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Bayesian Detection and Tracking for Joint Positioning and Multipath Mitigation in GNSS

Bernhard Krach, Michael Lentmaier, and Patrick Robertson

***Abstract*— A sequential Bayesian estimation algorithm for joint positioning and multipath mitigation in global navigation satellite systems is presented, with an underlying process model that is especially designed for dynamic user scenarios and dynamic channel conditions. In order to facilitate efficient integration into receivers it builds upon complexity reduction concepts that previously have been applied within maximum likelihood estimators. To demonstrate its capabilities simulation results are presented.**

***Index Terms*— Global Navigation Satellite Systems, Positioning, Time-of-Arrival, Signal Parameter Estimation, Bayesian Estimation, Multipath Mitigation, Synchronization.**

I. INTRODUCTION

Within global navigation satellite systems (GNSS), such as the Global Positioning System (GPS) or the future European satellite navigation system Galileo, the user position is determined based upon the code division multiplex access (CDMA) navigation signals received from different satellites using the time-of-arrival (TOA) method [1]. A major error source for positioning comes from multipath, the reception of additional signal replica due to reflections caused by the receiver environment. The reception of multipath introduces a bias into the time delay estimate of the delay lock loop (DLL) of a conventional navigation receiver, which finally leads to a bias in the receiver's position estimate.

For efficient removal of this bias it is possible to formulate advanced maximum likelihood (ML) estimators that incorporate the echoes into the signal model [2], [3], [4], [5], [6], [7] or to exploit the properties of the position domain [8], [9]. In case of static user and channel scenarios the ML approach is optimal and performs significantly better than other approaches as it is capable of achieving the theoretical limits given by the Cramer Rao bound. The drawback of ML estimator techniques is that the parameters are assumed to be constant during the time of observation. Independent estimates are obtained for successive observation intervals, whose length has to be adapted to the dynamics of the user and the channel. No explicit use of the user's and channel's temporal or spatial dynamics is made.

It has been proposed in [10] to consider the important practical case of a dynamic user and channel scenario. In this paper we take further advantage of the properties of the position domain likelihood [9] and formulate the line-of-sight (LOS) path delays through a transformation of the position and clock parameters, leading to a combined position, clock, and channel estimator, which allows to exploit available knowledge about the statistical properties of the user, clock, and the channel dynamics.

Our approach is based on Bayesian filtering, the optimal and well-known framework to address such dynamic state estimation problems. Sequential Monte Carlo (SMC) methods are used for computing the posterior probability density functions (PDFs) of the position and channel parameters. In contrast to existing sequential joint positioning and multipath mitigation approaches, which consider multipath as a bias parameter that can be tracked [8], we incorporate the multipath signals into our signal model. As the resulting position domain likelihood can be factorized into the contributions of each satellite [11] we propose to evaluate these factors via reduced complexity methods that previously have been applied within maximum likelihood estimators [12]. It is shown that for the proposed sequential estimator there is no need to assume the number of received multipath signals to be known a-priori as required for the ML approaches, because the number of received replica can be tracked implicitly along with all other user, clock, and channel parameters in a probabilistic fashion.

II. SIGNAL MODEL

Assume that the receiver provides M parallel channels to simultaneously process the signals arriving from the available satellites [1]. After coarse removal of the Doppler shifts, e.g. through a conventional phase lock loop (PLL), the complex valued baseband-equivalent received signal for the receiver channel j, $j = 1, \dots, M$, can be expressed as

$$z_j(t) = \sum_{i=1}^{N_m} e_{i,j}(t) \cdot a_{i,j}(t) \cdot [c_j(t) * g(t - \tau_{i,j}(t))] + n_j(t) \quad , \quad (1)$$

where $c_j(t)$ is a delta-train CDMA code sequence that is modulated on a pulse $g(t)$, N_m is the total number of allowed paths reaching the receiver (to restrict the modeling complexity), $e_{i,j}(t)$ is a binary function that controls the activity of the i'th path and $a_{i,j}(t)$ and $\tau_{i,j}(t)$ are their individual complex amplitudes and time delays, respectively. Neglecting CDMA interference the signal is disturbed by additive white Gaussian noise $n_j(t)$. Grouping blocks of L samples at times $(m + kL)T_s$, $m = 0, \dots, L-1$, together into vectors $\mathbf{z}_{j,k}$, $k = 0, 1, \dots$, and assuming the parameter functions $e_{i,j}(t)$, $a_{i,j}(t)$ and $\tau_{i,j}(t)$ to be constant within the corresponding time interval and equal to $e_{i,j,k}$, $a_{i,j,k}$ and $\tau_{i,j,k}$, the signal for block k can be rewritten as

$$\begin{aligned} \mathbf{z}_{j,k} &= \mathbf{C}_j \mathbf{G}(\boldsymbol{\tau}_{j,k}) \mathbf{E}_{j,k} \mathbf{a}_{j,k} + \mathbf{n}_{j,k} \\ &\widehat{=} \mathbf{s}_{j,k} + \mathbf{n}_{j,k} \quad . \end{aligned} \quad (2)$$

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In the compact form the samples of the delayed pulses $\mathbf{g}(\tau_{i,j,k})$ are stacked together as columns of the matrix $\mathbf{G}(\boldsymbol{\tau}_{j,k}) = [\mathbf{g}(\tau_{1,j,k}), \ldots, \mathbf{g}(\tau_{N_m,j,k})]$, $\mathbf{C}_j$ is a matrix representing the convolution with the code, and the delays and amplitudes are collected in the vectors $\boldsymbol{\tau}_{j,k} = [\tau_{1,j,k}, \ldots, \tau_{N_m,j,k}]^T$ and $\mathbf{a}_k = [a_{1,j,k}, \ldots, a_{N_m,j,k}]^T$ respectively. Furthermore, for concise notation we use $\mathbf{E}_{j,k} = \mathrm{diag}\,[\mathbf{e}_{j,k}]$, where the elements of the vector $\mathbf{e}_{j,k} = [e_{1,j,k}, \ldots, e_{N_m,j,k}]^T$, $e_{i,j,k} \in [0,1]$, determine whether the i'th path is active or not by being either $e_{i,j,k} = 1$ corresponding to an active path or $e_{i,j,k} = 0$ for a path that is currently not active. The term $\mathbf{s}_{j,k}$ denotes the signal hypothesis and is completely determined by the channel parameters $\boldsymbol{\tau}_{j,k}$, $\mathbf{a}_{j,k}$ and $\mathbf{e}_{j,k}$. The proper selection of the block size L is crucial and depends necessarily on the duration of the time interval for which the parameters can be assumed to be constant. This period is equivalent to the coherent integration time in a conventional navigation receiver, which commonly ranges from 1-20 ms. Using (2) we can write the associated *channel likelihood function* as

$$p(\mathbf{z}_{j,k}|\mathbf{s}_{j,k}) = \frac{1}{(2\pi)^L \sigma_j^{2L}} \cdot \exp\left[-\frac{1}{2\sigma_j^2}\left(\mathbf{z}_{j,k} - \mathbf{s}_{j,k}\right)^H \left(\mathbf{z}_{j,k} - \mathbf{s}_{j,k}\right)\right]. \quad (3)$$

The likelihood function will play a central role in the algorithm discussed in this paper; its purpose is to quantify the conditional probability of the received signal conditioned on the unknown signal (which depends finally on the position, clock and channel parameters as shown later).

