



# LUND UNIVERSITY

## Testing for Error Correction in Panel Data

Westerlund, Joakim

2005

[Link to publication](#)

*Citation for published version (APA):*

Westerlund, J. (2005). *Testing for Error Correction in Panel Data*. (Working Papers. Department of Economics, Lund University; No. 11). Department of Economics, Lund University.  
[http://swopec.hhs.se/lunewp/abs/lunewp2005\\_011.htm](http://swopec.hhs.se/lunewp/abs/lunewp2005_011.htm)

*Total number of authors:*

1

### General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00

# Testing for Error Correction in Panel Data\*

Joakim Westerlund<sup>†</sup>

January 26, 2005

## Abstract

This paper proposes four new tests for the null hypothesis of no cointegration in panel data that are based on the error correction parameter in a conditional error correction model. The limit distribution of the test statistics are derived and critical values are provided. Our Monte Carlo results suggest that the tests have reasonable size properties and good power relative to other popular residual-based cointegration tests. These differences arises because latter imposes a possibly invalid common factor restriction. In our empirical application, we present evidence suggesting that international health care expenditures and GDP are cointegrated once the possibility of an invalid common factor restriction has been accounted for.

**JEL Classification:** C12; C32; C33; O30.

**Keywords:** Panel Cointegration Test; Monte Carlo Simulation; Common Factor Restriction; International Health Care Expenditures.

## 1 Introduction

The use of panel cointegration techniques to test for the presence of long-run relationships among integrated variables with both a time series dimension  $t = 1, \dots, T$  and a cross-sectional dimension  $i = 1, \dots, N$  has received much attention recently. The literature concerned with the development of such tests has thus far taken two broad directions. The first consists of taking as the null hypothesis that of cointegration. This is the basis of the panel cointegration tests proposed by McCoskey and Kao (1998), and Westerlund (2004). The second approach is to take as null hypothesis that of no cointegration. Tests within this category are almost exclusively based on the methodology of Engle

---

\*The author would like to thank Michael Bergman, David Edgeton and other seminar participants at Lund University for helpful comments and suggestions. A GAUSS program that implements the test proposed in this paper is available from the author upon request.

<sup>†</sup>Department of Economics, Lund University, P. O. Box 7082, S-220 07 Lund, Sweden. Telephone: +46 46 222 4970; Fax: +46 46 222 4118; E-mail address: joakim.westerlund@nek.lu.se.

and Granger (1987) whereby a unit root statistic is employed to test for the existence of a unit root in the residuals of a static spurious regression. The most influential contributions within this category are those of Kao (1999) and Pedroni (1999; 2004). Pedroni (2004) introduces several test statistics that are appropriate for various cases of heterogeneous dynamics, endogenous regressors, and individual specific constants and trends. Tests are developed both for the case with a common autoregressive root under the alternative hypothesis as well as tests that permit heterogeneity of the autoregressive roots. In Pedroni (1999), critical values are also provided that facilitates the cointegration testing to be performed in situations characterized by multiple regressors. The study of Kao (1999) is similar but brings special attention to the case in which the autoregressive roots and the cointegration vectors are presumed homogenous.

The tests of Kao (1999) and Pedroni (1999; 2004) have attracted much interest in the empirical literature. Typical examples of common applications include studies of wage inequality, manufacturing growth and financial development, commercial security prices, the Balassa-Samuelson effect, current account dynamics, the Feldstein-Horioka puzzle, and international R&D spillovers, to mention a few.<sup>1</sup> The single most cited rationale for using these tests are the increased power that may be brought to bare on the cointegration hypothesis through the increased number of observations that derives from adding the individual time series. Yet, many studies fail to reject the null hypothesis, even in cases when cointegration is strongly suggested by theory (see, e.g. Ho, 2001). One plausible explanation for this derives from the common factor restriction that is implicitly superimposed when using residual-based tests of this sort. The common factor restriction being that the long- and short-run adjustment processes are equal. If this restriction is invalid, although still consistent, these tests may suffer from poor power properties in finite samples.

In this paper, we propose four new tests of the null hypothesis of no cointegration that does not impose any common factor restriction on the data and that uses the available information more efficiently than residual-based tests. The proposed tests are panel extensions of those proposed in the time series context by Banerjee *et al.* (1998). As such, they are designed to test the null hypothesis of no cointegration by inferring whether the error correction term in an conditional error correction model (ECM) is equal to zero. If the null hypothesis of no error correction is rejected, then the null hypothesis of no cointegration is also rejected. Each test is able to accommodate individual specific short-run dynamics, including serially correlated error terms and weakly exogenous regressors, individual specific intercept and trend terms, as well as individual specific slope parameters. It is shown that the tests have limiting normal distributions and that they are consistent. In our Monte Carlo study, we demonstrate that the ECM tests maintain nominal size reasonably well and that they are more powerful than other existing residual-based tests that ignores

---

<sup>1</sup>See Pedroni (1999) for references.

potentially valuable information by imposing a possibly invalid common factor restriction. In our empirical application, we provide evidence suggesting that international health care expenditures and GDP are cointegrated once the long- and short-run adjustment processes are allowed to differ.

The paper proceeds as follows. In the next section, we present the ECM test statistics. Section 3 concern itself with the asymptotic results, while Section 4 is devoted to the Monte Carlo study. Section 5 then contains the empirical application and Section 6 concludes the paper. For notational convenience, the Brownian motion  $B_i(r)$  defined on the unit interval  $r \in [0, 1]$  will be written as only  $B_i$  and integrals such as  $\int_0^1 W_i(r)dr$  will be written  $\int_0^1 W_i$ , and  $\int_0^1 W_i(r)dW(r)_i$  as  $\int_0^1 W_i dW_i$ . The symbol  $\Rightarrow$  will be used to signify respectively weak convergence,  $\xrightarrow{p}$  to signify convergence in probability and  $[z]$  to signify the largest integer less than  $z$ .

## 2 The ECM tests

Let  $z_{it} = (y_{it}, x'_{it})'$  be an  $K + 1$  dimensional vector of integrated variables that may be partitioned into a scalar variate  $y_{it}$  and a  $K$  dimensional vector  $x_{it}$ . The data generating process (DGP) of  $z_{it}$  can be described by the following conditional ECM system

$$\Delta y_{it} = \delta'_i d_t + \lambda'_i \Delta x_{it} + \gamma_i \beta'_i z_{it-1} + u_{it}, \quad (1)$$

$$\Delta x_{it} = v_{it}, \quad (2)$$

where the linear combination  $\beta'_i z_{it}$  is assumed to be stationary,  $\beta_i$  is the cointegration vector and  $\gamma_i$  contains the associated error correction parameters. The vector  $d_t$  contains the deterministic components. Typical elements of  $d_t$  include a constant and a linear time trend. To accommodate for this, we distinguish between three cases. In Case 1,  $d_t = \{\emptyset\}$ , in Case 2,  $d_t = 1$  and in Case 3,  $d_t = (1, t)'$ . To be able to derive the tests and their asymptotic distributions, we assume that the vector  $w_{it} = (u_{it}, v'_{it})'$  is cross-sectionally independent and that it follows a general linear process whose parameters satisfy the summability conditions of the following assumption.

**Assumption 1.** (Error process.) (i) The data is i.i.d. cross-sectionally; (ii) The vector  $w_{it}$  satisfies  $w_{it} = C_i(L)e_{it}$ , where  $L$  is the lag operator,  $C_i(L) = \sum_{j=0}^{\infty} C_{ij}L^j$ ,  $C_i(1) \neq 0$ ,  $\sum_{j=0}^{\infty} j^2 C_{ij}C_{ij} < \infty$  and  $e_{it}$  is a mean zero i.i.d. sequence with covariance matrix  $\Sigma_i$ ; (iii) The lower right  $K \times K$  submatrix of  $\Omega_i \equiv C_i(1)\Sigma_i C_i(1)$  is positive definite; (iv) The regressors are weakly exogenous with respect to  $(\delta'_i, \lambda'_i, \gamma_i, \beta'_i)'$ .

Assumption 1 provides us with the basic conditions for developing the panel cointegration tests. Assumption 1 (i) states that the individuals are i.i.d. over the cross-sectional dimension. This condition is convenient as it will allow us

to apply standard central limit theory in a relatively simple manner. Similarly, the linear process conditions of Assumption (ii) are convenient because they facilitate a straightforward asymptotic analysis by application of the methods developed by Phillips and Solo (1992). In particular, Assumption 1 (ii) ensures that a functional central limit theorem holds individually for each cross-section as  $T$  increases. Thus, we have  $T^{-1/2} \sum_{t=1}^{[Tr]} w_{it} \Rightarrow B_i \equiv LW_i$  as  $T \rightarrow \infty$  with  $N$  held fixed, where  $\Omega_i = L'L$ ,  $B_i = (B_{i1}, B'_{i2})'$  is a vector Brownian motion and  $W_i = (W_{i1}, W'_{i2})'$  is a vector standard Brownian motion with covariance matrix equal to identity. The asymptotic analysis of linear processes holds under a variety of conditions and can be generalized to different classes of time series innovations such as the class of all stationary autoregressive moving average processes. The asymptotic analysis is therefore widely applicable.

Assumption 1 (iii) and (iv) are concerned with the covariance matrix of  $B_i$ , equally the long-run covariance matrix of  $w_{it}$ . Specifically, Assumption 1 (iii) states that the lower right  $K \times K$  submatrix of  $\Omega_i$  is positive definite, which is tantamount to requiring that  $x_{it}$  is not cointegrated in case we have multiple regressors. Assumption 1 (iv) requires that the vector of regressors is weakly exogenous with respect to the parameters of interest, which is implicit in the formulation of the DGP given by (1) and (2) as the marginal model for  $x_{it}$  is not error correcting. The implication of this is that  $u_{it}$  is independent of all current and past realizations of  $v_{it}$  suggesting that  $E(u_{it}v_{ij}) = 0$  for all  $j < t$ . Apart from these assumptions, however, no further restrictions are placed on the long-run covariance of  $w_{it}$ . Notably, the fact that  $\Omega_i$  is permitted to vary between the individuals of the panel indicate that we are in effect allowing for a completely heterogeneous long-run covariance structure.

Assumption 1 is relatively weak and allow for quite general forms of error dynamics. In order to facilitate the construction of tests with simple enough structure, however, in this section we shall initially make some simplifying assumptions, which will subsequently be disregarded. Specifically, we strengthen Assumption 1 to the following set of conditions.

**Assumption 2.** (Error independence.) (i) The processes  $u_{it}$  and  $v_{kj}$  are mean zero and mutually independent for all  $i, k$  and  $t \neq j$ ; (ii) The covariance matrix of  $v_{it}$  is positive definite.

The parameter  $\gamma_i$  governs the error correction of the ECM. If  $\gamma_i < 0$ , then there is error correction, which imply that  $y_{it}$  and  $x_{it}$  will be cointegrated. Conversely, if  $\gamma_i = 0$ , then the error correction will be absent and there is no cointegration. In what follows, we shall propose four new test statistics that are based on the value taken by  $\gamma_i$ . Two of the statistics are based on pooling the information regarding the error correcting property of the data along the cross-sectional dimension of the panel. These are referred to as panel statistics. The second pair do not exploit this information and are referred to as group mean statistics. The relevance of this distinction lies in the formulation of

the alternative hypothesis. For the panel statistics, the null and alternative hypotheses are formulated as  $H_0 : \gamma_i = 1$  for all  $i$  versus  $H_1 : \gamma_i = \gamma < 1$  for all  $i$ . With this formulation, a rejection of the null should be taken as evidence in favor of cointegration for the panel as a whole. By contrast, for the group mean statistics,  $H_0$  is tested versus  $H_1 : \gamma_i < 1$  for at least some  $i$  suggesting that a rejection of the null should be taken as evidence in favor of cointegration for a nonzero fraction of the panel.

