	3.9430	1.0287	0.1367	3.5670	3.7192	1.0958	0.4376
Pooled	4.1674	12.9484	1.0492	0.7444	4.8284	24.7786	0.9360	−1.7180	4.9505	13.6832	0.9369	−0.8925

Notes:

(*i*) The DOLS estimator is based on two lags and leads. The FMOLS estimator is based on the Bartlett kernel.

(*ii*) The t-ratios are based on the Newey-West heteroskedasticity and autocorrelation consistent standard errors.

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