A. Complexity Reduction

In [3] a general concept for the efficient representation of the likelihood (3) was presented. The key idea of this concept is to formulate (3) through a vector $\mathbf{z}_{c,j,k}$ resulting from an orthonormal projection of the observed signal $\mathbf{z}_{j,k}$ onto a smaller vector space, so that $\mathbf{z}_{c,j,k}$ is a sufficient statistic according to the Neyman-Fisher factorization [13] and hence suitable for estimating $\mathbf{s}_{j,k}$. In other words the reduced signal comprises the same information as the original signal itself. In practice this concept becomes relevant as the projection can be achieved by processing the received signal (2) with a bank of correlators and a subsequent decorrelation of the correlator bank outputs. A variant of this very general concept, applied in [4], has also been referred to as the *Signal Compression Theorem* in [5]. The corresponding mathematical background will be briefly discussed below, including also interpolation of the likelihood and elimination of complex amplitudes as further methods for complexity reduction.

1) Data Compression: As explained above the large vector containing the received signal samples $\mathbf{z}_{j,k}$ is linearly transformed into a vector $\mathbf{z}_{c,j,k}$ of much smaller size. Following this approach the likelihood according to (3) can be rewritten as

$$\begin{aligned} p(\mathbf{z}_{j,k}|\mathbf{s}_{j,k}) &= \frac{1}{(2\pi)^L \sigma_j^{2L}} \exp\left[-\frac{\mathbf{z}_{j,k}^H \mathbf{z}_{j,k}}{2\sigma_j^2}\right] \\ &\quad \cdot \exp\left[\frac{\Re\{\mathbf{z}_{j,k}^H \mathbf{Q}_{c,j}\mathbf{Q}_{c,j}^H \mathbf{s}_{j,k}\}}{\sigma_j^2} - \frac{\mathbf{s}_{j,k}^H \mathbf{Q}_{c,j}\mathbf{Q}_{c,j}^H \mathbf{s}_{j,k}}{2\sigma_j^2}\right] \\ &= \frac{1}{(2\pi)^L \sigma_j^{2L}} \exp\left[-\frac{\mathbf{z}_{j,k}^H \mathbf{z}_{j,k}}{2\sigma_j^2}\right] \\ &\quad \cdot \exp\left[\frac{\Re\{\mathbf{z}_{c,j,k}^H \mathbf{s}_{c,j,k}\}}{\sigma_j^2} - \frac{\mathbf{s}_{c,j,k}^H \mathbf{s}_{c,j,k}}{2\sigma_j^2}\right], \end{aligned} \quad (4)$$

with the compressed received vector $\mathbf{z}_{c,j,k}$ and the compressed signal hypothesis $\mathbf{s}_{c,j,k}$:

$$\mathbf{z}_{c,j,k} = \mathbf{Q}_{c,j}^H \mathbf{z}_{j,k}, \quad \mathbf{s}_{c,j,k} = \mathbf{Q}_{c,j}^H \mathbf{s}_{j,k} \ , \quad (5)$$

and the orthonormal compression matrix $\mathbf{Q}_c$, which needs to fulfill

$$\mathbf{Q}_{c,j}\mathbf{Q}_{c,j}^H \approx \mathbf{I}, \quad \mathbf{Q}_{c,j}^H \mathbf{Q}_{c,j} \approx \mathbf{I} \ , \quad (6)$$

to minimize the compression loss. According to [3] the compression can be two-fold so that we can factorize

$$\mathbf{Q}_{c,j} = \mathbf{Q}_{cc,j}\mathbf{Q}_{pc,j} \quad (7)$$

into a *canonical component decomposition*, given by an $L \times N_{cc}$ matrix $\mathbf{Q}_{cc}$, and a *principal component decomposition*, given by an $N_{cc} \times N_{pc}$ matrix $\mathbf{Q}_{pc}$. In [3] two choices for $\mathbf{Q}_{cc,j}$ are proposed:

$$\mathbf{Q}_{cc,j} = \begin{cases} \mathbf{C}_j \mathbf{G}(\boldsymbol{\tau}_j^b)\mathbf{R}_{cc,j}^{-1} & \textit{Signal matched} \\ \mathbf{C}_j(\boldsymbol{\tau}_j^b)\mathbf{R}_{cc,j}^{-1} & \textit{Code matched} \end{cases} , \quad (8)$$

where the elements of the vector $\boldsymbol{\tau}_j^b$ define the positions of the individual correlators.

To decorrelate the bank outputs $(\mathbf{C}\mathbf{G}(\boldsymbol{\tau}^b))^H\mathbf{y}$ and $\mathbf{C}(\boldsymbol{\tau}^b)^H\mathbf{y}$ the whitening matrix $\mathbf{R}_{cc}$ can be obtained from a QR decomposition of $\mathbf{C}\mathbf{G}(\boldsymbol{\tau}^b)$ and $\mathbf{C}(\boldsymbol{\tau}^b)$ respectively. Apart from practical implementation issues both correlation methods given by (8) are equivalent from a conceptual point of view. For details on the compression through $\mathbf{Q}_{pc,j}$ the reader is referred to [3].