It is clear that the ECM test statistics of no cointegration must rely upon some estimate of  $\gamma_i$ . Although seemingly simple an exercise, the estimation of  $\gamma_i$  has proven extremely difficult. In fact, most time series test statistics based on the error correction parameter are not similar and depend on nuisance parameters (see, e.g. Banerjee *et al.*, 1986; Kremers *et al.*, 1992; Campos *et al.*, 1996), which is the main reason why ECM tests has not received much attention in the applied literature. Another problem is that most ECM tests require that the cointegration vector is known. Banerjee *et al.* (1998) suggest a straightforward solution to both these problems that is based on the following ordinary least squares (OLS) regression

$$\Delta y_{it} = \delta'_i d_t + \lambda'_i \Delta x_{it} + \gamma_i y_{it-1} + \varphi_i x_{it-1} + u_{it}. \quad (3)$$

Suppose that the cointegration vector  $\beta_i$  is normalized with respect to  $y_{it}$  so that  $\beta_i = (1, -\alpha'_i)'$ . In this case, the parameter on  $y_{it-1}$  in (3) is identically  $\gamma_i$ . Thus, a panel data test of  $H_0$  versus  $H_1$  may be constructed based on the OLS estimator of  $\gamma_i$  in (3) for each individual or for the panel as a whole. Notably, because the parameter on  $x_{it-1}$  is unrestricted and because the cointegration vector is implicitly estimated under the alternative hypothesis, as seen by writing  $\varphi_i = -\gamma_i \alpha_i$ , this means that it is possible to construct a test based on  $\gamma_i$  that is asymptotically similar and whose distribution is free of nuisance parameters. Following this, in this paper, we propose four new panel data tests of  $H_0$  versus  $H_1$  that are based on the value taken by  $\gamma_i$  in (3). The exact form of the test statistics is given as follows.

**Definition 1.** (The panel and group mean ECM test statistics.) Let  $e_{it} = y_{it} - \hat{\delta}'_i d_t - \hat{\lambda}'_i \Delta x_{it} - \hat{\varphi}_i x_{it-1}$ ,  $E_{it} = (e_{it-1}, \Delta e_{it})'$ ,  $E_i = \sum_{t=1}^T E_{it} E'_{it}$ ,  $\hat{\sigma}_i^2 = T^{-1} \sum_{t=1}^T \hat{u}_{it}^2$  and  $\hat{\sigma}^2 = N^{-1} \sum_{i=1}^N \hat{\sigma}_i^2$ . The panel and group mean ECM test statistics are defined as follows

$$\begin{aligned} EP_\gamma &\equiv \left( \sum_{i=1}^N E_{i11} \right)^{-1} \sum_{i=1}^N E_{i12}, & EP_t &\equiv \hat{\sigma}^{-1} \left( \sum_{i=1}^N E_{i11} \right)^{-1/2} \sum_{i=1}^N E_{i12}, \\ EG_\gamma &\equiv \sum_{i=1}^N E_{i11}^{-1} E_{i12} & \text{and} & EG_t &\equiv \sum_{i=1}^N \hat{\sigma}_i^{-1} E_{i11}^{-1/2} E_{i12}. \end{aligned}$$

Some remarks are in order. First, as will be shown in the next section, the limiting distributions of the above statistics are free of the nuisance parameters

associated with the underlying DGP. Once we allow for the possibility of nonzero constants and time trends in (1), however, the distributions of the statistics will no longer be invariant with respect to these nuisance parameters. Therefore, in order to obtain statistics that are asymptotically similar in Case 2, the data should be demeaned prior to using the above formulas. For Case 3, the data should be both demeaned and detrended to account for the linear trend appearing in (1). Thus, as in the case of a single time series, if a deterministic element is present but not accounted for when constructing the test statistics, the ensuing cointegration test will be inconsistent. Therefore, in order to obtain tests that are asymptotically similar, we use project  $y_{it}$  upon  $d_t$  when constructing the statistics.

Second, notice that the regression in (3) cannot be used to identify the underlying deterministic structure of the variables. In Case 1, (3) contains no deterministic component and there is no ambiguity. In this case, both  $x_{it}$  and  $y_{it}$  are pure unit root processes with no deterministic components. In Case 2, (3) is fitted with an individual specific constant as the deterministic component. This case captures both the situation when the variables are generated with a constant term as well as the situation when they are generated with both constant and trend terms but the trend is eliminated through the cointegration relationship. Similarly, Case 3 captures the situations when the variables are generated with constant and trend terms as well as when they are generated with a quadratic trend that is eliminated through the cointegration relation.

Third, the relaxation of Assumption 2 means that the error process may be serially correlated and the regressors weakly exogenous. This implies the ECM statistics are no longer asymptotically similar and that they need to be modified to account for the temporal dependence in the DGP. Under Assumption 1, this may be accomplished by simply augmenting the right-hand side of (1) with lagged values of  $\Delta y_{it}$  as well as lagged and leaded values of  $\Delta x_{it}$ . In so doing, it is necessary that the lag and lead order  $p$ , say, is chosen sufficiently large to whiten the errors and to make the regressors strictly exogenous. This suggests that in order to obtain similar test statistics, we should replace  $e_{it}$  in Definition 1 with the projection errors of  $y_{it}$  from  $p$  lags of  $\Delta y_{it}$  as well as  $p$  lags and leads of  $\Delta x_{it}$ . That is,  $e_{it} = y_{it} - \hat{\delta}'_i d_t - \hat{\lambda}'_i \Delta x_{it} - \hat{\varphi}_i x_{it-1}$  should be replaced with  $e_{it} = y_{it} - \hat{\delta}'_i d_t - \sum_{k=1}^p \hat{\theta}_{ik} \Delta y_{it-k} - \sum_{k=-p}^p \hat{\lambda}'_{ik} \Delta x_{it-k} - \hat{\varphi}_i x_{it-1}$ . Lags of  $\Delta y_{it}$  and  $\Delta x_{it}$  are required to accommodate for serial correlation while leads of  $\Delta x_{it}$  are needed to account for the effects of weakly exogenous regressors.

### 3 Asymptotic distribution

In this section, we study the asymptotic distribution of the ECM test statistics proposed in the previous section. In particular, it will be shown that all statistics converge to limiting normal distributions with moments based on the following

vector Brownian motion functionals

$$V_i \equiv \left( \int_0^1 Q_i^2, \int_0^1 Q_i dW_{i1} \right)' \quad \text{and} \quad K_i \equiv \left( V_{i2} V_{i1}^{-1}, V_{i2} V_{i1}^{-1/2} \right)',$$

where

$$Q_i = W_{i1} - \left( \int_0^1 W_{i1} \bar{W}'_{i2} \right) \left( \int_0^1 \bar{W}_{i2} \bar{W}'_{i2} \right)^{-1} \bar{W}_{i2}.$$

To succinctly express the limiting distributions of the ECM statistics when deterministic terms are added to the regression in (3), it is useful to let  $\bar{W}_{i2} = (d', W'_{i2})'$ , where  $d$  is the limiting trend function. Specifically, let  $D_T = \text{diag}(1, T)$  denote a matrix of normalizing orders that is conformable with  $d_t = (1, t)'$ , then  $D_T^{-1} d_{[Tr]} \Rightarrow d = (1, r)'$  as  $T \rightarrow \infty$ . It follows that  $\bar{W}_{i2} = W_{i2}$  in Case 1,  $\bar{W}_{i2} = (1, W'_{i2})'$  in Case 2 and  $\bar{W}_{i2} = (1, r, W'_{i2})'$  in Case 3. The vector  $\bar{W}_{i2}$  enters  $V_i$  and  $K_i$  through the Brownian motion functional  $Q_i$ , which is the Hilbert projection of  $W_{i1}$  onto the space orthogonal to the vector  $\bar{W}_{i2}$ . Also, it is convenient to let  $\Theta$  and  $\tilde{\Theta}$  denote the expected values of  $V_i$  and  $K_i$ , respectively. The variances of these functionals are written in an obvious notation as  $\Sigma$  and  $\tilde{\Sigma}$ . As indicated by the following theorem, when the ECM statistics are normalized by the appropriate values of  $T$  and  $N$ , then the asymptotic distributions only depend on the known values of  $\Theta$ ,  $\Sigma$ ,  $\tilde{\Theta}$  and  $\tilde{\Sigma}$ .

**Theorem 1.** (*Asymptotic distribution.*) Define  $\phi \equiv (-\Theta_2 \Theta_1^{-2}, \Theta_1^{-1})'$  and  $\varphi \equiv (-2^{-1} \Theta_2 \Theta_1^{-3/2}, \Theta_1^{-1/2})'$ . Under Assumption 1 and the null hypothesis of no cointegration, as  $T \rightarrow \infty$  prior to  $N$

$$\begin{aligned} TN^{1/2} EP_\gamma - N^{1/2} \Theta_2 \Theta_1^{-1} &\Rightarrow N(0, \phi' \Sigma \phi), \\ EP_t - N^{1/2} \Theta_2 \Theta_1^{-1/2} &\Rightarrow N(0, \varphi' \Sigma \varphi), \\ TN^{-1/2} EG_\gamma - N^{1/2} \tilde{\Theta}_1 &\Rightarrow N(0, \tilde{\Sigma}_{11}), \\ N^{-1/2} EG_t - N^{1/2} \tilde{\Theta}_2 &\Rightarrow N(0, \tilde{\Sigma}_{22}). \end{aligned}$$

The proof of Theorem 1 is outlined in the appendix but it is instructive to consider why it holds. The proof of the results for the group mean statistics is particularly simple and proceeds by showing that the intermediate limiting distribution of the normalized statistics passing  $T \rightarrow \infty$  while holding  $N$  fixed can be written entirely in terms of the elements of the vector Brownian motion functional  $K_i$ . Therefore, by subsequently passing  $N \rightarrow \infty$ , asymptotic normality follows by direct application of the Lindberg-Lévy central limit theorem to sums of  $N$  i.i.d. random variables. The proof for the panel statistics is similar. It proceeds by showing that the intermediate limiting distribution of the normalized statistics can be described in terms of differentiable functions of i.i.d. vector sequences to which the Delta method is applicable. Hence, taking the limit as  $N \rightarrow \infty$ , we obtain a limiting normal distribution for the panel test statistics.



Theorem 1 indicates that each of the normalized statistics, when standardized by the appropriate moments, converges to a standard normal distribution. Thus, to be able to make inference based on the normal distribution, we must first obtain the moments for each statistic. This can be done by Monte Carlo simulations. For this purpose, we make 10,000 draws of  $K$  independent scaled random walks of length  $T = 1,000$ . By using these random walks as simulated Brownian motions, we construct approximations of the vector Brownian motion functionals  $V_i$  and  $K_i$ . The means and the variances of these simulated functionals are then used to approximate the asymptotic moments. The results obtained from this exercise are reported for up to five regressors in Table 1.

In view of Table 1, note that, although the distributions of the statistics are free of nuisance parameters, they do depend upon the deterministic specification of the ECM in (1) and on the number of regressors as reflected by dependence of  $Q_i$  on  $\bar{W}_{i2}$ . Thus, the moments will also depend on the deterministic specification and on the number of regressors. Moreover, notice that the distributions are independent of the short-run dynamics of the DGP as captured by the first differences of the regressors. Thus, the statistics are asymptotically similar with respect to the short-run parameters of the ECM. In Table 1, therefore, we only report simulated moments for the different deterministic cases and for different number of regressors. There no need to tabulate separate moments for different lag and lead orders.

It is important that a statistical test is able to fully discriminate between the null and alternative hypotheses in large samples. The next theorem shows that the test statistics are consistent and that they are divergent under the alternative hypothesis.

**Theorem 2.** (*Test consistency.*) *Under Assumption 1 and the alternative hypothesis of no cointegration, then  $TN^{1/2}EP_\gamma$ ,  $EP_t$ ,  $TN^{-1/2}EG_\gamma$  and  $N^{-1/2}EG_t$  diverges to negative infinity as  $T \rightarrow \infty$  prior to  $N$ .*

The proof of Theorem 2 is provided in the appendix. Some remarks are in order though. First, the theorem establishes that the divergence occurs towards negative infinity. This suggests that the tests can be constructed as one-sided using only the left tail of the normal distribution to reject the null hypothesis. Therefore, to test the null hypothesis of no cointegration based on the moments from Table 1, one simply computes the value of the standardized test statistic so that it is in the form specified in Theorem 1. This value is then compared with the left tail of the normal distribution. Large negative values imply that the null hypothesis should be rejected.

Second, the proof of Theorem 2 uses the sequential limit theory developed by Phillips and Moon (1999). Although this allows for a relatively straightforward and tractable analysis, it cannot be used to obtain the joint rate of divergence, which is indicative of the relative power properties of the tests. It is, however, possible to establish the order of the statistics as  $T \rightarrow \infty$  for a fixed  $N$ . In this

case, it is shown in the appendix that  $TN^{1/2}EP_\gamma$  and  $TN^{-1/2}EG_\gamma$  are  $O_p(T)$  while  $EP_t$  and  $N^{-1/2}EG_t$  are  $O_p(T^{1/2})$ , which is in agreement with the results obtained for residual-based tests in the time series literature (see, e.g. Phillips and Ouliaris, 1990). Given their faster rate of divergence, it is likely that the  $EP_\gamma$  and  $EG_\gamma$  statistics have higher power than  $EP_t$  and  $EG_t$  in samples where  $T$  is substantially larger than  $N$ .

## 4 Monte Carlo simulations

In this section, we study some of the small-sample properties of the ECM tests relative to those of some of the popular residual-based tests recently proposed by Pedroni (2004). For this purpose, a large number of experiments were performed using the following process to generate the data

$$\Delta y_{it} = \lambda_i \Delta x_{it-1} + \gamma_i (y_{it-1} - \alpha_i x_{it-1}) + u_{it}, \quad (4)$$

$$x_{it} = \delta y_{it} + v_{it}, \quad (5)$$

$$v_{it} = v_{it-1} + w_{it}. \quad (6)$$

For the error process  $u_{it}$ , we have two scenarios. In the first,  $u_{it} = e_{it} + \theta e_{it-1}$  so  $u_{it}$  follows an MA(1) process. In the second,  $u_{it} = \phi u_{it-1} + e_{it}$  in which case  $u_{it}$  follows an AR(1) process. For the initiation of  $x_{it}$ ,  $y_{it}$ ,  $v_{it}$ ,  $u_{it}$  and  $e_{it}$ , we use the value zero. Moreover,  $\lambda_i \sim N(0, 1)$  and  $(e_{it}, w_{it})' \sim N(0, V)$ , where  $V$  is a positive definite matrix with  $V_{11} = 1$  and  $V_{12} = V_{21}$ . Data is generated for  $N \in \{10, 20\}$  individual and  $T \in \{50, 100\} + 50$  time series observations. To eliminate startup effects, we discard the first 50 observations for each series. The number of replications is 1,000.

In our basic DGP, we consider the parameter space  $(\delta \times \theta \times \phi \times V_{12} \times V_{22})$ , where  $\delta = (0, 1)$ ,  $\theta = (0, -0.2, -0.6)$ ,  $\phi = (0, 0.2, 0.6)$ ,  $V_{12} = 0.4$  and  $V_{22} = (1, 2, 4)$ . This gives us a total of 54 experiments for each combination of  $N$  and  $T$ . The parametrization is given as follows. The error correction of the ECM is governed by  $\gamma_i$ . Its value determines the extension to which the null hypothesis of no cointegration can be regarded as true. For brevity, we make the assumption that this parameter takes on a common value  $\gamma_i = \gamma$  for all  $i$ . Therefore, under the null hypothesis, we have  $\gamma = 1$ , while  $\gamma < 1$  under the alternative hypothesis. The remaining parameters  $\theta$ ,  $\phi$ ,  $\psi$ ,  $\delta$  and  $V$  introduces nuisance in the DGP. Specifically,  $\theta \neq 0$  imply that  $u_{it}$  will have an MA(1) component while  $\phi \neq 0$  imply that  $u_{it}$  will have an AR(1) component. The degree of endogeneity in the DGP is governed by  $\delta$  and  $V_{12}$ . The regressor is strictly exogenous if  $\delta = V_{12} = 0$  and it is weakly exogenous if  $\delta = 0$  and  $V_{12} \neq 0$ . If  $\delta \neq 0$ , then the regressor is fully endogenous. The choice of  $V_{12}$  did not affect the results and we therefore use  $V_{12} = 0.4$  throughout.

The parameters  $\lambda_i$ ,  $\alpha_i$  and  $V$  are especially interesting and their role in determining the relative power properties of the tests and will be examined

thoroughly. The reason for this is the following. Consider the conditional ECM in (4). The test regression for the residual-based tests of Pedroni (2004) can be derived from (4), thus establishing a relationship between them and the ECM tests. Specifically, let  $e_{it} = (\lambda_i - \alpha_i)\Delta x_{it} + u_{it}$  and subtract  $\alpha_i\Delta x_{it}$  from both sides of (4) and rearrange. This gives us the following expression

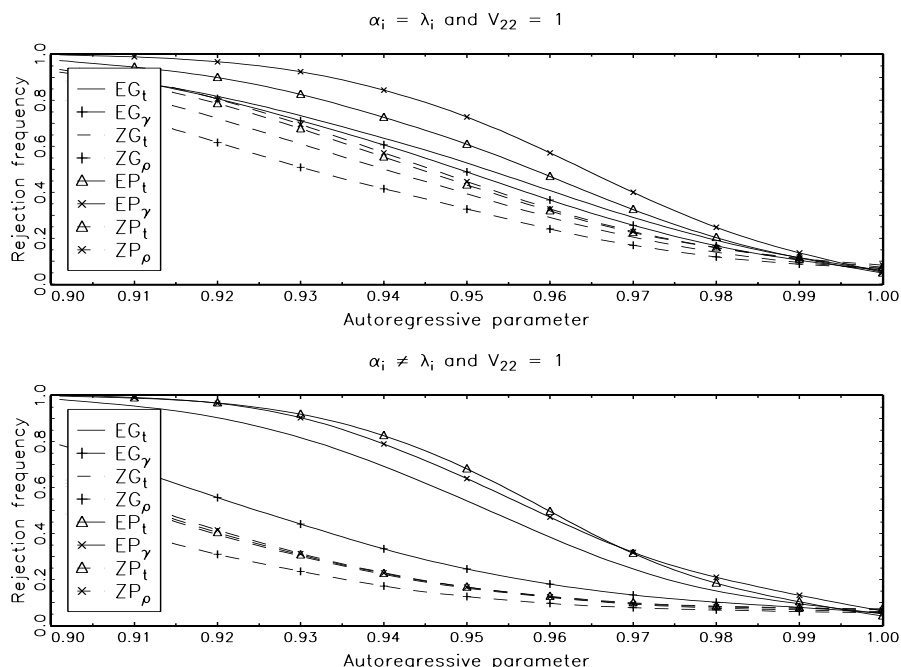
$$\Delta(y_{it} - \alpha_i x_{it}) = \gamma_i(y_{it-1} - \alpha_i x_{it-1}) + e_{it}. \quad (7)$$

The tests of Pedroni (2004) test the null hypothesis of no cointegration by inferring whether  $y_{it} - \alpha_i x_{it}$  has a unit root or, equivalently, whether  $\gamma_i$  in (7) is equal to zero. The problem with this approach is that it imposes a possibly invalid common factor restriction as seen by nothing that the two errors  $e_{it}$  and  $u_{it}$  are not equal unless  $\lambda_i = \alpha_i$ . To get an intuition on this, notice that the variance of  $e_{it}$  is given by  $V_{11} + (\lambda_i - \alpha_i)^2 V_{22}$ . Suppose that  $V_{11}$  is close to zero but that  $(\lambda_i - \alpha_i)^2 V_{22}$  is large. In this case, the ECM regression in (4) has nearly perfect fit with  $\gamma_i$  being estimated with excellent precision. The ECM test will therefore tend to have good power. By contrast, the estimation of  $\gamma_i$  in (7) will tend to be much more imprecise producing tests with low power. Thus, we expect the ECM tests to enjoy higher power whenever  $\alpha_i \neq \lambda_i$  and the signal-to-noise ratio of  $V_{22}$  to  $V_{11}$  is large. In our DGP,  $V_{11} = 1$  so the signal-to-noise ratio is given by  $V_{22}$ . Having drawn  $\lambda_i$  from  $N(0, 1)$ , we use  $\alpha_i$  to determine whether the common factor restriction is satisfied or not. If the restriction is satisfied, then  $\alpha_i = \lambda_i$ , whereas  $\alpha_i = 1$  otherwise. The degree of the violation is controlled by varying the value taken by  $V_{22}$ .

To evaluate these theoretical predictions, the ECM test statistics will be compared to four of the statistics developed by Pedroni (2004). To this end, we use  $ZG_t$  and  $ZG_\rho$  to denote his semiparametric group mean  $t$  and  $\rho$  test statistics. The corresponding panel statistics are denoted  $ZP_t$  and  $ZP_\rho$ , respectively. As with the ECM statistics, the panel and group mean statistics of Pedroni (2004) differ mainly because of the treatment of the autoregressive parameter  $\gamma_i$  in (7). In particular, while the panel statistics presume a common value  $\gamma_i = \gamma$  for all  $i$  under the alternative, the group mean statistics does not. To keep the amount of table space manageable, we present only the size-adjusted power and the empirical size on the five percent level when the critical value  $-1.645$  is used. The reported results are for Case 1 with no individual specific constant or trend terms. The results for the other cases did not change the conclusions and are therefore not included. All computations were performed in GAUSS.