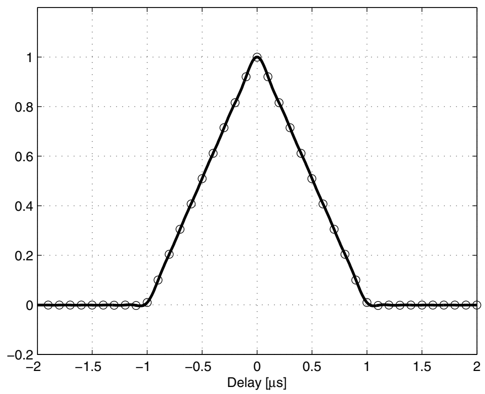
2) Interpolation: In order to compute (4) independently of the sampling grid, advantage can be made of interpolation techniques. Using the discrete Fourier transformation (DFT), with $\boldsymbol{\Psi}$ being the DFT matrix and $\boldsymbol{\Psi}^{-1}$ its inverse (IDFT), we get:

$$\begin{aligned} \mathbf{s}_{c,j,k} &= \mathbf{Q}_{c,j}^H \mathbf{C}_j \boldsymbol{\Psi}^{-1} \mathrm{diag}\left[\boldsymbol{\Psi}\mathbf{g}(0)\right] \boldsymbol{\Omega}(\boldsymbol{\tau}_{j,k})\mathbf{E}_{j,k}\mathbf{a}_{j,k} \\ &\widehat{=} \mathbf{M}_{s_c,j}\boldsymbol{\Omega}(\boldsymbol{\tau}_{j,k})\mathbf{E}_{j,k}\mathbf{a}_{j,k} \ , \end{aligned} \quad (9)$$

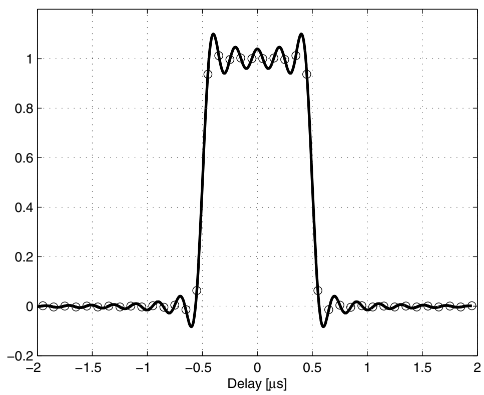
with $\boldsymbol{\Omega}(\boldsymbol{\tau}_{j,k})$ being a matrix of column-wise stacked vectors with Vandermonde structure [12], [3], such that the element at row p and column q computes with

$$\Re\left\{[\boldsymbol{\Omega}(\boldsymbol{\tau}_{j,k})]_{p,q}\right\} = \cos\Big(2\pi(p-1)\tau_{q,j,k}/(N_g T_s)\Big) \ , \quad (10)$$

$$\Im\left\{[\boldsymbol{\Omega}(\boldsymbol{\tau}_{j,k})]_{p,q}\right\} = -\sin\Big(2\pi(p-1)\tau_{q,j,k}/(N_g T_s)\Big) \ . \quad (11)$$



(a) BPSK signal matched



(b) BPSK code matched

Fig. 1. Output of two types of canonical component type correlator banks for BPSK usable for data size reduction according to (8).

N_g is the length of the pulse $\mathbf{g}$ in samples. The advantage of the interpolation is that it can take place in the reduced space. The most costly computations in (9) can be carried out in precalculations as the matrix $\mathbf{M}_{s_c,j}$, whose row dimension corresponds to the dimension of the reduced space and whose column dimension is N_g is constant.

3) Amplitude Elimination: In a further step we reduce the number of parameters by maximizing (4) for a given set of $\boldsymbol{\tau}_{j,k}$ and $\mathbf{e}_{j,k}$ with respect to the complex amplitudes $\mathbf{a}_{j,k}$, which can be achieved through a closed form solution. Using

$$\mathbf{S}_{c,k} = \mathbf{M}_{s_c,j}\boldsymbol{\Omega}(\boldsymbol{\tau}_{j,k})\mathbf{E}_{j,k} \tag{12}$$

and obtaining $\mathbf{S}^{+}_{c,j,k}$ by removing zero columns from $\mathbf{S}_{c,j,k}$ one yields the corresponding amplitude values of the active paths:

$$\hat{\mathbf{a}}^{+}_{j,k} = \left(\mathbf{S}^{+H}_{c,j,k}\mathbf{S}^{+}_{c,j,k}\right)^{-1}\mathbf{S}^{+H}_{c,j,k}\mathbf{z}_{c,j,k} \ . \tag{13}$$

For the active paths as indicated by $\mathbf{e}_{j,k}$ the vector $\hat{\mathbf{a}}^{+}_{j,k}$ is equal to the ML amplitude estimates for the time instance k. When evaluating (4) we substitute $\mathbf{s}_{c,j,k}$ and $\mathbf{s}_{j,k}$ respectively by

$$\tilde{\mathbf{s}}_{c,j,k} = \mathbf{S}_{c,j,k}\hat{\mathbf{a}}_{j,k} \quad \text{and} \quad \tilde{\mathbf{s}}_{j,k} = \mathbf{S}_{j,k}\hat{\mathbf{a}}_{j,k} \ , \tag{14}$$

whereby the elements of the vector $\hat{\mathbf{a}}_{j,k}$ that are indicated to have an active path ($a_{i,j,k} : i \rightarrow e_{i,j,k} = 1$) are set equal to the corresponding elements of $\hat{\mathbf{a}}^{+}_{j,k}$. All other elements ($a_{i,j,k} : i \rightarrow e_{i,j,k} = 0$) can be set arbitrarily as their influence is masked by the zero elements of $\mathbf{e}_{j,k}$. The elimination procedure introduced here is motivated further in section IV-C.

B. Parameter Transformation

For the given TOA estimation problem the LOS signal delays $\tau_{1,j,k}$ associated to the different satellites j are mutually dependent because of the common receiver position and clock offset. To exploit this fact we apply the following parameter transformations:

1) Delay Transformation: The line-of-sight delays are replaced by their navigation parameter equivalents using the TOA pseudorange equation [1]

$$\rho_{j,k} = \left|\mathbf{p}^{t,e}_{j,k} - \mathbf{p}^{r,e}_{k}\right| c^{-1} + \tau^{r}_{k}. \tag{15}$$

with the pseudorange $\rho_{j,k}$, the position of the transmitting satellite $\mathbf{p}^{t,e}_{j,k}$ in earth-centered earth-fixed (ECEF) coordinates, the receiver position $\mathbf{p}^{r,e}_{k}$ in ECEF coordinates, the receiver clock bias τ^{r}_{k} and the speed of light c. As the actual receiver delay estimate $\tau_{1,j,k}$ is affected by the transmitter clock offset $\tau^{t}_{j,k}$ and suffers additionally from the propagation through the atmosphere, we obtain $\tau_{1,j,k}$ from $\rho_{j,k}$ using the ionospheric correction $\tau^{\text{iono}}_{j,k}$ and the tropospheric correction $\tau^{\text{tropo}}_{j,k}$:

$$\tau_{1,j,k} = \rho_{j,k} + \tau^{t}_{j,k} + \tau^{\text{iono}}_{j,k} + \tau^{\text{tropo}}_{j,k} + \varepsilon_{j,k}. \tag{16}$$

Additional errors are assumed to be included in $\varepsilon_{j,k}$. Within this paper we assume $\mathbf{p}^{t,e}_{j,k}$, $\tau^{t}_{j,k}$, $\tau^{\text{iono}}_{j,k}$,$\tau^{\text{tropo}}_{j,k}$ and $\varepsilon_{j,k}$ to be known.