The purpose of this section is primarily to illustrate the common factor issue and the relative power of the ECM tests. For completeness, however, we first make a brief digression on the performance of the tests under the null hypothesis. To this effect, we have experimented with different selection rules for the lag and lead orders of the tests. Among these are information based rules such as the Akaike and the Schwarz Bayesian information criteria, and

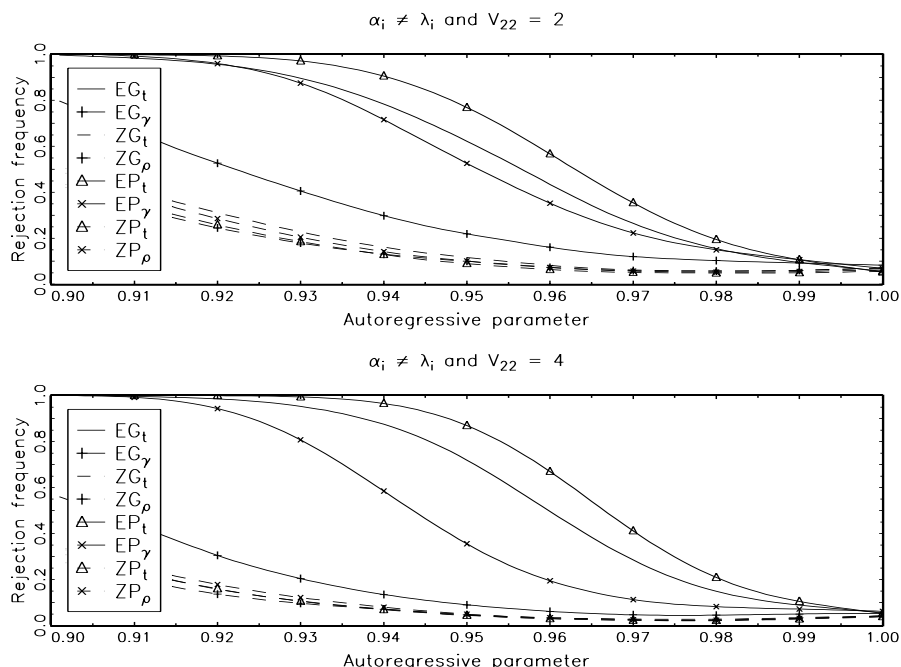
Figure 1: Size-adjusted power when  $N = 10$  and  $T = 50$ .



deterministic rules that chooses the lag order as a fixed function of  $T$ . Consistent with the results of Haug (1996), the results suggest that the information based rules tend to choose too parsimonious lag and lead orders, which generally result in size distortions. Selecting the order as a fixed function of  $T$  generally produces much more satisfactory results. To this end, we have performed a large number of experiments using different rules. Among these rules,  $[4(T/100)^{1/4}]$  generally performs best and we therefore only report the results for the tests based on this rule.

The results on the empirical size are reported in Table 2 for the case when the regressor is weakly exogenous and in Table 3 for the case when the regressor is endogenous. The data were generated with both MA(1) and AR(1) errors, which makes  $\theta$  and  $\phi$  convenient nuisance parameters to investigate. It has been well documented in the earlier literature that negative moving average structures may cause substantial size distortions when testing the null hypothesis of no cointegration (see, e.g. Haug, 1996; Kao, 1999). In agreement with these results, Table 2 and 3 show that all tests tend to reject the null hypothesis too frequently when  $\theta < 0$ . In fact, save for the  $EG_t$  and  $EP_t$  statistics, we see that a larger negative MA(1) component almost uniformly result in the size going to unity.

Figure 2: Size-adjusted power when  $N = 10$  and  $T = 50$ .

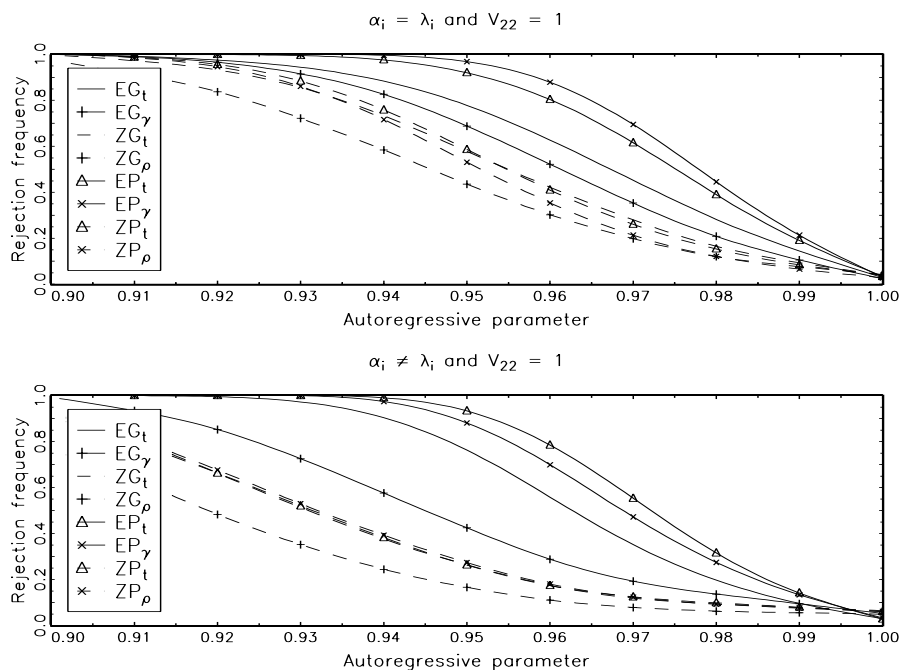


The results are very different when the errors are generated as an AR(1) process. In this case, a larger autoregressive parameter seem to result in the size going to zero. Thus, autoregressive errors causes an underrejection of the null thus leading to a more conservative test.

Overall, the simulations under the null hypothesis leads us to the conclusion that all tests performs reasonably well with the size being close to the nominal level in most experiments, which supports the asymptotic result that the distribution of the ECM test statistics should be free of nuisance parameters under the null hypothesis. Interestingly, since the performance of the tests appear to be unaffected by the introduction of endogenous regressors, this suggests that researchers may proceed with the cointegration testing as if the regressors are weakly exogenous with little or no loss of generality. Moreover, because there appear to be no large differences in performance between the ECM and the residual-based tests, the choice of test will depend to a large degree on their the performance under the alternative hypothesis.

Next, we continue to the results on the power properties of the tests. In this case,  $\theta = \phi = 0$  so the regression errors are generated as i.i.d. innovations. All results are adjusted for size so that each test has the same rejection frequency

Figure 3: Size-adjusted power when  $N = 20$  and  $T = 50$ .

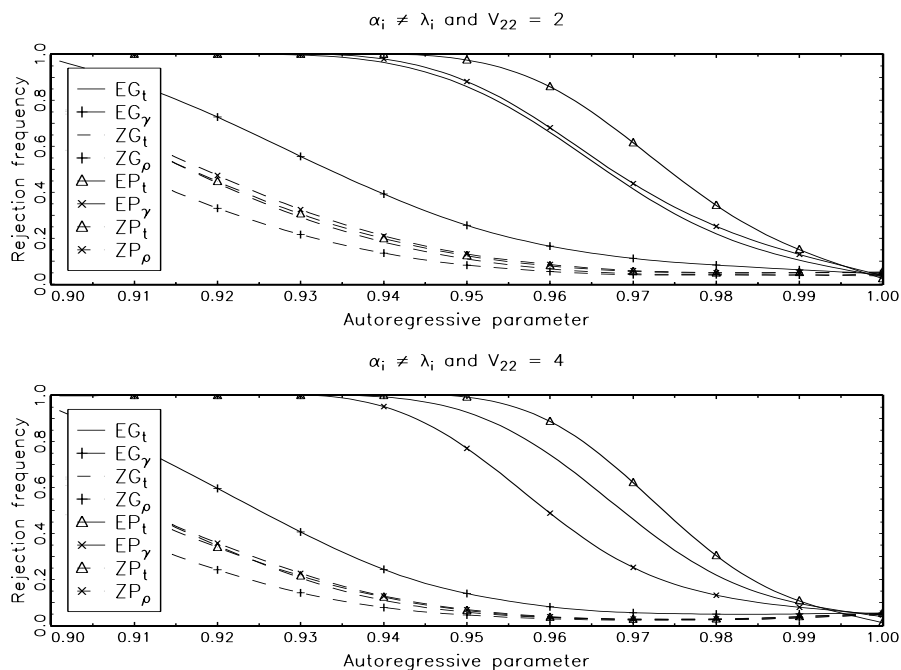


of five percent when the null hypothesis is true. The results are summarized in Figures 1 through 6.<sup>2</sup> The figures suggest that the ECM tests are uniformly more powerful than the residual-based tests. Notably, the ECM tests have highest power even though  $\lambda_i = \alpha_i$  and the common factor restriction is satisfied. Moreover, in accordance with our earlier discussion, we see that power of the ECM tests relative to that of the other tests increases monotonically as the signal-to-noise ratio  $V_{22}$  increases. This effect is further magnified by the fact that the power of the residual-based tests appear to be decreasing in  $V_{22}$ . This is to be expected as large values of  $V_{22}$  will tend to inflate the test regression in (7) with excess volatility and a loss of power. The implication is that the power advantages to the ECM tests may be substantial even though the signal-to-noise ratio is only slightly larger than one. Also, since these effects seem to materialize even in very small samples, they should be relevant in most empirical applications.

The panel tests have highest power. This is not surprising since they are constructed based on the pooled least squares estimator of the error correction

<sup>2</sup>In Figures 1 through 6, the curves representing the size-adjusted power of the test statistics have been smoothed by means of a least squares spline of neighboring points.

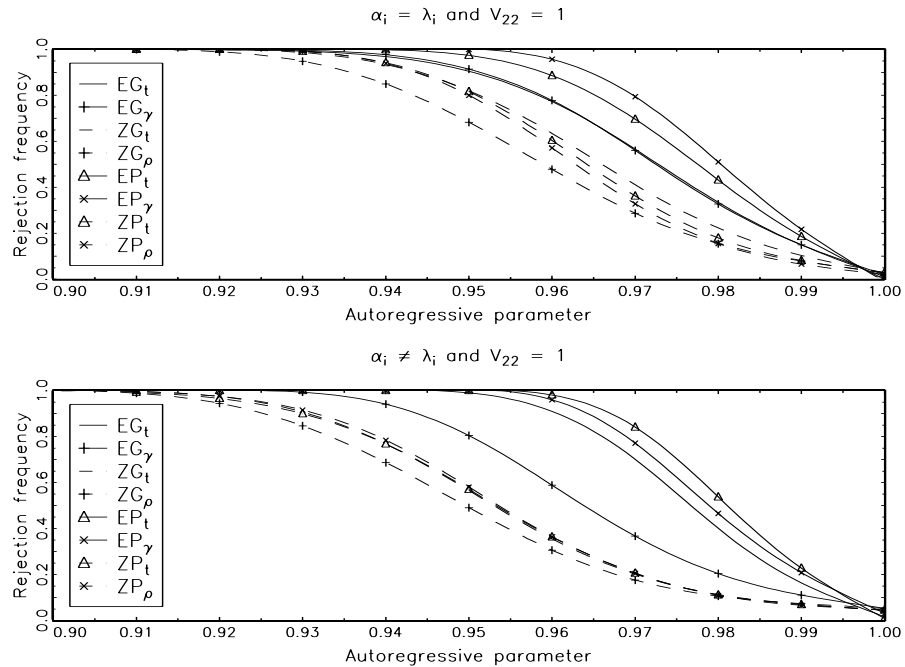
Figure 4: Size-adjusted power when  $N = 20$  and  $T = 50$ .



parameter and pooling is efficient under the homogenous alternative considered here. Among the panel tests, the figures suggest that the  $EP_\gamma$  test is most powerful when the common factor restriction is satisfied or the value of  $V_{22}$  is close to unity. As  $V_{22}$  increases, the power of the  $EP_t$  test raises relative to that of the  $EP_\gamma$  test. Among the group mean tests, the figures suggest that the  $EG_t$  test has highest power. As expected, we see that the power is increasing in  $N$  and  $T$ . We also see that the power increases as the autoregressive parameter departs from its hypothesized value of unity.

In summary, the simulation results suggest that the ECM tests generally perform well under the alternative hypothesis with good power in most panels. More importantly, when the common factor restriction is not satisfied by the data, as  $V_{22}$  increases, the power of the ECM tests increases significantly relative to that of the other tests. This result appears to be very robust and extends to all sample sizes examined and to the cases with demeaned and detrended data. The overall impression of the Monte Carlo evidence is therefore that the proposed tests compares favorably with the tests of Pedroni (2004).

Figure 5: Size-adjusted power when  $N = 10$  and  $T = 100$ .



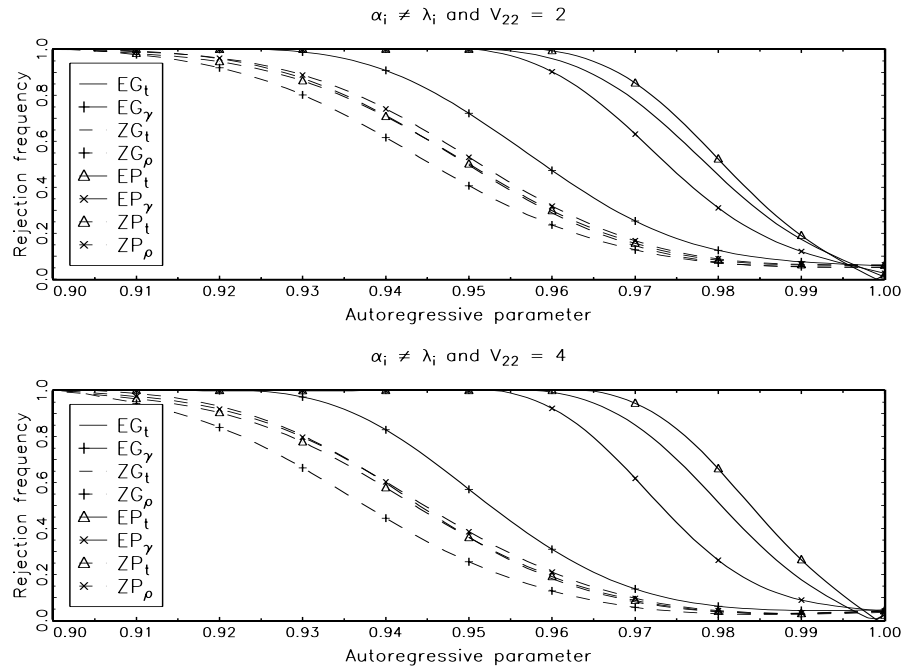
## 5 Health care expenditures and GDP

The relationship between health care expenditures (HCE) and GDP is the subject of a large literature in health economics. Many early contributions employed cross-sectional data to obtain estimates of this relationship. Without exception it has been found that most of the observed variation in HCE can be explained by variation in GDP. Many of these studies, however, have been criticized for the smallness of their data sets and for the assumption that HCE is homogeneously distributed across countries. More recent research have therefore resorted to panel data, which offers a number of advantages over pure cross-sectional data. For instance, using multiple years of data increases the sample size while simultaneously allowing researchers to control for a wide range of time invariant country characteristics through the inclusion of country specific constants and trends. In addition, with multiple time series observations for each country, this enables researchers to exploit the presence of unit roots and cointegration among HCE and GDP.

This avenue is taken by Hansen and King (1996), which examines a panel spanning the years 1960 to 1987 across 20 OECD member countries. They show



Figure 6: Size-adjusted power when  $N = 10$  and  $T = 100$ .



that, if one examines the time series for each of the countries separately, one can only rarely reject the unit root hypothesis for either HCE or GDP. Moreover, their county specific tests rarely reject the hypothesis of no cointegration. McCoskey and Selden (1998) uses the same data set as Hansen and King (1996). Based on the panel unit root proposed by Im *et al.* (2003), the authors are able to reject the presence of a unit root in both HCE and GDP. Once a linear time trend has been accommodated, however, the null hypothesis cannot be rejected. Hansen and King (1998) question the preference of McCoskey and Selden (1998) for omitting the time trend from their main results and argues that this may lead to misleading inference. Indeed, using a panel covering 24 OECD countries between 1960 and 1991, Blomqvist and Carter (1997) challenge the findings of McCoskey and Selden (1998). Drawing on a battery of tests, including the panel unit root test of Levin *et al.* (2002), the authors conclude that HCE and GDP both appear to be nonstationary and cointegrated. Gerdtham and Löthgren (2000) present confirmatory evidence using a panel of 21 OECD countries between 1960 and 1997. Similarly, using a panel of 10 OECD member countries over the period 1960 to 1993, Roberts (2000) found clear evidence suggesting that HCE and GDP are nonstationarity variables. The results on cointegration

were, however, not conclusive.

Apparently, although the evidence seem to support unit root hypothesis for HCE and GDP, it is less conclusive on the cointegration hypothesis. One possible explanation to the differing results may be the common factor restriction implicitly imposed when testing the null hypothesis of no cointegration using the two-step Engle and Granger (1987) procedure as in e.g. Hansen and King (1996).<sup>3</sup> In this section, we verify this conjuncture using a panel consisting of 20 OECD counties covering the period 1970 to 2001. For this purpose, data on annual frequency has been acquired through the OECD Health Data 2003 database. Both HCE and GDP are measured in per capita terms at constant 1995 prices and are transformed in logarithms. Moreover, since both variables are clearly trending, we follow the earlier literature and model HCE and GDP with a linear time trend in their levels. An obvious interpretation of such a trend is that it accounts, in part, for the impact of technological change. The basic model we postulate is the following simple log-linear relationship between HCE and GDP

$$\log HCE_{it} = \mu_i + \tau_i t + \lambda_i \log GDP_{it} + u_{it}. \quad (8)$$

The first step in our analysis of this relationship is to test whether the variables are nonstationary or not. To this effect, we employ the  $Z_{\bar{i}}$  and  $\tilde{Z}_{\bar{i}}$  statistics recently proposed by Im *et al.* (2003). Both statistics have limiting normal distributions under the null hypothesis of a unit root in the panel. The difference is that the tests have different distributional properties for a fixed  $T$  in which case the  $\tilde{Z}_{\bar{i}}$  statistic is analytically more manageable and is likely to lead to more accurate tests in small samples. The tests were constructed with both individual specific constant and trend terms in the level of the variables. The length of the lag augmentation is set equal to  $[4(T/100)^{2/9}]$ . Moreover, the appropriate moments needed to construct the  $Z_{\bar{i}}$  and  $\tilde{Z}_{\bar{i}}$  statistics for the model with a time trend are not available, and must therefore be obtained by means of Monte Carlo simulation. For this purpose, we make 10,000 draws of a single random walk of length  $T = 1,000$ , which is then used to compute the moments. The simulated mean and variance are  $-2.2208$  and  $0.5785$ , respectively. The calculated values of  $Z_{\bar{i}}$  and  $\tilde{Z}_{\bar{i}}$  for HCE based on these moments are  $-2.2972$  and  $-0.3671$ , respectively. The corresponding values for GDP are  $-1.3663$  and  $0.1886$ . Hence, compared to the lower tail of the normal distribution, we cannot reject the null hypothesis at the one percent significance level.

The tests of Im *et al.* (2003) are constructed as a sum of  $N$  individual unit root test statistics. In this sense, they are very similar to the group mean versions of the ECM statistic. The interpretation is therefore that a rejection should be taken as evidence in favor of a unit root for a nonempty subset of the panel. By contrast, the tests of Harris and Tzavalis (1999), and Levin *et al.*

---

<sup>3</sup>Since the data sets used in the previous studies are nearly identical, any differences in test results are not likely to be due to differences in the process generating the data.

(2002) are constructed as the panel ECM test statistics by pooling across the cross-sectional dimension. Hence, in this case, a rejection of the null should be taken as evidence in favor of a unit root for the panel as a whole. Given these differences, it is interesting to infer whether the same results are obtained using the panel type unit root tests. The computed value of the Harris and Tzavalis (1999) statistic for HCE and GDP are 1.3742 and  $-1.6162$ , respectively. The corresponding values of the Levin *et al.* (2002) statistic are 1.5859 and 0.2723. Hence, the results confirm those obtained using the tests of Im *et al.* (2003) and we therefore conclude that the variables are nonstationary.

The second step in our analysis is to test whether HCE and GDP are cointegrated. One way to do this is to follow the Engle and Granger (1987) procedure of subjecting the residuals from the OLS fit of (8) to a unit root test. As pointed out earlier, however, the prospect of imposing an invalid common factor restriction may well result in this procedure having very low power in samples as small as ours. In that case, the ECM test statistics may be able to produce more powerful tests. Our test results confirm this conjuncture. In agreement with the Monte Carlo results of the previous section, we set the lag and lead order of the tests to  $[4(T/100)^{2/9}]$ .

The calculated values of the  $ZG_t$  and  $ZG_\rho$  statistics with both a constant and a linear time trend are  $-0.4009$  and  $3.0643$ , respectively. For the  $ZG_t$  and  $ZG_\rho$  statistics, the calculated values are 1.5893 and 1.2991. Thus, based on the tests of Pedroni (2004), we cannot reject the null hypothesis of no cointegration. This conclusion is supported by the results presented in Table 4 on the individual test statistics abbreviated  $t_{EC}$  and  $\rho_{EC}$ . In fact, results suggest that the null hypothesis cannot be rejected based on the five percent significance level for any of the countries. We note, however, that the standard error of the individual test regressions for the residual-based tests are much larger than those of the corresponding ECM regressions, which is indicative of an invalid common factor restriction. Indeed, the individual  $F$ -statistics of the common factor hypothesis presented in Table 4 suggest that the restriction must be rejected at all conventional significance levels for all countries of the panel. The table also present the estimated signal-to-noise ratios, which are well above one in most cases.

The implication of these results is that the ECM test statistics may be more powerful. The calculated values of the  $EG_t$  and  $EG_\gamma$  statistics are  $-3.6743$  and  $0.2696$ , respectively. The corresponding values of the  $EP_t$  and  $EP_\gamma$  statistics are  $-2.3478$  and  $-0.7353$ . Hence, using the  $EG_t$  and  $EP_t$  tests, we are in fact able to reject the null hypothesis suggesting that HCE and GDP are cointegrated. In addition, based on the individual ECM test statistics presented in Table 4, we reject the null hypothesis at the five percent level on at least seven occasions, which reinforces this conclusion. By contrast, the null hypothesis cannot be rejected based on the  $EG_\gamma$  and  $EP_\gamma$  tests. This is not unexpected, however, given the simulation results of the previous section suggesting that the  $EG_t$  and

$EP_t$  statistics should be able to produce more powerful tests in the presence of an invalid common factor restriction. Hence, the evidence is interpreted as supportive of the hypothesis of cointegration between HCE and GDP.

## 6 Conclusions

In this paper, we propose four new panel ECM test, which is designed to test the null hypothesis of no cointegration by testing whether the error correction term in an conditional ECM is equal to zero. If the null hypothesis of no error correction is rejected, then the null hypothesis of no cointegration is also rejected. Each test is able to accommodate individual specific short-run dynamics, including serially correlated error terms and weakly exogenous regressors, individual specific intercept and trend terms, as well as individual specific slope parameters. Using sequential limit arguments, we are able to show that the tests have limiting normal distributions and that they are consistent. In our Monte Carlo study, we demonstrate that the ECM tests maintain nominal size reasonably well and that they are more powerful than the residual-based tests of Pedroni (2004). These differences in power arises because the latter statistics ignore potentially valuable information by imposing a possibly invalid common factor restriction. In our empirical application, we provide evidence suggesting that international health care expenditures and GDP are cointegrated once the short- and long-run dynamics are allowed to differ.