2) Coordinate Transformation: Furthermore we apply a coordinate transformation on the receiver position from ECEF coordinates to a local navigation coordinate system [1] that is suitable for characterization of the user/receiver dynamics:

$$\mathbf{p}^{r,e}_{k} \quad = \quad \mathbf{A}^{en}_{k}\mathbf{p}^{r,n}_{k} + \mathbf{p}^{o,e}_{k} \ . \tag{17}$$

The term $\mathbf{p}^{r,n}_{k}$ denotes the receiver position expressed in terms of the local navigation coordinates, the rotation matrix $\mathbf{A}^{en}_{k}$ and the vector $\mathbf{p}^{o,e}_{k}$ characterize the coordinate transformation and are, of course, assumed to be known.

III. PROCESS AND SYSTEM MODEL

The position of the user as well as the receiver clock parameters are known to be time varying but not independent from one time instance to the next, as physical restrictions impose constraints on their temporal evolution. We know from channel measurements that this is also valid for the multipath channel parameters; for example, an multipath echo usually experiences a "life-cycle" from its first occurrence, then a more or less gradual change in its delay and phase over time, until it disappears [14]. The purpose of the process model is to characterize the temporal dependencies of these parameters (introduced in Section II) in a probabilistic fashion. In our modeling approach we structure the entire system process, which is selected to have the properties of a Markovian process, into sub-processes, which are introduced now.

To model the temporal evolution of position and clock parameters we employ simple Gaussian transition models as typically applied within navigation Kalman filters [1]. The modeling of the multipath process is motivated by [14], [10].

A. User Model

The temporal evolution of the receiver position used in (17) can be characterized by a physical movement model of the user or vehicle that carries the receiver. Here we use a simple model given by

$$\mathbf{p}_k^{r,n} = \mathbf{p}_{k-1}^{r,n} + \dot{\mathbf{p}}_{k-1}^{r,n} \cdot T_s + \mathbf{n}_p \quad (18)$$

$$\dot{\mathbf{p}}_k^{r,n} = \dot{\mathbf{p}}_{k-1}^{r,n} + \mathbf{n}_{\dot{p}} \quad (19)$$

with $\dot{\mathbf{p}}_k^{r,n}$ being the temporal derivative of $\mathbf{p}_{k-1}^{r,n}$, and $\mathbf{n}_p$, $\mathbf{n}_{\dot{p}}$ being vectors of element-wise uncorrelated zero-mean white Gaussian noise, whose elements have a given variance of σ_x^2, σ_y^2, σ_z^2 and $\sigma_{\dot{x}}^2$, $\sigma_{\dot{y}}^2$, $\sigma_{\dot{z}}^2$, respectively.

B. Clock Model

The clock model is used to characterize the local receiver clock, in particular the evolution of the user clock offset τ_k^r and the user clock drift $\dot{\tau}_k^r$. We use this simple model:

$$\tau_k^r = \tau_{k-1}^r + \dot{\tau}_{k-1}^r \cdot T_s + n_\tau \ , \quad (20)$$

$$\dot{\tau}_k^r = \dot{\tau}_{k-1}^r + n_{\dot{\tau}}. \quad (21)$$

The noise terms n_τ and $n_{\dot{\tau}}$ are realizations of a zero-mean white Gaussian noise process of variance σ_τ^2 and $\sigma_{\dot{\tau}}^2$ respectively.

C. Multipath Channel Model

The multipath channel is determined by the parameters $e_{i,j,k}$ and $\tau_{i,j,k}$ with $i > 0$. According to [10] their temporal evolution is modeled by the following statistical processes:

1) Multipath Activity: According to (2) each path is either "on" or "off", as defined by channel parameter $e_{i,j,k} \in \{1 \equiv \text{"on"}, 0 \equiv \text{"off"}\}$, where $e_{i,j,k}$ is assumed to follow a simple two-state Markov process with asymmetric crossover and same-state probabilities:

$$p(e_{i,j,k} = 0 | e_{i,j,k-1} = 1) = p_{\text{onoff}} \ , \quad (22)$$

$$p(e_{i,j,k} = 1 | e_{i,j,k-1} = 0) = p_{\text{offon}} \ . \quad (23)$$

2) Multipath Delay: The associated delays of the multipath replica are characterized by

$$\boldsymbol{\tau}_k^{\text{mp}} = \boldsymbol{\tau}_{k-1}^{\text{mp}} + \dot{\boldsymbol{\tau}}_{k-1}^{\text{mp}} \cdot T_s + \mathbf{n}_{\text{mp}} \ , \quad (24)$$

$$\dot{\boldsymbol{\tau}}_k^{\text{mp}} = \dot{\boldsymbol{\tau}}_{k-1}^{\text{mp}} + \mathbf{n}_{\text{mip}} \ , \quad (25)$$

where for concise notation we have used

$$\boldsymbol{\tau}_k^{\text{mp}} \widehat{=} \{\boldsymbol{\tau}_{j,k}^{\text{mp}}, j = 1, \ldots, M\} \quad (26)$$

with $\boldsymbol{\tau}_{j,k}^{\text{mp}} = [\tau_{2,j,k}, \ldots, \tau_{N_m,j,k}]^T$. M is the total number of received satellites. The temporal derivative of $\boldsymbol{\tau}_k^{\text{mp}}$ is denoted by $\dot{\boldsymbol{\tau}}_k^{\text{mp}}$ and $\mathbf{n}_{\text{mp}}$, $\mathbf{n}_{\text{mip}}$ are vectors of element-wise uncorrelated zero-mean white Gaussian noise of variance σ_{mp}^2 and σ_{mip}^2 respectively. Due to physical constraints we restrict the multipath delay process to satisfy $\tau_{1,j,k} < \tau_{i,j,k}, i = 2, \ldots, N_m$.