## Appendix: Mathematical proofs

This appendix proves the limiting distributions of the ECM test statistics. For ease of exposure, we shall prove the results for Case 1 with no deterministic components. The proof uses the techniques of Banerjee *et al.* (1998) and hence only essential details are given.

**Proof of Theorem 1.** Under the null hypothesis of no cointegration,  $\gamma_i = 0$  in which case the ECM in (1) reduces to

$$\Delta y_{it} = \lambda_i' \Delta x_{it-1} + u_{it}. \quad (\text{A1})$$

For convenience in deriving the distributions under the null hypothesis, we introduce the following matrix notation. Define  $S_{it} \equiv \sum_{j=1}^t u_{ij}$  and  $R_{it} \equiv \sum_{j=1}^t v_{ij}$ , then we have  $S_i = (S_{i1}, \dots, S_{iT})'$ ,  $R_i = (R_{i1}, \dots, R_{iT})'$ ,  $V_i = (v_{i1}, \dots, v_{iT})'$  and  $U_i = (u_{i1}, \dots, u_{iT})'$ ,  $X_i = (R_{i,-1}, V_i)'$ ,  $H_i = (S_{i,-1}, R_{i,-1}, V_i, U_i)'$  and  $A_i = H_i H_i'$ . In addition, we define  $Q_i \equiv I_T - R_{i,-1} (R_{i,-1}' R_{i,-1})^{-1} R_{i,-1}'$  and  $P_i \equiv I_T - X_i (X_i' X_i)^{-1} X_i'$ . It follows that  $P_i = Q_i - V_i' Q_i (V_i' Q_i V_i)^{-1} V_i' Q_i$ , which implies that  $E_{i11}$  and  $E_{i12}$  can be expanded as

$$\begin{aligned} T^{-2} E_{i11} &= T^{-2} S_{i,-1}' P_i S_{i,-1} = T^{-2} S_{i,-1}' Q_i S_{i,-1} \\ &\quad - T^{-1} (T^{-1} S_{i,-1}' Q_i V_i) (T^{-1} V_i' Q_i V_i)^{-1} (T^{-1} V_i' Q_i S_{i,-1}), \quad (\text{A2}) \\ T^{-1} E_{i12} &= T^{-1} S_{i,-1}' P_i U_i = T^{-1} S_{i,-1}' Q_i U_i \\ &\quad - (T^{-1} S_{i,-1}' Q_i V_i) (T^{-1} V_i' Q_i V_i)^{-1} (T^{-1} V_i' Q_i U_i), \quad (\text{A3}) \end{aligned}$$

where

$$\begin{aligned} T^{-2} S_{i,-1}' Q_i S_{i,-1} &= T^{-2} A_{i11} - (T^{-2} A_{i12}) (T^{-2} A_{i22})^{-1} (T^{-2} A_{i21}), \\ T^{-1} S_{i,-1}' Q_i V_i &= T^{-1} A_{i13} - (T^{-2} A_{i12}) (T^{-2} A_{i22})^{-1} (T^{-1} A_{i23}), \\ T^{-1} V_i' Q_i V_i &= T^{-1} A_{i33} - T^{-1} (T^{-1} A_{i32}) (T^{-2} A_{i22})^{-1} (T^{-1} A_{i23}), \\ T^{-1} S_{i,-1}' Q_i U_i &= T^{-1} A_{i14} - (T^{-2} A_{i12}) (T^{-2} A_{i22})^{-1} (T^{-1} A_{i24}), \\ T^{-1} V_i' Q_i U_i &= T^{-1} A_{i34} - T^{-1} (T^{-1} A_{i32}) (T^{-2} A_{i22})^{-1} (T^{-1} A_{i24}). \end{aligned}$$

In these expressions, all normalized partitions of  $A_i$  but  $T^{-1} A_{i33}$  and  $T^{-1} A_{i34}$  are  $O_p(1)$  by standard limit theory. As for  $T^{-1} A_{i33}$ , notice that under Assumption 2 (iii),  $T^{-1} A_{i33} \xrightarrow{p} E(A_{i33}) > 0$  as  $T \rightarrow \infty$  so  $T^{-1} A_{i33} = O_p(1)$ . For  $T^{-1} A_{i34}$ , we have  $T^{-1} A_{i34} = 0$  under Assumption 2 (iv). Together, these results imply that the second term appearing in (A2) and (A3) are both  $o_p(1)$  and can be disregarded. Therefore, we obtain the following limits for  $T^{-2} E_{i11}$  and  $T^{-1} E_{i12}$  passing  $T \rightarrow \infty$  with  $N$  held fixed

$$T^{-2} E_{i11} = T^{-2} S_{i,-1}' Q_i S_{i,-1} + o_p(1) \Rightarrow \sigma_i^2 \int_0^1 Q_i^2, \quad (\text{A4})$$

$$T^{-1}E_{i12} = T^{-1}S'_{i,-1}Q_iU_i + o_p(1) \Rightarrow \sigma_i^2 \int_0^1 Q_i dW_{i1}. \quad (\text{A5})$$

Let  $F_i = I_T - Y_i(Y_i'Y_i)^{-1}Y_i'$ , where  $Y_i = (X_i', Y_{i,-1})'$  and  $Y_i = (y_{i1}, \dots, y_{iT})'$ . By the same arguments used in obtaining (A4) and (A5), it is possible to show that  $T^{-1}U_i'Y_i = O_p(1)$  and  $T^{-2}Y_i'Y_i = O_p(1)$ . This imply that the estimated variance can be written as

$$\begin{aligned} \hat{\sigma}_i^2 &= T^{-1}U_i'F_iU_i \\ &= T^{-1}U_i'U_i - T^{-1}(T^{-1}U_i'Y_i)(T^{-2}Y_i'Y_i)(T^{-1}Y_i'U_i) \\ &= T^{-1}U_i'U_i + o_p(1) \\ &= \sigma_i^2 + o_p(1). \end{aligned} \quad (\text{A6})$$

This imply that  $\hat{\sigma}_i^2$  is consistent for  $\sigma_i^2$ . Now, define the scaled random variables  $E_{i1} \equiv T^{-2}E_{i11}$  and  $E_{i2} \equiv T^{-1}E_{i12}$ , and let  $E_{i3} \equiv E_{i2}E_{i1}^{-1}$  and  $E_{i4} \equiv E_{i2}E_{i1}^{-1/2}$ . These definitions together with the results in (A4) and (A5) imply that  $TEG_\gamma$  and has the following limit as  $T \rightarrow \infty$  with  $N$  held fixed

$$\begin{aligned} TEG_\gamma &= \sum_{i=1}^N (T^{-1}E_{i11})^{-1} E_{i12} \\ &= \sum_{i=1}^N E_{i3} \\ &\Rightarrow \sum_{i=1}^N \left( \sigma_i^2 \int_0^1 Q_i^2 \right)^{-1} \sigma_i^2 \int_0^1 Q_i dW_{i1} \\ &= \sum_{i=1}^N \left( \int_0^1 Q_i^2 \right)^{-1} \int_0^1 Q_i dW_{i1}. \end{aligned} \quad (\text{A7})$$

For  $EG_t$ , we have the following limit

$$\begin{aligned} EG_t &= \sum_{i=1}^N \hat{\sigma}_i^{-1} E_{i11}^{-1/2} E_{i12} \\ &= \sum_{i=1}^N \hat{\sigma}_i^{-1} E_{i4} \\ &\Rightarrow \sum_{i=1}^N \sigma_i^{-1} \left( \sigma_i^2 \int_0^1 Q_i^2 \right)^{-1/2} \sigma_i^2 \int_0^1 Q_i dW_{i1} \\ &= \sum_{i=1}^N \left( \int_0^1 Q_i^2 \right)^{-1} \int_0^1 Q_i dW_{i1}. \end{aligned} \quad (\text{A8})$$

This shows that the limiting distribution of the group mean statistics are free of nuisance parameters under the null. Therefore, because the limiting distributions passing  $T \rightarrow \infty$  is i.i.d. over the cross-section, we deduce that  $E(K_i) = \tilde{\Theta}$

for all  $i$ . The variance of  $K_i$  may be decomposed as

$$\tilde{\Sigma} = \begin{pmatrix} \tilde{\Sigma}_{11} & \tilde{\Sigma}_{12} \\ \tilde{\Sigma}_{12} & \tilde{\Sigma}_{22} \end{pmatrix}.$$

To derive the sequential limiting distribution of  $EG_\gamma$  and  $EG_t$ , we expand the statistics in the following manner

$$TN^{-1/2}EG_\gamma - N^{1/2}\tilde{\Theta}_1 = N^{1/2} \left( N^{-1} \sum_{i=1}^N E_{i3} - \tilde{\Theta}_1 \right), \quad (\text{A9})$$

$$N^{-1/2}EG_t - N^{1/2}\tilde{\Theta}_2 = N^{1/2} \left( N^{-1} \sum_{i=1}^N \hat{\sigma}_i^{-1} E_{i4} - \tilde{\Theta}_2 \right). \quad (\text{A10})$$

It follows that  $TN^{-1/2}EG_\gamma - N^{1/2}\tilde{\Theta}_1 \Rightarrow N(0, \tilde{\Sigma}_{11})$  and  $N^{-1/2}EG_t - N^{1/2}\tilde{\Theta}_2 \Rightarrow N(0, \tilde{\Sigma}_{22})$  as  $T \rightarrow \infty$  prior to  $N$  by direct application of the Lindberg-Lévy central limit theorem. This establishes the limit distribution of the group mean statistics.

Consider next the limiting distribution of the panel statistics. Making use of the weak limits in (A2) and (A3), we may infer the following limit for  $TEP_\gamma$  as  $T \rightarrow \infty$

$$\begin{aligned} TEP_\gamma &= \left( \sum_{i=1}^N T^{-1} E_{i11} \right)^{-1} \sum_{i=1}^N E_{i12} \\ &= \left( \sum_{i=1}^N E_{i1} \right)^{-1} \sum_{i=1}^N E_{i2} \\ &\Rightarrow \left( \sum_{i=1}^N \sigma_i^2 \int_0^1 Q_i^2 \right)^{-1} \sum_{i=1}^N \sigma_i^2 \int_0^1 Q_i dW_{i1}. \end{aligned} \quad (\text{A11})$$

Similarly, for  $EP_t$ , we have the following limit

$$\begin{aligned} EP_t &= \hat{\sigma}^{-1/2} \left( \sum_{i=1}^N E_{i11} \right)^{-1} \sum_{i=1}^N E_{i12} \\ &= \hat{\sigma}^{-1} \left( \sum_{i=1}^N E_{i1} \right)^{-1/2} \sum_{i=1}^N E_{i2} \\ &\Rightarrow \sigma^{-1} \left( \sum_{i=1}^N \sigma_i^2 \int_0^1 Q_i^2 \right)^{-1/2} \sum_{i=1}^N \sigma_i^2 \int_0^1 Q_i dW_{i1}. \end{aligned} \quad (\text{A12})$$

To be able to infer the sequential limits of these expressions, we shall make use of the Delta method, which provides the limiting distribution for continuously

differentiable transformations of i.i.d. vector sequences. In so doing, we expand  $TN^{1/2}EP_\gamma$  as follows

$$\begin{aligned}
TN^{1/2}EP_\gamma &= N^{1/2}\Theta_2\Theta_1^{-1} \\
&= N^{1/2}\left(N^{-1}\sum_{i=1}^N E_{i2} - \sigma^2\Theta_2\right)\left(N^{-1}\sum_{i=1}^N E_{i1}\right)^{-1} \\
&= \sigma^2\Theta_2N^{1/2}\left(\left(N^{-1}\sum_{i=1}^N E_{i1}\right)^{-1} - (\sigma^2\Theta_1)^{-1}\right). \quad (\text{A13})
\end{aligned}$$

The expansion of  $N^{1/2}EP_t$  is as follows

$$\begin{aligned}
N^{1/2}EP_t &= N^{1/2}\Theta_2\Theta_1^{-1/2} \\
&= \hat{\sigma}^{-1}N^{1/2}\left(N^{-1}\sum_{i=1}^N E_{i2} - \sigma^2\Theta_2\right)\left(N^{-1}\sum_{i=1}^N E_{i1}\right)^{-1/2} \\
&= \sigma\Theta_2N^{1/2}\left(\left(N^{-1}\sum_{i=1}^N E_{i1}\right)^{-1/2} - (\sigma^2\Theta_1)^{-1/2}\right). \quad (\text{A14})
\end{aligned}$$

Let  $\sigma^2$  denote the expected value of  $\sigma_i^2$ . Thus, by a law of large numbers, we have that  $\hat{\sigma}^2 \xrightarrow{p} \sigma^2$  as  $T \rightarrow \infty$  and then  $N \rightarrow \infty$  sequentially. Consequently, by Corollary 1 of Phillips and Moon (1999), the terms appearing in (A13) and (A14) with normalizing order  $N^{-1}$  converges in probability to  $\sigma^2$  times the expectations of the corresponding random variable as  $T \rightarrow \infty$  prior to  $N$ . Hence,  $N^{-1}\sum_{i=1}^N R_{i1} \xrightarrow{p} \sigma^2\Theta_1$  and  $N^{-1}\sum_{i=1}^N R_{i2} \xrightarrow{p} \sigma^2\Theta_2$ . Moreover, by direct application of the Lindberg-Lévy central limit theorem,  $N^{1/2}(N^{-1}\sum_{i=1}^N E_{i2} - \sigma^2\Theta_2) \Rightarrow N(0, \sigma^4\Sigma_{22})$  passing  $T \rightarrow \infty$  and then  $N \rightarrow \infty$ . In deriving this result we use the fact that  $\Sigma$  may be decomposed as

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{pmatrix}.$$

The remaining expressions in (A13) and (A14) involves a continuously differentiable transformation of i.i.d. random variables. Thus, by the Delta method, as  $T \rightarrow \infty$  prior to  $N$

$$N^{1/2}\left(\left(N^{-1}\sum_{i=1}^N E_{i1}\right)^{-1} - (\sigma^2\Theta_1)^{-1}\right) \Rightarrow N(0, \sigma^{-4}\Theta_1^{-4}\Sigma_{11}), \quad (\text{A15})$$

$$N^{1/2}\left(\left(N^{-1}\sum_{i=1}^N E_{i1}\right)^{-1/2} - (\sigma^2\Theta_1)^{-1/2}\right) \Rightarrow N(0, 4^{-1}\sigma^{-2}\Theta_1^{-3}\Sigma_{11}). \quad (\text{A16})$$



This suggests that the limits of  $TN^{1/2}EP_\gamma - N^{1/2}\Theta_2\Theta_1^{-1}$  and  $N^{1/2}EP_t - N^{1/2}\Theta_2\Theta_1^{-1/2}$  may be rewritten in the following fashion

$$\begin{aligned} TN^{1/2}EP_\gamma - N^{1/2}\Theta_2\Theta_1^{-1} &\Rightarrow (\sigma^2\Theta_1)^{-1} N(0, \sigma^4\Sigma_{22}) \\ &\quad - \sigma^2\Theta_2 (\sigma^2\Theta_1)^{-2} N(0, \sigma^4\Sigma_{11}), \end{aligned} \quad (\text{A17})$$

$$\begin{aligned} N^{1/2}EP_t - N^{1/2}\Theta_2\Theta_1^{-1/2} &\Rightarrow \sigma^{-1} (\sigma^2\Theta_1)^{-1/2} N(0, \sigma^4\Sigma_{22}) \\ &\quad - \sigma\Theta_2 \left(4^{1/2}\sigma^3\Theta_1^{3/2}\right)^{-1} N(0, \sigma^4\Sigma_{11}). \end{aligned} \quad (\text{A18})$$

Using (A17) and (A18), it is straightforward to verify that the centered statistics  $TN^{1/2}EP_\gamma - N^{1/2}\Theta_2\Theta_1^{-1}$  and  $N^{1/2}EP_t - N^{1/2}\Theta_2\Theta_1^{-1/2}$  are mean zero with variances given by  $\phi'\Sigma\phi = \Theta_1^{-2}\Sigma_{22} - 2\Theta_1^{-3}\Theta_2\Sigma_{12} + \Theta_2^2\Theta_1^{-4}\Sigma_{11}$  and  $\varphi'\Sigma\varphi = \Theta_1^{-1}\Sigma_{22} - \Theta_1^{-2}\Theta_2\Sigma_{12} + 4^{-1}\Theta_2^2\Theta_1^{-3}\Sigma_{11}$ , respectively. This completes the proof.  $\blacksquare$

**Proof of Theorem 2.** Under the alternative hypothesis,  $\beta'_i z_{it}$  in (A1) is stationary. Moreover, if we denote by  $\hat{\beta}_i$  the OLS estimator of  $\beta_i$ , then  $\hat{\beta}_i - \beta_i = O_p(T^{-1})$ . Therefore, inference  $\gamma_i$  in (A3) is asymptotically equivalent to inference in the following regression

$$\Delta y_{it} = \lambda'_i \Delta x_{it-1} + \gamma_i w_{it-1} + u_{it}, \quad (\text{A19})$$

where  $w_{it} = \beta'_i z_{it}$  is the putative disequilibrium error. Let  $W_i = (w_{i1}, \dots, w_{iT})'$ ,  $Q_i \equiv I_T - V_i(V'_i V_i)^{-1} V'_i$ ,  $E_{i1} \equiv W'_{i,-1} Q_i W_{i,-1}$ ,  $E_{i2} \equiv W'_{i,-1} Q_i U_i$ ,  $H_i \equiv (W_{i,-1}, V_i, U_i)$  and  $B_i \equiv H'_i H_i$ . The normalized quantities  $T^{-1}E_{i1}$  and  $T^{-1}E_{i2}$  may be expanded as

$$T^{-1}E_{i1} = T^{-1}B_{i11} - (T^{-1}B_{i12}) (T^{-1}B_{i22})^{-1} (T^{-1}B_{i21}), \quad (\text{A20})$$

$$T^{-1}E_{i2} = T^{-1}B_{i13} - (T^{-1}B_{i12}) (T^{-1}B_{i22})^{-1} (T^{-1}B_{i23}). \quad (\text{A21})$$

By the stationarity of the regressors, all partitions of  $B_i$  normalized by  $T^{-1}$  are  $O_p(1)$ . Thus,  $T^{-1}E_{i1}$  and  $T^{-1}E_{i2}$  are  $O_p(1)$  too. These results imply that the ECM statistics have the following orders

$$TEG_\gamma = \sum_{i=1}^N T (T^{-1}E_{i1})^{-1} (T^{-1}E_{i2}) = O_p(T),$$

$$EG_t = \sum_{i=1}^N T^{1/2} (T^{-1}E_{i1})^{-1/2} (T^{-1}E_{i2}) = O_p(T^{1/2}),$$

$$TEP_\gamma = T \left( \sum_{i=1}^N T^{-1}E_{i1} \right)^{-1} \left( \sum_{i=1}^N T^{-1}E_{i2} \right) = O_p(T),$$

$$EP_t = T^{-1/2} \hat{\sigma}^{-1/2} \left( \sum_{i=1}^N T^{-1}E_{i1} \right)^{-1} \left( \sum_{i=1}^N T^{-1}E_{i2} \right) = O_p(T^{1/2}).$$

This establishes that each of the ECM test statistics diverges as  $T \rightarrow \infty$  and then  $N \rightarrow \infty$  sequentially. Moreover, as  $\gamma_i < 0$  under the alternative, the divergence occurs towards negative infinity. ■

Table 1: Simulated moments

Test	Case	Expected value					Variance				
		$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$
$EG_t$	1	-0.9763	-1.3816	-1.7093	-1.9789	-2.1985	1.0823	1.0981	1.0489	1.0576	1.0351
	2	-1.7776	-2.0349	-2.2332	-2.4453	-2.6462	0.8071	0.8481	0.8886	0.9119	0.9083
	3	-2.3664	-2.5284	-2.7040	-2.8639	-3.0146	0.6603	0.7070	0.7586	0.8228	0.8477
$EG_\gamma$	1	-3.8022	-5.8239	-7.8108	-9.8791	-11.7239	20.6868	29.9016	39.0109	50.5741	58.9595
	2	-7.1423	-9.1249	-10.9667	-12.9561	-14.9752	29.6336	39.3428	49.4880	58.7035	67.9499
	3	-12.0116	-13.6324	-15.5262	-17.3648	-19.2533	46.2420	53.7428	64.5591	74.7403	84.7990
$EP_t$	1	-0.5105	-0.9370	-1.3169	-1.6167	-1.8815	1.3624	1.7657	1.7177	1.6051	1.4935
	2	-1.4476	-1.7131	-1.9206	-2.1484	-2.3730	0.9885	1.0663	1.1168	1.1735	1.1684
	3	-2.1124	-2.2876	-2.4633	-2.6275	-2.7858	0.7649	0.8137	0.8857	0.9985	0.9918
$EP_\gamma$	1	-1.0263	-2.4988	-4.2699	-6.1141	-8.0317	8.3827	24.0223	39.8827	53.4518	63.2406
	2	-4.2303	-5.8650	-7.4599	-9.3057	-11.3152	19.7090	31.2637	42.9975	57.4844	69.4374
	3	-8.9326	-10.4874	-12.1672	-13.8889	-15.6815	37.5948	45.6890	57.9985	74.1258	81.3934

Notes:

- (i) Case 1 refers to the regression with no deterministic terms, Case 2 refers to the regression with a constant term, and Case 3 refers to the regression with a constant and a linear time trend.
- (ii) The value  $K$  refers to the number of regressors excluding any deterministic constant or trend terms.
- (iii) Usage: The table provides approximations of the test moments given in Theorem 1. For example, the expected value and the variance of  $EG_t$  in the table are the approximations of  $\Theta_2$  and  $\tilde{\Sigma}_{22}$ .