D. State Vector

Considering the proposed signal model including complexity reduction and the introduced process model we collect the relevant parameters into the state vector

$$\mathbf{x}_k = \{\mathbf{p}_k^{r,n}, \dot{\mathbf{p}}_k^{r,n}, \tau_k^r, \dot{\tau}_k^r, \boldsymbol{\tau}_k^{\text{mp}}, \dot{\boldsymbol{\tau}}_k^{\text{mp}}, \mathbf{e}_k\} \quad (27)$$

with

$$\mathbf{e}_k \widehat{=} \{\mathbf{e}_{j,k}, j = 1, \ldots, M\} \ . \quad (28)$$

Note that the model implicitly represents the number of paths per range

$$N_{m,j,k} = \sum_{i=1}^{N_m} e_{i,j,k} \quad (29)$$

as a time variant parameter.

E. Likelihood Factorization

So far we have introduced the channel likelihood (3) associated to the receiver channel j. The objective now is to calculate the likelihood that takes into account the observations of all receiver channels, namely $p(\mathbf{z}_k | \mathbf{x}_k)$ with

$$\mathbf{z}_k \widehat{=} \{\mathbf{z}_{j,k}, j = 1, \ldots, M\} \ , \quad (30)$$

Writing $\mathbf{z}_{j,k}^-$ for $\mathbf{z}_k$ after omitting $\mathbf{z}_{j,k}$, i.e. $\mathbf{z}_{j,k}^- = \mathbf{z}_k \backslash \mathbf{z}_{j,k}$, we assume independent noise realization for the receiver channels with

$$p(\mathbf{z}_{j,k} | \mathbf{x}_k, \mathbf{z}_{j,k}^-) = p(\mathbf{z}_{j,k} | \mathbf{x}_k) \ . \quad (31)$$

In this case the overall likelihood function can be written in product form according to the factorization of Bayes' rule [11] as

$$p(\mathbf{z}_k | \mathbf{x}_k) = C \cdot \prod_{j=1}^{M} p(\mathbf{z}_{j,k} | \tilde{\mathbf{s}}_{j,k}) \quad (32)$$

with C being a normalizing constant. Please note that according to (12), (13), (14), (15), (16), and (17) the signal hypothesis $\tilde{\mathbf{s}}_{j,k}$ is determined completely by $\mathbf{x}_k$ and $\mathbf{z}_{j,k}$ (for details see section IV-C).

IV. SEQUENTIAL ESTIMATION

To overcome the drawback of the ML approaches mentioned in Section I our objective here is to address the introduced estimation problem with a sequential estimator that is able to exploit not only a single set of observations $\mathbf{z}_k$ to estimate the hidden parameters $\mathbf{x}_k$ (via the likelihood function), but is also able to exploit our knowledge about the statistical dependencies between successive sets of position, clock and multipath channel parameters, in order to improve the performance of the estimator.

A. Optimal Solution

Given the models introduced in Section II and III the problem of positioning and multipath mitigation now becomes one of *sequential estimation of a hidden Markov process*: We want to estimate the unknown position, clock and multipath

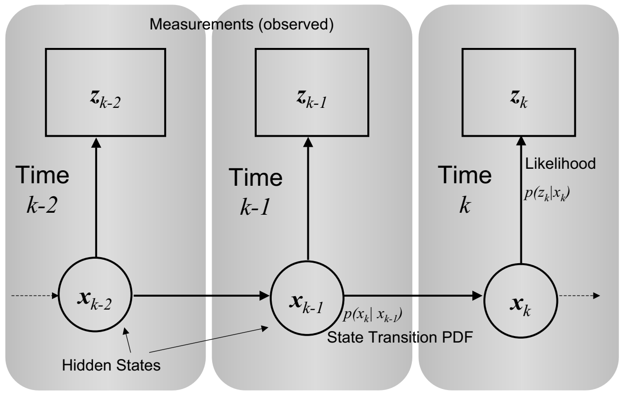


Fig. 2. Illustration of the hidden Markov estimation process for three time instances. Our measurements are the sequence $\mathbf{z}_q, q = 0, \ldots, k\}$, and the parameters to be estimated are $\mathbf{x}_q, q = 0, \ldots, k\}$

channel parameters, namely the hidden state $\mathbf{x}_k$ based on an evolving sequence of received noisy observations $\mathbf{z}_k$ (see Figure 2). According to Section III the user position and clock offset as well as the multipath channel process for each receiver channel are modeled as a first-order Markov process as future position, clock and multipath channel parameters given the present state of the position, clock and the channel and all past states, depend only on the present system state (and not on any past states). It is also assumed according to section II that the noise affecting successive channel outputs is independent of the past noise values; so *each observation depends only on the present channel state*.

Now that our major assumptions have been established we may apply the concept of *sequential Bayesian estimation*. The reader is referred to [15] which gives a derivation of the general framework for optimal estimation of temporally evolving (Markovian) parameters by means of inference; and we have chosen similar notation. The entire history of observations (over the temporal index k) can be written as

$$\mathbf{Z}_k \widehat{=} \{\mathbf{z}_q, q = 0, \ldots, k\} \ . \quad (33)$$

As $\mathbf{x}_k$ represents the characterization of the hidden state our goal is to determine the *posterior* probability density function (PDF) of every possible state characterization given all observations: $p(\mathbf{x}_k|\mathbf{Z}_k)$. Once we have evaluated this posterior PDF we can either determine the configuration that maximizes it - the so called maximum a-posteriori (MAP) estimate; or we can choose the expectation - equivalent to the minimum mean square error (MMSE) estimate. In addition, the posterior distribution itself contains all uncertainty about the current state and is thus the optimal measure in terms of reliability information.

It can be shown that the sequential estimation algorithm is recursive as illustrated in Figure 3, as it uses the posterior PDF computed for time instance $k-1$ to compute the posterior PDF for instance k. For a given posterior PDF at time instance $k-1$, $p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1})$, the *prior* PDF $p(\mathbf{x}_k|\mathbf{Z}_{k-1})$ is calculated

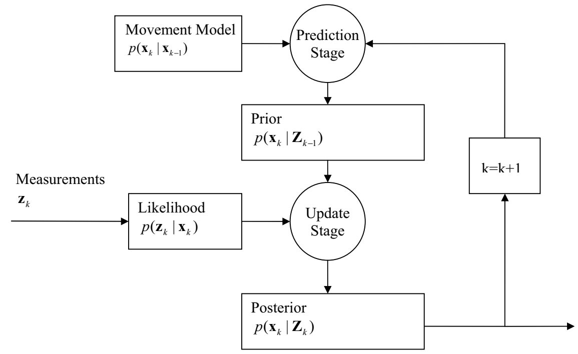


Fig. 3. Illustration of the recursive Bayesian estimator.

in the so-called *prediction step* by applying the Chapman-Kolmogorov equation:

$$p(\mathbf{x}_k|\mathbf{Z}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1})\mathrm{d}\mathbf{x}_{k-1} \ , \quad (34)$$

with $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ being the state transition PDF of the Markov process. In the *update step* the new posterior PDF for step k is obtained by applying Bayes' rule to $p(\mathbf{x}_k|\mathbf{z}_k, \mathbf{Z}_{k-1})$ yielding the normalized product of the likelihood $p(\mathbf{z}_k|\mathbf{x}_k)$ and the prior PDF:

$$\begin{aligned} p(\mathbf{x}_k|\mathbf{Z}_k) &= p(\mathbf{x}_k|\mathbf{z}_k, \mathbf{Z}_{k-1}) \\ &= \frac{p(\mathbf{z}_k|\mathbf{x}_k, \mathbf{Z}_{k-1})p(\mathbf{x}_k|\mathbf{Z}_{k-1})}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \\ &= \frac{p(\mathbf{z}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Z}_{k-1})}{p(\mathbf{z}_k|\mathbf{Z}_{k-1})} \ . \end{aligned} \quad (35)$$

The denominator of (35) does not depend on $\mathbf{x}_k$ and so it can be computed by integrating the numerator of (35) over the entire range of $\mathbf{x}_k$ (normalization).

To summarize so far, the entire process of prediction and update can be carried out recursively to calculate the posterior PDF (35) sequentially, based on an initial value of $p(\mathbf{x}_0|\mathbf{z}_0) = p(\mathbf{x}_0)$. The evaluation of the likelihood function $p(\mathbf{z}_k|\mathbf{x}_k)$ is the essence of the update step. Similarly, maximizing this likelihood function (i.e. ML estimation) would be equivalent to maximizing $p(\mathbf{x}_k|\mathbf{Z}_k)$ only in the case that the prior PDF $p(\mathbf{x}_k|\mathbf{Z}_{k-1})$ does not depend on $\mathbf{Z}_{k-1}$ and when all values of x_k are a-priori equally likely. Since these conditions are not met, evaluation of $p(\mathbf{x}_k|\mathbf{Z}_k)$ entails all the above steps.

B. Sequential Estimation using Particle Filters

The optimal estimation algorithm relies on evaluating the integral (34), which is usually a very difficult task, except for certain additional restrictions imposed on the model and the noise process. So very often a suboptimal realization of a Bayesian estimator has to be chosen for implementation. In this paper we use a Sequential Monte Carlo (SMC) filter, in particular a Sampling Importance Resampling Particle Filter SIR-PF according to [15]. In this algorithm the posterior density at step k is represented as a sum, and is specified by a set of N_p particles:

$$p(\mathbf{x}_k|\mathbf{Z}_k) \approx \sum_{\mu=1}^{N_p} w_k^\mu \cdot \delta(\mathbf{x}_k - \mathbf{x}_k^\mu) \ , \qquad (36)$$

where each particle with index μ has a state $\mathbf{x}_k^\mu$ and has a *weight* w_k^μ. The sum over all particles' weights is one. In SIR-PF, the weights are computed according to the principle of *Importance Sampling* where the so-called proposal density is chosen to be the state transition probability for the μ-th particle $p(\mathbf{x}_k|\mathbf{x}_{k-1} = \mathbf{x}_{k-1}^\mu)$, and with *resampling* at every time step. For $N_p \to \infty$ the approximate posterior approaches the true PDF.

The key step in which the *measurement* for instance k is incorporated, is in the calculation of the weight w_k^μ which for the SIR-PF can be shown to be the likelihood function: $p(\mathbf{z}_k|\mathbf{x}_k^\mu)$. The characterization of the *process* enters in the algorithm when at each time instance k, the state of each particle $\mathbf{x}_k^\mu$ is drawn randomly from the proposal distribution; i.e. from $p(\mathbf{x}_k|\mathbf{x}_{k-1}^\mu)$.

C. Exploiting Linear Substructures

If there exist linear substructures in the model, it is possible to reduce the computational complexity of the filter by means of marginalization over the linear state variables [16], also known as Rao-Blackwellization [17]. In our case, since the measurement $\mathbf{z}_{j,k}$ is a linear function of the complex amplitudes $\mathbf{a}_{j,k}$, we can estimate them analytically and marginalize:

$$\begin{aligned} p(\mathbf{z}_{j,k}|\boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}) &= p(\mathbf{z}_{j,k}|\mathbf{x}_k) \qquad (37) \\ &= \int_{\mathbf{a}_{j,k}} p(\mathbf{z}_{j,k}|\boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}, \mathbf{a}_{j,k}) \cdot p(\mathbf{a}_{j,k}|\boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}) \mathrm{d}\mathbf{a}_{j,k} \ . \end{aligned}$$

The term $p(\mathbf{a}_{j,k}|\boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k})$ is a constant here, since the amplitudes are assumed block-wise independent. As the channel likelihood function (3) can be written in product form as

$$\begin{aligned} p(\mathbf{z}_{j,k}|\mathbf{s}_{j,k}) &= p(\mathbf{z}_{j,k}|\boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}, \mathbf{a}_{j,k}) = \qquad (38) \\ & f(\mathbf{z}_{j,k}, \boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}) \cdot g(\mathbf{z}_{j,k}, \boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}, \mathbf{a}_{j,k}) \ , \end{aligned}$$

where $g(\mathbf{z}_{j,k}, \boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}, \mathbf{a}_{j,k})$ is Gaussian with respect to $\mathbf{a}_{j,k}$, using (37) and (38) leads to

$$p(\mathbf{z}_{j,k}|\boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}) \quad \propto \quad f(\mathbf{z}_{j,k}, \boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}) \ . \qquad (39)$$

Additionally it can be shown that $f(\mathbf{z}_{j,k}, \boldsymbol{\tau}_{j,k}, \mathbf{e}_{j,k}) \propto p(\mathbf{z}_{j,k}|\tilde{\mathbf{s}}_{j,k})$. Thus we may write

$$p(\mathbf{z}_{j,k}|\mathbf{x}_k) \propto p(\mathbf{z}_{j,k}|\mathbf{s}_{j,k} = \tilde{\mathbf{s}}_{j,k}) \ , \qquad (40)$$

and the weight factors of the SIR particle filter become $p(\mathbf{z}_{j,k}|\mathbf{x}_k^\mu) \propto p(\mathbf{z}_{j,k}|\mathbf{s}_{j,k} = \tilde{\mathbf{s}}_{j,k}^\mu)$. Hence the elimination procedure introduced in section II-A.3 leads to a simple marginalized estimator.