Table 2: Empirical size with weakly exogenous regressors

$\phi$	$\theta$	$N$	$T$	Group mean statistics			Panel statistics				
				$EG_t$	$EG_\gamma$	$ZG_t$	$ZG_\rho$	$EP_t$	$EP_\gamma$	$ZP_t$	$ZP_\rho$
0.0	0.0	10	50	0.082	0.102	0.092	0.012	0.084	0.172	0.062	0.080
0.0	0.0	10	100	0.072	0.080	0.062	0.012	0.086	0.152	0.034	0.068
0.0	0.0	20	50	0.076	0.104	0.102	0.002	0.096	0.154	0.056	0.068
0.0	0.0	20	100	0.068	0.104	0.086	0.002	0.070	0.120	0.044	0.048
0.2	0.0	10	50	0.092	0.022	0.022	0.000	0.084	0.086	0.012	0.014
0.2	0.0	10	100	0.088	0.034	0.022	0.000	0.074	0.076	0.014	0.024
0.2	0.0	20	50	0.084	0.018	0.012	0.000	0.090	0.054	0.014	0.020
0.2	0.0	20	100	0.070	0.018	0.022	0.000	0.068	0.040	0.008	0.018
0.6	0.0	10	50	0.064	0.000	0.000	0.000	0.078	0.002	0.010	0.010
0.6	0.0	10	100	0.050	0.000	0.000	0.000	0.068	0.002	0.002	0.008
0.6	0.0	20	50	0.066	0.000	0.000	0.000	0.068	0.000	0.004	0.004
0.6	0.0	20	100	0.052	0.000	0.002	0.000	0.056	0.000	0.002	0.002
0.0	-0.2	10	50	0.090	0.264	0.332	0.162	0.114	0.292	0.202	0.282
0.0	-0.2	10	100	0.080	0.292	0.288	0.280	0.086	0.262	0.206	0.294
0.0	-0.2	20	50	0.078	0.354	0.490	0.236	0.082	0.268	0.278	0.332
0.0	-0.2	20	100	0.080	0.346	0.380	0.314	0.076	0.260	0.268	0.350
0.0	-0.6	10	50	0.568	0.968	1.000	1.000	0.444	0.848	0.990	0.998
0.0	-0.6	10	100	0.346	0.948	1.000	1.000	0.264	0.810	0.988	0.996
0.0	-0.6	20	50	0.786	1.000	1.000	1.000	0.596	0.962	1.000	1.000
0.0	-0.6	20	100	0.524	1.000	1.000	1.000	0.366	0.908	0.998	1.000

Table 3: Empirical size with endogenous regressors

$\phi$	$\theta$	$N$	$T$	Group mean statistics			Panel statistics				
				$EG_t$	$EG_\gamma$	$ZG_t$	$ZG_\rho$	$EP_t$	$EP_\gamma$	$ZP_t$	$ZP_\rho$
0.0	0.0	10	50	0.078	0.096	0.090	0.000	0.092	0.166	0.072	0.090
0.0	0.0	10	100	0.066	0.088	0.064	0.016	0.068	0.110	0.034	0.068
0.0	0.0	20	50	0.088	0.112	0.102	0.004	0.084	0.134	0.054	0.068
0.0	0.0	20	100	0.086	0.098	0.110	0.006	0.106	0.156	0.058	0.080
0.2	0.0	10	50	0.086	0.028	0.026	0.000	0.094	0.104	0.006	0.016
0.2	0.0	10	100	0.084	0.018	0.030	0.000	0.098	0.096	0.016	0.028
0.2	0.0	20	50	0.088	0.018	0.014	0.000	0.054	0.046	0.008	0.010
0.2	0.0	20	100	0.070	0.008	0.018	0.000	0.076	0.052	0.006	0.008
0.6	0.0	10	50	0.082	0.000	0.000	0.000	0.078	0.002	0.000	0.000
0.6	0.0	10	100	0.076	0.000	0.000	0.000	0.096	0.000	0.002	0.004
0.6	0.0	20	50	0.062	0.000	0.000	0.000	0.074	0.000	0.000	0.002
0.6	0.0	20	100	0.050	0.000	0.000	0.000	0.080	0.000	0.002	0.002
0.0	-0.2	10	50	0.110	0.278	0.346	0.176	0.118	0.280	0.234	0.310
0.0	-0.2	10	100	0.080	0.298	0.296	0.292	0.102	0.276	0.226	0.322
0.0	-0.2	20	50	0.094	0.362	0.508	0.230	0.110	0.260	0.284	0.344
0.0	-0.2	20	100	0.096	0.338	0.408	0.352	0.092	0.264	0.290	0.374
0.0	-0.6	10	50	0.550	0.964	1.000	1.000	0.422	0.856	0.980	0.990
0.0	-0.6	10	100	0.342	0.958	1.000	1.000	0.274	0.818	0.994	0.998
0.0	-0.6	20	50	0.816	0.996	1.000	1.000	0.608	0.966	1.000	1.000
0.0	-0.6	20	100	0.472	0.994	1.000	1.000	0.336	0.884	1.000	1.000

Table 4: Country specific test statistics

Country	$t_{ECM}$	$\gamma_{ECM}$	$t_{EG}$	$\rho_{EG}$	$SN$	$F_{COMF}$
Australia	-4.6858	-14.0851	-2.3205	-6.7425	1.1009	12.0856
Austria	-2.9665	-12.7742	-2.7261	-11.0941	1.5644	16.7759
Belgium	-3.6164	-11.6763	-2.3254	-9.7256	1.5586	107.7053
Canada	-2.9827	-6.9541	-1.9592	-4.7615	2.4904	21.6730
Denmark	-2.6486	-10.1772	-2.2761	-8.2464	2.9986	9.2030
Finland	-1.2624	-3.5524	-1.6016	-3.7843	1.5465	68.4147
Germany	-4.6412	-10.8999	-3.4844	-12.0944	1.1896	132.7105
Iceland	-2.9639	-15.2907	-2.9582	-14.8821	2.8438	31.8930
Ireland	-3.3967	-11.6916	-3.3115	-9.1965	2.6383	16.2727
Japan	-2.3934	-4.0364	-2.0804	-4.1806	2.5626	42.7549
Luxembourg	-1.7553	-7.6790	-2.4192	-8.0706	2.0777	9.0761
Netherlands	-4.9915	-26.0161	-3.4908	-14.0679	3.2189	33.0150
New Zealand	-1.1904	-4.1305	-1.9545	-5.5628	0.4271	4.8109
Norway	-1.8287	-10.4927	-3.1842	-14.0203	1.7195	24.2765
Portugal	-4.4761	-24.0592	-3.6218	-12.6986	1.0101	16.9359
Spain	-2.0632	-13.9547	-2.6695	-7.7966	1.5551	6.7591
Sweden	-2.3528	-4.4077	-2.1590	-4.6508	1.3850	24.6726
Switzerland	-3.4925	-12.9242	-3.3841	-11.2136	2.3383	16.1373
United Kingdom	-4.4140	-21.2605	-2.8328	-11.3774	4.3440	8.7129
United States	-2.1945	-3.8733	-1.1822	-1.0543	4.1947	28.6096

Notes:

- (i) The column labelled  $SN$  contains the estimated signal-to-noise ratios.
- (ii) The  $F_{COMF}$  statistics for the common factor restriction have a limiting  $F$ -distribution under the null hypothesis. The five and one percent critical values for one and 25 degrees of freedom are 4.24 and 7.77, respectively.
- (iii) The critical values for the individual cointegration test statistics have been obtained through Monte Carlo simulation. The five percent critical values for the  $t_{ECM}$  and  $\gamma_{ECM}$  statistics are  $-3.5824$  and  $-24.8003$ , respectively. The corresponding values for  $t_{EG}$  and  $\rho_{EG}$  are  $-3.6752$  and  $-26.8896$ .

## References

- [1] Banerjee, A., J. J. Dolado, D. Hendry and G. Smith, 1986, Exploring Equilibrium Relationships in Econometrics Through Static Models: Some Monte-Carlo Evidence, *Oxford Bulletin of Economics and Statistics*, Vol. 48, pp. 253-277.
- [2] Banerjee, A., J. J. Dolado and R. Mestre, 1998, Error-Correction Mechanism Tests for Cointegration in a Single-Equation Framework, *Journal of Time Series Analysis*, Vol. 19, No. 3, pp. 267-283.
- [3] Blomqvist, A. G., and R. A. L. Carter, 1997, Is Health Care Really Luxury? *Journal of Health Economics*, No. 16, pp. 207-229.
- [4] Engle, R., and C. Granger, 1987, Cointegration and Error Correction: Representation, Estimation and Testing, *Econometrica*, No. 55, pp. 251-276.
- [5] Gerdttham, U. G., and M. Löthgren, 2000, On Stationarity and Cointegration of International Health Expenditure and GDP, *Journal of Health Economics*, No. 19, pp. 461-475.
- [6] Hansen, P., and A. King, 1996, The Determinants of Health Care Expenditures: A Cointegration Approach, *Journal of Health Economics*, No. 15, pp. 127-137.
- [7] Hansen, P., and A. King, 1998, Health Care Expenditures and GDP: Panel Data Unit Root Test Results - Comment, *Journal of Health Economics*, No. 17, pp. 377-381.
- [8] Harris, R. D. F., E. Tzavalis, 1999, Inference for Unit Roots in Dynamic Panels where the Time Dimension is Fixed, *Journal of Econometrics*, No. 91, pp. 201-226.
- [9] Haug, A. A., 1996, Tests for Cointegration: A Monte Carlo Comparison, *Journal of Econometrics*, Vol. 71, pp. 891-915.
- [10] Ho, T., 2002, A Panel Cointegration Approach to the Saving-Investment Correlation, *Empirical Economics*, No. 27, pp. 91-100.
- [11] Im, K. S., M. Peseran and Y. Shin, 2003, Testing for Unit Roots in Heterogeneous Panels, *Journal of Econometrics*, No. 115, pp. 53-74.
- [12] Kremers, J., N. Ericsson and J. Dolado, 1992, The Power of Cointegration Tests, *Oxford Bulletin of Economics and Statistics*, Vol. 54, No. 3, pp. 325-348.
- [13] Levin, A., C-F. Lin and C-S. J. Chu (2002). 'Unit Root Tests in Panel Data: Asymptotic and Finite-Sample Properties', *Journal of Econometrics*, Vol. 108, pp. 1-24.

- [14] McCoskey, S., and C. Kao, 1998, A Residual-Based Test of the Null of Cointegration in Panel Data, *Econometric Reviews*, No. 17, pp. 57-84.
- [15] McCoskey, S., and T. Selden, 1998, Health Care Expenditures and GDP: Panel Data Unit Root Test Results, *Journal of Health Economics*, No. 17, pp. 369-376.
- [16] Pedroni, P., 1999, Critical Values for Cointegration Tests in Heterogeneous Panels with Multiple Regressors, *Oxford Bulletin of Economics and Statistics*, No. 61, pp. 653-670.
- [17] Pedroni, P., 2004, Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests with an Application to the PPP Hypothesis, *Econometric Theory*, Vol. 3, pp. 579-625.
- [18] Phillips, P. C. B., and H. R. Moon, 1999, Linear Regression Limit Theory of Nonstationary Panel Data, *Econometrica*, No. 67, pp. 1057-1111.
- [19] Phillips, P. C. B., and S. Ouliaris, 1990, Asymptotic Properties of Residual Based Tests for Cointegration, *Econometrica*, No. 58, pp. 165-193.
- [20] Phillips, P. C. B., and V. Solo, 1992, Asymptotics for Linear Processes, *Annals of Statistics*, Vol. 20, No. 2, pp. 971-1001.
- [21] Roberts, J., 2000, Spurious Regression Problems in the Determinants of Health Care Expenditure: A Comment on Hitiris (1997), *Applied Economics Letters*, No. 7, pp. 279-283.
- [22] Zivot, E., 2000, The Power of Single Equation Tests for Cointegration when the Cointegration Vector is Prespecified, *Econometric Theory*, No. 16, pp. 407-439.
- [23] Westerlund, J., 2004, A Panel CUSUM Test of the Null of Cointegration, *Oxford Bulletin of Economics and Statistics*, forthcoming.