D. Model Matching

It is important to point out that a sequential estimator is only as good as its state transition model matches the real world situation. The state model needs to capture *all* relevant hidden states with memory and needs to correctly model their dependencies, while adhering to the first order Markov condition. Furthermore, any memory of the measurement noise affecting the likelihood function $p(\mathbf{z}_k|\mathbf{x}_k)$ must be explicitly contained as additional states of the model $\mathbf{x}$, so that the measurement noise is i.i.d.

The multipath channel state model according to Section III-C is motivated by channel modeling work for multipath prone environments such as the urban satellite navigation channel [14] [18]. In fact the process of constructing a channel model in order to characterize the channel for signal level simulations and receiver evaluation comes close to our task of building a first order Markov process for sequential estimation. For particle filtering, the model needs to satisfy the condition that one can draw states with relatively low computational complexity. Adapting the model structure and the model parameters to the real channel environment is a task for current and future work.

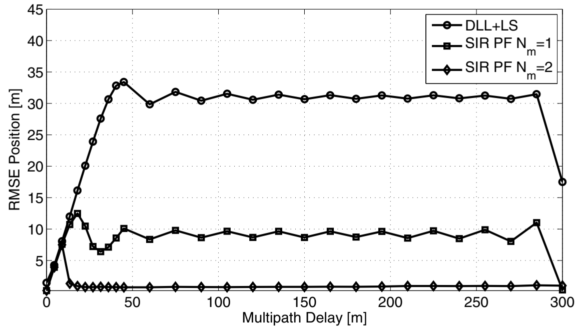
V. Performance Evaluation

To demonstrate the capabilities of the proposed estimator simulations were carried out. The employed navigation signal is a BPSK modulated GPS C/A code signal having a two-sided bandwidth of 20 MHz. In the simulations it is assumed that four satellites are received with a C/N_0 of 50 dB-Hz respectively. The geometry of the four transmitting satellites is 58, 65, 135 and 195 degrees for the azimuth values and 67, 27, 51 and 39 degrees for the elevation values. The SIR PF runs with an observation period of 10 ms and signal compression is applied with $N_{cc} = 25$ (code-matched correlators), $N_{pc} = 25$. The user, clock and channel parameters σ_x^2, σ_y^2, σ_z^2, $\sigma_{\dot{y}}^2$, $\sigma_{\dot{x}}^2$, $\sigma_{\dot{z}}^2$, σ_τ^2, $\sigma_{\dot{\tau}}^2$, σ_{mp}^2, $\sigma_{\dot{\mathrm{mp}}}^2$, and $p(e_{i,j,k}|e_{i,j,k-1})$ are selected to approximate the statistics of a measured channel according to [14]. The SIR PF uses the minimum mean square error (MMSE) criterion to estimate the parameters $\mathbf{x}_k$ from the posterior. As reference the SIR PF results are shown together with results obtained based upon conventional signal tracking and least squares (LS) position estimation [1] with a non-coherent delay lock loop with 0.15 chip early/late correlator spacing and 2 Hz tracking loop bandwidth.

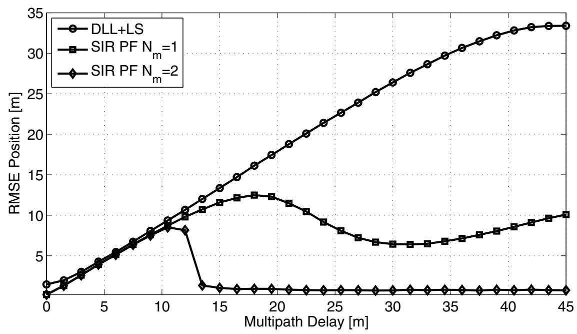
A. Static Multipath Channel

In Figure 4(a) and 4(b) the performance of the SIR PF is shown by means of the root mean square error (RMSE) of the minimum mean square error (MMSE) position estimates obtained from the posterior as a function of the multipath delay for a static multipath on the signal associated to the satellite channel $j = 1$ only. The other channels do not suffer from multipath in this simulation. It can be observed that the proposed algorithm performs significantly better than the conventional DLL-based LS positioning even without the estimator modeling multipath ($N_m = 1$). It can be also observed from the simulation results that further improvement is possible, if the multipath is taken into account by the SIR PF ($N_m = 2$).

From the posterior it is possible to calculate the estimated average probability $p(N_{m,j,k} = 2|\mathbf{Z}_k)$ of a two path model, which is shown in Figure 5 and indicates the transition



(a)



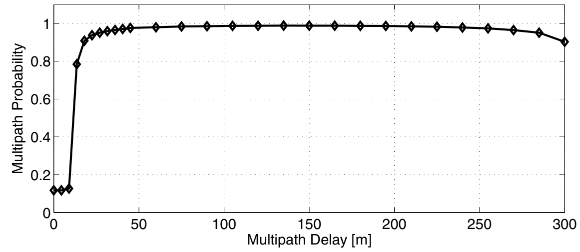
(b)

Fig. 4. Static multipath scenario on range 1: Performance of DLL+LS, SIR PF with single path model and SIR PF with path activity tracking as function of relative multipath delay. 4(b) is a detailed view of 4(a) for 0-45 meters.

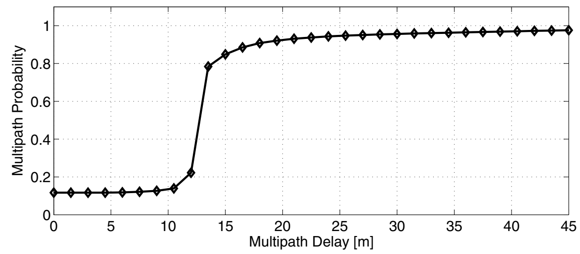
between the models: for small delays the two paths essentially merge to a single one. Note that in these simulations the model parameters of the sequential estimator are still the ones designed for the dynamic channel and not optimal for this static scenario.

B. Dynamic Multipath Channel

Furthermore we have carried out simulations under a dynamic multipath scenario. Results for a randomly chosen dynamic channel are depicted in Figure 6 and Figure 7 for two kinds of SIR PFs, one using $N_m = 1$, corresponding to a conventional receiver using a sequential positioning algorithm, and the other using $N_m = 2$, both running with 20 000 particles, respectively. The SIR PF results show the magnitude of the error of the MMSE position estimate. Figure 8 shows the multipath channels affecting the four received satellite signals including the MMSE estimates of the path delays. To consider two different types of echoes the amplitude of the echoes in the simulation is either picked randomly from 0.1 up to 0.2 times the amplitude of the direct path (weak echo) or picked randomly from 0.6 up to 0.8 times the amplitude of the direct path (strong echo). The performance of the DLL+LS approach suffers significantly from the multipath reception (RMSE = 17.97 m) and the SIR PF using $N_m = 1$ (RMSE = 4.31 m) is able to outperform it, as it exploits the properties of the position domain likelihood as well as the position and clock parameter movement models. Further improvement is achieved with the SIR PF with $N_m = 2$ (RMSE = 1.42 m).



(a)



(b)

Fig. 5. Static multipath scenario: Average probability of a two path model for the estimator with path activity tracking. 5(b) is a detailed view of 5(a) for 0-45 meters.

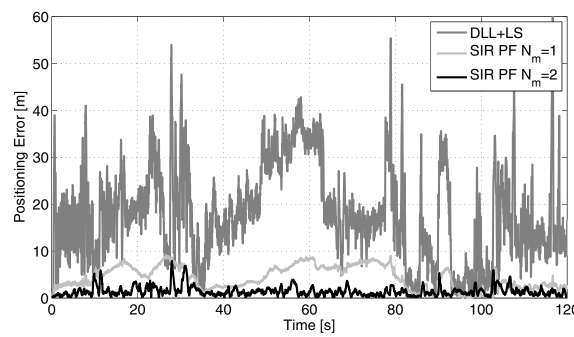


Fig. 6. Performance of DLL+LS based positioning, SIR PF with single path model and SIR PF with path activity tracking. Simulation with multipath environment according to Figure 8

Despite up to three echoes being active simultaneously and the estimators restriction to two paths it can be observed that the SIR PF tracks predominantly the strong multipath signals as illustrated in Figure 8.

VI. CONCLUSIONS

We have demonstrated how sequential Bayesian estimation techniques can be applied to the combined positioning and multipath mitigation problem in a navigation receiver. The proposed approach is characterized by complexity reduction techniques for efficient likelihood computation in combination with a particle filter realization of the prediction and update recursion. The considered movement model has been adapted to dynamic user and multipath channel scenarios and incorporates the number of echoes as a time variant hidden channel state variable that is tracked together with the position and clock parameters in a probabilistic fashion. A promising advantage compared to existing ML estimation approaches is

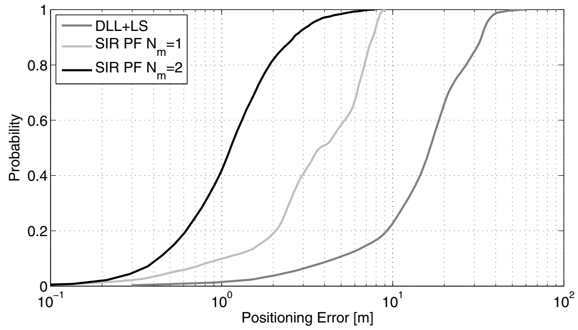
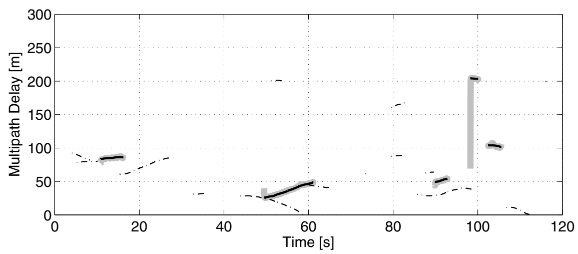
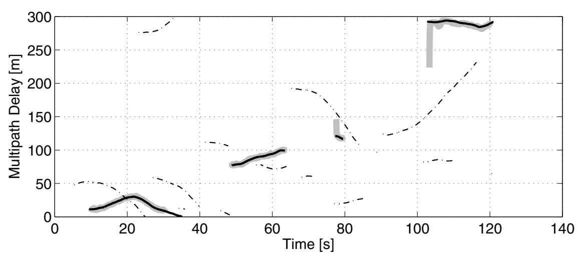


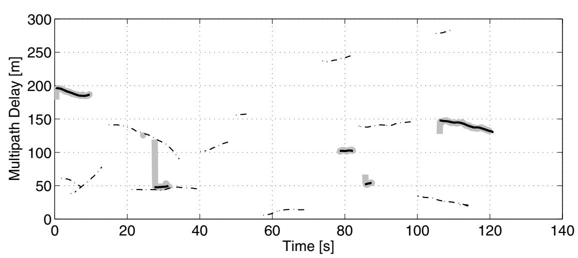
Fig. 7. Normalized cumulative histogram of the simulation errors shown in Figure 6



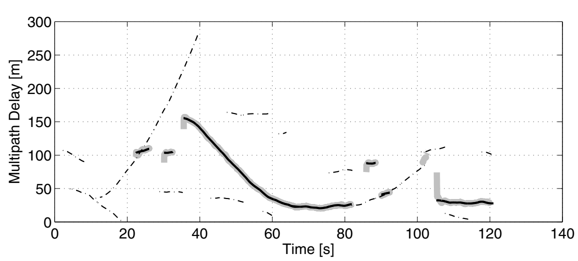
(a) Channel satellite 1



(b) Channel satellite 2



(c) Channel satellite 3



(d) Channel satellite 4

Fig. 8. Multipath channel with weak (dash-dotted line) and strong (bold line) echoes. Estimated echo tracks (grey) shown if $p(N_{m,j,k} > 1|\mathbf{Z}_k) > 0.8$.

that the posterior PDF at the output of the estimator represents reliability information about the desired parameters and preserves the ambiguities and multiple modes that may occur within the likelihood function. Simulation results for a GPS-like positioning scenario show that the proposed sequential estimator can achieve significant improvements compared to the conventional tracking and positioning approach, that uses a DLL with narrow correlator spacing and LS position estimation. A detailed comparison with respect to performance and complexity of the proposed algorithm against other approaches (see Section I) will be a subject of future work.

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